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Fast excitation of EGAM by NBI

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Abstract
A new mechanism for the fast excitation of the energetic geodesic acoustic mode (EGAM) is proposed to explain the recent experiment in DIII-D (Nazikian *et al* 2008 *Phys. Rev. Lett.* **101** 185001), where the mode turns on in less than a millisecond after the turn-on of the neutral beam injection. The existence of loss boundary in pitch angle when beam particles are injected counter to the plasma current is crucial to the formation of negative energy EGAM mode. The resonance of this negative energy wave with energetic particles, whose distribution decreases with energy, destabilizes the mode. We find that when there is a loss region, the onset time of instability can be significantly shorter than it would be if the injected particles had no loss region.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

A feature of the energetic geodesic acoustic mode (EGAM) excitation in DIII-D is the apparent immediate turn-on of the instability as neutral beams are injected [1]. The previous analysis of the EGAM [2] used a slowing down distribution with a finite spread of pitch angle [3, 4]. The experimental result indicates that the ionized neutral beam confined in the tokamak will not have a chance to slow down when instability is first excited. Hence, it is appropriate to consider a nearly mono-energetic velocity distribution to model the response of the energetic particles.

Another feature of the DIII-D experiment is that rapid turn-on occurs when the beams are injected counter to the tokamak current where a substantial fraction of the neutral beams is ionized in the loss region [5, 6]. On the other hand, co-injected neutral beams do not as easily trigger the EGAM instability. We would like to investigate why there is a difference in the response of the two configurations.

Here we present a simplified model for a distribution function, shown in figure 1 to attempt to explain the rapid turn-on of instability and why the experimental situation favours the excitation of counter-injected modes. We shall consider a distribution of the form

$$F(u, \Lambda, r) = n_{\mathrm{h}}(r) f(u) g(\Lambda), \tag{1}$$

$$f(u) = \frac{\delta(u-u_0)}{u_0^2}, \tag{2}$$

$$g(\Lambda) = \frac{3}{8\pi\Delta\Lambda}\left[1-\left(\frac{\Lambda-\Lambda_0}{\Delta\Lambda}\right)^2\right] \cdot\Theta(\Lambda-\Lambda_0+\Delta\Lambda)\Theta(\Lambda_0+\Delta\Lambda-\Lambda)\Theta(\Lambda-\Lambda_c), \tag{3}$$

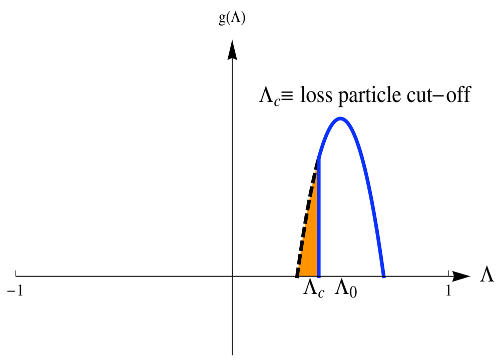

Figure 1. Clipped Λ distribution.

where $\Lambda \equiv u_\parallel/u$ is the pitch angle, Λ_{c} is the loss boundary to mock up the loss of energetic particles, $\Lambda_0 - \Delta\Lambda < \Lambda_{\mathrm{c}} < \Lambda_0 + \Delta\Lambda$. We define $n_{\mathrm{h}}(r)$ ($\mathrm{d}n_{\mathrm{h}}(r)/\mathrm{d}r < 1$ is assumed) to be the hot particle density when the entire distribution is in the confinement region so that $\Lambda_{\mathrm{c}} = \Lambda - \Delta\Lambda$. When in phase space there is a cut-off region, $\Lambda_0 - \Delta\Lambda < \Lambda < \Lambda_{\mathrm{c}}$, the actual hot particle density is less than $n_{\mathrm{h}}(r)$. However, we will see that the lower density distribution with a sharp cut-off is the more unstable one.

In this case the energetic particle source is centred at Λ_0 with a spread $\Delta\Lambda$ taken to be $\Delta\Lambda \ll \Lambda_0$. We assume $\Delta u/u_0 \ll \Delta\Lambda/\Lambda_0$ as the beam is injected at fixed energy, while there is an intrinsic pitch angle spread of the source due to the processes of beam focusing and beam ionization within the radius of the injection region occurring at different flux surfaces leading to a distribution with finite width of pitch angles. Hence, although $\Delta\Lambda/\Lambda_0 \ll 1$, it still should be larger than the corresponding spread of the injected speed. Thus for early stages we can take the velocity distribution to be a delta function, and only deal with the finite injection pitch angle.

The loss region arises from particles with smaller values of Λ, which have large enough orbit excursions so as not to be confined in the tokamak. In DIII-D it is the counter-injected particles that tend to be lost due to this large orbit effect, while the co-injected particles tend to remain confined. Here we show that the onset of instability is apparently shortened due to this loss boundary, which is only present in experiment where beams are injected counter to the plasma current.

In the last section we will discuss the reasonableness of this model for explaining the experimental data.

2. Marginal stability conditions

The local EGAM dispersion relation at radius r is given by [7]

$$1-\frac{\omega_{\mathrm{GAM}}^2}{\omega^2}+\frac{St}{n_\mathrm{i}R_0^2}\int\frac{\mathrm{d}^3u}{n_\mathrm{h}}\left[\left[\frac{\partial F(u_\parallel^2,u_\perp^2,r)}{\partial u_\parallel^2}+\mathrm{sgn}\frac{R_0q}{2r\omega_{\mathrm{ci}}u_\parallel}\times\frac{\partial F(u_\parallel^2,u_\perp^2,r)}{\partial r}\right]\left(u_\parallel^2+\frac{u_\perp^2}{2}\right)^2\right][\omega^2-\omega_\mathrm{b}^2]^{-1}=0, \quad (4)$$

where ω_{GAM} is the GAM frequency of thermal species and is given by

$$\omega_{\mathrm{GAM}}^2=\frac{2}{m_\mathrm{i}R_0^2}\left[\frac{7T_\mathrm{i}}{4}\left(1+O\left(\frac{1}{q^2}\right)\right)+T_\mathrm{e}\left(1+\frac{1}{2q^2}\right)\right].$$

This expression is valid in the large safety factor, q, limit when ions with temperature T_i and mass m_i are treated fluid-like with the assumption that their transit frequencies are much smaller than the mode frequency whereas electrons with temperature T_e are treated adiabatically under the assumption that their transit frequencies are much larger than the mode frequency, in the large aspect ratio limit. The transit frequency for passing particles, ω_b, is assumed to be sufficiently far from the trapped-passing boundary (i.e. $u_\parallel/u_\perp(R_0/r)^{\frac{1}{2}}\gg 1$), so that the transit frequency is approximately given by $\omega_\mathrm{b}=u\Lambda/(qR_0)$. In equation (4) ω_{ci} is the beam ion's cyclotron frequency. The term containing a gradient of energetic particle density emerges when the energetic particle distribution is chosen to be a function of canonical angular momentum subtracted by the bounce average of its mechanical component, which is a function of energy, and therefore has to be accounted for when $\partial F(E,\ \mu,\ P_\phi-m_\mathrm{i}\overline{Ru_\phi})/\partial E|_{\mu,P_\phi}$ is evaluated. Such a dependence allows for the co- (sgn $=1$) and counter- (sgn $=-1$) injected particles to be loaded in the same radial position to within a drift orbit width $\Delta_\mathrm{b}\approx qu_\parallel/\omega_{\mathrm{ci}}$. If the subtraction of $m_\mathrm{i}\overline{Ru_\phi}$ is omitted, the correction needed is $O(\Delta_\mathrm{b}R/r^2)$, which may not be small for energetic particles in tokamaks. However, if the subtraction is made the error is much smaller, i.e. $O(\Delta_\mathrm{b}/r)$. The quantity S is proportional to the rate at which neutral beam particles are injected into the plasma. The beam particle density, at time t, is $n_\mathrm{h}=St$ if all the ionized neutral beam particles are confined. If only a fraction η are confined, the energetic particle density would be $n_\mathrm{h}=\eta St$, but equation (4) would remain independent of η [3] except that the distribution would vanish in the part of phase space where the beam particles are lost, i.e. the cut-off Λ_c takes care of the fraction of lost beam particles.

It is interesting to note that with sgn $=0$, $\omega\gg\omega_\mathrm{b}$ and F being Maxwellian, the integral goes back to $7T_\mathrm{i}/2m_\mathrm{i}$, a typical contribution to GAM frequency by thermal ions [8, 9].

Equation (4) can be rewritten in terms of the speed, u, and pitch angle, Λ, as

$$D(\omega)=1-\frac{\omega_{\mathrm{GAM}}^2}{\omega^2}+\frac{St}{2n_\mathrm{i}R_0^2}\int\frac{\mathrm{d}^3uu^3}{n_\mathrm{h}}\left[\left[\frac{\partial F(u,\Lambda,r)}{\partial u}+\frac{1-\Lambda^2}{u\Lambda}\frac{\partial F(u,\Lambda,r)}{\partial\Lambda}+\mathrm{sgn}\frac{R_0q}{r\omega_{\mathrm{ci}}\Lambda}\frac{\partial F(u,\Lambda,r)}{\partial r}\right]\times\left(\frac{1+\Lambda^2}{2}\right)^2\right][\omega^2-\omega_\mathrm{b}^2]^{-1}=0. \quad (5)$$

We now make the assumption that the distribution function is finite only in a narrow region of u, i.e. Δu peaked at $u=u_0$, and a narrow region of Λ, i.e. $\Delta\Lambda$, centred about Λ_0 ($\Lambda_0-\Delta\Lambda<\Lambda<\Lambda_0-\Delta\Lambda$). We look for a root when ω is close to the transit frequency in the narrow phase space region occupied by the energetic particles. Then the frequency in the second term in equation (5) can be approximated by the transit frequency of particles at the centre of the distribution, i.e. $\omega_{\mathrm{b0}}\equiv u_0\Lambda_0/qR_0$, and the integrals in equation (5) can be approximated by

$$1-\frac{\omega_{\mathrm{GAM}}^2}{\omega_{\mathrm{b0}}^2}+\frac{q^2St\omega_{\mathrm{b0}}u_0}{16n_\mathrm{i}\Lambda_0^2}(1+\Lambda_0^2)^2\times\int\frac{\mathrm{d}^3u}{n_\mathrm{h}}\left[\left[\frac{\partial F(u,\Lambda,r)}{\partial u}+\frac{1-\Lambda^2}{u\Lambda}\frac{\partial F(u,\Lambda,r)}{\partial\Lambda}+\mathrm{sgn}\frac{R_0q}{r\omega_{\mathrm{ci}}\Lambda}\frac{\partial F(u,\Lambda,r)}{\partial r}\right]\right]\left[\omega-\frac{u\Lambda}{qR_0}\right]^{-1}=0. \quad (6)$$

Using the result which can be verified by integrating by parts,

$$u_0\int\mathrm{d}u\,\mathrm{d}\Lambda\frac{\frac{\partial F}{\partial u}}{\omega-\frac{u\Lambda}{qR_0}}=\Lambda_0\int\mathrm{d}u\,\mathrm{d}\Lambda\frac{\frac{\partial F}{\partial\Lambda}}{\omega-\frac{u\Lambda}{qR_0}},$$

we can combine the first two terms inside the square brackets under the integral in equation (6),

$$1-\frac{\omega_{\mathrm{GAM}}^2}{\omega_{\mathrm{b0}}^2}+\frac{q^2St\omega_{\mathrm{b0}}}{16n_\mathrm{i}\Lambda_0^3}(1+\Lambda_0^2)^2\times\int\frac{\mathrm{d}^3u}{n_\mathrm{h}}\frac{\frac{\partial F(u,\Lambda,r)}{\partial\Lambda}+\mathrm{sgn}\frac{R_0qu_0}{r\omega_{\mathrm{ci}}}\frac{\partial F(u,\Lambda,r)}{\partial r}}{\omega-\frac{u\Lambda}{qR_0}}=0. \quad (7)$$

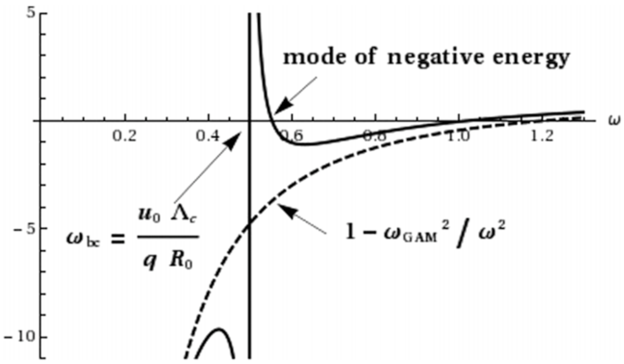

Figure 2. Negative energy mode.

Upon inserting the distribution in equation (1) into equation (7), the dispersion function reduces to

$$1-\frac{\omega_{\rm GAM}^2}{\omega_{\rm b0}^2}+\frac{3q^2St\omega_{\rm b0}}{32n_{\rm i}\Delta\Lambda^2\Lambda_0}\left(\frac{1+\Lambda_0^2}{\Lambda_0}\right)^2$$
$$\times\left\{\left[1-\left(\frac{\Lambda_{\rm c}-\Lambda_0}{\Delta\Lambda}\right)^2\right]\frac{\Delta\Lambda}{2\left(\omega-\dfrac{u_0\Lambda_{\rm c}}{qR_0}\right)}\right.$$
$$+\int_{\Lambda_{\rm c}}^{\Lambda_0+\Delta\Lambda}{\rm d}\Lambda\left\{-\frac{\Lambda-\Lambda_0}{\Delta\Lambda}+{\rm sgn}\frac{R_0qu_0\Delta\Lambda\dfrac{{\rm d}n_{\rm h}}{{\rm d}r}}{2r\omega_{\rm ci}n_{\rm h}}\right.$$
$$\left.\left.\times\left[1-\left(\frac{\Lambda-\Lambda_0}{\Delta\Lambda}\right)^2\right]\right\}\frac{1}{\omega-\dfrac{u_0\Lambda}{qR_0}}\right\}=0, \qquad (8)$$

where the first term inside the curly brackets originates from integrating a Λ-dependent function multiplied by a delta function, $\delta(\Lambda-\Lambda_{\rm c})$, which is the sharp gradient of distribution function $g(\Lambda)$ at the loss boundary. Together with the first two terms, a negative energy mode is established, whose frequency is between $\omega_{\rm bc}\equiv u_0\Lambda_{\rm c}/qR_0$ and $\omega_{\rm GAM}$ (see figure 3 where the dashed blue curve is the dispersion relation of thermal species alone while the addition of energetic particle terms produces the solid red curve. The contribution from the rapid increase in the energetic particle distribution leads to a new root of negative slope, which implies that the wave associated with the root is a negative energy mode).

With the following definitions

$$\kappa\equiv{\rm sgn}\frac{qu_0\Delta\Lambda}{2\omega_{\rm ci}}\frac{R_0}{r}\frac{\dfrac{{\rm d}n_{\rm h}}{{\rm d}r}}{n_{\rm h}};$$

$$\tau\equiv\frac{3q^2St}{32n_{\rm i}\Delta\Lambda^2(\omega_{\rm GAM}^2/\omega_{\rm b0}^2-1)}\left(\frac{1+\Lambda_0^2}{\Lambda_0}\right)^2;$$

$$\sigma\equiv\frac{qR_0\Delta\omega}{u_0\Delta\Lambda};\qquad \Delta\omega\equiv\omega-\omega_{\rm b0};\qquad \delta\Lambda_{\rm c}\equiv\frac{\Lambda_0-\Lambda_{\rm c}}{\Delta\Lambda};$$

$$\delta\Lambda\equiv\frac{\Lambda-\Lambda_0}{\Delta\Lambda},$$

the dispersion function can be simplified further to the following:

$$-\frac{1}{\tau}+\frac{1-\delta\Lambda_{\rm c}^2}{2(\sigma+\delta\Lambda_{\rm c})}+\int_{-\delta\Lambda_{\rm c}}^{1}{\rm d}\delta\Lambda\frac{-\delta\Lambda+\kappa(1-\delta\Lambda^2)}{\sigma-\delta\Lambda}=0.$$

We assume that $-\delta\Lambda_{\rm c}<\sigma<1$ and then perform the integral to obtain

$$\frac{1}{\tau}-\frac{1-\delta\Lambda_{\rm c}^2}{2(\sigma+\delta\Lambda_{\rm c})}=(1+\kappa\sigma)(1+\delta\Lambda_{\rm c})+\frac{\kappa}{2}(1-\delta\Lambda_{\rm c}^2)$$
$$+[\sigma-\kappa(1-\sigma^2)]\left[\ln\left|\frac{1-\sigma}{-\delta\Lambda_{\rm c}-\sigma}\right|+{\rm i}\pi\right]. \qquad (9)$$

First let us assume κ is small and can be neglected. We then consider

$$\frac{1}{\tau}-\frac{1-\delta\Lambda_{\rm c}^2}{2(\sigma+\delta\Lambda_{\rm c})}=1+\delta\Lambda_{\rm c}+\sigma\left[\ln\left|\frac{1-\sigma}{-\delta\Lambda_{\rm c}-\sigma}\right|+{\rm i}\pi\right]. \qquad (10)$$

There are three interesting limits to study for the root of this equation. The first case is where $\delta\Lambda_{\rm c}=1$ so that the distribution is not cut off. The second case is when $0<\delta\Lambda_{\rm c}<1$ and the third is $-1<\delta\Lambda_{\rm c}<0$.

For the first two cases we note that the imaginary part of equation (10) vanishes for $\sigma=0$. The time for the onset of instability, $t=t_{\rm c}$, is then determined by when the real part of equation (10) vanishes simultaneously. Hence we find

$$\frac{1}{\tau}-\frac{1-\delta\Lambda_{\rm c}^2}{2(\sigma+\delta\Lambda_{\rm c})}=1+\delta\Lambda_{\rm c}. \qquad (11)$$

When $\delta\Lambda_{\rm c}=1$, the case where there is no cut-off in the distribution function, we find that the critical condition for the onset of instability, $\tau=\tau_{\rm c}$ or alternatively the time $t=t_{\rm c}$ for the onset of instability, is determined from

$$\tau_{\rm c}=\frac{1}{2},\qquad \text{or}$$
$$t_{\rm c}=\frac{16n_{\rm i}(\Delta\Lambda)^2}{3q^2S}\left(\frac{\Lambda_0}{1+\Lambda_0^2}\right)^2\left(\frac{\omega_{\rm GAM}^2}{\omega_{\rm b0}^2}-1\right). \qquad (12)$$

For the case when $1>\delta\Lambda_{\rm c}>0$, the instability condition is given by

$$\tau_{\rm c}=\frac{2\delta\Lambda_{\rm c}}{(1+\delta\Lambda_{\rm c})^2}$$

or equivalently

$$t_{\rm c}=\frac{64n_{\rm i}(\Delta\Lambda)^2}{3q^2S}\left(\frac{\Lambda_0}{1+\Lambda_0^2}\right)^2\left(\frac{\omega_{\rm GAM}^2}{\omega_{\rm b0}^2}-1\right)\frac{\delta\Lambda_{\rm c}}{(1+\delta\Lambda_{\rm c})^2}. \qquad (13)$$

Note that equation (13) predicts that $t_{\rm c}$ decreases as $\delta\Lambda_{\rm c}$ decreases and $t_{\rm c}\to0$ as $\delta\Lambda_{\rm c}\to0$, i.e. the more particles are ionized in the loss region, the earlier the instability turns on. In addition at $\delta\Lambda_{\rm c}=1$, the factor $\delta\Lambda_{\rm c}/(1+\delta\Lambda_{\rm c})^2$ achieves its maximum. Thus the turn-on time is longest when $\delta\Lambda_{\rm c}=1$ where there is no abrupt discontinuity in the distribution due to the boundary. In the other limit, where the loss boundary is adjacent to the peak of the input beam distribution, so that $\delta\Lambda_{\rm c}\to0$, the instability is triggered instantaneously.

For the third case where $-1<\delta\Lambda_{\rm c}<0$, instability sets in immediately after turn-on.

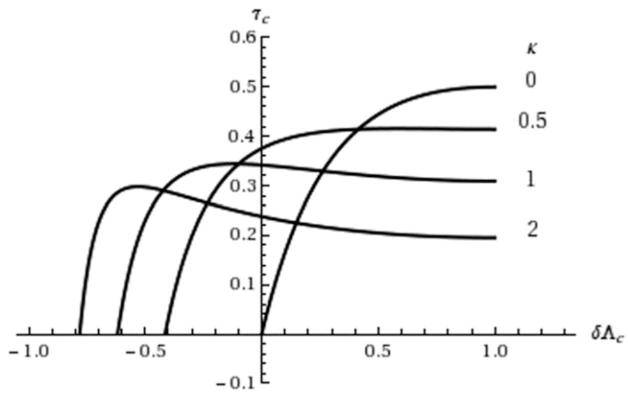

Figure 3. τ_c versus $\delta\Lambda_c$.

Solving equation (10) for the normalized frequency σ perturbatively yields

$$\sigma - |\delta\Lambda_c| \approx \frac{\tau}{2}(1-\delta\Lambda_c^2)\left(1 + i\pi\frac{\tau|\delta\Lambda_c|}{1+(\pi\tau\delta\Lambda_c)^2}\right). \tag{14}$$

Now we return to dispersion relation in equation (9) that includes the κ term, which accounts for the effect of co- or counter-injected particles. Then we find the critical frequency of the marginal unstable point from the vanishing of the imaginary part of equation (9),

$$\sigma_c = \frac{\kappa}{|\kappa|}\sqrt{1+\frac{1}{4\kappa^2}} - \frac{1}{2\kappa} \underset{\kappa\to 0}{\longrightarrow} \kappa. \tag{15}$$

With σ_c given in terms of κ, the condition for the onset of instability is given by

$$\frac{1}{\tau_c} = \frac{1-\delta\Lambda_c^2}{2(\sigma_c+\delta\Lambda_c)} + (1+\delta\Lambda_c)(1+\kappa\sigma_c) + \frac{\kappa}{2}(1-\delta\Lambda_c^2) \tag{16}$$

with $1 > \delta\Lambda_c > 1/2\kappa - \kappa/|\kappa|\sqrt{1+1/4\kappa^2}$.

For the case $\delta\Lambda_c = 1$ (the case where the distribution has a continuous edge), the marginal instability condition is given by

$$\tau = \frac{1}{1+\sqrt{1+4\kappa^2}}.$$

Because τ is proportional to the turn-on time, we see that in this model, the turn-on time is reduced by the same amount for either co ($\kappa < 0$) or counter ($\kappa > 0$)-injection.

Figure 3 shows the numerical solutions of equation (16) for the critical onset parameter, τ_c, as a function of $\delta\Lambda_c$, for various values of κ.

It should also be noted that the rapid turn-on does not arise if $\omega_{b0}^2 > \omega_{GAM}^2$, as τ_c is then negative and the marginal stability condition is not satisfied. This would be an interesting issue to test in experiment.

3. Interpretation of results with respect to DIII-D

To interpret the results in section 2, we will for the most part assume κ to be small and neglect it (typically $\kappa < 0.3$). In the previous section we noted that there is an enhancement of the mode turn-on time if part of the ionized beam particles is injected into a loss region of the tokamak. Then the distribution function will have an abrupt jump in its value at the loss boundary, which we saw led to the formation of a mode even at extremely small τ. This mode is a negative energy wave and its frequency lies just above the frequency $\omega_{bc} = u_0\Lambda_c/qR_0$. This wave resonates with the particles of the distribution function and produces either damping or growth. If $\Lambda_c < \Lambda_0$, as it is at early enough times where τ is small, the resonance will be with particles that produce inverse dissipation, leading to the damping of the negative energy wave. However, as the beam density builds up, the frequency shift moves closer to the point in phase space where the inverse dissipation goes to zero, which when reached is the marginal point for instability. For higher density, the resonance leads to positive dissipation and growth of the negative energy wave.

Should the loss region in the phase space be so large as to penetrate past the region in pitch angle in which neutral beam source is peaked, then except for the discontinuity in $F(\Lambda)$, we have that $\partial F(\Lambda)/\partial\Lambda < 0$. Then even at arbitrarily small density, the resonance leads to positive dissipation which will destabilize the negative energy wave whose frequency is just above ω_{bc}. This is the property exhibited in equation (14), which predicts instability for any finite time.

In practice the sharp jump is smoothed out by pitch angle scattering, and the width of this diffusion will be $\delta\Lambda \approx \sqrt{\nu_\perp t}$ [3], where $\nu_\perp$ [10] is the pitch angle scattering rate of the energetic ions. This pitch angle scattering means that the description of the negative energy wave obtained using a step function is valid only after time t satisfying the condition

$$\frac{\omega(t)-\omega_{bc}}{\omega_{b0}} = \frac{3q^2St}{64n_i\Lambda_0\Delta\Lambda}\left[1-\left(\frac{\Lambda_c-\Lambda_0}{\Delta\Lambda}\right)^2\right] \times\left(\frac{\omega_{GAM}^2}{\omega_{b0}^2}-1\right)^{-1} > \sqrt{\nu_\perp t}. \tag{17}$$

Clearly, at very early times, equation (17) cannot be satisfied. However, for our theory to have meaning it suffices to show that equation (17) is satisfied at the instability onset time t_c given by equation (13). Then substituting equation (13) into equation (17) yields for the condition of relevance for our theory,

$$\frac{\Delta\Lambda\Lambda_0}{(1+\Lambda_0^2)^2}\frac{\delta\Lambda_c(1-\delta\Lambda_c)}{1+\delta\Lambda_c} > \sqrt{\nu_\perp t_c}. \tag{18}$$

Now experimentally, $\nu_\perp \approx 1\ \mathrm{s}^{-1}$ and $t_c \leqslant 1$ ms. Hence, if $\delta\Lambda_c$ is not too small, equation (18) is typically satisfied, thereby justifying our model.

By accounting for pitch angle scattering, it follows when $\delta\Lambda_c < 0$, that the instability onset time is no longer predicted to be immediate, but gets deferred to a time comparable to t_{c1}, the time when equation (17) is an equality. For $t < t_{c1}$ we expect a stable response because close to the boundary of the loss region, we would have a negative energy wave resonating with particles of which the distribution function is rising with increasing Λ to create negative dissipation that stabilizes the wave. However, for $t > t_{c1}$, the shift of the wave frequency is large enough that resonant particles are part of a distribution that is decreasing with increasing Λ to yield positive dissipation that destabilizes the negative energy wave.

We now examine how well the turn-on predictions apply to the DIII-D experiment. The neutral beam source S is given by

$$\frac{S}{n_{\mathrm{i}}} = \frac{10^6 P(\mathrm{MW})}{2\pi^2 R_0 r^2 E_{\mathrm{b}}(\mathrm{keV}) n_{\mathrm{i}} 1.6 \times 10^{-16}}, \tag{19}$$

where P is the neutral beam power injected at its highest energy component, E_{b}, R is the major radius and πr^2 is the area enclosed by the target neutral beam. For the base parameters we choose values that are applicable to the DIII-D experiment: $E_{\mathrm{b}} = 75\,\mathrm{keV}$ of deuterons, $P = 1\,\mathrm{MW}$, $R_0 = 1.7\,\mathrm{m}$, $n_{\mathrm{i}} = 10^{13}\,\mathrm{cm}^{-3}$ and $r = 0.1\,\mathrm{m}$. Additional base values taken are $q = 4$, $\Lambda_0^2 = 0.5$, $\Delta\Lambda = 0.2$ and $\omega_{\mathrm{b0}}^2/\omega_{\mathrm{GAM}}^2 = 0.5$. The turn-on time for the case where all the injected particles are confined is given by equation (12), which leads to $t_{\mathrm{c}} \approx 0.45\,\mathrm{ms}$.

A turn-on time of $\sim 0.45\,\mathrm{ms}$ is well within the compatibility of the experimentally observed turn-on time of less than 1 ms for counter injection. However, this turn-on mechanism would also work for the co-injection case for which the choice $\delta\Lambda_{\mathrm{c}} = 1$ is appropriate but where experimentally the excitation of EGAM is more difficult to observe. We could obtain a clearer separation between the predicted co- and counter turn-on times, if our characteristic choices of experimental parameters were somewhat different. Note that it follows from equations (13) and (19) that the turn-on time is proportional to $(r\Delta\Lambda\Lambda)^2$. Thus an increase of a factor 2 in r, a factor of 1.5 in $\Delta\Lambda$ and a factor of $7/5$ in Λ, would increase the theoretically predicted onset time by nearly a factor of 20. Then the onset for the $\delta\Lambda_{\mathrm{c}} = 1$ case would be $\sim 8\,\mathrm{ms}$ which could be detected experimentally, while if there were a significant loss region for counter injection, e.g. $\delta\Lambda_{\mathrm{c}} \approx 0.1$, the predicted onset time would be less than 1 ms, and there would be compatibility of this theory with experiment. Another uncertainty is related to the nonlocal properties of the mode, which will lead to a spread of q as it is a function of radius. This effect may lead to significant shifts in the predicted instability onset time.

Thus given the uncertainty of absolute time predictions, our theory can be best viewed as a qualitative effect. The theory shows that the case when there is a loss region on the small Λ side is considerably more unstable than the case without a loss region. Indeed in the DIII-D experiment counter-injection particles have a much larger loss region than co-injected particles. Further, if the loss region is large enough, the onset of instability is predicted to start immediately. The only effect that would prevent immediate turn-on would be the violation of the inequality in equation (17).

We have also left out consideration of the small damping effects of the background plasma. Interestingly enough, when there is a negative energy wave present due to the presence of a significant loss region of the injected energetic beams, the background dissipation would be a destabilizing influence.

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