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Convective transport of fast particles in dissipative plasmas near an instability threshold

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Abstract
We demonstrate that a marginally unstable energetic particle population in a dissipative plasma can change globally due to the act of a single wave–particle resonance. The resonance serves as a seed for the continuous production of nonlinear holes and clumps, whose convective motion in phase-space results in substantial flattening of the fast particle distribution function. The holes and clumps can emerge recurrently without any particle source or collisional relaxation process that would restore the particle distribution function at the resonance. A bump-on-tail instability is considered as an example in a single-mode limit as well as in the quasilinear regime. The convective hole-clump transport tends to be more significant near the instability threshold than quasilinear diffusion.

(Some figures may appear in colour only in the online journal)

1. Introduction

Energetic particle driven instabilities in burning plasmas are frequently viewed as a risk factor for fusion reactor performance, because the excited waves have the potential to degrade alpha particle confinement. However, the free energy source for these instabilities is notably weak, suggesting that the excited waves should saturate at a relatively low level. A single saturated wave will therefore only affect a tiny fraction of the energetic particle population, which indicates that many perturbations are essential for global transport. Such perturbations can create a wide-ranging tangle of wave–particle resonances in phase space, the average result of which is a global diffusion of the energetic particles over the overlapped resonances. This mechanism is known as quasilinear diffusion. Its theoretical description commonly implies that the perturbations are linear eigenmodes of the bulk plasma. Consequently, the frequencies and spatial structures of these eigenmodes are determined robustly by the properties of the bulk plasma. Only the growth rates and the nonlinear saturation levels for such (perturbative) modes are then affected by the energetic particles.

A plausible alternative to this diffusive transport is a convective transport mechanism that allows particles to travel long distances in phase space by being locked into a single resonance that itself evolves in time. The convective mechanism requires non-perturbative waves, that is waves whose existence relies on the nonlinear response of the resonant particles themselves rather than just the linear response of the background plasma. The number of resonant particles, which scales roughly as the square root of the wave amplitude, is typically small compared with the entire energetic particle population. A sequence of many convective events would therefore be required to produce a global change in the energetic particle population.

The non-perturbative modes of interest are nonlinear BGK-type waves that represent hole/clump phase-space structures. Such modes can form spontaneously in dissipative plasmas when the energetic particle population is only marginally unstable, as shown in [1, 2] using a bump-on-tail model. The model captures resonant particle physics in more general multidimensional problems, such as radial transport of fast particle in tokamaks, since the particle motion is known to be effectively one dimensional in the vicinity of an isolated nonlinear resonance, once expressed in proper action-angle variables [3, 4]. The dissipation also forces the modes to have time-dependent phase velocities that allow particles to be convected away from the resonance. Moreover, it was recently discovered [5] that these holes and clumps are in fact generated continuously at the wave–particle resonance in the absence of any particle source.

This non-perturbative transport scenario is in stark contrast with the near-threshold quasilinear case in which global relaxation is actually prevented by the presence of dissipation. The pertubrative modes are stabilized after only a slight modification of the fast particle distribution function,

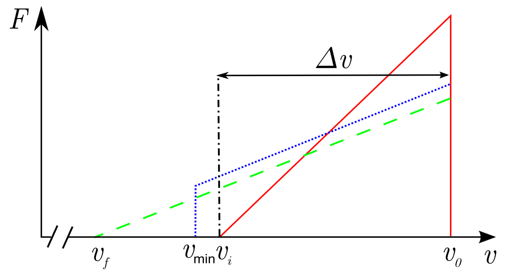

Figure 1. Snapshots of the distribution function undergoing quasilinear relaxation. Initial, intermediate and final states are shown by the solid, dotted and dashed lines, respectively.

at which point the quasilinear diffusion is suppressed. As a result, convective motion of phase-space holes and clumps may provide a more efficient transport mechanism than quasilinear diffusion. In what follows, we justify this conjecture and confirm our previous speculation [5] that the production of holes and clumps can lead to global relaxation of the fast particle population.

2. Quasilinear transport

We first illustrate a distinct feature of the near-threshold regime in quasilinear theory. We consider a simple one-dimensional bump-on-tail problem in which a weakly unstable distribution of energetic electrons evolves due to the excitation of electron plasma waves (with frequency ω_p) in a dissipative plasma. To be specific, we assume that the initial distribution is localized within a small velocity interval $\Delta v = v_0 - v_i \ll v_0$ and has a triangular shape shown in figure 1 by the solid line.

The positive slope of this distribution provides an instability drive, so that, in the absence of dissipation, the instability growth rate would be

$$\gamma_L = a^2 \frac{\partial F_0}{\partial v}, \tag{1}$$

where $a^2 \equiv 2\pi^2 e^2 v_0^2/\omega_p$ (the assumption that the particle distribution is sufficiently narrow ($\Delta v = v_0 - v_i \ll v_0$) allows us to treat a^2 as a constant instead of using an exact velocity-dependent expression, $a^2 \equiv 2\pi^2 e^2 v^2/\omega_p$, and to treat the wave damping rate γ_d as a constant). The actual growth rate in a dissipative plasma is much smaller than γ_L, because we choose the value of γ_L to be only slightly greater than the damping rate γ_d.

The one-dimensional quasilinear equations for the bump-on-tail problem can be written as [6]

$$\frac{\partial F}{\partial t} = \frac{\partial}{\partial v} D \frac{\partial F}{\partial v}, \tag{2}$$

$$\frac{\partial D}{\partial t} = 2D\left(a^2 \frac{\partial F}{\partial v} - \gamma_d\right), \tag{3}$$

where the diffusion coefficient $D(v, t)$ is proportional to the spectral energy density of the waves with phase velocity v . It follows from equations (2) and (3) that the waves tend to reduce the slope of the distribution function until the slope comes down to the critical value $\partial F/\partial v = \gamma_d/a^2$. Any further reduction of the slope will result in an exponential decay of the excited waves. On the other hand, the presence of dissipation requires that all the waves eventually damp, because the energetic electrons have only a finite amount of free energy, whereas a non-zero level of waves in the final state would require a constant energy source to balance the dissipated power. It is also clear that the high-velocity edge of the distribution function stays at its initial location, because there is no excitation of waves at the negative slope of the distribution, and the diffusion coefficient vanishes there as a result. These features suggest that the slope of the distribution function "freezes" at the critical level after the waves damp. Consequently, the distribution function extends somewhat to lower velocities to reduce the slope and form a new triangular profile shown in figure 1 by the dashed line. This scenario differs considerably from the non-dissipative case in which the particles form a quasilinear plateau and the wave energy remains finite in the final state.

Taking into account that the number of particles is conserved during the relaxation process, we obtain the following expression for the lower boundary v_f of the final distribution:

$$\gamma_d (v_0 - v_f)^2 = \gamma_L (v_0 - v_i)^2 = \gamma_L (\Delta v)^2,$$

$$v_f = v_0 - \Delta v \sqrt{\gamma_L/\gamma_d}. \tag{4}$$

The final distribution has a lower energy than the initial one. The relative difference between the two energies is given by

$$\frac{\varepsilon_i - \varepsilon_f}{\varepsilon_i} = \frac{(v_i - v_f)(v_i + v_f + 2v_0)}{3\left(v_0^2 + v_i^2\right) + 2v_i (v_0 - v_i)} \approx \frac{2(v_i - v_f)}{3v_0} = \frac{2\Delta v}{3v_0}\left(\sqrt{\gamma_L/\gamma_d} - 1\right). \tag{5}$$

As seen from equation (5), the particles can release only a small fraction of their energy if the initial distribution function is only slightly above the instability threshold so that $(\gamma_L - \gamma_d)/\gamma_d \ll 1$.

In addition to finding the final distribution function, for completeness, we now describe the time-evolution of the system by solving the quasilinear equations within a large logarithm approximation developed in [7]. This approximation implies that the waves maintain a marginally stable slope of the distribution function everywhere except near the steep downward moving edge. The waves grow quickly at the edge from the noise level to a much higher level that causes quasilinear diffusion. With regard to equations (2) and (3) this approximation means that every snapshot of the time-dependent distribution function has a trapezoidal form with a fixed (marginal) slope, as shown in figure 1 by the dotted line. As the edge of this distribution (v_{min}) moves to lower velocities, the height of the edge decreases and the trapezoid evolves into the final triangle (dashed line in figure 1).

Conservation of particles gives the following relation between the height of the edge (F_{min}) and its location:

$$F_{min}(v_0 - v_{min}) + \gamma_d \frac{(v_0 - v_{min})^2}{2a^2} = \gamma_L \frac{(v_0 - v_i)^2}{2a^2}. \tag{6}$$

We now use equation (3) to obtain an evolution equation for v_{min}. We rewrite equation (3) as

$$\frac{\partial \ln D}{\partial t} = 2\left(a^2 \frac{\partial F}{\partial v} - \gamma_d\right) \tag{7}$$

and integrate it over a small velocity interval around the moving edge ($v_{\min} - 0 < v < v_{\min} + 0$) to find

$$\Lambda \frac{\mathrm{d}v_{\min}}{\mathrm{d}t} = -2\left(a^2 F_{\min}\right), \tag{8}$$

where $\Lambda \equiv \ln[D(v_{\min}+0)/D(v_{\min}-0)]$ is a logarithm of the large ratio between the diffusion coefficients on the two sides of the edge. The value of Λ is relatively insensitive to $D(v_{\min}+0)$, which makes it allowable to treat Λ as a constant in the large logarithm approximation. Taken together, equations (6) and (8) give the following time dependence for $v_{\min}$:

$$(v_0 - v_{\min})^2 = \frac{\gamma_L}{\gamma_d}(v_0 - v_i)^2\left[1 - \left(1 - \frac{\gamma_d}{\gamma_L}\right)\exp\left(-2\frac{\gamma_d}{\Lambda}t\right)\right]. \tag{9}$$

We next use a trapezoidal profile of the distribution function,

$$F = \gamma_L \frac{(v_0 - v_i)^2}{2a^2(v_0 - v_{\min})} + \frac{\gamma_d}{2a^2}(2v - v_0 - v_{\min}) \tag{10}$$

to obtain the diffusion coefficient from equation (2):

$$D = \left[\frac{\gamma_L}{\gamma_d}\frac{(v_0 - v_i)^2}{(v_0 - v_{\min})^2} - 1\right]\frac{(v - v_0)}{2}\frac{\mathrm{d}v_{\min}}{\mathrm{d}t}. \tag{11}$$

Equations (9)–(11) give a complete description of the quasi-linear relaxation within the large logarithm approximation.

3. Convective transport

We now demonstrate that the energetic particle distribution can change globally via the generation of non-perturbative modes (holes and clumps). In order to explain the underlying mechanism we consider the same initial distribution function that is shown by the solid line in figure 1, but we introduce a periodicity constraint that discretizes the spectrum of unstable waves. This constraint can mimic the discreteness of modes excited by energetic particles in tokamaks. We will discuss the simplest case of only one linearly unstable mode whose phase velocity $v_p \equiv \omega_p/k$ is in the middle of the initial distribution function.

Away from the threshold of a weak instability, the nonlinear evolution consists of an initial wave saturation followed by a gradual decay [8], leaving behind a narrow plateau in the distribution function around the resonance. However, the near-threshold regime exhibits a very different scenario in which the unstable resonance gives birth to nonlinear travelling waves with time-dependent frequencies [1, 2]. These waves form holes and clumps in the fast particle distribution function. They represent particles trapped in the field of the wave that are convected in phase space as the wave frequency changes. It is noteworthy that these BGK waves were not seen in the near-threshold quasilinear picture because they are known to develop unstable side-bands in the presence of a dense spectrum of linear modes with different spatial periods [9, 10].

Without particle sources to restore the initially unstable distribution function one might expect the system to stabilize on a time-scale associated with the production of a hole or clump. In contrast, our recent simulations show that these holes and clumps are in fact generated continuously close to the resonance if the initial distribution function has a constant slope [5]. This process is shown in figure 2. Each spectral line in the figure corresponds to a wave whose frequency shift $\delta\omega$ from the initial plasma frequency changes in time. In the absence of any particle source, and with very low numerical dissipation, (evidenced by the long lived features seen in the simulation) this continuous production of holes and clumps seems surprising. It can however be understood by recognizing that since the phase space is incompressible a number of particles must be displaced during the motion of a hole or clump, which leads to a slight excess behind a hole and a depletion behind a clump (see figure 3). There is thus a tendency for the gradient in the distribution function to steepen, making the system susceptible to another instability. This mechanism is qualitatively different from collisional reconstitution, which requires a fairly long recovery time in cases of interest. It is appropriate to look back at [1] in that regard. The numerical results of [1] can now be viewed retrospectively as a hint that there is a fast collisionless recovery mechanism that gives a short sequence of holes and clumps. However, the presence of significant collisions in [1] (see also [11, 12]) has made it difficult to identify the nature of the recovery mechanism and to recognize its ability to change the particle distribution globally. Our new simulations with much lower collisionality are free from this difficulty.

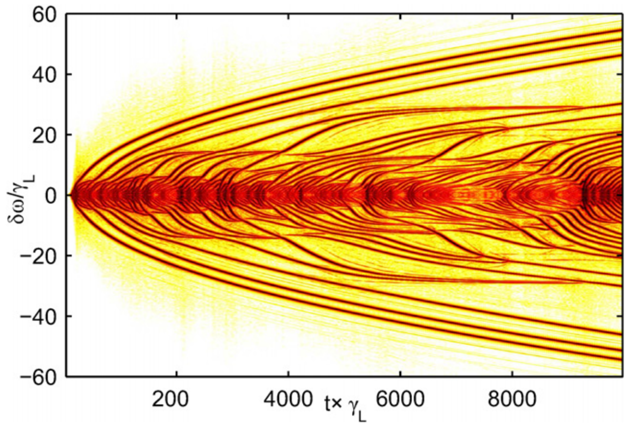

Figure 2. Modes with time-dependent frequencies in nonlinear simulations of the bump-on-tail instability [5]. Fourier analysis of the mode amplitude shows that waves are continuously produced at the resonance $v_p \equiv \omega_p/k$ if the initial distribution function has a constant slope.

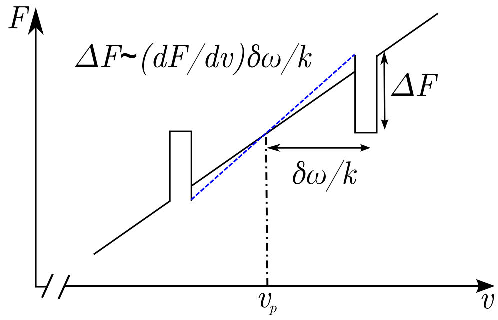

Figure 3. Cartoon illustrating the motion of holes and clumps and the wake (dotted line) that acts to steepen the distribution function, creating a favourable environment for instability.

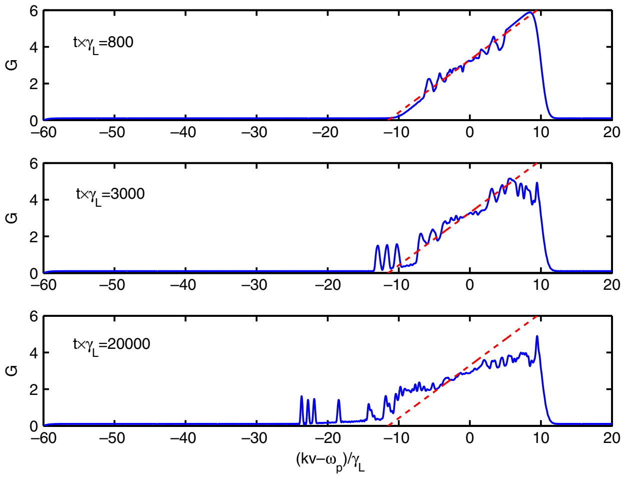

Figure 4. Snapshots of the normalized distribution function $G = (2\pi e^2\omega_p/m_e\gamma_L^2 k)F$ from a bump-on-tail simulation, using the BOT code [5, 13], shows plateau formation above the original resonant velocity. The dashed line indicates the critical slope. Simulation parameters relating to [5, 13] are $\gamma_d/\gamma_L = 0.9$, $N = 10$, $s_{max} = 10$, $\Delta\tau = \Delta s = 0.02$.

In reality, the distribution function is finite in extent. This limits the number of holes and clumps that can be generated, since an upward chirping hole cannot move into a region of lower phase-space density than its own. Eventually the holes will 'stack up' next to one another at the boundary of the distribution. The clumps on the other hand are free to move down in velocity as long as the electric field of the travelling wave survives. Figure 4 shows how the aforementioned process leads to a significant modification of the particle distribution.

The global relaxation of the fast particle distribution releases a significant fraction of the particle energy. An upper limit for this energy release can be estimated in the following way. We assume that each hole and clump evolves independently of any others, in which case, referring to figure 5, the resonance effectively transforms the upper half of the triangle (shaded region) into a series of clumps that move down the slope to lower velocities. By treating the clumps as having moved down sufficiently far so that their contribution to the kinetic energy can be neglected, the energy release from the distribution per unit volume can be estimated as simply the energy of the particles in the shaded region, i.e.

$$\frac{\varepsilon_i - \varepsilon_f}{\varepsilon_i} = \frac{1}{4}\frac{3\left(v_0^2 + v_p^2\right) + 2v_p\left(v_0 - v_p\right)}{3\left(v_0^2 + v_i^2\right) + 2v_i\left(v_0 - v_i\right)}. \qquad (12)$$

Equation (12) and figure 5 show that this convective process releases about a quarter of the total energy, which is much more than in the quasilinear case.

4. Concluding remarks

The notion that a marginally unstable resonance can produce such a dramatic change in the fast particle population is at first glance surprising, but it in fact reflects a deep difference between the perturbative (soft) and non-perturbative (hard)

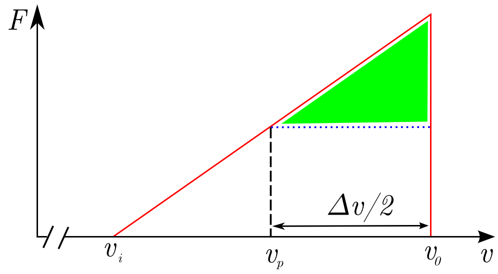

Figure 5. Distribution function before (solid triangle) and after (dotted trapezoid) hole-clump relaxation. Shaded particles are transformed into clumps during the evolution.

regimes of the near-threshold instability. The quasilinear model, which involves only perturbative modes, is apparently designed to describe the soft scenario. This model forces the particle distribution to become marginally stable, but it cannot push the distribution far below the linear instability threshold because such a distribution would suppress all eigenmodes. In this way particle relaxation is restricted. In contrast, the continuously produced non-perturbative holes and clumps, which represent the hard nonlinear regime, provide a new channel for particle relaxation when linear modes are suppressed. Furthermore, many plasmas of interest exhibit a discrete spectrum of waves that are excited close to the instability threshold. It is therefore pertinent to consider how the convective transport described in this paper will manifest itself in experiments. This consideration is an interesting topic for future work that should involve particle sources and sinks as well as multiple wave–particle resonances.

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[13] Lilley M.K. 2010 Source code available from code.google.com/p/bump-on-tail