Magnetic reconnection in weakly collisional highly magnetized electron-ion plasmas

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(Received 17 August 2009; accepted 9 March 2010; published online 12 April 2010)

A reduced three-field model of two-dimensional magnetic reconnection in a weakly collisional, highly magnetized plasma consisting of isothermal electrons and cold ions is derived from a set of Braginskii-like fluid equations. The model is then used to calculate the linear growth rate of the reconnecting instability in collisionless and semicollisional parameter regimes. © *2010 American Institute of Physics*. [doi:10.1063/1.3374427]

I. INTRODUCTION

Magnetic reconnection is a fundamental physical phenomenon which occurs in magnetized plasmas found, for example, in magnetic fusion experiments,[1] the solar corona,[2] and the Earth's magnetotail.[3] The reconnection process produces a change in magnetic field-line topology with an accompanying release of magnetic energy. Conventional collisional magnetohydrodynamical (MHD) fluid theory is capable of accounting for magnetic reconnection, but generally predicts reconnection rates which are many orders of magnitude smaller than those seen in high temperature plasmas.[4] On the other hand, more sophisticated plasma models that neglect collisions (since these are comparatively weak in high temperature plasmas), and treat electrons and ions as *separate* fluids, yield much faster reconnection rates that are fairly consistent with observations.[5,6]

A simple *reduced*[7] two-fluid model of two-dimensional collisionless magnetic reconnection in a highly magnetized plasma consisting of isothermal electrons and cold ions was derived in Ref. 8. However, there has recently been some debate in the literature as to whether certain of the terms (i.e., the so-called density diffusion terms, or, alternatively, the parallel magnetic field diffusion terms[9]) appearing in this model are spurious in nature.[10,11] Under certain circumstances, the terms in question can affect the linear growth-rate of the reconnecting instability.[8–12] The primary aim of this paper is to resolve any uncertainty in this regard by rederiving the model in as self-consistent and rigorous a manner as possible. A secondary aim is to slightly extend the model to allow the plasma to be weakly collisional.

II. DERIVATION OF MODEL

A. Introduction

The purpose of this section is to derive a self-consistent set of reduced fluid equations governing two-dimensional magnetic reconnection in a highly magnetized, weakly collisional, quasineutral plasma consisting of *isothermal* electrons and *cold* (singly charged) ions.

B. Geometry

For the sake of simplicity, we shall work in *slab geometry*. Let us adopt the standard right-handed Cartesian coordinates, x, y, and z. It is assumed that there is no variation in quantities in the z-direction: i.e., $\partial/\partial z \equiv 0$. Moreover, the dominant magnetic field is taken to be uniform, parallel to the z-axis, and of strength B_0.

C. Fundamental equations

Our starting point is a conventional *two-fluid* treatment of the plasma dynamics which takes the following form:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{V}_e) = 0, \tag{1}$$

$$m_e n_e \left(\frac{\partial}{\partial t} + \mathbf{V}_e \cdot \nabla\right)\mathbf{V}_e + \nabla \cdot \mathbf{\Pi}_e = -T_e \nabla n_e - e n_e (\mathbf{E} + \mathbf{V}_e \times \mathbf{B}) + e n_e [\eta_\parallel (J_z - J_z^{(0)})\mathbf{e}_z + \eta_\perp \mathbf{J}_\perp], \tag{2}$$

$$m_i n_e \left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla\right)\mathbf{V} = -T_e \nabla n_e + \mathbf{J} \times \mathbf{B}, \tag{3}$$

where

$$\mathbf{J} = n_e e (\mathbf{V} - \mathbf{V}_e), \tag{4}$$

and $\mathbf{J}_\perp = \mathbf{J} - J_z \mathbf{e}_z$. Here, e is the magnitude of the electron charge, m_e is the electron mass, m_i is the ion mass, T_e is the (uniform and constant) electron temperature, n_e is the electron number density, $\mathbf{V}_e$ is the electron fluid velocity, $\mathbf{V}$ is the center of mass (i.e., ion) fluid velocity, $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, $\mathbf{J}$ is the electric current density, $J_z^{(0)}\mathbf{e}_z$ is the equilibrium current density, $\mathbf{\Pi}_e$ is the electron viscosity tensor, $\eta_\parallel$ is the parallel (to the magnetic field) electrical resistivity, and $\eta_\perp$ is the perpendicular resistivity. Of course, Eqs. (1)–(3) are, respectively, the *continuity equation*, the *electron fluid equation of motion*, and the *ion fluid equation of motion* (or, to be more exact, the sum of the electron and the ion fluid equations of motion). Incidentally, in writing Eq. (3), we have neglected electron inertia with respect to ion inertia. Furthermore, the assumption of isothermal electrons

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and cold ions negates the need for an energy equation, and for cross-terms involving ∇T_e in Eq. (2).

As is well-known, Eqs. (1)–(3) are appropriate to a *highly magnetized* electron-ion plasma: i.e., a plasma in which the *electron gyroradius*, $\rho_e = v_e/\Omega_e$, is *much smaller* than the typical variation length-scale, L, of the principal electron fluid moments, and in which $\Omega_e \tau_e \gg 1$. Here, $v_e = (T_e/m_e)^{1/2}$ is the *electron thermal velocity*, $\Omega_e = eB_0/m_e$ the *electron gyrofrequency*, and τ_e the *electron-ion collision time*. Now, in a highly magnetized plasma, closed expressions for $\eta_\parallel$, $\eta_\perp$, and $\mathbf{\Pi}_e$ can be obtained via a Chapman–Enskog[13] expansion which exploits the smallness of ρ_e/L and $(\Omega_e\tau_e)^{-1}$. In fact, according to Braginskii,[14] $\eta_\parallel = 0.51 m_e/(n_e e^2 \tau_e)$, $\eta_\perp = m_e/(n_e e^2 \tau_e)$, plus

$$\mathbf{\Pi}_e = \mathbf{\Pi}_\parallel + \mathbf{\Pi}_g, \tag{5}$$

where

$$\nabla \cdot \mathbf{\Pi}_\parallel = -\nabla(0.24 n_e T_e \tau_e \nabla \cdot \mathbf{V}_e), \tag{6}$$

and the nonzero elements of $\mathbf{\Pi}_g$ are $\Pi_{gxx} = -\Pi_{gyy} = (\eta_4/2) W_{xy}$, $\Pi_{gxy} = \Pi_{gyx} = -(\eta_4/4)(W_{xx} - W_{yy})$, $\Pi_{gxz} = \Pi_{gzx} = \eta_4 W_{yz}$, and $\Pi_{gyz} = \Pi_{gzy} = -\eta_4 W_{xz}$. Here, $\eta_4 = n_e T_e/\Omega_e$, and $\mathbf{W} = \nabla \mathbf{V}_e + (\nabla \mathbf{V}_e)^T - (2/3)\nabla \cdot \mathbf{V}_e \mathbf{I}$, where $\mathbf{I}$ is the identity tensor. Incidentally, in writing Eq. (5), we have neglected the *perpendicular* electron viscosity tensor with respect to the corresponding *parallel* viscosity tensor, $\mathbf{\Pi}_\parallel$, and gyroviscosity tensor, $\mathbf{\Pi}_g$. As is well-known, this is a reasonable approximation in the limit that $\Omega_e \tau_e \gg 1$.[14]

It turns out that the expression (6) for (minus) the momentum flux due to parallel electron viscosity is only valid when the additional constraint $L \gg \lambda_e$ is satisfied, where $\lambda_e = v_e \tau_e$ is the *electron mean-free-path*.[14] Assuming that $L \sim d_e$, where d_e is the collisionless electron skin-depth (see Sec. II D), we conclude that Eq. (6) only holds in the relatively narrow collisionality range

$$1 \ll \Omega_e \tau_e \ll \frac{d_e}{\rho_e}. \tag{7}$$

Note, however, that expression (6) is fairly generic in form, and can be derived, for instance, from a standard Chew–Goldberger–Low *ansatz*.[15] Moreover, there is a limit to how large the viscous momentum flux can become, as τ_e increases, since electron momentum obviously cannot diffuse significantly faster than the electron thermal velocity. Such considerations lead us to replace Eq. (6) with the *flux-limited* expression

$$\nabla \cdot \mathbf{\Pi}_\parallel = -\nabla\left(\frac{0.24 n_e T_e \tau_e}{1 + \zeta' \Omega_e \tau_e \rho_e/d_e} \nabla \cdot \mathbf{V}_e\right), \tag{8}$$

where ζ' is an $\mathcal{O}(1)$ positive constant. The above expression is asymptotically correct in the intermediate collisionality limit (7), and is, at least, the right order of magnitude in the low collisionality limit

$$\Omega_e \tau_e \gg \frac{d_e}{\rho_e}. \tag{9}$$

Our set of fundamental equations is completed by Maxwell's equations: i.e.,

$$\nabla \cdot \mathbf{B} = 0, \tag{10}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{11}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}. \tag{12}$$

D. Normalized equations

Let us adopt the following normalization scheme: $\hat{\nabla} = d_e \nabla$, $\partial/\partial\hat{t} = (d_e/\sqrt{\beta_e} V_{De})\,\partial/\partial t$, $\hat{\mathbf{V}}_e = \mathbf{V}_e/V_{De}$, $\hat{\mathbf{V}} = \mathbf{V}/V_{De}$, $\hat{n} = n/n_0$, $\hat{\mathbf{B}} = \mathbf{B}/B_0$, $\hat{\mathbf{E}} = \mathbf{E}/(V_{De} B_0)$, $\hat{\mathbf{J}} = (\mu_0 d_e/\beta_e B_0)\mathbf{J}$, $\hat{\mathbf{\Pi}}_{\parallel,g} = \mathbf{\Pi}_{\parallel,g}/(\beta_e n_0 T_e)$, and $\hat{\tau}_e = (n_e/n_0)\tau_e$. Note that $\hat{\tau}_e$ only depends *logarithmically* on n, and can consequently be treated as a uniform constant to a good approximation. Here, n_0 is the equilibrium electron number density, and

$$d_e = \left(\frac{m_e}{n_0 e^2 \mu_0}\right)^{1/2} = \frac{\rho_e}{\sqrt{\beta_e}}, \tag{13}$$

$$V_{De} = \frac{T_e}{e B_0 d_e} = \sqrt{\beta_e}\, v_e, \tag{14}$$

$$\beta_e = \frac{\mu_0 n_0 T_e}{B_0^2}, \tag{15}$$

are the *collisionless electron skin-depth*, the *electron drift-velocity*, and the *electron beta*, respectively. In essence, our normalization scheme assumes that the typical variation length-scale of electron fluid quantities is d_e, whereas the typical electron fluid velocity is V_{De}. Incidentally, the fundamental ordering $L \gg \rho_e$, which is adopted so as to permit the use of the standard Braginskii expressions for $\eta_\parallel$, $\eta_\perp$, and $\mathbf{\Pi}_e$, requires that $d_e \gg \rho_e = \sqrt{\beta_e} d_e$. Obviously, this is only possible in the low-beta limit $\sqrt{\beta_e} \ll 1$.

The normalized versions of the equations presented in Sec. II C are

$$\sqrt{\beta_e}\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}_e) = 0, \tag{16}$$

$$\beta_e\left[\left(\sqrt{\beta_e}\frac{\partial}{\partial t} + \mathbf{V}_e \cdot \nabla\right)\mathbf{V}_e + n^{-1}\nabla \cdot \mathbf{\Pi}_\parallel + n^{-1}\nabla \cdot \mathbf{\Pi}_g\right] = -\nabla \ln n - \mathbf{E} - \mathbf{V}_e \times \mathbf{B} + (\Omega_e \tau_e)^{-1} \times [0.51(J_z - J_z^{(0)})\mathbf{e}_z + \eta_\perp \mathbf{J}_\perp], \tag{17}$$

$$\mu^{-1}\beta_e\left(\sqrt{\beta_e}\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla\right)\mathbf{V} = -\nabla \ln n + n^{-1}\mathbf{J} \times \mathbf{B}, \tag{18}$$

where

$$\mu = \frac{m_e}{m_i} \ll 1, \tag{19}$$

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$$\nabla \cdot \mathbf{\Pi}_{\parallel} = -\nabla\left(\frac{0.24\Omega_e\tau_e}{1+\zeta'\sqrt{\beta_e}\Omega_e\tau_e}\nabla\cdot\mathbf{V}_e\right), \quad (20)$$

$$\mathbf{J} = n(\mathbf{V}-\mathbf{V}_e), \quad (21)$$

$$\nabla\cdot\mathbf{B} = 0, \quad (22)$$

$$\nabla\times\mathbf{E} = -\sqrt{\beta_e}\frac{\partial\mathbf{B}}{\partial t}, \quad (23)$$

$$\nabla\times\mathbf{B} = \beta_e\mathbf{J}, \quad (24)$$

and any hats on normalized quantities have been omitted for ease of notation. Furthermore, the nonzero elements of the normalized gyro viscosity tensor are $\Pi_{gxx}=-\Pi_{gyy}=(n/2)W_{xy}$, $\Pi_{gxy}=\Pi_{gyx}=-(n/4)(W_{xx}-W_{yy})$, $\Pi_{gxz}=\Pi_{gzx}=nW_{yz}$, and $\Pi_{gyz}=\Pi_{gzy}=-nW_{xz}$, where $\mathbf{W}=\nabla\mathbf{V}_e+(\nabla\mathbf{V}_e)^T-(2/3)\nabla\cdot\mathbf{V}_e\mathbf{I}$.

E. Ordering assumptions

Let us assume that

$$\sqrt{\beta_e}\sim\sqrt{\mu}, \quad (25)$$

and

$$\Omega_e\tau_e \gtrsim \beta_e^{-3/2}. \quad (26)$$

In the following, the small quantity $\sqrt{\beta_e}$ is employed as an expansion parameter.

F. Preliminary analysis

It is consistent with Eq. (22) to write

$$\mathbf{B} = \beta_e\nabla\psi\times\mathbf{e}_z + (1+\beta_e^{3/2}b_z)\mathbf{e}_z, \quad (27)$$

where ψ, $b_z\sim\mathcal{O}(1)$. Thus, it follows from Eq. (24) that

$$\mathbf{J} = \sqrt{\beta_e}\nabla b_z\times\mathbf{e}_z + J_z\mathbf{e}_z, \quad (28)$$

where

$$J_z = -\nabla^2\psi. \quad (29)$$

It is also consistent with Eqs. (23) and (27) to write

$$\mathbf{E} = -\sqrt{\mu}\nabla\Phi - \beta_e^{3/2}\frac{\partial\psi}{\partial t}\mathbf{e}_z + \mathcal{O}(\beta_e^2), \quad (30)$$

where $\Phi\sim\mathcal{O}(1)$. Let us represent $\mathbf{V}$ in the completely general form

$$\mathbf{V} = \sqrt{\mu}\nabla\phi\times\mathbf{e}_z + \beta_e\nabla\chi + \sqrt{\mu\beta_e}V_z\mathbf{e}_z, \quad (31)$$

where ϕ, χ, $V_z\sim\mathcal{O}(1)$. Thus, it follows from Eqs. (21) and (28) that

$$\mathbf{V}_e = -(\sqrt{\beta_e}n^{-1}\nabla b_z - \sqrt{\mu}\nabla\phi)\times\mathbf{e}_z + \beta_e\nabla\chi + V_{ez}\mathbf{e}_z, \quad (32)$$

where

$$V_{ez} = n^{-1}\nabla^2\psi + \sqrt{\mu\beta_e}V_z. \quad (33)$$

G. Ion fluid equation of motion

The ion fluid equation of motion, Eq. (18), reduces to

$$0 = n^{-1}\nabla(n+\sqrt{\beta_e}b_z) + \beta_e\left\{\nabla\left(\frac{V_\perp^2}{2\mu}\right) + \left(\frac{\beta_e}{\mu}\right)^{1/2}\nabla\left(\frac{\partial\phi}{\partial t}\right)\times\mathbf{e}_z - \nabla^2\phi\nabla\phi + n^{-1}\nabla^2\psi\nabla\psi\right\} + \beta_e^{3/2}\left\{\left(\frac{\beta_e}{\mu}\right)^{1/2}\frac{\partial V_z}{\partial t} - [\phi,V_z] - n^{-1}[b_z,\psi]\right\}\mathbf{e}_z + \beta_e^{3/2}\left(\frac{\beta_e}{\mu}\right)^{1/2}\left\{\left(\frac{\beta_e}{\mu}\right)^{1/2}\nabla\left(\frac{\partial\chi}{\partial t}\right) + \nabla^2\phi\nabla\chi\times\mathbf{e}_z\right\} + \mathcal{O}(\beta_e^2), \quad (34)$$

where $[A,B]\equiv\nabla A\times\nabla B\cdot\mathbf{e}_z$, and use has been made of Eqs. (27), (28), and (31).

To lowest order in $\sqrt{\beta_e}$, the above equation yields

$$n = 1 - \sqrt{\beta_e}b_z. \quad (35)$$

It follows from Eq. (32) that

$$\mathbf{V}_e = \sqrt{\beta_e}\nabla\phi_e\times\mathbf{e}_z + \beta_e\nabla\chi + V_{ez}\mathbf{e}_z, \quad (36)$$

where

$$\phi_e = -Z + \left(\frac{\mu}{\beta_e}\right)^{1/2}\phi, \quad (37)$$

and

$$\ln n = -\sqrt{\beta_e}Z. \quad (38)$$

Incidentally, Eqs. (35) and (38) imply that

$$Z = b_z + \mathcal{O}(\sqrt{\beta_e}). \quad (39)$$

The z-component of Eq. (34) yields the *ion equation of parallel motion*,

$$\frac{\partial V_z}{\partial t} = \left(\frac{\mu}{\beta_e}\right)^{1/2}[\phi,V_z] + \left(\frac{\mu}{\beta_e}\right)^{1/2}[Z,\psi] + \mathcal{O}(\sqrt{\beta_e}), \quad (40)$$

whereas the z-component of its curl gives the *ion vorticity equation*,

$$\frac{\partial\nabla^2\phi}{\partial t} = \left(\frac{\mu}{\beta_e}\right)^{1/2}[\phi,\nabla^2\phi] + \left(\frac{\mu}{\beta_e}\right)^{1/2}[\nabla^2\psi,\psi] + \mathcal{O}(\sqrt{\beta_e}). \quad (41)$$

H. Continuity equation

The continuity equation, Eq. (16), reduces to

$$\frac{\partial Z}{\partial t} = \left(\frac{\mu}{\beta_e}\right)^{1/2}[\phi,Z] + \nabla^2\chi + \mathcal{O}(\sqrt{\beta_e}), \quad (42)$$

where use has been made of Eqs. (36)–(38).

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I. Electron fluid equation of motion

The electron fluid equation of motion, Eq. (17), can be written as

$$\beta_e \mathbf{F} = \sqrt{\mu}\,\nabla(\Phi+\phi) - \beta_e\{n^{-1}\nabla^2\psi\nabla\psi + \nabla\chi\times\mathbf{e}_z\} + \beta_e^{3/2}\left\{\frac{\partial\psi}{\partial t} + [Z,\psi] - \left(\frac{\mu}{\beta_e}\right)^{1/2}[\phi,\psi] - \alpha\nabla^2(\psi-\psi^{(0)})\right\}\mathbf{e}_z + \mathcal{O}(\beta_e^2), \quad (43)$$

where

$$\mathbf{F} = \left(\sqrt{\beta_e}\frac{\partial}{\partial t} + \mathbf{V}_e\cdot\nabla\right)\mathbf{V}_e + n^{-1}\nabla\cdot\mathbf{\Pi}_\parallel + n^{-1}\nabla\cdot\mathbf{\Pi}_g, \quad (44)$$

and $\alpha = 0.51(\beta_e^{3/2}\Omega_e\tau_e)^{-1} \lesssim \mathcal{O}(1)$. Here, use has been made of Eqs. (27)–(30) and (36)–(39).

Assuming that $|\mathbf{F}| \lesssim \mathcal{O}(1)$ (see Sec. II J), Eq. (43) yields

$$\phi = -\Phi \quad (45)$$

to lowest order in $\sqrt{\beta_e}$. Furthermore, the z-component of this equation gives

$$F_z = \sqrt{\beta_e}\left\{\frac{\partial\psi}{\partial t} + [Z,\psi] - \left(\frac{\mu}{\beta_e}\right)^{1/2}[\phi,\psi] - \alpha\nabla^2(\psi-\psi^{(0)})\right\} + \mathcal{O}(\beta_e), \quad (46)$$

whereas the z-component of its curl reduces to

$$(\nabla\times\mathbf{F})\cdot\mathbf{e}_z = -[\nabla^2\psi,\psi] + \nabla^2\chi + \mathcal{O}(\sqrt{\beta_e}). \quad (47)$$

J. Evaluation of F

It is easily demonstrated from Eqs. (36) and (37) that

$$\left(\sqrt{\beta_e}\frac{\partial}{\partial t} + \mathbf{V}_e\cdot\nabla\right)\mathbf{V}_e = \frac{1}{2}\nabla(V_{\perp e}^2) + \sqrt{\beta_e}\left\{\frac{\partial\nabla^2\psi}{\partial t} + [Z,\nabla^2\psi] - \left(\frac{\mu}{\beta_e}\right)^{1/2}[\phi,\nabla^2\psi]\right\}\mathbf{e}_z + \mathcal{O}(\beta_e). \quad (48)$$

Moreover, it follows from Eqs. (20), (26), (36), and (38) that

$$n^{-1}\nabla\cdot\mathbf{\Pi}_\parallel = -\zeta\sqrt{\beta_e}\,\nabla(\nabla^2\chi) + \mathcal{O}(\beta_e), \quad (49)$$

where $\zeta = 0.24/\zeta'$. Finally, it can be shown that

$$n^{-1}\nabla\cdot\mathbf{\Pi}_g = \tfrac{1}{2}\sqrt{\beta_e}\,\nabla(\nabla^2 Z) - \sqrt{\beta_e}[Z,\nabla^2\psi]\mathbf{e}_z + \mathcal{O}(\beta_e). \quad (50)$$

Thus, from Eq. (44),

$$\mathbf{F} = \nabla\left(\frac{1}{2}V_{\perp e}^2 + \frac{1}{2}\sqrt{\beta_e}\nabla^2 Z - \zeta\sqrt{\beta_e}\nabla^2\chi\right) + \sqrt{\beta_e}\left\{\frac{\partial\nabla^2\psi}{\partial t} - \left(\frac{\mu}{\beta_e}\right)^{1/2}[\phi,\nabla^2\psi]\right\}\mathbf{e}_z + \mathcal{O}(\beta_e). \quad (51)$$

Note that the two terms involving $[Z,\nabla^2\psi]$ in Eqs. (48) and (50) cancel one another *exactly* in Eq. (51). This is a manifestation of the well-known *gyroviscous cancellation*.[16]

Equations (46) and (51) yield the *Ohm's law*,

$$\frac{\partial\psi_e}{\partial t} = \left(\frac{\mu}{\beta_e}\right)^{1/2}[\phi,\psi_e] - [Z,\psi] + \alpha\nabla^2(\psi-\psi^{(0)}) + \mathcal{O}(\sqrt{\beta_e}), \quad (52)$$

where

$$\psi_e = \psi - \nabla^2\psi. \quad (53)$$

Finally, Eqs. (42), (47), and (51) reduce to the *density evolution equation*,

$$\frac{\partial Z}{\partial t} = \left(\frac{\mu}{\beta_e}\right)^{1/2}[\phi,Z] + [\nabla^2\psi,\psi] + \mathcal{O}(\sqrt{\beta_e}). \quad (54)$$

K. Discussion

Our final set of reduced equations takes the form

$$\frac{\partial\psi_e}{\partial t} = \left(\frac{\mu}{\beta_e}\right)^{1/2}[\phi,\psi_e] - [Z,\psi] + 0.51(\beta_e^{3/2}\Omega_e\tau_e)^{-1}\nabla^2 \times(\psi-\psi^{(0)}) + \mathcal{O}(\sqrt{\beta_e}), \quad (55)$$

$$\frac{\partial Z}{\partial t} = \left(\frac{\mu}{\beta_e}\right)^{1/2}[\phi,Z] + [\nabla^2\psi,\psi] + \mathcal{O}(\sqrt{\beta_e}), \quad (56)$$

$$\frac{\partial\nabla^2\phi}{\partial t} = \left(\frac{\mu}{\beta_e}\right)^{1/2}[\phi,\nabla^2\phi] + \left(\frac{\mu}{\beta_e}\right)^{1/2}[\nabla^2\psi,\psi] + \mathcal{O}(\sqrt{\beta_e}), \quad (57)$$

where $\psi_e = \psi - \nabla^2\psi$. We can now relax the somewhat restrictive ordering assumption (25), to allow for higher values of β_e, provided that the $\mathcal{O}(\sqrt{\mu/\beta_e})$ terms appearing in the above equations remain significantly larger than the $\mathcal{O}(\sqrt{\beta_e})$ terms which have been neglected. This implies that Eqs. (55)–(57) are valid for β_e in the range

$$\beta_e \ll \sqrt{\mu}. \quad (58)$$

Equations (55)–(57) involve *three fields*: i.e., the magnetic flux function, ψ; the perturbed density {or, alternatively, the perturbed parallel magnetic field [see Eqs. (38) and (39)]}, Z; and the ion stream function, ϕ. There is a fourth field—namely, the ion parallel velocity, V_z—which is coupled to these three fields via the equation

$$\frac{\partial V_z}{\partial t} = \left(\frac{\mu}{\beta_e}\right)^{1/2}[\phi,V_z] + \left(\frac{\mu}{\beta_e}\right)^{1/2}[Z,\psi] + \mathcal{O}(\sqrt{\beta_e}), \quad (59)$$

but does not explicitly appear in Eqs. (55)–(57).

The three-field reduced model (55)–(57) differs in two important respects from the four-field reduced model derived in Ref. 8. The first difference is that the parallel ion velocity, V_z, is decoupled from the other three fields, ψ, Z, and ϕ. In fact, the coupling term is $\mathcal{O}(\sqrt{\mu\beta_e})$, and it would therefore be inconsistent to retain it in the model while neglecting other

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much larger $\mathcal{O}(\sqrt{\beta_e})$ terms. The second difference is the absence of density diffusion terms in Eq. (56). There are two such terms—the *collisionless density diffusion term*, $\beta_e \nabla^2 \partial Z/\partial t$, and the *collisional density diffusion term*, $\beta_e \alpha \nabla^2 Z$— both of which would appear on the right-hand side of Eq. (56). However, these terms are omitted because they are $\mathcal{O}(\beta_e)$, and it would be inconsistent to include them while neglecting other much larger $\mathcal{O}(\sqrt{\beta_e})$ terms. It was erroneously stated in Ref. 10 that the *collisionless* density diffusion term is cancelled out when the electron gyro viscosity tensor is included in the analysis. In fact, the cancellation is only partial.[11] Thus, the true reason for the omission of the density diffusion terms is the low-β_e ordering used to derive Eqs. (55)–(57).

In the collisionless limit, $\Omega_e \tau_e \to \infty$, the three-field model Eqs. (55)–(57) can be shown to *conserve energy*, as ought to be the case, since the collisionless model is ultimately derivable from the energy conserving Vlasov equation. In fact, the (normalized) conserved energy takes the form[17]

$$\mathcal{E} = \frac{1}{2}\int\int [(\nabla^2\psi)^2 + |\nabla\psi|^2 + Z^2 + |\nabla\phi|^2]dxdy. \tag{60}$$

Incidentally, the gyro viscous cancellation, mentioned in Sec. II J, plays a vital role in ensuring that the collisionless three-field model is energy conserving. In fact, if the cancellation is neglected then the $-[Z,\psi]$ term on the right-hand side of Eq. (55) is converted into $-[Z,\psi_e]$, and, instead of $d\mathcal{E}/dt=0$, we obtain

$$\frac{d\mathcal{E}}{dt} = -\int\int [Z,\psi-\psi_e]\psi dxdy. \tag{61}$$

In other words, if the gyro viscous cancellation is neglected then the collisionless three-field model is not energy conserving. This highlights the importance of retaining the electron gyroviscosity tensor in the electron fluid equation of motion.

III. LINEAR STABILITY ANALYSIS

A. Introduction

As an illustration of its utility, let us employ the three-field model derived in Sec. II to calculate the linear growth-rate of the reconnecting instability. In the following, it is assumed that equilibrium quantities only vary in the x-direction, that the system is periodic in the y-direction, with periodicity length L_y, and that the small y-directed reconnecting magnetic field reverses sign at $x=0$.

B. Analysis

Let

$$\psi(x,y,t) = -\frac{1}{2}\frac{x^2}{\beta_e L_s} + \tilde{\psi}(x)e^{i(ky+\gamma t)}, \tag{62}$$

$$Z(x,y,t) = \tilde{Z}(x)e^{i(ky+\gamma t)}, \tag{63}$$

$$\phi(x,y,t) = \tilde{\phi}(x)e^{i(ky+\gamma t)}, \tag{64}$$

where $k=2\pi/L_y$, and $\tilde{}$ denotes a perturbed quantity. (Here, L_y is normalized with respect to d_e.) Moreover, we are neglecting any equilibrium shear flows, density gradients, and current gradients, for the sake of simplicity. The expression for the (normalized) equilibrium magnetic field is $\mathbf{B}^{(0)}=(0,x/L_s,1)$. Thus, L_s is the shear-length of the reconnecting magnetic field (normalized with respect to d_e).

At large-$|x|$, we expect the plasma to be governed by the equations of ideal-MHD.[18] Moreover, in the limit $|x|\to 0$, the ideal-MHD solution for a reconnecting instability takes the standard form

$$\tilde{\psi}(x) = \tilde{\psi}_0\left[\frac{\Delta'}{2}|x| + 1 + \mathcal{O}\left(\frac{1}{x}\right)\right], \tag{65}$$

where $\tilde{\psi}_0$ is a constant, and Δ' is the *linear tearing stability index* (normalized to d_e).[18] The above solution must be asymptotically matched to the solution of the linearized three-field model at the edge of a narrow reconnecting layer centered on $x=0$.

Linearizing Eqs. (55)–(57), and making the long wavelength ordering $k\ll 1$, we obtain the following layer equations:

$$g\left[1-\left(1+\frac{\epsilon'}{g}\right)\frac{d^2}{dx^2}\right]\tilde{\psi} = \frac{-ix}{\beta_e L_s}\left[\tilde{Z} - \left(\frac{\mu}{\beta_e}\right)^{1/2}\tilde{\phi}\right], \tag{66}$$

$$g\tilde{Z} = \frac{ix}{\beta_e L_s}\frac{d^2\tilde{\psi}}{dx^2}, \tag{67}$$

$$g\frac{d^2\tilde{\phi}}{dx^2} = \left(\frac{\mu}{\beta_e}\right)^{1/2}\frac{ix}{\beta_e L_s}\frac{d^2\tilde{\psi}}{dx^2}, \tag{68}$$

where $g=\gamma/k$ and $\epsilon'=0.51(\beta_e^{3/2}\Omega_e\tau_e)^{-1}/k$. It is assumed that $\tilde{\psi}(x)$ is even in x, while $\tilde{Z}(x)$ and $\tilde{\phi}(x)$ are both odd [which is consistent with Eq. (65)].

Suppose that

$$\bar{\psi}(p) = \int_{-\infty}^{\infty}\tilde{\psi}(x)e^{-ipx}dx, \tag{69}$$

etc. The above Fourier transform should be understood in the generalized sense, since $\psi(x)$ is not square-integrable. The Fourier transformed layer equations can be combined to give

$$\frac{d}{dq}\left(\frac{q^2}{1+q^2}\frac{dY}{dq}\right) - Q^2\left(\frac{q^2}{q^2+\lambda^2}\right)Y = 0, \tag{70}$$

where $Q=g\beta_e L_s$, $q=(1+\epsilon/Q)^{1/2}p$, $\epsilon=\epsilon'\beta_e L_s$, and $\lambda=(\mu/\beta_e)^{1/2}(1+\epsilon/Q)^{1/2}$, and

$$\frac{dY}{dq} \propto (1+q^2)\bar{\psi}. \tag{71}$$

The boundary conditions on Eq. (70) are that [see Eqs. (65) and (71)]

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$$Y(q\to 0)\to Y_0\left[\frac{1}{q}+\frac{\pi}{\Delta'(1+\epsilon/Q)^{1/2}}+\mathcal{O}(q)\right], \tag{72}$$

where Y_0 is a constant, and that $Y(q\to\infty)$ be well-behaved.

Assuming that Q, $\lambda\ll 1$, Eq. (70) can be solved in three overlapping regions. In the first of these regions, $q\sim\lambda\ll 1$, Eq. (70) reduces to

$$\frac{d^2F}{dq^2}-\left(\frac{Q^2}{q^2+\lambda^2}\right)F\simeq 0, \tag{73}$$

where $F=qY$. To zeroth order in Q^2, this equation yields

$$\frac{d^2F^{(0)}}{dq^2}\simeq 0. \tag{74}$$

The solution which satisfies the boundary condition Eq. (72) is

$$F^{(0)}=Y_0\left\{1+\left[\frac{\pi}{\Delta'(1+\epsilon/Q)^{1/2}}\right]q\right\}. \tag{75}$$

Moreover, the first-order correction to this solution is written

$$\frac{d^2F^{(1)}}{dq^2}\simeq\left(\frac{Q^2}{q^2+\lambda^2}\right)F^{(0)}. \tag{76}$$

Now, the above correction is only important at comparatively high values of the normalized growth-rate, Q. However, high values of Q correspond to low values of $1/\Delta'$ (see Table II). Hence, we can safely assume that $F^{(0)}$ is dominated by the first term appearing within the square brackets in Eq. (75) (the validity of this approximation is easily established *a posteriori*), which implies that

$$\frac{d^2F^{(1)}}{dq^2}\simeq\left(\frac{Q^2}{q^2+\lambda^2}\right)Y_0. \tag{77}$$

Thus, integrating once in q, we obtain

$$\frac{dF^{(1)}}{dq}\simeq\frac{Y_0Q^2}{\lambda}\tan^{-1}\left(\frac{q}{\lambda}\right). \tag{78}$$

In the region $\lambda\ll q\ll 1$, this expression yields

$$\frac{dF^{(1)}}{dq}\simeq\frac{Y_0Q^2}{\lambda}\frac{\pi}{2}. \tag{79}$$

Integrating once more in q, we get

$$F^{(1)}\simeq\frac{Y_0Q^2}{\lambda}\frac{\pi}{2}q. \tag{80}$$

Hence, in the region $\lambda\ll q\ll 1$, the solution to Eq. (70) takes the general form

$$Y\simeq Y_0\left\{\frac{1}{q}+\left[\frac{Q^2}{\lambda}\frac{\pi}{2}+\frac{\pi}{\Delta'(1+\epsilon/Q)^{1/2}}\right]+\mathcal{O}(q)\right\}. \tag{81}$$

Next, let us examine the region $q\sim 1$. Here, to lowest order in Q^2, Eq. (70) simplifies to

$$\frac{d}{dq}\left(\frac{q^2}{1+q^2}\frac{dY}{dq}\right)\simeq 0. \tag{82}$$

Matching the solution of the above equation to expression (81), in the region $\lambda\ll q\ll 1$, we obtain

$$Y\simeq Y_0\left\{\frac{1}{q}+\left[\frac{Q^2}{\lambda}\frac{\pi}{2}+\frac{\pi}{\Delta'(1+\epsilon/Q)^{1/2}}\right]-q+\mathcal{O}(q^2)\right\}. \tag{83}$$

Let us now consider the region $q\sim Q^{-1}\gg 1$. Here, Eq. (70) reduces to

$$\frac{d^2Y}{dq^2}-Q^2Y\simeq 0. \tag{84}$$

The solution to the above equation which is well-behaved as $q\to\infty$ is

$$Y=Y_1e^{-Qq}, \tag{85}$$

where Y_1 is a constant. Hence, in the region $1\ll q\ll Q^{-1}$, we get

$$Y\simeq Y_1[1-Qq+\mathcal{O}(q^2)]. \tag{86}$$

Finally, matching this expression to Eq. (83), in the region $1\ll q\ll Q^{-1}$, we obtain the dispersion relation

$$\frac{\pi}{\Delta'}=\frac{(1+\epsilon/Q)^{1/2}}{Q}-\frac{\pi}{2}\left(\frac{\beta_e}{\mu}\right)^{1/2}Q^2. \tag{87}$$

C. Discussion

The above dispersion relation can be written as[19]

$$\frac{\pi}{\hat{\Delta}}=\frac{(1+\epsilon/\hat{\gamma})^{1/2}}{\hat{\gamma}}-\frac{\pi}{2}\left(\frac{\beta_e}{\mu}\right)^{1/2}\hat{\gamma}^2, \tag{88}$$

where (in terms of un-normalized quantities)

$$\hat{\gamma}=\frac{\gamma L_s}{\sqrt{\beta_e}\Omega_e k d_e^2}, \tag{89}$$

$$\epsilon=\frac{0.51L_s}{\sqrt{\beta_e}\Omega_e\tau_e k d_e^2}, \tag{90}$$

$$\hat{\Delta}=\Delta' d_e. \tag{91}$$

Equation (88) is valid provided $Q\ll 1$ (which turns out to always be the case), and $\lambda\ll 1$, or

$$(1+\epsilon/\hat{\gamma})^{1/2}\ll(\beta_e/\mu)^{1/2}. \tag{92}$$

Criterion (92) is equivalent to the constraint that the collision-broadened width of the current channel in un-normalized x-space, $\delta_e=(1+\epsilon/\hat{\gamma})^{1/2}d_e$, be much less than $\rho_s=(\beta_e/\mu)^{1/2}d_e$, where ρ_s is the ion gyroradius calculated with the electron temperature. According to the well-known classification of Drake and Lee,[20] this constraint implies that the above dispersion relation only describes collisionless and *semicollisional* reconnection regimes. In fact, assuming that Eq. (92) is satisfied, a collisionless regime corresponds to $\hat{\gamma}\gg\epsilon$, and a semicollisional regime to $\hat{\gamma}\ll\epsilon$.

Recall that the three-field model (55)–(57), from which Eq. (88) was obtained, was itself derived under the assumption that all significant length-scales greatly exceed the electron gyroradius, $\rho_e=\sqrt{\beta_e}d_e$. Now, the shortest length-scale in the reconnecting layer is Q^{-1} in q-space, and

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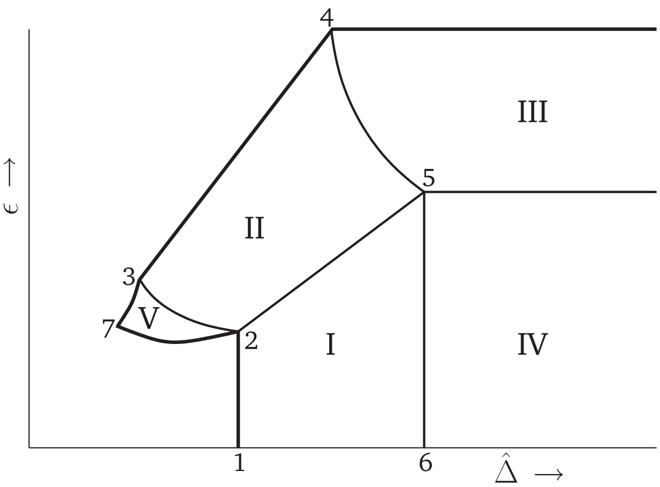

FIG. 1. Boundaries in $\hat{\Delta}$, ϵ space of the various linear collisionless/semicollisional reconnection regimes described in Table II. The coordinates of the various numbered points are given in Table I.

$\delta_0 = \hat{\gamma}(1+\epsilon/\hat{\gamma})^{1/2} d_e$ in (un-normalized) x-space. Thus, an additional criterion for the validity of the dispersion relation (88) is $\rho_e \ll \delta_0$, or

$$\hat{\gamma}(1+\epsilon/\hat{\gamma})^{1/2} \gg \sqrt{\beta_e}. \quad (93)$$

Finally, since we have relaxed the strict ordering, Eq. (25), used to derive the three-field model, it is necessary to check again that the neglect of the density diffusion terms remains valid. In fact, it is easily shown that the terms in question are negligible provided

$$\hat{\gamma} \gg \sqrt{\beta_e}. \quad (94)$$

Note that in the collisionless limit, $\hat{\gamma} \gg \epsilon$, the above constraint is identical with the earlier constraint (93). This suggests that the collisionless density diffusion term is negligible throughout the whole of the region of parameter space in which a Braginskii-like fluid treatment of the electrons is valid. However, in the semicollisional limit, $\hat{\gamma} \gg \epsilon$, the constraint (94) is more stringent than the constraint (93). This suggests the *collisional* density diffusion term is *not* negligible through the whole of the region of parameter space in which a Braginskii-like fluid treatment of the electrons is valid.

Figure 1 and Table I specify the boundaries in $\hat{\Delta}$, ϵ space of the various linear reconnection regimes predicted by the

TABLE I. Coordinates in $\hat{\Delta}$, ϵ space of the various numbered points appearing in Fig. 1.

No.	$\hat{\Delta}$	ϵ
1	$\beta_e^{1/2}$	0
2	$\beta_e^{1/2}$	$\beta_e^{1/2}$
3	$\mu^{1/2}$	$\mu^{-1}\beta_e^{3/2}$
4	$\mu^{1/2}\beta_e^{-1/2}$	$\mu^{-1}\beta_e$
5	$\mu^{1/6}\beta_e^{-1/6}$	$\mu^{1/6}\beta_e^{-1/6}$
6	$\mu^{1/6}\beta_e^{-1/6}$	0
7	$\mu^{3/4}\beta_e^{-1/4}$	$\mu^{-1/2}\beta_e$

TABLE II. Normalized linear growth rate of the reconnecting instability in the various collisionless/semicollisional reconnection regimes shown in Fig. 1. Here, $c=\Gamma(1/4)/[2\pi\Gamma(3/4)]$.

Regime	$\hat{\gamma}$
I	$\pi^{-1}\hat{\Delta}$
II	$\pi^{-2/3}\hat{\Delta}^{2/3}\epsilon^{1/3}$
III	$(2/\pi)^{2/7}\epsilon^{1/7}\mu^{1/7}\beta_e^{-1/7}$
IV	$(2/\pi)^{1/3}\mu^{1/6}\beta_e^{-1/6}$
V	$0.51^{1/4}c\hat{\Delta}\epsilon^{1/2}\beta_e^{-1/4}$

dispersion relation (88). It can be seen that there are *four* distinct regimes, which are labeled I to IV. Note that if ϵ is too large then the constraint (92) ceases to hold. Likewise, if $\hat{\Delta}$ becomes too small then the constraint (94) is no longer satisfied. This accounts for the fact that the reconnection regimes I–IV do not occupy all of $\hat{\Delta}$, ϵ space. Incidentally, the dispersion relation (88) is well-known, and was first derived in Ref. 19. The novel aspect of the derivation presented above is the determination of the region of $\hat{\Delta}$, ϵ space over which Eq. (88) is valid.

Expressions for the normalized growth-rate, $\hat{\gamma}$, of the reconnecting instability in all four of the regimes shown in Fig. 1 are given in Table II. It can be seen that two of the regimes, I and IV, are collisionless in nature: i.e., $\hat{\gamma}$ is *independent* of the collisionality factor, ϵ. Actually, in these regimes, there is no collisional broadening of the current channel, whose width is consequently of order the collisionless electron skin-depth, d_e, and the magnetic reconnection is solely mediated by *electron inertia*. The other two regimes, II and III, are semicollisional in nature. This follows because $\hat{\gamma} \to 0$ as $\epsilon \to 0$, indicating that the magnetic reconnection is mediated by plasma *resistivity*. In these regimes, the current channel is broadened by collisions, but its width, δ_e, still remains less than ρ_s, so that $d_e \ll \delta_e \ll \rho_s$.

For the sake of completeness, the tables and figure also include a semicollisional reconnection regime, labeled V, in which the collisional density diffusion term plays an important role. The growth-rate in this regime is taken from Eq. (55) of Ref. 9 (except that we have generalized this expression to take into account the fact that $\eta_\parallel \neq \eta_\perp$, which is the origin of the factor $0.51^{1/4}$ in Table II), and its boundaries in parameter space are determined by the constraints (92)–(94). Note that if the strict ordering, Eq. (25), from which the three-field model was derived, holds then points 2, 3, and 7 in Fig. 1 merge together. This suggests that the appearance of regime V, which implies a breakdown of our ordering scheme in the relevant region of parameter space, is a consequence of the relaxation of the strict ordering (25).

The collisionless and semicollisional reconnection regimes specified in Tables I and II, and Fig. 1, are similar to those described in Refs. 8–12, apart from the absence of a collisionless low-Δ' regime in which $\gamma \propto \Delta'^2$. It turns out that the collisionless density diffusion term, which has been ordered out of the density evolution equation (56) (see Sec. II K), plays an important role in this regime. If the aforemen-

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tioned term was included in Eq. (56), despite this being inconsistent with our ordering scheme, then the $\gamma \propto \Delta'^2$ regime would occupy a region in Fig. 1 to the left of regime I and below regime V. However, in this region of parameter space the shortest length-scale in the reconnecting layer falls below the electron gyroradius, ρ_e. Unfortunately, this invalidates the Braginskii expression for the electron gyro viscosity tensor which was used to derive the three-field model. (Recall that the gyro viscosity tensor must be included in the analysis, otherwise the collisionless three-field model does not conserve energy.) Hence, we conclude that the $\gamma \propto \Delta'^2$ regime is *spurious*, at least within the context of a Braginskii-like *fluid* treatment of the electron dynamics.

IV. SUMMARY

In Sec. II, we derive a reduced three-field model [Eqs. (55)–(57)] of electron-ion plasma dynamics which is suitable for investigating two-dimensional magnetic reconnection in a weakly collisional, highly magnetized plasma consisting of isothermal electrons and cold ions. The starting point for the derivation is a set of two-fluid equations which are closed by standard Braginskii expressions for the electron gyro viscosity tensor, and the parallel and perpendicular electrical resistivity. Note that the gyro viscosity tensor must be included in the analysis so as to ensure that the final model is energy conserving in the collisionless limit. As is well-known, the aforementioned Braginskii expressions are obtained from a Chapman–Enskog expansion which relies on the comparative smallness of the electron gyroradius, ρ_e, compared to the typical variation length-scale, L, of the principal electron fluid moments. In other words, the expressions are only valid in the limit $\rho_e \ll L$. Now, for collisionless reconnection, $L \sim d_e$, where $d_e = \rho_e / \sqrt{\beta_e}$ is the collisionless electron skin-depth. Thus, the requirement $\rho_e \ll L \sim d_e$ limits our analysis to comparatively small values of the electron beta, β_e. In fact, our derivation of the three-field model is only valid in the limit $\beta_e \ll \sqrt{\mu}$, where $\mu = m_e / m_i$. The three-field model does not include the so-called collisionless and collisional density diffusion terms[9] in the density evolution equation [Eq. (56)] because these terms are $\mathcal{O}(\beta_e)$, and, in the course of our derivation, we have neglected any terms which are $\mathcal{O}(\sqrt{\beta_e})$ and smaller. Note that it was erroneously stated in Ref. 10 that the collisionless density diffusion term is cancelled out when the electron gyro viscosity tensor is included in the analysis. Actually, this is not the case.[11] The term is, in fact, always present, but is too small to play a significant role in the electron dynamics in the region of parameter space in which a Braginskii-like fluid treatment of the electron dynamics is valid.

In Sec. III, we employ the three-field model derived in Sec. II to calculate the linear growth-rate of the reconnecting instability in collisionless and semicollisional parameter regimes. Within the context of a Braginskii-like fluid treatment of the electron dynamics, we find that there is no weakly growing collisionless regime in which the growth-rate depends quadratically on the tearing stability index. Of course, this does not preclude the existence of such a regime in regions of parameter space in which Braginskii-like fluid equations are invalid, and a *kinetic* treatment of the electron dynamics is instead required. However, we note that no such regime is apparent from the results of the computer simulations recently performed by Rogers *et al.*[21]

ACKNOWLEDGMENTS

The author would like to thank R. D. Hazeltine (University of Texas at Austin), F. L. Hinton (Hinton Associates, Escondido, CA), E. Tassi (Université de Provence, France), D. Grasso (Politecnico di Torino, Italy), G. Vekstein (University of Manchester, U.K.), B. N. Rogers (Dartmouth College), V. V. Mirnov (University of Wisconsin-Madison), P. J. Catto (MIT), A. N. Simakov (LANL), and R. J. Hastie (Culham Laboratory, U.K.) for helpful discussions during the preparation of this paper.

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