JOURNAL OF GEOPHYSICAL RESEARCH, VOL. ???, XXXX, DOI:10.1029/,

Theory of Magnetized Rossby Waves in the Ionospheric E-layer

T.D. Kaladze, G.D. Aburjania, O.A. Kharshiladze

I. Vekua Institute of Applied Mathematics, Tbilisi State University, 2

University str., 0143 Tbilisi, Georgia
W. Horton and O. Sharia

Department of Physics and Institute for Fusion Studies, The University of

Texas at Austin, Austin, TX 78712 USA

T.D. Kaladze, G.D. Aburjania, and O.A. Kharshiladze, I. Vekua Institute of Applied Mathematics, Tbilisi State University, 2 University str., 0143 Tbilisi, Georgia.

W. Horton and O. Sharia, Department of Physics and Institute for Fusion Studies, The University of Texas at Austin, Austin, TX 78712 USA. (e-mail: horton@physics.utexas.edu)

Abstract. For the weakly-ionized E-layer plasma, a generalized Charney-Obukhov equation for the magnetized Rossby waves is derived. This magnetized Rossby wave is produced by the dynamo electric field and represents the ionospheric generalization of the tropospheric Rossby waves in a rotating atmosphere by the spatially inhomogeneous geomagnetic field. The basic characteristics of the wave are given. The modified Rossby velocity and Rossby-Obukhov radius are introduced. The mechanism of self-organization into solitary vortical nonlinear structures is examined. The mechanism of a self-organization of solitary structures is the result of the mutual compensation of wave dispersion and interaction through the scalar and Poisson bracket convective nonlinearities in the nonlinear wave equation. As a result, the solitary structures are anisotropic containing a circular vortex superimposed on a dipole perturbation. The degree of anisotropy sharply increases when the vortex size approaches the so-called intermediate geostrophic size.

1. Introduction

Planetary Rossby waves are of interest because of their significant influence on the global atmospheric circulation [*Pedlosky, 1987; Petviashvili and Pokhotelov, 1992*]. It is known that similar perturbations can exist in the ionospheric conductive layers. The majority of the ionospheric phenomena such as super-rotation of the Earth's atmosphere [*Rishbeth, 1972*], the ionospheric precursors of some extraordinary phenomena [*Haykowicz, 1991; Liperovsky et al., 1992*], and the ionospheric response on the anthropogenic activity [*Pokhotelov et al., 1995; Shaefer et al., 1999*] are in the frequency range of planetary waves. In reality, these perturbations display themselves as background oscillations. Recent observations show that forced oscillations of that kind appear in impulsive impacts on the ionosphere or during magnetospheric storms [*Haykowicz, 1991*]. They may also arise from earthquakes, volcano eruptions or man made activities [*Liperovsky et al., 1992; Cheng and Huang, 1992*]. In the latter case the corresponding perturbations appear in the form of solitary wave structures.

Large-scale planetary Rossby waves generated in the tropo-stratosphere from natural conditions penetrate to the ionospheric heights. The theoretical investigations [*Charney and Drazin, 1961; Dickinson, 1968*] of propagation of planetary scale wave perturbation from the lower layers to upper atmosphere show that the system of stable zonal winds partially screens (especially in summer) the upper atmosphere from the influence of these waves. The equinox periods when the turn of zonal winds takes place are the most propitious conditions for the upward penetration of the long (with the wavenumbers 1 or 2) planetary waves [*Charney and Drazin, 1961; Dickinson, 1969*].

The study of the generation and dynamics of planetary Rossby waves that are induced by the spatial inhomogeneity of the Earth's angular velocity in the ionospheric plasma has accordingly been a subject of a great deal of theoretical and experimental investigations in recent years. But the presence of charged particles in the ionosphere substantially enriches the class of possible low frequency wave modes in the magnetized ionosphere.

Dokuchaev [1959] first pointed out the importance of the induction electric current interaction with the Earth's magnetic field. He demonstrated that the action of the electromagnetic (EM) forces results in a deviation of the zonal flows from their geostrophic values. In the Earth's ionosphere the latitudinal gradient of the geomagnetic field can lead to such a deviation. *Tolstoy* [1967] includes this effect when considering Rossby type waves in the Earth's ionosphere. The SC waves were termed hydromagnetic gradient (HMG) waves. Tolstoy suggested that HMG waves may display themselves as traveling perturbations of the Sq current system and that they can produce strong variations of the geomagnetic field. Recent publications by the Kaladze group [*Kaladze and Tsamalashvili, 1997; Kaladze, 1998; Kaladze, 1999; Kaladze et al., 1999*] have developed the picture of nonlinear solitary vortex structures in the ionosphere arising from the latitudinal inhomogeneity of the geomagnetic field. Immersed in the geomagnetic field, these E-layer waves acquire an additional degree of freedom, and unlike of the usual Rossby waves can propagate either westward or eastward. In addition the inhomogeneity of the geomagnetic field results in the same facts as the topography of the bottom of the system: 1) decrease of the Rossby velocity and 2) opposes the Coriolis force vorticity. In order to distinguish them from the HMG waves they were termed magnetized Rossby (MR) waves [*Kaladze and Tsamalashvili, 1997; Kaladze, 1998; Kaladze, 1999*].

The mechanism of self-organization of the MR waves into vortical structures is produced by the"vectorial" nonlinearity or Poisson bracket nonlinearity from the convection of the vorticity. The nonlinear MR equations give the geomagnetic field modified analog of the Charney-Obukhov (ChO) equation describing the nonlinear Rossby waves in the geophysical hydrodynamics. The ChO equation describes nonlinear structures of a size that is less than order of the Rossby-Obukhov radius r_R, i.e.

$$a \leq r_R, \tag{1}$$

where

$$r_R = c_s/f = (gH_0)^{1/2}/f, \tag{2}$$

with f-Coriolis parameter, g-constant of gravity, H_0-equivalent depth of medium (atmosphere, ocean, shallow water in the experiments) measured along the normal to the free surface, c_s-equivalent sound speed (in atmosphere $H_0 = T/Mg$, where T is the temperature, M is the mass of molecules, and $c_s = (gH_0)^{1/2}$). The ChO equation describes only solitary (exponentially shielded in space perturbation) small-scale structures in the from of dipole vortex, i.e., a cyclone-anticyclone pair. This corresponds to the quasi-geostrophic approximation in geophysical hydrodynamics when the structures are being considered as purely two-dimensional, i.e., a perturbation of free surface of liquid is being considered either absent or negligibly small. Naturally, these nonlinear equations are symmetric with respect to the transformation of cyclone into anticyclone. Besides, in order to be long-lived, i.e., not to radiate the linear Rossby waves, the solitary vortical structure should propagate with the velocity lying out of the range of linear Rossby waves

velocities. It should be also noted that solitary monopole vortices (cyclone or anticyclone) are absent within the framework of ChO equation.

But for the planetary Rossby waves (with the wavelengths more them 1000 km) in the E-layer of ionosphere opposite to inequality (1), i.e.,

$$a \geq r_R, \tag{3}$$

is satisfied. The numerical value for the ionosphere $r_R \approx 1000\,\mathrm{km}$. These larger scale structures are more complex with an additional scalar or KdV nonlinearity.

These larger scale structures are considered in *Nycander* [1991]; *Su et al.* [1991]; *Nycander and Sutyrin* [1992]; *Nezlin and Snezhkin* [1993]; *Sutyrin* [1994]; *Chernikov* [1994]; see also *Nezlin and Chernikov* [1995] and cited references. The restriction to small scales was removed and large-scaled structures with sizes greater than Rossby-Obukhov radius were considered. This corresponds to the intermediate geostrophic (IG) approximation in the geophysical hydrodynamics where the perturbation of free surface of liquid is essential. The new IG-equations break the symmetry between cyclones and anticyclones and predict the existence of solitary waves (solitons) with monopole structure and only of a definite sign: either cyclones or anticyclones. The phenomenon is called as cyclone-anticyclone asymmetry. Solitary structures at first was found at the laboratory modeling of solitary Rossby vortices [*Antipov et al., 1982*].

Another phenomenon called "twisting" should be mentioned, and was described by *Williams and Yamagata* [1984] and *Romanova and Ceitlin* [1984]. This "twisting" condition requires that the structure must rotate faster than it propagates as the whole. This means that in current lines system moving with the vortex, a separatrix appears inside of fluid particles of medium are captured. In other words, the structure turns into a "real

vortex" carrying with the medium's particles. The properties of structures change radically, namely the characteristic transverse size of structure ceases to be connected with an amplitude (in contrast with the planar soliton of Korteweg-de Vries (KdV) type, which width is inversely proportional of amplitude) and collisions of the same structures became nonelastic.

With two types of nonlinearity working to balance wave dispersion, the standard KdV soliton picture changes. In a solitary wave (soliton) of the KdV type, dispersive spreading is balanced by a nonlinear steepening. The nonlinear steepening of wave packet is stopped by dispersion. This balance yields the definite relation between the characteristic width of packet and its amplitude: the larger the amplitude is (nonlinearity is stronger), the smaller width is (dispersion is stronger). The laboratory experiment [*Antipov et al. 1982*] did not show this feature. The situation in the laboratory experiments [*Antipov et al. 1982*] showed that the characteristic size of the structures turned out to be independent of the amplitude. This observation points to a more complicated dynamics of the structure formation than the simple compensation of Rossby wave dispersion by the nonlinearity of KdV type. The solitary vortical structure can be self-organized by the new type of mechanism when in the process the wave dispersion is competing with two nonlinearities: the "scalar" (KdV nonlinearity) and the "vectorial" nonlinearity [*Nezlin and Chernikov, 1995*]. Namely, if at a fixed size of the structure its amplitude is "too" high, i.e., scalar nonlinearity predominates over dispersion then the missing dispersive compensation of the KdV nonlinearity is provided by the "vectorial" nonlinearity which prevents the structure under consideration from an unrestricted nonlinear steepening. In this case vectorial nonlinearity "works" against scalar one. At other case when at fixed size of structure

its amplitude turned out too small so that the dispersion predominates over the scalar nonlinearity then the missing compensation of dispersion by scalar nonlinearity is covering by the vectorial nonlinearity. In this case vectorial nonlinearity works together with scalar one.

So it is evident that large-scale nonlinear solitary vortical structures have the new properties. Obtaining of magnetized Rossby (MR) equation for waves in the E-layer of the ionosphere and investigation of their dynamic conditions is the main goal of this work.

2. Physical Modeling of the Ionospheric Motions in the E-layer

Let us consider the weakly ionized ionospheric gas of the E-layer consisting of electrons, ions and neutral particles. The behavior of such a weakly-ionized gas on the whole is determined by its massive neutral component because of the strong collisional coupling between the ionized component n with the neutral component N. In the E-layer, the gas satisfies the condition $n/N \ll 1$ where n and N are the equilibrium number densities of charged particles and neutrals, respectively. The presence of charged particles produces the electrical conductivity of the medium. For the typical ionization fraction in the E-layer, the Lorentz force will be comparable to the Coriolis force acting on the weakly ionized gas. In this case we must take into account both the influence of spatially inhomogeneous geomagnetic field $\boldsymbol{B}(\boldsymbol{x})$ and vertical component of rotation $\boldsymbol{\Omega}$ of the Earth, which are inherent to the ionosphere.

We describe the dynamics of the ionospheric gas by the following hydrodynamic equation of motion,

$$\frac{d\boldsymbol{v}}{dt} \equiv \frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\frac{\nabla p}{\rho} + \frac{\boldsymbol{F}_A}{\rho} + 2\boldsymbol{v}\times\boldsymbol{\Omega} + \boldsymbol{g} + \nu\nabla^2\boldsymbol{v} \tag{4}$$

derived adding the momentum balance equations for the strongly-coupled neutral component and the ionization component. Here $\boldsymbol{v}$ is velocity of the neutral gas, $\rho = Nm_N$ is the neutral gas mass density, m_N is the mean mass of the neutrals, p is the gas pressure, $\boldsymbol{g}$ is the gravitational acceleration and ν is the kinematic viscosity. In (4) $\boldsymbol{F}_A = \boldsymbol{j} \times \boldsymbol{B}$ stands for the Ampere force, where $\boldsymbol{j}$ is the inductive electric current density and $\boldsymbol{B} = \boldsymbol{B}_0 + \delta\boldsymbol{B}$ is the total geomagnetic field consisting of the spatially inhomogeneous equilibrium $\boldsymbol{B}_0$ field and its perturbation $\delta\boldsymbol{B}$. Accounting of the electromagnetic Ampere force $\boldsymbol{F}_A$ is the principal physical effect in the problem, since it significantly influences the character of the ionospheric motions. In (4) the Coriolis acceleration is $2\boldsymbol{v} \times \boldsymbol{\Omega}$, where $\boldsymbol{\Omega}$ is the angular velocity of the Earth's rotation.

It is known that the momentum equations regulate the effects of the E-region neutral winds upon the ionospheric currents. When the charged particle pressure gradients and gravity forces are neglected, one obtains for the ion and electron drifts [cf. *Kamide, 1988*]

$$\boldsymbol{v}_i = \frac{\omega_{ci}^2}{\omega_{ci}^2 + \nu_i^2}\left(\frac{\boldsymbol{E}' \times \boldsymbol{B}}{B^2} + \frac{\nu_i}{\omega_{ci}}\frac{\boldsymbol{E}'}{B}\right) + \frac{\omega_{ci}}{\nu_i}\frac{(\boldsymbol{E}' \cdot \boldsymbol{B})\boldsymbol{B}}{B^3} + \boldsymbol{v}, \tag{5}$$

$$\boldsymbol{v}_e = \frac{\omega_{ce}^2}{\omega_{ce}^2 + \nu_e^2}\left(\frac{\boldsymbol{E}' \times \boldsymbol{B}}{B^2} - \frac{\nu_e}{\omega_{ce}}\frac{\boldsymbol{E}'}{B}\right) - \frac{\omega_{ce}}{\nu_e}\frac{(\boldsymbol{E}' \cdot \boldsymbol{B})\boldsymbol{B}}{B^3} + \boldsymbol{v}, \tag{6}$$

where $\boldsymbol{v}_i$ and $\boldsymbol{v}_e$ are the electron and ion velocities, μ_e and μ_i are the electron-neutral and ion-neutral collision frequencies, $\omega_{ci,e} = eB/m_{i,e}$ is the (ion or electron) cyclotron frequency, e is the magnitude of the electron charge, $m_{i,e}$ is the (ion or electron) mass. The electron-ion collisions are of little importance and thus neglected. When taking into account the motion of medium the electric field is defined by the following expression

$$\boldsymbol{E}' = \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = \boldsymbol{E} + \boldsymbol{E}_d, \tag{7}$$

where $\boldsymbol{E}$ is the electric field and $\boldsymbol{E}_d = \boldsymbol{v} \times \boldsymbol{B}$ is called a dynamo-field.

The electric current induced by the relative motion of ions and electrons in the ionosphere has the well known form [*Kelley, 1989*]

$$\boldsymbol{j} = en(\boldsymbol{v}_i - \boldsymbol{v}_e) = \sigma_\parallel \frac{(\boldsymbol{E}' \cdot \boldsymbol{B})\boldsymbol{B}}{B^2} + \sigma_P \boldsymbol{E}'_\perp + \sigma_H \frac{\boldsymbol{B} \times \boldsymbol{E}'}{B}, \tag{8}$$

where σ_P and σ_H are Pedersen and Hall conductivities

$$\sigma_\parallel = \frac{en}{B}\left(\frac{\omega_{ci}}{\nu_i} + \frac{\omega_{ce}}{\nu_e}\right), \tag{9}$$

$$\sigma_P = \frac{en}{B}\left(\frac{\omega_{ce}\nu_e}{\omega_{ce}^2 + \nu_e^2} + \frac{\omega_{ci}\nu_i}{\omega_{ci}^2 + \nu_i^2}\right), \tag{10}$$

and

$$\sigma_H = \frac{en}{B}\left(\frac{\omega_{ce}^2}{\omega_{ce}^2 + \nu_e^2} - \frac{\omega_{ci}^2}{\omega_{ci}^2 + \nu_i^2}\right). \tag{11}$$

The plasma conditions in the E-region allow us to make some simplifications of the expression for the electric current. First, $\omega_{ci}/\nu_i \ll 1$, and thus the ions can be considered as unmagnetized. According to (5), their velocities across the magnetic field coincide with the wind velocity, $\boldsymbol{v}_i = \boldsymbol{v}$, and thus the ions are completely dragged by the ionospheric winds. In this limiting case the Hall conductivity is $\sigma_P \simeq \sigma_H \omega_{ci}/\nu_i \ll \sigma_H$. This allows us in the E-layer to neglect the ion friction caused by the Pedersen conductivity. On the contrary, the electrons are magnetized, $\omega_{ce}/\nu_e \gg 1$, and thus they are frozen in the external magnetic field. The electrons undergo solely the drift perpendicular to the magnetic field, i.e., $\boldsymbol{v}_e = \boldsymbol{v}_E = (\boldsymbol{E} \times \boldsymbol{B})/B^2$.

Due to the high mobility of the electrons along the magnetic field lines the parallel conductivity of the plasma is high, $\sigma_\parallel \approx \sigma_P \omega_{ce}/\nu_e \gg \sigma_P$. That condition serves to shortcut the electric field along the magnetic field $\boldsymbol{B}$ and imposes a vanishing field-aligned electric current. The latter condition results in a rather simple expression for the electric current

in the E-layer

$$\boldsymbol{j} = en(\boldsymbol{v} - \boldsymbol{v}_E). \tag{12}$$

Let us estimate the value of vortical part of the field $\boldsymbol{E}$ in (7). Introducing the characteristic scale of variation L, the characteristic time T and the characteristic scale of velocity U from the Faraday's law $\nabla \times \boldsymbol{E} = -\partial\delta\boldsymbol{B}/\partial t$ we find that

$$E \sim \frac{L\delta B}{T} \sim U\delta B. \tag{13}$$

Since $\delta B \ll B_0$, then the rotational (vortical) part of $\boldsymbol{E}$ is much less than the quantity $\boldsymbol{E}_d = \boldsymbol{v} \times \boldsymbol{B}$ in (7). Thus, we can derive $\boldsymbol{E} = -\nabla\varphi$.

As to the electrostatic part $\boldsymbol{E} = -\nabla\varphi$, it differs from zero only in the presence of free charges which cannot exist in conductive medium for a long time. Therefore, ionospheric plasma always may be considered with great degree of accuracy as quasineutral. Thus we neglect the electrostatic and vortical parts of the electric field leaving $\boldsymbol{E}_d$ in Eq. (7). In (8) the electric field will be equal to the dynamo-field, i.e., [*Dokuchaev, 1959*]

$$\boldsymbol{E}' = \boldsymbol{E}_d = \boldsymbol{v} \times \boldsymbol{B}_0, \tag{14}$$

Accordingly for the current density (12) in the E-layer we obtain

$$\boldsymbol{j} = en\boldsymbol{v}. \tag{15}$$

Of course we get the same result for $\boldsymbol{j}$ when using expressions (8)-(11) under conditions of the ionospheric E-layer.

The approximation developed here is known as the noninductive approximation when the magnetic Reynolds number $R_m = LU\mu_0\sigma_H \ll 1$ where μ_0 is the permeability of free space. This condition is fulfilled sufficiently well in the ionospheric E-layer. Indeed

for the E-layer with perturbations $L \sim 10^3\,\mathrm{km}$, $U \sim 10^2\,\mathrm{ms}^{-1}$, $\sigma_H \cong 3 \cdot 10^{-4}$ S/m, $\mu_0 = 4\pi \cdot 10^{-7}\,\mathrm{N/A}^2$ we get $R_m \sim 10^{-2}$. In the noninductive approximation it is enough to consider only currents arising in the gas neglecting self-generated magnetic fields. That is, the magnetic field is equal to the external geomagnetic field $\boldsymbol{B}_0$.

Let us now estimate the ratio of volumetric electromagnetic $\boldsymbol{j} \times \boldsymbol{B}$ force compared with the viscous friction force which is characterized by the square of the Hartmann number

$$\mathrm{Ha}^2 = \frac{\sigma_H B_0^2 L^2}{\eta}, \tag{16}$$

where $\eta = \nu\rho$ is a dynamic viscosity. For the ionospheric E-layer taking $B_0 \cong 0.5 \cdot 10^{-4}\,\mathrm{T}$ and $\eta \cong 10^{-5}\,\mathrm{kg/s{\cdot}m}$ we get $\mathrm{Ha}^2 \approx 10^5$. Thus, one may consider that for large-scale motions electromagnetic force is greater than viscous force $\eta\nabla^2\boldsymbol{v}$. Thus, we neglect the last term in (4).

Thus, under the noninductive approximation for the ionospheric E-layer, the equation of motion (4) reduces to the following form:

$$\frac{d\boldsymbol{v}}{dt} \cong \frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{\nabla p}{\rho} + \frac{en}{\rho}(\boldsymbol{v} \times \boldsymbol{B}_0) + 2(\boldsymbol{v} \times \boldsymbol{\Omega}) + \boldsymbol{g} \tag{17}$$

where the Lorentz force on the ionized component couples through the strong ion-neutral collisions to the neutral gas component. In Eq. (17) we see that the influence of the geomagnetic field comes to the following additive replacement of the vectorial Coriolis parameter

$$2\boldsymbol{\Omega} \to 2\boldsymbol{\Omega} + \frac{en}{\rho}\boldsymbol{B}_0 \tag{18}$$

to produce a more complex acceleration, depending on the degree of ionization. For the E-layer the ionization fraction is $n/N \sim 10^{-7} \div 10^{-6}$, and thus $(en/\rho)B_0 \approx (n/N)10^2/s \simeq 10^{-4}/s$ comparable to $2\Omega_0 \sim 10^{-4}s^{-1}$. The E-layer, Eq. (17), indicates that in the

ionospheric plasma new branches of large-scale oscillations owing to the spatial inhomogeneity of the geomagnetic field $\boldsymbol{B}_0$ are expected as the usual planetary Rossby waves owing to the inhomogeneity of the Earth's vertical rotation.

Since the motions under consideration are slow compared with sound waves, we add the condition of incompressibility to the system.

The vertical component of Eq. (17) is restricted to the E-layer where the ionization and collisionality are such as to justify the derivation of the equation. We assume that the E-layer is a stably stratified layer with a Brunt-Väisälä buoyancy frequency $N^2 = -(g/\rho)(d\rho/dz)$ that is fast compared with the large-scale horizontal motion of the magnetized Rossby wave. This condition allows the condition of three-dimensional incompressibility on the long-time scales of the magnetized Rossby waves to be expressed as in shallow water theory with the effect height $H(x, y, t)$ of shallow water theory replaced by the thickness of the E-layer. Derivation of the equivalence is beyond the scope of the present work, but involves time averaging over the rapid vertical oscillations given by

$$\frac{\partial^2 w}{\partial t^2} + N^2 w = -\frac{1}{\rho}\frac{\partial^2 \delta p}{\partial z \partial t} \tag{19}$$

where $w = dH/dt$ is the vertical velocity and δp is the perturbed pressure (*Gill, pp. 129 and 529, 1982*). For large horizontal scales $k_x \sim k_y \ll 1/H_E$ where $H_E \lesssim 10\,\mathrm{km}$ the thickness of the E-layer, the decoupling of the fast vertical oscillations from the large-scale horizontal waves is a good approximation.

3. Generalized Wave Equation for the Magnetized Rossby Waves

In this chapter we will derive approximate equations for large-scale vortices containing new effects in comparison with the "quasi-geostrophic" equations, namely: the existence

of monopole solitary structures from the cyclone-anticyclone asymmetry. Their value consist in simple physical interpretation of sharply expressed prevalence of anticyclones in system of large-scale, long-lived vortices on giant planets where there is enough size for them. We give here simple derivation of the generalized magnetized Rossby wave equation starting from the approximation of geostrophic velocities.

Let us consider the ionospheric plasma which is immersed in a dipole magnetic field $\boldsymbol{B}_0$, assuming that the wave motions in the E-layer are localized in the vicinity of a given latitude $\lambda = \lambda_0$. We introduce a local Cartesian system of coordinates (x, y, z) with latitudinal, $y = (\lambda - \lambda_0)R$, and longitudinal, $x = \varphi R \cos\lambda_0$, coordinates. The z-axis of our system coincides with the local vertical direction. Furthermore, φ is the longitude and R is distance from the Earth's center. In this coordinate system the x-axis is directed from the west to the east and the y-axis is directed from the south to the north. The corresponding derivatives with respect to the horizontal coordinates are given by $\partial/\partial\varphi = R\cos\lambda_0 \partial/\partial x$ and $\partial/\partial\lambda = R\partial/\partial y$. The modulus of the dipole geomagnetic field is defined by the well-known expression $B_0 = B_{\rm eq}(1 + 3\sin^2\lambda)^{1/2}$, whereas, the local components of the geomagnetic field vector are

$$B_{0x} = 0 \tag{20}$$

$$B_{0y} = B_{\rm eq}\left(1 - \frac{2z}{R}\right)\cos\lambda, \tag{21}$$

and

$$B_{0z} = -2B_{\rm eq}\left(1 - \frac{z}{2R}\right)\sin\lambda, \tag{22}$$

where $B_{\rm eq}$ is the equatorial value of the geomagnetic field at a distance R from the Earth's center. Using (20)-(22), one can easily verify that $\nabla \cdot \boldsymbol{B}_0 = \nabla \times \boldsymbol{B}_0 = 0$. Equations (20)-

(22) are obtained by expansion of the dipole magnetic field in powers of z/R. Thus, these expressions are local formulas valid when $z/R \ll 1$.

As to the angular, velocity of the Earth's rotation $\mathbf{\Omega}$ in the local system of coordinates, we have

$$\mathbf{\Omega} = (0, \Omega_{0y},\ \Omega_{0z}) = (0, \Omega_0 \cos\lambda,\ \Omega_0 \sin\lambda)\,. \tag{23}$$

The vertical component of the rotation frequency, Ω_{0z}, has the northward gradient $\partial_t \Omega_{0z}/\partial y = \Omega_0 \cos\lambda/R$.

From equations (17) and (18) we can get the well-known Ertel Theorem of potential vorticity conservation on shallow water [*Pedlosky, 1987*], which after the using of the replacement (19) will have the following form

$$\frac{d}{dt}\left(\frac{\zeta_z + 2\Omega_{0z} + \frac{en}{\rho}B_{0z}}{H}\right) = 0. \tag{24}$$

Here

$$\zeta_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{25}$$

is the z-component of the velocity $\zeta = \hat{\boldsymbol{z}}\nabla \times \boldsymbol{v}, u$ and v are the horizontal velocity components along the coordinate axes x and y. The operator ∇ is two-dimensional in the x-y plane, i.e.

$$\nabla = \hat{\boldsymbol{x}}\frac{\partial}{\partial x} + \hat{\boldsymbol{y}}\frac{\partial}{\partial y},$$

and the operator d/dt is also spatially two-dimensional and means

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}.$$

Furthermore H is the effective height of E-gas layer which has a displacement in the vertical ($\hat{\boldsymbol{z}}$) direction that follows from the height-integrated mass continuity equation.

Mass conservation then yields the relation for the horizontal divergence of the flow related to the charge thickness of the layer

$$\nabla \cdot \boldsymbol{v} = -\frac{d}{dt} \ln H, \tag{26}$$

The quantity H also represents the surface mass density of the E-layer.

Consider a shallow E-layer having an uniform thickness H_0 in its stationary state. Assume that a perturbation of a free surface occurs, so that the total depth of the gas varies horizontally as

$$H(x, y, t) = H_0 + \delta H(x, y, t) \equiv H_0(1 + h). \tag{27}$$

Furthermore in the vicinity of the latitude $\lambda = \lambda_0$ we use the so-called "β-approximation" and represent the Coriolis parameter as

$$f = 2\Omega_{0z} = 2\Omega_0 \sin\lambda = 2\Omega_0 \left(\sin\lambda_0 + \Delta\lambda \cos\lambda_0\right) = f_0 + \beta y \tag{28}$$

$$f_0 = 2\Omega_0 \sin\lambda_0 > 0, \qquad \beta = \frac{\partial f}{\partial y} = \frac{1}{R}\frac{\partial f}{\partial \lambda} = \frac{2\Omega_0 \cos\lambda_0}{R} > 0, \tag{29}$$

and analogously the geomagnetic field parameter varies as

$$\gamma = \frac{en}{\rho} B_{0z} = -\frac{2en}{\rho} B_{\mathrm{eq}} \sin\lambda = \gamma_0 + \alpha y, \tag{30}$$

$$\gamma_0 = -\frac{2en}{\rho} B_{\mathrm{eq}} \sin\lambda_0 < 0, \quad \alpha = \frac{\partial \gamma}{\partial y} = \frac{en}{\rho}\left(\frac{\partial B_{0z}}{\partial y}\right) = -\frac{2enB_{\mathrm{eq}}}{\rho R} \cos\lambda_0 < 0. \tag{31}$$

The quantities α, β, f_0 and γ_0 are related to the latitude $\lambda = \lambda_0$ where assumed $y = 0$.

Using the expressions (27)-(31) we obtain from (24)

$$(1 + h)\frac{d\zeta_z}{dt} + (\alpha + \beta)v(1 + h) - \left[\zeta_z + f_0 + \gamma_0 + (\alpha + \beta)y\right]\frac{dh}{dt} = 0. \tag{32}$$

where $h = \delta H/H_0$. To summarize (28)-(3.12), give $\alpha\beta < 0$ and $f_0\gamma_0 < 0$. Regions where $f_0 + \gamma_0 \approx 0$ are excluded.

The value $(\alpha + \beta)$ in (32) represents the generalized Rossby parameter where α corresponds to the contribution from the Hall effect. It should be noted that in the dynamo-region the α and β terms have opposite sign but are comparable in value, i.e. $\beta \approx -\alpha \approx 10^{-11} m^{-1} s^{-1}$.

Now, for simplicity we consider a geostrophic equilibrium (i.e., the Rossby regime when the Rossby number R_0 is small):

$$u = -\frac{g}{f_0 + \gamma_0} \frac{\partial \delta H}{\partial y}, \qquad v = \frac{g}{f_0 + \gamma_0} \frac{\partial \delta H}{\partial x}, \tag{33}$$

and, consequently

$$\zeta_z = \frac{g}{f_0 + \gamma_0} \nabla^2 (\delta H), \tag{34}$$

where ∇^2 is a also two-dimensional Laplacian in the x-y plane, g is the free-fall acceleration. Within the Rossby geostrophic flow regime (33)

$$\frac{dh}{dt} = \frac{\partial h}{\partial t}$$

and, according to (33) and (34) we obtain

$$\frac{d}{dt} \zeta_z = \frac{g H_0}{f_0 + \gamma_0} \frac{\partial}{\partial t} (\nabla^2 h) + \left(\frac{g H_0}{f_0 + \gamma_0} \right)^2 J(h, \nabla^2 h), \tag{35}$$

where

$$J(a, b) = \frac{\partial a}{\partial x} \frac{\partial b}{\partial y} - \frac{\partial a}{\partial y} \frac{\partial b}{\partial x}$$

is Jacobian. The origin of $J(a, b)$ is the convection by Eq. (33) of the vorticity given by (34). Since the flow in Eq. (33) has the Hamiltonian structure with p, q-canonical coordinates $= x, y$, the convective derivative is the Poisson bracket with effective Hamiltonian $\delta H = H_0 \, h$.

Using the obtained expressions from (32) we get the following

$$\frac{\partial}{\partial t}\left[\nabla^2 h - \frac{(f+\gamma)(f_0+\gamma_0)}{gH_0}h\right] + (\alpha+\beta)\frac{\partial h}{\partial x} + (\alpha+\beta)h\frac{\partial h}{\partial x} + \frac{gH_0}{f_0+\gamma_0}J(h,\nabla^2 h)$$
$$+ h\frac{\partial}{\partial t}(\nabla^2 h) - (\nabla^2 h)\frac{\partial h}{\partial t}\frac{gH_0}{f_0+\gamma_0}hJ(h,\nabla^2 h) = 0. \tag{36}$$

Equation (36) is the analog of the neutral fluid generalized Charney-Obukhov equation (cf. with the analogous equations in *Nezlin* [1994]; *Nezlin and Chernikov* [1995]).

We will now analyze this equation. First of all we note that the last term in (36) is cubic with respect to h and thus small in comparison with the analogous, but the quadratic last term in the first line of (36). We neglect also the first two terms in the second line of (36) in comparison with the first term of the first line. Strictly speaking neglecting of these terms is not quite correct. But on the other hand these terms are not related with the $(\alpha+\beta)$-effect when the paper deals with structures dependent on the existence of the $(\alpha+\beta)$-effect.

Thus the generalized magnetized Rossby wave equation for the magnetized Rossby waves has the following form

$$\frac{\partial}{\partial t}\left(\nabla^2 h - \frac{1}{r_R^2}h\right) + (\alpha+\beta)\frac{\partial h}{\partial x} + (\alpha+\beta)h\frac{\partial h}{\partial x} + \frac{gH_0}{f_0+\gamma_0}J(h,\nabla^2 h) = 0, \tag{37}$$

or

$$\frac{\partial}{\partial t}\left(h - r_R^2\nabla^2 h\right) + v_R\frac{\partial h}{\partial x} + v_R h\frac{\partial h}{\partial x} - \frac{gH_0}{f_0+\gamma_0}r_R^2 J(h,\nabla^2 h) = 0. \tag{38}$$

Here

$$r_R(y) = \left[\frac{gH_0}{(f+\gamma)(f_0+\gamma_0)}\right]^{1/2}, \tag{39}$$

is the modified by geomagnetic field the barotropic Rossby-Obukhov radius for the ionospheric E-layer and

$$v_R(y) = -(\alpha + \beta) r_R^2 = -r_R^2 \frac{\partial}{\partial y}(f + \gamma) \tag{40}$$

is the respective Rossby velocity for the ionospheric E-layer. It should be noted that first the generalized Charney-Obukhov equation (37) for the Rossby waves ($\alpha = \gamma = 0$) was derived in *Petviashvili* [1980].

It is useful to elucidate the origin of last two quadratic terms in (37). As seen from (24) the first nonlinear $(\alpha + \beta) h \partial h / \partial x$ term, which we call the "scalar" or the "KdV-type" nonlinearity, arises from the product of the free-surface elevation h and $(\alpha + \beta)$-effect. As to the second nonlinear $gH_0 J(h, \nabla^2 h)/(f_0 + \gamma_0)$, term, which we call the "vectorial" or "Jacobian" nonlinearity, is implied by the expression $(\boldsymbol{v} \cdot \nabla)\boldsymbol{v}$ in the Euler equation and occurs always for a rotating fluid even in case of the rigid surface. Both types of nonlinearities have been extensively studied and are known to be characteristic of Hamiltonian field equations [*Morrison, 1982*].

It should be noted that the equation (37) and (38) are similar to the corresponding ones obtained in *Nezlin and Chernikov* [1995] on account of inclusion of the topography of system's bottom except for the coefficient at the "scalar" nonlinearity $h \partial h / \partial x$. This difference is very essential when determining the type of vortical structure (see chapter 5).

The generalized MR Equation (37) is valid for the arbitrary relations between the characteristic size of considered nonlinear structures a and the Rossby-Obukhov radius r_R. In case of small-scale structures when the inequality (1) is valid from (37) we get the

usual quasi-geostrophic equation

$$\frac{\partial}{\partial t}\nabla^2 h + (\alpha+\beta)\frac{\partial h}{\partial x} + \frac{gH_0}{f_0+\gamma_0}J(h,\nabla^2 h) = 0, \tag{41}$$

which describes the solitary dipole vortices in the ionospheric E-layer. In case of large-scale structures (3) Eq. (37) yields to

$$\frac{\partial}{\partial t}\left(\nabla^2 h - \frac{1}{r_R^2}h\right) + (\alpha+\beta)\frac{\partial h}{\partial x} + (\alpha+\beta)h\frac{\partial h}{\partial x} = 0, \tag{42}$$

which describes the solitary monopole vortices of the ionospheric E-layer.

In Eq. (37) the coefficient r_R^{-2} is dependent on the meridional coordinate y. So the maximal velocity of the linear Rossby waves (Rossby velocity) also will depend on y [see (40)]. Therefore that anti-twisting condition should be found when the inhomogeneity of coefficient in Eq. (37) cannot influence on the possibility of existence of stationary propagating localized structure.

From Eq. (37) we find the following dispersion equation of the linear magnetized Rossby waves

$$\frac{\omega}{k_x} = -\frac{\alpha+\beta}{k_x^2+k_y^2+\frac{1}{r_R^2}}, \tag{43}$$

where ω is the wave frequency, k_x and k_y are components of wave vector $\boldsymbol{k}$ along x and y axes accordingly; Rossby-Obukhov radius r_R is defined by (39). Thus the Rossby velocity v_R (40) is defined by the relation

$$\left(\frac{\omega}{k_x}\right)_{\max} \equiv v_R. \tag{44}$$

According to (40), v_R is not constant:

$$\begin{aligned} v_R(y) &= (\alpha+\beta)\frac{gH_0}{(f+\gamma)(f_0+\gamma_0)} = -\frac{(\alpha+\beta)gH_0}{(f_0+\gamma_0)(f_0+\gamma_0+\alpha y+\beta y)} \\ &= -\frac{(\alpha+\beta)gH_0}{(f_0+\gamma_0)^2}\left(1-\frac{\alpha+\beta}{f_0+\gamma_0}y\right) = v_R(0)\left(1-\frac{\alpha+\beta}{f_0+\gamma_0}y\right). \end{aligned} \tag{45}$$

This phenomenon shearing of the Rossby speed is called "twisting" by *Williams and Yamagata* [1984] and *Romanova and Ceitlin* [1984]. Here

$$v_R(0) = -(\alpha+\beta)r_R^2(0), \qquad r_R(0) = \frac{(gH_0)^{1/2}}{|f_0+\gamma_0|}, \tag{46}$$

and

$$\frac{dv_R(0)}{dy} = -v_R(0)(\alpha+\beta)/(f_0+\gamma_0).$$

are Rossby velocity and Rossby-Obukhov radius at the latitude $\lambda = \lambda_0$ (at the point $y = 0$), respectively.

In order for the nonlinear solitary drift structure not to be in the Cerenkov resonance with linear waves at no point of structure it is necessary that its drift velocity to exceed the Rossby velocity $v_R(y)$ at all points within the structure. Practically for a structure of meridional extent $|y| = 2a$, condition is satisfied in $v_R(-a < y < a)$ it is enough that two characteristic size a, i.e., from the structure's center. In addition when an amplitude of perturbation in structure falls exponentially the Cerenkov resonance will be possible only in the exponential "tail" of the structure [*Meiss and Horton, 1983*].

Let us see what the pointed requirement means. The drift velocity of structure increases with its amplitude and is connected with the maximal relative perturbation h_0 of the level of free surface with the relation

$$v_D = v_R(0)\,(1 + kh_0)\,. \tag{47}$$

Here according to *Nezlin and Chernikov* [1995], the coefficient $k = 0.5$ may be taken for obvious physical reasons.

Then the condition of absence of the Cerenkov resonance with linear magnetized Rossby waves takes the from

$$|v_R(0)\,(1+0.5h_0)| > \left|v_R(0)\left(1-\frac{\alpha+\beta}{f_0+\gamma_0}2a\right)\right|. \tag{48}$$

We will take into account also the estimation (33) for velocity of liquid rotation in the structure being under consideration

$$v_{rt} = \frac{gH_0}{|f_0+\gamma_0|a}h_0.$$

Then from (48) we obtain the negligible twisting condition of the following form

$$v_{rt} > 4\,|v_R(0)|\,. \tag{49}$$

According to what was mentioned above on structures satisfying this condition, the inconstancy of coefficients in Eqs. (37) and (38) doesn't influence. All our examinations concern just such structures and therefore Eqs. (37) and (38) further are considered as equations with constant coefficients (significance of magnitudes r_R and v_R are taken at the point $y = 0$ i.e., at the latitude $\lambda = \lambda_0$). From the condition (49) is seen that the characteristic (maximal) velocity of structure's rotation should several times exceed the Rossby velocity $v_R(0)$. Strictly speaking the maximal velocity of rotation on velocity profile of structure must exceed the drift velocity of structure as whole. Namely, this condition is called "anti-twisting" condition. But when at any place of structure's profile a rotation velocity exceed a drift velocity of structure as whole then, as it known, in system of streamlines of structure the separatrix will appear inside of which the streamlines become closed. Structures satisfying the condition (49) may be called as "real" vortices. Such objects are not only carriers of vorticity, but in addition carry with captured fluid particles of the medium.

From the expressions of the geostrophic equilibrium (33) we can rewrite the anti-twisting condition (49) as

$$h \gg \frac{|\alpha+\beta|}{|f_0+\gamma_0|}a, \tag{50}$$

i.e., only sufficiently high amplitude vortices satisfy the anti-twisting condition (50). Necessity of stationary drift structure to be "real vortex" and in addition to satisfy pointed anti-twisting condition (when $\alpha = \gamma_0 = 0$) first was shown in the laboratory experiments [*Antipov et al., 1982; Nezlin et al., 1990; Nezlin and Snezhkin, 1993*].

Now we can introduce the following dimensionless variables

$$|f_0+\gamma_0|t \equiv t, \tag{51}$$

$$\frac{(x,y)}{r_R(0)} \equiv (x,y), \tag{52}$$

$$\frac{v_R(0)}{c_g} \equiv v_R, \tag{53}$$

where $c_g = (gH_0)^{1/2}$ is the gravity wave speed. Then the generalized Charney-Obukhov equation (38) for the magnetized Rossby waves in dimensionless variables has the following form

$$\frac{\partial}{\partial t}(h-\nabla^2 h) + v_R\frac{\partial h}{\partial x} + v_R h\frac{\partial h}{\partial x} - \mathrm{sgn}(f_0+\gamma_0)J(h,\nabla^2 h) \equiv 0. \tag{54}$$

From (33) follows the estimation

$$h = Ro\,\frac{a}{r_R^2}, \tag{55}$$

where $\boldsymbol{Ro}$ is the Rossby number. This means that when $a \ll r_R$, the effect from free surface becomes negligible and the generalized equation (37) for the magnetized Rossby

waves reduces to the classic equation

$$\frac{\partial}{\partial t}\nabla^2\psi + \Big(\alpha + \beta\Big)\frac{\partial\psi}{\partial x} + J(\psi, \nabla^2\psi) = 0, \tag{56}$$

where ψ is the stream function:

$$\psi = \frac{gH_0}{f_0 + \gamma_0}h, \quad u = -\frac{\partial\psi}{\partial y}, \quad v = \frac{\partial\psi}{\partial x}. \tag{57}$$

In particular, if a liquid surface has a rigid cover [*Meyers et al., 1989*] so that the gravity wave speed $c_g \to \infty$ and according to (46) and (55) $r_R \to \infty$ and $h = 0$, then only two last relations of (57) along with (56) have sense of consideration [*Kaladze and Tsamalashvili, 1997; Kaladze, 1998; Kaladze, 1999*].

4. Linear Magnetized Rossby Waves in the Ionospheric *E*-layer

The dispersion relation for the linear magnetized Rossby waves is given by (43). One can easily verify that in the absence of the term $\propto r_R^{-2}$ i.e., when the free surface is not taken into account this dispersion relation is identical to that for the hydromagnetic gradient (HMG) mode [*Tolstoy, 1967*] and to those for the so-called magnetized Rossby waves [*Kaladze and Tsamalashvili, 1997; Kaladze, 1998; Kaladze, 1999*]. It is seen that the linear magnetized Rossby waves possess negative dispersion: their phase velocity ω/k_x decreases with an increase in the wavenumber $|\boldsymbol{k}|$ as shown in Fig. 1. Their maximal phase velocity is equal to the modified by geomagnetic field Rossby velocity for the ionospheric E-layer [see (44), and (46)]. It should be noted that the dispersion relation (43) is valid for the wave lengths from the interval $H_0 \ll k_\perp^{-1} \ll R$, where R is the Earth's radius. The dispersion equation (43) is illustrated by Fig. 1.

Fig. 1

According to (46) the modified Rossby velocity

$$v_R = v_R(0) = -(\beta + \alpha) r_R^2(0) = v_{R1} + v_{R2} = gH_0 \frac{d}{dy}\left(\frac{1}{f+\gamma}\right) \tag{58}$$

consist of two parts, where

$$v_{R1} = -\beta r_R < 0 \tag{59}$$

is the Rossby velocity produced by the gradient of Coriolis parameter f [see (29)], and

$$v_{R2} = -\alpha r_R^2 > 0 \tag{60}$$

is the Rossby velocity part stipulated by the gradient of geomagnetic field parameter [see (31)]. The analogous part occurs when considering the topography of system's bottom [*Nezlin and Chernikov, 1995*].

The ratio of these terms gives

$$\frac{|v_{R2}|}{|v_{R1}|} = \frac{|\alpha|}{\beta} = \frac{|\partial\gamma/\partial y|}{\partial f/\partial y} = \frac{n}{N}\frac{|\omega_{BN}|}{f}\frac{|\partial \ln B_{0z}/\partial y|}{\partial \ln f/\partial y}, \tag{61}$$

where $|\omega_{BN}| = e|B_{0z}|/m_N$ is the cyclotron frequency computed with average mass m_N of the neutral gas. As it was mentioned above, the parameters α and β are comparable in magnitude ($\beta \approx -\alpha \approx 10^{-11} m^{-1} s^{-1}$) for the ionospheric E-layer and depending on a value of the ratio of charged particles number density to neutrals one, i.e., n/N. The ionization fraction is sharply different for night- and day-sides of the Earth the ratio in Eq. (61) may be more or less one.

The sign $(-)$ in (59) means that in the "usual" regime (when the interaction with ionized particles is not significant, i.e. $\alpha = 0$), owing to $\beta > 0$ the linear Rossby waves are propagating westward, opposite to the Earth's rotation. Since the geomagnetic field has a strong latitudinal gradient, and the ratio (61) is more than one, the total modified Rossby

velocity (58) changes the direction and magnetized Rossby waves can propagate eastward, parallel to the Earth's rotation direction. Thus, the Hall effect (the α-effect) due to the interaction with the ionized ionospheric component brings additional freedom of motion. Generally, the waves propagate more slowly than the tropospheric Rossby waves.

In addition it should be noted that the Ampere force opposes the Coriolis force vorticity and therefore partially or fully compensates the Coriolis deviation by the magnetic one is possible. At the inversion point $\alpha + \beta = 0$ the direction of the linear magnetized Rossby waves changes. The period of the magnetized Rossby waves is of the order of tenths of hours. The waves can propagate westward or eastward along the parallel and can be excited by the dynamo electric field. The frequency of the modes varies in the range $(10^{-6} \div 10^{-5}) s^{-1}$ whereas its wavelength is of the order 10^3 km and longer, and the phase velocity is of the order of the velocity of the local winds, i.e., $\sim (10 - 100)$ m/s.

In Fig. 2 we show a typical state of the magnetized Rossby wave vortex turbulence described by Eq. (54) with a driving term and a hyperviscosity. There is a sea of monopolar vortices of various sizes and a few dipole vortices, for example, in the upper left corner and lower center. The simulation is created with a 2D pseudo-spectral code on a $(512)^2$-grid in a box $(2\pi\, r_G)^2$ using the Cray Parallel Vector Supercomputer (SV1). In Fig. 2a the contour levels are evenly spaced with $\Delta h =$ const. In Fig. 2b the same data is shown with the contour levels h_n spaced at the increasing intervals with $(2.5)^n \Delta h_1$, so that the low-lying complex waves between the vortices are now visible. The simulation method is given by *Horton and Ichikawa* (1996).

5. General Nonlinear Properties and Results of the Generalized Equation for the Magnetized Rossby Waves

Now we consider the physical role of the KdV term.

First of all we should note that Eq. (37) conserves the following total energy W

$$\frac{\partial W}{\partial t} \equiv \frac{\partial}{\partial t}\frac{1}{2}\int\int dxdy\left[(\nabla h)^2 + \frac{1}{r_R^2}h^2\right] = 0. \tag{62}$$

It is well known that the quasi-geostrophic Charney-Obukhov equation (41) admits solution in form of solitary vortex only of the dipole structure type [*Kaladze and Tsamalashvili, 1997; Kaladze, 1998; Kaladze, 1999*]. This equation does not allow the long-time existence of solitary monopole vortices.

Let us discuss the question of splitting of dipolar vortex into monopoles. By combining the coefficients of the $\partial h/\partial x$ terms in Eq. (54) we see that the local spread of propagation of a structure is

$$\frac{dx}{dt} = v_R(1+h) \tag{63}$$

so that regions with $h > 0$ propagate faster than v_R, while regions with $h < 0$ propagate slower than v_R.

Considering the dipole vortex solution of (41) or (56) we now see that the positive and negative vortex lobes propagate with different speeds. Studies by *Su et al.*. [1991] show that the speeds of the positive and negative centers are $u_\pm \cong v_R(1+h)(1+|h_{dp}|/4.8)$ so that in time Δt the relative speeds $u_+ - u_-$ separate the two dipole lobes by its own diameter $2r_0$ where

$$\Delta t = \left(\frac{4.8}{2}\right)\frac{r_0}{|v_R||h_{dp}|}, \tag{64}$$

After time Δt the dipoles are split into monopoles and both signs will propagate for a period depending on the strength of the vortices. But due to dependence of the Rossby velocity v_R on the generalized Rossby parameter $(\alpha + \beta)$ [see (46)] there is a preferred sign for vortex self-binding. So after some number of rotation periods only the preferred sign monopole will survive. In order to pass to consideration of such vortices on the basis of generalized Eq. (37) we give the determination of cyclones and anticyclones.

A cyclone is a vortex rotating in the direction of system's rotation [see Ertel's equation (24)]. A vortex rotating in the opposite direction is called anticyclone. We have shown that the geomagnetic field causes opposite to the Coriolis (vectorial) parameter rotation. When the total rotation of our system (in the rotating frame) is directed to the East the linear magnetized Rossby waves are propagating westward [see Eq. (43)], opposite to the system's rotation. At the Rossby regime the total force consisting of Coriolis and Ampere forces is directed to the center of anticyclone and is balanced by the gradient force connected with the increasing of pressure at the center of vortex. Therefore we have the magnetized Rossby anticyclone with the elevation of free surface of the liquid. In other cases, when the total rotation of the system is directed to the west, the linear magnetized Rossby waves are propagating eastward, also opposite to the system's rotation. The total force is directed to the center of anticyclone, and represents the elevation of free surface of the liquid. (In the case of cyclones, total force is directed away from the center of rotation, and we have a depression on the free surface of the liquid).

Let have a look at our equations obtained. First it should be noted that according to what has been said the "transformation of cyclone to anticyclone" means

$$x \to x, \qquad y \to -y, \qquad h \to -h. \tag{65}$$

With such a transformation, the direction of propagation is conserved, but the directions of vortex rotation and perturbation of a level of free surface of shallow water are changing the sign. In this connection all terms of the quasi-geostrophic Charney-Obukhov equation (41) change sign, and the equation remains the same.

We have a different situation for the generalized equations (37) and (38). Indeed, at the transformation (65), the "KdV-type" nonlinearity (i.e., the term proportional to hh_x) conserves its sign, i.e., violates the cyclone-anticyclone symmetry of equations. Let us see what such cyclone-anticyclone asymmetry results in.

From Eqs. (37) and (54) we obtain the following nonlinear phase velocity for the long magnetized Rossby waves varying with amplitude, as given in Eq. (63). In addition, we know that the existence of a solitary structure requires that the structure not radiate waves by the Čerenkov resonance. This means that the velocity of structure should exceed the maximal velocity of linear waves. According to (63), it is possible only at $h > 0$, i.e., only for an anticyclone. Relationship (63) elucidates why we keep the term $(\alpha + \beta)hh_x$, in spite of the assumption $h \ll 1$ and, hence, $(\alpha + \beta)hh_x$: if we neglect h, the solitary wave solution of Eq. (37) will be lost.

6. The mechanism of Self-organization of Solitary Magnetized Rossby Vortices

Here we will follow the new conception of self-organization mechanism of solitary vortices elucidated in *Horton and Hasegawa* [1994] and *Nezlin and Chernikov* [1995].

As it was indicated in the introduction, the scalar nonlinearity of KdV type cannot be only one compensating a dispersion and ensuring formation of solitary Rossby vortices. Thus, we come to a conclusion that in the dispersion-nonlinear equilibrium, the second vectorial nonlinearity presented in Eqs. (37) and (38) by the Jacobian, should play an

essential role. It means that the structure cannot be axially symmetrical (when Jacobian is zero). In other words, the structure should be a superposition of an axially symmetric vortex and certain underlying dipole structure. We can determine the spatial structure of this perturbation from the following arguments.

Let us draw isolines of the function hh_x, for instance, for anticyclone ($h > 0$). They will have the form of ovals with positive values on the leading edge of vortex profile, and negative on the trailing edge, as shown in Fig. 3. Consequently, the system of isolines has a dipole structure with the moment directed along the vortex propagation direction. Now, in order to explain the method of evaluation of necessary non-isotropic addition to axially symmetric vortex, let us consider that special case when the scalar nonlinearity of the KdV type is considerably stronger than the dispersion of Rossby waves, and consequently, vectorial nonlinearity should essentially compensate scalar one. In this case, the Jacobian should have approximately the same structure of isolines as in Fig. 2, but the sign of Jacobian should be opposite to the sign of scalar nonlinearity. But in order for Jacobian to have the pointed dipole structure, the vortex itself should have the dipole perturbation oriented perpendicularly to a propagation direction of vortex, as shown in Fig. 4.

Fig. 3

Fig. 4

Now, let us make some estimations. First, we determine the condition when mutual compensation, being under consideration of scalar and vectorial nonlinearities, is possible in principle. The maximal value of the Jacobian in order of magnitude is

$$J\left(h, \nabla^2 h\right) \sim \frac{h^2}{a^4},$$

and consequently the vectorial nonlinearity in Eq. (37)

$$\frac{gH_0}{f_0 + \gamma_0} J\left(h, \nabla^2 h\right) \sim \frac{|f_0 + \gamma_0| r_R^2 h^2}{a^4}.$$

On the other hand, the scalar nonlinearity in the β-plane approximation is of the order

$$(\alpha+\beta)h\frac{\partial h}{\partial x} \sim \frac{|f_0+\gamma_0|h^2}{Ra},$$

where a is the "radius" of vortex and R is the curvature radius of system. Consequently, the vectorial nonlinearity can compensate the scalar one if

$$\frac{|f_0+\gamma_0|r_R^2h^2}{a^4} > \frac{|f_0+\gamma_0|h^2}{Ra},$$

i.e., when

$$a < \left(r_R^2 R\right)^{1/3}. \tag{66}$$

It means that the radius of vortex should be less than the so-called intermediate geostrophic radius. Respectively, the vectorial nonlinearity can compensate the wave dispersion when

$$a < \left(r_R^2 R h^2\right)^{1/3}. \tag{67}$$

Let us now formalize what has been said above. If the structure drifts with the constant velocity v_D, then $\partial/\partial t = -v_D\,\partial/\partial x$, and Eq. (54) has the form

$$\begin{aligned} &-v_D\left(h-\nabla^2 h\right)_x + v_R h_x + v_R h h_x - \mathrm{sgn}(f_0+\gamma_0)J(h,\nabla^2 h) = 0,\\ &\frac{\partial}{\partial x}\left(v_D\nabla^2 h - v_D h + v_R h + v_R\frac{h^2}{2}\right) - \mathrm{sgn}(f_0+\gamma_0)J(h,\nabla^2 h) = 0. \end{aligned} \tag{68}$$

If the quantity in parentheses is zero, it means the compensation of the KdV type nonlinearity by the dispersion of magnetized Rossby waves. In this case

$$\nabla^2 h = \left(1-\frac{v_R}{v_D}\right)h - \frac{v_R}{v_D}\frac{h^2}{2} \tag{69}$$

and $J(h,\nabla^2 h) = 0$.

Equation (69) defines the spatial profile of the above-mentioned "scalar" soliton of KdV type, which is being formed due to the presence of the spatial inhomogeneity of the

geomagnetic field, and the Earth's angular velocity. Now, the spatial inhomogeneity of the structure under consideration is such that the quantity

$$\frac{\partial}{\partial x}\left(v_D\nabla^2 h - v_D h + v_R h + v_R\frac{h^2}{2}\right)$$

is strongly different from zero, for instance, owing to large scalar nonlinearity $v_R\frac{h^2}{2}$, and according to (68) the Jacobian also differs from zero. In order to ensure the dispersion-nonlinear compensation, it is necessary in the given case, that the Jacobian have the same spatial structure as the term proportional to hh_x, and be negative. For example, let us consider approximately "round" vortex with $h_0(r)$ in the polar system of coordinates $x = r\cos\vartheta$ and $y = r\sin\vartheta$. Then isolines of the function $v_R h_0 h_{0x}$ will have the spatial structure of dipole oriented with its "moment" along the x-axis: $h_0 h_{0x} \sim \cos\vartheta$. According to what has been said, if a dipole perturbation of relative deviation of a level of free surface $h_0(r)$ of the "round" monopole vortex will have the form $v_R h'(r)\sin\theta$ i.e., will represent dipole oriented with its moment along the y-axis, then the vectorial nonlinearity will compensate the scalar one. It follows from that the Jacobian $J(h,\nabla^2 h)$ at $h = h_0(r) + v_R h'(r)\sin\vartheta$ is determined by the expression

$$\begin{aligned} J(h,\nabla^2 h) &= J\left(h_0(r) + v_R\, h'(r)\sin\vartheta, \nabla^2\left(h_0(r) + v_R'(r)\sin\vartheta\right)\right) \\ &\approx v_R\, J\left(h_0(r), \nabla^2\left(h'(r)\sin\vartheta\right)\right) + v_R J\left(h'(r)\sin\vartheta, \nabla^2 h_0(r)\right) \sim \cos\vartheta, \end{aligned}$$

because when differentiation in the Jacobian $\sin\vartheta$ pass into $\cos\vartheta$; in other words the Jacobian has the required dipole structure oriented along the x-axis. The full compensation of the nonlinearities under consideration will take place if the quantity $h'(r)$ will have required sign and numerical value. The quantity $h'(r)$ is being defined as the solution of

the following ordinary differential equation

$$h_0 \frac{dh_0}{dr} - \mathrm{sgn}(f_0 + \gamma_0) \frac{1}{4} \frac{dh_0}{dr} \left(\frac{d^2 h'}{dr^2} + \frac{1}{r} \frac{dh'}{dr} - \frac{h'}{r^2} \right) - \frac{h'}{r} \frac{d}{dr} \left(\frac{d^2 h_0}{dr^2} + \frac{1}{r} \frac{dh_0}{dr} \right) = 0.$$

The numerical calculations of concrete spatial structure of dipole perturbations leading out initial axially symmetric anticyclone on the regime of stationary propagating vortex are carried out in [*Sutyrin and Yushina, 1988; Nycander, 1991; Nycander and Sutyrin, 1992; Sutyrin, 1994*]. There is shown also that the initial axially symmetric vortex with the corresponding dipole perturbation begins drift at once with constant velocity. If on the initial axially symmetric vortex the necessary dipole perturbation is absent an evolution of vortex turns out to be more complicated. In this case the transitional period precedes drift with constant velocity during of which the necessary dipole perturbation grows on the vortex.

7. Discussion

We have shown that in the E-layer of the ionosphere there occur magnetized Rossby waves that give rise to a mixture of waves and vortex structures. These waves do not perturb the geomagnetic field significantly and are induced by the latitudinal inhomogeneity of both the Earth's angular velocity (β-effect) and geomagnetic field (α-effect). They are excited solely by the ionospheric dynamo electric field when the Hall effect due to the interaction with the ionized ionospheric component in the E-layer is included. The dynamics of propagation is essentially depend on the alternating generalized Rossby parameter ($\alpha + \beta$) and the modified Rossby-Obukhov radius. Immersed in the spatially inhomogeneous geomagnetic field magnetized Rossby waves acquire an additional degree of freedom and unlike of the usual Rossby waves can propagate either westward or east-

ward along the parallel. In addition it should be noted that the Ampere force opposes the Coriolis force vorticity and therefore partial or full compensation of the Coriolis deviation by the magnetic one is possible. Correspondingly the propagation phase velocity of linear waves also decreases. The period of these waves is of order of tenths of hours. The frequency of magnetized Rossby waves varies in the range $(10^{-6} - 10^{-5})s^{-1}$, whereas its wavelength is of the order 10^3 km and longer, and the phase velocity is of the order the velocity of the local winds, i.e. $\sim (10 - 100)$ m/s.

It is well known that the nonlinear properties of Rossby waves are described by the classical Charney-Obyukhov equation which is valid for small-scale vortical structures of dipole type with size less than the Rossby-Obukhov radius. Magnetized Rossby waves typically have wavelengths greater than the Rossby-Obukhov radius ($r_R \sim 1000 - 3000$ km). The nonlinear equation for the magnetized Rossby waves is in the form of the generalized Charney-Obukhov equation is given in Eq. (37). This equation corresponds to the intermediate geostrophic approximation in the geophysical hydrodynamics where the perturbation of free surface of the atmosphere is taken into account.

The obtained generalized Charney-Obukhov equation for the magnetized Rossby waves contains both the "scalar" and "vectorial" nonlinearities and describes solitary vortical nonlinear structures of arbitrary size. This generalized equation breaks the symmetry between cyclones and anticyclones. The mechanism of self-organization of the corresponding solitary vortical structures is examined. According to the new understanding of this mechanism a self-organization of solitary structures is a result of the mutual compensation of wave dispersion and interactions from both the scalar and vectorial nonlinearities. As a result, generally a solitary structure is intrinsically anisotropic containing a circular vor-

tex superimposed on a dipole perturbation. The degree of anisotropy sharply increases as the size approaches the vortex scale of the so-called intermediate geostrophic size. A large-scale dipole vortex splits in two monopoles (a cyclone and an anticyclone) where a vortex of one polarity turns out to be long-lived but the vortex of opposite polarity disperses. In case of the magnetized Rossby waves only anticyclones survive propagating faster then the maximal velocity of corresponding linear waves what just should be in case of solitary structures. The trapping condition necessary for the long-time existence of solitary structure also is obtained. The latter condition means that the structure should rotate faster than it propagates as whole. Then in current lines system moving with the vortex, the separatrix appears inside of which captured particles of medium are trapped. These are "real vortices" that carry the medium particles of the medium.

An important remaining question is what classes of events are most important in exciting these vortices. The structures may occur as driven vortices from horizontal or vertical wind stresses in neighboring layers, from thermal gradients or from instabilities driven by thermal gradients and wind shears to name a few possibilities. Strong thermal gradients may be produced by intense Ohmic dissipation in the lower E-layer of the ionosphere carrying the current surges from substorm and storm time magnetosphere-ionosphere couplings. For the vertical upwelling and down falling driving mechanisms one would like to gain knowledge of the vertical eigenmode problem. Here the local time-dependent profile of the buoyancy frequency in the E-layer and the neighboring layers is required. Global computer simulations of the E-layer dynamics with the day-night ionization asymmetry would be most useful for gaining a better understanding of this complex ionospheric-weather system.

Acknowledgments. The research described in this publication was made possible in part by Award No. 3317 of the Georgian Research and Development Foundation (GRDF) and the U.S. Civilian Research & Development Foundation for the Independent states of the Former Soviet Union (CRDF).

References

Antipov, S.V., M.V. Nezlin, E.N. Snezhkin, and A.S. Trubnikov, Rossby soliton in the laboratory, Sov. Phys. JETP *55*, 85, 1982.

Cheng, K.Y., and N. Huang, Ionospheric disturbances observed during the period of Mount Pinatubo eruptions in June 1991, J. Geophys. Res. *97*, 16, 995, 1992.

Charney, J.G., and P.G. Drazin, Propagation of planetary-scale disturbances from the lower into upper atmosphere, J. Geophys. Res. *66*, 83, 1961.

Chernikov, G.P., Rossby solitons-cyclones in the rotating shallow water, Dokl. Akad. Nauk SSSR *344*, 65, 1994.

Dickinson, R.E., Planetary Rossby wave propagating vertically through weak westerly wind wave guides, J. Atmos. Sci. *25*, 984, 1968.

Dickinson, R.E., Theory of planetary wave-zonal flow interaction, J. Atmos. Sci. *26*, 73, 1969.

Dokuchaev, V.P., Influence of the Earth's magnetic field on the ionospheric winds, Izvestia AN SSSR, Seria Geophysica *5*, 783, 1959.

Gill, A.E., *Atmosphere-Ocean Dynamics* Academic Press, Harcourt Brace & Co., San Diego, CA, 1982).

Haykowicz, L.A., Global onset and propagation of large-scale traveling ionospheric disturbance as a result of the great storm of 13 March 1989, Planet. Space Sci. *39*, 583, 1991.

W. Horton and A. Hasegawa, Quasi-two-dimensional dynamics of Plasmas and Fluids, Chaos 4, 227-251 (1994).

W. Horton and Y-H Ichikawa, Chaos and Structures in Nonlinear Plasmas, World Scientific Publishing Co., Singapore, ISBN 9810226365, 1996.

Kaladze, T.D., and L.V. Tsamalashvili, Solitary dipole vortices in the Earth's ionosphere, Phys. Lett. A *232*, 269, 1997.

Kaladze, T.D., Nonlinear vortical structures in the Earth's ionosphere, Physica Scripta *T75*, 153, 1998.

Kaladze, T.D., Magnetized Rossby waves in the Earth's ionosphere, Plasma Phys. Rep. *25*, 284, 1999.

Kaladze, T.D., I.N. Keshelashvili, and L.V. Tsamalashvili, Inductive damping of nonlinear solitary Rossby vortices in the ionospheric F-layer, Plasma Phys. Rep. *25*, 625, 1999.

Kamide, Y., *Electrodynamic Processes in the Earth's Ionosphere and Magnetosphere*, Kyoto Sangyo University Press, Kyoto, 1988.

Kelley, M.C., *The Earth's Ionosphere*, Academic Press, Inc., San Diego, 1989.

Liperovsky, V.A., O.A. Pokhotelov, and S.L. Shalimov, Ionospheric earthquake precursors, Nauka, Moscow, 1992.

Meiss, J., and W. Horton, Phys. Fluids *26*, 990, 1983

Meyers, S.D., J. Sommeria, and H.L. Swinney, Laboratory study of the dynamics of Jovian-type vortices, Physica D *37*, 515, 1989.

Morrison, P.J., "Poisson brackets for fluids and plasmas," in *Mathematical Methods in Hydrodynamics and Integrability in Dynamical Systems*, edited by M.Tabor and Y. Treve, AIP Conf. Proc. *88*, AIP, New York, 1982.

Nezlin, M.V., and E.N. Snezhkin, *Rossby vortices, Spiral Structures, Solitons*, Springer-Verlag, Heidelberg, 1993.

Nezlin, M.N., and G.P. Chernikov, Analogy of drift vortices in plasmas and geophysical hydrodynamics, Plasma Phys. Rep. *21*, 975, 1995.

Nezlin, M.V., Rossby solitary vortices on giant planets and in the laboratory, Chaos *4*, 187, 1994.

Nezlin, M.V., A.Yu. Rylov, A.S.Trubnikov, and A.V. Khutoretsky, Cyclonic-anti-cyclonic asymmetry and a new soliton concept for Rossby vortices in the laboratory, oceans and the atmospheres of giant planets, Geophys. Astrophys. Fluid Dyn. *52*, 211, 1990.

Nycander, J., Stationary drift vortices with large amplitude, Phys. Fluids B *3*, 931, 1991.

Nycander, J., and G.G.Sutyrin, Steadily translating anticyclones on the beta plane, Dyn. Atmos. Oceans *16*, 473, 1992.

Pedlosky, J., *Geophysical Fluid Dynamics*, Springer-Verlag, New York, 1987.

Petviashvili, V.I., and O.A. Pokhotelov, *Solitary waves in plasmas and in the atmosphere*, Gordon and Breach, Reading, MA, 1992.

Petviashvili, V.I., Red Spot of Jupiter and drift soliton in a plasma, JETP Lett. *32*, 619, 1980.

Pokhotelov, O.A., Parrot, M., Fedorov, E.N., Pilipenko, V.A., Surkov, V.V., and Gladychev, V.A., Response of the ionosphere to natural and man-made acoustic sources, Annales Geophysical *13*, 1197, 1995.

Rishbeth, H., Super-rotation of the upper atmosphere, Rev. Geophys. Space Phys.*25*, 799, 1972.

Romanova, N.N., and V.Yu. Tseitlin, On quasi-geostrophic motions in barotropic and baroclinic fluid, Izv. Atmos. Ocean Phys. *20*, 85, 1984.

Shaefer, L.D., D.R. Rock, J.P. Lewis *et al.*, Detection of explosive events by monitoring acoustically-induced geomagnetic perturbations, Lawrence Livermore Laboratory, Livermore, CA 94550, p. 63, 1999.

Su, X.N., W. Horton, and P.J. Morrison, Drift vortices in inhomogeneous plasma, Phys. Fluids B *3*, 921-930, 1991.

Sutyrin, G.G., Long-lived planetary vortices and their evolution: Conservative intermediate geostrophic model, Chaos *4*, 203, 1994.

Sutyrin, G.G., and I.G.Yushina, Formation of the vortical soliton, Sov. Phys. Dokl. *33*,179, 1988.

Tolstoy, I., Hydromagnetic gradient waves in the ionosphere, J. Geophys. Res. *72*, 1435, 1967.

Williams, G.P., and T. Yamagata, Geostrophic regimes, intermediate solitary vortices and Jovian eddies, J. Atmos. Sci. *41*, 453, 1984.

FIG. 1. Dispersion relations and region of coherent vortex structures in the $\omega - k_x$ space. (a) dispersion relation (43) from Eq. (42) for large horizontal wavenumbers where the free surface of E-layer are significantly perturbed; (b) dispersion relation for Eq. (41) for smaller-scale structures. In both cases the coherent vortex structures propagate faster than the small amplitude waves.

FIG. 2. A typical steady state of vortex-wave turbulence generated from Eq. (41) with a narrow-band linear growth rate and a hyperviscosity. The box size is $2\pi r_G$ corresponding to approximately 18000 km for $r_G = 3000\,\mathrm{km}$. The length of the simulation depends on the value of $f_0 + \gamma_0$ and corresponds to the approximate range of 5 to 10 days. Both large- and small-scale vortices are formed and mix with lower amplitude waves (a) corresponds to equal contour intervals which emphasizes the vortices and (b) is the same state with contours at $\Delta h_1 = (2.5)^n h$, so that the small amplitude waves are clearly visible.

FIG. 3 Contours of the KdV nonlinear term hh_x evaluated for a typical lowest order circular vortex. This structure generates the first order underlying dipole required by the vector nonlinearity to form a long-lived vortex.

FIG. 4 Schematic structures of the composite circular and dipolar vortex structure generated by the full or generalized magnetized Rossby wave equation.

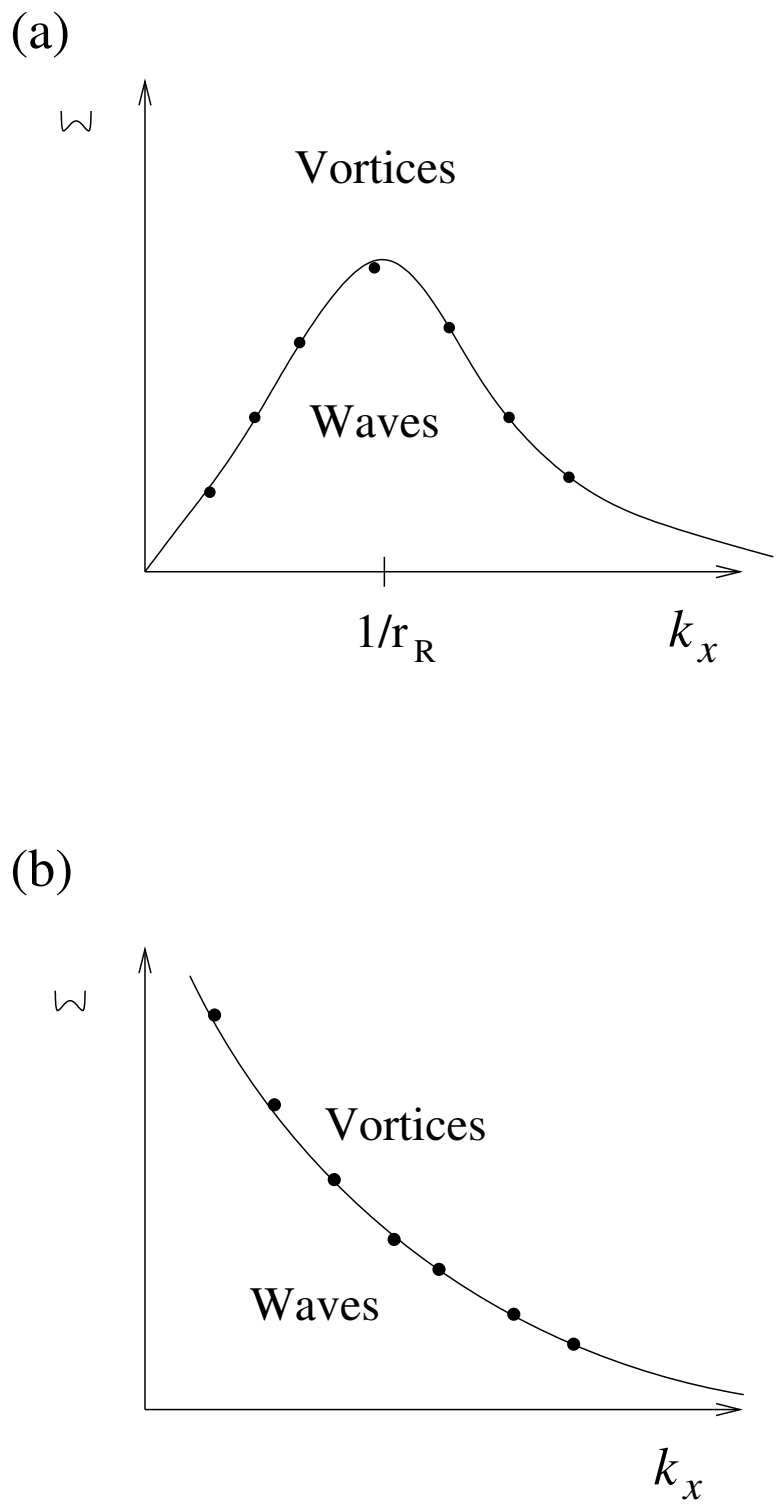

Figure 1.

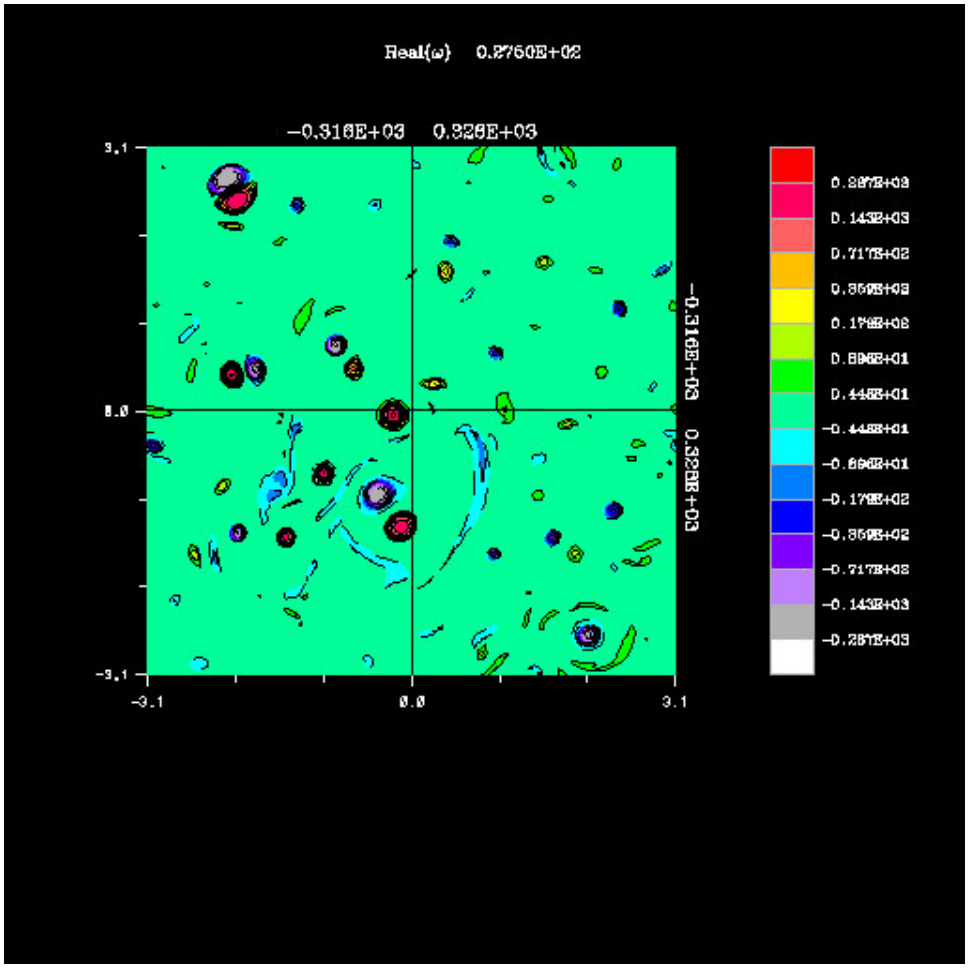
Real(ω) 0.2760E+02
−0.316E+03 0.326E+03
3.1
0.0
-3.1
-3.1
0.0
3.1
0.287E+03
0.143E+03
0.717E+02
0.359E+02
0.179E+02
0.896E+01
0.448E+01
−0.448E+01
−0.896E+01
−0.179E+02
−0.359E+02
−0.717E+02
−0.143E+03
−0.287E+03
−0.316E+03
0.326E+03

Figure 2a.

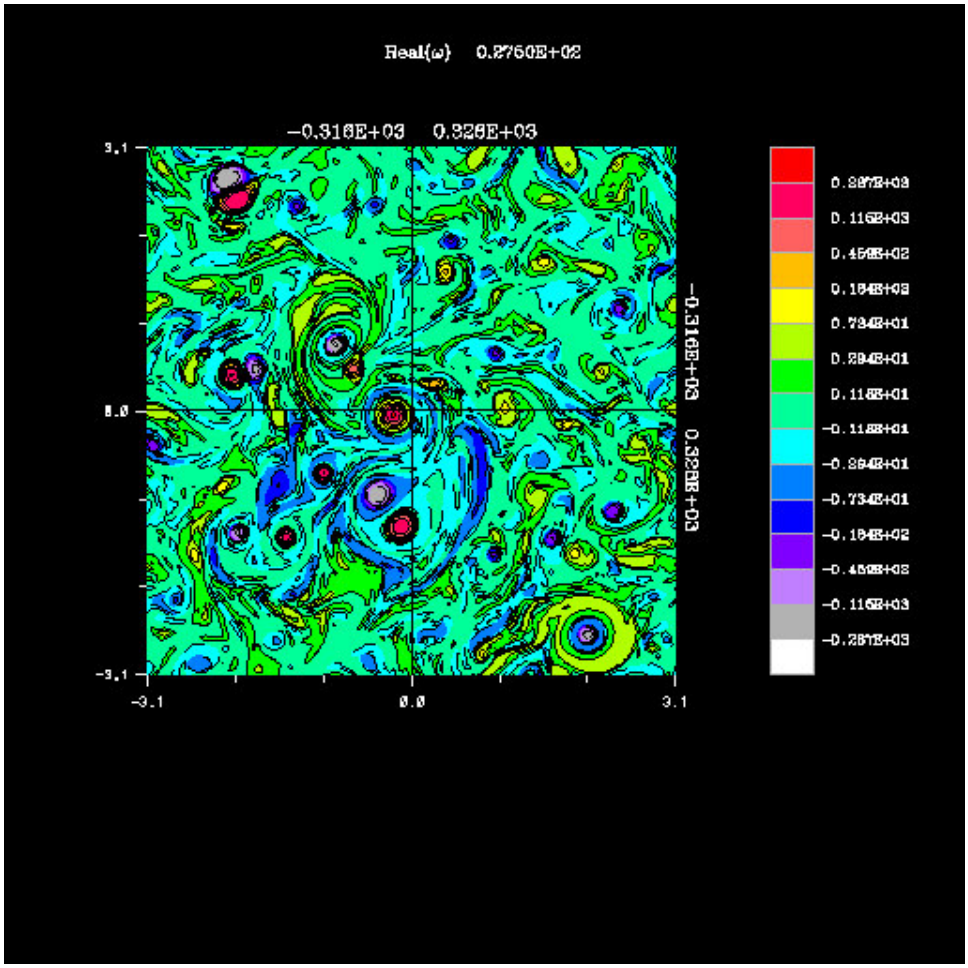
Real(ω) 0.2760E+02
−0.316E+03 0.326E+03
3.1
0.0
−3.1
−3.1
0.0
3.1
−0.316E+03
0.326E+03

Figure 2b.

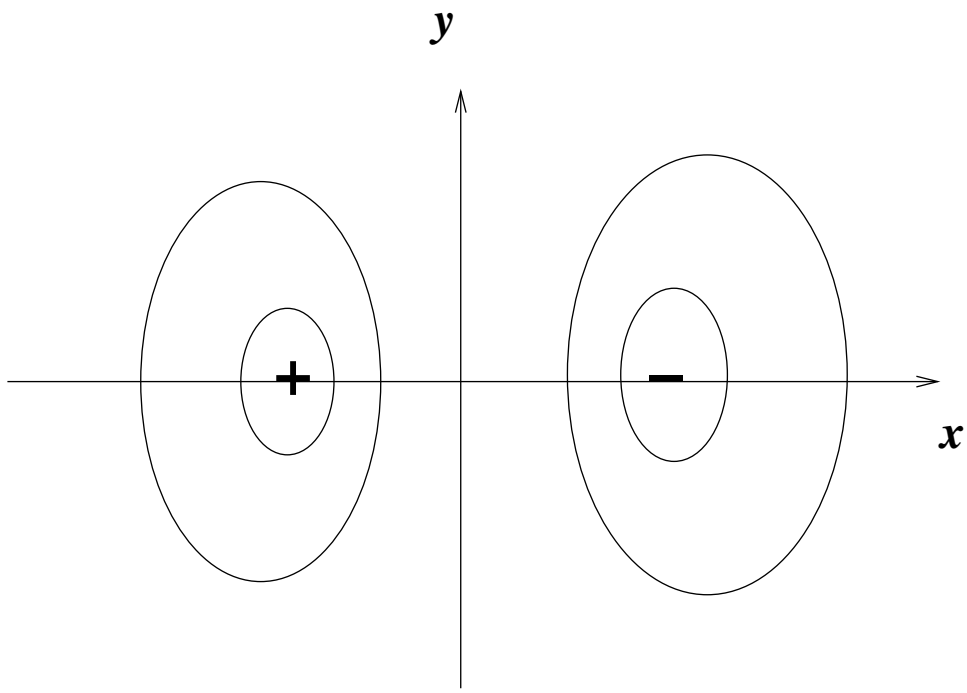

Figure 3.

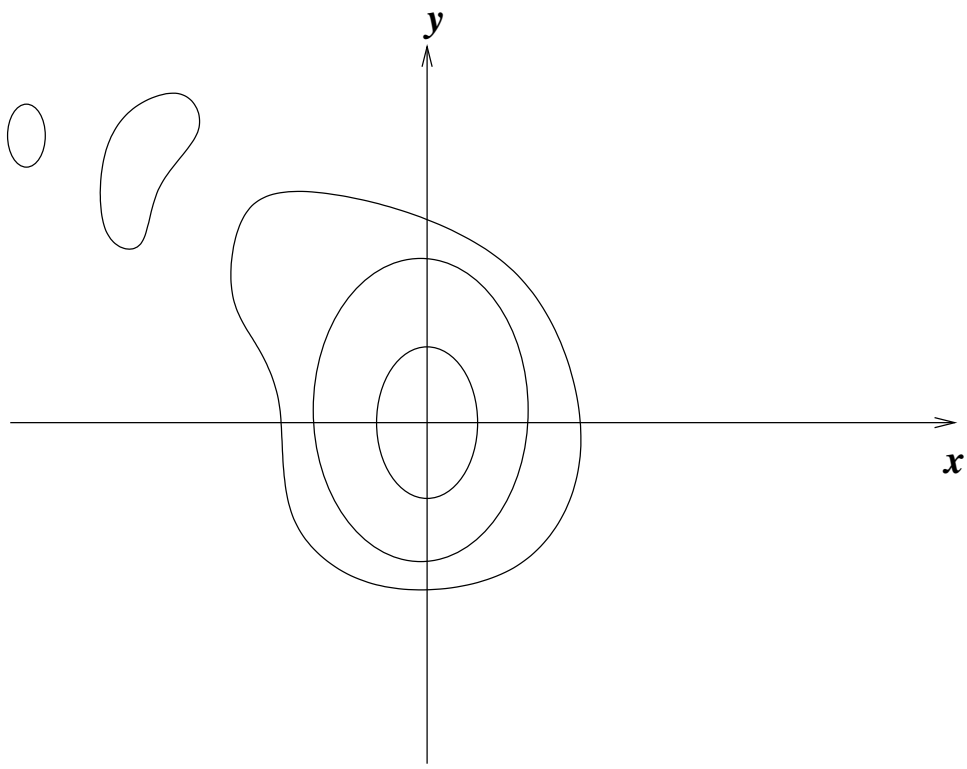

Figure 4.