ELECTRON CONFINEMENT AND HEATING IN LASER-IRRADIATED MICRO-CLUSTERS

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Abstract

It is shown that a two component electron distribution can emerge in a cluster under an intense laser pulse. A bulk of internal electrons adjusts adiabatically to the laser field whereas a smaller electron population at the cluster edge can undergo stochastic heating. Self-consistent equilibrium has been found and collective modes discussed for the confined electrons.

I. INTRODUCTION

This work has been motivated by recent desktop laser fusion experiments, in which fusion reactions were produced by irradiation of deuterium micro-clusters with a very short and intense laser beam [1–3]. It has been immediately recognized that the observed phenomenon is associated with ion acceleration in the process of cluster explosion. There are two conceivable scenarios for such an explosion: an electrical one (Coulomb explosion) and a thermal one. The electrical scenario implies that the laser field quickly pulls the electrons out of the cluster, and the unneutralized ions are then accelerated by the electric field of their space charge. The thermal scenario emphasizes electron heating that leads to cluster expansion due to high electron pressure. Although both scenarios involve the space-charge electric field as an accelerating force for the ions, the essential difference between the two is that the thermal scenario preserves quasi-neutrality whereas the electrical scenario does not.

The picture of the electrical explosion is particularly simple when the laser field is sufficiently strong to extract all the electrons from the cluster. It is apparently more challenging to describe the case when the extraction is incomplete. This particular problem is the main topic of our paper.

In what follows, we will first formulate our basic assumptions that link the experimental situation to a properly idealized physics model. The key point here is a qualitative distinction between the confined and the extracted electrons, which is reflected in the separate treatment of these two groups. Next, we construct a simple analytical solution for an electrical explosion of an initially uniform spherical cluster. This solution is presented in Section II. In Section III, we develop a numerical procedure for calculating the space-charge electric field together with the electron density distribution inside the cluster for an arbitrary axially symmetric cluster. This procedure is designed to facilitate simulations of the ion dynamics by eliminating the electron time-scales from the problem. However, the actual modelling of the ion motion goes beyond the scope of this paper. Section IV describes collective electron oscillations in a cluster. Finally, Section V deals with electron reflection from the cluster boundary, which causes phase mixing and gives rise to stochastic electron heating.

We will limit our consideration to the case in which the cluster radius (R) is much smaller than the laser wavelength. This implies that the electric field of the laser beam can be treated as a spatially uniform albeit time-dependent external field. This external field oscillates at a laser frequency ω that is typically smaller than the electron plasma frequency ω_{pe} at a solid state density of the cluster. For the sake of simplicity, we will hereon assume that $\omega \ll \omega_{pe}$, which will allow us to treat the confined electrons adiabatically. As for the extracted electrons, we assume that they quickly move far away from the cluster and, therefore, do not contribute to the local space charge density. The cluster will thus become positively charged, which occurs so fast that the ions hardly have time to move anywhere. We thus formulate a problem of finding an equilibrium configuration of the confined electrons in the self-consistent field that is a superposition of the external electric field and the space-charge field. This problem needs to be solved for a given spatial distribution of ions. Clearly, the total electric field and the space-charge density should vanish in the region occupied by the confined electrons provided that these electrons remain cold, which is indeed the case as long as one can neglect binary collisions. In other words, the confined electrons behave as a perfectly conducting fluid. What we need to find, when we look for an equilibrium, is the boundary of that fluid.

II. UNIFORM SPHERICAL CLUSTER

It is interesting that the electron equilibrium can be found analytically for a spherical cluster with a uniform ion density. In order to do that, we note that the electric field created by the ion background alone is given by

$$\mathbf{E} = \frac{4\pi n_i |e|}{3}\mathbf{r} \tag{1}$$

for $r \leq R$, where the radius $\mathbf{r}$ is measured from the cluster center as shown in Fig. 1. Similarly, a uniformly charged spherical volume of electrons (with the opposite charge density) creates the field

$$\mathbf{E} = -\frac{4\pi n_i |e|}{3}\mathbf{r}_1 \tag{2}$$

inside the volume, where the radius $\mathbf{r}_1$ is measured from the center of the electron sphere. If we now superimpose the electron and the ion spheres, then the total space-charge field in

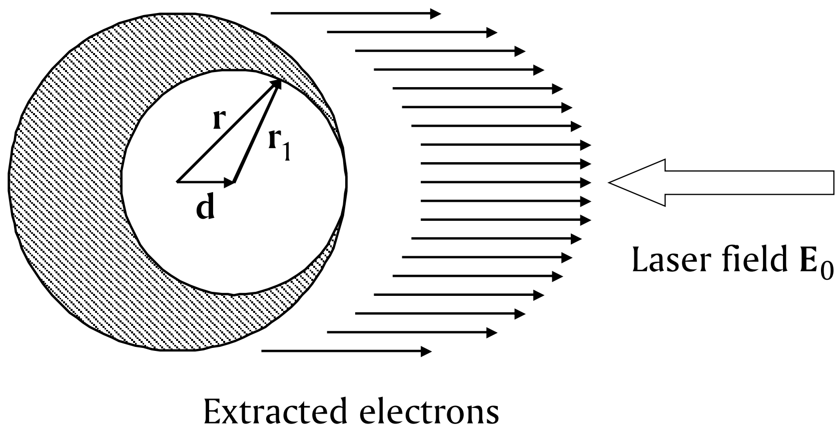

FIG. 1: Confined electron equilibrium in a uniform spherical cluster in the presence of a laser field. The outer circle marks the edge of the ion background. The inner circle marks the boundary of the confined electron population. The shaded area represents the uncompensated ion space charge. The inner circle stays at the edge of the cluster when the electric field $\mathbf{E}_0$ increases monotonically in time. It detaches from the edge and shifts inward when $\mathbf{E}_0$ decreases after achieving its maximum value.

the region where the two volumes overlap will be

$$\mathbf{E} = \frac{4\pi n_i|e|}{3}\mathbf{r} - \frac{4\pi n_i|e|}{3}\mathbf{r}_1 = \frac{4\pi n_i|e|}{3}\mathbf{d}, \tag{3}$$

where $\mathbf{d}$ is the distance between the centers. It is noteworthy that this field is *uniform*, which allows us to choose the displacement $\mathbf{d}$ in such a way that $\mathbf{E}$ cancels the external electric field $\mathbf{E}_0$ to satisfy the requirement that the total electric field vanishes in the region occupied by the confined electrons. We then obtain the following expression for $\mathbf{d}$:

$$\mathbf{d} = -\frac{3}{4\pi n_i|e|}\mathbf{E}_0. \tag{4}$$

For this solution to be consistent, the entire electron sphere needs to be located inside the ion sphere (see Fig. 1). It may seem that, having found the displacement $\mathbf{d}$, we still have the freedom to choose the radius of the electron sphere anywhere between zero and $R - d$. However, this freedom is deceptive. The radius of the electron sphere (R_e) in our problem is actually determined by the time history of the external electric field. First, it is clear that R_e should be equal to $R - d$ if the external electric field grows monotonically from an initially zero value. Indeed, the condition $R_e = R - d$ ensures that the potential well for the electrons has a small leak at the pole of the cluster where the electron sphere touches the ion sphere. The need for such a leak is obviously due to the fact that the cluster is initially neutral and that any monotonic increase in d in this case requires a monotonic decrease in R_e. The situation changes when the external electric field passes through its maximum. If the field goes down, then the electron sphere just shifts closer to the center of the ion sphere with R_e being constant, i.e. without a leak. Furthermore, if the next cycle of the laser field has a larger amplitude, then there will be a further reduction in R_e when the displacement d brings the electron sphere to the surface of the ion sphere. This description shows that the value of R_e at the end of the laser pulse is

$$R_e = R - \frac{3}{4\pi n_i |e|} \left|\mathbf{E}_0\right|_{max}, \tag{5}$$

where $\left|\mathbf{E}_0\right|_{max}$ is the maximum value of the laser electric field during the pulse. It is also apparent that the electron and the ion spheres remain concentric after the pulse, i.e. the cluster consists of a neutral core surrounded by a positive ion shell. The shell will then expand radially outwards on an ion time-scale whereas the core will remain at rest. This expansion is described by the following equation of motion

$$M\frac{d^2 r}{dt^2} = \frac{4\pi n_0 e^2 \left[r_0^3 - (R-d)^3\right]}{3}\frac{1}{r^2}, \tag{6}$$

where r_0 is the initial position of the ion in the shell ($R \geq r_0 \geq R-d$). This equation has an energy integral that relates the ion kinetic energy after the expansion to the initial position r_0:

$$\frac{MV_\infty^2}{2} = \frac{4\pi n_0 e^2 \left[r_0^3 - (R-d)^3\right]}{3r_0}. \tag{7}$$

III. ARBITRARY CLUSTERS

The analytical solution presented above suggests a numerical procedure that allows us to quickly find an equilibrium configuration of the confined electrons for an arbitrary shaped cluster. Instead of solving the actual electron equation of motion, which would involve many particles and require a very small time step, we use electron boundary dynamics as described below.

We introduce a boundary of the electron population and we evolve this boundary assuming that its instantaneous velocity at any point is proportional to the local value of the electric field. We take the initial electron boundary at the edge of the cluster, so that the cluster is initially neutral, and we freeze the value of the external field $\mathbf{E}_0$ and the ion space charge density when we evolve the electron boundary. If this dynamics takes any part of the electron boundary outside the ion population, we simply eliminate the protrusion by forcing the appropriate segment of the electron boundary to be at the cluster edge. We also take the total charge density to be zero everywhere inside the electron boundary, which means that we calculate the instantaneous electric field as a sum of the external field and the electrostatic field created by the exterior ions (those that are outside the electron boundary). We have implemented this algorithm in a 2D code that allows us to find the electron equilibrium for an arbitrary axisymmetric ion density profile. For a uniform spherical cluster, the code shows rapid convergence to the analytical solution described above.

IV. COLLECTIVE ELECTRON MODES

It is apparent that the equilibrium of the cold electron core of the cluster is stable with respect to small perturbations as long as the ions can be treated as a fixed positive background. It is therefore appropriate to look for collective oscillations of the electron core. Such oscillations are described by the following eigenmode equation for the perturbed electrostatic potential φ:

$$div\left[(\omega^2 - \omega_{pe}^2)\nabla\varphi\right] = 0, \tag{8}$$

where ω is the eigenfrequency and ω_{pe} is the local value of the electron plasma frequency. This equation follows immediately from the Poisson equation combined with the linearized electron fluid equations. Apart from the continuum of singular modes with $\omega = \omega_{pe}$, where ω_{pe} is taken at the point of the mode location, Eq. (8) also gives a discrete set of global modes. These discrete modes are likely to be more robust than the modes in the continuum, since the continuum modes are subject to fine-scale mixing [4] due to either density inhomogeneity or finite amplitude. The discrete modes are particularly easy to find in the case of constant electron density within the core. We will assume that the cluster is spherically symmetric. Then Eq. (8) translates into a set of radial equations for uncoupled spherical harmonics φ_l:

$$\frac{1}{r^2}\frac{\partial}{\partial r}r^2\left[\omega^2 - \omega_{pe}^2(r)\right]\frac{\partial}{\partial r}\varphi_l - \frac{l(l+1)}{r^2}\left[\omega^2 - \omega_{pe}^2(r)\right]\varphi_l = 0, \tag{9}$$

where l is the harmonic number, and

$$\omega_{pe}^2(r) = \begin{cases} \omega_{pe}^2(0), \text{ for } r < R_e; \\ 0, \text{ for } r \geq R_e. \end{cases} \tag{10}$$

The inner and the outer solutions for φ_l have to be matched at $r = R_e$, where R_e is the radius of the electron core. The matching conditions require continuity of φ_l and $\left[\omega^2 - \omega_{pe}^2(r)\right] \partial\varphi_l/\partial r$, which leads to the dispersion relation

$$\omega^2 = \omega_{pe}^2(0)\frac{l}{2l+1}. \tag{11}$$

It is noteworthy that the frequency of the dipole mode ($l = 1$), which corresponds to a rigid displacement of the electron core, remains equal to $\omega_{pe}(0)/\sqrt{3}$ even for finite displacements as long as none of the electrons go beyond the radius of the ion background. This feature comes from the observation that rigid displacement of the electron core produces a constant field inside the core, as given by Eq. (3). It should also be noted that the existence of a discrete spectrum is not limited to the case of constant electron density. The same matching procedure should give a properly modified dispersion relation for any other radial profile of $\omega_{pe}^2(r)$, although it may be necessary to use a numerical procedure to find the inner solution for φ_l.

V. EXTRACTED ELECTRONS

Once an electron leaves the cluster, it accelerates in the laser field until its kinetic energy reaches roughly $\frac{m}{2}\left(\frac{eE_0}{m\omega}\right)^2$ (assuming that the electron is non-relativistic). The corresponding radial excursion of such an electron is of the order of $\frac{eE_0}{m\omega^2}$. Under the condition $\omega \ll \omega_{pe}$, the excursion is much larger than the thickness of the ion shell

$$|\mathbf{d}| = \frac{3}{4\pi n_i|e|}|\mathbf{E}_0|_{max}, \tag{12}$$

so that the extracted electrons contribute very little to the space charge density in the shell. However, the condition $\omega \ll \omega_{pe}$ alone may not be sufficient to neglect the extracted electrons. The latter would require the electron excursion to exceed the cluster radius (not just the shell thickness), i.e.

$$\frac{eE_0}{m\omega^2} \gg R. \tag{13}$$

This inequality presents an applicability condition for the equilibrium solutions described in Sections II and III. Equation (13) means that, once extracted, the electron typically does not return to the cluster. It also means that the characteristic energy of an extracted electron, $\frac{m}{2}\left(\frac{eE_0}{m\omega}\right)^2$, is larger than the final energy of an accelerated ion from the shell (see Eq. (7)). The expanding ion shell of the cluster never reaches quasineutrality under condition (13). The shell radius always remains smaller than the Debye length corresponding to the extracted electrons.

The situation is qualitatively different in the opposite limiting case:

$$\frac{eE_0}{m\omega^2} \ll R, \tag{14}$$

in which most of the extracted electrons become trapped by the ion space charge and oscillate within the cluster. As fast electrons from the edge layer move inward, they take up a larger volume. Their density at the cluster edge decreases, which allows some cold electrons to come close enough to the edge to be extracted by the laser field during its next period. We therefore conclude that each period of the laser pulse should convert a small portion of cold electrons into warm electrons. The converted fraction (per period) is roughly

$$\frac{|\mathbf{d}|}{R} = \frac{3|\mathbf{E}_0|}{4\pi n_i|e|R}, \tag{15}$$

with the typical electron energy from the initial kick being $\frac{m}{2}\left(\frac{eE_0}{m\omega}\right)^2$. This is essentially the effect of vacuum heating [5–7]. However, the vacuum heating in a small size cluster has an additional aspect compared to that described in Refs. [5–7]. In the cluster case, the laser field can stochastically heat the earlier generations of the extracted electrons as these electrons return to the edge in their oscillatory motion in the ion potential well. The bounce period in the well can be estimated as R/v, where v is the electron velocity. Each bounce oscillation adds roughly $\frac{m}{2}\left(\frac{eE_0}{m\omega}\right)^2$ to the electron energy, so that the rate of energy increase is

$$\frac{d}{dt}\frac{mv^2}{2} = \frac{v}{R}\frac{m}{2}\left(\frac{eE_0}{m\omega}\right)^2 \tag{16}$$

and the resulting electron energy scales as

$$\frac{mv^2}{2} = m\left(\frac{\tau}{R}\right)^2 \left(\frac{eE_0}{m\omega}\right)^4 \tag{17}$$

where τ is the laser pulse duration, unless the heated electrons escape before the end of the pulse. It should be emphasized that this process of consecutive extraction and heating only applies to the electrons that cross the cluster edge. The electrons that are confined inside the cluster remain relatively cold as they adjust to the laser field adiabatically. We therefore observe a trend for creating a two-component electron distribution under condition (14).

If the laser pulse is sufficiently short, then there will always remain a core of cold electrons in the cluster, and the corresponding number of core ions will not be involved in the cluster expansion. The ion shell will expand after the pulse under the hot electron pressure, and this will continue until the hot electrons cool down adiabatically. The final energy per single-charged ion will be roughly equal to the average energy of a heated electron.

Acknowledgments

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