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Spontaneous healing and growth of locked magnetic island chains in toroidal plasmas

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Recent experiments have demonstrated that locked magnetic island chains in stellarator plasmas spontaneously heal under certain conditions, and spontaneously grow under others. A formalism initially developed to study magnetic island dynamics in tokamak plasmas is employed to investigate this phenomenon. It is found that island healing/growth transitions can be caused either by a breakdown in torque balance in the vicinity of the island chain, or by an imbalance between the various terms in the island width evolution equation. The scaling of the healing/growth thresholds with the standard dimensionless plasma parameters β, ν_*, and ρ_* is determined. In accordance with the experimental data, it is found that island healing generally occurs at high β and low ν_*, and island growth at low β and high ν_*. In further agreement, it is found that island healing is accompanied an ion poloidal velocity shift in the electron diamagnetic direction, and island growth by a velocity shift in the ion diamagnetic direction. Finally, it is found that there is considerable hysteresis in the healing/growth cycle, as is also seen experimentally. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4766582]

I. INTRODUCTION

Magnetic island physics is a topic of considerable interest to the stellarator community. (By "stellarator," we mean any significantly non-axisymmetric toroidal magnetic confinement device.) The main reason for this interest is that, unlike an axisymmetric plasma equilibrium, a general three-dimensional (3D) equilibrium does not necessarily possess a set of nested magnetic flux-surfaces. In the worst-case scenario, overlapping helical magnetic island chains completely disrupt the surfaces, thereby severely compromising their energy and particle confinement properties. For this reason, the elimination of magnetic island chains is often used as a criterion for the optimization of stellarator designs. Unfortunately, at zero β, even the most highly optimized designs still generally retain a few, narrow, non-overlapping, locked (i.e., non-rotating) island chains (this is particularly the case for 3D configurations, such as heliotrons and heliacs, that possess significant magnetic shear). However, recent studies on the large helical device (LHD)[1–4] (which is a heliotron) and TJ-II[4] (which is a heliac) have indicated that, under certain circumstances, a locked magnetic island chain in a *finite*-β stellarator equilibrium can *spontaneously heal*, leaving behind perfectly nested magnetic flux-surfaces with optimal confinement properties. Conversely, under different circumstances, a locked island chain can *spontaneously grow* in an equilibrium with perfectly nested flux-surfaces, thereby causing a confinement degradation.[5] Generally speaking, spontaneous island healing is associated with *large magnetic shear, high-*β, and *low collisionality*, whereas spontaneous island growth is associated with *small magnetic shear, low-*β, and *high collisionality*. In addition, the ion poloidal flow velocity in the island region is observed to shift in the electron/ion diamagnetic direction during the island healing/growth process. Finally, the island healing/growth cycle exhibits considerable hysteresis.

One mechanism by which finite pressure effects could, in principle, modify magnetic island widths in stellarators is via *resonant Pfirsch-Schlüter currents*.[6–8] To be more exact, if the resonant magnetic field produced by such currents is in anti-phase with the island-generating vacuum magnetic field then finite pressure effects counteract the vacuum island chain, leading to exact cancelation of the island-generating magnetic field, and consequent island healing, at a critical β value. However, at higher β values, finite pressure effects inevitably overcompensate the island-generating vacuum field, producing an island chain whose helical phase is flipped (by 180°) with respect to the vacuum chain. This prediction is in conflict with the LHD and TJ-II observations, which indicate that exceeding the critical β required to heal an island chain does not lead to the reappearance of the chain with a different phase. Hence, resonant Pfirsch-Schlüter currents seem unable to account for the existing experimental data.

Another mechanism by which finite pressure effects could, in principle, modify magnetic island widths is via the *perturbed bootstrap current*, which is expected to have a stabilizing effect in stellarators (with the correct choice of the bootstrap current direction, and the sign of the magnetic shear).[9,10] Unfortunately, attempts to account for the existing experimental data on the basis of this effect alone have not been particularly successful.[2]

In this paper, following the lead of Ida *et al.*,[1] Nishimura *et al.*,[11] and Hegna,[12] we shall investigate whether the LHD and TJ-II data can be explained by means of a formalism for magnetic island dynamics that was originally developed for tokamaks.[13–18] In fact, we shall concentrate our attention on two main effects that occur within this formalism. First, the *drag torque* acts on a locked island chain in a rotating plasma, due to the combined effect of neoclassical flow damping and ion perpendicular viscosity. Second, the influence of the *ion polarization current* on island stability. In principle, both of these effects could lead to spontaneous island healing.[1,11,12] In order to make use of the tokamak formalism, it is necessary to view a stellarator plasma as an axisymmetric toroidal plasma that is perturbed by a relatively small amplitude, static, externally generated, helical magnetic field. Obviously, this is not a particularly good approximation, since the helical field in stellarators is not generally small. (It is certainly not small in the LHD and TJ-II devices.) Hence, the analysis presented in this paper can only be regarded as a first step to producing an accurate theory of magnetic island dynamics in stellarator plasmas.

II. DERIVATION OF NEOCLASSICAL FOUR-FIELD MODEL

A. Coordinates

Consider a large aspect-ratio, low-β, circular cross-section, axisymmetric, toroidal plasma equilibrium. Let us adopt the standard right-handed toroidal coordinates (r, θ, φ), where r is the magnetic flux-surface minor radius, θ the poloidal angle, and φ the toroidal angle. Note that $|\nabla\varphi| = 1/R_0$, where R_0 is the plasma major radius. In the following, $\mathbf{e}_\theta$ denotes a unit vector pointing in the θ-direction, etc. Suppose that the aforementioned plasma equilibrium is slightly perturbed by a static, externally generated, helical magnetic field of poloidal mode number m_θ, and toroidal mode number n_φ. Consider a constant-ψ,[19] helical magnetic island chain, with m_θ poloidal periods, and n_φ toroidal periods, embedded in this modified equilibrium. The chain is assumed to be *radially localized* in the vicinity of the resonant magnetic flux-surface, $r = r_s$, where $q(r_s) \equiv q_s = m_\theta/n_\varphi$. Here, $q(r)$ is the equilibrium safety-factor (i.e., the inverse of the rotational transform) profile. It is also assumed that $\epsilon_s \equiv r_s/R_0 \ll 1$ and $q_s \sim \mathcal{O}(1)$.

B. Outer and inner regions

The plasma is conveniently divided into an "inner region," which comprises the plasma in the immediate vicinity of the resonant surface, and an "outer region," which comprises the remainder of the plasma. As is well-known, standard, linear, ideal-MHD analysis is perfectly adequate in the outer region, whereas nonlinear, nonideal, drift-MHD analysis is generally required in the inner region. Let us assume that a conventional linear ideal-MHD solution has been found in the outer region. In the absence of the external perturbation, such a solution is characterized by a single real parameter, Δ', known as the *tearing stability index*, which is defined as the jump in logarithmic derivative of the radial component (of the resonant harmonic) of the perturbed magnetic field across the inner region.[19] The tearing stability index measures the *free energy* available in the outer region to cause a spontaneous change in the island chain's radial width.[20,21] This free energy acts to increase the width if $\Delta' > 0$, and vice versa. It remains to obtain a nonlinear, nonideal, drift-MHD solution in the inner region, and then to asymptotically match this solution to the aforementioned linear, ideal-MHD solution at the boundary between the inner and the outer regions.

C. Drift-MHD model

In the inner region, our starting point is the following drift-MHD fluid model of the plasma dynamics, which is adapted from Braginskii[22] and Hazeltine and Meiss:[23]

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_i\cdot\nabla\right) n + \nabla\cdot\mathbf{V}_i\, n = D_\perp \nabla^2 n, \tag{1}$$

$$\mathbf{E} + \mathbf{V}_e\times\mathbf{B} + \frac{1}{e\,n}\,\nabla P + 0.71\,\frac{\nabla_\parallel T_e}{e} = \eta_\parallel\,\mathbf{J}_\parallel + \eta_\perp\left(\mathbf{J}_\perp - \frac{3}{2}\frac{n}{B}\,\mathbf{e}_\parallel\times\nabla T_e\right), \tag{2}$$

$$m_i\, n\left[\left(\frac{\partial}{\partial t} + \mathbf{V}_i\cdot\nabla\right)\mathbf{V}_E + \left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right)\mathbf{V}_\parallel\right] = \mathbf{J}\times\mathbf{B} - \nabla(P-\Xi) + \nabla_\parallel\Xi - \nabla\cdot\pi_i + \mu_{\perp\,i}\,\nabla^2\mathbf{V}_i. \tag{3}$$

Here, $\mathbf{E}$ is the electric field-strength, $\mathbf{B}$ the magnetic field-strength, $\mathbf{e}_\parallel = \mathbf{B}/B$, $\mathbf{J}$ is the electric current density, $\mathbf{J}_\parallel = (\mathbf{e}_\parallel\cdot\mathbf{J})\,\mathbf{e}_\parallel$, $\mathbf{J}_\perp = \mathbf{J} - \mathbf{J}_\parallel$, $n = n_0\,(1 + \delta n/n_0)$ is the electron number density, $T_e = T_{e\,0}\,(1 + \eta_e\,\delta n/n_0)$ the electron temperature, $T_i = T_{i\,0}\,(1 + \eta_i\,\delta n/n_0)$ the ion temperature, $P = n\,(T_e + T_i)$ the total plasma pressure, $P_i = n\,T_i$ the ion pressure, π_i the neoclassical ion stress tensor, $\mathbf{V}_E = \mathbf{E}\times\mathbf{B}/B^2$, $\mathbf{V}_\parallel = V_\parallel\,\mathbf{e}_\parallel$ is the parallel (to the equilibrium magnetic field) guiding center velocity, $\mathbf{V} = \mathbf{V}_E + \mathbf{V}_\parallel$ the total guiding center velocity, $\mathbf{V}_i = \mathbf{V} + \mathbf{V}_{*\,i}$ the ion fluid velocity, $\mathbf{V}_{*\,i} = -\nabla P_i\times\mathbf{e}_\parallel/e\,n B$ the ion diamagnetic velocity, $\mathbf{V}_e = \mathbf{V}_i - \mathbf{J}/n\,e$ the electron fluid velocity, $\nabla_\parallel A \equiv (\mathbf{e}_\parallel\cdot\nabla A)\,\mathbf{e}_\parallel$, $\Xi = (P_i/2\Omega_i)\,\mathbf{e}_\parallel\cdot\nabla\times(\mathbf{V}_E + \mathbf{V}_{*\,i})$, and $\Omega_i = e B/m_i$. The constants $n_0, T_{e0}, T_{i0}, \eta_e$, and η_i specify the unperturbed (by the island chain) equilibrium electron number density, electron temperature, ion temperature, electron temperature gradient (relative to the density gradient), and ion temperature gradient (relative to the density gradient), respectively, at the resonant surface. In addition, $Z = 1$ is the ion charge number, m_i the ion mass, and e the magnitude of the electron charge. The quantities $\eta_\parallel$ and $\eta_\perp$ are the parallel and perpendicular plasma resistivities, respectively, whereas $D_\perp$ is a phenomenological perpendicular particle diffusivity (due to small-scale turbulence), and $\mu_{\perp\,i}$ a phenomenological perpendicular ion viscosity (likewise, due to small-scale turbulence). All four of these quantities are evaluated at $r = r_s$, and are assumed to be constant across the inner region. Note that Eq. (3) implicitly includes the contribution of the ion gyroviscous tensor.[23] Furthermore, Eqs. (2) and (3) neglect electron inertia, the neoclassical electron stress tensor (which implies the neglect of the bootstrap current), and charged particle drifts due to magnetic field-line curvature. It is assumed that $|\delta n|/n_0 \ll 1$ in the inner region. Moreover, η_e and η_i are both taken to be *positive*, as is generally the case in a conventional stellarator plasma. Finally, the terms involving Ξ that appear in Eq. (3) are related to the ion vorticity gradient. These terms are usually neglected in the drift-MHD model,[23] and are only retained here for the sake of completeness. (Actually, they have no influence on island dynamics.)

Our model neoclassical ion stress tensor takes the form

$$\nabla\cdot\pi_i \simeq m_i\,n[\nu_{\theta\,i}\,V_{\theta\,i}^{nc}\,\mathbf{e}_\theta+\nu_{\perp\,e}\,V_{\perp\,i}^{nc}\,\mathbf{e}_\perp], \tag{4}$$

where $\mathbf{e}_\perp=\mathbf{e}_\parallel\times\mathbf{e}_r$, and

$$V_{\theta\,i}^{nc}=\mathbf{e}_\theta\cdot\left(\mathbf{V}_i+\lambda_{\theta\,i}\,\frac{\eta_i}{1+\eta_i}\,\mathbf{V}_{*\,i}\right), \tag{5}$$

$$V_{\perp\,i}^{nc}=\mathbf{e}_\perp\cdot\left(\mathbf{V}_i+\lambda_{\perp\,i}\,\frac{\eta_i}{1+\eta_i}\,\mathbf{V}_{*\,i}\right). \tag{6}$$

Expression (4) is a somewhat simplified version of the stress tensor derived in Refs. 24–27 (on the assumption that the plasma equilibrium is only weakly non-axisymmetric). In this expression, $\nu_{\theta\,i}$ is the neoclassical *poloidal flow damping rate*, evaluated at the resonant surface. Poloidal flow damping is caused by *collisional friction* between trapped and passing ions, and acts to relax the ion poloidal flow velocity in such a manner than $V_{\theta\,i}^{nc}\to 0$.[24,28] Furthermore, $\nu_{\perp\,i}$ is the neoclassical perpendicular flow damping rate, evaluated at the resonant surface. Perpendicular flow damping is due to *non-ambipolar* cross-flux-surface ion transport induced by the applied non-axisymmetric magnetic perturbation, and acts to relax the ion perpendicular flow velocity in such a manner that $V_{\perp\,i}^{nc}\to 0$.[25] Note that, in the presence of strong neoclassical poloidal flow-damping, neoclassical perpendicular flow damping effectively relaxes the ion toroidal flow velocity. For this reason, such flow damping is usually referred to as *neoclassical toroidal flow damping*. The quantities $\nu_{\theta\,i}$, $\nu_{\perp\,i}$, $\lambda_{\theta\,i}$, and $\lambda_{\perp\,i}$ are all assumed to be constant across the inner region.

Finally, our model is completed by Maxwell's equations: $\nabla\cdot\mathbf{B}=0$, $\nabla\times\mathbf{E}=-\partial\mathbf{B}/\partial t$, and $\nabla\times\mathbf{B}=\mu_0\,\mathbf{J}$.

D. Single helicity approximation

By definition, the inner region is radially localized in the vicinity of the resonant flux-surface, $r=r_s$. All fields in this region are assumed to be functions of the radial variable $x=r-r_s$, and the helical angle $\zeta=m_\theta\,\theta-n_\varphi\,\varphi-\phi_v$, where ϕ_v is a helical phase (see Sec. III C). The magnetic and electric fields in the inner region take the form $\mathbf{B}\simeq(B_0+b_\parallel)\,\mathbf{e}_\parallel+\nabla A_\parallel\times\mathbf{e}_\parallel$ and $\mathbf{E}\simeq(E_0-\partial_t\,A_\parallel)\,\mathbf{e}_\parallel+\partial_t\nabla\chi\times\mathbf{e}_\parallel-\nabla\Phi$, respectively, where $\mathbf{e}_\parallel\simeq(\epsilon_s/q_s)\,\mathbf{e}_\theta+\mathbf{e}_\varphi$, and $\nabla^2\chi=b_\parallel$. Here, $\partial_t\equiv\partial/\partial_t$, B_0 is the equilibrium toroidal magnetic field-strength, and E_0 is a constant. Lowest-order force balance reveals that $b_\parallel\simeq-\mu_0\,(1+\eta_i)\,(1+\tau)\,T_{i\,0}\,\delta n/B_0$, where

$$\tau=\frac{T_{e\,0}}{T_{i\,0}}\left(\frac{1+\eta_e}{1+\eta_i}\right). \tag{7}$$

Finally, $\nabla\zeta\simeq k_\theta\,\mathbf{e}_\perp$, where $k_\theta=m_\theta/r_s$, and $\mathbf{e}_\perp\simeq\mathbf{e}_\theta-(\epsilon_s/q_s)\,\mathbf{e}_\varphi$. Note that $\mathbf{e}_\parallel\cdot\nabla\equiv 0$.

E. Basic definitions

Let $4w$ be the full radial width of the island chain separatrix. It is assumed that $w/r_s\ll 1$. Suppose that

$$\frac{\delta n}{n_0}\to-\frac{x}{L_n}, \tag{8}$$

$$A_\parallel\to\frac{B_0}{L_s}\left(\frac{1}{2}\,x^2+w^2\cos\zeta\right), \tag{9}$$

as $|x|/w\to\infty$. Here, L_n and $L_s=R_0\,q_s/(d\ln q/d\ln r)_{r_s}$ are termed the equilibrium *density gradient scale-length*, and the *magnetic shear-length*, respectively, at the resonant surface. The density gradient scale-length is assumed to be *positive*, corresponding to a conventional stellarator equilibrium in which the density decreases radially outward. However, the magnetic shear-length is taken to be *negative*, as is generally the case in stellarators (whose rotational transform profiles usually increase radially outward).

It is helpful to define the *ion diamagnetic velocity*

$$V_{*\,i}=\frac{T_{i\,0}\,(1+\eta_i)}{e\,B_0\,L_n}, \tag{10}$$

the effective ion gyroradius

$$\rho_i=\left[\frac{T_{i\,0}\,(1+\eta_i)}{m_i}\right]^{1/2}\left(\frac{m_i}{e\,B_0}\right), \tag{11}$$

and the effective ion beta

$$\beta_i=\frac{\mu_0\,n_0\,T_{i\,0}\,(1+\eta_i)}{B_0^2}. \tag{12}$$

Furthermore, it is assumed that $\beta_i\ll 1$.

Parallel resistivity, cross-flux-surface particle transport, and cross-flux-surface ion momentum transport in the inner region are conveniently parameterized in terms of the dimensionless quantities

$$\eta=\frac{\eta_\parallel}{\mu_0\,k_\theta\,V_{*\,i}\,w^2}, \tag{13}$$

$$D=\left[D_\perp+\beta_i\,(1+\tau)\,\frac{\eta_\perp}{\mu_0}\left(1-\frac{3}{2}\,\frac{\eta_e}{1+\eta_e}\,\frac{\tau}{1+\tau}\right)\right]\frac{1}{k_\theta\,V_{*\,i}\,w^2}, \tag{14}$$

$$\mu=\frac{\mu_{\perp\,i}}{n_0\,m_i\,k_\theta\,V_{*\,i}\,w^2}, \tag{15}$$

respectively. Likewise, neoclassical poloidal and toroidal flow damping in the inner region are parameterized in terms of the dimensionless quantities

$$\hat{\nu}_{\theta\,i}=\left(\frac{\epsilon_s}{q_s}\right)^2\left(\frac{\nu_{\theta\,i}}{k_\theta\,V_{*\,i}}\right), \tag{16}$$

$$\hat{\nu}_{\perp\,i}=\left(\frac{\epsilon_s}{q_s}\right)^2\left(\frac{\nu_{\perp\,i}}{k_\theta\,V_{*\,i}}\right), \tag{17}$$

respectively.

F. Neoclassical four-field model

Employing a procedure similar to that discussed in Ref. 15, we can eliminate the compressional Alfvén wave from

Eqs. (1)–(3), to obtain the following *neoclassical four-field model* for the plasma in the inner region

$$0 = [\phi + \tau N, \psi] + \eta\,\beta J, \tag{18}$$

$$0 = [\phi, N] - \rho\,[\alpha V + J, \psi] + D\,\partial_X^2 N, \tag{19}$$

$$0 = [\phi, V] - (1+\tau)\,\alpha\,[N, \psi] + \mu\,\partial_X^2 V - \hat{\nu}_{\theta\,i}\{V - \partial_X\,[\phi + (v_\theta - 1)\,N]\}, \tag{20}$$

$$0 = \epsilon\,\partial_X[\phi - N, \partial_X\phi] + [J, \psi] + \epsilon\,\mu\,\partial_X^4(\phi - N) + \hat{\nu}_{\theta\,i}\,\partial_X\{V - \partial_X\,[\phi + (v_\theta - 1)\,N]\} + \hat{\nu}_{\perp\,i}\,\partial_X\{-\partial_X\,[\phi + (v_\perp - 1)\,N]\}. \tag{21}$$

Here,

$$X = \frac{x}{w}, \tag{22}$$

$$[A, B] \equiv \partial_X A\;\partial_\zeta B - \partial_\zeta A\;\partial_X B, \tag{23}$$

and

$$\psi(X, \zeta) = \frac{L_s}{B_0\,w^2}\,A_\parallel, \tag{24}$$

$$N(X, \zeta) = \frac{L_n}{w}\,\frac{\delta n}{n_0}, \tag{25}$$

$$\phi(X, \zeta) = -\frac{\Phi}{w\,V_{*\,i}\,B_0} + v\,X, \tag{26}$$

$$V(X, \zeta) = \left(\frac{\epsilon_s}{q_s}\right)\left(\frac{V_\parallel}{V_{*\,i}}\right), \tag{27}$$

with

$$J(X, \zeta) = \beta^{-1}(-1 + \partial_X^2\psi), \tag{28}$$

and

$$v = \frac{1}{k_\theta\,V_{*\,i}}\,\frac{d\phi_v}{dt}. \tag{29}$$

Also,

$$\epsilon = \left(\frac{\epsilon_s}{q_s}\right)^2, \tag{30}$$

$$\rho = \left(\frac{q_s}{\epsilon_s}\right)^2\left(\frac{\rho_i}{w}\right)^2, \tag{31}$$

$$\alpha = \left(\frac{\epsilon_s}{q_s}\right)\left(\frac{L_n}{L_s}\right)\left(\frac{w}{\rho_i}\right)^2, \tag{32}$$

$$\beta = \beta_i\left(\frac{q_s}{\epsilon_s}\right)^2\left(\frac{L_s}{L_n}\right)^2\left(\frac{\rho_i}{w}\right)^2. \tag{33}$$

Finally,

$$E_0 = -\frac{\eta_\parallel}{\mu_0}\,\frac{B_0}{L_s}, \tag{34}$$

$$v_\theta = -\lambda_{\theta\,i}\,\frac{\eta_i}{1 + \eta_i}, \tag{35}$$

$$v_\perp = -\lambda_{\perp\,i}\,\frac{\eta_i}{1 + \eta_i} + v. \tag{36}$$

The four fundamental fields, ψ, N, ϕ, and V, as well as the auxilary field J, are subject to the following boundary conditions:

$$\psi(X, \zeta) \to \frac{1}{2}\,X^2 + \cos\zeta, \tag{37}$$

$$N(X, \zeta) \to -X, \tag{38}$$

$$\phi(X, \zeta) \to -(1 - v_\perp)\,X, \tag{39}$$

$$V(X, \zeta) \to v_\perp - v_\theta, \tag{40}$$

$$J(X, \zeta) \to 0, \tag{41}$$

as $|X| \to \infty$.

Note, incidentally, that Eq. (18) is the parallel component of Eq. (2). Moreover, Eq. (19) is obtained by eliminating $\nabla\cdot\mathbf{V}$ between Eq. (1) and the parallel component of the curl of Eq. (2). Finally, Eq. (20) is the parallel component of Eq. (3), and Eq. (21) the parallel component of the curl of Eq. (3).

The neoclassical four-field model derived in this paper is an improvement on that described in Refs. 16–18 in two respects. First, it takes into account the fact that the flux-surface averaged neoclassical toroidal damping force is actually directed parallel to $\mathbf{e}_\perp$, rather than $\mathbf{e}_\varphi$.[29] Second, it takes into account the fact that the neoclassical poloidal and toroidal flow damping forces relax the ion poloidal and toroidal velocities, respectively, toward values that are proportional to the local ion temperature gradient, which is modified in the presence of the island chain.

III. DERIVATION OF ISLAND EVOLUTION EQUATIONS

A. Island geometry

To lowest order, we expect that

$$\psi(X, \zeta) = \Omega(X, \zeta) \equiv \frac{1}{2}\,X^2 + \cos\zeta \tag{42}$$

in the inner region. The contours of $\Omega(X, \zeta)$ map out the magnetic flux-surfaces of a constant-ψ, helical magnetic island chain whose O-points are located at $X = 0$ and $\zeta = \pi$, and whose X-points are located at $X = 0$ and $\zeta = 0$. The magnetic separatrix corresponds to $\Omega = 1$, the region enclosed by the separatrix to $-1 \le \Omega \le 1$, and the region outside the separatrix to $\Omega > 1$. A general field $A(X, \zeta)$, in the island region, can also be written $A(s, \Omega, \zeta)$, where $s = \mathrm{sgn}(X)$.

B. Flux-surface average operator

It is convenient to define the *flux-surface average operator*

$$\langle A(s,\Omega,\zeta)\rangle = \begin{cases} \oint \frac{A(s,\Omega,\zeta)}{[2\,(\Omega-\cos\zeta)]^{1/2}}\frac{d\zeta}{2\pi} & 1<\Omega \\ \int_{-\zeta_0}^{\zeta_0} \frac{A(s,\Omega,\zeta)+A(-s,\Omega,\zeta)}{2\,[2\,(\Omega-\cos\zeta)]^{1/2}}\frac{d\zeta}{2\pi} & -1\leq\Omega\leq 1 \end{cases}, \tag{43}$$

where $\zeta_0 = \cos^{-1}(\Omega)$. It follows that $\langle[A,\Omega]\rangle \equiv 0$, for any field $A(s,\Omega,\zeta)$. It is also helpful to define the operator $\tilde{A} \equiv A - \langle A\rangle/\langle 1\rangle$. Note that $\langle\tilde{A}\rangle \equiv 0$, for any field A.

C. Asymptotic matching

Standard asymptotic matching between the inner and outer regions[21] yields the *island width evolution equation*

$$4I_1\,\tau_R\,\frac{d}{dt}\left(\frac{w}{r_s}\right) = \Delta'\,r_s + 2m_\theta\left(\frac{w_v}{w}\right)^2\cos\phi_v + J_c\,\beta_i\left(\frac{q_s}{\epsilon_s}\right)^2\left(\frac{L_s}{L_n}\right)^2\left(\frac{\rho_i}{w}\right)^3, \tag{44}$$

and the island phase evolution equation

$$2m_\theta\left(\frac{w_v}{r_s}\right)^2\left(\frac{w}{r_s}\right)^2\sin\phi_v = J_s\,\beta_i\left(\frac{q_s}{\epsilon_s}\right)^2\left(\frac{L_s}{L_n}\right)^2\left(\frac{\rho_i}{r_s}\right)^2\left(\frac{w}{r_s}\right), \tag{45}$$

where $I_1 = 0.8227$, $\tau_R = \mu_0\,r_s^2/\eta_\parallel$, and

$$J_c = -2\int_{-\infty}^{\infty}\oint J\cos\zeta\,dX\,\frac{d\zeta}{2\pi} = -4\int_{-1}^{\infty}\langle J_+\cos\zeta\rangle\,d\Omega, \tag{46}$$

$$J_s = -2\int_{-\infty}^{\infty}\oint J\sin\zeta\,dX\,\frac{d\zeta}{2\pi} = -4\int_{-1}^{\infty}\langle X\,[J_+,\Omega]\rangle\,d\Omega, \tag{47}$$

with $J_+(\Omega,\zeta) \equiv (1/2)\,[J(s,\Omega,\zeta)+J(-s,\Omega,\zeta)]$. Here, $4w_v$ is the full radial width of the vacuum island chain (i.e., the island chain obtained by naively superimposing the vacuum magnetic perturbation onto the unperturbed plasma equilibrium), and ϕ_v is the helical phase-shift between the true island chain and the vacuum chain.

The first term on the right-hand side of Eq. (44) governs the intrinsic stability of the island chain. (The chain is intrinsically stable if $\Delta' < 0$, and vice versa.) The second term represents the destabilizing effect of the resonant component of the external perturbation (which is responsible for generating the vacuum island chain). The final term represents the stabilizing or destabilizing (depending on whether the integral J_c is negative or positive, respectively) effect of helical currents flowing in the inner region.

The left-hand side of Eq. (45) represents the electromagnetic locking torque exerted on the plasma in the inner region by the resonant component of the external perturbation, whereas the right-hand side represents the drag torque due to the combined effects of poloidal and toroidal flow damping, and perpendicular ion viscosity. The former torque acts to reduce the helical phase-shift between the island chain and the vacuum chain to zero, whereas the latter torque acts to increase this phase-shift. The resulting phase-shift is in the *electron* diamagnetic direction if the integral J_s is *positive*, and in the *ion* diamagnetic direction otherwise.

IV. SOLUTION OF FOUR-FIELD EQUATIONS

A. Expansion procedure

Equations (18)–(21) and (28) are solved, subject to the boundary conditions (37)–(41), via an expansion in *two* small parameters, Δ and δ, where $\Delta \lll \delta \ll 1$.[30] The expansion procedure is as follows. First, the coordinates X and ζ are assumed to be $\mathcal{O}(\Delta^0\,\delta^0)$. Next, some particular ordering scheme is adopted for the twelve physics parameters v_θ, $v_\perp$, τ, α, ϵ, ρ, β, $\hat{\nu}_{\theta\,i}$, $\hat{\nu}_{\perp\,i}$, η, D, and μ. The fields ψ, N, ϕ, V, and J are then expanded in the form $\psi(X,\zeta) = \sum_{i,j=0,\infty}\psi_{i,j}(X,\zeta)$, etc., where $\psi_{i,j} \sim \mathcal{O}(\Delta^i\,\delta^j)$. Finally, Eqs. (18)–(21) and (28) are solved order by order, subject to the boundary conditions (37)–(41).

B. Ordering scheme

The particular ordering scheme adopted in this paper is as follows:

$$\Delta^0\,\delta^0: \quad v_\theta,\ v_\perp,\ \tau,\ \alpha,$$

$$\Delta^0\,\delta^1: \quad \epsilon,\ \rho,\ \beta,$$

$$\Delta^1\,\delta^0: \quad \hat{\nu}_{\theta\,i},\ \hat{\nu}_{\perp\,i},\ \eta,\ D,\ \mu.$$

This scheme, which is similar to the *intermediate poloidal flow damping* scheme described in Refs. 17 and 18, is appropriate to a *relatively wide* island chain—i.e., a chain whose radial width satisfies $w \gg (q_s/\epsilon_s)\,\rho_i$—embedded in a relatively *low collisionality* plasma—i.e., a plasma for which $\nu_i \ll k_\theta\,V_{*\,i}/f_t$ at the resonant surface, where ν_i is the ion-ion collision frequency, and $f_t \simeq 1.46\,\epsilon_s^{1/2}$ the trapped particle fraction.[24]

C. Zeroth-order solution

To lowest order in the primary and secondary expansions (i.e., to order $\Delta^0\,\delta^0$), Eqs. (18)–(21), and (28) yield

$$0 = [\phi_{0,0} + \tau\,N_{0,0},\psi_{0,0}], \tag{48}$$

$$0 = [\phi_{0,0}, N_{0,0}], \tag{49}$$

$$0 = [\phi_{0,0}, V_{0,0}] - (1+\tau)\,\alpha\,[N_{0,0},\psi_{0,0}], \tag{50}$$

$$0 = [J_{0,0},\psi_{0,0}], \tag{51}$$

$$0 = \partial_X^2\psi_{0,0} - 1, \tag{52}$$

respectively.

Equations (37) and (52) give

$$\psi_{0,0} = \Omega(X,\zeta). \tag{53}$$

Hence, as was previously assumed in Sec. III A, the lowest-order magnetic flux-function maps out a constant-ψ magnetic island chain.

Equations (38), (39), (48), and (49) yield

$$\phi_{0,0} = s\,\phi_0(\Omega), \tag{54}$$

$$N_{0,0} = s\,N_0(\Omega). \tag{55}$$

It follows that the lowest-order electrostatic potential and density profiles are *odd* (in X) *magnetic flux-surface functions*. This implies that, to lowest order, the ion and electron fluids (whose stream-functions are $\phi - N$ and $\phi + \tau N$, respectively) do not cross magnetic flux-surfaces. Let

$$M(\Omega) = -\frac{d\phi_0}{d\Omega}, \tag{56}$$

$$L(\Omega) = -\frac{dN_0}{d\Omega}. \tag{57}$$

Note that $\phi_0 = N_0 = M = L = 0$ within the magnetic separatrix (i.e., $-1 \leq \Omega < 1$), since it is impossible to have an odd flux-surface function in this region. This means that the density profile—and, hence, the electron and ion temperature profiles—are *flattened* inside the magnetic separatrix.[31] Moreover, the boundary conditions (38) and (39) reduce to

$$M(\Omega \to \infty) \to \frac{1 - v_\perp}{\sqrt{2\Omega}}, \tag{58}$$

$$L(\Omega \to \infty) \to \frac{1}{\sqrt{2\Omega}}. \tag{59}$$

Incidentally, because the density and temperature profiles are flattened inside the island separatrix, the electron diamagnetic term in Eq. (2) does not (directly) give rise to island rotation.

Equations (40), (50), (54), and (55), give

$$V_{0,0} = \bar{V}(\Omega), \tag{60}$$

where

$$\bar{V}(\Omega \to \infty) \to v_\perp - v_\theta. \tag{61}$$

It follows that the lowest-order ion parallel flow is even (in X), and lies within magnetic flux-surfaces.

Finally, Eq. (51) implies that $\tilde{J}_{0,0} = 0$. Thus, assuming that $\langle J_{0,0}\rangle = 0$, which turns out to be the case, we obtain

$$J_{0,0} = 0. \tag{62}$$

To zeroth order in the primary expansion, and first order in the secondary expansion (i.e., to order $\Delta^0\,\delta^1$), Eqs. (21) and (62) give

$$[J_{0,1}, \psi_{0,0}] = -\epsilon\,\partial_X[\phi_{0,0} - N_{0,0}, \partial_X\phi_{0,0}], \tag{63}$$

which implies that

$$J_{0,1} = (\epsilon/2)\,d_\Omega[M\,(M-L)]\,\tilde{X^2} + \bar{J}(\Omega). \tag{64}$$

Finally, the lowest-order flux-surface average of Eq. (18) yields

$$\bar{J} = 0. \tag{65}$$

D. First-order solution

To first order in the primary expansion, and zeroth order in the secondary expansion (i.e., to order $\Delta^1\,\delta^0$), Eqs. (18)–(21), and (28) yield

$$0 = [\phi_{1,0} + \tau N_{1,0}, \psi_{0,0}] + [\phi_{0,0} + \tau N_{0,0}, \psi_{1,0}], \tag{66}$$

$$0 = [\phi_{1,0}, N_{0,0}] + [\phi_{0,0}, N_{1,0}] + D\,\partial_X^2 N_{0,0}, \tag{67}$$

$$\begin{aligned}0 = {} & [\phi_{1,0}, V_{0,0}] + [\phi_{0,0}, V_{1,0}] - (1+\tau)\,\alpha\,[N_{1,0}, \psi_{0,0}] \\ & - (1+\tau)\,\alpha\,[N_{0,0}, \psi_{1,0}] + \mu\,\partial_X^2 V_{0,0} \\ & - \hat{\nu}_{\theta\,i}\{V_{0,0} - \partial_X[\phi_{0,0} + (v_\theta - 1)\,N_{0,0}]\},\end{aligned} \tag{68}$$

$$\begin{aligned}0 = {} & [J_{1,0}, \psi_{0,0}] + \hat{\nu}_{\theta\,i}\,\partial_X\{V_{0,0} - \partial_X[\phi_{0,0} + (v_\theta - 1)\,N_{0,0}]\} \\ & + \hat{\nu}_{\perp\,i}\,\partial_X\{-\partial_X[\phi_{0,0} + (v_\perp - 1)\,N_{0,0}]\},\end{aligned} \tag{69}$$

$$0 = \partial_X^2\psi_{1,0}, \tag{70}$$

respectively. Moreover, Eq. (70), (66)–(69) reduce to

$$\psi_{1,0} = 0, \tag{71}$$

$$\phi_{1,0} = -\tau\,N_{1,0}, \tag{72}$$

$$[N_{1,0}, \Omega] = \frac{D\,(X^2\,d_\Omega L + L)}{M + \tau\,L}, \tag{73}$$

$$\begin{aligned}& \tau\,[N_{1,0}, \bar{V}] - s\,M\,[V_{1,0}, \Omega] + (1+\tau)\,\alpha\,[N_{1,0}, \Omega] \\ & \quad = \mu\,\partial_X^2\bar{V} - \hat{\nu}_{\theta\,i}\{\bar{V} + |X|\,[M + (v_\theta - 1)\,L]\},\end{aligned} \tag{74}$$

$$\begin{aligned}[J_{1,0}, \Omega] = {} & -\hat{\nu}_{\theta\,i}\,\partial_X\{\bar{V} + |X|\,[M + (v_\theta - 1)\,L]\} \\ & - \hat{\nu}_{\perp\,i}\,\partial_X\{|X|\,[M + (v_\perp - 1)\,L]\},\end{aligned} \tag{75}$$

respectively.

The flux-surface average of Eq. (73), combined with the boundary condition (59), yields

$$L(\Omega) = \frac{1}{\langle X^2\rangle} \tag{76}$$

outside the magnetic separatrix.

Equation (74) reduces to

$$\bar{V}(\Omega) = 0 \tag{77}$$

inside the separatrix, assuming that $N_{1,0} = V_{1,0} = 0$ in this region [as Eqs. (72) and (73) imply]. Moreover, the flux-surface average of Eq. (75), combined with the boundary condition (61), gives

$$\bar{V}(\Omega) = -\left(\frac{\hat{\nu}_{\theta\,i} + \hat{\nu}_{\perp\,i}}{\hat{\nu}_{\theta\,i}}\right)(\langle X^2\rangle\,F + \bar{v}) \tag{78}$$

outside the separatrix, where $F(\Omega) = M(\Omega) - L(\Omega)$ is an ion stream-function, and

$$\bar{v} = \frac{\hat{\nu}_{\theta\,i}\,v_\theta + \hat{\nu}_{\perp\,i}\,v_\perp}{\hat{\nu}_{\theta\,i} + \hat{\nu}_{\perp\,i}}. \tag{79}$$

Assuming that $\bar{V}(\Omega)$ is continuous across the separatrix [because of the perpendicular diffusion operator acting on $\bar{V}$ in Eq. (74)[16]], we obtain

$$F(\Omega \to 1) \to -\frac{\pi}{4}\,\bar{v}. \tag{80}$$

Likewise, the boundary conditions (58) and (59) yield

$$F(\Omega \to \infty) \to -\frac{v_\perp}{\sqrt{2\Omega}}. \tag{81}$$

Of course, $F=0$ inside the separatrix. The discontinuity in $F(\Omega)$ across the separatrix is resolved by a thin boundary layer of typical radial width ρ_i.[14] The fact that $L = F = \bar{V} = 0$ within the separatrix implies that the ion poloidal velocity and the $\mathbf{E}\times\mathbf{B}$ velocity are both reduced to *zero* in the region lying within the separatrix of a *locked* magnetic island chain. Note, however, that both velocities are non-zero immediately outside the separatrix.

Now, the flux-surface average of Eq. (74) gives

$$0 = \hat{\mu}\, d_\Omega[\langle X^2\rangle\, d_\Omega(\langle X^2\rangle F)] - \hat{\nu}_{\theta\, i}\,(\langle X^2\rangle\langle 1\rangle - 1)(F + v_\theta/\langle X^2\rangle) - \hat{\nu}_{\perp\, i}(\langle X^2\rangle F + v_\perp)\langle 1\rangle \tag{82}$$

outside the separatrix, where

$$\hat{\mu} = \left(\frac{\hat{\nu}_{\theta\, i} + \hat{\nu}_{\perp\, i}}{\hat{\nu}_{\theta\, i}}\right)\mu. \tag{83}$$

Finally, the flux-surface average of X times Eq. (75) gives

$$\langle X\,[J_{1,0},\Omega]\rangle = -d_\Omega\langle X^2\, G\rangle + \langle G\rangle, \tag{84}$$

where

$$G(\Omega,\zeta) = \begin{cases} 0 & -1 \le \Omega \le 1 \\ -(\hat{\nu}_{\theta\, i} + \hat{\nu}_{\perp\, i})(\langle X^2\rangle - |X|)(F + \bar{v}/\langle X^2\rangle) & 1 < \Omega \end{cases}. \tag{85}$$

E. Determination of ion stream-function

It is convenient to define a new flux-surface label $k = [(1+\Omega)/2]^{1/2}$. Thus, $k=0$ corresponds to the O-points of the island chain, $k=1$ to the X-points and the magnetic separatrix, and $k\to\infty$ to $|X|\to\infty$. It is also helpful to define the complete elliptic integrals

$$E(x) \equiv \int_0^{\pi/2} (1 - x^2\sin^2 u)^{1/2}\, du, \tag{86}$$

$$K(x) \equiv \int_0^{\pi/2} (1 - x^2\sin^2 u)^{-1/2}\, du, \tag{87}$$

for $0 \le x \le 1$. It is easily demonstrated that

$$\langle 1\rangle = K(1/k)/(k\,\pi), \tag{88}$$

$$\langle X^2\rangle = (4k/\pi)\, E(1/k), \tag{89}$$

$$\langle |X^3|\rangle = 2\,(2k^2 - 1), \tag{90}$$

$$\langle X^4\rangle = (16k/3\pi)[2\,(2k^2-1)\,E(1/k) - (k^2-1)\,K(1/k)], \tag{91}$$

in the region $1 < k < \infty$.

Equation (82) reduces to

$$0 = \frac{\hat{\mu}}{4}\, d_k\{\mathcal{A}(k)\, d_k[\mathcal{A}(k)\, Y(k)]\} - \hat{\nu}_{\theta\, i}\,[\mathcal{A}(k)\,\mathcal{B}(k) - 1]\, Y(k) - \hat{\nu}_{\perp\, i}\,[\mathcal{A}(k)\, Y(k) - 1]\,\mathcal{B}(k), \tag{92}$$

where $Y(k) = -[2k\,F(k) + v_\theta/\mathcal{A}(k)]/(v_\perp - v_\theta)$, $\mathcal{A}(k) = \langle X^2\rangle/(2k) = (2/\pi)\,E(1/k)$, and $\mathcal{B}(k) = 2k\,\langle 1\rangle = (2/\pi)\,K(1/k)$. The above equation describes how the normalized ion stream-function, $Y(k)$, is determined via a competition between cross flux-surface momentum transport due to perpendicular ion viscosity (first term on the right-hand side), neoclassical poloidal flow damping (second term on the right-hand side), and neoclassical toroidal flow damping (third term on the right-hand side). Equation (92) must be solved for $Y(k)$ in the region $1 < k < \infty$, subject to the boundary conditions [see Eqs. (80) and (81)]

$$Y(k\to 1) \to \frac{\pi}{2}\left(\frac{\hat{\nu}_{\perp\, i}}{\hat{\nu}_{\theta\, i} + \hat{\nu}_{\perp\, i}}\right), \tag{93}$$

$$Y(k\to\infty) \to 1. \tag{94}$$

This procedure fully specifies $Y(k)$ [and, hence, $F(\Omega)$].

F. Evaluation of cosine integral

The lowest-order contribution to the cosine integral, J_c, comes from $J_{0,1}$. In fact, Eqs. (42), (46), (64), (65), (76)–(78), and (82) yield

$$J_c = \epsilon\, I_{pol}, \tag{95}$$

where

$$I_{pol} = \frac{2\pi}{3}\,\bar{v}\,(\bar{v}-1) + v_\theta\,(v_\theta - 1)\int_1^\infty d_k[\mathcal{A}^{-2}\,k^{-2}]\,2k^3 \times\left(\mathcal{C} - \frac{\mathcal{A}^2}{\mathcal{B}}\right)dk + (v_\perp - v_\theta)\int_1^\infty d_k[Y([v_\perp - v_\theta]\,Y + [2v_\theta - 1]\,\mathcal{A}^{-1})k^{-2}]\,2k^3\left(\mathcal{C} - \frac{\mathcal{A}^2}{\mathcal{B}}\right)dk, \tag{96}$$

and $\mathcal{C}(k) = \langle X^4\rangle/8k^3$. Here, the first term on the right-hand side emanates from the boundary layer on the island separatrix,[14] whereas the final two terms emanate from the region outside the separatrix. There is no contribution to J_c from the region inside the separatrix.

Giving $\Delta' r_s$ the vacuum value $-2m_\theta$ (which implies that the island chain is intrinsically stable), the island width evolution equation, (44), reduces to

$$\frac{2I_1\,\tau_R}{m_\theta}\,\frac{d}{dt}\left(\frac{w}{r_s}\right) = -1 + \left(\frac{w_v}{w}\right)^2\cos\phi_v + I_{pol}\left(\frac{\beta_i}{2m_\theta}\right)\left(\frac{L_s}{L_n}\right)^2\left(\frac{\rho_i}{w}\right)^3. \tag{97}$$

Here, the final term on the right-hand side describes the effect of the *ion polarization current* on the stability of the island chain.[14,32]

G. Evaluation of sine integral

The lowest-order contribution to the sine integral, J_s, comes from $J_{1,0}$. In fact, Eqs. (47), (84), (85), and (92) yield

$$J_s = -\hat{\nu}_{\perp\,i}\,(v_\perp - v_\theta)\int_1^\infty 8\,(\mathcal{A}^{-1} - Y)\,dk. \tag{98}$$

Note that the only contribution to J_s comes from the region outside the island separatrix. Thus, the island phase evolution equation, (47), reduces to

$$\left(\frac{w}{w_v}\right)\sin\phi_v = J_s\left(\frac{\beta_i}{2m_\theta}\right)\left(\frac{q_s}{\epsilon_s}\right)^2\left(\frac{L_s}{L_n}\right)^2\left(\frac{\rho_i}{w_v}\right)^3. \tag{99}$$

Incidentally, we have not made use of the so-called *no slip* approximation to determine the island phase evolution.[13] Rather, we have calculated this evolution in a self-consistent manner from Eqs. (1)–(3).

H. Island regimes

Equations (92)–(94) can be solved analytically in *four* different regimes, whose extents in $\hat{\nu}_{\perp\,i}$–$\hat{\nu}_{\theta\,i}$ space are sketched in Figure 1.

In regime I, which corresponds to $\hat{\nu}_{\theta\,i} \ll \hat{\nu}_{\perp\,i}$ and $\hat{\nu}_{\theta\,i} \gg \mu$,

$$Y(k) \simeq \frac{1}{\mathcal{A}}\left[1 - \frac{\hat{\nu}_{\theta\,i}}{\hat{\nu}_{\perp\,i}}\left(1 - \frac{1}{\mathcal{A}\,\mathcal{B}}\right)\right]. \tag{100}$$

It follows from Eq. (96) that

$$I_{pol} \simeq I_{pol}^{(I)}\,v_\perp\,(v_\perp - 1), \tag{101}$$

where

$$I_{pol}^{(I)} = \frac{2\pi}{3} - \int_1^\infty \frac{4}{\mathcal{A}}\left(\frac{\mathcal{C}\,\mathcal{B}}{\mathcal{A}^2} - 1\right)dk = 1.38. \tag{102}$$

Moreover, it follows from Eq. (98) that

$$J_s \simeq -I_s^{(I)}\,\hat{\nu}_{\theta\,i}\,(v_\perp - v_\theta), \tag{103}$$

where

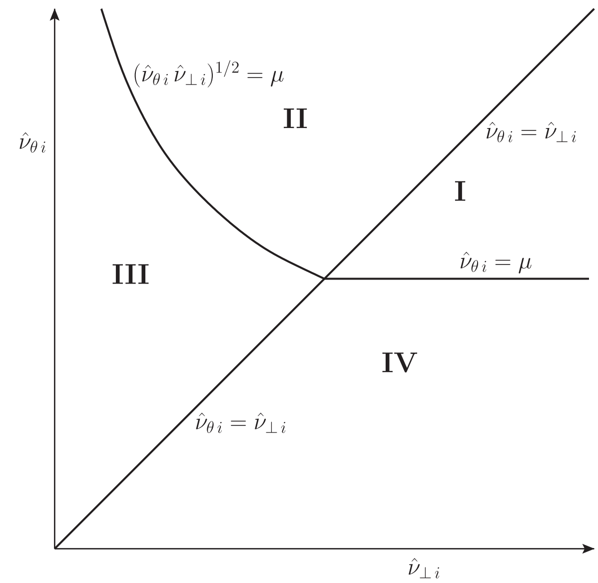

FIG. 1. Extents of the various island regimes in the $\hat{\nu}_{\perp\,i} - \hat{\nu}_{\theta\,i}$ plane.

$$I_s^{(I)} = \int_1^\infty \frac{8}{\mathcal{A}}\left(1 - \frac{1}{\mathcal{A}\,\mathcal{B}}\right)dk = 0.357. \tag{104}$$

In regime II, which corresponds to $\hat{\nu}_{\theta\,i} \gg \hat{\nu}_{\perp\,i}$ and $(\hat{\nu}_{\theta\,i}\,\hat{\nu}_{\perp\,i})^{1/2} \gg \mu$,

$$Y(k) \simeq \frac{\hat{\nu}_{\perp\,i}}{\hat{\nu}_{\theta\,i}}\,\frac{\mathcal{B}}{\mathcal{A}\,\mathcal{B}\,(1 + \hat{\nu}_{\perp\,i}/\hat{\nu}_{\theta\,i}) - 1}. \tag{105}$$

It follows that

$$I_{pol} \simeq I_{pol}^{(I)}\,v_\theta\,(v_\theta - 1), \tag{106}$$

and

$$J_s \simeq -I_s^{(II)}\,\hat{\nu}_{\theta\,i}^{\,1/4}\,\hat{\nu}_{\perp\,i}^{\,3/4}\,(v_\perp - v_\theta), \tag{107}$$

where

$$I_s^{(II)} = \frac{4}{2^{1/4}}\int_0^\infty \frac{dx}{1 + x^4} = 3.74. \tag{108}$$

In regime III, which corresponds to $\hat{\nu}_{\theta\,i} \gg \hat{\nu}_{\perp\,i}$ and $(\hat{\nu}_{\theta\,i}\,\hat{\nu}_{\perp\,i})^{1/2} \ll \mu$,

$$Y(k) \simeq \begin{cases} \dfrac{\hat{\nu}_{\perp\,i}}{\hat{\nu}_{\theta\,i}}\,\dfrac{\mathcal{B}}{\mathcal{A}\,\mathcal{B}\,(1 + \hat{\nu}_{\perp\,i}/\hat{\nu}_{\theta\,i}) - 1} & 1 < k < k_1 \\ 1 - (1 + k_1/k_2)\,\mathrm{e}^{-k/k_2} & k > k_1 \end{cases}, \tag{109}$$

when $\hat{\nu}_{\theta\,i} \gg \mu$, and

$$Y(k) \simeq \begin{cases} (\hat{\nu}_{\perp\,i}/\hat{\nu}_{\theta\,i})\,\mathcal{A}^{-1} & 1 < k < k_3 \\ 1 - (1 + k_3/k_2)\,\mathrm{e}^{-k/k_2} & k > k_3 \end{cases}, \tag{110}$$

when $\hat{\nu}_{\theta\,i} \ll \mu$. Here, $k_1 = (\hat{\nu}_{\theta\,i}/8\,\mu)^{1/2}$, $k_2 = (\mu/4\,\hat{\nu}_{\perp\,i})^{1/2}$, and $k_3 = (\mu/4\,\hat{\nu}_{\theta\,i})^{1/2}$. It follows that

$$I_{pol} \simeq I_{pol}^{(I)}\, v_\theta\,(v_\theta - 1), \tag{111}$$

and

$$J_s \simeq -I_s^{(III)}\,(\hat{\nu}_{\perp\,i}\,\mu)^{1/2}\,(v_\perp - v_\theta), \tag{112}$$

where $I_s^{(III)} = 4$.

In regime IV, which corresponds to $\hat{\nu}_{\theta\,i} \ll \hat{\nu}_{\perp\,i}$ and $\hat{\nu}_{\theta\,i} \ll \mu$,

$$Y(k) \simeq \frac{1}{\mathcal{A}}\left(1 - \frac{\hat{\nu}_{\theta\,i}}{\hat{\nu}_{\perp\,i}}\,{\rm e}^{-k/k_3}\right). \tag{113}$$

It follows that

$$I_{pol} \simeq I_{pol}^{(I)}\, v_\perp\,(v_\perp - 1), \tag{114}$$

and

$$J_s \simeq -I_s^{(III)}\,(\hat{\nu}_{\theta\,i}\,\mu)^{1/2}\,(v_\perp - v_\theta). \tag{115}$$

The properties of the various island regimes are summarized in Table I. Note that these regimes differ somewhat from those described in Refs. 17 and 18 because of our improved expression for the neoclassical ion stress tensor.

V. SPONTANEOUS ISLAND HEALING AND GROWTH

A. Introduction

It this section, we shall examine solutions to the island evolution equations, (97) and (99), with a view to identifying scenarios in which a locked magnetic island chain in a stellarator plasma might spontaneously heal or grow. However, before we can do this, we must adopt specific expressions for the neoclassical poloidal and toroidal flow damping rates.

B. Flow damping rates

According to Ref. 24, the poloidal flow damping rate in the inner region is written

$$\nu_{\theta\,i} = f_t\,\nu_i\left(\frac{q_s}{\epsilon_s}\right)^2, \tag{116}$$

and the parameter $\lambda_{\theta\,i}$ takes the value 1.17 (assuming that $Z = 1$). The above expression is valid provided $\nu_* \ll 1$, where $\nu_* = \nu_i/(\epsilon_s^{3/2}\,\omega_{tr\,i})$ and $\omega_{tr\,i} = (T_{i\,0}/m_i)^{1/2}/(R_0\,q_s)$. This constraint is equivalent to the requirement of *collisionless* banana trapped ions. Although expression (116) was originally derived for an axisymmetric plasma equilibrium, we shall assume that it also holds approximately in a weakly non-axisymmetric equilibrium.

TABLE I. Properties of the various island regimes.

Regime	I_{pol}	J_s
I	$1.38\,v_\perp\,(v_\perp - 1)$	$-0.36\,\hat{\nu}_{\theta\,i}\,(v_\perp - v_\theta)$
II	$1.38\,v_\theta\,(v_\theta - 1)$	$-3.74\,\hat{\nu}_{\theta\,i}^{1/4}\,\hat{\nu}_{\perp\,i}^{3/4}\,(v_\perp - v_\theta)$
III	$1.38\,v_\theta\,(v_\theta - 1)$	$-4.00\,(\hat{\nu}_{\perp\,i}\,\mu)^{1/2}\,(v_\perp - v_\theta)$
IV	$1.38\,v_\perp\,(v_\perp - 1)$	$-4.00\,(\hat{\nu}_{\theta\,i}\,\mu)^{1/2}\,(v_\perp - v_\theta)$

According to Refs. 25, 26, and 33, the toroidal flow damping rate in the inner region is written

$$\nu_{\perp\,i} \simeq f_t\,\nu_i\left(\frac{q_s}{\epsilon_s}\right)^4\left(\frac{n_\varphi}{\nu_*}\right)^2\left(\frac{w}{R_0}\right)^2, \tag{117}$$

and the parameter $\lambda_{\perp\,i}$ takes the value 2.37 (assuming that $Z = 1$, and that the damping is predominately caused by the resonant component of $\delta B/B$). The above expression is valid provided $(w/r_s)^{3/2} \ll \nu_* \ll 1$. This constraint is equivalent to the requirement of *collisionless* banana trapped ions, but *collisional* ripple trapped ions (where the ripple is caused by the dominant $\delta B/B \simeq w/R_0$ variation around magnetic flux-surfaces induced by the island chain). Expression (117) is also only valid when $\nu_i \gg q_s\,\omega_E$, where $\omega_E = V_E/R_0$, and V_E is the typical $\mathbf{E}\times\mathbf{B}$ velocity just outside the island separatrix. This constraint ensures that the flow damping lies in the so-called "$1/\nu$ regime." Note, incidentally, that although $V_E = 0$ within the magnetic separatrix of a *locked* magnetic island chain, $V_E \sim V_{*\,i}$ immediately outside the separatrix. Moreover, the drag torque on the island chain induced by toroidal flow damping originates entirely from the region outside the separatrix. Thus, the flow damping in this region lies in the $1/\nu$ regime provided $\nu_i \gg q_s\,\omega_E$, which is equivalent to $\nu_* \gg f_t\,(q_s/\epsilon_s)\,(\rho_i/L_n)$.

Finally, if the ion collisionality in the inner region becomes sufficiently low that $\nu_* \ll f_t\,(q_s/\epsilon_s)\,(\rho_s/L_n)$ then the toroidal flow damping enters the so-called "ν regime." In this regime,[25,26]

$$\nu_{\perp\,i} \simeq f_t\,\nu_i\left(\frac{q_s}{\epsilon_s}\right)^4\left(\frac{n_\varphi}{\nu_*}\right)^2\left(\frac{\nu_i}{q_s\,\omega_E}\right)^2\left(\frac{w}{R_0}\right)^2, \tag{118}$$

and the parameter $\lambda_{\perp\,i}$ takes the value -0.25 (assuming that $Z = 1$).

C. Torque-driven transitions

Suppose that the island chain lies in regime III (see Table I and Fig. 1), and the toroidal flow damping lies in the $1/\nu$ regime. This implies that the normalized toroidal flow damping rate, $\hat{\nu}_{\perp\,i}$, is significantly less than the normalized poloidal flow damping rate, $\hat{\nu}_{\theta\,i}$, which is most likely to be the case when the ion collisionality is relatively high and/or the island width is relatively small (since $\hat{\nu}_{\perp\,i} \propto w^2/\nu_i$ and $\hat{\nu}_{\theta\,i} \propto \nu_i$). The island evolution equations, (97) and (99), take the form

$$\alpha_1\,\frac{d}{d\hat{t}}\left(\frac{w}{w_v}\right) = -1 + \left(\frac{w_v}{w}\right)^2\cos\phi_v + \alpha_2\,v_0\,(1 + v_0)\left(\frac{w_v}{w}\right)^3, \tag{119}$$

$$\left(\frac{w}{w_v}\right)\sin\phi_v = \alpha_3\left(v_1 - \frac{d\phi_v}{d\hat{t}}\right), \tag{120}$$

where $\hat{t} = t\,k_\theta\,V_{*\,i}$, and

$$\alpha_1 = \frac{2I_1\,\tau_R\,k_\theta\,V_{*\,i}}{m_\theta}\,\frac{w_v}{r_s}, \tag{121}$$

$$\alpha_2 = 1.38\left(\frac{\beta_i}{2m_\theta}\right)\left(\frac{L_s}{L_n}\right)^2\left(\frac{\rho_i}{w_v}\right)^3, \tag{122}$$

$$\alpha_3 = 4\,(\hat{\nu}_{\perp\,i}\,\mu)^{1/2}\left(\frac{q_s}{\epsilon_s}\right)^2\left(\frac{\beta_i}{2m_\theta}\right)\left(\frac{L_s}{L_n}\right)^2\left(\frac{\rho_i}{w_v}\right)^3, \tag{123}$$

$$v_0 = 1.17\,\frac{\eta_i}{1+\eta_i}, \tag{124}$$

$$v_1 = 1.20\,\frac{\eta_i}{1+\eta_i}. \tag{125}$$

Note that

$$\hat{\nu}_{\perp\,i}\,\mu \simeq f_t\left(\frac{\nu_i}{k_\theta\,V_{*\,i}}\right)\left(\frac{q_s}{\epsilon_s}\right)^2\left(\frac{n_\varphi}{\nu_*}\right)^2\left(\frac{\mu_{\perp\,i}}{n_0\,m_i\,k_\theta\,V_{*\,i}\,R_0^2}\right) \tag{126}$$

is independent of the island width.

Suppose that the destabilizing ion polarization term (i.e., the final term on the right-hand side) in Eq. (119) is *negligible*. Let us search for *steady-state* (i.e., locked) solutions of Eqs. (119) and (120). We obtain

$$\frac{w}{w_v} \simeq (\cos\phi_v)^{1/2}, \tag{127}$$

and

$$(\cos\phi_v)^{1/2}\sin\phi_v \simeq \alpha_3\,v_1. \tag{128}$$

The previous equation can only be satisfied when $0 \leq \phi_v < \cos^{-1}(1/\sqrt{3})$ and $\alpha_3 < (4/27)^{1/4}/v_1$. If α_3 exceeds the critical value $(4/27)^{1/4}/v_1$ then the drag torque on the right-hand side of the equation can no longer be balanced by the electromagnetic locking torque on the left-hand side. In this situation, we would expect the island chain to *unlock* from the external magnetic perturbation, spin up, and decay to small amplitude (see the next paragraph).[12,13] It follows that the drag torque causes *spontaneous healing* of a locked magnetic island chain of finite width when β_i exceeds the critical value $\beta_{heal} = (4/27)^{1/4}\,(\beta_i/\alpha_3\,v_1)$, or, equivalently,

$$\beta_{heal} = \frac{1}{(108)^{1/4}}\left(\frac{m_\theta}{v_1}\right)\left(\frac{1}{\hat{\nu}_{\perp\,i}\,\mu}\right)^{1/2}\left(\frac{\epsilon_s}{q_s}\right)^2\left(\frac{L_n}{L_s}\right)^2\left(\frac{w_v}{\rho_i}\right)^3. \tag{129}$$

The spontaneous healing transition is associated with a shift in the ion poloidal velocity at the rational surface from a value of zero, when the island is present, to a value of $1.17\,\eta_i\,V_{*\,i}/(1+\eta_i)$ when the island is absent. Since the latter value is *positive* (assuming that $\eta_i > 0$), the shift is in the *electron* diamagnetic direction.

In order to investigate the spontaneous growth of a locked island chain, we need to consider time varying solutions of Eqs. (119) and (120) (again, neglecting the ion polarization term in the former equation). We find that

$$\frac{\alpha_1\,v}{3}\,\frac{dy}{d\phi_v} \simeq -y^{2/3} + \cos\phi_v, \tag{130}$$

$$y^{1/3}\sin\phi_v = \alpha_3\,(v_1 - v), \tag{131}$$

where $y = (w/w_v)^3$ and $v = d\phi_v/d\hat{t}$. Assuming that $|y| \ll 1$, Eq. (130) can be solved to give

$$y \simeq \left(\frac{3}{v\,\alpha_1}\right)\sin\phi_v. \tag{132}$$

The previous assumption is valid provided $\alpha_1 \gg 1$ (i.e., provided the resistive diffusion rate across the vacuum island width is much smaller than the diamagnetic frequency, which is almost certainly the case in a high temperature plasma). Note that the above expression holds only for $0 \leq \phi_v \leq \pi$, since the island width cannot be negative. In fact, when ϕ_v approaches π, and the island width simultaneously approaches zero, we expect the helical phase of the island chain to shift abruptly by π radians (with an accompanying transformation of O-points to X-points, and vice versa), so that the island width remains positive. In other words, the island width *pulsates* in time, periodically falling to zero, because the island chain cannot maintain a destabilizing phase relation with respect to the external perturbation due to the plasma rotation in the inner region.[34,35] Consequently, the island width remains much smaller than the vacuum island width. Equation (131) yields

$$(\sin\phi_v)^{4/3} \simeq \alpha_4\left(\frac{v}{v_1}\right)^{1/3}\left(1 - \frac{v}{v_1}\right), \tag{133}$$

where

$$\alpha_4 = v_1\,\alpha_3\left(\frac{v_1\,\alpha_1}{3}\right)^{1/3}. \tag{134}$$

Now, Eq. (133) must be satisfied for all island phases in the range $0 \leq \phi_v \leq \pi$. However, the left-hand side takes values in the range $0 \leftrightarrow 1$, as ϕ_v varies, whereas the right-hand side can only take values in the range $0 \leftrightarrow (27/256)^{1/3}\,\alpha_4$. The former value corresponds to $v/v_1 = 1$, and the latter to $v/v_1 = 1/4$. Hence, Eq. (133) can only be satisfied when $\alpha_4 > (256/27)^{1/3}$. If α_4 falls below the critical value $(256/27)^{1/3}$, then the electromagnetic locking torque on the left-hand side of Eq. (133) can no longer be balanced by the drag torque on the right-hand side. In this situation, we would expect the island chain to *lock* to the external magnetic perturbation in a destabilizing phase, and subsequently grow to a width comparable with the vacuum island width.[13,34] It follows that the electromagnetic torque triggers *spontaneous growth* of a locked magnetic island chain when β_i falls below the critical value $\beta_{grow} = (\beta_i/v_1\,\alpha_3)^{3/2}\,(9\,v_1\,\alpha_1\,\beta_i/256)^{1/2}$, or, equivalently,

$$\beta_{grow} = [(108)^{1/4}\,\beta_{heal}]^{3/2}\left(\frac{16}{9}\,\frac{m_\theta}{I_1\,\tau_R\,k_\theta\,V_{*\,i}\,\beta_i\,v_1}\,\frac{r_s}{w_v}\right)^{1/2}. \tag{135}$$

In writing the above expression, we are anticipating the fact that $\tau_R\,k_\theta\,V_{*\,i}$ scales as β_i^{-1}. The spontaneous growth of the island chain is associated with a shift in the ion poloidal velocity at the rational surface from a value of $1.17\,\eta_i\,V_{*\,i}/(1+\eta_i)$ to a value of zero. Thus, the shift is in the *ion* diamagnetic direction.

Moreover, since β_{grow} is generally less than β_{heal}, there is considerable *hysteresis* in the island healing/growth cycle.

Let us consider the scaling of β_{heal} and β_{grow} with the standard dimensionless plasma parameters $\beta \sim \beta_i$, ν_*, and $\rho_* = \rho_i/R_0$. Assuming ohmic power balance, we have

$$\tau_R \sim \left(\frac{r_s}{L_s}\right)^2 \frac{\tau_E}{\beta_i}, \tag{136}$$

where τ_E is the energy confinement timescale. Assuming gyro-Bohm energy and momentum confinement, we obtain

$$\tau_E^{-1} \sim \tau_M^{-1} \sim \left(\frac{\rho_i}{r_s}\right)^3 \Omega_i, \tag{137}$$

where $\tau_M = n_0\, m_i\, r_s^2/\mu_{\perp\, i}$ is the momentum confinement timescale. It follows that

$$\beta_{heal} \sim |s_s|^{1/2} \left(\frac{\epsilon_s}{q_s}\right)^4 \frac{\nu_*^{1/2}}{\rho_*^3} \left(\frac{b_v}{B_0}\right)^{3/2}, \tag{138}$$

$$\beta_{grow} \sim \left(\frac{\epsilon_s}{q_s}\right)^{19/4} \frac{\nu_*^{3/4}}{\rho_*^4} \left(\frac{b_v}{B_0}\right)^2, \tag{139}$$

where $s_s = R_0\, q_s/L_s$ is the magnetic shear at the resonant surface,

$$\frac{w_v}{R_0} = \left(\frac{b_v}{B_0}\,\frac{\epsilon_s}{q_s}\,\frac{1}{|s_s|}\right)^{1/2}, \tag{140}$$

and b_v is the resonant harmonic of the *vacuum* radial magnetic field (due to the external perturbation) evaluated at $r = r_s$.

Figure 2 shows the locations of the torque-driven island healing/growth transition boundaries in the $\beta - \nu_*$ plane. In region I, a locked island chain is always present. In region III, a locked island chain is always absent. A locked island chain is present in region II when it is entered from the left, and absent when it is entered from the right. Finally, the island healing transition takes place when the boundary between regions II and III is crossed from left to right, whereas an island growth transition takes place when the boundary between regions I and II is crossed from right to left.

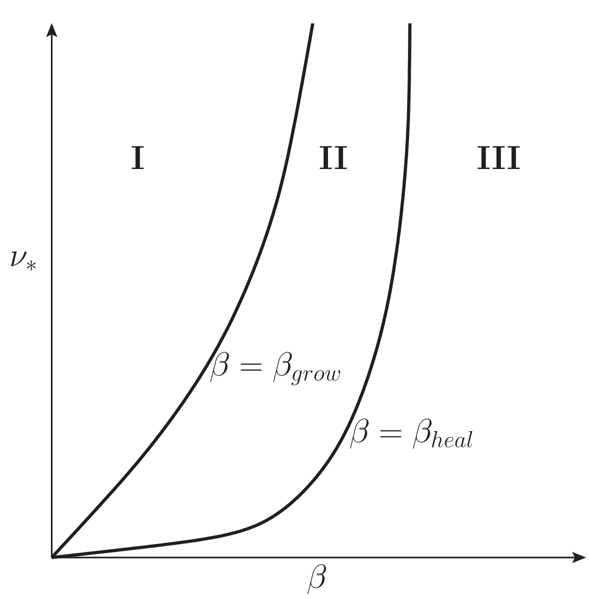

FIG. 2. Torque-driven island healing/growth transitions in the $\beta - \nu_*$ plane.

D. Stability-driven transitions

In the previous section, we examined spontaneous island healing/growth transitions caused by the breakdown of torque balance in the island phase evolution equation, (120). In this section, we shall investigate transitions caused by an imbalance between the various terms in the island stability equation, (119).

Suppose that the drag torque term on the right-hand side of (120) is *negligible*. It immediately follows that $\phi_v = 0$. Suppose that the island chain lies in regimes I or IV (see Table I and Fig. 1), and that the toroidal flow damping lies at the boundary between the $1/\nu$ and ν regimes. This implies that the normalized toroidal flow damping rate, $\hat{\nu}_{\perp\, i}$, is comparable to the normalized poloidal flow damping rate, $\hat{\nu}_{\theta\, i}$, which is most likely to be the case when the ion collisionality is relatively low and/or the island width is relatively large (since $\hat{\nu}_{\perp\, i} \propto w^2/\nu_i$ and $\hat{\nu}_{\theta\, i} \propto \nu_i$). Equation (119) becomes

$$\frac{\alpha_1}{4}\frac{dy^4}{d\hat{t}} = -y^3 + y + \alpha_2\, v_0\,(1 + v_0), \tag{141}$$

where $y = w/w_v$, and (interpolating between the $1/\nu$ and ν regimes)

$$v_0 \simeq 2.37\left(\frac{\nu_* - \nu_{*\,c}}{\nu_* + 9.48\,\nu_{*\,c}}\right), \tag{142}$$

where $\nu_{*\,c} \simeq 0.105 f_t\,(q_s/\epsilon_s)\,(\rho_i/L_n)$. Note that v_0 is *positive* in the $1/\nu$ regime (i.e., when $\nu_* > \nu_{*\,c}$), but *negative* in the ν regime (i.e., when $\nu_* < \nu_{*\,c}$). It follows that the ion polarization term (i.e., the last term on the right-hand side) in Eq. (141) is *destabilizing* in the $1/\nu$ regime, and *stabilizing* in the ν regime. When $\nu_* < \nu_{*\,c}$ and $\beta_i > \beta_{heal}$, Eq. (141) exhibits a discontinuous transition in which a locked island chain of finite width spontaneously heals. This occurs as a stable (i.e., $dy^4/d\hat{t} = 0$, $d^2y^4/d\hat{t}^2 < 0$) and an unstable (i.e., $dy^4/d\hat{t} = 0$, $d^2y^4/d\hat{t}^2 > 0$) steady-state solution merge and annihilate, leaving $dy^4/d\hat{t} < 0$ for all $y^4 > 0$. Here, $\beta_{heal} = (2/\sqrt{27})\,(\beta_i/\alpha_2)\,[1/(-v_0)\,(1+v_0)]$, or, equivalently,

$$\beta_{heal} = \left(\frac{4}{1.38\,\sqrt{27}}\right)\left[\frac{m_\theta}{(-v_0)\,(1+v_0)}\right]\left(\frac{L_n}{L_s}\right)^2\left(\frac{w_v}{\rho_i}\right)^3. \tag{143}$$

Moreover, the scaling of β_{heal} with dimensionless parameters is as follows:

$$\beta_{heal} \sim |s_s|^{1/2}\left(\frac{\epsilon_s}{q_s}\right)^{7/2} \frac{F(\nu_*/\nu_{*\,c})}{\rho_*^3}\left(\frac{b_v}{B_0}\right)^{3/2}, \tag{144}$$

where

$$F(x) = \frac{(9.48 + x)^2}{2.37\,(1-x)\,(7.11 + 3.37\,x)}. \tag{145}$$

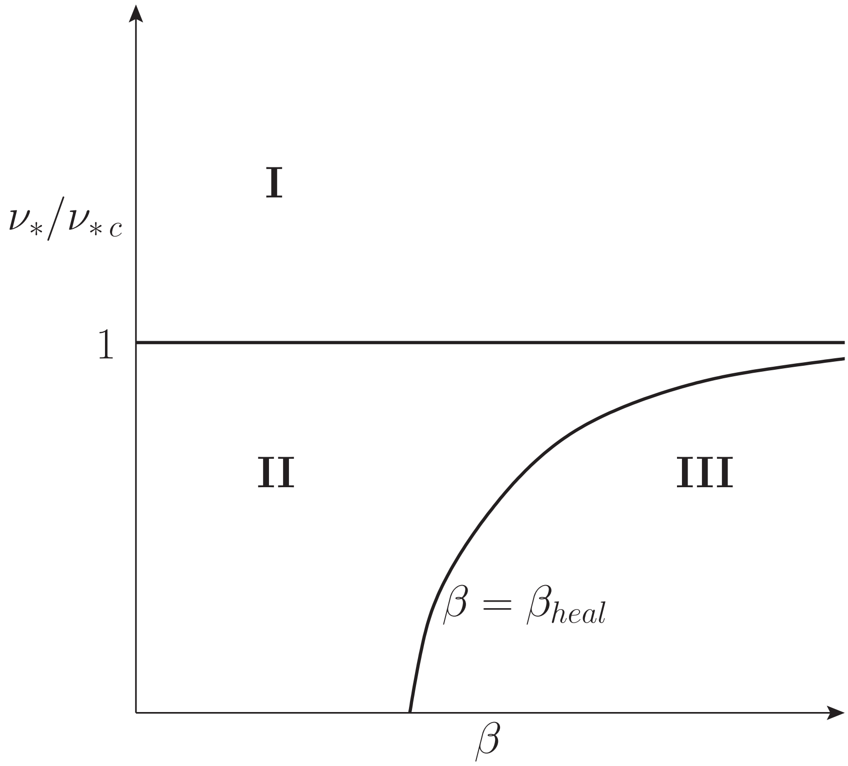

FIG. 3. Stability-driven island healing/growth transitions in the $\beta - \nu_*$ plane.

The healing transition occurs because the stabilizing ion polarization term in Eq. (141) can no longer be balanced by the other terms on the right-hand side of the equation. To be more exact, the transition occurs as $dy^4/d\hat{t}|_{y=0}$ changes sign from negative to positive. We would expect the island chain to remain locked to the external perturbation as it heals (since the drag torque is assumed to be negligible). Moreover, as discussed in the previous subsection, the healing process is accompanied by an ion poloidal velocity shift at the resonant surface in the electron diamagnetic direction. When $\nu_* > \nu_{*c}$ and $\beta_i > 0$, Eq. (141) exhibits a discontinuous transition in which a locked island chain spontaneously grows to a width of order the vacuum island width. This transition occurs because there is no barrier to the spontaneous growth of a locked island chain when the ion polarization term in Eq. (141) is destabilizing. As discussed in the previous subsection, the healing process is accompanied by an ion poloidal velocity shift at the resonant surface in the ion diamagnetic direction.

Figure 3 shows the locations of the stability-driven island healing/growth transition boundaries in the $\beta - \nu_*$ plane. In region I, a locked island chain is always present. In region III, a locked island chain is always absent. A locked island chain is present in region II when it is entered from above, and absent when it is entered from the right. Finally, the island healing transition takes place when the boundary between regions II and III is crossed from left to right, whereas an island growth transition takes place when the boundary between regions I and II is crossed from below.

VI. SUMMARY AND DISCUSSION

In this paper, we have employed a formalism originally developed to investigate magnetic island dynamics in tokamak plasmas to study the spontaneous healing and growth of a locked magnetic island chain in a stellarator plasma. (It should be noted, however, that the formalism assumes that the helical component of the magnetic field is small compared to the axisymmetric component, which is not a good approximation in stellarators.) Two healing/growth scenarios have been identified. In the first scenario, discussed in Sec. V C, healing/growth transitions are caused by a breakdown in torque balance in the vicinity of the island chain.[12] In the second scenario, discussed in Sec. V D, transitions are caused by an imbalance between the various terms in the island width evolution equation. The first type of transitions is termed "torque-driven," whilst the second is termed "stability-driven."

Torque-driven island healing transitions occur at high β, and low collisionality, when the drag torque acting on the island chain, due to the combined effect of neoclassical flow damping and ion viscosity, overwhelms the electromagnetic locking torque due to the resonant external magnetic perturbation (that is ultimately responsible for maintaining the island chain in the plasma), causing the chain to unlock from the perturbation, spin-up, and decay to small amplitude. On the other hand, torque-driven island growth transitions occur at low β, and high collisionality, when the electromagnetic locking torque acting on a small amplitude island chain, whose width is suppressed by plasma rotation, overwhelms the drag torque, causing the chain to lock to the external perturbation, and grow to large amplitude.

Stability-driven island healing transitions occur at high β, and low collisionality, when the ion polarization term in the island width evolution equation, which is stabilizing at low collisionality, overwhelms the destabilizing term due to the resonant external perturbation, causing the island to decay away without necessarily unlocking from the perturbation. Stability-driven island growth transitions occur as soon as the collisionality becomes sufficiently large to render the ion polarization term destabilizing.

In both the torque- and stability-driven cases, the healing transition is accompanied by an ion (or $\mathbf{E} \times \mathbf{B}$) poloidal velocity shift in the electron diamagnetic direction, whereas the growth transition is accompanied by a velocity shift in the ion diamagnetic direction. Furthermore, in both cases, the island healing/growth cycle exhibits considerable hysteresis.

The phenomenology of torque- and stability-driven island healing/growth transitions that is described above is broadly consistent with experimental observations of the spontaneous growth and healing of locked island chains in stellarator plasmas. However, a detailed comparison between theory and observations is left to a future work.

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