Anomalous Ion Thermal Transport from Toroidal Ion Temperature Gradient Mode

J-Y. Kim and Y. Kishimoto

Naka Fusion Research Establishment, Japan Atomic Energy Research Institute

Naka, Ibaraki 311-01, Japan

W. Horton

Institute for Fusion Studies, The University of Texas at Austin

Austin, Texas 78712 USA

and

M. Wakatani

Plasma Physics Laboratory, Kyoto University, Uji 611, Japan

Abstract

It is shown that the observed large edge fluctuation and thermal conductivity can be explained in terms of the toroidal ion temperature gradient mode. The very weak nonlinear interaction rate due to the large radial width of the radially extended toroidal modes is found to be an essential factor. The Bohm-type diffusivity and the near-marginality in core region are also predicted by the same property.

The anomalous particle and energy transport in magnetic fusion plasma systems is widely believed to occur by some microinstabilities, but establishing the correlation between the transport properties and a particular instability remains a difficult problem. The ion temperature gradient (ITG) mode[1−3] has received steady attention as a candidate, particularly, responsible for the ion transport. The mode provides a proper explanation for some experimental features, like the confinement improvements in peaked-density supershot or hot ion plasmas.[4] However, the mode has a significant difficulty in explaining the observed large fluctuation or thermal conductivity in edge region [for example, see Refs.[5]-[7]. The mode also predicts the ion thermal conductivity of gyro-Bohm type, unlike many recent experimental results showing Bohm-like diffusivity.[8]

An important point to note here is that these difficulties arise typically for the *slab* ITG mode,[1] while for the *toroidal* ITG mode[2,3] their occurrence is still unclear related to the complicated structure of the mode. Significant works have thus been performed recently to understand more clearly the property of toroidal ITG mode, through analytical or numerical simulations studies. Some important new facts have been found; among them, particularly interesting are the direct particle simulation results[9] showing the clear excitation of toroidal

ITG modes with a large radial width and ballooning structure, and the analytical results[10] showing that the toroidal ITG mode may exist at all plasma radii, not only at the maximum equilibrium gradient point, and its typical radial width is order $(L\rho_i)^{1/2}$ where L is the equilibrium scale length and ρ_i is the ion gyro-radius. The purpose of this work is to show that the above mentioned difficulties can be resolved well in terms of the toroidal ITG mode, when we utilize these new facts.

We first note that the observed radial increase of ion thermal conductivity $\chi_i(r)$ is expected from the ion heat balance equation,

$$\frac{\partial T_i}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\chi_i\frac{\partial T_i}{\partial r}\right) + P_{\mathrm{i}}, \tag{1}$$

where $P_i(r)$ is the ion input power per unit density. In steady state limit, where most of transport analyses are actually performed, Eq. (1) gives

$$\chi_i(r) = -\frac{1}{r\partial T_i/\partial r}\int_o^r r' P_i(r')dr'. \tag{2}$$

We see that $\chi_i(r)$ should increase radially by the usual radially decreasing temperature ($\propto 1/T_i$) [and partially by the radially accumulated input power ($\propto \int_0^r r' P_i(r')dr'/r$)].

The usual belief that this radially increasing transport occurs by fluctuation generated by some microinstability is supported from the corresponding radial increase in observed fluctuation amplitude.[11] However, no microinstability theory appears to provide a proper explanation of this radial increase in fluctuation amplitude or $\chi_i(r)$, particularly, near edge region.[5−7] A typical example is given in Fig. 1, which shows the $\chi_i(r)$ profile of ITG mode for the TFTR supershot 44669A discharge, calculated using the well-known mixing length formula

$$\chi_i(r) = \gamma_{\max}/k_r^2, \tag{3}$$

with the usual assumption $k_r \sim k_\theta$, where $\gamma_{\max}$ is the maximum growth rate and k_r and k_θ are its radial and poloidal wavenumber, respectively.[12] We see that the calculated χ_i is too small in edge region, by approximately one order of magnitude compared with the observed value, even though Eq. (3) shows some agreement in the core region.

This result in edge region can be interpreted physically as due to that for the slab-like modes (with $k_r \sim k_\theta$), the nonlinear saturation force is too strong and this limits the saturation amplitude and thus χ_i significantly. We now show that for the toroidal ITG mode the nonlinear saturation force is much smaller and the large edge fluctuations and χ_i then become possible.

We first note that the nonlinear saturation force γ_N has in general the dependence

$$\gamma_N \propto k_r^2. \tag{4}$$

This k_r dependence of γ_N may be easily understood from the usual nonlinear interaction term,[2]

$$\mathbf{b} \times \boldsymbol{\nabla}\phi \cdot \boldsymbol{\nabla} f, \tag{5}$$

where $\mathbf{b} = \mathbf{B}/B$, ϕ is the electrostatic potential and f is some fluctuating quantity. The form (5) indicates that the nonlinear interaction rate through beat waves is proportional to

$$\left(\frac{\partial}{\partial r}\right)^2 \sim k_r^2, \tag{6}$$

[note the nonlinear term $\phi_k^{(3)} \propto (\partial\phi_{k'}^{(1)}/\partial r)\phi_{k-k'}^{(2)}$ where the beat wave $\phi_{k-k'}^{(2)} \propto (\partial\phi_k^{(1)}/\partial r)\phi_{-k'}^{(1)}$ so that $\phi_k^{(3)} \propto (\partial/\partial r)^2$]. This dependence of γ_N is also implied in the form of the mixing length formula (3). Writing first the nonlinear saturation force in the form $\gamma_N = \widehat{\gamma}_N E$, as expected from the above nonlinear interaction term, the saturation amplitude of the normalized fluctuation amplitude $\sqrt{E} = |e\phi/T_i|$ is estimated from the steady state force balance condition, $\gamma_L = \gamma_N$, as

$$E = \frac{\gamma_L}{\widehat{\gamma}_N}, \tag{7}$$

with the linear growth rate γ_L. When we substitute this into the usual quasilinear formula $\chi_i = \chi_0 E$, the mixing length form (3) is obtained if

$$\widehat{\gamma}_N = \chi_0 k_r^2, \tag{8}$$

which shows a more explicit form of γ_N ($\equiv \widehat{\gamma}_N E$).

The form (4) or (8) shows the critical importance of the radial mode width in relation to the nonlinear saturation force. Let us now compare the mode widths between the slab and toroidal ITG modes. For the slab ITG mode it is well-recognized that the mode width is order of ρ_i, so

$$k_r \sim k_\theta \sim \frac{1}{\rho_i}. \tag{9}$$

[For example, 3D slab simulations and theory show $\Delta x = \rho_i\ (L_s/L_{T_i})^{1/2}$] On the other hand, for the toroidal ITG mode there has been some ambiguity over the exact radial width, but a series of recent works[10] make it clear that the global radial mode width of toroidal ITG mode (in general equilibrium profiles) is proportional to $\sqrt{\rho_i L}$, so that

$$k_r^2 \sim \frac{1}{\rho_i\, L}. \tag{10}$$

These radial mode widths are also observed from the direct particle simulations of toroidal ITG mode.[9] An example of such a result is shown in Fig. 2. The iso-potential contours in the saturated state show the extended radial structure predicted by toroidal mode theory.

With the radial wavenumbers (9) and (10), we now see that the nonlinear saturation force (8) becomes much smaller for the toroidal mode, roughly by ρ_i/L, compared with the slab mode. Noting typically $\rho_i/L \simeq 0.01 - 0.1$ in edge region, an increase by one order of magnitude in the fluctuation level E, and thus in χ_i, is expected from Eq. (7) and $\chi_i = \chi_0 E$.

An explicit example is given in Fig. 1 which shows the re-calculated χ_i using the width (10). We see that the χ_i has now a reasonable agreement with the observed value in edge region. (The behavior of χ_i in core region will be discussed later). In addition, we note that with the width (10) the mixing length χ_i now takes the Bohm-diffusion form, in good agreement with many recent transport analysis results.[8]

The above successful explanations in terms of the toroidal ITG mode depends crucially on the radial mode width (10) and the resulting very weak nonlinear saturation force. It is thus worthwhile giving here a more detailed discussion of the width (10) and related issues.

(i) We may suspect that the expression (10) is not valid over all θ (poloidal) region. In fact, the typical eigenfunction form of toroidal ITG mode

$$\phi(x,\theta) \propto e^{-\epsilon x^2 + isk_\theta x(\theta-\theta_o) + im\theta - \sigma(\theta-\theta_o)^2}, \tag{11}$$

indicates that the small $k_r \sim \epsilon^{1/2}$ is valid only near $\theta \sim \theta_o$ region, where $\epsilon \sim 1/L\rho_i$, $x = r - r_o$ with the mode center r_o, s is shear, θ_o is the Bloch shift parameter, m is the main poloidal mode number, and $\sigma \sim 1$. Due to the second term in the exponent, which describes the twisted streamline structure in Fig. 2, the k_r averaged over θ becomes[13]

$$k_r \sim sk_\theta \Delta\theta, \tag{12}$$

with $\Delta\theta = 1/\sqrt{\sigma}$, the mode width in the ballooning θ space. This wavenumber (12) is much larger than the value (10) and actually the same order as the slab value (9) (note typically $\Delta\theta \sim 1$). We argue that this is the reason why many previous work of toroidal ITG mode, which used the form (12), failed to explain the large χ_i in edge, like the slab mode case.

We now show that as far as we are concerned with the nonlinear saturation force the form (10) can be used over most θ region. We need just to redefine k_r as the wavenumber or mode size in the tilted *streamline* direction, rather than the *radial* direction. To see this, we first note from the vector form (5) that the nonlinear interaction rate between streamlines depends only on its anisotropic structure itself, but independent of its direction in (r, θ) space. In other words, the nonlinear interaction rate does not vary when we change only the orientation of the streamline structures (for example, by rotation), keeping its anisotropic structure and size the same. We then note that the eigenmode structure shown in Fig. 2 corresponds almost to this case where the local streamline direction is changed (or twisted) with increasing $|\theta - \theta_o|$, but its anisotropic structure is nearly kept. The local nonlinear interaction rate is thus expected not to vary with θ but almost the same as that estimated using the width (10) at $\theta \sim \theta_o$. This means that the form (10) is effectively valid over most θ region, even though we need now to interpret k_r as the wavenumber along the rotating streamline direction (i.e., as k_r' in Fig. 2), rather than the fixed radial direction.

(ii) One may argue that the width (10) is valid only in the linear or quasilinear regime, but not in the strong turbulent regime where the global structure of toroidal mode might be disintegrated into the slab mode-like structures there. Once the disintegration occurs, the nonlinear saturation force will increase rapidly to that of the slab mode. An important point to note here is, however, that by causality this nonlinear disintegration can occur only after the linear mode has grown to a large amplitude so that a finite nonlinear interaction between them becomes possible. The large fluctuation amplitude, as expected from the width (10), is thus still possible in time-averaged sense, even when a small space-small amplitude secondary turbulence is present.

(iii) Due to the long radial width Δr, a large overlap of toroidal modes centered at adjacent rational surfaces may occur. The number of such overlapping modes at a radial position is roughly estimated as $\Delta r/(sk_y)^{-1} \sim sk_y/k_x \gg 1$ (where $(sk_y)^{-1}$ is the rational surface interval). The nonlinear interaction rate will then increase by this number, reducing the saturation amplitude of a single toroidal mode. However, we note that the total amplitude, which will be actually observed and is given by the single mode amplitude times the number of overlapping modes, is not changed. The fluctuation amplitude or χ_i, estimated from Eq. (10), is thus still valid even for the case of strong overlapping of modes.

(iv) The present model is different from that by Mattor *et al.*[14] where the large edge fluctuation is proposed to originate from the wave propagation from core. A basic assumption of the model by Mattor *et al.* is that the global mode width of a toroidal ITG mode is order L, i.e., the equilibrium scale length so that a nonlocal propagation from core to edge can occur. A series of recent works[10] show, however, that the usual eigenmodes with a finite growth rate have the mode width of order $(L\rho_i)^{1/2}$, much smaller than L, so that the assumption by Mattor *et al.* may not be realized, except for a rather special circumstance. The nonlocal wave propagation does not occur (or is negligible) for the localized toroidal modes, while the large edge fluctuation is possible from the weak nonlinear saturation force, as shown in this work.

Finally, we discuss another important feature expected from the very weak nonlinear saturation force of the toroidal ITG mode; the core region will have a strongly marginal temperature profile. When we assume the usual form $\gamma_L = \widehat{\gamma}_L(\eta_i - \eta_c)$, the force balance equation (8) indeed gives

$$\eta_i - \eta_c = \frac{\widehat{\gamma}_N}{\widehat{\gamma}_L} E, \tag{13}$$

where $\eta_i = d \ln T_i(r)/d \ln n(r)$ and η_c is its threshold value. We see that $\eta_i - \eta_c$ becomes much smaller for the toroidal mode, compared with the slab mode case, for a given E which is mainly determined by the heat balance equation (2). We note that this feature is indeed in good agreement with many recent results,[5,6,15] indicating that with a more advanced calculation of η_c the ion temperature profile in the core region appears to be near-marginal. Related to the (still finite) diagnostic uncertainty in the measurements of quantities like T_i, q, impurity and shear flow profiles etc., however, a more rigorous check of this strong marginality has some limitation. This limitation also applies to the exact calculation of χ_i in core region. With the very small k_r^2, the χ_i in Eq. (3) becomes highly sensitive to the growth rate or $\eta_i - \eta_c$, particularly in the core region with high temperature. A small error in these quantities can result in a significantly wrong χ_i. Also note the simple mixing length formula (3) might become incorrect in the near-marginal stability regime.

Even though the exact calculation of χ_i has thus a difficulty in the core region, the steady state core ion energy confinement property can be defined relatively well from the near-marginality which means that the core ion temperature profile is almost determined by the η_c (or $\eta_i \simeq \eta_c$). Noting most of ion energy is confined in the core region, the confinement time scaling of ion energy is then expected to follow closely the scaling of η_c. This emphasizes the importance of knowing the exact properties of η_c, as also noted in a recent work.[15]

In summary, we have shown that the long-standing problem, the observed large edge fluctuation or ion thermal conductivity can be explained in terms of the toroidal ITG mode. The very weak nonlinear saturation force due to the large radial width of the mode is found to be an essential factor. The observed Bohm-like diffusivity and the near-marginality of core ion temperature profile are also predicted by the same property. These present results suggest strongly that the toroidal ITG mode is the mode responsible for the anomalous ion thermal transport over most plasma region in high ion temperature plasmas, as also suggested from the recent experimental observations[16] showing the anisotropic fluctuation spectrum ($k_r \ll k_\theta$) and the in-out asymmetric heat flux. A remaining important problem is to explain the observed confinement scaling, particularly on the current and isotope. The present work suggests that this scaling is closely related to the property of η_c, rather than the usual χ_i, and a more detailed consideration will be given in the future. Finally, we note that the present ion transport model may be also be applied to the electron thermal transport, when we assume the electron transport occurs dominantly by the trapped electron mode or the toroidal η_e mode which have the similar properties with the toroidal ITG mode.

We would like to thank Dr. Azumi for useful discussions. This work was supported by Japanese Science and Technology Agency and the U.S. Department of Energy contract no. DE-FG05-80ET-53088.

REFERENCES

[1]L.I. Rudakov and R.Z. Sagdeev, Sov. Phys. Dokl. **6**, 415 (1961); B. Coppi, M.N. Rosenbluth, and R.Z. Sagdeev, Phys. Fluids **10**, 582 (1967).

[2]W. Horton, D.I. Choi, and W.M. Tang, Phys. Fluids **24**, 1077 (1981).

[3]W. Horton, M. Wakatani, and A. Wootton, eds., *Ion Temperature Gradient Driven Turbulent Transport* (AIP Conference Proceedings, No. **284**, New York 1994).

[4]S.M. Wolfe and M. Greenwald, Nucl. Fusion **26**, 329 (1986); F.X. Soldner *et al.*, Phys. Rev. Lett. **61**, 1105 (1988); S.D. Scott *et al.*, Phys. Rev. Lett. **64**, 531 (1990); S.D. Scott *et al.*, Phys. Fluids B **2**, 1300 (1990).

[5]M. Kotschenreuther *et al.* , in *Plasma Phys. and Controlled Nucl. Fusion Research 1990* (IAEA, Vienna, 1991), Vol. 2, p. 361.

[6]W. Horton, D. Lindberg, J-Y. Kim, J.Q. Dong, G.W. Hammett, S.D. Scott, M.C. Zarnstorff, and S. Hamaguchi, Phys. Fluids B **4**, 953 (1992); J-Y. Kim *et al.*, Phys. Fluids B **5**, 4030 (1993).

[7]J.W. Connor *et al.*, Plasma Phys. Control. Fusion **35**, 757 (1993); J.W. Connor and H.R. Wilson, *ibid* **36**, 719 (1994).

[8]F. Perkins *et al.*, Phys. Fluids B **5**, 477 (1993); J.P. Christiansen *et al.*, Nucl. Fusion **33**, 863 (1993); C.C. Petty *et al.*, Phys. Rev. Lett. **74**, 1763 (1995); H. Shirai *et al.*, to appear in J. Phys. Soc. Jpn. (1995).

[9]M.L. Lebrun *et al.*, Phys. Fluids B **5**, 752 (1993); S.E. Parker *et al.*, Phys. Rev. Lett. **71**, 2042 (1993); Y. Kishimoto *et al.*, to be published in Phys. Plasmas (1995).

[10]J-Y. Kim and M. Wakatani, Phys. Rev. Lett. **73**, 2200 (1994); J.B. Taylor and H.R. Wilson, UKAEA FUS Report No. 279 (1995).

[11]A.J. Wootton *et al.*, Phys. Fluids B **2**, 2879 (1990).

[12]Adopted from Fig. 8 in the paper by J-Y. Kim *et al.* in Ref. 6.

[13]D.I. Choi and W. Horton, Phys. Fluids **23**, 356 (1980).

[14]N. Mattor and P.H. Diamond, Phys. Rev. Lett. **72**, 486 (1994).

[15]M. Kotschenreuther *et al.*, Phys. Plasmas **2**, 2381 (1995).

[16]R.J. Fonck *et al.*, Phys. Rev. Lett. **70**, 3736 (1993); E. Mazzucatto, and R. Nazikian, Phys. Rev. Lett. **71**, 1840 (1993).

FIGURE CAPTIONS

FIG. 1. The observed and calculated ion thermal conductivities for the TFTR supershot 44669 discharge.

FIG. 2. The typical linear eigenmode structure of toroidal ITG mode, obtained from the particle simulation by Kishimoto *et al.* in Ref. 9.