

# Frequency up-conversion and trapping of ultrashort laser pulses in semiconductor plasmas

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## Abstract

It is shown that the interaction of ultrashort laser pulses with nonstationary semiconductor plasmas can, under appropriate conditions, lead to a variety of interesting phenomena including controlled upshifting of the laser frequency leading to the possibility of tunable lasers in a wide range of frequencies, and trapping (nonpropagation) of a substantial part of the incident pulse.

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The availability of high power ultrashort laser pulses has opened up new vistas in the physics of electromagnetic (EM) radiation propagation in nonstationary dispersive media. A temporal variation in the properties of the medium implies that the solutions of Maxwell's equations cannot be frequency eigenstates. This phenomenon is the appropriate counterpart of what happens in spatially inhomogeneous media for which a wave with a definite wavenumber can not be an eigensolution. The temporal inhomogeneity can lead to quite spectacular consequences in the propagation dynamics of pico(femto) second pulses: for example a pulse can be frequency shifted as it passes through a medium in which the dielectric properties vary as a function of time [1]. Since the magnitude of the frequency shift can be readily controlled, this pulse-nonstationary medium interaction can lead, among other things, to the creation of tunable laser sources.

Gaseous media, undergoing rapid ionization induced by an external source, or by a propagating strong EM pulse (multiphoton (or tunneling) absorption), are obvious systems where manifestations of this phenomenon can be studied. A representative example of such a study is the paper by Wilks *et al.* [2] dealing with the propagation of a linearly polarized EM wave through an instantly ionized (by an external agent) gas. Due to the electron plasma created by ionization, there is a sudden reduction in the refractive index of the gas. Since the spatial homogeneity of the medium does not allow a change in the wavelength, the waves (both reflected and transmitted) must instantly increase their frequency to compensate for the change in the refractive index so as to satisfy the new dispersion relation,  $\omega = (\omega_0^2 + \omega_p^2)^{1/2}$ , where  $\omega_0 = ck_0$  is the initial frequency of the EM radiation and  $\omega_p$  is the plasma frequency corresponding to the newly created plasma. The value of  $\omega_p$  can be controlled externally and consequently the suggested mechanism can be used for an effective control of the frequency upshift of the EM radiation [3]. Wilks *et al.* [2] have also deduced an interesting additional consequence; the appearance of a time independent residual magnetic field with the characteristic wavelength equal to the vacuum wavelength of the passing radiation. Recent theory and experiment have conclusively demonstrated that a powerful short laser pulse propagating in a gas can experience a strong frequency blueshift caused

entirely by the plasma generated by the pulse itself [4].

In this paper, we explore a new kind of a collisionless plasma made possible by the action of ultrashort pulses on semiconductor materials. We will concentrate on two basic problems: 1) we shall demonstrate that semiconductor materials can become excellent sources for the creation of highly effective, efficient, and dependable tunable lasers in an extremely broad frequency range, and 2) we shall investigate the possible consequences of the interaction of the pulse with a moving ionization front (the instantly created ionization front has infinite speed of propagation). Although the interaction of laser pulses with semiconductor plasmas have been studied for a long time [5], their collision dominated dynamics had little in common with the motions associated with ordinary collisionless gaseous plasmas. To observe the collisionless collective phenomena in semiconductors, the pulse duration  $\tau_L$  must be less than the characteristic relaxation time of the carriers, which is typically  $\tau_R \approx 10^{-12} - 10^{-13}$  sec. The ready availability of femtosecond pulses, therefore, has ushered in a new era in the physics of semiconductors; it has now become possible to reproduce a variety of physical processes normally associated with high temperature gaseous plasmas [6]. In fact, the semiconductor plasmas, as experimental systems, have several advantages over the gaseous ones: automatic confinement, homogeneity, and an easy density control over a large range coupled with the possibility of carrying out relatively inexpensive table-top experiments. The implications for physics as well as for the resulting technology are enormous.

One- or two-photon absorption of the impinging EM radiation is a very efficient method of producing a high density cold electron-hole plasma in a semiconductor. The optical response of the medium is rather sensitive to the optically induced redistribution of carriers between bands. In our recent studies [7], for example, we demonstrated that an intense, short ( $\tau_L \approx 100$  fs) laser pulse tuned near the two-photon resonance in an InSb semiconductor waveguide, first generates large densities of excess carriers via two-photon absorption (TPA), and then accelerates them creating a large nonlinear rapidly varying current in the electron-hole plasma. It was further shown that for intensities of order  $I > 10^9$  W/cm<sup>2</sup>, the dynamics of the laser pulses is dominated by the pulse-current interaction that leads to a

large frequency blueshift. The phenomenon pertains for a wide variety of semiconductors as long as the energy gained by the carriers from the laser pulse is greater than the energy needed (by TPA) to generate the free carriers. This is the principal point of departure of the above-mentioned study from the conventional approach where the TPA merely contributes to the imaginary part of the third-order susceptibility [8].

There is, however, a serious constraint on the working of the above-mentioned mechanism. Since the pulse has first to create a plasma of high enough density and then accelerate it, significant blueshift can result only for rather high intensity pulses (typically  $\sim 10^9 \text{W/cm}^2$ ). How can we, then, blue shift a pulse of moderate or low intensity? To answer this question, we investigate the dynamics of a short, moderate (to low) intensity laser pulse (to be called the Pulse) propagating in a semiconductor in which another short but intense pulse (to be called the Source-Pulse) is used for the rapid creation of the required electron-hole plasma. To avoid complications, let us assume that the Pulse propagates in a planar semiconductor waveguide and its frequency is below the band-gap resonance, i.e.  $\hbar\omega_o < E_g$ , where  $E_g$  is the gap energy separating the bottom of the conduction band from the top of the valence band.

We can imagine the following experimental scenario: The plane of an appropriate semiconductor sample is uniformly illuminated with the Source-Pulse whose frequency,  $\omega_s$ , is tuned near a one-photon resonance,  $\hbar\omega_s \approx E_g$ . An electron-hole plasma is suddenly but almost uniformly created in the wave guide with a thickness typically of the order of a few microns. The propagating Pulse encounters a time varying medium (with increasing plasma density) and responds by upshifting its frequency.

The carrier density generated by the Source-Pulse is determined by

$$\frac{\partial n}{\partial t} = \frac{\alpha_s}{\hbar\omega_s} I_s \quad (1)$$

where  $I_s$  is the intensity of the source, and  $\alpha_s$  is an appropriate resonant absorption coefficient. Since we are interesting in femtosecond time scales, we have neglected [in Eq. (1)] the nano(pico) second processes like the radiative and nonradiative recombination of electron-

hole pairs, and of carrier diffusion [5].

Frequency upshift of the Pulse can be estimated by the relation  $\omega(t) = \omega_0(1+n(t)/n_c)^{1/2}$ , where  $n_c = m^* \epsilon_0 \omega_0^2 / 4\pi e^2$  is the critical plasma density (corresponding to the initial frequency  $\omega_0$ ),  $m^*$  is the effective carrier mass, and  $\epsilon_0$  is the dielectric constant. For sufficiently intense, femtosecond, Source-Pulses ( $I_s \approx 10^9 - 10^{12} \text{W/cm}^2$ ), plasma densities as high as  $n_{\text{max}} \approx 10^{18} - 10^{20} \text{cm}^{-3}$  can be excited without damaging the semiconductor samples. These densities are in the range 1-100  $n_c$ , the critical density associated with the mid infrared Pulses with vacuum wavelength  $\lambda > 10\mu$  ( $n_c < 10^{18} \text{cm}^{-3}$ ). Consequently frequency upshifts, as high as  $\omega_{\text{max}} \approx 10\omega_0$ , can be affected by such an arrangement. Naturally the Pulse duration should be short (femtosecond range) to avoid strong absorption losses. If the frequency of the Pulse is larger than the band gap ( $\hbar\omega_m > E_g$ ), additional absorption will result due to the electron-hole generation by the Pulse itself, and the Pulse will damp out in a micron or so. Due to two-photon absorption, the Pulse would suffer some attenuation (absorption) even for lower frequencies ( $\hbar\omega_m > E_g/2$ ). However, because of the strong intensity dependence of this process, for sufficiently short propagation lengths, and at the intensities we are interested in, the two-photon absorption can be arranged to be negligible. For mid-infrared Pulses (see below), both the one-photon and the two-photon absorption processes are energetically forbidden.

In the mid-infrared range ( $\lambda \approx 10\mu$ ), pulses as short as 130 fs, and containing only four optical cycles, were created decades ago [9] by the use of semiconductor switching. The generation of short Pulses with longer wavelength (in mid(far) infrared range with  $\lambda_0 > 10\mu$ ) is still not commonplace (see the current state of art in [10]). Because of the requirement of lower free-carrier density for the same amount of upshifting, the proposed mechanism will be even more effective for pulses in the far-infrared range.

We now examine the workability of the mechanism for a concrete physical system. Consider an undoped GaAs semiconductor waveguide with a room temperature band gap energy  $E_g = 1.43 \text{eV}$ , an effective electron mass  $m^* = 0.07m_e$ , and the lattice constant  $\epsilon_0 = 12$ . On illuminating the plane of the waveguide by a 200 femtosecond Source-Pulse ( $\tau_s = 200 \text{fs}$

with wave length  $\lambda_s = 0.86\mu$  (i.e.  $\hbar\omega_s = 1.44\text{ eV}$ ), and intensity  $I_s = 1.5 \times 10^{11}\text{ W/cm}^2$ , an electron-hole plasma with  $n_{\text{max}} = 8 \times 10^{19}\text{ cm}^{-3}$  can be generated. The thickness of the plasma layer, defined by the inverse of the absorption coefficient, is  $\alpha_s^{-1} \approx 16\mu$ . At this level, the fluence of the laser is  $0.3\text{ KJ/m}^2$ , which is below the surface damage threshold ( $\approx 1\text{ KJ/m}^2$ ) [11]. Let us use this setup to upshift a  $\text{CO}_2$  Pulse with a wavelength  $\lambda_0 = 10.6\mu$  ( $\hbar\omega_0 = 0.117\text{ eV}$ ) and duration  $\tau_L = \tau_s = 200\text{ fs}$ . For this Pulse the corresponding critical density is  $n_c = 8.3 \times 10^{18}\text{ cm}^{-3}$ , and consequently frequency upshift can be estimated to be  $\omega_{\text{max}} \approx 3.3\omega_0$ . The upshifting takes place during the electron-hole plasma formation ( $\sim 200\text{ fs}$ ), while the Pulse travels a distance  $L = 2v_g\tau_L = 11\mu$  ( $v_g = \epsilon_0^{-1/2}c\omega_0/\omega_{\text{max}}$  is the group velocity of the pulse). This distance is much smaller than the collision length  $L_{\text{coll}} = (\tau_R/\tau_L)L \approx 100\mu$ , allowing us to construct a waveguide in which the collisional effects can be safely neglected. It is worth while to remark that for waveguides with lengths comparable to  $L_{\text{coll}}$ , the collisions can lead, in addition to absorption, to a frequency spectral breaking of the Pulse [12], i.e. the spectrum of the transmitted Pulse breaks up into two peaks; one has an upshifted central frequency while the other has a downshifted central frequency.

During this process of the major spectral transformation of the pulse, another equally spectacular phenomenon is expected to take place in the interior of the medium. Simple calculations yield that a static magnetic field with a strength equal to the magnetic field of the laser Pulse, and with a spatial wavelength  $\lambda = \epsilon_0^{-1/2}\lambda_0 = 3\mu$ , will be generated. The implications of this high magnetic field, which can be higher than a  $1\text{ KG}$  (and which would decay in a “relaxation time” of the carriers ( $\sim 1\text{ ps}$ )), are yet unexplored. The effects it would this have on the transport properties of the carriers, or on the crystalline structure of the medium and other relevant phenomena will be a fascinating problem to study.

The preceding qualitative reasoning strongly indicates that for an appropriately chosen system consisting of a semiconductor, an ultra short Source-Pulse, and an ultrashort Pulse we expect to find: 1) a strong frequency upshifting of the ultrashort Pulse, and 2) a simultaneous generation of a strong, static but spatially periodic magnetic field. We now develop a

quantitative framework to investigate the verity of these notions; we will present the results of an analytic as well as a numerical simulation of the processes described earlier. Since we are dealing with ultrashort pulses in semiconductors, our simulations will deal with the dynamics of pulses rather than plane waves. We remind the reader that the plane wave analysis is the standard (and relevant) mode of treating similar phenomena in the interaction of microwave pulses with gaseous plasmas.

To describe the dynamics of the laser Pulse, we use the full wave equation (the basic hyperbolic system) rather than its conventional parabolic approximation. Our formalism enables us to retain effects (like the wave reflection) which are not accessible to the truncated treatment. In any case, the parabolic assumption breaks down when the pulse width becomes comparable to the wavelength; the full description then becomes a zeroth order necessity. The suggested geometry of the waveguide allows a one-dimensional description; we can assume that all physical quantities vary only along the Pulse propagation direction  $z$ , i.e. along the waveguide. The electric field of the Pulse is assumed to be linearly polarized  $\mathbf{E} = \hat{\mathbf{x}}E(z, t)$ , and the equation governing its propagation can be written as:

$$\frac{\partial^2 E}{\partial t^2} - \frac{c^2}{\epsilon_0} \frac{\partial^2 E}{\partial z^2} + \frac{4\pi}{\epsilon_0} \frac{\partial J}{\partial t} = 0 \quad (2)$$

where  $J$ , the current density of free carriers, satisfies [13]:

$$\frac{\partial J}{\partial t} + \frac{1}{\tau_R} J = \frac{e^2}{m^*} n E \quad (3)$$

Here  $n$  is the electron density produced by the Source-Pulse [see Eq. (1)],  $\tau_R$  is the relaxation time, and it is assumed that the free electrons are born in the conduction band with zero initial velocities, and are distributed isotropically. Due to their heavier mass, the hole contribution to the current can be neglected without any loss of generality.

Assuming that the wave processes under consideration take place in times less than the carrier relaxation time ( $\sim 1ps$ ), Eqs. (2)-(3) imply

$$\frac{\partial^2 E}{\partial t^2} - \frac{c^2}{\epsilon_0} \frac{\partial^2 E}{\partial z^2} + \omega_0^2 \frac{n}{n_c} E = 0. \quad (4)$$

An equivalent equation for the magnetic field follows by using the relation  $\partial E/\partial z = -(1/c)\partial B/\partial t$  ( $B = B_y$ ),

$$\frac{\partial^2 B}{\partial t^2} - \frac{c^2}{\epsilon_0} \frac{\partial^2 B}{\partial z^2} + \omega_0^2 \int_{t_0}^t dt' \frac{n(t')}{n_c} \frac{\partial B}{\partial t'} = 0. \quad (5)$$

In deriving Eq. (5), we assumed that the plasma generation begins at time  $t = t_0$ , precisely the time at which the Source pulse is switched on to illuminate the plane of the waveguide. Note that since we are considering undoped samples, the contribution of the intrinsic free carriers is negligible ( $n_c \gg n_{int}$ ) so that  $n(t) = 0$  for  $t < t_0$ .

The vacuum solution for the Pulse ( $t < t_0$ ) can be represented as

$$E = E_0(z, t) \exp(-i\omega_0 t + ik_0 z) + c.c. \quad (6)$$

where  $E_0$  is slowly varying amplitude of the pulse,  $\omega_0 \gg T^{-1}$ ,  $k_0 \gg L^{-1}$ , where  $T, L$  are respectively the duration and spatial extent of the pulse, and the wavenumber  $k_0 = (\epsilon_0)^{1/2}\omega_0/c$ . For an instantaneously produced electron-hole plasma  $n(t) = n_{\max}H(t - t_0)$ , where  $H(t)$  is the Heaviside function.

Equation (4) tells us that just after the birth of the carriers, the frequency of the field  $\omega = -\partial\phi/\partial t$  ( $E = |E| \exp i\phi$ ) shifts, while its wavenumber remains unchanged,  $k = \partial\phi/\partial z = k_0$ . The new frequency is given by  $\omega = \omega_0(1 + n_{\max}/n_c)^{1/2}$ . Similarly from Eq. (5), one can show that a static magnetic field

$$B_{st} = \frac{n_{\max}}{n_c + n_{\max}} B(t < t_0, z) \exp ik_0 z \quad (7)$$

is also generated. Notice that for  $n_{\max} \gg n_c$ , the static magnetic field in the material approaches the value of vacuum (or of the initial) magnetic field associated with the pulse ( $= B(t < t_0)$ ). The formula given in (7) is really valid for a long pulse. For shorter pulses, (7) conveys only a qualitative picture, and the right-hand side must be replaced by an integral.

To confirm and refine the results of these simple estimates, we numerically solve the full wave equation coupled with the equation giving the current carried by the new carriers. From this we will simulate the frequency upshifting, and the magnetic field generation.

The simulation employs an initial Gaussian Pulse  $|E| = E_m \exp[-(z - \epsilon_0^{-1/2} ct)^2/T^2]$  with  $T\omega_0 = 10$  propagating in the waveguide for  $t < t_0 = 0$ ; At time  $t_0 = 0$ , the Source-Pulse is switched on and generates a uniformly distribution of free carriers. Without loss of generality, we assume that the density of the carriers grows linearly,  $n = n_{\max}t/T$  for  $0 < t < T$ , to a value  $n = n_{\max} = 10n_c$  at  $t = T$  (and then stays constant for  $t > T$ ) when the Source is switched off. The spatio-temporal behavior of the Pulse electric field in dimensionless coordinates  $t \rightarrow t/T$ ,  $z \rightarrow (\epsilon_0^{1/2}/cT)z$  is presented in Fig. 1. One can see that, immediately after switching the Source, the Pulse experiences a sudden decrease in intensity. This loss is due to the energy expended in the carrier acceleration (that are born with zero velocity) and partially to the energy carried away by the reflected pulse. After the Source is switched off, the evolution of the transmitted and the reflected pulses follows the familiar dynamics of pulse propagation in dispersive media, i.e. they spread out developing a frequency chirp. In Fig. 2, we display the spectral content of the initial, the reflected and the transmitted pulses. We see that both pulses are upshifted considerably in conformity with the simple estimates given earlier. In Fig. 3, we plot the evolution of the magnetic field. Between the transmitted (moving to the right) and the reflected magnetic pulses (moving to the left, very low amplitude), we can see the spatially varying but temporally static magnetic field with almost the same spatial period as that of the incident Pulse.

The preceding example of the Pulse propagation in a uniformly produced electron-hole plasma is, in fact, a particular case of the interesting wave processes that take place in the media which support a propagating (moving) interface between regions with different dielectric and polarization properties. Phenomena arising in the course of the interaction of electromagnetic waves with a moving ionization or recombination front (separating the neutral gas from the plasma) have attracted considerable attention in the past [14]. Due to recent technological advances in producing short ultrastrong laser pulses, there is a resurgence of interest in this old problem [15]. The reasons are obvious; Such pulses can easily produce moving ionization fronts and what was essentially an academic pursuit, can become

practical and useful.

Due to strong absorption losses, it is difficult to produce moving, self-supported and long-lived “ionization” fronts in bulk semiconductors. However on illuminating the plane of a waveguide with a Source-Pulse incident on a angle to the surface, the front of the electron-hole plasma will move with superluminal velocity  $v = c/\sin\beta$ , where  $\beta$  is an angle of incidence. Note that for normal incidence, and this corresponds to the case previously considered,  $v \rightarrow \infty$ . The dynamics of superluminal propagation (but with finite velocity) of the front exhibits essentially the same features as the dynamics associated with the infinite velocity front (corresponding to the uniform electron-hole production): similarly frequency upshifting of the reflected and the transmitted waves as well as the generation of stationary magnetic field takes place. However, now the strength of the observed effects will depend on one more parameters  $v$ . A remarkable new effect, however, also appears on the scene. It seems that for a range of velocities (of the front propagation), it becomes possible to trap a part of the incident radiation leading to the formation of spatially localized, high frequency, oscillating structures; the group velocity of the wave packet is close to zero.

In order to appropriately describe the physics of moving fronts, we must modify our formalism by modeling the electron-hole plasma density as  $n = n(t + z/v)$ , i.e. a front “moving” towards the incident Pulse. Since the conventional tool of Lorentz Boost is not available for the superluminal case, we plan to formulate the problem by introducing the following transformation of variables,  $\tau = t + z/v$ , and  $\xi = z$ . In these variables, the equations for the electric and the magnetic fields take the form:

$$\frac{\partial^2 E}{\partial \tau^2} - \frac{c^2}{\epsilon_0} \left( \frac{\partial}{\partial \xi} + \frac{1}{v} \frac{\partial}{\partial \tau} \right)^2 E + \omega_0^2 \frac{n(\tau)}{n_c} E = 0, \quad (8)$$

and

$$\frac{\partial^2 B}{\partial \tau^2} - \frac{c^2}{\epsilon_0} \left( \frac{\partial}{\partial \xi} + \frac{1}{v} \frac{\partial}{\partial \tau} \right)^2 B + \frac{\omega_0^2}{n_c} \int_{\tau_0}^{\tau} d\tau' \left( n(\tau') \frac{\partial B}{\partial \tau'} - \frac{c}{v} E \frac{\partial n}{\partial \tau'} \right) = 0. \quad (9)$$

Before interaction (i.e.  $\tau < \tau_0$ ), the Pulse, propagating towards the “ionization” front (carrying the electron-hole plasma), can be represented as

$$E = E_0(\xi, \tau) \exp(-i\Omega_\tau^{(0)}\tau + ik_\xi^{(0)}\xi) \quad (10)$$

where  $E_0$  is the slowly varying amplitude of the Pulse and  $\Omega_\tau^{(0)}$ , and  $k_\xi^{(0)}$  are the frequency and wavenumber in  $(\xi, \tau)$  space. Note that  $\omega$  and  $k$ , the frequency and wavenumber in laboratory variables (i.e.  $(z, t)$ ), can be found by using the relations  $\omega = \Omega_\tau$  and  $k = k_\xi - \Omega_\tau/v$ . For simplicity let us assume that the moving boundary is sharp, and is represented by a step function,  $n = n_{\max}H(\tau - \tau_0)$ . From Eqs. (8),(9) one can immediately derive that the frequency,  $\Omega_\tau = -\partial\phi/\partial\tau$ , will be upshifted, while the wavenumber remains unaffected,  $k_\xi = k_\xi^{(0)}$ . The corresponding dispersion relation reads

$$\Omega_\tau^2 = \omega_0^2 \frac{n_{\max}}{n_c} + k_\xi^2 c^2 \epsilon_0^{-1}. \quad (11)$$

After simple algebra, we find that, in the laboratory variables, the characteristic frequencies and the wavenumbers of the two waves are

$$\frac{\omega^\pm}{\omega_0} = \mp \frac{1}{V-1} \left[ V \left( 1 + \frac{n_{\max}}{n_c} \frac{(V-1)}{(V+1)} \right)^{1/2} \pm 1 \right] \quad (12)$$

and

$$\frac{k^\pm}{k_0} = \frac{1}{V-1} \left[ V \pm \left( 1 + \frac{n_{\max}}{n_c} \frac{(V-1)}{(V+1)} \right)^{1/2} \right] \quad (13)$$

where  $\omega_0 = \epsilon_0^{-1/2}ck_0$ , and  $V = \epsilon_0^{1/2}v/c$ .

Since  $V > 1$ , it is clear that  $\omega^+ < 0$ , and  $k^+ > 0$ . Consequently this branch, to be called the reflected wave, propagates in the same direction as the density step. Note, however, that this nomenclature is purely formal; the reflected Pulse can never catch up with the density step, which propagates with superluminal velocity ( $V > 1$ ), and the group velocity of the reflected wave is (naturally) subluminal,  $v_g^+ < c/\epsilon_0^{1/2}$ . This wave is some kind of a photon wake produced by the moving ionization step during interaction with the incident wave.

For the second branch  $\omega^- > 0$ , but  $k^-$  can have either sign depending on the value of  $V$ . For  $V > V_c = (n_{\max}/n_c)^{1/2} - 1$ ,  $k^- > 0$ . The wave, then, can be called the transmitted wave, and will propagate in the direction opposite to that of the density step. Alternatively, for

$V < V_c$ ,  $k^- < 0$  implying that even this branch is reflected. However with a group velocity given by  $v_g^- < v_g^+$ , it can not catch up with the first branch (not to speak of catching up with the front). A most interesting consequence of the departure from the instant ionization case is, that when  $V \approx V_c$ , the group velocity of this branch tend to zero. This means that the EM field is trapped inside the semiconductor waveguide forming a localized, high-frequency, oscillating structure. One can simply generalize this result for the case when the density step moves in the opposite direction, i.e. it overtakes the Pulse. In this case, the high-frequency field trapping takes place if  $V = V_c \approx (n_{\max}/n_c)^{1/2} + 1$ .

The generation of the static magnetic field takes place just as before. Indeed the form of the third term in Eq. (9), which can be written as

$$\frac{\omega_0^2}{n_c} \int_{\tau_0}^{\tau} d\tau' \left( n(\tau') \frac{\partial B}{\partial \tau'} - \frac{c}{v} E \frac{\partial n}{\partial \tau'} \right) = \omega_0^2 \frac{n_{\max}}{n_c} [B(\xi, \tau) - B(\xi, \tau_0) + \frac{c}{v} E(\xi, \tau_0)], \quad (14)$$

reveals that a static component, determined by the incident wave, will be excited. The Static magnetic field for the Pulse (for the slowly varying envelope) is found to be

$$B_{st} = \frac{n_{\max} \left(1 + \frac{1}{V}\right)}{n_c \left(1 + \frac{1}{V}\right)^2 + n_{\max}} B(\tau_0, z) \exp[ik_0(1 + 1/V)z] \quad (15)$$

For  $V \rightarrow \infty$ , this formula reduces to Eq. (7), derived for the uniformly produced plasma. For other speeds ( $V > 1$ ), the strength and wavelength of the magnetic field is somewhat modified.

In Fig. 4, we show the results of numerical simulation when the ionization front moves with a velocity  $V \approx V_c$  towards the Pulse. Each plot is at a time later than the preceding one. All other parameters are exactly the same as in previous simulations. Notice that the peak of the Pulse moves very slowly towards the left; it is essentially trapped and diffuses as the time goes on.

The interaction of ultrashort laser pulses with plasma created in semiconductor waveguides, thus, can lead to a variety of physically interesting phenomena ranging from wave-trapping, creation of large static magnetic fields, to the blueshifting of the pulses. we believe

that the collisionless solid state plasmas are ripe for a thorough investigation; the results are likely to be exciting and useful.

## REFERENCES

- [1] Y.A. Kravtsov and Y.I. Orlov, *Geometrical Optics of Nonhomogeneous Media* (Moscow: Nauka, 1980).
- [2] S.C. Wilks, J.M. Dawson, and W.B. Mori, Phys. Rev. Lett. **61**, 337 (1988).
- [3] V.C.J. Joshi, IEEE Trans. Plasma Sci. **18**, 814 (1990).
- [4] W.M. Wood, G. Focht, and M.C. Downer, Opt. Lett. **13**, 984 (1988); M.C. Downer, W.M. Wood, and J.I. Trisnadi, Phys. Rev. Lett. **65**, 2832 (1990); W.M. Wood, C.W. Siders, and M.C. Downer, Phys. Rev. Lett. **67**, 3523 (1991).
- [5] A.A. Borshch, M. Brodin, and V. Volkov, in *International Handbook Laser Science and Technology* (Harwood Academic Publishers, Chur, 1990); A. Miller, in *Nonlinear Optics in Signal Processing* (Chapman and Hall, 1993).
- [6] V.I. Berezhiani and S.M. Mahajan, Phys. Rev. Lett. **73**, 1837 (1994); V.I. Berezhiani and S.M. Mahajan, Phys. Rev. B **55**, 9247 (1997); V.I. Berezhiani, V. Skarka, and R. Miklaszewski, Phys. Rev. B **57**, 6251 (1998).
- [7] V.I. Berezhiani, S.M. Mahajan, and I.G. Murusidze, Phys. Rev. A **56**, 5147 (1997).
- [8] A. Miller, D.A.B. Miller, and S.D. Smith, Adv. Phys. **30**, 697 (1981); E.A. Smith, R.G. McDuff, and N.R. Heckenmerg, J. Opt. Soc. Am. B **12**, 393 (1995); A.S. Rodrigues, M. Santagiustina, and E.M. Wright, Phys. Rev. A **52**, 3231 (1995).
- [9] C. Rolland and P.B. Corkum, J. Opt. Soc. Am. B **3**, 1625 (1986).
- [10] G.M.H. Knippels *et al.*, Phys. Rev. Lett. **75**, 1755 (1995); M.K. Reed, M.K. Steiner-Shepard, and D.K. Negus, Optics Letters **19**, 1855 (1994).
- [11] M.C. Downer, R.L. Fork, and C.V. Shank, J. Opt. Soc. Am. B **2**, 595 (1985); L. Huang, J.P. Callan, E.N. Glezer, and E. Mazur, Phys. Rev. Lett. **80**, 185 (1997).

- [12] S.P. Kuo, A. Ren, and G. Schmidt, Phys. Rev. E **49**, 3310 (1994).
- [13] G.I. Freidman, Zn. Eksp. Teor. Fiz. **41**, 226 (1961) [Sov. Phys. JETP **14**, 165 (1962)]; F. Brunel, J. Opt. Soc. Am. B **7**, 521 (1990); A. Banos, Jr., W.B. Mori, and J.M. Dawson, IEEE Trans. Plasma Sci. **21**, 57 (1993).
- [14] V.I. Semenova, Izv. Vuzov Radiofiz. **10**, 1077 (1967) [Sov. Radiophys. **10**, 599 (1967)]; L.A. Ostrovskii, Usp. Fiz. Nauk **116**, 315 (1975) [Sov. Phys. Usp. **18**, 452 (1975)]; M. Lampe, E. Ott, and J.H. Walker, Phys. Fluid **21**, 42 (1978).
- [15] R.L. Savage, W.B. Mori, and C. Joshi, Phys. Rev. Lett. **68**, 946 (1992); W.B. Mori *et al.*, Phys. Rev. Lett. **74**, 544 (1995); C.H. Lai *et al.*, Phys. Rev. Lett. **77**, 4764 (1996).

## FIGURES

FIG. 1. The plot of the electric field intensity  $E$  (normalized to its initial value) versus  $z$  and  $t$ . The maximum density  $n = n_{\max} = 10n_c$ . One can see the splitting into a reflected (relatively small) and a transmitted pulse.

FIG. 2. Spectral contents of the reflected and the transmitted Pulses shown in Fig. 1.

FIG. 3. Normalized magnetic field  $B$  versus  $z$  and  $t$ . One can see three distinct “pulses”: two propagating (the reflected and the transmitted), and the other static in time.

FIG. 4. A plot of the electric field illustrating the Pulse interaction with a superluminal ionization front moving with a  $V$  close to  $V_c = 2$ . The high frequency field is trapped in space.