Stability and nonlinear dynamics aspects of a model for collisionless magnetic reconnection

Emanuele TASSI, Daniela GRASSO, Francesco PEGORARO[1] and P.J. MORRISON[2]

Burning Plasma Research Group, Dipartimento di Energetica and CNISM, Politecnico di Torino, Corso Duca degli Abruzzi 24, Torino, Italy

[1] *Dipartimento di Fisica E. Fermi and CNISM, Pisa, Italy*

[2] *Department of Physics and Institute for Fusion Studies, University of Texas, Austin, USA*

(Received: 29 August 2008 / Accepted: 10 November 2008)

A Hamiltonian 4-field fluid model describing magnetic reconnection in collisionless plasmas is investigated both analytically and numerically. The noncanonical Hamiltonian structure of the model is used in order to derive equilibrium equations and sufficient conditions for stability of equilibria in the presence of toroidal flow. Numerical simulations of the model equations are then used in order to investigate the vorticity evolution in the nonlinear regime. The coexistence of vortex-sheet-like and filamented structures is observed, which had no counterpart in a previously investigated 2-field model. Such evolution of the vorticity field is explained using the Casimir functionals of the system. Comments on the dependence of the vorticity structure on the value of the electron skin depth are also given.

Keywords: magnetic reconnection, collisionless plasmas, noncanonical Hamiltonian systems, stability

1. Introduction

The rearrangement of the connectivity of magnetic field lines, denoted as magnetic reconnection, is believed to be responsible for many catastrophic events occurring in laboratory and astrophysical plasmas. Paradigmatic examples of phenomena that are most probably related to magnetic reconnection events are solar flares, magnetospheric substorms and sawtooth oscillations in tokamaks [1, 2]. Magnetic reconnection in plasmas can take place only if some physical mechanism, such as e.g. collisions, electron inertia or turbulence, is present, that, in a fluid description, violates the frozen-in condition and allows plasma volumes not to remain linked to the same magnetic field line during the evolution of the system. In high temperature plasmas, such as those present in tokamaks, it is likely that electron inertia, that prevents the plasma from behaving as a perfect conductor, can effectively break the frozen-in condition and allow magnetic reconnection to take place.

In order to describe reconnection induced by electron inertia in collisionless plasmas, a number of reduced fluid models have been adopted (e.g. [3, 4, 5, 6, 7]). In recent years a $2\frac{1}{2}$ dimensional 4-field model for describing reconnection in the presence of plasma velocity and magnetic perturbations parallel to a guide field, has been derived by Fitzpatrick and Porcelli [8]. This model can be seen as an extension to higher β regimes (where β is the ratio between a constant equilibrium plasma pressure and the magnetic pressure exerted by the guide field) of the model derived by Schep et al. [4]. An investigation, by means of analytical methods, of the properties of this 4-field model was carried out in Refs. [9, 10], where the noncanonical Hamiltonian structure of the system was derived, spectral and linear stability of homogeneous equilibria were discussed and a derivation of the tearing mode growth rate, making use of a collisionless conductivity, was presented. The 4-field model has also been investigated with numerical tools and the corresponding results have been shown mainly in Refs. [11, 12], where a first qualitative analysis of the nonlinear structures observed in the numerical simulations was carried out. The present contribution resides in this line of investigation of the 4-field model and shows both analytical and numerical new results. In particular we extend here the stability analysis provided in [10] by applying the so called Energy-Casimir method, which makes it possible to obtain sufficient condition for the stability of equilibria of the system. Moreover we present results obtained from numerical solutions of the model equations, that show the time evolution of the parallel vorticity after the saturation of the reconnection process, and how it is influenced by the value of the electron skin depth.

The paper is organized as follows: after reviewing the model equations and their Hamiltonian structure, we derive equilibrium equations by means of a variational principle. Subsequently, the Energy-Casimir method is applied in order to obtain the conditions for stability. In Sec.3 the numerical results are shown and qualitative features of the vorticity evolution governed by the 4-field model are discussed. Section 4 is devoted to conclusions.

author's e-mail: emanuele.tassi@polito.it

2. The model equations: Hamiltonian formulation, equilibria and their stability

Let us consider a Cartesian coordinate system (x, y, z). The model investigated in this contribution consists, in a dimensionless form, of the following set of equations:

$$\frac{\partial(\psi - d_e^2\nabla^2\psi)}{\partial t} + [\varphi, \psi - d_e^2\nabla^2\psi] - d_\beta[\psi, Z] = 0, \quad (1)$$

$$\frac{\partial Z}{\partial t} + [\varphi, Z] - c_\beta[v, \psi] - d_\beta[\nabla^2\psi, \psi] = 0, \quad (2)$$

$$\frac{\partial \nabla^2\varphi}{\partial t} + [\varphi, \nabla^2\varphi] + [\nabla^2\psi, \psi] = 0, \quad (3)$$

$$\frac{\partial v}{\partial t} + [\varphi, v] - c_\beta[Z, \psi] = 0. \quad (4)$$

The four fields ψ, Z, φ and v are functions of x, y and time t, and are related to the magnetic field $\mathbf{B}$ and to the plasma fluid velocity $\mathbf{v}$ by the relations $\mathbf{B}(x, y, t) = \nabla\psi \times \hat{\mathbf{z}} + (B^{(0)} + c_\beta Z)\hat{\mathbf{z}}$ and $\mathbf{v}(x, y, t) = -\nabla\varphi \times \hat{\mathbf{z}} + v\hat{\mathbf{z}}$, where $B^{(0)}$ is a constant guide field, $c_\beta = \sqrt{\beta/(1 + \beta)}$, $d_\beta = d_i c_\beta$, while d_i and d_e indicate the ion and electron skin depth, respectively. The symbol $[,]$ indicates the canonical Poisson bracket, so that $[f, g] = (\nabla f \times \nabla g) \cdot \hat{\mathbf{z}}$, for generic fields f and g. The model (1)-(4) can be derived from the standard two-fluid description of a plasma. Eqs.(1) and (2) can be obtained from the parallel components of the electron momentum equation and electron vorticity equation, respectively, whereas (3) and (4) originate from the parallel component of the average vorticity and of the plasma fluid velocity equations, respectively.

2.1 Hamiltonian formulation

Given the absence of dissipative terms, the set of equations (1)-(4) is a natural candidate for being a Hamiltonian system. Indeed, the noncanonical Hamiltonian structure of the system has been derived in [9] and thoroughly discussed in [10]. The derivation of such structure follows from having realized that the functional

$$H = \frac{1}{2}\int_{\mathcal{D}} d^2x\,(d_e^2 J^2 + |\nabla\psi|^2 + |\nabla\varphi|^2 + v^2 + Z^2) \quad (5)$$

is a constant of motion for the system. In (5) $J = -\nabla^2\psi$ is the parallel current density whereas $\mathcal{D}$ is the domain of integration. Periodic boundary conditions are imposed. The functional H represents the total energy of the system, which includes both kinetic and magnetic contributions. The derivation of the Hamiltonian structure of an n-field system is completed (see, e.g. [13]) when a suitable antisymmetric bilinear quantity $\{,\}$ (noncanonical Poisson bracket), satisfying the Jacobi identity is found, such that the model equations can be written in the form

$$\frac{\partial \xi_i}{\partial t} = \{\xi_i, H\}, \qquad i = 1, \cdots, n, \quad (6)$$

with ξ_i indicating a set of field variables. The expression for the Poisson bracket of the 4-field model (which has Lie-Poisson form [13]) in the original physical variables is lengthy and can be found in [10]. Noncanonical Poisson brackets have associated Casimir functionals C [13], which are constants of motion characterized by the property $\{f, C\} = 0$, for every f. In the case of the 4-field model, four infinite families of Casimirs have been found, namely,

$$C_1 = \int_{\mathcal{D}} d^2x\,\omega\mathcal{F}(D)\,, \quad (7)$$

$$C_2 = \int_{\mathcal{D}} d^2x\,\mathcal{K}(D)\,, \quad (8)$$

$$C_\pm = \int_{\mathcal{D}} d^2x\,g_\pm(T_\pm)\,. \quad (9)$$

In the above, $\mathcal{F}$, $\mathcal{K}$, g_+ and g_- are arbitrary functions and we introduced the variables

$$D = \psi - d_e^2\nabla^2\psi + d_i v\,, \quad (10)$$

$$\omega = U + \frac{d_i}{c_\beta d^2}Z\,, \quad (11)$$

$$T_\pm = \pm\frac{d_i}{2c_\beta d^3 d_e}\left(d_i\psi - d_i d_e^2\nabla^2\psi - d_e^2 v \mp d d_e Z\right), \quad (12)$$

where $d = \sqrt{d_i^2 + d_e^2}$ and $U = \nabla^2\varphi$ is the parallel vorticity. Note that, by making use of the variables suggested by the Casimirs, the set (1)-(4) can be rewritten in the much more compact form

$$\frac{\partial D}{\partial t} = -[\varphi, D], \quad (13)$$

$$\frac{\partial \omega}{\partial t} = -[\varphi, \omega] + d^{-2}[D, \psi], \quad (14)$$

$$\frac{\partial T_\pm}{\partial t} = -[\varphi_\pm, T_\pm]\,, \quad (15)$$

where, for convenience, we have defined

$$\varphi_\pm = \varphi \pm \frac{c_\beta d}{d_e}\psi\,. \quad (16)$$

This form makes it evident that the fields D, T_+ and T_- are Lagrangian invariants of the system advected by the incompressible flows associated with the stream functions φ, φ_+ and φ_-, respectively.

2.2 Equilibria

Noncanonical Hamiltonian systems, such as the Fitzpatrick-Porcelli 4-field model, possess a built-in method for obtaining equilibrium solutions and investigating their stability (see, e.g. [13, 14, 15]). Indeed, unlike canonical Hamiltonian systems, for which equilibria correspond to extremals of the Hamiltonian, in the noncanonical case equilibrium solutions can be found by setting to zero the first variation of the free energy functional F obtained from a linear combination of the Hamiltonian functional with the Casimirs of the system. In the case of the 4-field model such functional reads

$$F[D,\omega,T_+,T_-] = \int_{\mathcal{D}} d^2x \left[\frac{c_\beta^2 d^4}{d_i^2}(T_+^2+T_-^2)+ \right.$$
$$\frac{D^2}{2d^2} - \frac{1}{2}\left(\omega+T_++T_-\right)\nabla^{-2}\left(\omega+T_++T_-\right)$$
$$-\frac{1}{2}\left(\frac{d_e}{d^2}D + c_\beta d(T_+-T_-)\right)$$
$$\mathcal{L}\left(\frac{d_e}{d^2}D + c_\beta d(T_+-T_-)\right)$$
$$\left. +\mathcal{K}(D)+\omega\mathcal{F}(D)+\mathcal{G}_+(T_+)+\mathcal{G}_-(T_-)\right], \tag{17}$$

where ∇^{-2} is the inverse Laplacian and $\mathcal{L}$ is an operator such that $\mathcal{L}(f - d_e^2\nabla^2 f) = f$, for a function f. Note that the first four lines of Eq.(17) contain the expression for the Hamiltonian (5), rewritten in terms of the variables D, ω, $T_\pm$, which turn out to be more convenient for this analysis.
The first variation of (17) is given by

$$\delta F = \int_{\mathcal{D}} d^2x \left[\left(\frac{D}{d^2} - \frac{d_e}{d^2}\mathcal{L}\left(\frac{d_e}{d^2}D + \right.\right.\right.$$
$$\left.\left. c_\beta d\left(T_+-T_-\right)\right) + \mathcal{K}'(D) + \omega\mathcal{F}'(D)\right)\delta D +$$
$$\left(-\nabla^{-2}(\omega+T_++T_-)+\mathcal{F}(D)\right)\delta\omega +$$
$$\left(2\frac{c_\beta^2 d^4}{d_i^2}T_+ - \nabla^{-2}(\omega+T_++T_-) - \right.$$
$$\left. c_\beta d\mathcal{L}\left(\frac{d_e}{d^2}D + c_\beta d(T_+-T_-)\right) + g_+'(T_+)\right)\delta T_+ +$$
$$\left(2\frac{c_\beta^2 d^4}{d_i^2}T_- - \nabla^{-2}(\omega+T_++T_-) + \right.$$
$$\left.\left. c_\beta d\mathcal{L}\left(\frac{d_e}{d^2}D + c_\beta d(T_+-T_-)\right) + g_-'(T_-)\right)\delta T_- \right]. \tag{18}$$

Therefore, equilibrium solutions for the 4-field model can be found by solving the system

$$\frac{D}{d^2} - \frac{d_e}{d^2}\mathcal{L}\left(\frac{d_e}{d^2}D + c_\beta d(T_+-T_-)\right) +$$
$$\mathcal{K}'(D) + \omega\mathcal{F}'(D) = 0, \tag{19}$$

$$-\nabla^{-2}(\omega+T_++T_-)+\mathcal{F}(D) = 0, \tag{20}$$

$$2\frac{c_\beta^2 d^4}{d_i^2}T_\pm - \nabla^{-2}(\omega+T_++T_-) \mp$$
$$c_\beta d\mathcal{L}\left(\frac{d_e}{d^2}D + c_\beta d(T_+-T_-)\right) +$$
$$g_\pm'(T_\pm) = 0. \tag{21}$$

Different choices for the functions $\mathcal{F}$, $\mathcal{K}$, $g_\pm$ lead to different classes of equilibria. In particular, from (20), going back to the original variables, one sees that fixing $\mathcal{F}(D)$ corresponds to choosing the equilibrium poloidal flow. As an example, if one chooses

$$\mathcal{K}(D) = \frac{A_D}{2}D^2, \qquad \mathcal{F}(D) = A_\omega D,$$
$$g_\pm(T_\pm) = \frac{A_\pm}{2}T_\pm^2, \tag{22}$$

with constants A_D, A_ω, $A_\pm$ then the resulting equilibrium equations contain as solutions the homogeneous equilibria

$$\psi = \alpha_\psi x, \qquad \varphi = 0,$$
$$Z = \alpha_Z x, \qquad v = \alpha_v x \tag{23}$$

where α_ψ, α_Z and α_v are constant. The same choice includes also the dipole vortex solutions that, in polar coordinates (r,θ), have the form

$$\chi_i(r,\theta) = A_i J_1(\sqrt{\lambda_i}r)\cos\theta, \qquad i=1,2 \tag{24}$$

where χ_1 and χ_2 are linear combinations of ψ and φ, whereas $A_{1,2}$ and $\lambda_{1,2}$ are constant. The corresponding solutions for v and Z are linear combinations of φ and ψ [10].
The equilibrium equations (19)-(21) can also be written in the following form:

$$\nabla^2\psi = S(\psi,\varphi), \tag{25}$$
$$\nabla^2\varphi = P(\psi,\varphi), \tag{26}$$
$$v(\psi,\varphi) = d_i a(\varphi)/d^2 -$$
$$c_\beta d_e d(t_+(\varphi_+)-t_-(\varphi_-))/d_i; \tag{27}$$
$$Z(\psi,\varphi) = -\frac{c_\beta d^2}{d_i}(t_+(\varphi_+)+t_-(\varphi_-)), \tag{28}$$

where

$$S(\psi,\varphi) := \frac{\psi}{d_e^2} - \frac{a(\varphi)}{d^2} -$$
$$\frac{c_\beta d}{d_e}\Big[t_+(\varphi_+)-t_-(\varphi_-)\Big], \tag{29}$$
$$P(\psi,\varphi) := b(\varphi)+t_+(\varphi_+)+t_-(\varphi_-)$$
$$+a'(\varphi)\frac{\psi}{d^2}. \tag{30}$$

and $t_+(\varphi_+)$, $t_-(\varphi_-)$, $a(\varphi)$, $b(\varphi)$ represent arbitrary invertible functions. The problem of finding equilibrium solutions amounts to solving the coupled system given by (25) and (26) for the unknowns ψ and

φ. In particular, Eq. (25) is a generalized Grad-Shafranov equation that accounts also for an equilibrium flow, whereas (26) governs the equilibrium vorticity. Once these two coupled equations are solved, the corresponding solutions for v and Z can then be easily found using (27) and (28).

2.3 Stability

Sufficient conditions for the stability of the equilibria of the 4-field model can be derived by making use of the well known Energy-Casimir method [16, 17, 18, 19, 20, 21, 13]. According to this method, equilibria of a noncanonical Hamiltonian systems are stable to infinitesimal perturbations if the second variation of the corresponding free energy functional has a definite sign. For the system under consideration the second variation of the free energy functional (H_L in Ref.[10]) reads

$$\begin{aligned}\delta^2 F = \int_{\mathcal{D}} d^2x \Bigg[&\left(\frac{2c_\beta^2 d^4}{d_i^2} + g_+^{''}(T_+)\right)|\delta T_+|^2 + \\ &\left(\frac{2c_\beta^2 d^4}{d_i^2} + g_-^{''}(T_-)\right)|\delta T_-|^2 + \left(\mathcal{K}^{''}(D) + \frac{1}{d^2} + \right.\\ &\left.\omega\mathcal{F}^{''}(D)\right)|\delta D|^2 \\ &-\delta(\omega + T_+ + T_-)\nabla^{-2}\delta(\omega + T_+ + T_-) - \\ &\delta\left(\frac{d_e}{d^2}D + c_\beta d(T_+ - T_-)\right) \\ &\mathcal{L}\delta\left(\frac{d_e}{d^2}D + c_\beta d(T_+ - T_-)\right) + 2\mathcal{F}^{'}(D)\delta\omega\delta D\Bigg]. \end{aligned} \tag{31}$$

It is convenient then to express the perturbations in terms of the variables $\psi_e = \psi - d_e^2\nabla^2\psi$, U, Z and v. By making use of integration by parts and of the definition of the operators ∇^{-2} and $\mathcal{L}$, the resulting expression for the second variation of the free energy functional is the following:

$$\begin{aligned}\delta^2 F = \int_{\mathcal{D}} d^2x \Bigg[&\left(\frac{2c_\beta^2 d^4}{d_i^2} + g_+^{''}(T_+)\right) \\ &\left|\delta\left(\frac{d_i}{2c_\beta d^3 d_e}\left(d_i\psi_e - d_e^2 v - dd_e Z\right)\right)\right|^2 + \\ &\left(\frac{2c_\beta^2 d^4}{d_i^2} + g_-^{''}(T_-)\right) \\ &\left|\delta\left(-\frac{d_i}{2c_\beta d^3 d_e}\left(d_i\psi_e - d_e^2 v + dd_e Z\right)\right)\right|^2 + \\ &\left(\mathcal{K}^{''}(D) + \frac{1}{d^2} + \omega\mathcal{F}^{''}(D)\right)|\delta(\psi_e + d_i v)|^2 + \\ &|\nabla\delta\varphi|^2 + |\delta\psi|^2 + d_e^2|\nabla\delta\psi|^2 + \\ &2\mathcal{F}^{'}(D)\delta\left(U + \frac{d_i}{c_\beta d^2}Z\right)\delta\left(\psi_e + d_i v\right)\Bigg]. \end{aligned} \tag{32}$$

Therefore, the positive definiteness of $\delta^2 F$ is obtained if

$$2\frac{c_\beta^2 d^4}{d_i^2} + g_+^{''}(T_+) > 0, \tag{33}$$

$$2\frac{c_\beta^2 d^4}{d_i^2} + g_-^{''}(T_-) > 0, \tag{34}$$

$$\mathcal{F}^{'}(D) = \mathcal{F}^{''}(D) = 0, \tag{35}$$

$$\mathcal{K}^{''}(D) + \frac{1}{d^2} > 0. \tag{36}$$

If (33)-(36) are satisfied, then $\delta^2 F = 0$ if and only if $\delta T_\pm$, δD and $\delta\omega$ are identically zero. According to the Energy-Casimir method, an equilibrium of the 4-field model satisfying (33)-(36) is linearly stable and with some minor technical limitation on the Casimir functions it can usually be shown to be (conditionally) nonlinearly stable. Note that (35) implies that the plasma fluid poloidal velocity be zero at equilibrium. As a simple application of this result one could consider the classes of equilibria originated by quadratic Casimirs mentioned in Sec.2.2. Those equilibria turn out to be linearly stable if

$$2\frac{c_\beta^2 d^4}{d_i^2} + A_\pm > 0, \tag{37}$$

$$A_\omega = 0, \tag{38}$$

$$A_D + \frac{1}{d^2} > 0. \tag{39}$$

In particular, if one focuses on the homogeneous equilibria, from the conditions (37)-(39) one can retrieve the stability conditions that were derived in Ref.[10] from an analysis of the dispersion relation. More precisely, by applying (37)-(39) to homogeneous equilibria one finds

$$d_i - d_e^2\frac{\alpha_v}{\alpha_\psi} \mp dd_e\frac{\alpha_Z}{\alpha_\psi} > 0, \tag{40}$$

$$1 + d_i\frac{\alpha_v}{\alpha_\psi} > 0. \tag{41}$$

as stability conditions. These corresponds namely to the conditions derived in [10] (cf. pp. 18-21), that guarantee stability for all the four branches of the dispersion relation. Note that, in the absence of sources of free energy coming from gradients in the equilibrium parallel magnetic and velocity (i.e. $\alpha_Z = \alpha_v = 0$), the equilibrium is stable. Indeed such gradients can excite drift-like modes and shear-flow instabilities.

3. Numerical simulations

In order to investigate the nonlinear evolution of the fields described by the 4-field model, the equations (1)-(4) have been solved numerically on a rectangular domain of 1024×1024 gridpoints with periodic boundary conditions. The chosen initial equilibrium is given by $\psi = \mathrm{sech}^2(x)$, $Z = 0$, $v = 0$, $\varphi = 0$ and is known to be unstable to reconnecting perturbations [22]. The perturbation applied to the equilibrium current density and is of the form $\delta j(x)\cos(2\pi m y/L_y)$, where $\delta j(x)$ is a function localized around the reconnection layer centered at $x = 0$ and with a width of order d_e, whereas m is an integer specifying the mode of the perturbation and L_y is the length of the simulation box along the y direction. For these simulations the mode $m = 1$, which is the most unstable, has been chosen.

The evolution of the magnetic field in the simulations, on macroscopic scales, is qualitatively very similar to that observed in previous simulations (e.g. [23], although initialized with a different equilibrium magnetic field): the field lines reconnect around the neutral line $x = 0$ forming a magnetic island whose width grows in time until a saturation phase is reached and a macroscopic steady state follows.

If, on the other hand, we focus on the evolution of the vorticity, after the magnetic island has reached saturation, we observe that two vertical vortex sheets form along the $x = 0$ line and move toward each other (see Fig. 1). These vortex sheets eventually collide and form pairs of vortices that subsequently move in opposite directions along the $y = 0$ axis. At a later stage, the vertical vortex sheets are seen to become unstable to a Kelvin-Helmholtz type instability [11]. This feature was already observed in simulations of a low-β 2-field model for cold electrons [25, 26]. However, the novel feature of the 4-field model is that the process of vortex sheet formation, and its subsequent destabilization, occurs in the presence of filamented structures, (enclosed in the magnetic island and most visible at $t = 35$), which are suppressed in the 2-field model, in the absence of electron compressibility. These structures are formed as a consequence of the stretching of the fields T_+ and T_-, advected by the "velocity" fields associated with φ_+ and φ_-, which rotate in opposite directions (a similar mechanism was described in Ref. [24] for a 2-field model). Indeed, the vorticity field U can be written as

$$U = \frac{T_+ - T_-}{2 d_e d_\beta} + \omega. \tag{42}$$

This expression shows that the vorticity can be decomposed into two contributions: the first due to the difference between the fields T_+ and T_- and the second coming from ω. The former is responsible for the presence of the filamented structures, whereas the latter (which has no counterpart in the 2-field model with electron compressibility) gives rise to the vortex sheet formation and their subsequent dynamics [12]. Note that this coexistence of filamented and vortex-sheet-like structures, observed in the 4-field model, occurs

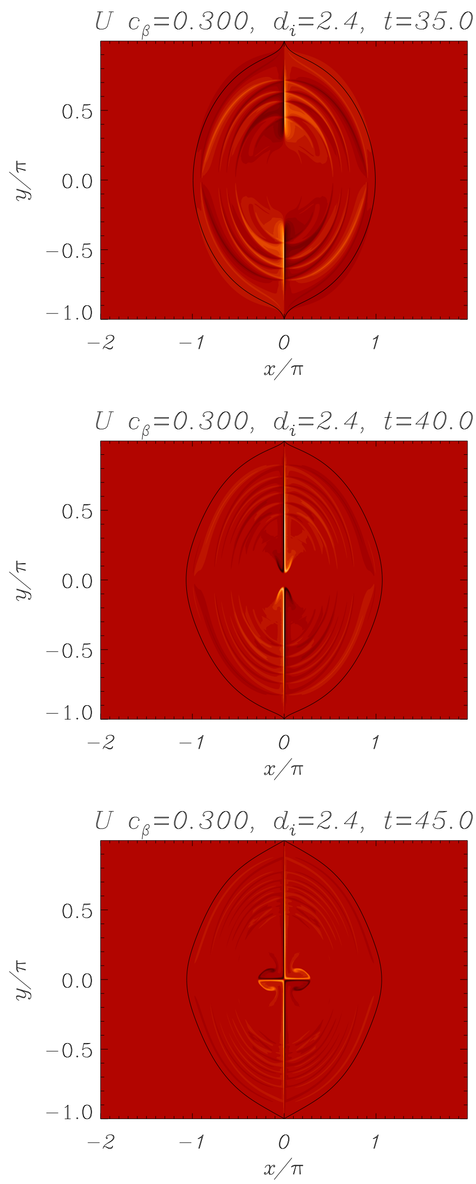

Fig. 1 Contour plots at $t = 35, 40, 45$ of the parallel vorticity field U for $c_\beta = 0.3$, $d_\beta = 0.72$ and $d_e = 0.24$.

only in a regime of non-negligible β. Indeed, for $\beta \simeq 0$, one consistently retrieves either the vortex-sheet-like structures or the vorticity filamentation, depending on whether electron compressibility is negligible or not.

Finally we would like to note that the onset of Kelvin-Helmholtz-like instability, that causes the breaking of the vortex sheet structures, is sensitive not only to the values of β and to the electron compressibility but also to the value of the electron skin depth.

Indeed, by looking at Fig. 2, which refers to a lower value of d_e, one can see that, although the central vortex pair just formed as result of the vortex sheet collisions, the two vertical structures in the vicinity of the filamented regions, already broke up, as a consequence of the instability. The more rapid

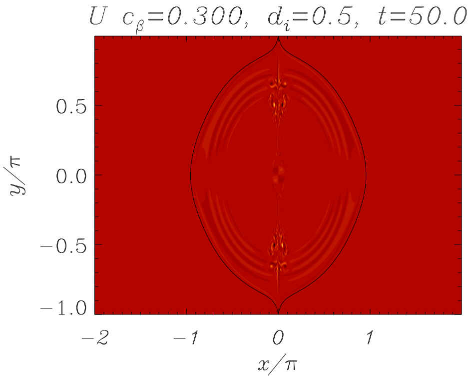

Fig. 2 Contour plot at $t = 50$ of the parallel vorticity field U for $c_\beta = 0.3$, $d_\beta = 0.15$ and $d_e = 0.05$.

occurrence of the instability as a consequence of a decrease in the value of d_e was also observed in the 2-field model [26] and can be ascribed to the consequent thinning of the vertical vorticity layers, whose width grows with d_e. Thinner layers, indeed, imply stronger velocity gradients, which favour the onset of the Kelvin-Helmholtz-type instability.

4. Conclusions

In this contribution some properties of a 4-field model for describing magnetic reconnection in collisionless plasmas have been analyzed analytically and numerically. After reviewing the Hamiltonian structure of the model equations, a variational principle has been adopted to derive equilibrium equations that allow the classification of equilibria in terms of different choices of the Casimir functionals of the model. The second variation of the free energy functional has been used in order to derive sufficient conditions for stability of generic relative equilibria. These conditions consistently coincide with those derived in Ref. [10] when the specific case of homogeneous equilibria is considered.

Numerical simulations have been used in order to carry out a qualitative analysis of the parallel vorticity dynamics. Coexistence of vortex-sheet-like and filamented structures in the contour plots of U at finite β has been shown. This coexistence, prevented in previously investigated low-β 2-field models, is explained in terms of the decomposition of the vorticity field into a first component, related to the Lagrangian invariants T_+ and T_-, that is responsible for the filamentation on small scales, and to a second component, corresponding to the field ω, that accounts for the vortex sheet formation and dynamics. The Kelvin-Helmholtz-type instability that causes the breaking of the vortex sheet structures is seen to occur more rapidly when decreasing the value of d_e, similarly to what happens in the low-β limit in the absence of electron compressibility. This behavior can be ascribed to the steepening of the velocity gradients that occurs when d_e is decreased and that accelerates the destabilization process.

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