Collisionless Damping of Perpendicular Magnetosonic Waves in a Two-Ion-Species Plasma

Daiju Dogen and Mieko Toida

Department of Physics, Nagoya University, Nagoya 464-01, Japan

and

Yukiharu Ohsawa[a)]

Institute for Fusion Studies, The University of Texas at Austin

Austin, Texas 78712 USA

Abstract

Propagation of finite-amplitude magnetosonic waves in a collisionless plasma containing two ion species is studied with a one-dimensional, fully electromagnetic code based on a three-fluid model. It is found that perpendicular magnetosonic waves are damped in a two-ion-species plasma; a magnetosonic pulse accelerates heavy ions in the direction parallel to the wave front, which results in the excitation of a longer wavelength perturbation behind the pulse. The damping due to the energy transfer from the original pulse to the longer wavelength perturbation occurs even if the plasma is collisionless and the pulse amplitude is small. The theoretically obtained damping rate is in agreement with the simulation result.

Keywords: collisionless damping, magnetosonic wave, two-ion-species plasma, heavy ion acceleration, soliton

[a)]Permanent address: Department of Physics, Nagoya University, Nagoya, Japan

Recently it has been recognized that linear and nonlinear magnetosonic waves in a multi-ion-species plasma behave quite differently from those in a single-ion-species plasma.[1–3] First of all, the magnetosonic wave is divided into two modes in a two-ion-species plasma. The lower frequency mode has an ion-ion hybrid resonance frequency ω_{-r}.[4] The higher frequency mode has a finite cut-off frequency ω_{+o}. (The dispersion curves and mathematical expressions can be found in Ref. 1.) If we denote the cyclotron frequency of the lighter ions by Ω_a (the subscript a refers to the lighter ion species) and that of heavier ions by Ω_b (b designating the heavier ions), then these frequencies are ordered as follows:

$$\Omega_b < \omega_{-r} < \omega_{+o} < \Omega_a. \tag{1}$$

The phase velocity of the low frequency mode is about the Alfvén speed, v_A, in the long-wavelength region. On the other hand, for the wavenumber range

$$(m_e/m_i)^{1/2} \ll c^2k^2/\omega_{pe}^2 \ll 1, \tag{2}$$

the frequencies of the high frequency mode are given by

$$\omega_+ = v_h k \; [1 - c^2k^2/(2\omega_{pe}^2)], \tag{3}$$

where v_h is defined as

$$v_h = v_A \left[1 + \frac{\omega_{pa}^2 \omega_{pb}^2}{\omega_{pe}^4} \Omega_e^2 \left(\frac{1}{\Omega_a} - \frac{1}{\Omega_b} \right)^2 \right]^{1/2}. \tag{4}$$

Here, ω_{pj} is the plasma frequency for particle species j. The speed v_h is slightly higher than the Alfvén speed v_A, and in a single-ion-species plasma it reduces to v_A. The dispersion curves for both the low- and high-frequency modes bend sharply at the wavenumber k_c, where k_c is defined as $k_c = \omega_{-r}/v_A$. Even though the high-frequency mode has a finite cut-off frequency, both the low- and high-frequency modes have been shown to be described by KdV equations.[1]

In the following we will discuss the high-frequency mode; the terminology "magnetosonic wave" will designate this mode when it is used for two-ion-species plasmas. Indeed, its frequency range is much wider than that of the low-frequency mode, and it is believed that in large-amplitude waves, such as shock waves, the high-frequency mode plays a more important role than the low-frequency mode.[2]

In a collisionless, single-ion-species plasma, magnetosonic waves propagating perpendicular to a magnetic field do not suffer Landau damping. Thus they can propagate without damping. (Large-amplitude waves can be damped, because they accelerate some fraction of ions[5−.9]) In this Letter, however, we will show, using a one-dimensional electromagnetic simulation code based on a three-fluid model, that even in a collisionless plasma, perpendicular magnetosonic waves are damped in a two-ion-species plasma. Even when the wave amplitude is small, heavy ions are accelerated in the direction parallel to the wave front by the transverse electric field; this generates a longer wavelength perturbation and causes the original pulse to be damped.

We consider waves propagating in the x-direction ($\partial/\partial y = \partial/\partial z = 0$) in a magnetic field $\boldsymbol{B}$ that points in the z-direction. To study the space-time evolution of finite amplitude waves, we carry out numerical simulations of the three-fluid model, employing the pseudo spectral method[10]:

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j v_{jx}) = 0, \tag{5}$$

$$m_j \left(\frac{\partial}{\partial t} + v_{jx} \frac{\partial}{\partial x} \right) \boldsymbol{v}_j = q_j \boldsymbol{E} + \frac{q_j}{c} \boldsymbol{v}_j \times \boldsymbol{B}, \tag{6}$$

$$\frac{1}{c}\frac{\partial B}{\partial t} = -\frac{\partial E_y}{\partial x}, \tag{7}$$

$$\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \boldsymbol{B} - \frac{4\pi}{c}\sum_j q_j n_j \boldsymbol{v}_j, \tag{8}$$

where the subscript j refers to the ion species (species a or b) or the electrons (j =e); m_j is the mass, q_j the charge, n_j the number density, and $\boldsymbol{v}_j$ the velocity. We assume periodic boundary conditions. As the initial wave profiles, we use the solitary wave solutions obtained from the KdV equation for the high-frequency mode[1] and observe their evolution.

We simulate a hydrogen-helium plasma. Thus we have chosen the mass and charge ratios between heavy and light ions as $m_b/m_a = 4$ and $q_b/q_a = 2$, respectively. The density ratio is $n_b/n_a = 0.1$, as in space plasmas. The mass and charge ratios between light ions and electrons are $m_a/m_e = 1000$ and $q_a/q_e = -1$. The magnetic field strength is $|\Omega_e|/\omega_{pe} = 0.5$, so that $c/v_A = 68.3$ and $v_A/v_h = 0.967$.

In Fig. 1 we show profiles of the magnetic field at various times for a solitary wave with the initial amplitude $B_n(0) = 0.1$, where B_n is the amplitude (or the maximum value) of the perturbed magnetic field normalized to the external field, B_o. Figure 2 shows the profiles of v_{by}, the y component of the velocity of the heavy ions. Although the magnetic field nearly keeps its initial profile, the heavy-ion velocity v_{by}, which is quite small at $t = 0$, increases with time in the pulse region; in this case it reaches its steady state value, $v_{by} \simeq 0.01 v_h$, at about $\omega_{pe}t = 1500$. That is, even in the fluid model, the heavy ions are accelerated in the direction parallel to the wave front. (Here, initially we imposed a small-amplitude solitary wave. If the original pulse is a large-amplitude shock-like wave, the acceleration would be much stronger.[3]) The heavy-ion motion across the magnetic field produces a long-wavelength perturbation behind the pulse region. This is the high-frequency mode, with the wavelength $\lambda \sim 2\pi/k_c$: more precisely, $\lambda = 1.2(2\pi/k_c)$ (and hence $\omega = 1.053\omega_{+o}$) for this case.

Because of the excitation of the long-wavelength perturbation, the amplitude of the main pulse gradually decreases. Figure 3 shows the time variation of the amplitude of the magnetic field; the straight line represents the theoretical prediction, which will be described below.

The energy of the original pulse is gradually transferred to the long-wavelength perturbation through the acceleration of the heavy ion. Thus, the damping rate of the original main pulse can be obtained from the following equation:

$$\frac{dE_w}{dt} = -w(\omega) v_{bym}^2 M v_h, \tag{9}$$

where E_w is the wave energy of the original pulse, v_{bym} is the maximum speed that the heavy ions gain from the original pulse, $w(\omega)v_{bym}^2$ is the wave-energy density of the long-wavelength perturbation with amplitude v_{bym} and frequency ω, and Mv_h is the propagation speed of the original pulse (M is the Mach number). The right-hand side of Eq. (9) is the energy that the long-wavelength perturbation gains per unit time.

Because the quantity $\int B dx$ is conserved in this system, we have the relation $\int (B - B_o)^2 dx = \int B^2 dx + \text{const}$. Thus the total energy of the original pulse can be given by

$$E_w = \int \left(\frac{(B - B_o)^2 + E^2}{8\pi} + \sum_j \frac{m_j n_j v_j^2}{2} \right) dx. \tag{10}$$

Substituting the soliton solution into this equation, we can obtain the specific form for E_w of the original pulse as a function of the normalized amplitude B_n. The wave energy E_w is proportional to $B_n^{3/2}$, because the wave-energy density is proportional to B_n^2 and the soliton width is proportional $B_n^{-1/2}$.

Also, from the theory of heavy-ion acceleration,[3] we know that the maximum speed v_{bym} is given by

$$v_{bym} = g_{vB} B_n^{1/2}, \tag{11}$$

where the coefficient g_{vB} is defined as

$$g_{vB} = \frac{4\alpha'^{1/2}}{\eta^3} \frac{\Omega_b \omega_{pa}^2}{|\Omega_e| \omega_{pe}^2} \left(1 - \frac{\Omega_b}{\Omega_a} \right) v_h. \tag{12}$$

Here α' is an order-unity quantity, and η is of the order of m_e/m_a; for their precise expressions, see Ref. 3. Equation (11) is valid when the original wave is a soliton-like pulse. From the linear theory for the wave, we have $w(\omega)$ given as

$$w(\omega) = \frac{1}{2} \frac{m_b n_{bo}}{m_a n_{ao}} (m_a n_{ao} + m_b n_{bo}) \left(\frac{\omega_{-r}^2}{\omega^2} + \frac{(\omega^2 - \omega_{-r}^2)^2}{\omega^2 (\omega_{+o}^2 - \omega_{-r}^2)} \right). \tag{13}$$

We therefore obtain a differential equation for the amplitude of the magnetic field

$$dB_n^{3/2}/dt = -(3/2)\gamma B_n(1 + B_n/2), \tag{14}$$

where γ is

$$\gamma = \frac{\omega_{pe} v_h}{8(\alpha')^{1/2} c} \left(\frac{w(\omega) g_{vB}^2}{B_o^2/(8\pi)} \right). \tag{15}$$

If we neglect the second term on the right-hand side of Eq. (14), we have

$$B_n(t) = B_n(0)[1 - \gamma B_n(0)^{-1/2} t/2]^2. \tag{16}$$

When the second term in the square bracket on the right-hand side of Eq. (16) is smaller than unity, this equation can be approximated as

$$B_n(t) = B_n(0)[1 - \gamma B_n(0)^{-1/2} t]. \tag{17}$$

The damping rate, γ_d, is thus given by

$$\gamma_d = \gamma B_n^{-1/2}(0). \tag{18}$$

Note that it decreases with increasing initial amplitude $B_n(0)$. (The soliton theory for this mode is valid for amplitudes $(m_e/m_i)^{1/2} \ll B_n \ll 1$.[1,2]) If we retain the second term on the right-hand side of Eq. (14), we have

$$B_n(t) = 2\tan^2[C - \gamma t/(2\sqrt{2})], \tag{19}$$

where C is given by $C = \arctan[B_n(0)/2]^{1/2}$. When t and B_n are small, Eq. (19) reduces to Eq. (16).

From the simulation results in Fig. 2, we find the wavenumber of the long-wavelength perturbation, $k = k_c/1.2$, and hence the frequency, $\omega = 1.053\omega_{+o}$. Substituting this in Eq. (15), we obtain the damping rate for the case when $B_n = 0.1$. As Fig. 3 shows, the theoretical damping rate for the original pulse agrees well with the simulation result.

In summary, by using a one-dimensional, fully electromagnetic simulation code based on the three-fluid model, we have studied the propagation of magnetosonic waves in a two-ion-species plasma. As a magnetosonic pulse propagates, it accelerates the heavy ions in the direction parallel to the wave front. The cross-field heavy-ion motion then causes the generation of a long-wavelength perturbation with wavenumber $k \sim k_c$ (frequency near the cut-off frequency, $\omega \simeq \omega_{+o}$). Therefore, the original pulse is gradually damped. By equating the energy loss rate of the original pulse to the energy gain rate of the long-wavelength perturbation, we theoretically obtained the damping rate of the original pulse, which was found to be in good agreement with the simulation result.

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FIGURE CAPTIONS

FIG. 1. Magnetic field profiles of a pulse at various times. The initial amplitude is $B_n(0) = 0.1$.

FIG. 2. Profiles of the heavy ion velocity v_{by} at various times.

FIG. 3. Time variation of the amplitude of the original pulse. The dots are simulation results. The straight line shows the theoretical prediction.