Collisionless transport parallel to the magnetic field in a toroidal plasma

R. D. Hazeltine

Institute for Fusion Studies

The University of Texas at Austin

Austin, TX 78712 USA

July 28, 1998

Abstract

Transport parallel to the magnetic field of a toroidal plasma confinement system is investigated through kinetic theory, with emphasis on the long mean-free path limit. The crucial differences between transport on rational and irrational (ergodic) magnetic surfaces is discussed in detail. A collisionless transport law, involving a non-local operator that accounts for toroidal topology, is derived for parallel heat conduction on irrational magnetic surfaces. In the rational surface case, perpendicular diffusion is included in the kinetic equation to avoid singularity; this allows a calculation of the width and amplitude of resonant temperature perturbations that will be excited by heat sources with sufficiently broad Fourier spectra.

I Introduction

We study transport parallel to the magnetic field of a toroidal confinement system, at low collisionality. The analysis is similar to, and based on, a previous study in slab geometry, hereafter refered to as **I** [**?**]. We allow for arbitrary mean-free path, λ, but emphasize the long mean-free path limit:

$$\lambda \gg L,$$

where L is a gradient scale length. Our objective is to clarify the variation of temperature on a toroidal magnetic surface in the presence of heat sources—such as injected pellets, or the throat of a divertor—that vary on the surface. We note that a similarly motivated treatment by Fitzpatrick[**?**] emphasized temperature variation near magnetic islands; the present treatment assumes that any islands present have negligible thickness.

We begin with the drift kinetic equation

$$v_\parallel \nabla_\parallel f - C(f) = S \tag{1}$$

where f is the distribution function, C the collision operator and S a source term. The kinetic equation is linearized by assuming that S is relatively small:

$$S \ll \omega_t \sim \nu \tag{2}$$

Here ω_t and ν are the transit and collision frequencies respectively, measuring the sizes of the two terms on the left-hand side of (**??**). Thus we expand the distribution function in terms of the small parameter S/ω_t:

$$f = f_0 + f_1 + \cdots$$

From the lowest order equation

$$v_\parallel \nabla_\parallel f_0 - C(f_0) = 0$$

we see that f_0 is Maxwellian and constant along the magnetic field:

$$f_0 = f_M \equiv \frac{N_0}{\pi^{3/2} v_t^3} e^{-v^2/v_t^2}, \tag{3}$$

$$\nabla_\parallel f_M = 0,$$

where v is the (magnitude of the) velocity coordinate and

$$v_t \equiv \sqrt{2T_0/M}$$

is the thermal speed, with T_0 the lowest-order temperature and M the particle mass.

We have been up to now repeating the argument of **I**; at this point we depart from the previous work to consider (axisymmetric) toroidal geometry. Thus let (r, θ, ζ) be toroidal coordinates, with r constant on flux surfaces and (θ, ζ) appropriate poloidal and toroidal angle coordinates. The corresponding contravariant components of the magnetic field $\mathbf{B}$ are

$$B^\theta \equiv \mathbf{B} \cdot \nabla\theta$$

and

$$B^\zeta \equiv \mathbf{B} \cdot \nabla\zeta.$$

We use the safety factor

$$q(r) \equiv B^\zeta / B^\theta$$

to express the parallel gradient as

$$\nabla_\parallel = \frac{B^\theta}{B}\left(\frac{\partial}{\partial\theta} + q\frac{\partial}{\partial\zeta}\right) \tag{4}$$

Thus the linearized kinetic equation takes the form

$$v_\parallel \frac{B^\theta}{B}\left(\frac{\partial f_1}{\partial\theta} + q\frac{\partial f_1}{\partial\zeta}\right) - C(f_1) = S(r,\theta,\zeta,\mathbf{v}) \tag{5}$$

While the safety factor depends only on radius, the coefficient B^θ/B will generally vary on the flux surface, due to toroidal curvature. We study the effects of toroidal topology without curvature and ignore this variation. It then becomes convenient to express the coefficient as

$$\frac{B^\theta}{B} = \frac{1}{qR}$$

where R is the major radius of the magnetic axis.

The key distinction of toroidal geometry is of course the periodicity requirement,

$$F(\theta + 2\pi m, \zeta + 2\pi n) = F(\theta, \zeta) \tag{6}$$

for any physical quantity F. It follows that the distribution function has a Fourier series representation,

$$f_1(r,\theta,\zeta) = \sum_{m,n} e^{-i(m\theta - n\zeta)} f_{mn}(r) \tag{7}$$

and that the linearized kinetic equation can be expressed as

$$-i\frac{v_\parallel}{qR}(m-nq)f_{mn} - C(f_{mn}) = S_{mn} \tag{8}$$

Comparing this equation to the one-dimensional slab version of **I** we see that they differ essentially in the replacement

$$k \to (m-nq)/qR \tag{9}$$

Hence most of the results of **I** are easily adopted to the present case. However we must keep in mind the following distinctions:

1. Because the toroidal surface is two dimensional, we have two transform indices, m and n, instead of only k. Similarly the one-dimensional velocity variable of **I** has been replaced by a three-dimensional velocity space, with $v_\parallel \neq v$.

2. The inverse transform in the present case involves a discrete sum, (**??**), rather than a Fourier integral.

3. While the one-dimensional transform variable k vanishes only at a point, its toroidal analog can vanish everywhere on the flux surface—a set of finite measure—if the safety factor is rational. In other words, toroidal transport is complicated by the presence of rational flux surfaces.

II Rational magnetic surfaces

A magnetic differential equation has the form (when magnetic curvature is neglected)

$$\nabla_{\|} F = S$$

where the source, S, is prescribed and F to be determined. Solutions to the magnetic differential equation can become singular on rational magnetic surfaces, where the parallel gradient of resonant Fourier harmonics vanishes. Newcomb[?] observed, from (??), that on such surfaces the resonant component of the source term must vanish:

$$S_{m_0\, n_0}(\mathbf{v}) = 0, \tag{10}$$

when $q = m_0/n_0$. Because of the collision term, our kinetic equation is not formally a magnetic differential equation. However various velocity moments of (??) annihilate the collision term, so that the Newcomb singularity is pertinent, even at large collisionality.

Consider for example the energy moment of (??). If we neglect energy exchange between species, and introduce the parallel heat flow,

$$h \equiv \frac{1}{T_0} \int d^3 v v_{\|} \left(\frac{1}{2} M v^2 - \frac{5}{2} \right) f_1 \tag{11}$$

then (??) implies that

$$\nabla_{\|} h = \sigma_2 \tag{12}$$

where

$$\sigma_2(r, \theta, \zeta) \equiv \int d^3 v \left(\frac{1}{2} M v^2 - \frac{5}{2} \right) S(r, \theta, \zeta, \mathbf{v}) \tag{13}$$

measures the local rate at which the source adds heat. Equation (**??**) states simply that the added heat is distributed by flow along the lines of force, without temporal change. The steady state obviously requires

$$\oint d\theta \oint d\zeta \sigma_2 = 0; \tag{14}$$

that is, there must be heat sinks as well as sources on the surface, so that the net input vanishes. On a typical, irrational flux surface, this condition guarantees, for physically well-behaved sources, a nonsingular solution for the heat flow h. However, as Newcomb emphasized, (**??**) does not guarantee regular solutions on a rational surface, where we must enforce the much stronger Newcomb condition, (**??**).

The physical issue is easily understood. Irrational surfaces are egodically covered by a single field line, so that heat added anywhere will eventually find its balancing sink. But the field lines on rational flux surfaces are closed and distinct, allowing the sources and sinks to be thermally isolated from each other. Any field lines in contact with the source alone would be continually heated, while those touching only the sink would be cooled, leading to a growing "flute" perturbation without steady state. It is clear that the Newcomb condition simply rules out this possibility, requiring the balance of source and sink on each closed field line.

It is physically clear that no rational-surface singularity can occur for a poloidally or toroidally symmetric (axisymmetric) source. In the case of poloidal symmetry, for example, sources and sinks are arranged on a ring encircling the magnetic axis, every point of which is evidently connected to every other by field lines (for finite q). This is also clear from (**??**), since the components S_{0n} and S_{m0} have no corresponding rational surfaces. We study the axisymmetric case in the following section.

The most dangerous case (ignoring the physically implausible arrangement of sources along a single, helical field line) is that of sources highly localized in both θ and ζ: "spots" of heat input or outflow, as might be associated with pellet injection. (Singularity occurs if *either* the source or the sink is sufficiently localized.) On a low-order rational surface,

$$q = m_0/n_0, \tag{15}$$

with n_0 a small integer (less than 5, say), it is not hard to achieve unbalanced heating of certain field lines, even when the total source satisfies (**??**). For $q > 1$ the distance between successive passes of a single field line is approximately $2\pi r/qn_0 = 2\pi r/m_0$; a spot less than half this size would lead to singularity. That small spots are dangerous is also clear from (**??**), since the corresponding Fourier spectrum is broad:

$$m_{max}\Delta\theta \sim n_{max}\Delta\zeta \sim 1 \tag{16}$$

where m_{max} (n_{max}) is the largest poloidal (toroidal) harmonic with appreciable S_{mn} and $(\Delta\theta, \Delta\zeta)$ is the angular spot width.

When rational surface singularity occurs we must replace the kinetic equation (**??**) with a more accurate version, including terms to disperse the heat.

In so far as these terms are actually small—so that (**??**) is a good approximation on irrational surfaces—the steady-state result is strong flute-like perturbation of the temperature on the rational surface. We compute this perturbation, using perpendicular diffusion to resolve the singularity, in Section **??**.

There is a mathematical aspect of the singularity that is also worth mentioning. Note that resonance corresponds, in the one-dimensional case, to the point $k = 0$, and indeed the analysis of **I** shows that h_k is proportional to $1/k$ when k is small. [This circumstance is also clear from (**??**). The transport relation between h_k and the Fourier transform of the temperature gradient is not singular at $k = 0$ because the transformed temperature gradient has the same $1/k$ singularity.] The singularity is harmless, corresponding to a finite heat flow; it introduces only an *ambiguity* in the inverse transform: which integration path should one take around the pole? Indeed, the obvious one-dimensional solution to (**??**),

$$h = \int^{x_\parallel} dx_\parallel \sigma_2$$

displays the same ambiguity, in terms of freedom in choosing the lower limit of integration. This is resolved by an appropriate (often implicit) boundary condition, such as $h(x \to -\infty) = 0$.

The toroidal situation is very different because of discreteness of the sum, (**??**), the resonant terms of which become meaningless. Thus resolving the rational-surface singularity is not just a matter of finding an appropriate boundary condition to address ambiguity; one must introduce additional physics to avoid unbounded heating of the closed field lines.

III Axisymmetric sources

Here we treat the non-resonant case of axisymmetric sources, as might correspond to a conventional axisymmetric divertor. It is natural in this case to seek axisymmetric solutions and therefore to set the toroidal mode number $n = 0$ throughout. The kinetic equation (**??**) then becomes

$$-ik_m v_\parallel f_m - C(f_m) = S_m, \tag{17}$$

where we have introduced the abbreviation

$$k_m \equiv m/qR.$$

This is essentially the equation that was solved in **I**, using a Krook collision operator and the assumption of Maxwellian sources:

$$S = \widehat{S}\widehat{f}_M. \tag{18}$$

Here

$$\widehat{f}_M = f_M/N_0$$

is a Maxwellian normalized to unit density. It was shown in **I** that the heat flux could be related to the temperature profile, without explicit reference to the source, for any collisionality. In the present case this relation takes the form

$$h_m = -\frac{1}{2}\nu\lambda^2 N_0 Q(\xi_m)(\nabla_\parallel \log T)_m \tag{19}$$

where

$$\xi_m \equiv \frac{-i}{k_m \lambda} \tag{20}$$

and Q is a complicated function, defined in **I**, with the following limits:

$$\lim_{\xi \to \infty} Q(\xi) = 3/2, \tag{21}$$

$$\lim_{\xi \to 0} Q(\xi) = \frac{2}{\sqrt{\pi}|k_m|\lambda}. \tag{22}$$

We take note of the collisional limit, (**??**), to confirm that the expected collisional heat flux, proportional to the local temperature gradient, is reproduced in this case.

From here on we focus on the collisionless limit of (**??**), thus writing the heat flux as

$$h(\theta) = -\frac{N_0 v_t q R}{\sqrt{\pi}} \sum_m e^{-im\theta} \frac{(\nabla_\parallel \log T)_m}{|m|} \tag{23}$$

The appearance of $|m|$ in the denominator of the sum shows that h is not proportional to the local gradient, but depends on a mix of its Fourier components. This manifests the nonlocal character of heat transport in the absence of collisions.

It is helpful to express h in terms of the temperature perturbation, ΔT, where

$$(\nabla_\parallel \log T)_m = -ik_m \frac{(\Delta T)_m}{T_0}$$

Thus

$$h_m = \frac{iN_0 v_t}{\sqrt{\pi} T_0} \frac{m}{|m|} (\Delta T)_m \tag{24}$$

whence

$$\begin{aligned} h(\theta) &= \frac{iN_0 v_t}{\sqrt{\pi}T_0}\sum_m e^{-im\theta}\frac{m}{|m|}(\Delta T)_m \\ &= \frac{iv_t N_0}{\sqrt{\pi}T_0}\oint\frac{d\theta'}{2\pi}\Delta T(\theta')\sum_m e^{-im(\theta-\theta')}\frac{m}{|m|}, \end{aligned}$$

or after performing the sum,

$$h(\theta) = -\frac{N_0 v_t}{2\pi^{3/2}T_0}\oint d\theta'\Delta T(\theta')\cot\left(\frac{\theta'-\theta}{2}\right). \tag{25}$$

Here the singularity in $\cot x$ at $x = 0$ is to be interpreted in the principal value sense[**?**].

Equation (**??**) is the toroidal version of the Hammett-Perkins [**?**, **?**] formula,

$$h(x) = -\frac{N_0 v_t}{\pi^{3/2}T_0}\int_{-\infty}^{\infty} dx'\Delta T(x')P\left(\frac{1}{x'-x}\right),$$

for slab geometry. The toroidal version is rather different, because of periodicity, but displays a similar dependence of collisionless heat flow on the entire temperature profile. Of course both versions involve principal value integrals.

One remarkable difference is most visible in (**??**): the collisionless heat flux becomes local, similar to the collisional case, for the case of an $|m| = 1$ temperature perturbation:

$$h = \frac{N_0 v_t qR}{\sqrt{\pi}}\nabla_\parallel \log T, \quad for\ |m| = 1.$$

For the opposite extreme we consider a narrow temperature perturbation,

$$\Delta T(\theta) = \tau T_0 \delta(\theta).$$

Perturbations approaching this form can occur just inside the separatrix of a divertor tokamak. The corresponding heat flow,

$$h = -\frac{\tau N_0 v_t}{2\pi^{3/2}} \cot \frac{\theta}{2}, \tag{26}$$

is clearly not proportional to any local temperature gradient.

Note that using a delta-function in (**??**) is mathematically improper, since the principal value is not a smooth function (that is, the product of two generalized functions is not well defined). This impropriety is responsible for the singularity of (**??**) at $\theta = 0$; physically the heat source has finite width and the heat flow cusp is correspondingly blunted.

Finally we note that the generalization of (**??**) to non-axisymmetric, non-resonant, perturbations is straightforward; one finds that

$$h(\theta, \zeta) = -\frac{N_0 v_t}{2\pi^{3/2} T_0} \oint d\theta' \Delta T(\theta', \zeta + [q](\theta' - \theta)) \cot \left(\frac{\theta' - \theta}{2} \right). \tag{27}$$

where $[q]$ is the largest integer contained in q. While this formula is of some interest, it applies only on ergodic surfaces, or on rational surfaces where the Newcomb condition is satisfied. Since rational surfaces are dense, and since the Newcomb condition is unlikely to be satisfied by a generic source, we do not pursue (**??**) here, nor give the details of its derivation.

IV Thermal perturbation of a rational surface

Layer width

When the safety factor is rational,

$$q = m_0/n_0$$

the parallel gradient term is absent in the kinetic equation for $f_{m_0\,n_0}$, which therefore would be insoluble. A natural mechanism for resolving the singularity—allowing thermal access to the closed field lines—is spatial perpendicular diffusion, with diffusion coefficient D. For simplicity the diffusion coefficient is assumed to be approximately constant on scales of interest. Thus (**??**) is replaced by

$$v_\parallel \nabla_\parallel f - D\nabla_\perp^2 f - C(f) = S \tag{28}$$

The occurence of spatial diffusion directly in the kinetic equation may appear unusual but in fact can occur through various means, including classical Coulomb collisions[**?**].

Since D is relatively small, diffusion matters only in a thin neighborhood of the rational surface. We let r_0 denote the minor radius of that surface, and use the dimensionless radial variable

$$x \equiv (r - r_0)/r_0.$$

Then $q \cong m_0/n_0 + q'(r_0)r_0 x$ and the relevant Fourier component of (**??**) has the form

$$-i\kappa\omega_t u x f_{m_0\,n_0} - (D/r_0^2) f''_{m_0\,n_0} - C(f_{m_0\,n_0}) = S_{m_0\,n_0} \tag{29}$$

where ω_t is the nominal transit frequency,

$$\omega_t \equiv \frac{v_t}{q(r_0)R},$$

the primes indicate x-derivatives and we have introduced the abbreviations $\kappa = n_0 q'(r_0)$ and $u = v_\parallel/v_t$. We have also omitted perpendicular diffusion in the flux surface, keeping only the radial terms in $\nabla_\perp^2$. This approximation is easily seen to be sensible unless the source is extremely narrow; recall (**??**). We denote the (dimensionless) width of the layer by w, requiring that diffusion balance parallel streaming for $x \leq w$, and find from (**??**) that

$$w^3 \sim \frac{D}{\kappa\omega_t r_0^2}. \tag{30}$$

A more perspicuous version is obtained by writing the diffusion coefficient as

$$D = A\nu\rho_p^2 \tag{31}$$

where ρ_p is the poloidal gyroradius and $A > 1$ is an anomaly factor. ($A = 1$ corresponds roughly to neoclassical transport.) It is convenient to use the resulting form of w as a definition:

$$w \equiv A^{1/3} \left(\frac{\nu}{\kappa\omega_t}\right)^{1/3} \left(\frac{\rho_p}{r_0}\right)^{2/3} \tag{32}$$

Here the first factor exceeds unity (A might exceed 10^2 for electrons, although it appears to be much closer to unity for ions), but the succeeding factors are both quite small, of order 10^{-2} is typical tokamaks. Thus the resonant layer is indeed thin.

We next use (**??**) to estimate the importance of the collision term in (**??**). The ratio of the collision term to the diffusion term is evidently

$$\frac{w^2}{A\rho_p^2} \sim \alpha^{2/3}$$

where

$$\alpha \equiv \frac{\nu}{\kappa\omega_t}\frac{r_0}{\sqrt{A}\rho_p}. \qquad (33)$$

A survey of present and planned tokamak parameters shows that α is consistently small for ions (no larger than 10^{-2}) and usually quite small for electrons, although the electron version can approach unity in some devices. We find sufficient excuse to neglect the collision term in (**??**).

Distribution function

The kinetic equation has become

$$-i\kappa\omega_t uxf - (D/r_0^2)f'' = S$$

where indices are suppressed. We introduce the scaled radial variable

$$y \equiv x/w$$

to obtain

$$-iuyf - f_{yy} = S/(\kappa\omega_t w), \qquad (34)$$

where the derivatives are indicated by subscripts. This driven version of Airy's differential equation can be solved by Fourier transformation,

$$f(y) = \frac{1}{2\pi}\int dk e^{-iky} f_k,$$

since joining to the nonresonant region requires $f(y)$ to become small outside the resonant layer. Thus

$$u\frac{df_k}{dk} + k^2 f_k = \frac{S_k}{\kappa\omega_t w}$$

which implies

$$f_k = \frac{1}{\kappa\omega_t u w} e^{-(k^3/3u)} \int_{-\sigma\infty}^{k} dk' e^{(k'^3/3u)} S_{k'}, \tag{35}$$

where

$$\sigma \equiv u/|u|. \tag{36}$$

In order to proceed we need to know the radial dependence of the source. We consider the generically important case in which $S(x)$ is nearly constant across the layer, implying

$$S_k = 2\pi\bar{S}\delta(k)$$

and therefore

$$f_k = \frac{2\pi\bar{S}}{\kappa\omega_t|u|w} e^{-(k^3/3u)}\Theta(\sigma k), \tag{37}$$

where

$$\begin{aligned}\Theta(t) &= 1,\ for\ t \geq 0,\\ &= 0,\ for\ t < 0.\end{aligned}$$

is a step-function. We suppress the overbar to write the distribution function for the resonant layer as

$$f(x,\mathbf{v}) = \frac{S}{\kappa\omega_t|u|w}\int_0^\infty dk e^{ik\sigma x}e^{-(k^3/3|u|)} \tag{38}$$

Temperature perturbation

Equation (**??**) describes an Airy function, peaked at the origin and decaying over the scale–width w; its structure—although somewhat modified by the dependence on σ—is well-known[**?**]. Here we are content to examine the amplitude of the corresponding temperature perturbation:

$$\Delta T_{m_0\,n_0}(0) = T_0\int d^3v\frac{v^2}{v_t^2}f(0,\mathbf{v}).$$

We substitute for $f(0,\mathbf{v})$ from (**??**), use (**??**) for the source, and integrate over the perpendicular velocity components. The result is

$$\frac{\Delta T_{m_0\,n_0}}{T_0} = \frac{2\widehat{S}_{m_0\,n_0}}{\sqrt{\pi}\kappa\omega w}\int_0^\infty du I(u)e^{-u^2}\left(\frac{1+u^2}{u}\right) \tag{39}$$

where

$$I(u) \equiv \int_0^\infty dk e^{-k^3/3|u|} = 0.89(3|u|)^{1/3}.$$

Hence we obtain the resonant temperature perturbation

$$\begin{aligned}\frac{\Delta T_{m_0\,n_0}}{T_0} &= 4.72\frac{\widehat{S}_{m_0\,n_0}}{\kappa\omega_t w}\\ &= 4.72\frac{\widehat{S}_{m_0\,n_0}r_0^{2/3}}{(A\nu)^{1/3}(\kappa\omega_t\rho_p)^{2/3}}\end{aligned} \tag{40}$$

Here we recall that $m_0 = q(r_0)n_0$ describes the Fourier harmonic on the resonant surface at r_0; $\widehat{S}_{m_0\,n_0}$, which has the dimensions of frequency, is the resonant component of the source, normalized according to (??); $\kappa = n_0 r_0 dq/dr$ measures the shear at the resonant surface; $\omega_t = v_t/qR$ and ν are respectively the transit and collision frequencies; A is the transport anomaly factor defined by (??); and ρ_p is the poloidal gyroradius.

It is instructive to express the result (??) in terms of the parameter α, defined by (??). We find that

$$\frac{\Delta T_{m_0\,n_0}}{T_0} = 4.72 \frac{\widehat{S}_{m_0\,n_0}}{\nu} \alpha^{2/3}. \tag{41}$$

Our perturbation theory is consistent only if the left-hand side of this equation is smaller than unity. The first factor on the right-hand side, comparing the heating rate to the collision rate, can exceed unity in the realistic regime of small ν/ω_t. (Our analysis assumes $\widehat{S}$ to be smaller than either ν or ω, but allows $\nu \ll \widehat{S} \ll \omega_t$.) It follows that our previous assumption of small α is indeed necessary. Of course the smallness of α in typical tokamaks also makes (??) plausibly relevant to tokamak experiments.

It is not hard to see that resonant perturbation becomes weaker with increasing n_0—that is, on higher–order rational surfaces. First, the κ in the denominator of (??) is proportional to n_0; second, for a reasonable smooth source, $\widehat{S}_{m_0\,n_0}$ becomes small for large n_0; and third, perpendicular diffusion on the flux surface, which we have omitted, is proportional to m_0^2 and will thoroughly smear high-order perturbations over the surface. As noted in Section ??, only resonance with low–order rationals is likely to be observed.

In this regard it is instructive to compare $\Delta T_{m_0\,n_0}$ to its nonresonant version, ΔT_{mn}. Assuming that the corresponding components of the source are comparable,

$$\widehat{S}_{m_0\,n_0} \sim \widehat{S}_{mn},$$

energy balance suggests that

$$\frac{\Delta T_{m_0\,n_0}}{\Delta T_{mn}} \sim \frac{1}{w}$$

We confirm this estimate by using (**??**) to infer

$$\frac{\Delta T_{mn}}{T_0} \sim \frac{\widehat{S}_{mn}}{m\omega_t},$$

and then using (**??**) to compute

$$\frac{\Delta T_{m_0\,n_0}}{\Delta T_{mn}} \sim A^{-1/3} \left(\frac{r_0}{\rho_p}\right)^{2/3} \left(\frac{\omega_t}{\nu}\right)^{1/3} \sim \frac{1}{w}. \tag{42}$$

We have already remarked that this ratio is large in all major tokamaks. Closed (and therefore nearly insulated), field lines on low-order rational surfaces are selectively perturbed, as discussed in Section **??**; a heat pulse with a reasonably broad Fourier spectrum affects such surfaces in a sharply distinctive way.

V Summary

The distribution of heat on the flux surfaces of a nearly collisionless toroidal system depends critically on whether the surface is rational or ergodic. Parallel heat conduction on a rational surface, discussed in Section **??**, vividly illustrates the need for a Newcomb condition on heat source terms. In the case of non-resonant flux surfaces, illustrated by the axisymmetric source of Section **??**, a periodic version of the collisionless transport law of Hammett and Perkins [**?**] pertains. Equation (**??**) and its generalization (**??**) show that collisionless heat conduction in a torus depends upon the temperature profile over the entire magnetic surface, with some surprising features. The thermal response on resonant rational surfaces is controlled, under typical circumstances, by radial diffusion. The width of the resonant layer is small, allowing an asymptotic calculation of the resonant temperature perturbation. Given by (**??**), this perturbation is larger by orders of magnitude than the nonresonant version.

Acknowledgments

This work was supported by the US Department of Energy Contact No. DE-FG03-96ER-54346.

References

[1] R. D. Hazeltine, Phys. Plasmas, to be published (1998).

[2] R. Fitzpatrick, Phys. Plasmas **2**, 825 (1995).

[3] W. A. Newcomb, Phys. Fluids **2**, 362 (1959).

[4] I. M. Gel'fand and G. E. Shilov, *Generalized Functions*, (Academic Press, 1964) vol. 1.

[5] G. W. Hammett and F. W. Perkins, Phys. Rev. Lett. **64**, 3019 (1990).

[6] R. D. Hazeltine and P. J. Catto, Phys. Plasmas **3**, 2365 (1996).

[7] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, (National Bureau of Standards, 1964).