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COMMENT

Comment on 'Magnetic topology effects on Alcator C-Mod scrape-off layer flow'

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In their recent paper [1] Simakov *et al* (hereafter referred to as the authors) draw attention to certain differences between their work and mine [2] and claim that my results contradict an earlier work by Cohen and Ryutov [3] while theirs are in agreement, thus questioning the validity of my work. The authors are wrong in their assertion. My work is correct and in agreement with the relevant portions of Cohen and Ryutov's. There are, however, serious errors in the two primary authors' earlier work [4] and its erratum [5] (the erratum itself is in error) on which this paper [1] is based.

The subject here is the symmetry properties of tokamak plasmas under various transformations, in particular the question of what happens to the plasma flows when the toroidal field is reversed. For a magnetohydrodynamic (MHD) flow in an axisymmetric system (an assumption made by all works referenced in this comment[1]) my initial, boundary value calculations using the CTD code (see [2] and the references therein) show that the toroidal field reversal leads to reversal of the toroidal flow, whereas the authors claim that the poloidal flow should reverse.

In order to determine the correct symmetry under reversal of the toroidal field, let us examine a representative subset of equation (11) from Cohen and Ryutov [3].

$$
\begin{aligned}
\frac{\mathrm{d}\rho}{\mathrm{d}t} &= -\rho \nabla \cdot \boldsymbol{v}, \\
\rho \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} &= \boldsymbol{J} \times \boldsymbol{B}, \\
\frac{\partial \boldsymbol{B}}{\partial t} &= \nabla \times \boldsymbol{v} \times \boldsymbol{B}.
\end{aligned}
\tag{1}
$$

Using the cylindrical coordinate system (R, ϕ, Z) as in [3], and letting $B_\phi \to -B_\phi$, we get $J_R \to -J_R$, $J_\phi \to +J_\phi$, $J_Z \to -J_Z$ and $(\boldsymbol{J}\times\boldsymbol{B})_R \to +(\boldsymbol{J}\times\boldsymbol{B})_R$, $(\boldsymbol{J}\times\boldsymbol{B})_\phi \to -(\boldsymbol{J}\times\boldsymbol{B})_\phi$, $(\boldsymbol{J}\times\boldsymbol{B})_Z \to +(\boldsymbol{J}\times\boldsymbol{B})_Z$. Then the momentum equation leads $v_R \to +v_R$, $v_\phi \to -v_\phi$, $v_Z \to +v_Z$, i.e. the toroidal velocity is reversed, leaving the poloidal components intact. These transformations have to be consistent with the Maxwell equations (Faraday's law) also. We see that $E_R = -(\boldsymbol{v}\times\boldsymbol{B})_R \to -E_R$, $E_\phi \to +E_\phi$, $E_Z \to -E_Z$, which leaves the poloidal components of $\partial\boldsymbol{B}/\partial t$ invariant while flipping the sign of the toroidal component, as expected. Thus, this set of equations is invariant under the transformation $(B_\phi \to -B_\phi, v_\phi \to -v_\phi)$.

[1] Cohen and Ryutov assume axisymmetry of the external fields and structures but not of the plasma dynamics.

It is trivial to show that this is a symmetry, not just of this subset, but also of the full set of equations in equation (11) of Cohen and Ryutov. Note also that the derivation of this symmetry involved no geometric assumptions other than that of axisymmetry; thus, it is valid both for up–down symmetric double-null and asymmetric single-null magnetic topologies.

Now let us examine the authors' symmetry claim, $(B_\phi \to -B_\phi, v_R \to -v_R, v_Z \to -v_Z)$. It is trivial to show that this is *not* a symmetry of the MHD equations, either the full set in equation (11) of [3] or any of its meaningful subsets that include the momentum equation and Faraday's law, such as equation (1). The momentum equation still leads to $v_\phi \to -v_\phi$, and Faraday's law now requires the poloidal fields to reverse, inconsistent with the assumptions.

Since the authors' claim of being in agreement with Cohen and Ryutov seems to be based on their interpretation of Cohen and Ryutov results for the Darwin Lagrangian and the kinetic models, let us examine these in a little more detail.

Reproducing the Darwin Lagrangian from [3], we have

$$L = \sum_a \frac{m_a v_a^2}{2} - \sum_{a>b} \frac{e_a e_b}{r_{ab}} + \sum_{a>b} \frac{e_a e_b}{2c^2 r_{ab}} \left[\boldsymbol{v}_a \cdot \boldsymbol{v}_b + \frac{(\boldsymbol{v}_a \cdot \boldsymbol{r}_{ab})(\boldsymbol{v}_b \cdot \boldsymbol{r}_{ab})}{r_{ab}^2} \right] + \sum_a \frac{e_a}{c} (\boldsymbol{A}_0 \cdot \boldsymbol{v}_a). \tag{2}$$

The external axisymmetric vector potential in cylindrical coordinates (R, ϕ, Z) is assumed to be of the form

$$\boldsymbol{A}_0 = [0, A_{0\phi}(R, Z), A_{0Z}(R)]. \tag{3}$$

Summations are over all particles, and $e_a, \boldsymbol{r}_a, \boldsymbol{v}_a$ are the charge, radius-vector and velocity of a particle a, respectively.

Now let us examine the symmetries of this system under toroidal field reversal or $A_{0Z} \to -A_{0Z}$. As pointed out by Cohen and Ryutov [3], this Lagrangian is invariant under the transformation $(A_{0Z} \to -A_{0Z}, v_Z \to -v_Z, Z \to -Z)$, if we also assume up–down symmetry of the external fields, $A_{0\phi}(R, Z) = A_{0\phi}(R, -Z)$. Note that there is *no* invariance without the reversal of the coordinate Z. In order to reverse the diamagnetic/paramagnetic contributions to the toroidal field, all poloidal currents must be reversed, not just the vertical components. Thus, we need to supplement this transformation with $v_R \to -v_R, R \to -R$, with the understanding that the latter is equivalent to a rotation by π, i.e. $(R \to R, \phi \to \phi + \pi)$. Because of the assumed axisymmetry of the external fields, this rotation has a null effect. Thus, the full symmetry operation is $(A_{0Z} \to -A_{0Z}, v_Z \to -v_Z, Z \to -Z, v_R \to -v_R, R \to -R)$. The authors' transformation $(B_\phi \to -B_\phi, v_Z \to -v_Z, v_R \to -v_R)$ is not equivalent to this, and it is not a symmetry of this Lagrangian and the associated Euler–Lagrange equation. Since there seems to be some confusion in Cohen and Ryutov about this, it is instructive to see what this set of transformations accomplishes for an up–down symmetric tokamak:

- $Z \to -Z$ flips the tokamak upside down. Since it is up–down symmetric, we now have the same tokamak but with all flows, currents and fields reversed.
- $A_{0Z} \to -A_{0Z}$ reverses back the external toroidal field.
- $v_R \to -v_R, v_Z \to -v_Z$. These reverse back the poloidal flows and currents, thus also reversing the diamagnetic/paramagnetic modifications of the toroidal field.

At the end of these transformations, the new state differs from the original up–down symmetric tokamak in that only the toroidal flows and toroidal current are reversed, i.e. these transformations do not represent a new symmetry. With the up–down symmetry caveat, they are equivalent to another well-known symmetry, invariance under reversal of the toroidal flows and toroidal current $(A_{0\phi} \to -A_{0\phi}, v_\phi \to -v_\phi, \phi \to -\phi)$, which is valid for all three systems discussed by Cohen and Ryutov, without the up–down symmetry requirement [3].

Finally, let us examine the kinetic (Vlasov) model. Again reproducing from Cohen and Ryutov their equation (6) without the collision and source terms, we have

$$\frac{\partial f_\alpha}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f_\alpha}{\partial \boldsymbol{r}} + \frac{e_\alpha}{m_\alpha} \left(\boldsymbol{E} + \frac{1}{c} \boldsymbol{v} \times \boldsymbol{B} \right) \cdot \frac{\partial f_\alpha}{\partial \boldsymbol{v}} = 0. \tag{4}$$

The transformation ($B_\phi \to -B_\phi, v_R \to -v_R, v_Z \to -v_Z$) leads to $(\boldsymbol{v} \times \boldsymbol{B})_R = v_\phi B_Z - v_Z B_\phi \to +(\boldsymbol{v} \times \boldsymbol{B})_R$, $(\boldsymbol{v} \times \boldsymbol{B})_\phi \to -(\boldsymbol{v} \times \boldsymbol{B})_\phi$, $(\boldsymbol{v} \times \boldsymbol{B})_Z \to +(\boldsymbol{v} \times \boldsymbol{B})_Z$. Thus if the electric field $\boldsymbol{E}$ also has the same symmetry, $E_R \to +E_R$, $E_\phi \to -E_\phi$, $E_Z \to +E_Z$, then the last term in the equation changes sign. If the distribution function is axisymmetric, then the second term also flips sign. But the first term, the time derivative, does not change; thus, the equation is not invariant under this transformation, irrespective of whether the system is up–down symmetric or not. As mentioned earlier in the MHD discussion, a second problem with this transformation is that this set of transformations of the electric and magnetic fields is not consistent with Faraday's law, $\partial \boldsymbol{B}/\partial t = -\nabla \times \boldsymbol{E}$.

From the above discussion it is clear that the symmetry claimed by the authors is not valid for any of the time-dependent models in Cohen and Ryutov [3]. However, steady-state conditions may exhibit more symmetry than those evolving in time, and the authors' symmetry *is* valid for the time-*in*dependent Vlasov and MHD equations, although not for the particle system with the Darwin Lagrangian. The Authors may have assumed that symmetry for a time-independent model is sufficient because they apply their results only to systems in equilibrium. But this assumption is incorrect for the following reason: we do these symmetry analyses not in the abstract but to understand tokamak behavior and answer specific questions like 'What happens to the discharge if we reverse the toroidal field?' Two discharges that start with the same initial and boundary conditions, except for those differences that lead to $+B_\phi$ in one and $-B_\phi$ in the other (e.g. reversal of the current in the toroidal field coils) will evolve towards two different equilibrium states. To the extent that MHD describes the evolution and the final state of that plasma, those two equilibria will have opposite toroidal flows (assuming of course there are no external momentum sources). However, since reversal of the poloidal flow is not a symmetry of any of the time-dependent models, it will not be part of the dynamics and a final state with that symmetry will not be accessible from those initial conditions, even though the equilibrium equations do allow that symmetry. In short, if the intermediate (time-varying) path connecting the initial and final states does not allow the symmetry, you cannot reach an equilibrium with that symmetry starting from those initial conditions.

In summary, the symmetry that I observe in my numerical calculations with the CTD code [2], $B_\phi \to -B_\phi$, $v_\phi \to -v_\phi$, is correct and in agreement with Cohen and Ryutov's results on MHD symmetries [3]. On the other hand, the symmetry claimed by Simakov *et al* [1] and Catto and Simakov [4, 5] ($B_\phi \to -B_\phi, v_Z \to -v_Z, v_R \to -v_R$) is incorrect since it is not a valid symmetry of *any* of the time-dependent models discussed by Cohen and Ryutov, making it irrelevant for making predictions about the behavior of tokamak discharges.

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