Generation of Flows in the Solar Atmosphere Due to Magnetofluid Coupling

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\section*{ABSTRACT}

It is shown that a generalized magneto–Bernoulli mechanism can effectively generate high velocity flows in the Solar subcoronal regions; sharp amplification of the flow speed is accompanied by a significant density fall.

\textit{Subject headings:} Sun: atmosphere — Sun: chromosphere— Sun: corona — Sun: magnetic fields — Sun: transition region

\section{1. Introduction}

Recent observations, strongly fortified by immensely improved measuring and interpretive capabilities, have convincingly demonstrated that the solar corona is a highly dynamic

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arena replete with multiple-scale spatiotemporal structures (Aschwanden et al. 2001a). A major new advance is the discovery that strong flows are found everywhere — in the subcoronal (chromosphere) as well as in the coronal regions (see e.g. (Schrijver et al. 1999; Winebarger, LeLuca and Golub 2001; Wilhelm 2001; Aschwanden et al. 2001a; Aschwanden et al. 2001b; Seaton et al. 2001; Winebarger et al. 2002) and references therein). Equally important is the growing belief and realization that the plasma flows may complement the abilities of the magnetic field in the creation of the amazing richness observed in the coronal structures.

Interestingly enough, even before the observational mandate, theoretical efforts in harnessing the plasma flows to solve some of the riddles of solar physics had already begun. In particular the dynamics of flow-based structure creation and heating was the subject of Mahajan et al. (1999, 2001); in this model the flows provided the basic material as well as energy for the primary heating of the coronal loops. A systematic treatment of loop models that include flows was also developed by Orlando, Peres and Serio (1995a, 1995b), and by Mahajan et al. (1999, 2001).

If flows are to play a crucial role in determining the dynamics and structure of the solar corona, we must immediately look for the sources and mechanisms for their creation. Catastrophic models of flow production in which the magnetic energy is suddenly converted into bulk kinetic energy (and thermal energy) are rather well-known; various forms of magnetic reconnection (flares, micro and nanoflares) schemes permeate the literature (see e.g. Wilhelm 2001; Christopoulou, Georgakilas and Koutchmy 2001 for chromosphere up-flow generations). A few other mechanism of this genre also exist: Uchida et al. (2001) proposed that the major part of the supply of energy and mass to the active regions of the corona may come from a dynamical leakage of magnetic twists produced in the subphotospheric convection layer; Ohsaki, Shatashvili, Yoshida and Mahajan (2001, 2002) have shown how a slowly evolving closed structure (modelled as a Double Beltrami two fluid equilibrium) may experience, under appropriate conditions, a sudden loss of equilibrium with the initial magnetic energy appearing as the mass flow energy. Another mechanism, based on loop interactions and fragmentations and explaining the formation of loop threads, was given in Sakai and Furusawa (2002). These mechanisms, though extremely interesting, are an unlikely source to account for the observed ubiquity of plasma flows. One should, perhaps, look for relatively gentler, more widespread, and steadier mechanisms.

To exploring new avenues for flow generation, we seek guidance from phenomenology. Based on estimates of energy fluxes required to heat the chromosphere and the corona, Goodman (2001) has shown that the mechanism which transports mechanical energy from the convection zone to the chromosphere (to sustain its heating rate) could also supply the
energy to heat the corona, and accelerate the solar wind (SW). The coronal heating problem, in this context, is shifted to the problem of the dynamic energization of the chromosphere for which the flows are found to be critical as warranted by the following observations made in soft X-rays and extreme ultraviolet (EUV), and recent findings from the Transition Region and Coronal Explorer (TRACE): the over-density of coronal loops, the chromospheric up-flows of heated plasma, and the localization of the heating function in the lower corona (e.g. Schrijver, et al. 1999; Aschwanden et al. 2001a; Aschwanden 2001b) and references therein. The main message then, is that the coronal heating problem may only be solved by including processes (including the flow dynamics) in the chromosphere and the transition region (TR). The challenge, therefore, is to develop a semi steady state theory of flow generation in these subcoronal regions.

The most obvious process for flow generation could be the conversion of magnetic and/or the thermal energy to plasma kinetic energy. We have already mentioned a few examples of the magnetically driven transient but sudden flow-generation. For a more quiescent pathway, we could turn to the Bernoulli mechanism converting thermal energy into kinetic energy, or to the general magnetofluid rearrangement of a relatively constant kinetic energy, i.e. going from an initial high density–low velocity to a low density–high velocity state. We will soon show that the Double–Beltrami–Bernoulli states accessible to a two–fluid systems in which the velocity field is formally treated at par with the magnetic field (Mahajan and Yoshida 1998), can readily provide the necessary framework.

2. Model for the generation of flows in subcoronal regions

In astrophysics, the plasma flow could be assigned at least two distinct connotations: 1) the flow is a primary object whose dynamics bears critically on the phenomena under investigation. The problems of the formation and the original heating of the coronal structure, the creation of channels for particle escape, for instance, fall in this category, 2) the flow is a secondary feature, possibly created as a by product (e.g. see Ohsaki et al. (2002)) and/or used to drive or suppress an instability. Since the generation of flows which will eventually create the coronal loops is the theme of this effort, the flows here are fundamental.

Our theoretical model is based on a straightforward application of recently developed magnetofluid theory. We plan to restrict ourselves to almost steady state considerations (for a steady and continuous supply of plasma flows emerging from the subcoronal regions). Very near the photospheric surface, the influence of neutrals and ionization (and processes of flux emergence etc.) would not permit a quasi-equilibrium approach. A little farther distance ($\Delta r \geq 2000$ km) from the surface, however, we expect that there exist fully ionized and
magnetized plasma structures such that the quasi-equilibrium, two-fluid model will capture the essential physics of flow generation.

From recent observational data (see e.g. Goodman (2000); Aschwanden et al. (2001a); Socas-Navarro and Almeida (2002) and references therein) we could obtain the following average plasma density and temperature at \( \sim (500 - 5000) \) km: \( n \sim (10^{14} - 10^{11}) \text{cm}^{-3}; \) \( T \sim (1 - 6) \text{eV} \) (for simplicity we will assume equal electron and ion temperatures). The information about the magnetic field is a little harder to extract because of the low sensitivity and lack of high spatial resolution of the measurements coupled with the inhomogeneity and coexistence of small- and large-scale structures with different temperatures in nearby regions. At these distances we have different values for the network and for the internetwork fields. The network plasmas have typically short-scale fields in the range \( B_0 \sim (700 - 1500) \text{ G}, \) have more or less uniform density and will be prone to explosive/eruptive analysis of the kind carried out in Ohsaki et al. (2002). The internetwork fields, on the other hand, are generally smaller (with some exceptions (Socas-Navarro and Almeida (2002))) \( - B_0 \lesssim 500 \text{ G}, \) and are embedded in larger-scale plasma structures with inhomogeneous densities. The theory of steady creation of flows in the subcoronal regions will be based on these latter objects.

We consider the simplest two-fluid equilibria with \( T = \text{const} \) leading to \( n^{-1} \nabla p \to T \nabla \ln n \). The generalization to a homentropic fluid with \( p = \text{const} \cdot n^{\gamma} \) is straightforward and is done in the numerical work. The dimensionless equations describing the model equilibrium can be read off from Mahajan et al. (2001)

\[
\begin{align*}
\frac{1}{n} \nabla \times \mathbf{b} \times \mathbf{b} + \nabla \left( \frac{r_{A0}}{r} - \beta_0 \ln \left( n - \frac{V^2}{2} \right) \right) + \mathbf{V} \times (\nabla \times \mathbf{V}) &= 0, \quad (1) \\
\nabla \times \left[ \left( \mathbf{V} - \frac{\alpha_0}{n} \nabla \times \mathbf{b} \right) \times \mathbf{b} \right] &= 0, \quad (2) \\
\nabla \cdot (n \mathbf{V}) &= 0, \quad (3) \\
\n\nabla \cdot \mathbf{b} &= 0, \quad (4)
\end{align*}
\]

where the notation is standard with the following normalizations: the density \( n \) to \( n_0 \), the density at some appropriate distance from the solar surface, the magnetic field to the ambient field strength at the same distance, and velocities to the Alfvén velocity \( V_{A0} \). The parameters \( r_{A0} = GM_\odot/V_{A0}^2 R_\odot = 2\beta_0 r_\odot, \alpha_0 = \lambda_{i0}/R_\odot, \beta_0 = c_{s0}/V_{A0}^2 \) are defined with \( n_0, T_0, B_0 \). Here \( c_{s0} \) is a sound speed, \( R_\odot \) is the solar radius and \( \lambda_{i0} = c/\omega_{i0} \) is the collisionless skin depth.

The double Beltrami solutions, expressing the simple physics that the electrons follow the field lines, while the ions, due to their inertia, follow the field lines modified by the fluid vorticity, are

\[
\begin{align*}
\mathbf{b} + \alpha_0 \nabla \times \mathbf{V} &= d \ n \ \mathbf{V}, \\
\mathbf{b} &= a \ n \left[ \mathbf{V} - \frac{\alpha_0}{n} \nabla \times \mathbf{b} \right],
\end{align*}
\]
where $a$ and $d$ are dimensionless constants related to the two ideal invariants, the magnetic and the generalized helicities (Mahajan and Yoshida 1998; Mahajan et al. 2001), $h_1 = \int (A \cdot \mathbf{b}) \, d^3x$, $h_2 = \int (A + \mathbf{V}) \cdot (\mathbf{b} + \nabla \times \mathbf{V}) \, d^3x$. On substituting this set of equations into Eqs. (1)–(2) one obtains the Bernoulli condition

$$
\nabla \left( \frac{2\beta_0 r \sigma_0}{r} - \beta_0 \ln n - \frac{V^2}{2} \right) = 0,
$$

relating the density with the flow kinetic energy, and solar gravity. Equations (1), (5) and (6) represent a close system, and may be easily manipulated to yield ($g(r) = r_{a0}/r$)

$$
\frac{\sigma_0^2}{n} \nabla \times \nabla \times \mathbf{V} + \sigma_0 \nabla \times \left[ \left( \frac{1}{a n} - d \right) n \mathbf{V} \right] + \left( 1 - \frac{d}{a} \right) \mathbf{V} = 0,
$$

$$
\sigma_0^3 \nabla \times \left( \frac{1}{n} \nabla \times \mathbf{b} \right) + \sigma_0 \nabla \times \left[ \left( \frac{1}{a n} - d \right) \mathbf{b} \right] + \left( 1 - \frac{d}{a} \right) \mathbf{b} = 0,
$$

$$
n = \exp \left( - \frac{2g_0 - \frac{V_0^2}{2\beta_0} - 2g + \frac{V^2}{2\beta_0}}{2} \right),
$$
a set ready to be solved for the density, the velocity and the magnetic field. We must point out that this time-independent set is not suitable for studying chromospheric heating processes (primary heating at lower heights). The main thrust of this paper is to uncover mechanisms which create flows in the subcoronal regions — the flows that will supply matter and energy needed to create the coronal structures, and provide their primary heating. The creation and heating problem, of course, requires a fully time-dependent (Mahajan et al. 2001) treatment with multi-species.

We have carried out a 1D simulation (the relevant dimension being the height “$Z$” from the center of Sun; $Z_0 = R_\odot + \Delta r$ is the surface at which the boundary conditions are applied) of the coupled non-linear system (7), (8), and (9) for a variety of boundary conditions. The height $Z_0$ of the boundary surface is chosen to lie higher than $(1 + 2.8 \cdot 10^{-3}) R_\odot$ so that the influence of ionization can be neglected.

The simulation results are presented in Figs. 1–2. These are the plots of various physical quantities as functions of the height. The first figure consists of three frames (a–b, c–d, and e–f) each consisting of two pictures – one for the density and the magnetic field and the other for the velocity field. The parameters defining different frames are (we will give them in the order ($n_0$; $B_0$; $T_0$; $V_{a0}$)): (1) a–b frame: ($10^{12}$ cm$^{-3}$; 200 G; 2 eV; 440 km/s) implying $\beta_0 \sim 0.002 < 1$ and $r_{a0} = 225$; 2) c–d frame: ($10^{11}$ cm$^{-3}$; 100 G; 5 eV; 600 km/s) implying $\beta_0 \sim 0.007 < 1$ and $r_{a0} = 40$; 3) e–f frame: ($10^{11}$ cm$^{-3}$; 50 G; 6 eV; 330 km/s) implying $\beta_0 \sim 0.04 < 1$ and $r_{a0} = 30$. In each frame there are three sets of curves labelled by $\sigma_0$.
(1–2–3 corresponding respectively to $\alpha_0 = 0.000013; \ 0.005; \ 0.1$), the measure of the strength of the two-fluid Hall currents.

For all our runs the boundary conditions, $|b_0| = 1, \ V_0 = 0.01 V_{40}$ (with $V_{40} = V_{90} = V_{90}$) were imposed; we begin with just a small residual flow speed. The choice, $d \sim a \sim 100$ and $(a - d)/a^2 \sim 10^{-6}$ for the parameters characterizing the double Beltrami state, reflects the physical constraint that we are dealing with a sub-Alfvénic flow with a very small $\alpha_0$ (Mahajan et al. 1999). Due to the limitations of the code, the values of $\alpha_0$ chosen for the simulation are much larger than their actual values ($\sim 10^{-8}$ for corona and smaller for subcoronal regions); we hope to do better in future.

The most remarkable result of the simulation is that for small $\alpha_0$ (curves labelled 1), there exists some height where the density begins to drop precipitously with a corresponding sharp rise in the flow speed. The effect is stronger for low beta (a-b are the lowest beta frames) plasmas. It is also obvious that at very short distances, the stratification is practically due to gravity but as we approach the velocity “blow-up” height, the self-consistent magneto-Bernoulli processes take over and control the density (and hence the velocity) stratification.

An examination of the Bernoulli condition (9) readily yields an indirect estimate for the height at which the observed shock-formation may take place. For a low beta plasma, the sharp fall in density is expected to occur when (this is true for all $\alpha_0$), i.e,

$$|V|^2 - V_0^2 > 2 \beta_0.$$  \hspace{1cm} (10)

For the current simulation, at $\beta_0 = 0.04$, it occurs approximately at $|V|^2 > 0.08$ or at $|V| \sim 0.28$; This analytically predicted value is very close to the simulation result (see Fig. 2(b)). Simulation results also confirm that the velocity blow-up distance depends mainly on $\beta_0$, and that the final velocity is greater for greater $T_0$ (Fig. 2). The data presented in Fig. 1 and Fig. 2 corresponds to a uniform temperature plasma. For this case, the variations in plasma pressure are entirely due to the variations in density. Since the magnetic energy remains practically uniform over the distance, (though the magnetic field components change self-consistently assuring the varying magnetization properties) sharp decrease in density with a corresponding sharp rise in the flow-speed (of the order of $n^{-1/2}$) is nothing but the expression of Bernoulli constraint imposed by the magneto-fluid equilibrium. If we allow the temperature to vary by using a polytropic equation of state, the general nature of the results remain unchanged. The final parameters, naturally, depend upon the adiabaticity index $\gamma$; the temperature, typically, changes by an order of magnitude over the distance covered.

To check whether the generated flows are predominantly radial or somewhat more isotropic (to explain the observational constraints) we studied in detail different $\beta_0$ and $\alpha_0$ cases (fixing $\beta_0$ is quite difficult due to complications like ionization) and found that
the flows tend to be mostly radial only for large \( \alpha_0 \) (see, for example, plots labeled 2 and 3 in Fig. 1(b,d,f)). The situation could change considerably when we deal with a more inclusive time-dependent dynamical model with dissipation. Plasma heating, then, could result from the dissipation of the perpendicular energy so that at larger distances, the flows would have larger radial components. Heating would also keep \( \beta(\mathbf{r}, t) \) large at upper heights shifting the velocity blow-up distance further or eliminating it altogether; we know from Fig.1 that as \( \beta_0 \) goes up, the density fall (velocity amplification) becomes smoother. These issues will be dealt with later in a more detailed work. Notice, that final velocities go up with \( V_0 [km/s] \sim d^{-1} V_{A0} \). An initial flow with speed 3.3 km/s (e-f frame of Fig. 1) ends up acquiring a high speed \( \sim 100 km/s \) at the height \( (Z - Z_0) \sim 0.09 R_\odot \) but at a lower density \( \sim 10^{9.5} \text{cm}^{-3} \).

The general nature of the solution is determined by the values of dimensionless input parameters. Since \( \beta_0 \) determines whether the density fall (velocity amplification) will be sharp or smooth, lowering the magnetic field and keeping other parameters fixed will drive the system towards a smoother change. On the other hand if density is adjusted to bring the system back to the same \( \beta_0 \), the results will be exactly like that of the high field cases we have discussed above — region of sharp changes will persist.

If one were to ignore the flow term in (6) (a totally wrong assumption commonly used in many studies), we will end up finding essentially radial flows. The magnitude of these flows, however, remains small; there is no region of sharp rise (10), and the generated flows achieve reasonable energies at heights typically 10 times greater than the heights at which the correct Bernoulli condition would do the trick.

3. Conclusions and Summary

We have shown a possible pathway for a steady generation of flows in the quasi-equilibrium structures established in the subcoronal regions. These structures consist of fully ionized two-species plasma trapped in magnetic fields. The suggested mechanism is a straightforward application of the recently developed magnetofluid model (Mahajan and Yoshida 1998; Mahajan et al. 1999, 2001); a generalized Bernoulli mechanism (a necessary condition for the double-beltrami magnetofluid equilibrium) allows the conversion of thermal energy into kinetic energy and/or a readjustment of the kinetic energy from a high density–low velocity to a low density–high velocity plasma. Numerical results show a significant density fall with a sharp amplification of the flow speed. In the presence of dissipation, these flows are likely to play a fundamental role in the heating of the upper chromosphere and TR, although our explicit purpose in this paper was to create a steady source of matter
and energy for the formation and primary heating of the corona. Our preliminary results agree with the observational data, and lend promise to attempts, based on the existence of subcoronal flows, to tackle unresolved problems like the coronal heating and origin of the Solar wind.

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Fig. 1.— Plots of density, magnetic fields and velocity versus height for values of $\alpha_0$ and $\beta_0$. Sub-figures (a) and (b) are for $\beta_0 = 0.001$, $r_{c0} = 225$; (c) and (d) are for $\beta_0 = 0.007$, $r_{c0} = 40$; (e) and (f) are for $\beta_0 = 0.014$, $r_{c0} = 30$. The numbers 1, 2, 3 represent $\alpha_0 = 0.000013$; 0.005; 0.1, respectively. $V_y$ is not displayed since its behavior is practically similar to $V_x$. The velocity blow-up is controlled by $\beta_0$. For a bigger (unrealistic) $\alpha_0$ there is a splitting of the velocity components — at the end the radial component is dominant. Magnetic field energy does not change much on these distances.

Fig. 2.— The “blow-up” distance (a) and velocity (b) versus $\alpha_0$. The smaller the $\beta_0$, the smaller the “blow-up” distance and smaller the velocity at “blow-up” (compare with (10)). For fixed $\beta_0$, the process is less sensitive to changes in $\alpha_0$. 