Effects of negative magnetic shear on toroidicity induced eigenmode instability in tokamaks

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Abstract

The effects of negative magnetic shear on the toroidicity induced (TI) eigenmode are studied with a numerical WKB shooting scheme. The ion temperature gradient (ITG or η_i), and the parallel velocity shear (PVS) of the ions are included. It is found that for a given set of plasma parameters, the negative magnetic shear causes much stronger damping than the positive shear of equal magnitude. It is also shown that PVS tends to destabilize the TI mode.

In the late 70s and early 80s, the magnetic shear induced damping was a major topic of investigation in the studies on the drift instability. Recent results from experiments on the Tokamak Fusion Test Reactor (TFTR)[1] show a considerable improvement in particle and energy confinement in the regions of negative magnetic shear. A possible implication is that the negative shear may have a strongly stabilizing influence on the instabilities responsible for the anomalous transport. The DIII-D[2] and the Joint European Torus (JET)[3] experiments

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have also shown results similar to TFTR's. These important experimental observations have led to a renewed interest in the role of magnetic shear in the instabilities occuring in toroidal plasmas.

It is well known from early work[4,5] that negative magnetic shear could exert a stabilizing influence on the ideal and dissipative ballooning instabilities. Recently, Kessel *et al.*[6] demonstrated that the reversed shear, resulting from bootstrap currents, in the core of a tokamak plasma has a surprisingly strong stabilizing impact on the non-ideal modes.

A physical picture showing the effect of negative shear on the curvature driven instabilities is given by Antonsen, Jr. *et al.*,[7] and the influence of the negative shear on resistive ballooning modes has been recently studied by Drake *et al.*[8] A systematic fluid and kinetic investigations of the ITG (ion temperature gradient) and PVS (parallel velocity shear) driven modes in plasmas with negative magnetic shear has also been recently performed.[9] It was demonstrated[9] that the instabilities peculiar to the toroidal geometry, especially those driven by the geodesic curvature of the magnetic lines, are quite sensitive to the sign of the magnetic shear. In contrast, the characteristics of the ITG and the PVS instabilities, in a sheared slab configuration, are quite insensitive to the sign of the magnetic shear.

It was found in early studies that although the standard slab-like drift wave was fully stabilized by magnetic shear (these studies were limited to positive magnetic shear), there exists in toroidal plasmas a toroidicity induced (TI) branch[10] which resisted shear stabilization. This branch, induced by the finite toroidal coupling, has no counterpart in the slab. Our recent experience (mentioned in the preceding paragraph) indicates that eigenmode instabilities, that are fundamentally toroidal, seem to be more sensitive to the sign of the magnetic shear. It is natural, therefore, to re-examine the nature and importance (its role in the confinement of tokamak plasmas, for example) of the TI modes when the magnetic shear is negative.

We present here a numerical study (using a WKB shooting code) of The TI modes in plasmas with negative magnetic shear. We employ a fluid model which is valid in the $\omega_{Di}/\omega \ll 1$ and $b_\theta \ll 1$ limit. Here, ω_{Di} and ω are respectively the ion toroidal drift, and the eigenmode frequencies, and b_θ (defined bellow) is proportional to the square of the poloidal wave number of the perturbations. Finite ion temperature gradients are taken into account. On DIII-D tokamak,[2] high toroidal velocity shear is observed in the regimes where the confinement improves with the negative magnetic shear. This important experimental observation is incorporated in the model by including an equilibrium PVS for the ions.

The toridicity induced mode is described by the following eigenmode equation for the perturbed electrostatic potential $\Phi(\kappa)$,[9]

$$\frac{d^2\Phi}{d\kappa^2} + \frac{\widehat{\omega}^2 q^2 b_\theta}{\epsilon_n^2} Q(\widehat{\omega}, \kappa)\Phi(\kappa) = 0 \tag{1}$$

with

$$Q = \frac{v_\parallel'^2}{4(\widehat{\omega}+K)^2} + \frac{(\widehat{\omega}-1)}{(\widehat{\omega}+K)} + b_\theta(1+\widehat{S}^2\kappa^2) + \frac{2\epsilon_n}{\widehat{\omega}}(\cos\kappa + \widehat{S}\kappa\sin\kappa), \tag{2}$$

where

$$K = \frac{(1+\eta_i)}{\tau}, \quad \widehat{\omega} = \frac{\omega}{\omega_{*e}}, \quad \tau = \frac{T_e}{T_i},$$
$$\widehat{S} = \frac{rdq}{qdr}, \quad \epsilon_n = \frac{L_n}{R}, \quad b_\theta = \widehat{k}_\theta^2\rho_s^2,$$
$$\rho_s = \frac{c_s}{\omega_{ci}}, \quad c_s = \left(\frac{T_e}{m_i}\right)^{\frac{1}{2}}, \quad v_\parallel' = \frac{L_n dv_\parallel}{c_s dr},$$

ω_{*e} is the electron diamagnetic drift frequency, ω_{ci} is ion gyro-frequency, R is the major radius of plasma column, r is the coordinate in the radial direction, L_n is the density scale length, and η_i is the ion temperature gradient parameter. The variable κ is the extended poloidal angle in ballooning representation, which can be regarded as the coordinate along field lines.

Notice that Eq. (1) is a generalization of similar equations used before. It reduces to Eq. (13) of Ref. 11 for $v_\parallel' = 0$. On removing the last term on the left hand side, representing the toroidal coupling effects, Eq. (1) is just the Fourier transform of Eq. (23) in Ref. 12, and

Eq. (7) in Ref. 13. A equation similar to but with a few differences from Eq. (1) is derived in Ref. 14.

Taking into account the parallel velocity shear $v'_{\|}$, the toroidal coupling (finite ϵ_n) and η_i effects, Eq. (1) is solved numerically with a shooting code. Numerically, it is more difficult to study the basic TI modes than the ITG and PVS modes. Without the ITG or the PVS effects, the TI modes are marginally stable. Even with these effects, the growth rates remain rather low. In order to benchmark our numerical results, we compare them with the results of Ref. 11 for positive shear. We calculate the eigenvalue (real frequency and growth rate) for the parameters $\epsilon_n = 0.1$, $b_\theta = 0.1$, $q = 1.0$, $\tau = \infty$, when $\widehat{S}$ varies from 0.2 to 3.0, and find an exact match with Fig. 6 of Ref. 11.

Unless stated otherwise, the following reference parameters are used throughout this study: $\epsilon_n = 0.1$, $b_\theta = 0.1$, $q = \tau = 1.0$ and $\eta_i = 1$.

The normalized growth rate and the real frequency of the mode are plotted in Figs. 1(a) and (b) as a function of the magnetic shear $\widehat{S}$ for two values of the toroidicity paramater: $\epsilon_n = .25$ and $.4$.

In Fig. 1(a), it is shown that the normalized growth rate of the modes for both values of ϵ_n decreases monotonically for $\widehat{S} > 0$. For $\widehat{S} < 0$, on the other hand, it initially increases and then monotonically decreases as the shear magnitude is increased. The growth rate is higher for $\epsilon_n = 0.4$ than it is for $\epsilon_n = 0.25$ for the same value of the magnetic shear, indicating that the mode is driven by toroidicity. The behavior of the growth rate versus magnetic shear are approximately symmetric with respect to the middle plane $\widehat{S} = 0$ for $|\widehat{S}| > 0.5$. However, the growth rate scaling is quite asymmetric for low (close to zero) magnitudes of the magnetic shear. For example, when $\epsilon_n = 0.25$, the growth rates are $\gamma/\omega_{*e} = 0.0135$ (0.006)for $\widehat{S} = 0.1(-0.1)$ are quite different. The real frequency in Fig. 1(b), however, does not show any such dramatic difference even in the neighborhood of zero shear. The mode rotates in the electron drift direction and the frequency increases with decreasing of the magnetic shear magnitude.

Figure 2, where the normalized growth rate and the real frequency are plotted versus ϵ_n (for $\widehat{S} = 0.1$ and -0.1), clearly shows that we are dealing with the toroidicity driven mode. The growth rate increases with ϵ_n approximately linearly as $\gamma \sim \alpha_1 \epsilon_n$ with $\alpha_1 = 0.08$ and 0.03 for $\widehat{S} = 0.1$ and -0.1, respectively. Not only the proportionality coefficient α_1 but also the overall value of the growth rate for $\widehat{S} = 0.1$ is higher than that for $\widehat{S} = -0.1$. For example, at $\epsilon_n = 0.15(0.4)$, γ/ω_{*e} is 0.0025 (0.011) for $\widehat{S} = -0.1$ while it is 0.005(0.026) for $\widehat{S} = 0.1$. The growth rate for $\widehat{S} = 0.1$ is more than twice of that for $\widehat{S} = -0.1$. The real frequency increases linearly with ϵ_n for both $\widehat{S} = 0.1$ and -0.1, and changes only slightly when the magnetic shear is reversed [Fig. 2(b)].

Figure 3 shows the effects of the ion temprature gradient (η_i) on the TI modes. In Ref. 11, it was pointed out (without any data) that the growth rate γ/ω_{*e} should have rather weak dependence on η_i for positive shear $\widehat{S} > 0$. That is, indeed, the case. The influence of η_i on the TI modes is even weaker for negative shear. In this case, the curves of γ/ω_{*e} versus η_i are almost completely flat. The real frequency [Fig. 3(b)] is reduced by the ion temperature gradient, as it should be. Again, the damping from negative magnetic shear is much stronger than it is from positive shear of same magnitude.

The TI modes in the presence of PVS were not studied in previous papers.[10,11] The growth rate γ/ω_{*e} versus parallel velocity shear $v'_{\parallel}$ is given in Figs. 4(a) ($\epsilon_n = 0.25$) and 4(b) ($\epsilon_n = 0.4$) for $\eta_i = 1$ and 3, and $\widehat{S} = 0.1$ and -0.1. PVS has negligible influence on the growth rate of the TI modes for $\eta_i = 3$. However, for smaller values of η_i, the PVS tends to destabilize the mode both for positive and negative small values of the magnetic shear. For $\widehat{S} = 0.1$ or -0.1, and $\eta_i = 1$, the growth rates at $v'_{\parallel} = 2$ are approximate twice of that at $v'_{\parallel} = 0$. The real frequency is pushed down by PVS in all cases and the results for $\epsilon_n = 0.4$ are presented in Fig. 4(c). Again, the PVS effects on the real frequency are negligibly small for $\eta_i = 3$ while they are significant for $\eta_i = 1$.

In summary, the toroidicity induced (TI) eigenmode instability is numerically studied in fluid plasmas with positive and negative magnetic shear. The ITG effects are included, and the PVS dynamics is introduced in the TI mode study for the first time.

It is found that for strong magnetic shear $|\widehat{S}| > 0.5$, the damping caused by the positive and negative shear of the same magnitude are comparable. However, for weak magnetic shear $|\widehat{S}| < 0.5$, the damping mechanism is much stronger and the TI mode growth rate, for given plasma parametrs, is much lower for the negative shear than it is for the positive shear of same magnitude. It is also demonstarted that for low shear (positive or negative) and low η_i, PVS destabilizes the TI mode.

We conclude the paper by reminding the readers that electrons are considered to be adiabatic in the derivation of Eq. (1). The TI modes are known to be destabilized by electron dissipation,[14] and for a more complete theory one must include these effects.

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FIGURE CAPTIONS

FIG. 1. (a) Mode growth rate and (b) real frequency as functions of magnetic shear (for both of $\widehat{S} > 0$ and $\widehat{S} < 0$) for $\epsilon_n = 0.4$ and 0.25. The other parameters are $q = \tau = \eta_i = 1$, $b_\theta = 0.1$ and $v_\parallel' = 0$.

FIG. 2. (a) Mode growth rate and (b) real frequency as functions of ϵ_n for $\widehat{S} = -0.1$ and 0.1. The other parameters are the same as Fig. 1.

FIG. 3. (a) Mode growth rate and (b) real frequency versus η_i for $\widehat{S} = -0.1$, 0.1 and $\epsilon_n = 0.4$, 0.25. The other parameters are $q = \tau = 1$, $b_\theta = 0.1$ and $v_\parallel' = 0$.

FIG. 4. Mode growth rate: (a) $\epsilon_n = 0.25$; (b) $\epsilon_n = 0.4$ and (c) real frequency ($\epsilon_n = 0.4$) versus $v_\parallel'$ for $\widehat{S} = -0.1$, 0.1 and $\eta_i = 1.0$, 3.0. The other parameters are $q = \tau = 1$, $b_\theta = 0.1$.