Escaping Radio Emission from Pulsars

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Abstract

It is demonstrated that the velocity shear, intrinsic to the e^+e^- plasma present in the pulsar magnetosphere, can efficiently convert the nonescaping longitudinal Langmuir waves (produced by some kind of a beam or stream instability) into propagating (escaping) electromagnetic waves. It is suggested that this shear induced transformation may be the basic mechanism needed for the eventual generation of the observed pulsar radio emission.

It is generally believed that the radio emission from a pulsar has its origin in the processes occurring in its magnetospheric plasma. These processes generate a variety of waves some of which propagate out of the magnetosphere, travel through the interstellar medium and are seen as radio emission by a distant observer (Ginzburg and Zheleznyakov 1970). Over the years, several different models for the pulsar radio emission (Ginzburg and Zheleznyakov 1975; for the most recent and comprehensive review see, e.g., Melrose 1995) have been suggested, and certain aspects of the phenomenon, like the polarization properties of the emission, are rather well understood (Kazbegi *et al.* 1991; Kazbegi, Machabeli and Melikidze 1991; Kazbegi *et al.* 1996). However, there are still many unanswered questions. One of the most significant and puzzling problems is the delineation of a satisfactory mechanism for the conversion of potential waves (like the Langmuir waves), readily generated in the magnetosphere, into escaping radio waves. In this *letter* we propose that the velocity shear inherent in the magnetospheric e^+e^- plasma can provide the desired conversion mechanism; this may lead to a more comprehensive theory for the generation of the observed radio emission.

The relativistic medium forming a pulsar magnetosphere has two main constituents: an ultrarelativistic (primary) beam, and a relativistic (secondary) e^+e^- plasma, created via the *pair cascade process* (Sturrock 1971). It is generally surmised that the original source of pulsar radio emission lies in the differential dynamics of these constituents. What is needed, however, is an acceptable scenario for the generation and escape of the electromagnetic radiation.

The first step in this process, perhaps, is the excitation of Langmuir waves by some kind of a beam or two-stream instability (Ruderman and Sutherland 1975; Cheng and Ruderman 1980; Asseo, Pellat, and Rosado 1980; Asseo, Pellat, and Sol 1983). Initially, the instability was attributed to the primary ultrarelativistic electron or positron beam. However, the beam has too low a density and too large a Lorentz factor, so that the characteristic growth time turns out to be a few times more than the time needed for the beam particles to escape the pulsar magnetosphere (Benford and Buschauer 1977; Egorenkov, Lominadze, and Mamradze 1983). In order to overcome this difficulty Usov (1987) (see also Ursov and Usov 1988) suggested the interesting idea of a *nonstationary plasma flow.* According to this model clouds of e^+e^- plasma are injected into the pulsar magnetosphere from time to time (with small enough intervals). Fast particles from the following clump overtake slower ones from the preceding clump creating favorable conditions for the development of a two-stream instability leading to the generation of Langmuir waves propagating along the magnetic field lines. In this model the instability is attributed to the dense and low Lorentz factor e^+e^- plasma, and its growth rate is found to be large enough. Thus it appears that by either Usov's or through an alternative mechanism, it should be possible to produce Langmuir waves of sufficient intensity.

The second crucial step in the development of a model for understanding the observed radio emission is to pinpoint a mechanism(s) which will convert the energy "accumulated" in the longitudinal Langmuir waves into the energy of such waves that can escape out of a pulsar magnetosphere.

There seem to be a variety of physical processes which could mediate mode conversion: induced wave scattering (Machabeli 1983), wave-wave interaction (Gedalin and Machabeli 1983; Mamradze, Machabeli, and Melikidze 1980), and *mode couplings due to some kind of a plasma inhomogeneity* (Melrose 1995). In the latter class, however, an extremely important inhomogeneity, i.e., the inhomogeneity of the velocity field (velocity *shear*) has, until recently, attracted very little attention (Scharlemann, Arons, and Fawley 1978; Arons and Scharlemann 1979; Kaladze and Mamradze 1984) inspite of the fact that Arons and Smith (1979) had, long ago, outlined a basic mechanism of an electrostatic instability of a sheared stream of charged particles flowing along a strong magnetic field. They conjectured that the energy may be liberated indirectly "through coupling of electrostatic modes generated by the instability to propagating electromagnetic modes." (Arons and Smith 1979, p. 728).

In this *letter* we intend to prove that this hypothesis of Arons and Smith is highly plausible. In particular, we shall demonstrate that the velocity shear of the relativistic e^+e^- plasma flow can mediate an efficient conversion of the longitudinal, nonescaping waves (Langmuir waves) into the desired electromagnetic waves which can propagate outwards.

This physical mechanism underlying mode conversion has been recently explored by Chagelishvili, Rogava and Tsiklauri (1996). It was shown that in inhomogeneous plasmas, which can sustain a variety of wave motions, the velocity shear can induce a mutual transformation of the waves with corresponding energy transfer between them. This phenomena is quite general, and is found to be efficient in a number of different physical situations (see, e.g., Chagelishvili and Chkhetiani 1995; Rogava and Mahajan 1996; Rogava, Mahajan, and Berezhiani 1996).

We consider a collisionless, viscosity-free and *cold* e^+e^- plasma. Following Arons and Smith (1979), we neglect the plasma pressure, and model the flow by the following, relativistic two-fluid equations:

$$\partial_t n^{\pm} + \nabla \cdot \left(n^{\pm} \mathbf{V}^{\pm} \right) = 0, \tag{1}$$

$$\left[\partial_t + (\mathbf{V}^{\pm}, \nabla) \right] \mathbf{P}^{\pm} = \pm e \left(\mathbf{E} + \mathbf{V}^{\pm} \times \mathbf{B} \right), \tag{2}$$

$$\nabla \cdot \mathbf{E} = 4\pi e [n^+ - n^-], \tag{3}$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \tag{4}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{5}$$

$$\nabla \times \mathbf{B} = 4\pi e [n^+ \mathbf{V}^+ - n^- \mathbf{V}^-] + \partial_t \mathbf{E}. \tag{6}$$

where the notation is standard with the speed of light taken to be unity. The equilibrium velocity of electrons and positrons in the sheared stream will be modelled by

$$\mathbf{V}_0^{\pm} \equiv \mathbf{V}_0 = \{U_0 + Ay,\ 0,\ 0\}, \tag{7}$$

where A measures the strength of the shear. It will be assumed that the stream is weekly sheared in the sense that Ay is much smaller than the average part U_0. The resulting momentum becomes

$$P_{x0}(y) \simeq P_0 + ay, \tag{8}$$

where $a \equiv mA\gamma_0^3$, $P_0 \equiv m\gamma_0 U_0$ and $\gamma_0 \equiv (1-U_0^2)^{-1/2}$ is the average Lorentz factor. In our model, the equilibrium velocities of electrons and positrons are equally sheared ($A^+ = A^- = A$) and there is no mutual streaming of the two species ($\gamma_0^+ = \gamma_0^-$).

Let us place the x axis along the magnetic axis of the pulsar dipole magnetic field. For perturbations with characteristic parallel (to $\mathbf{B}_0$) length scale small compared to $(r-R_*)/3$, we can take $\mathbf{B}_0 = \text{const} = B_{x0}$ (Arons and Smith 1979). Here r is a radial distance from the stellar center and R_* is the stellar radius. The plasma is assumed to be quasineutral ($n_0^\pm \equiv n_0$) with an equilibrium (one fluid) mass density $\rho_0 \equiv 2mn_0$.

In order to delineate the basic features of shear-induced mode conversion, we shall assume that the equilibrium magnetic field is infinite in strength. The superstrong magnetic fields of pulsars readily allow this simplifying assumption. In the super strong field the e^+e^- plasma becomes a quasi-one-dimensional system because it is forced to move strictly along the field lines. In other words, $\mathbf{B}_0$ is assumed to be so strong that any perpendicular momentum is rapidly lost to synchrotron radiation. We can, therefore, neglect perpendicular dynamics altogether and deal with the perturbed motion only along the x axis (zeroth order guiding centre approximation).

Further simplification is attained by restricting the wave vectors of the perturbations to lie in the X-Y plane (the plane defined by the direction of $\mathbf{B}_0$ ($\mathbf{U}_0$) and by the direction of the velocity shear). We consider, from now, the *ℓt waves* for which the electric field vector $\mathbf{E}$ lies in the X-Y plane, and the magnetic field perturbation is constrained to be along the z-axis, $\mathbf{B} = \{0\ ,0\ ,B_z\}$.

Within the framework of the preceding discussion, the relevant motion of the magnetospheric plasma can be described by the following set of linearized equations:

$$D_t\rho_q + \partial_x J_x = 0, \tag{9}$$

$$D_t J_x = (\omega_p^2/4\pi\gamma_0^3)E_x, \tag{10}$$

$$\partial_x E_x + \partial_y E_y = 4\pi\rho_q, \tag{11}$$

$$\partial_t B_z = \partial_y E_x - \partial_x E_y, \tag{12}$$

$$\partial_t E_y = -\partial_x B_z, \tag{13}$$

where $D_t \equiv \partial_t + (U_0 + Ay)\partial_x$, $\omega_p^2 \equiv 8\pi e^2 n_0/m$, and we have used the *one fluid* variables: $\rho_q \equiv e(n^+ - n^-)$, the perturbed charge density, and $\mathbf{J} \equiv en_0(\mathbf{u}^+ - \mathbf{u}^-)$, the perturbed current density.

Note that (12)–(13) contain the usual time derivative, while in (9)–(10) we have the convective derivative D_t. Since it is assumed that $Ay \ll U_0$ we can approximate $\partial_t \simeq D_t - U_0\partial_x$. The advantage of the resulting system is that it may be handled by the standard method of "Kelvin modes" (see, e.g., Marcus and Press 1977, Criminale and Drazin 1990). This method requires the change of variables:

$$x_1 = x - (U_0 + Ay)t; \quad y_1 = y; \quad t_1 = t \tag{14}$$

that leads to a substantial simplification in the solution of the initial-value problem. The differential operators, appearing in the above equations, are so transformed,

$$D_t \equiv \partial_t + (U_0 + Ay)\partial_x = \partial_{t_1}, \tag{15a}$$

$$\partial_x = \partial_{x_1}, \tag{15b}$$

$$\partial_y = \partial_{y_1} - At_1\partial_{x_1}, \qquad (15c),$$

that an initial inhomogeneity in space (y) is exchanged for a new inhomogeneity in time. The Fourier transform in the new spatial variables,

$$F = \int dk_{x_1} dk_{y_1} \hat{F}(k_{x_1}, k_{y_1}, t_1) \exp[i(k_{x_1}x_1 + k_{y_1}y_1)], \tag{16}$$

now converts (9)–(10) and (12)–(13) to a set of first order, ordinary differential equations (ODE's) for the evolution of the spatial Fourier harmonics (SFH) (see, e.g., Chagelishvili, Rogava, and Segal 1994). The wave vector components may also be written in the original $(x\ ,y\ ,t)$ coordinates: $k_x = k_{x_1}$ and $k_y(t) = k_{y_1} - Atk_{x_1}$. It is of principal importance to note that the velocity shear induces linear drifts of SFH so that initially longitudinal modes can become eventually oblique.

By introducing the dimensionless quantities: $\mathcal{D} \equiv \widehat{\rho}_q/en_0$, $J \equiv \widehat{J}_x/en_0$, $e_{x,y} \equiv (k_{x_1}/en_0)\widehat{E}_{x,y}$, $b_z \equiv (k_{x_1}/en_0)\widehat{B}_z$, $\sigma \equiv \omega_p^2/4\pi k_{x_1}^2$, $\tau \equiv k_{x_1}t_1$, $R \equiv A/k_{x_1}$, $\beta_0 \equiv k_{y_1}/k_{x_1}$, $\beta(\tau) \equiv \beta_0 - R\tau$, we can reduce the original system to the following complete set of dimensionless equations:

$$\partial_\tau \mathcal{D} = -iJ, \tag{17}$$

$$\partial_\tau J = -(2\sigma/\gamma_0^3)[4i\pi\mathcal{D} + \beta(\tau)e_y], \tag{18}$$

$$(\partial_\tau - iU_0)e_y = -ib_z, \tag{19}$$

$$(\partial_\tau - iU_0)b_z = -i[1 + \beta^2(\tau)]e_y + 4\pi\beta(\tau)\mathcal{D}, \tag{20}$$

where we have eliminated e_x by using the dimensionless form of (11): $e_x + \beta(\tau)e_y = -4\pi i\mathcal{D}$.

The system (17)–(20) describes the temporal evolution of SFH of the obliquely propagating ℓt-modes in the relativistic e^+e^- plasma.

In this letter we shall investigate the evolution of those modes for which the initial perturbations are purely longitudinal ($|k| = k_x$ and $\beta_0 = 0$). This is indeed the most important case because purely longitudinal Langmuir waves are the easiest to excite in a pulsar magnetosphere. For further analysis, it is convenient and revealing to combine (17)–(20) to obtain two equations for the variables $E(\tau) \equiv e^{-iU_0\tau}e_y$ and $D(\tau) \equiv -4\pi i\mathcal{D}$,

$$\partial_\tau^2 D + W^2 D = -W^2 R\tau e^{iU_0\tau}E, \tag{21}$$

$$\partial_\tau^2 E + (1 + R^2\tau^2)E = -R\tau e^{-iU_0\tau}D, \tag{22}$$

where $W^2 \equiv 8\pi\sigma/\gamma_0^3$.

It is now apparent that in the absence of shear ($R = 0$), (21) and (22) decouple and describe purely potential, longitudinal Langmuir oscillations (with phase velocity $\omega/k = W$), and purely transverse electromagnetic waves (with phase velocity $\omega/k = 1$), respectively.

It is equally clear that the shear ($R \neq 0$) couples these two modes with each other. The system of Eqs. (21)–(22) is mathematically equivalent to a system of coupled linear oscillators with eigenfrequencies $\omega_1 = W$ and $\omega_2 = [1 + R^2\tau^2]^{1/2}$, and coupling coefficients $c_1 = W^2 R\tau e^{iU_0\tau}$ and $c_2 = R\tau e^{-iU_0\tau}$. Both of the coupling coefficients and one of the eigenfrequencies vary in time. The time dependence is entirely due to the nonzero shear, and is slow or *adiabatic* for $R \ll 1$.

Though, the physical meaning of Eqs. (21)–(22) is transparent enough, it is instructive to look at some representative solutions. In Fig. 1, we plot the functions $e_x(\tau)$, $e_y(\tau)$, $b_z(\tau)$, and $e_y + R\tau e_x$ (the latter function measures in dimensionless notations value of $\nabla \times \mathbf{E}$) obtained by a numerical integration of the defining equations. For this case, the values of parameters are $R = 4 \times 10^{-2}$, $\sigma = 1$, $\gamma_0 = 10$, and the initial perturbation is taken to be a pure longitudinal Langmuir wave ($e_x(0) \neq 0$, while $e_y = b_z = 0$). We see that as time progresses, the fields $e_y(\tau)$ and $b_z(\tau)$ are excited and the wave becomes more and more nonpotential ($e_y + R\tau e_x$ is increasing). In other words, the initial perturbation (longitudinal and purely potential Langmuir wave) begins to acquire transverse "features" as it evolves.

It would seem that we have now identified both pieces of the puzzle: 1) a reasonable mechanism (some kind of a beam or stream instability) for generating longitudinal, potential Langmuir waves (with $\mathbf{k}(0) \| \mathbf{B}_0$) in the e^+e^- plasma in the pulsar magnetosphere, and 2) an effective shear induced coupling to transform these nonescaping waves into the longitudinal-transversal, nonpotential waves which are perfectly capable of escaping the stellar environment.

We now propose a comprehensive model. We do this by incorporating Usov's (1987) nonstationary injection hypothesis into our model. Let us now consider two streams of e^+e^- plasma with average Lorentz factors γ_1, and γ_2 ($\gamma_2 > \gamma_1$). The problem reduces to the following set of *three*, second order coupled ODE's:

$$\partial_\tau^2 D_1 + W_1^2(D_1 + D_2) = -W_1^2 R\tau e^{iU_1\tau} E, \tag{23}$$

$$(\partial_\tau + i\Delta U)^2 D_2 + W_2^2(D_1 + D_2) = -W_2^2 R\tau e^{iU_1\tau} E, \tag{24}$$

$$\partial_\tau^2 E + (1 + R^2\tau^2)E = -R\tau e^{-iU_1\tau}(D_1 + D_2), \tag{25}$$

where $\Delta U \equiv U_2 - U_1$, $W_1^2 \equiv 8\pi\sigma/\gamma_1^3$, and $W_2^2 \equiv 8\pi\sigma/\gamma_2^3$.

In this case the equations explicitly encompass both of the essential processes leading to the pulsar radio emission: The onset and amplification of Langmuir oscillations due to a built-in two-stream instability, and the subsequent conversion of these oscillations into escaping radiation. Corresponding plots are presented on Fig. 2 for two streams with $\gamma_1 = 10$ and $\gamma_2 = 10^2$ ($\sigma = 1$, as above). Figures 2(a) and 2(c) represent the zero shear case ($R = 0$), while for Figs. 2(b) and 2(d), $R = 2 \times 10^{-3}$. In the former case, the two-stream instability is "switched on," and the amplitude $e_x(\tau)$ increases with time. But $e_y(\tau) = 0$ for all times, and the wave remains potential. In the latter case, however, the presence of nonzero shear changes the situation drastically: the wave becomes nonpotential and the electromagnetic component $e_y(\tau)$ is strongly excited.

The transformation of purely longitudinal, non-propagating modes into the electromagnetic waves is just one of the many mode transformation process that can actually happen in the magnetospheric plasma. Velocity shear induced mode conversion is a very general phenomenon, and is likely to play an important part in the overall dynamics of the pulsar plasma. For example, Rogava, Mahajan, and Berezhiani (1996) recently showed that shear-mediated efficient mutual transformation can take place between the shear and compressional Alfv́en modes in cold, nonrelativistic as well as weakly sheared relativistic plasmas. A detailed and comprehensive paper dealing with the interactions of various linear waves sustained by an e^+e^- plasma (see, for review, e.g., Volokitin, Krasnoselskikh, and Machabeli 1983; Lominadze *et al.* 1986; Lyubarsky 1995) is under preparation.

In summary, we have demonstrated in this *letter* that the mode coupling induced by velocity shear could be a vital link in the chain of processes which must be invoked in order to solve the puzzle behind the pulsar radio emission. We must also remember that one of the most severe criterion, imposed on possible pulsar radio emission models is that the *bona fide* mechanism must apply to both the weak-B_0 (millisecond) and the strong-B_0 (young, fast) pulsars (Melrose 1995). In other words this criterion demands that the true mechanism must apply in a range of four to five orders of magnitude in B_0. The velocity-shear based mechanism of mode conversion seems to be tailor-made to satisfy this requirement.

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FIGURE CAPTIONS

FIG. 1. Evolutionary plots ("one stream case") for real parts of $e_x(\tau)$ [(a)], $b_z(\tau)$ [(b)], $e_y(\tau)$ [(c)], and $e_y(\tau) + R\tau e_x(\tau)$ [(d)]. Time is measured in dimensionless units $\tau \equiv k_{x_1} t$. $R = 4 \times 10^{-2}$, $\sigma = 1$, and $\gamma_0 = 10$.

FIG. 2. Evolutionary plots (real parts of $e_x(\tau)$ and $e_y(\tau)$) for two streams of e^+e^- plasma with average Lorentz factors $\gamma_1 = 10$, and $\gamma_2 = 10^2$ ($\sigma = 1$). Figs. 2(a) and 2(c) are plotted for the "zero shear" ($R = 0$) case, while for Figs. 2(b) and 2(d), $R = 2 \times 10^{-3}$.