Bootstrap Current Close to Magnetic Axis in Tokamaks

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Abstract

It is shown that the bootstrap current density close to the magnetic axis in tokamaks does not vanish in simple electron-ion plasmas because the fraction of the trapped particles is finite. The magnitude of the current density could be comparable to that in the outer core region. This will reduce or eliminate the need of the seed current.

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It is well known from conventional neoclassical theory that a steady-state tokamak cannot be sustained by the bootstrap current alone without a seed current on the magnetic axis.[1−3] The main reason is that as r, the minor radius, approaches zero, the bootstrap current also vanishes. A seed current is thus required to maintain the equilibrium safety factor q profile. However, because particle orbit topology in the region close to the magnetic axis deviates from that employed in the conventional theory, the prediction of the bootstrap current based on the conventional theory becomes questionable in that region.[4,5] Indeed, a parallel flow close to the magnetic axis for the fusion alpha particles is calculated in Ref. 6 by taking into account the proper orbit topology. This parallel alpha flow contributes to the bootstrap current on the magnetic axis. It is also argued that because the fraction of the trapped ions does not vanish when $r \to 0$, the bootstrap current should be finite there.[7] Here, we calculate parallel plasma viscosities and find from the solution of the parallel balance equations for simple electron-ion plasmas that the bootstrap current close to the magnetic axis does not vanish. The magnitude of the current density can be comparable to that in the outer core region. This will reduce or eliminate the need of the seed current.

The proper linearized drift kinetic equation is[8]

$$\left(v_\parallel \widehat{n} + \mathbf{v}_d\right) \cdot \boldsymbol{\nabla} f - C(f) = 2\,\frac{v^2}{v_t^2}\left(\frac{1}{2} - \frac{3}{2}\,\frac{v_\parallel^2}{v^2}\right) f_M$$
$$\times \left[\mathbf{V} \cdot \boldsymbol{\nabla} \ell n\, B + \left(\frac{v^2}{v_t^2} - \frac{5}{2}\right)\frac{2}{5}\,\frac{\mathbf{q} \cdot \boldsymbol{\nabla} \ell n\, B}{p}\right], \tag{1}$$

where $v_\parallel$ is the parallel (to the magnetic field $\mathbf{B}$) particle speed, $\mathbf{v}_d$ is the drift velocity, f is the perturbed particle distribution function, v is the particle speed, v_t is the thermal speed, $\mathbf{V}$ is the mass flow velocity, $\mathbf{q}$ is the heat flow, p is the plasma pressure, $B = |\mathbf{B}|$, $C(f)$ is the Coulomb collision operator, and f_M is the Maxwellian distribution. The independent variables in Eq. (1) are (E, μ, ψ, θ) where $E = v^2/2$, $\mu = v_\perp^2/2B$, ψ is the poloidal flux function, θ is the poloidal angle, and $v_\perp^2 = (v^2 - v_\parallel^2)/2$ is the perpendicular (to $\mathbf{B}$) speed. For

simplicity, we neglect the effects of orbit squeezing here. The basic assumptions for Eq. (1) are that the equilibrium gradient scale length is larger than the width of the orbit, inverse aspect ratio $\epsilon < 1$, and that all the relevant flow velocities are subsonic (so that the plasma is incompressible).

To solve Eq. (1), we need to know the particle trajectory shown in Fig. 1. The particle trajectory close to the magnetic axis is determined by three constants of motion: toroidal canonical momentum $P_\zeta = \psi - Iv_\parallel/\Omega$, magnetic moment μ, and energy $v^2/2$. Here, $I = R^2\boldsymbol{\nabla}\zeta \cdot \mathbf{B}$, R is the major radius, ζ is the toroidal angle, and Ω is the gyrofrequency. For particles in the vicinity of the magnetic axis, the deviation from the magnetic axis $\psi_0 = 0$ can be described as

$$x^3 + 2\,\frac{Iv_{\parallel 0}}{\Omega_0}x - \frac{2I^2C_1}{\Omega^2}\left(v_{\parallel 0}^2 + \mu B_0\right)\cos\theta = 0, \tag{2}$$

where θ is the poloidal angle, $x = \sqrt{\psi}$, the subscript "0" indicates evaluation at $\psi_0 = 0$ and $\theta = \theta_0$, and $C_1 = \sqrt{2q/IR}$. To obtain Eq. (2), we have used a large aspect ratio expansion, i.e., $\epsilon \ll 1$, and assumed that there is no magnetic shear for simplicity.

The solution to Eq. (2) is characterized by the effective pitch angle parameter $\kappa = (8/27)(I|v_{\parallel 0}|/\Omega_0)^3/[(I^2C_1/\Omega_0^2)^2(v_{\parallel 0}^2 + \mu B_0)^2]$. For simplicity, we assume Ω_0 is positive. For circulating particles, $-\infty < \sigma\kappa < -1$, and $0 < \sigma\kappa < \infty$, where $\sigma = v_{\parallel 0}/|v_{\parallel 0}|$. For trapped particles $-1 \leq \sigma\kappa \leq 0$. Trapped particles are defined as particles that have turning points, namely, poloidal angles at which poloidal angular speed $\omega = (v_\parallel \widehat{n} + \mathbf{v}_d)\cdot\boldsymbol{\nabla}\theta/(\widehat{n}\cdot\boldsymbol{\nabla}\theta) = 0$, on their trajectories. Note that if Ω_0 is negative, the orbit trajectory is the same as that of the positive Ω_0 as long as the sign of σ is also changed simultaneously.

The real positive solutions to Eq. (2) have the general form

$$x = 2\widehat{x}T, \tag{3}$$

where $\widehat{x} = [(I^2C_1/\Omega_0^2)(v_{\parallel 0}^2 + \mu B_0)]^{1/3}(|\sigma\kappa|)^{1/6}$, and T is one of the following functions:

$\cos(\beta/3)$, $\sin(\pi/6 \pm \beta/3)$, $\sinh(\beta/3)$, and $\cosh(\beta/3)$. Because orbit trajectories are up-down symmetric in poloidal angle θ, we only describe trajectories in the first and the second quadrants. There are two classes of circulating particles with $-\infty < \sigma\kappa < -1$. One class is described by $T = \cos(\beta/3)$ with $\cos\beta = \cos\theta/\sqrt{|\sigma\kappa|}$ for $0 \leq \theta \leq \pi/2$, and $T = \sin(\pi/6 + \beta/3)$ with $\cos\beta = |\cos\theta|/\sqrt{|\sigma\kappa|}$ for $\pi/2 \leq \theta \leq \pi$. The other class is described by $T = \sin(\pi/6 - \beta/3)$ with $\cos\beta = |\cos\theta|/\sqrt{|\sigma\kappa|}$ for $\pi/2 \leq \theta \leq \pi$. This class of circulating particles intersect the magnetic axis; thus the poloidal angle span is $\pi/2 \leq \theta \leq \pi$. There is only one class of circulating particles with $0 < \sigma\kappa < \infty$. They can be described as $T = \sinh(\beta/3)$ with $\sinh\beta = \cos\theta/\sqrt{|\sigma\kappa|}$ for $0 \leq \theta \leq \pi/2$. Note this class of circulating particles also intersect the magnetic axis. For trapped particles, there exists a critical angle θ_c defined by the solution of the equation $\sigma\kappa + \cos^2\theta_c = 0$. The turning point is $\theta_t = \pi - \theta_c$. There are two branches for a trapped particle trajectory separated by θ_t. The inner branch that intersects the magnetic axis is described by $T = \sin(\pi/6 - \beta/3)$ with $\cos\beta = |\cos\theta|/\sqrt{|\sigma\kappa|}$ for $\pi/2 \leq \theta \leq \pi - \theta_t$. The outer branch is described by $T = \sin(\pi/6 + \beta/3)$ with $\cos\beta = |\cos\theta|/\sqrt{|\sigma\kappa|}$ for $\pi/2 \leq \theta \leq \pi - \theta_t$, $T = \cos(\beta/3)$ with $\cos\beta = |\cos\theta|/\sqrt{|\sigma\kappa|}$ for $\theta_c \leq \theta \leq \pi/2$, and $T = \cosh(\beta/3)$ with $\cosh\beta = \cos\theta/\sqrt{|\sigma\kappa|}$ for $0 \leq \theta \leq \theta_c$.

Employing the constants of motion and the definition of ω, we find[9]

$$\omega = \frac{3}{4}\frac{\Omega_0}{I}\left(\psi + \frac{2}{3}\frac{I v_{\|0}}{\Omega_0}\right), \tag{4}$$

if $\epsilon \ll 1$. Note that $\psi = x^2$ and x is given in Eq. (3). The poloidal angular speed ω can be written as

$$\omega = \widehat{\omega}(4T^2 + \sigma), \tag{5}$$

where $\widehat{\omega} = (3\Omega_0/4I)[(I^2 C_1/\Omega_0^2)(v_\|^2 + \mu B_0)\sqrt{|\sigma\kappa|}\,]^{2/3}$. It is straightforward to show that $\omega = 0$ at $\theta = \theta_t$ as expected.

The fraction of trapped particles f_t can be estimated from $\kappa \simeq 1$ to obtain $f_t \simeq$

$(Iv_tC_1^2/\Omega_0)^{1/3}$, by approximating $v_{\|0}^2 + \mu B_0 \simeq v^2/2 \simeq v_t^2/2$.

The trapped particle bounce frequency ω_b can be found by approximating $|\sigma\kappa| \sim 1$ in $\widehat{\omega}$ and is $\omega_b \approx (Iv_tC_1^2/\Omega_0)^{1/3}(v_t/Rq) = v_t f_t/Rq$.

We are interested in the collisionless regime where $\nu/(f_t^2) < \omega_b$ with ν the collision frequency. The perturbed distribution function f can be expanded as $f = f_1 + f_2 + ...$ with small parameter $\nu/(f_t^2\omega_b)$. The leading order equation is

$$\left(v_\| \widehat{n} + \mathbf{v}_d\right) \cdot \boldsymbol{\nabla} f_1 = 2\,\frac{v^2}{v_t^2}\left(\frac{1}{2} - \frac{3}{2}\,\frac{v_\|^2}{v^2}\right) f_M (\widehat{n}\cdot\boldsymbol{\nabla}\theta)\frac{\partial B}{\partial\theta}$$
$$\times\left[K + \left(\frac{v^2}{v_t^2} - \frac{5}{2}\right)\frac{2H}{5p}\right], \tag{6}$$

where $K = \mathbf{V}\cdot\boldsymbol{\nabla}\theta/\mathbf{B}\cdot\boldsymbol{\nabla}\theta$ and $H = \mathbf{q}\cdot\boldsymbol{\nabla}\theta/\mathbf{B}\cdot\boldsymbol{\nabla}\theta$. The next order equation is

$$(v_\| \widehat{n} + \mathbf{v}_d)\cdot\boldsymbol{\nabla} f_2 = C(f_1). \tag{7}$$

Equation (6) can be solved approximately by neglecting the curvature drift and $(3v_\|^2/2v^2)$ term on the right side of Eq. (6). Note that neglecting the curvature drift and $(v_\|/v)^2$ terms are justified because transport fluxes in a large aspect ratio tokamak, including bootstrap current, are dominated by either trapped particles or *barely* circulating particles where $\omega \approx 0$.[10] Both ω and $\mathbf{v}_d\cdot\boldsymbol{\nabla}\psi$ can be expressed in terms of the gradients of P_ζ, namely, $\omega = -(I/\Omega)(\partial P_\zeta/\partial\psi)/(\partial P_\zeta/\partial E)$ and $\mathbf{v}_d\cdot\boldsymbol{\nabla}\psi = (I/\Omega)[(\partial P_\zeta/\partial\theta)/(\partial P_\zeta/\partial E)]\widehat{n}\cdot\boldsymbol{\nabla}\theta$. Because $v_\|^2/v^2 \sim f_t^2 \ll 1$, the driving term on the right side of Eq. (6) can be written in terms of $\mathbf{v}_d\cdot\boldsymbol{\nabla}\psi$ approximately to become

$$(v_\| \widehat{n} + \mathbf{v}_d)\cdot\boldsymbol{\nabla} f_1 = -\frac{I}{\Omega}\,\frac{\partial P_\zeta/\partial\theta}{\partial P_\zeta/\partial E}\,\widehat{n}\cdot\boldsymbol{\nabla}\theta\cdot\mathcal{D}, \tag{8}$$

where $\mathcal{D} = (2/v_t^2)(\Omega_0 B_0/I) f_M[K + (v^2/v_t^2 - 5/2)2H/(5p)]$. Changing variables from (E,μ,ψ,θ) to (E,μ,P_ζ,θ), and utilizing Eq. (4) we solve Eq. (8) for f_1

$$f_1 = -\frac{4}{3}\,\frac{I\omega}{\Omega_0}\,\mathcal{D} + g(E,\mu,P_\zeta), \tag{9}$$

where g is the integration constant to be determined from Eq. (7).[11] The pitch angle scattering operator in Eq. (7) can be simplified by noting that the collision process is dominated by pitch angle scattering across the $\omega \approx 0$ boundary:

$$C(f_1) \approx \nu_D E \, \frac{\partial^2 f_1}{\partial \omega^2}. \tag{10}$$

Substituting Eqs. (9) and (10) into Eq. (7), annihilating the left side of Eq. (7) by averaging over the particle trajectory and employing reflection boundary condition for trapped particles and periodic boundary condition for circulating particles, we find

$$\frac{\partial g}{\partial \kappa} = \frac{C}{\frac{\kappa}{\widehat{\omega}^2} \oint \frac{d\theta}{\hat{n}\cdot\boldsymbol{\nabla}\theta} \, \omega}. \tag{11}$$

where C is an integration constant. To obtain Eq. (10) we have employed the relation $d\omega \approx (4/3)[\widehat{\omega}^2/(2\kappa\omega)]d\kappa$. The average integral $\oint A d\theta$ in Eq. (11) is defined as $\oint d\theta A = \int_0^T d\theta A/T$ for each class of the circulating particles where T can be either $T_1 = \pi$ or $T_2 = \pi/2$ depending on whether they encircle or pass the magnetic axis, and $\oint d\theta A = (\int_0^{\theta_t} d\theta |A|)/T_1 + (\int_0^{\theta_t} d\theta |A|)/T_2$ for trapped particles. The constant C is determined by the condition that $\frac{\partial f_1}{\partial \omega}$ vanishes when $|\sigma\kappa| \to \infty$ for circulating particles and the even (in ω) part of $\partial g/\partial \kappa$ continuous across the circulating/trapping boundary.[11] This yields

$$\frac{\partial f_1}{\partial \omega} = -\frac{4}{3} \frac{I}{\Omega_0} \mathcal{D} \left(1 - \frac{|\omega|}{\langle |\omega| \rangle_\theta} H \right). \tag{12}$$

where $\langle |\omega| \rangle_\theta = \oint d\theta |\omega|$, and $H = 1$ for circulating particles and $H = 0$ for trapped particles. To calculate the parallel plasma viscosity, $\partial f_1/\partial \omega$ is adequate.

The parallel plasma viscosity is defined as[12]

$$\langle \mathbf{B} \cdot \boldsymbol{\nabla} \cdot \boldsymbol{T}_j \rangle = \left\langle \int d^3v \, M v^2 \Sigma_j \left(\frac{1}{2} - \frac{3}{2} \frac{v_\parallel^2}{v^2} \right) f \hat{n} \cdot \boldsymbol{\nabla}\theta \, \frac{\partial B}{\partial \theta} \right\rangle, \tag{13}$$

where $\boldsymbol{T}_1 = \boldsymbol{\pi}$, the viscous tensor, $\boldsymbol{T}_2 = \boldsymbol{\Theta}$, the heat viscous tensor, $\Sigma_1 = 1$, $\Sigma_2 = v^2/v_t^2 - 5/2$, M is the mass, and the angular brackets denote both radial average and flux surface average

as defined in Refs. 8 and 11. With Eq. (6), $\langle \mathbf{B}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{T}_j\rangle$ can be expressed in terms of collision operator

$$\langle \mathbf{B}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{T}_j\rangle = -\left\langle \int d^3v\, M f_1 C(f_1)\mathcal{D}_1^{-1}\right\rangle, \tag{14}$$

where $\mathcal{D}_1^{-1} = (v_t^2/2)/\{f_M[K + (v^2/v_t^2 - 5/2)(2H/5p)]\}$. Integrating by parts, and changing variables from $d\mu$ to $d\omega$, we find

$$\langle \mathbf{B}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{T}_j\rangle \simeq \left\langle \int d^3v\, M\mathcal{D}_1^{-1}\nu_E E\left(\frac{\partial f_1}{\partial\omega}\right)^2\right\rangle. \tag{15}$$

$\langle \mathbf{B}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{T}_j\rangle$ can be evaluated by employing Eq. (12) in Eq. (15) and is

$$\langle \mathbf{B}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{T}_j\rangle = 1.12 I_p \frac{NMB_0^2}{\sqrt{\pi}}\,\nu C_1^{2/3}\left(\frac{Iv_t}{\Omega_0}\right)^{1/3}$$
$$\times \int_0^\infty dx\, x^{5/3}\frac{\nu_D}{\nu}\, e^{-x}\Sigma_j\left[K + (x-5/2)\frac{2H}{5p}\right], \tag{16}$$

where $I_p = \Sigma_\alpha \int_0^\infty (d\kappa/\kappa^{2/3}(\langle\widehat{\omega}/|\omega|\rangle_\theta - H\widehat{\omega}/\langle|\omega|\rangle_\theta) = 2.77$, $\alpha = \omega/|\omega|$, and ν is the self-collision frequency. A set of viscous coefficients can be defined

$$\mu_j = 1.12 I_p \frac{\nu}{\sqrt{\pi}}\left(\frac{Iv_t}{\Omega_0}\right)^{1/3} C_1^{2/3}\int_0^\infty dx\, x^{5/3}\sigma_j\frac{\nu_D}{\nu}\, e^{-x} \tag{17}$$

for $j = 1-3, \sigma_1 = 1, \sigma_2 = x - 5/2$, and $\sigma_3 = (x-5/2)^2$. The parallel viscosities then become

$$\begin{pmatrix}\langle \mathbf{B}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\pi}\rangle \\ \langle \mathbf{B}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\Theta}\rangle\end{pmatrix} = NMB_0^2\begin{pmatrix}\mu_1 & \mu_2 \\ \mu_2 & \mu_3\end{pmatrix}\begin{pmatrix}K \\ \dfrac{2H}{5p}\end{pmatrix}. \tag{18}$$

Note that μ_j is proportional to the fraction of the trapped particles $f_t = C_1^{2/3}(Iv_t/\Omega_0)^{1/3}$ similar to the viscous coefficient in the conventional theory.[12]

To calculate the bootstrap current in simple electron-ion plasmas, we solve the parallel force balance equations for electrons and ions

$$\begin{aligned}\langle \mathbf{B}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\pi}_j\rangle &= \langle BF_{1j}\rangle, \\ \langle \mathbf{B}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\Theta}_j\rangle &= \langle BF_{2j}\rangle,\end{aligned} \tag{19}$$

where $j = 1$ for ions and $j = e$ for electrons. $F_{1e} = -F_{1i} = \ell_{11}^e(V_{i\|} - V_{e\|}) + (2/5)\ell_{12}^e(q_{e\|}/p_e)$, $F_{2e} = -\ell_{12}^e(V_{i\|} - V_{e\|}) - (2/5)\ell_{22}^e(q_{e\|}/p_e)$, $F_{2i} = -(2/5)\ell_{22}^i(q_{i\|}/p_i)$, $\ell_{11}^e = N_e M_e \nu_{ee}$, $\ell_{12}^e = 1.5\ell_{11}^e$, $\ell_{22}^e = 4.66\ell_{11}^e$ and $\ell_{12}^i = \sqrt{2}\, N_i M_i \nu_{ii}$.[12] Equation (19) has the same form as that in the conventional theory, the bootstrap current has thus the familiar form[12]

$$\langle J_b B\rangle = -\sigma_{\rm eff}\left(\frac{M_e\mu_{1e}}{Ne^2}\right) Ic\left[\left(1 + \frac{\mu_{2e}}{\mu_{1e}}\frac{\ell_{12}^{eb}}{\ell_{22}^{eb}}\right)P' + \left(1 + \frac{\mu_{2e}}{\mu_{1e}}\frac{\ell_{12}^{eb}}{\ell_{22}^{eb}}\right)\frac{\mu_{2i}}{\mu_{1i}}NT_i' + \frac{\mu_{2e}}{\mu_{1e}}\left(1 + \frac{\mu_{3e}}{\mu_{2e}}\frac{\ell_{12}^{eb}}{\ell_{22}^{eb}}\right)NT_e'\right], \tag{20}$$

except for different viscous coefficients which are given in Eq. (17). The notations in Eq. (20) are: J_b is the bootstrap current, $\ell_{11}^{eb} = \ell_{11}^e + NM_e\mu_{1e}$, $\ell_{12}^{eb} = \ell_{12}^e - NM_e\mu_{2e}$, $\ell_{22}^{eb} = \ell_{22}^e + NM_e\mu_{3e}$, prime denotes $d/d\psi$, $P' = p_i' + p_e'$, c is the speed of light, e is the ion charge, and electric conductivity close to the magnetic axis $\sigma_{\rm eff}$ is

$$\sigma_{\rm eff} = (Ne)^2\,\frac{\ell_{22}^{eb}}{\ell_{11}^{eb}\ell_{22}^{eb} - (\ell_{12}^{eb})^2}. \tag{21}$$

Thus, the electric conductivity is not classical as $\psi \to 0$.

Note that for a parabolic profile in r, $dP/d\psi$, and $dT/d\psi$ are finite as $\psi \to 0$. Also $\mu_j \propto f_t$ are also finite as $\psi \to 0$. We have thus shown that $\langle J_b B\rangle$ remains finite as $\psi \to 0$. The physical reason is obvious: the fraction of trapped particles does not vanish as $\psi \to 0$, because of the nature of orbit topology close to the magnetic axis. The magnitude of the bootstrap current density in Eq. (20) can be comparable to that of the conventional theory in the core region. For example, for parameters in enhanced-reversed-shear (ERS) mode in TFTR.[13]: $T_e = 5\,\mathrm{keV}$, $B = 4.6\,\mathrm{T}$, $R = 2.6\,\mathrm{m}$, $q(\psi = 0) = 3$, and minor radius $a = 94\,\mathrm{cm}$, the ratio $\mathcal{R}$ of the bootstrap current density at $\psi \to 0$ to that at $r/a = 0.5$ is about 18% if we assume $dP/d\psi$ at $\psi \to 0$ is the same as that at $r/a = 0.5$ and employ the large aspect ratio expression of the bootstrap current at $r/a = 0.5$. Similarly, for DIII-D parameters[14]: $T_e = 4\,\mathrm{keV}$, $B = 2.1\,\mathrm{T}$, $R = 1.6\,\mathrm{m}$, $q(\psi = 0) = 3$, and $a = 60\,\mathrm{cm}$, the same ratio is

about 27%. The corresponding fraction of trapped particles is $f_t \simeq 6 \times 10^{-2}$ for DIII-D and $f_t \simeq 4 \times 10^{-2}$ for TFTR. The width of the trapped electron orbits discussed here is about $r_M^e = R(2\rho_e q/R)^{2/3}$ where ρ_e is the electron gyroradius.[5] For TFTR, $r_M^e = 0.62\,\mathrm{cm}$, and for DIII-D, $r_M^e = 0.83\,\mathrm{cm}$.[15] Because the bootstrap current density calculated from the conventional theory and the theory developed here should have similar magnitude at $r \sim r_M^e$, we can also estimate $\mathcal{R}$ as $\mathcal{R}_c \sim \sqrt{r_M^e/R}/\sqrt{\epsilon|_{r/a=0.5}}$ based on the conventional theory. For TFTR, $\mathcal{R}_c \simeq 11.5\%$ and for DIII-D, $\mathcal{R}_c \simeq 16.6\%$. The difference between $\mathcal{R}$ and $\mathcal{R}_c$ results from the detailed numerical viscous coefficients in Eq. (17) and is an indication that the transition of the bootstrap current from the conventional theory to the theory presented here is at about $r \sim 2r_M^e > 1\,\mathrm{cm}$ for both TFTR and DIII-D.

The bootstrap current presented here could be narrow in radius. However, without such a current, we can never have tokamaks with 100% bootstrap current. Thus the narrowness of the bootstrap current density is not an issue. The *existence* of the bootstrap current density on the magnetic axis is crucial, however. Indeed, it has been shown that conventional tokamak equilibria do not exist if plasma current density vanishes on the magnetic axis.[16] Because bootstrap current density vanishes on the magnetic axis based on the conventional neoclassical theory, on needs seed current to maintain tokamak plasma equilibria. The amount of the seed current that is required can be very small, but finite nevertheless. Stable tokamak plasma equilibria with bootstrap current fraction as high as 99.7%, but not 100%, have been found based on the conventional neoclassical theory.[17] Employing the theory developed here, we can find stable tokamak equilibria with 100% bootstrap current by following the procedure used in Ref. 17. Namely, we first choose a $dP/d\psi$ profile which has a finite value on the magnetic axis. Next, we calculate both diamagnetic current density profile and bootstrap current density profile from the standard neoclassical theory and the theory presented here. We then solve Grad-Shafranov equation to find the equilibrium which now only has bootstrap current and diamagnetic current. Finally, we examine the stability

property of the equilibrium.

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REFERENCES

[1]R.J. Bickerton, J.W. Connor, and J.B. Taylor, Nat. Phys. Sci. **229**, 110 (1971).

[2]A.A. Galeev and R.Z. Sagdeev, JETP. Lett. **13**, 113 (1971).

[3]F.L. Hinton and R.D. Hazeltine, Rev. Mod. Phys. **48**, 239 (1976).

[4]T.H. Stix, Plasma Phys. **14**, 367 (1972).

[5]T.E. Stringer, Plasma Phys. **16**, 651 (1974).

[6]V. Ya Goloborod'ko, Ya.I. Kolesnichenko, and V.A. Yavorskij, Nucl. Fusion **23**, 399 (1982).

[7]P.A. Politzer, Bull. Am. Phys. Soc. **40**, 1787 (1995).

[8]K.C. Shaing, C.T. Hsu, and N. Dominguez, Phys. Plasmas **1**, 1168 (1994).

[9]K.C. Shaing, R.D. Hazeltine, and M.C. Zarnstorff, Phys. Plasmas **4**, 771 (1997).

[10]Note that the large aspect ratio limit of the conventional neoclassical theory can be reproduced *exactly* by neglecting the curvature drift and $(v_\parallel/v)^2$ terms for the same reason that transport fluxes, including bootstrap current, are dominated by trapped and barely circulating particles with $v_\parallel \approx 0$.

[11]K.C. Shaing and R.D. Hazeltine, Phys. Fluids B **4**, 2547 (1992).

[12]S.P. Hirshman and D.J. Sigmar, Nucl. Fusion **21**, 1079 (1981).

[13]F.M. Levington, M.C. Zarnstorff, S.H. Batha, M. Bell, R.E. Bell, R.V. Budny, C.E. Bush, Z. Chang, E.D. Fredrickson, A. Janos, J. Manickam, A.T. Ramsey, G.L. Schmidt, E.J. Synakowski, and G. Taylor, Phys. Rev. Lett. **75**, 4417 (1995).

[14]E.J. Strait, L.L. Lao, M.E. Mauel, B.W. Rice, T.S. Taylor, K.H. Burrell, M.S. Chu, E.A. Lazarus, T.H. Osborne, S.J. Thompson, and A.D. Turnbull, Phys. Rev. Lett. **75**, 4421 (1995).

[15]The corresponding trapped ion orbit width is $r_M^i = 7.6\,\mathrm{cm}$ for TFTR and $r_M^i = 10\,\mathrm{cm}$ for DIII-D. However, because of the orbit squeezing in the ERS mode, the trapped ion orbit width is smaller than r_M^i so that the trapped ion orbit width is smaller than the scale length of the plasma pressure. This issue is discussed in detail in K.C. Shaing, R.D. Hazeltine, and M.C. Zarnstorff, "Effects of orbit squeezing on ion transport processes close to magnetic axis," accepted for publication (1997).

[16]T.H. Jensen, R.L. Miller, and Y.R. Lin-Liu, Phys. Fluids **3**, 1656 (1996).

[17]R.L. Miller, Y.R. Lin-Liu, A.D. Turnbull, V.S. Chan, L.D. Pearlstein, O. Sauter, and L. Villard, Phys. Plasmas **4**, 1062 (1997).

FIGURE CAPTIONS

FIG. 1. Schematic diagrams of the circulating and trapped particle orbits for (i) $-\infty < \sigma\kappa < -1$, (ii) $0 < \sigma\kappa < \infty$, and (iii) $-1 < \sigma\kappa < 0$.