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MAGNETIC RECONNECTION DRIVEN BY THE COALESCENCE INSTABILITY

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Abstract

We present a detailed study of magnetic reconnection at an x-point driven by the coalescence instability. In particular, we have explored the effects of plasma compressibility and the magnitude of the toroidal field on the rate of reconnection. For large toroidal fields, the plasma is almost incompressible and the destruction of magnetic flux proceeds linearly with time. However, when the attractive force between two neighboring islands is strong, and when the poloidal and the toroidal fields are comparable in magnitude, the reconnection rate is found to be faster. Implications for the steady-state models of magnetic reconnection are discussed.

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I. INTRODUCTION

The coalescence instability is expected to occur in a magnetohydrodynamic (MHD) equilibrium consisting of a chain of magnetic islands. The instability originates from the tendency of two parallel current channels to attract one another. Consequently, two neighboring magnetic islands sharing an x-point tend to merge to form one island. If the plasma is ideal, the process of coalescence is eventually halted because magnetic flux accumulates in the vicinity of the x-point and produces magnetic pressure which opposes further merging of the islands. However, if the plasma is slightly nonideal, the topology of field lines is allowed to change, magnetic flux is destroyed, and the coalescence proceeds uninhibited until the two islands have merged to form one larger island. That the final state is of lower energy than the initial state was speculated originally by Furth, Rutherford and Selberg.[1] Finn and Kaw[2] proved the existence of the linear, ideal instability from the Energy Principle of ideal MHD. The linear dynamics of the instability and the nonlinear process of merging of two islands in the presence of resistivity was demonstrated at first numerically by Pritchett and Wu.[3]

The coalescence instability may occur in any toroidal plasma configuration, which either by design or as a result of other MHD instabilities, has evolved into a state consisting of a chain of magnetic islands. It has been invoked[2] as an explanation for the experimental observation that during the initial stage of a tokamak discharge[4], the high m- , n- modes on a particular rational surface disappear, leaving behind only the low m- , n- modes commensurate with the value of q on the rational surface. It should also be directly observable in a Doublet equilibrium if the external poloidal field-shaping coils are turned off. The phenomenon of coalescence is clearly important because it provides a mechanism by which a large fraction of the energy in the magnetic field may be released as kinetic energy of the fluid. In point of fact, Shafranov[5] has recently suggested that this mechanism may be exploited for heating a laboratory plasma. The same mechanism may also play a crucial role in triggering solar flares and intense local heating in the Earth's magnetosphere, both of which are cosmic events in which magnetic energy is unleashed as thermal and kinetic energy of particles.[6]

The process of reconnection driven by the coalescence instability has some distinctive physical characteristics. The attractive force between two neighboring current-carrying lobes is essentially an ideal MHD effect, and it is therefore not surprising that the linear growth rate of the instability is independent of plasma resistivity.[2,3] Resistivity, however, is crucial to the completion of the process, and provides the mechanism for the conversion of magnetic energy to kinetic energy of the fluid. Reconnection at the x-point resulting from the nonlinear evolution of the instability is richer and more complex than the simple phenomenon of magnetic field annihilation at a null point in as much as the former is driven by global, ideal MHD forces, while the dynamics of the latter is essentially localized near the neutral layer.

The purpose of this paper is to study the nonlinear evolution of the coalescence instability in a resistive plasma. In particular, we make a detailed study of the effects of plasma compressibility and the magnitude of the toroidal field on the rate of magnetic reconnection at the x-point. We begin with an exact class of MHD equilibria due to

Fadeev, Kvartshava and Komarov.[7] Each of these equilibria are characterized by a periodic chain of magnetic islands within which the spatial distribution of the plasma current may be controlled to give strongly peaked or flat current profiles. For a given total current, the attractive force between two neighboring islands is stronger for current distributions which are more peaked at the centers of the islands.

We find, not surprisingly, that in the limit of large toroidal fields, the plasma is almost incompressible, and the rate of reconnection is roughly of the same order of magnitude as that observed in earlier work[8] which has made explicit use of the incompressibility condition. However, for smaller values of the toroidal field (comparable in magnitude with the poloidal field), and when the current channels in two neighboring, coalescing islands are strongly peaked, we find that plasma compressibility makes a qualitative difference by introducing a new nonlinear phase during which the destruction of flux at the x-point proceeds at a much faster rate. The new phase is triggered by the large accumulation of magnetic flux in the vicinity of the x-point, and is hindered, in general, by the presence of a large toroidal field.

The plan of the paper is as follows. In Section II, we describe the MHD equilibria which provide the beginning point for the initial-value calculation. Section III contains a detailed discussion of our numerical results. In Section IV, we discuss implications for the steady-state models of externally driven reconnection.

II. EXACT MHD EQUILIBRIA

In rectangular geometry (x,y,z), assuming that equilibrium quantities depend only on x and y, the magnetic field $\vec{B}$ may be represented as

$$\vec{B} = \vec{\nabla}\Psi(x,y) \times \vec{\nabla} z + B_z(x,y)\vec{\nabla} z \qquad (1)$$

where $\Psi(x,y)$ is a poloidal flux function. From the z-component of Ampere's law

$$\vec{\nabla}\times\vec{B} = \vec{J} \ , \qquad (2)$$

we get

$$\nabla^2\Psi(x,y) = -J_z(x,y) \ . \qquad (3)$$

We begin with the well-known class of MHD equilibria due to Fadeev et al.[7] These equilibria, which are characterized by a periodic chain of magnetic islands, are produced by a toroidal current distribution given by

$$J_z = B_{ox}k(1-\varepsilon^2)\exp(2k\Psi/B_{ox}) \qquad (4)$$

where $0 \leqslant \varepsilon \leqslant 1$, and B_{ox} is a normalization constant. Equation (3) may be solved for Ψ to obtain

$$\Psi = -\frac{B_{ox}}{k}\,\ell n(\cosh ky + \varepsilon \cos kx) \qquad (5)$$

The magnetostatic equation

$$\vec{J} \times \vec{B} = \vec{\nabla} p \tag{6}$$

which, in the present case, reduces to

$$\frac{d}{d\Psi}\left[p(\Psi) + \frac{B_z^2(\Psi)}{2} \right] = J_z(\Psi) \tag{7}$$

may be trivially integrated to give

$$p + \frac{B_z^2}{2} = \frac{B_{ox}{}^2}{2}(1-\varepsilon^2)\ \exp\left(\frac{2k\Psi}{B_{ox}}\right) + \frac{B_{oz}{}^2}{2} \tag{8}$$

where $B_{oz}{}^2$ is an additive constant.

In the present study, we employ two subclasses of the general solution (8). One class consists of force-free equilibria with

$$\frac{dp}{d\Psi} = 0\ , \tag{9a}$$

and

$$B_z{}^2 = B_{ox}{}^2(1-\varepsilon^2)\ \exp\left(\frac{2k\Psi}{B_{ox}}\right) + B_{oz}{}^2 \tag{9b}$$

For the other class of solutions, the magnetic field is purely poloidal with B_z=0, and p given by Eq. (8). For both of these subclasses

$$B_x = \frac{\partial \Psi}{\partial y} = -B_{0x} \frac{\sinh ky}{\cosh ky + \varepsilon \cos kx} \tag{10a}$$

and

$$B_y = -\frac{\partial \Psi}{\partial x} = -B_{0x} \frac{\varepsilon \sin kx}{\cosh ky + \varepsilon \sin kx} \quad . \tag{10b}$$

We note, from Eqs. (4) and (5), that

$$J_z = \frac{B_{0x} k\left(1 - \varepsilon^2\right)}{\left(\cosh ky + \varepsilon \cos kx\right)^2} \quad . \tag{11}$$

The system consists of a chain of islands, and is periodic in x. We see that if all other constants are held fixed, the parameter ε determines the magnitude, and the spatial distribution of the toroidal current within an island. As $\varepsilon \to 1$, $J_z \to \infty$ at the centers of the islands. It is practically impossible to resolve numerically the singular current sheets at the centers of the islands for values of ε close to unity. In what follows, we have considered equilibria with values of ε up to 0.7, for which the poloidal magnetic flux-tubes within each island is strongly bunched near its center. We emphasize that in order to isolate the effect of plasma compressibility, it is important to consider high values of ε for which the attractive force between two neighboring, strongly peaked current channels is large. It is for these cases that compressibility becomes important as density is piled up rapidly near the x-point, and we observe fast rates of connection.

III. NUMERICAL RESULTS

The numerical result in this section are obtained from the integration of the primitive set of resistive MHD equations

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0 \qquad (12)$$

$$\rho \frac{d\vec{v}}{dt} = - \nabla p + \rho_0 (\nabla \times B) \times \vec{B} , \qquad (13)$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) + \eta \nabla^2 \vec{B} \qquad (14)$$

and

$$\frac{d}{dt} \left(\frac{p}{\rho^\gamma}\right) = 0 \qquad (15)$$

where the quantities $\vec{v}$, ρ, p, and B are the normalized fluid velocity, mass density, pressure and magnetic field respectively, and γ is the adiabatic constant. [The normalization is such that the unit of length is the spacing between grid points and the sound velocity is unity for $\rho = \rho_0$]. Equations (12) to (15) are pushed using a 2 1/2 D (2 spatial dimensions and 3 velocity components) MHD-particle code in which the particles represent fluid elements.[9,10] The runs are done on a 128×64 mesh with periodic boundary conditions in x and perfectly conducting walls in y. The conducting walls are placed at values of y large enough that the initial equilibrium is only slightly modified. We begin with a system of two islands initially in equilibrium according to Eqs. (4) - (11) and apply a small velocity perturbation.

The coalescence instability, and the destruction of magnetic flux at the x-point, proceeds generically in three stages. In the first stage, the two islands approach one another, driven by the attractive force between the two current channels. The end of this stage, and the beginning of the second stage, is marked by the development of a neutral layer where the flux-lines are concentrated. The destruction of flux, which is rather slow in the first stage, takes place predominantly in the second stage, which is the most interesting stage in the study of magnetic reconnection driven by instabilities. The third stage appears when the instability begins to saturate, and the rate of destruction of flux levels off. The demarcation between these three stages is not always sharp, and is sensitively dependent on the Reynolds number.

In Fig. 1, we show a typical sequence of frames following the time evolution of the poloidal flux function Ψ , and in Fig. 2, a typical flow field in the x-y plane.

We now discuss our numerical results on the dynamics of the coalescence instability evolving from equilibria for which $\varepsilon=0.3$ (also studied in Refs. 3 and 8 for incompressible plasmas) and $\varepsilon=0.7$. In what follows, when we refer to the scaling with time of the rate of reconnection, it should be interpreted as the rate of destruction of magnetic flux in the "second stage" of the evolution of the instability.

(i) ε=0.3

In Fig. 3 we compare the rates of reconnection as we vary the magnetic Reynolds number for a given number of the subclass of equilibria given by Eq. 9 with $B_{0x} = 3.5$, $B_{0z} = 0$, and $\varepsilon = 0.3$. For all values of the Reynolds number ($s = 5\times10^2$ to 2×10^3), the destruction of flux proceeds linearly with time. The reconnection process does exhibit self-similar characteristics as the Reynolds number varies, in qualitative agreement with the results of Biskamp and Welter.[8] As mentioned earlier, the separation in time of the three stages of evolution of the coalescence process is less sharp for larger than for smaller values of the resistivity. The explanation for this observation is that for larger values of resistivity, the destruction of flux is effective even in the first stage of the instability and blurs the separation between the first and the second stages.

We have computed the destruction of flux for a member of the second subclass of equilibria , with zero toroidal field, given by Eqs. (9) with $B_{0x} = 3.5$, $B_z = 0$. Again, as shown in Fig. 4, the destruction of flux proceeds approximately linearly in time. It is seen thus, that for $\varepsilon = 0.3$, the rate of reconnection is insensitive to the magnitude of the toroidal field, and the effect of plasma compressibility is negligible. We remark parenthetically that our numerical results for this case with zero toroidal field are qualitatively different from those of Biskamp[11] in one important aspect. Biskamp has found that the secondary tearing modes excited in the neutral layer during the merging process substantially reduce the rate of reconnection. We believe that Biskamp's conclusion may have been due to the fact that in order to achieve higher numerical resolution, he has constrained the flow-field to be "up-down" symmetric, and eliminated perturbations which would otherwise sweep the secondary islands along the neutral line away from the reconnection region. We have found that the secondary islands are always removed from the neutral layer by the flow field, though the process is inhibited to some extent by the presence of large toroidal fields.

(ii) $\varepsilon = 0.7$

For equilibria with $\varepsilon = 0.7$, plasma compressibility plays an important role in the dynamics of the instability. We present numerical results for the subclass of force-free equilibria given by Eq. (9). In the first set of runs we choose $B_{oz}=0$, and compute the rate of reconnection for equilbria obtained by varying B_{ox} (or the poloidal field). At $\varepsilon = 0.7$, there is very little residual B_z in the neighborhood of the current sheet. B_z does not play any role in the reconnection process, and serves only to maintain the pressure equilibrium inside the 0-point. In Fig. 5 we plot the magnitude of flux destroyed and find that as values of B_{ox} increases from 0.25 to 3.5, the exponent m in t^m varies from 1.1 to 1.9; this is shown more explicitly in Table I. At low B_{ox}, the poloidal magnetic pressure is much less than the background plasma pressure. Therefore, the attracting force between the current channels is not strong enough to compress the plasma in the current sheet, and the plasma behaves as if it were incompressible. But for high values of B_{ox} the faster reconnection rate observed (for $B_{ox}=3.5$, say) is primarily due to the pile-up of flux density caused by plasma compressibility. The phenomenon is similar to that described in earlier work[12,13] where a reconnection rate $\dot{\Psi} \propto t^m$ is obtained with $m = \rho_i/\rho_e$, where ρ_i and ρ_e are respectively the density inside and outside the current sheet. Evidence of the accumulation of density in the current sheet is shown in Fig. 6.

In the second set of runs, we study the consequences of increasing the toroidal field on the rate of reconnection. We keep B_{ox} at a relatively high value in order to maximize the role of plasma compressibility at low values of B_{oz}. Intuitively, the plasma is expected to become more incompressible as the value of B_{oz} rises. In Fig. 7, we plot the flux destroyed in time for $B_{ox} = 3.5$ and $B_{oz} = 5.0$ and indeed observe a decrease in the exponent m from 1.9 (B_{oz}=0) to 1.5 ($B_{oz} = 5.0$). However, we have not observed numerically, for sizable values of B_{oz}, the expected reduction of the exponent to the value $m \simeq 1$. In addition to the physical diffusion, the Lax-Wendroff scheme used to integrate the magnetic field also contains a higher order numerical diffusion. For very thin current sheets, the diffusion of B_z can be as important as that of B_x, and plasma compressibility continues to be a strong effect.

In order to alleviate the effects of numerical diffusion and resolve the singular layer to an acceptable order of accuracy, we model the region in the vicinity of the singular layer between two merging islands as a sharp-boundary plasma column in which is embedded a unidirectional, homogenous magnetic field with opposite sense. This set-up has been previously used in Ref. 12. The two strongly peaked current channels in neighboring islands are modelled as current filaments external to the plasma column, and the merging of field lines is brought about by the pinching action of the filaments. In Fig. 8 we plot the flux destroyed $\Delta\Psi$ as a function of time t with and without a toroidal magnetic field. As B_z is reduced to zero, the exponent m in t^m

increases for 0.9 to 1.4, demonstrated that for large toroidal fields, reconnection proceeds at approximately the same rate as in an incompressible plasma. Among other features, we also observe that as the toroidal magnetic field B_{oz} is increased, the flow pattern for v_z is such as to indicate a tilting of the toroidal field B_z in the poloidal direction. We interpret this to correspond to a transfer of flux from the toroidal component of the magnetic field to the poloidal component, and may be related to the scenario given by Hamieri.[14] When this transfer of flux is important, it may reduce the toroidal field in the sheet and make the plasma more compressible during coalescence. However, in the cases studied, this phenomenon is seen to be important only at the beginning when the plasma is driven together either by the coalescence instability or the external coils, and disappears after some time, when fast reconnection proceeds. It seems also that this rotation of the magnetic field extends to a region much broader than the current sheet itself. In the case of the pinch, however, the rotation of the magnetic field did not seem to increase or have any effect on the rate of reconnection, since for strong B_z we observe slow reconnection rates, with $m \simeq 1$.

IV. COMMENTS IN RELATION TO THE STEADY-STATE MODELS OF EXTERNALLY-DRIVEN RECONNECTION

In studies of the solar-flare phenomenon, the two steady-state models[15] due to Sweet-Parker and Petschek have attracted considerable attention. In both these models, the plasma is externally driven, and the balance between the internal forces of the plasma and the external forces is required to lead to a dynamical steady-state. The crucial physical factor distinguishing the two models is the width of the dissipation region. In the Sweet-Parker model, this width is comparable to the overall dimension of the opposite fields. In the Petschek model, however, aided by somewhat exceptional boundary conditions which may produce sharp Alfvénic shocks near the neutral layer, the width of the dissipation region may be made much smaller, with the consequence that reconnection may proceed at a rate proportional to η^0 , which is larger than the Sweet-Parker rate proportional to $\eta^{1/2}$.

The question of the relationship between externally-driven reconnection, and reconnection driven by instabilities which are self-consistently realized in a plasma has been the subject of considerable interest recently[6,11,12,13], and motivated to an extent the present study. Our first observation on this question is that, once perturbed from equilibrium, a plasma driven by instabilities is in a state of dynamical non-equilibrium until the instability saturates, and the assumption of a steady-state, crucial to the Sweet-Parker or the Petschek models, is not strictly valid. However, a comparison with those regimes remains possible during which the rate of reconnection is constant. For equilibria with $\varepsilon = 0.3$, we find that the rate of reconnection is approximately constant (Fig. 3), and may be seen to conform to the Sweet-Parker scaling $\eta^{1/2}$. For equilibria with $\varepsilon = 0.7$, the destruction of flux is almost quadratic in time. As stated earlier, the mechanism of this faster process is qualitatively similar to that in the recent work of Brunel, Tajima, and Dawson.[12,13]

We may further conjecture that as compressiblity becomes a stronger effect, it may serve to provide a transition from Sweet-Parker to Petschek rates (since the reconnection rate accelerates in time). Such a transition stage is physically realizable, and is somewhat less artificial than the origianl steady-state model due to Petschek, in which exceptional boundary conditions are stipulated, and not shown to be an outcome of the dynamics. As far as we can see, this natural dynamic transition does not seem possible if the plasma is incompressible.

Acknowledgments

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References

1. H. P. Furth, P. H. Rutherford, and H. Selberg, Phys. Fluids 16, 1054 (1973).
2. J. M. Finn and P. K. Kaw, Phys. Fluids 20, 72 (1977).
3. P. L. Pritchett and C. C. Wu, Phys. Fluids 22, 2140 (1979).
4. S. V. Mirnov and I. B. Semenov, in Plasma Physics and Controlled Nuclear Fusion Research, (International Atomic Energy Agency, Vienna, 1971), Vol. II, p. 401.
5. V. D. Shafranov, Nucl. Fusion 19, 187 (1979).
6. T. Tajima, F. Brunel and J. Sakai, Ap. J. 245, L45 (1982).
7. V. M. Fadeev, I. F. Kvartskhava, and N. N. Komarov, Nucl. Fusion 5, 202 (1965).
8. D. Biskamp and H. Welter, Phys. Rev. Lett. 44, 1069 (1980).
9. J. N. Leboeuf, T. Tajima and J. M. Dawson, J. Comput. Phys. 31, 379 (1979).
10. F. Brunel, J. N. Leboeuf, T. Tajima and J. M. Dawson, J. Comput. Phys. 43, 268 (1981).
11. D. Biskamp, Phys. Lett. 87A, 357 (1982).
12. F. Brunel, T. Tajima, and J. M. Dawson, Phys. Rev. Lett. 49,323 (1982).
13. T. Tajima, in Fusion Energy - 1981 (International Centre for Theoretical Physics, Trieste, 1982), p. 403.
14. E. Hamieri, J. Plasma Physics 22, 245 (1979).

15. For a detailed discussion, the reader is referred to E. N. Parker, Cosmical Magnetic Fields: Their Origin and Their Activity, (Clarendon Press, New York, 1979).

Figure Captions

1. Typical sequence of frames following the time evolution of the poloidal flux function Ψ. Fig. 1(a) displays the initial, and Fig. 1(d) the final configuration.

2. Plasma flow field in the x-y plane at some instant of time during the evolution of the coalescence instability.

3. Poloidal flux destroyed ($\Delta\Psi$) as a function of time (t) for different values of the Reynolds number. The initial configuration is force-free, with $\varepsilon = 0.3$, $B_{ox} = 3.5$, $B_{oz} = 0$.

4. Plot of $\Delta\Psi$ versus t. The initial configuration has $B_z = 0$, with $\varepsilon = 0.3$, $B_{ox} = 3.5$, and $p(\Psi)$ given by Eq. (8). The value of $s = 10^3$.

5. Plots of $\Delta\Psi$ versus t.The initial configuration force-free with $\varepsilon = 0.7$, $B_{oz} = 0$ and four different values of B_{ox}. [(a)=3.5, (b)=0.7, (c)=0.5, (d)=0.25]. The exponent m decreases from 1.9 to 1.1 as B_{ox} decreases from 3.5 to 0.25.

6. Contours of constant density in the x-y plane at a particular instant during the evolution of the instability from an initial configuration with $\varepsilon = 0.7$.

7. Plot of $\Delta\Psi$ versus t.The initial configuration is force-free, with $\varepsilon = 0.7$, $B_{ox} = 3.5$ and $B_{oz} = 5.0$.

8. Plots of $\Delta\Psi$ versus t for the model sharp-boundary plasma column as B_z is varied. The exponent m decreases from 1.4 to 0.9 as B_z increases. In (b), the additional line of slope 1.65 corresponds to transients as the current is ramped to its maximum value.

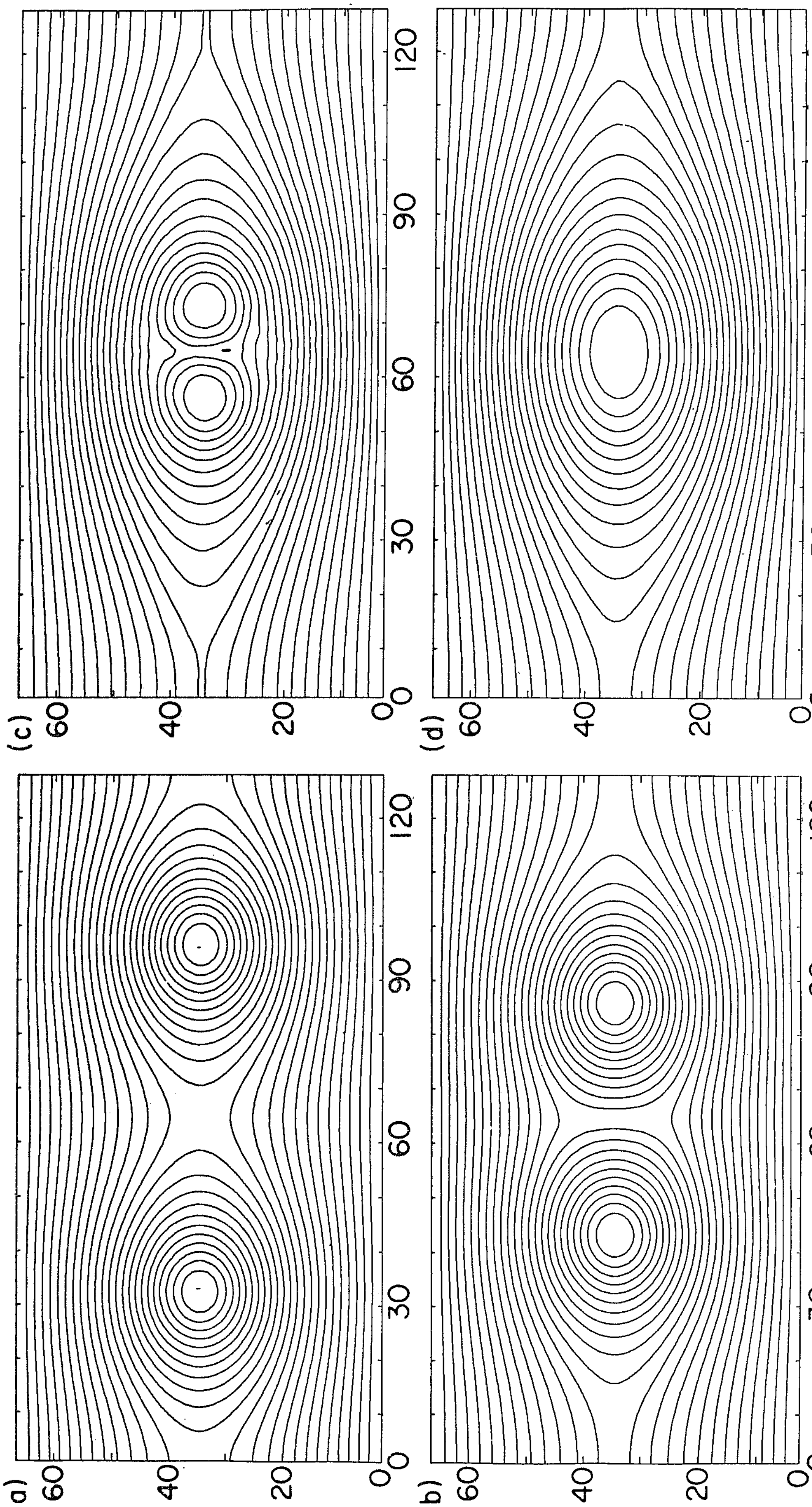

Fig. 1

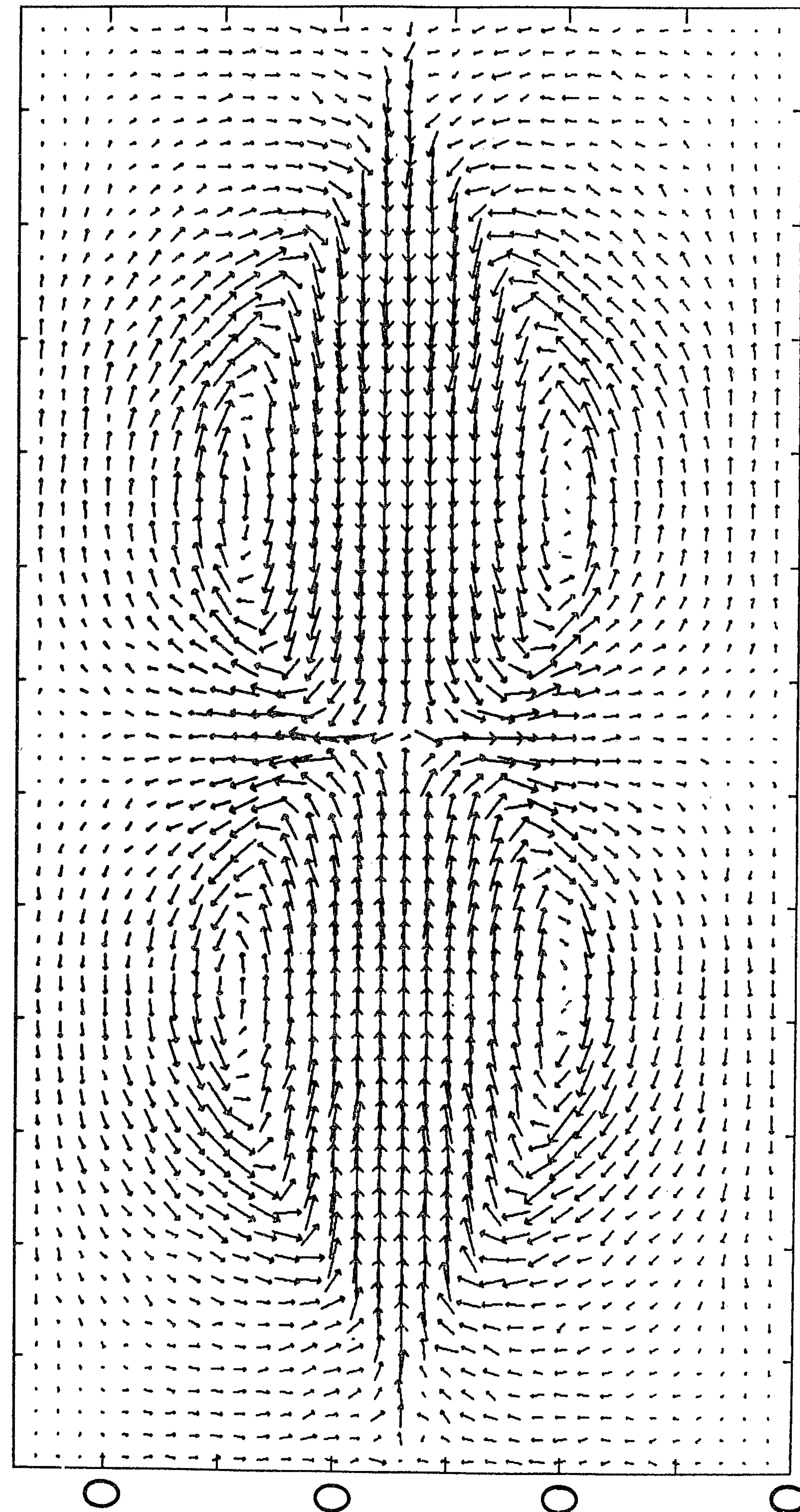

Fig. 2

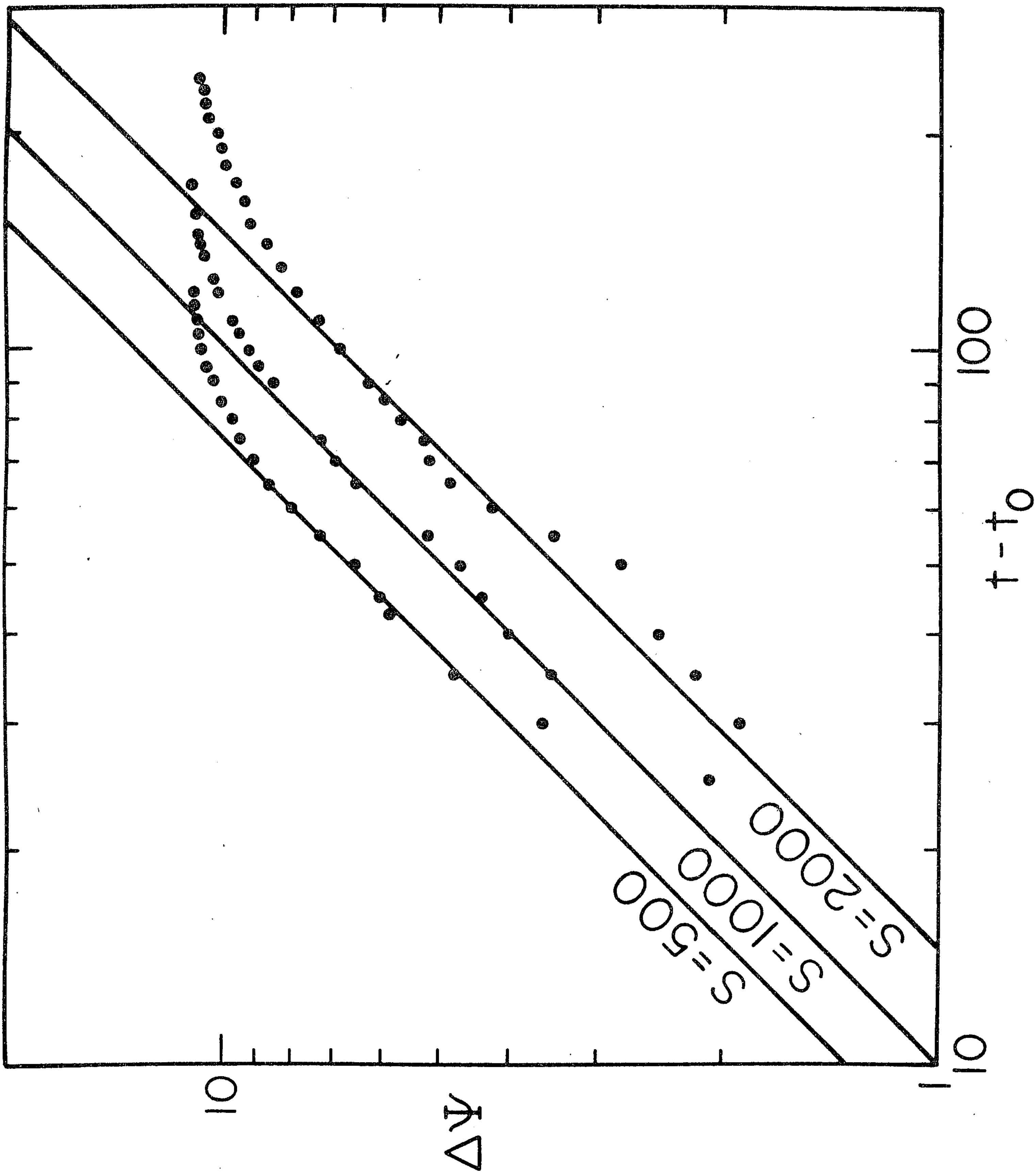

Fig. 3

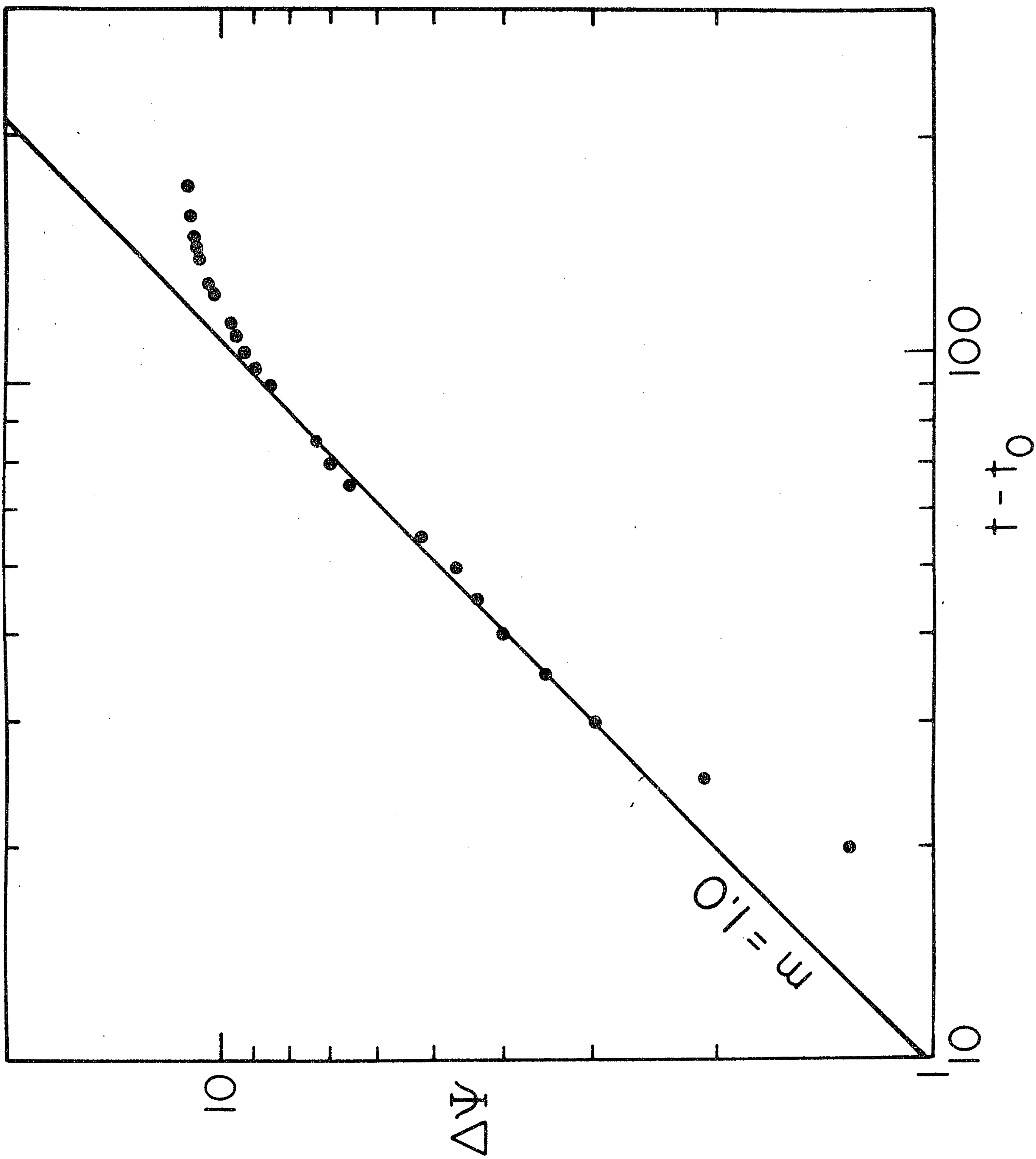

Fig. 4

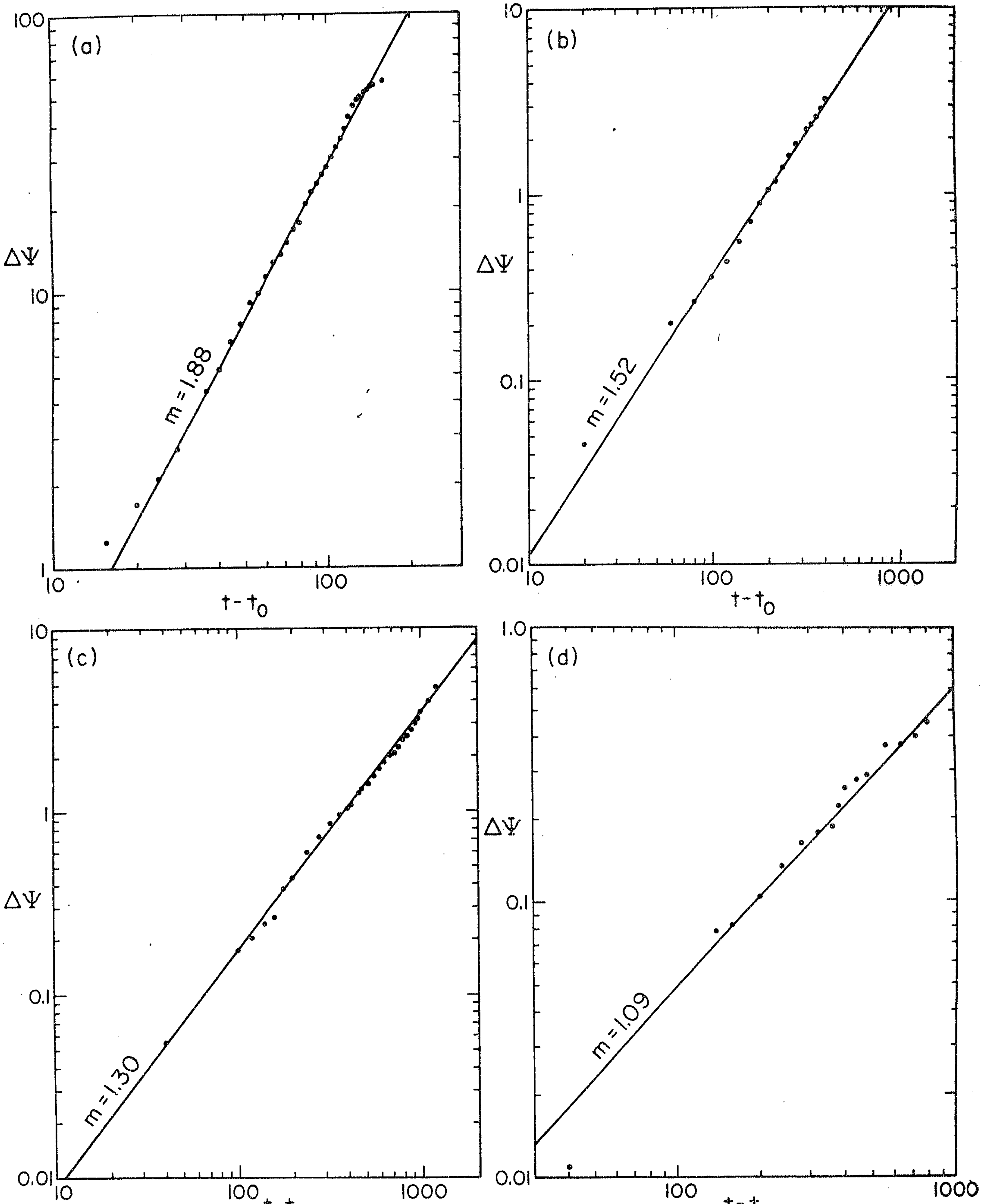

Fig. 5

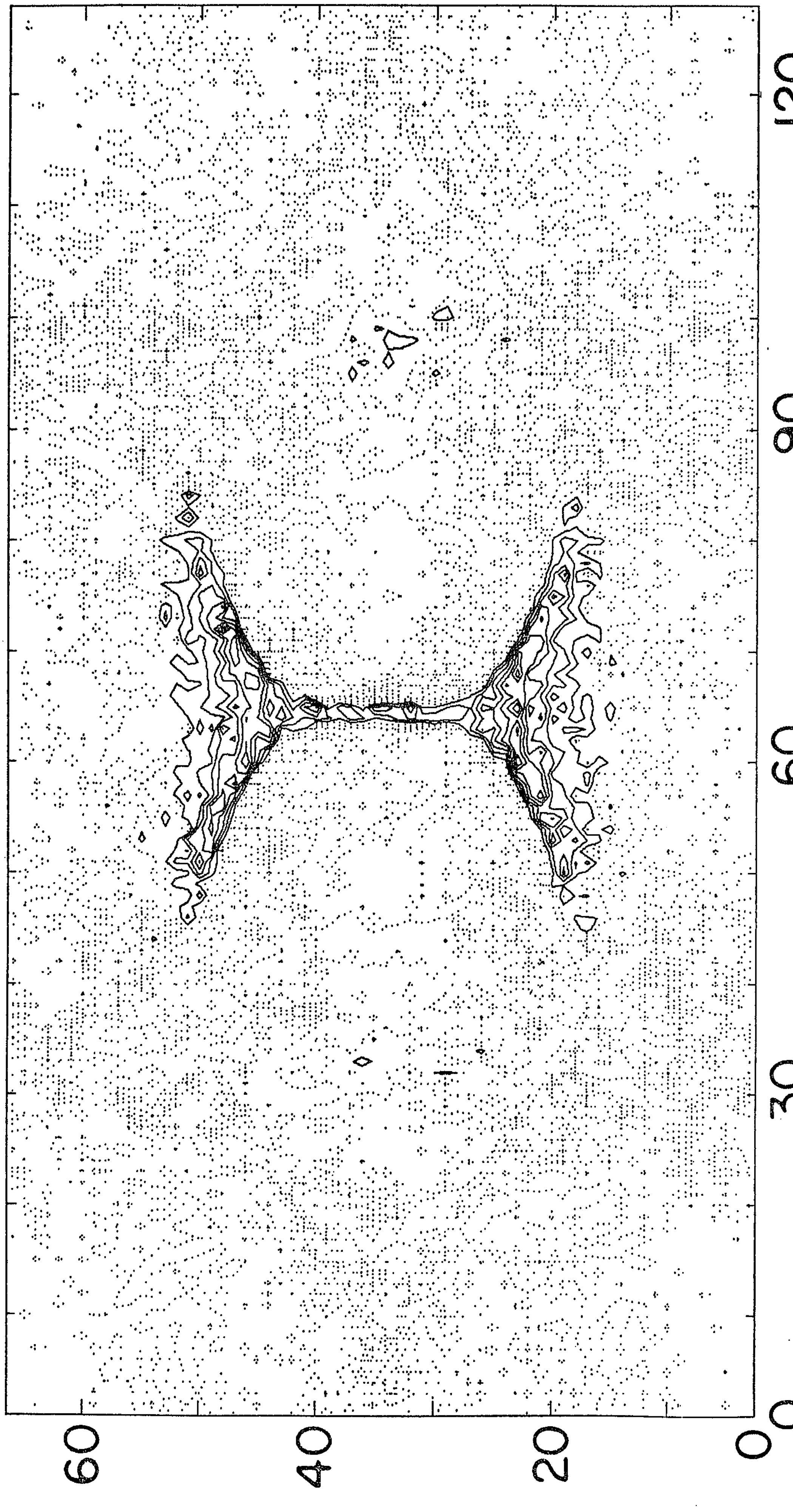

Fig. 6

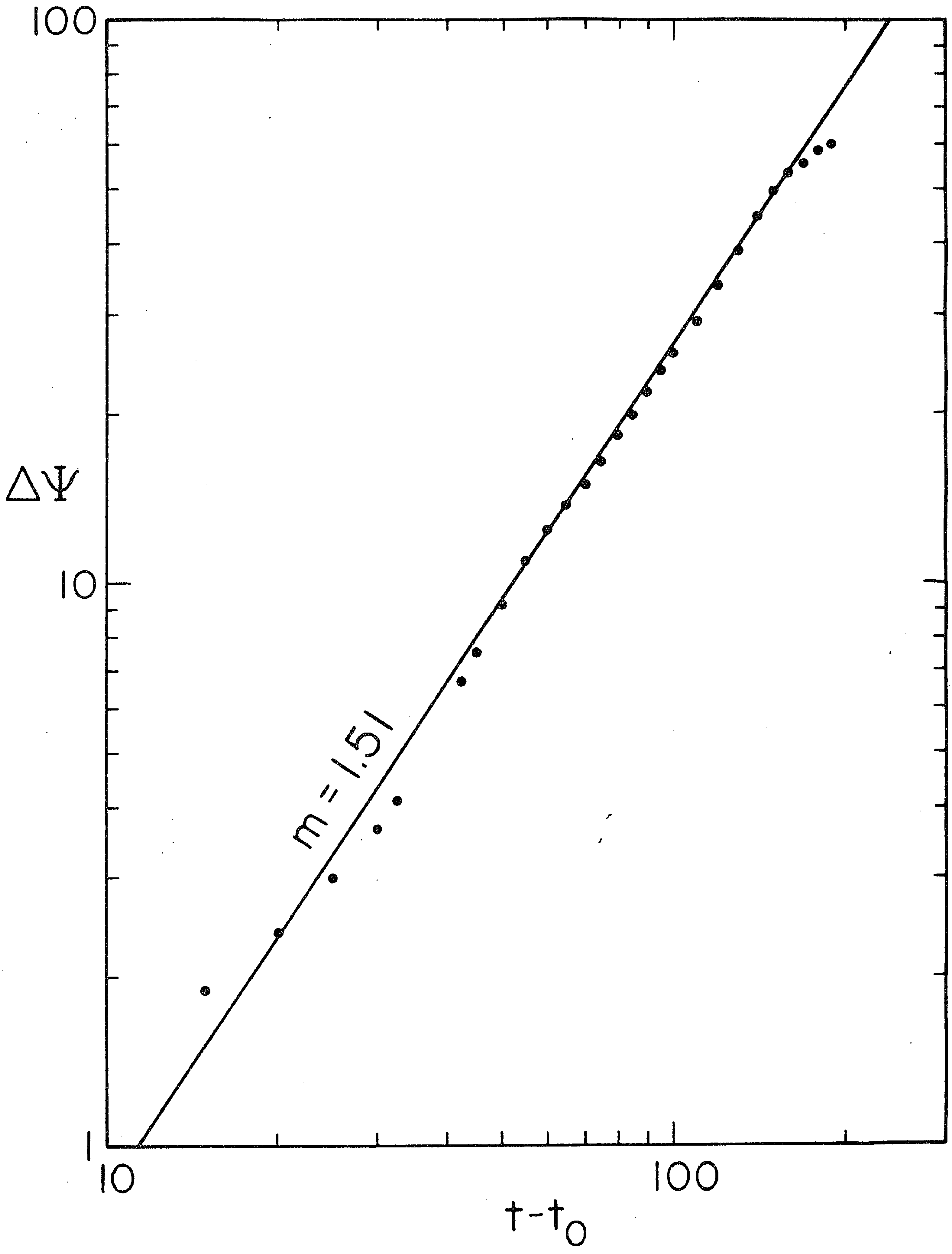

Fig. 7

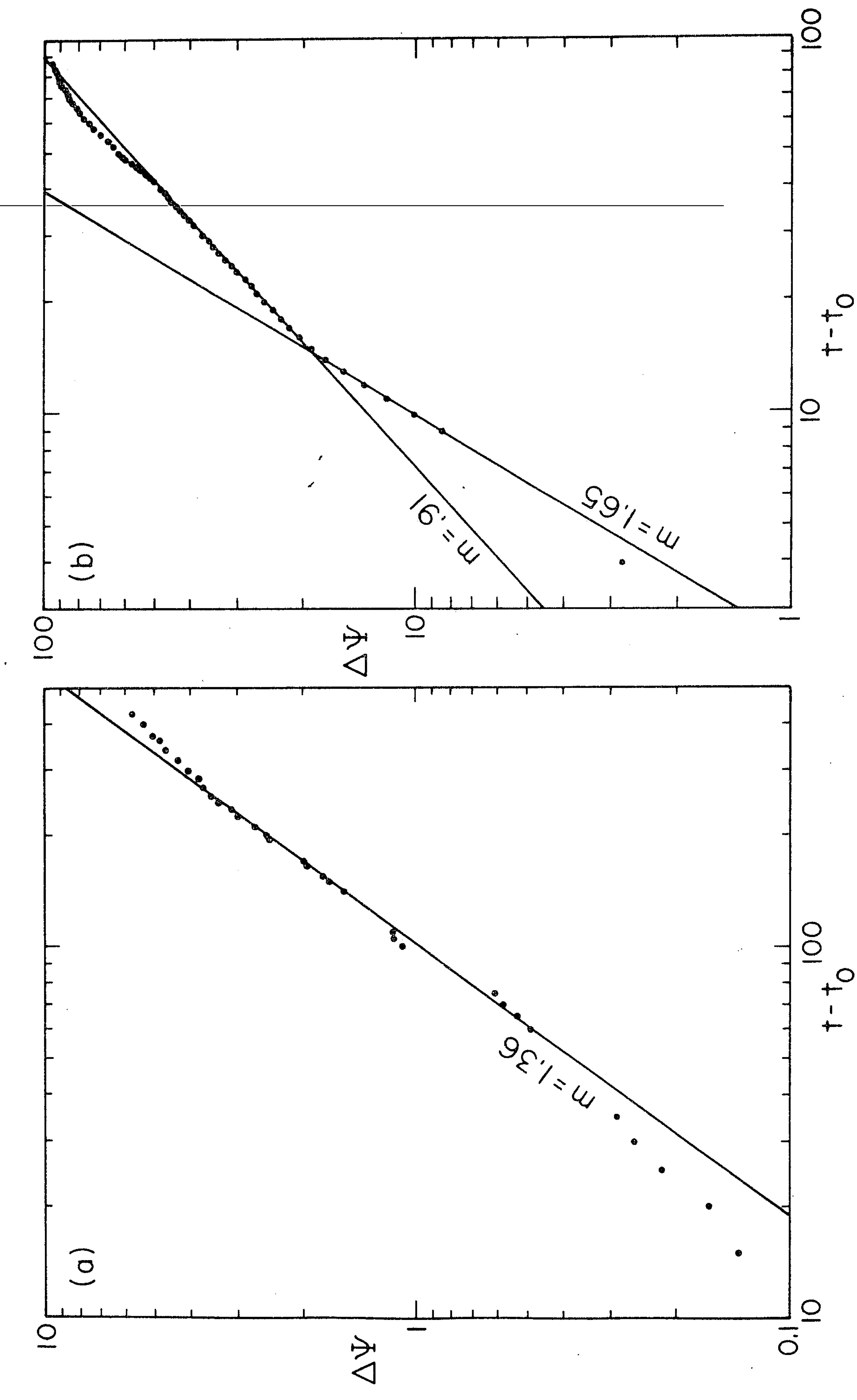
(a)
(b)
m = 1.36
m = .91
m = 1.65
$\Delta\Psi$
$\Delta\Psi$
$t-t_o$
$t-t_o$
10
1
0.1
10
100
100
10
1
10
100

Fig. 8

$B_{\odot x}$	m
0.25	1.09
0.50	1.30
0.70	1.52
1.00	1.55
3.50	1.89

Table I