

Self-trapping of strong electromagnetic beams in relativistic plasmas

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Abstract

Interaction of an intense electromagnetic (EM) beam with hot relativistic plasma is investigated. It is shown that the thermal pressure brings about a fundamental change in the dynamics — localized, high amplitude, EM field structures, not accessible to a cold (but relativistic) plasma, can now be formed under well- defined conditions. Examples of the trapping of EM beams in self-guiding regimes to form stable 2D solitonic structures in a pure e - p plasma are worked out.

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I. INTRODUCTION

The problem of electromagnetic (EM) wave propagation and related phenomena in relativistic plasmas has attracted considerable attention in the recent past. From the nonthermal emission of the high-energy radiation coming from a variety of compact astrophysical objects it has become possible to deduce the presence of a population of relativistic electrons in the plasma created in the dense radiation fields of those sources [1]. The principal components of these plasmas could be either relativistic electrons and nonrelativistic ions (protons), or relativistic electron-positron ($e-p$) pairs.

Relativistic $e-p$ dominated plasmas may be created in a variety of astrophysical situations. Electron-positron pair production cascades are believed to occur in pulsar magnetospheres [2]. The $e-p$ plasmas are also likely to be found in the bipolar outflows (jets) in Active Galactic Nuclei (AGN) [3], and at the center of our own Galaxy [4]. In AGNs, the observations of superluminal motions are commonly attributed to the expansion of relativistic $e-p$ beams in a pervading subrelativistic medium. This model implies copious pair production via $\gamma - \gamma$ interactions creating an $e-p$ atmosphere around the source. The actual production of $e-p$ pairs due to photon-photon interactions occurs in the coronas of AGN accretion disks, which upscatter the soft photons emitted by the accretion disks by inverse Compton scattering. The presence of $e-p$ plasma is also argued in the MeV epoch of the early Universe. In the standard cosmological model, temperatures in the MeV range ($T \sim 10^{10} K - 1 \text{ MeV}$) prevail up to times $t = 1 \text{ s}$ after the Big Bang [5]. In this epoch, the main constituent of the Universe is the relativistic $e-p$ plasma in equilibrium with photons, neutrinos, antineutrinos, and a minority population of heavier ions.

Contemporary progress in the development of super strong laser pulses with intensities $I \sim 10^{21-23} \text{ W/cm}^2$ has also made it possible to create relativistic plasmas in the laboratory by a host of experimental techniques [6]. At the focus of an ultrastrong laser pulses, the electrons can acquire velocities close to the speed of light opening the possibility of simulating in the laboratory the conditions and phenomena that, generally, belong in the astrophysical

realm [7].

Elucidation of the electromagnetic wave dynamics in a relativistic plasmas will, perhaps, be an essential tool for understanding the radiation properties of astrophysical objects as well as of the media exposed to the field of superstrong laser radiation. Although the study of wave propagation in relativistic plasmas has been in vogue for some time, it is only in the recent years that the nonlinear dynamics of EM radiation in $e-p$ dominated plasmas [8] has come into focus. The enhanced interest stems from two facts: 1) $e-p$ plasmas seem to be essential constituents of the universe, and 2) under certain conditions, even an ultrarelativistic electron-proton plasma can behave akin to an $e-p$ plasma [9].

Wave self-modulation and soliton-formation is, perhaps, one of the more interesting and significant signatures of the overall plasma dynamics. The existence of stable localized envelop solitons of EM radiation has been suggested as a potential mechanism for the production of micropulses in AGN and pulsars [10]. In the early universe localized solitons are strong candidates to explain the observed inhomogeneities of the visible universe [11,12].

In $e-p$ plasmas, there does not exist a general satisfactory theory for the generation of soliton like structures by ultrastrong high-frequency EM fields of arbitrary spatio-temporal shape. Relative complexity of the fully relativistic equations (hydrodynamic or kinetic) has limited their solutions essentially to one-dimensional systems . A summary of the salient results is : in unmagnetized $e-p$ plasmas, high-frequency pressure of the EM pulse pushes $e-p$ pairs in the direction of its propagation thus creating a density hump in the region of the field localization. The effective refractive index of medium decreases in this region and as a result localized soliton-formation is not supported by the medium. (The increase in the refractive index due to the relativistic nonlinearity related with the particles' high-frequency motion is not enough to cope with the decrease caused by the above mentioned effect.) In Refs. [11, 13] it has been argued that localized solitons can be formed if the interaction between the EM field and acoustic phonons is taken into account- the envelope solitons propagating with subsonic velocities may, then, emerge. In magnetized $e-p$ plasma, larger classes of soliton solutions are possible-typical examples may be found in Refs. [14]. However, it is conceivable

that soliton solutions obtained in a one-dimensional formulation will turn out to be unstable in higher dimensions.

It is, therefore, a matter of utmost priority that we explore the possibility of finding stable multidimensional soliton solutions in e - p plasmas. Dynamics of 3D envelope solitons of arbitrary strong EM fields in e - p plasma with a small fraction of heavy ions has been analyzed in Ref. [12]. It was shown that, in a transparent e - p plasma, EM pulses with $L_{\parallel} \ll L_{\perp}$ (where L_{\parallel} and L_{\perp} respectively, the characteristic longitudinal and transverse spatial dimensions of the field), may propagate as stable, nondiffracting and nondispersing objects (light bullets) with large density bunching. It was further shown in Ref. [15] that these bullets are exceptionally robust: they can emerge from a large variety of initial field distributions, and are remarkably stable. In these studies, the EM field is pulse-like with longitudinal localization much stronger than the transverse; the localization is brought about by the charge separation electric field (usually absent in a pure e - p plasma) created by the presence of a small population of ions.

In the present paper we explore another mechanism for localization - we will show that in the pure e - p plasma, the thermal pressure can provide the confining “glue” just as the charge separation electric field did for an e - p plasma contaminated with ions. We will also deal with a complimentary manifestation of the radiation field — the beam ($L_{\perp} \ll L_{\parallel}$) rather than the pulse ($L_{\perp} \gg L_{\parallel}$). Assuming the plasma to be transparent to the beam, and applying a fully relativistic hydrodynamical model, we demonstrate the possibility of beam self-trapping leading to the formation of stable 2D solitonic structures. The high-frequency pressure force of the EM field (tending to completely expel the pairs radially from the region of localization) is overwhelmed by the thermal pressure force which opposes the radial expansion of the plasma creating conditions for the formation of the stationary self-guiding regime of beam propagation.

II. BASIC EQUATIONS

In this paper the word “relativistic” connotes two distinct regimes: the plasma becomes relativistic when either the directed fluid velocity approaches the speed of light or the thermal energy per particle is of the order of, or larger than the rest mass energy. Since both these paths to relativity are encountered in the astrophysical as well as laboratory plasmas (produced and accelerated by intense laser pulses), we will investigate a fully relativistic hydrodynamical model.

If the velocity distribution of the particles of species α ($= e, p, i, \dots$ where e, p , and i denote respectively electrons, positrons and heavy ions (protons)) is taken to be a local relativistic Maxwellian, the hydrodynamics of such fluids is described by [16]:

$$\frac{\partial}{\partial x_k}(U_\alpha^i U_{\alpha k} W_\alpha) - \frac{\partial}{\partial x_i} P_\alpha = \frac{1}{c} F^{ik} J_{\alpha k} \quad (1)$$

where $U_\alpha^i = [\gamma_\alpha, \gamma_\alpha \mathbf{u}_\alpha / c]$ is the hydrodynamic four-velocity with \mathbf{u}_α as the three-velocity, $\gamma_\alpha = (1 - u_\alpha^2/c^2)^{-1/2}$ is the relativistic factor, $J_{\alpha k}$ is the four-current, F^{ik} is the electromagnetic field tensor, and W_α is the enthalpy per unit volume

$$W_\alpha = \frac{n_\alpha}{\gamma_\alpha} m_{0\alpha} c^2 G_\alpha \left(\frac{T_\alpha}{m_{0\alpha} c^2} \right). \quad (2)$$

Here $m_{0\alpha}$ and T_α are the particle invariant rest mass and temperature, respectively, n_α is the density in the laboratory frame of the fluid of species α . The pressure $P_\alpha = n_\alpha T_\alpha / \gamma_\alpha$, and $G_\alpha(z_\alpha) = K_3(1/z_\alpha) / K_2(1/z_\alpha)$, where K_2 and K_3 are, respectively, modified Bessel functions of the second and third order and $z_\alpha = T_\alpha / m_{0\alpha} c^2$. The factor $G_\alpha(z_\alpha)$ has the following asymptotes, $G_\alpha \approx 1 + 5z_\alpha/2$ for $z_\alpha \ll 1$ (nonrelativistic) and $G_\alpha \approx 4z_\alpha$ for $z_\alpha \gg 1$ (highly relativistic).

The set of equations (1)-(2) may be written in the standard form :

$$\frac{d_\alpha}{dt}(m_{0\alpha} c^2 G_\alpha \gamma_\alpha) - \frac{1}{n_\alpha} \frac{\partial P_\alpha}{\partial t} = e_\alpha \mathbf{u}_\alpha \cdot \mathbf{E} \quad (3)$$

$$\frac{d_\alpha}{dt}(G_\alpha \mathbf{p}_\alpha) + \frac{1}{n_\alpha} \nabla P_\alpha = e_\alpha \mathbf{E} + \frac{e_\alpha}{c} (\mathbf{u}_\alpha \times \mathbf{B}) \quad (4)$$

where $\mathbf{p}_\alpha = \gamma_\alpha m_{0\alpha} \mathbf{u}_\alpha$ is the hydrodynamical momentum, \mathbf{E} and \mathbf{B} are the electric and magnetic fields, and $d_\alpha/dt = \partial/\partial t + \mathbf{u}_\alpha \cdot \nabla$ is the comoving derivative. The hydrodynamical velocity \mathbf{u}_α and the relativistic γ_α are related to the momentum by the standard relations: $\mathbf{u}_\alpha = \mathbf{p}_\alpha/m_{0\alpha}\gamma_\alpha$ and $\gamma_\alpha = (1 + \mathbf{p}_\alpha^2/m_{0\alpha}^2 c^2)^{1/2}$. It is interesting to note from Eqs. (3)-(4) that the fluid inertia is modified by the temperature; the expression $M_{\text{eff}}^\alpha = m_{0\alpha} G_\alpha(z_\alpha)$ denotes the effective mass of the particle. For ultrarelativistic temperatures ($T_\alpha \gg m_{0\alpha} c^2$), the effective mass turns out to be $M_{\text{eff}}^\alpha = 4T_\alpha/c^2 \gg m_{0\alpha}$. Thus the particles “forget” their rest mass and the plasma turns into a kind of “photon” gas. If an ultrarelativistic plasma is in thermodynamical equilibrium with the high-frequency photon gas ($\hbar\omega \sim T$), one should also take into account the radiation pressure $P_R = \sigma T^4$ ($\sigma = \pi/45 h^3 c^3$) [17]. In this paper this effect will be neglected. We must also bear in mind that for extremely relativistic regimes, the model Eqs. (3)-(4) fail to adequately describe the plasma dynamics since heavy particle production has been neglected. This shortcoming will impose an upper limit on the temperature for the validity of the model. Note that in the context of early universe, the epoch in which the e - p plasma is dominant, has a characteristic temperature $T_\alpha \sim 1$ MeV and $M_{\text{eff}} \sim 4m_{0e}$. Since the particle masses are just a few times larger than their rest mass at these temperatures, the e - p plasma can still be considered as a two component fluid rather than a photon gas.

The equation of state directly follows from the self-consistency of Eqs. (3) and (4). Taking the scalar product of Eq. (4) with \mathbf{u}_α and comparing it with Eq. (3), we can derive $d_\alpha \ln P_\alpha/dt = z_\alpha d_\alpha G_\alpha/dt$. After straightforward manipulation, the equation can be easily integrated to yield

$$P_\alpha = C \frac{K_2(z_\alpha)}{z_\alpha} \exp(z_\alpha G_\alpha) \quad (5)$$

where the arbitrary constant C must be defined by the initial state. Using $P_\alpha = n_\alpha T_\alpha/\gamma_\alpha$ Eq. (5) reduces to the adiabatic equation of “state”:

$$\frac{n_\alpha}{\gamma_\alpha} \frac{z_\alpha}{K_2(z_\alpha)} \exp(-G_\alpha z_\alpha) = \text{const.} \quad (6)$$

For nonrelativistic temperatures, Eq. (6) yields the usual result for a monoatomic ideal gas ($n_\alpha^r/T_\alpha^{3/2} = \text{const}$, where $n_\alpha^r = n_\alpha/\gamma_\alpha$ is the density in the rest frame of fluid element) and for ultrarelativistic temperatures one obtains the equation of state for the photon gas ($n_\alpha^r/T_\alpha^3 = \text{const}$). We would like to emphasize that the γ_α factor appearing in Eq. (6) is related to the coherent or directed motion of fluid elements whose origin may lie either in an initial macroscopic flow or in the motion imparted by intense EM radiation.

To complete the picture we must couple the plasma equations of motion with Maxwell equations:

$$c\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + 4\pi \mathbf{J} \quad (7)$$

$$c\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (8)$$

$$\nabla \cdot \mathbf{E} = 4\pi \rho \quad (9)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (10)$$

where

$$\rho = \sum_\alpha e_\alpha n_\alpha, \quad \mathbf{J} = \sum_\alpha e_\alpha n_\alpha \mathbf{u}_\alpha \quad (11)$$

are, respectively, the charge and current densities. The system of Eqs. (3)-(11) along with the continuity equation (for each species)

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{u}_\alpha) = 0, \quad (12)$$

represents a closed set of equations which describe propagation of EM radiation in relativistic multicomponent plasmas.

The above system can be manipulated further to reveal interesting structural properties. To begin with, Eqs. (5)-(6) can be cast in the form

$$\frac{1}{n_\alpha} \nabla P_\alpha = \frac{m_{0\alpha} c^2}{\gamma_\alpha} \nabla G_\alpha \quad (13)$$

which, when substituted into Eq. (4), converts it to

$$\frac{\partial}{\partial t}(G_\alpha \mathbf{p}_\alpha) + m_{0\alpha} c^2 \nabla(G_\alpha \gamma_\alpha) = e_\alpha \mathbf{E} + [\mathbf{u}_\alpha \times \boldsymbol{\Omega}_\alpha] \quad (14)$$

where

$$\boldsymbol{\Omega}_\alpha = \frac{e_\alpha}{c} \mathbf{B} + \nabla \times (G_\alpha \mathbf{p}_\alpha) \quad (15)$$

is the so-called generalized vorticity. Taking the curl of Eq. (14), we find that the evolution equation for $\boldsymbol{\Omega}_\alpha$

$$\frac{\partial \boldsymbol{\Omega}_\alpha}{\partial t} = \nabla \times [\mathbf{u}_\alpha \times \boldsymbol{\Omega}_\alpha] \quad (16)$$

is of the standard vortex dynamics form. Although the system of Eqs. (14)-(16) can be traced to early publications (see for instance Ref. [18]), their consequences are yet to be fully worked out. An immediate consequence, for instance, is the appropriate equivalent of Kelvin's theorem — the flux of generalized vorticity $\boldsymbol{\Omega}_\alpha$ is frozen-in through a co-moving area.

The system yields the following set of relativistic Beltrami- Bernoulli equations for equilibrium [19]:

$$\boldsymbol{\Omega}_\alpha = a_\alpha \mathbf{u}_\alpha \quad (17)$$

$$G_\alpha \gamma_\alpha + \frac{e_\alpha \phi}{m_{0\alpha} c^2} = \text{const} \quad (18)$$

where a_α are constants and ϕ is the scalar potential. The relevance of these equilibria for astrophysics is the subject matter of a forthcoming paper.

For the current effort, we apply Eqs. (14)-(16) for wave processes in an unmagnetized plasma. From Eq. (16) it follows that if the generalized vorticity is initially zero ($\boldsymbol{\Omega}_\alpha = 0$) everywhere in space, it remains zero for all subsequent times. We assume that before the EM radiation is “switched on,” the generalized vorticity of the system is zero. In this case the equation of motion may be written as:

$$\frac{\partial}{\partial t}\mathbf{\Pi}_\alpha + m_{0\alpha}c^2\nabla\Gamma_\alpha = e_\alpha\mathbf{E} \quad (19)$$

where the temperature dependent momentum $\mathbf{\Pi}_\alpha$ and Γ_α are defined by:

$$\mathbf{\Pi}_\alpha = G_\alpha\mathbf{p}_\alpha \quad (20)$$

$$\Gamma_\alpha = G_\alpha\gamma_\alpha = \sqrt{G_\alpha^2 + (\mathbf{\Pi}_\alpha/m_{0\alpha}c)^2}. \quad (21)$$

The condition of vanishing generalized vorticity connects $\mathbf{\Pi}_\alpha$ with the magnetic field:

$$\mathbf{B} = -\frac{c}{e_\alpha}\nabla \times \mathbf{\Pi}_\alpha. \quad (22)$$

It is remarkable that in Eq. (19) the magnetic part of the Lorentz force is formally absent; this fact greatly simplifies analytical manipulations. It is equally remarkable that our equations which describe the dynamics of a hot relativistic plasma are structurally similar to equations used in the theoretical treatment of different aspects of ultrastrong laser interaction with a cold plasma [20]. This similarity becomes even more evident when we study the interaction of short EM pulse with relativistic electron-ion plasmas. If the pulse is assumed to be shorter than the characteristic time for ion response (i.e. inverse of ion Langmuir frequency), the ion motion may be ignored, and the electric field may be found from the electron part of Eq. (19),

$$e\mathbf{E} = -\frac{\partial\mathbf{\Pi}_e}{\partial t} - m_{0e}c^2\nabla\Gamma_e. \quad (23)$$

Substituting this expression into Poisson's Eq. (3) (which now reads as $\nabla \cdot \mathbf{E} = 4\pi e(n_{0i} - n_e)$, where n_{0i} is the equilibrium ion density) we find the electron density

$$\frac{n_e}{n_{0i}} = 1 + \frac{1}{m_{0e}\omega_e^2}\frac{\partial}{\partial t}\mathbf{\Pi}_e + \frac{c^2}{\omega_e^2}\Delta\Gamma_e \quad (24)$$

where $\omega_e = (4\pi e^2 n_{0i}/m_{0e})^{1/2}$ is the plasma frequency. Using Eqs. (22)-(24), Eq. (7) reduces to :

$$c^2\nabla \times \nabla \times \mathbf{\Pi}_e + \frac{\partial^2\mathbf{\Pi}_e}{\partial t^2} + m_{0e}c^2\frac{\partial\nabla\Gamma_e}{\partial t} + \omega_e^2\frac{\mathbf{\Pi}_e}{\Gamma_e} \left[1 + \frac{1}{m_{0e}\omega_e^2}\frac{\partial}{\partial t}\mathbf{\Pi}_e + \frac{c^2}{\omega_e^2}\Delta\Gamma_e \right] = 0 \quad (25)$$

which, along with the equation of state ($z_e = m_{0e}c^2/T_e$)

$$\frac{n_\alpha G_e}{\Gamma_e} \frac{z_e}{K_2(z_e)} \exp(-G_e z_e) = \text{const}, \quad (26)$$

constitutes the simplified system to which the entire set of Maxwell and relativistic hydrodynamic equations for an electron-ion plasma have been reduced. An equation similar to Eq. (25) was derived in the cold plasma limit in Ref. [21]. However, there are important differences : a) Due to the temperature dependence of G_e in Eq. (21), the factor Γ_e and the momentum Π_e are no more related by simple relations as they do for a “cold” case, and b) to incorporate the temperature variation in the system we must add the equation of state (26).

Though Eqs. (25)-(26) form a more complicated system than what we have for the cold plasma, we believe that many results obtained in the cold plasma limit can find appropriate analogies in the hot relativistic-temperature case. Detailed studies of the nonlinear dynamics of the electron-ion plasma is beyond the intended scope of the current paper and some new results will be presented separately elsewhere. In the next part of the current paper, we concentrate on the nonlinear dynamics of EM beams in e - p -dominated plasmas.

III. THE ELECTRON-POSITRON DOMINATED PLASMA

In this section we apply our general formulation to the problem of self-trapping of EM beams in pure e - p plasmas with relativistic temperatures. For notational convenience, we replace the subscripts (e) and (p) by superscripts ($-$), and ($+$). We assume that the equilibrium state of the plasma is characterized by an overall charge neutrality $n_\infty^- = n_\infty^+ = n_\infty$, where n_∞^- and n_∞^+ are the unperturbed number densities of the electrons and positrons in the far region of the EM beam localization. In most mechanisms for creating e - p plasmas, the pairs appear simultaneously and due to the symmetry of the problem it is natural to assume that $T_\infty^- = T_\infty^+ = T_\infty$, where T_∞^- and T_∞^+ are the respective equilibrium temperatures.

We shall assume that for the radiation field of interest, the plasma is underdense and transparent, i.e., $\epsilon = \omega_e/\omega \ll 1$, where ω is the mean frequency of EM radiation and

$\omega_e = (4\pi e^2 n_\infty / m_{0e})^{1/2}$ is the plasma frequency. Since both species are mobile, the e - p dynamics can not be reduced to just one vector equation similar to Eq. (25). We will display the entire set in terms of potentials (the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$ will be used),

$$\mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} - \nabla \phi, \quad \mathbf{B} = \nabla \times \mathbf{A}, \quad (27)$$

and the dimensionless quantities $\tilde{t} = \omega t$, $\tilde{\mathbf{r}} = (\omega/c)\mathbf{r}$, $\tilde{T}^\pm = T^\pm / m_{0e} c^2$, $\tilde{\mathbf{A}} = e\mathbf{A} / (m_{0e} c^2)$, $\tilde{\phi} = e\phi / m_{0e} c^2$, $\tilde{\mathbf{\Pi}}^\pm = \mathbf{\Pi}^\pm / (m_{0e} c)$, and $\tilde{n}^\pm = n^\pm / n_\infty$. Suppressing the label — “tilde,” we may arrive at the dimensionless equations,

$$\frac{\partial \mathbf{\Pi}^\pm}{\partial t} + \nabla \Gamma^\pm = \mp \frac{\partial \mathbf{A}}{\partial t} \mp \nabla \phi \quad (28)$$

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} - \Delta \mathbf{A} + \frac{\partial}{\partial t} \nabla \phi - \epsilon^2 (\mathbf{J}^+ - \mathbf{J}^-) = 0 \quad (29)$$

$$\Delta \phi = \epsilon^2 (n^- - n^+) \quad (30)$$

$$\nabla \cdot \mathbf{A} = 0 \quad (31)$$

$$\frac{\partial n^\pm}{\partial t} + \nabla \mathbf{J}^\pm = 0 \quad (32)$$

with $\mathbf{J}^\pm = n^\pm \mathbf{\Pi}^\pm / \Gamma^\pm$ and $\Gamma^\pm = \sqrt{(G^\pm)^2 + (\mathbf{\Pi}^\pm)^2}$. The species equation of state is:

$$\frac{n^\pm}{\Gamma^\pm f(T^\pm)} = \frac{1}{\Gamma_\infty^\pm f(T_\infty)} \quad (33)$$

where

$$f(T^\pm) = \frac{K_2 (1/T^\pm) T^\pm}{G^\pm} \exp[G^\pm / T^\pm]. \quad (34)$$

Of various techniques that could be invoked to investigate Eqs. (28)-(34) to study the self-trapping of high-frequency EM radiation propagating along the z -axis, we choose the method presented in the excellent paper by Sun *et al.* [22]. The method is based on the multiple scale expansion of the equations in the small parameter ϵ . Assuming that all variations are slow compared to the variation in $\xi = z - at$, we expand all quantities $Q = (\mathbf{A}, \phi, \mathbf{\Pi}^\pm, n^\pm, \dots)$ as

$$Q = Q_0(\xi, x_1, y_1, z_2) + \epsilon Q_1(\xi, x_1, y_1, z_2) \quad (35)$$

where $(x_1, y_1, z_2) = (\epsilon x, \epsilon y, \epsilon^2 z)$ denote the directions of slow change, and $a_1 = (a^2 - 1)/\epsilon^2 \sim 1$. We further assume that the high-frequency EM field is circularly polarized,

$$\mathbf{A}_{0\perp} = \frac{1}{2}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})A \exp(i\xi/a) + \text{c.c.} . \quad (36)$$

Here A is the slowly varying envelope of the EM beam, $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ denote unit vectors, and c.c. is the complex conjugate. We now give a short summary of the steps in the standard multiple-scale methodology (Ref. [22]). To the lowest order in ϵ , we obtain the following. The transverse (to the direction of EM wave propagation z) component of Eq. (28) reduces to

$$\mathbf{\Pi}_{0\perp}^\pm = \mp \mathbf{A}_{0\perp} \quad (37)$$

and for the longitudinal components we get:

$$-a \frac{\partial \Pi_{0z}^\pm}{\partial \xi} + \frac{\partial \Gamma_0^\pm}{\partial \xi} = \mp (-a) \frac{\partial A_{0z}}{\partial \xi} \mp \frac{\partial \phi_0}{\partial \xi}. \quad (38)$$

Equations (29)-(31) yield $\partial_\xi \nabla_\perp \phi_0 = \partial_\xi^2 \phi_0 = \partial_\xi A_{z0} = 0$, where ∇_\perp is the perpendicular Laplacian in (x_1, y_1) . These relations imply that ϕ and A_{0z} do not depend on the fast variable ξ . For the self-trapping problem, we can assume that $A_{0z} = \Pi_{0z}^\pm = 0$ [22]. From Eq. (38), and from the lowest order continuity Eq. (32), we obtain: $\partial_\xi \Gamma_0^\pm = \partial_\xi n_0^\pm = 0$, i.e., Γ_0^\pm and n_0^\pm also do not depend on the fast variable ξ .

To the next order (in ϵ), the transverse component of Eq. (28) reads :

$$-a \frac{\partial \mathbf{\Pi}_{1\perp}^\pm}{\partial \xi} + \nabla_\perp \Gamma_0^\pm = \mp (-a) \frac{\partial \mathbf{A}_{1\perp}}{\partial \xi} \mp \nabla_\perp \phi_0. \quad (39)$$

Averaging over the fast variable ξ we obtain $\nabla_\perp \Gamma_0^\pm = \mp \nabla_\perp \phi_0$ yielding the trivial solution $\phi_0 = 0$ and $\Gamma_0^\pm = \Gamma_0 = \text{const}$. Note that from Eqs. (30) and (33), we can deduce that $n_0^+ = n_0^- = n_0$ and $T_0^+ = T_0^- = T_0$.

Thus, as one would expect, the low frequency motion of the e - p plasma is driven by the ponderomotive pressure ($\sim \mathbf{\Pi}_{0\perp}^2$) of the high-frequency EM field and this force, being same for the electrons and positrons, does not cause charge separation. It is also evident that

due to the symmetry between the electron and positron fluids, their temperatures, being initially equal, will remain equal during the evolution of the system. The relation between the EM field and the temperature can be found using the equation $\Gamma_0 = \text{const}$ obtained above. Using Eqs. (36)-(37) and by choosing the *const* by requiring that at infinity $A \rightarrow 0$ and $T_0 \rightarrow T_\infty$, we derive

$$G^2(T_0) = G^2(T_\infty) - |A|^2. \quad (40)$$

It follows from Eq. (40) that the present hydrodynamical model, which describes the nonlinear waves in e - p plasma, is valid for $|A|^2/G_\infty^2 \leq 1$. In our opinion the origin of this restriction lies in the inadequacy of the basic model, and is not solely due to a failure of the perturbation technique used above. When this condition is violated, the EM waves are overturned and they will cause multistream motion of the plasma (i.e. wave breaking takes place). In such a situation, one must resort to kinetic description for studying the nonlinear wave motion. Notice however that the function $G(T_\infty) \rightarrow 1$ if $T_\infty \rightarrow 0$ but rapidly increases with increase of T_∞ thus providing room for $|A|_{\text{max}}$ to reach from weak to relativistic values.

We are now ready to deal with the equation for the slowly varying envelope A of the EM beam. To the lowest order in ϵ , one finds from Eq. (29)

$$a_1 \frac{\partial^2 \mathbf{A}_{0\perp}}{\partial \xi^2} - \nabla_\perp^2 \mathbf{A}_{0\perp} 2 \frac{\partial^2 \mathbf{A}_{0\perp}}{\partial \xi \partial z_2} + 2 \frac{n_0(T_0)}{G_\infty} \mathbf{A}_{0\perp} = 0. \quad (41)$$

For deriving this equation, we used the relation $\Gamma_0 = \sqrt{G^2(T_0) + |A|^2} = G_\infty$, and

$$n_0(T_0) = \frac{f(T_0)}{f(T_\infty)} \quad (42)$$

which follows from Eq. (33). Substituting Eq. (36) into Eq. (41) we find:

$$2i \frac{\partial A}{\partial z} + \nabla_\perp^2 A + \frac{2}{G_\infty} [1 - n(T_0)] A = 0 \quad (43)$$

where subscripts for variables (z_2, x_1, y_1) are dropped for simplicity. We also assumed without loss of generality that $(a^2 - 1)/\epsilon^2 a^2 = 2/G_\infty$, which in dimensional units coincides with the linear dispersion relation of the EM wave in an e - p plasma, namely: $\omega^2 = 2\omega_e^2/G_\infty + k^2 c^2$ provided that $a = \omega/kc$.

Thus, the dynamics of EM beams in hot relativistic e - p plasma has become accessible within the context of a generalized nonlinear Schrödinger equation (NSE) (43).

IV. THE SELF-TRAPPED BEAMS IN E - P PLASMA

In this section we seek the localized 2D soliton solutions of Eq. (43), and analyze the stability of such solutions. Making the self-evident re-normalization of variables $z \rightarrow zG_\infty$, $r_\perp \rightarrow r_\perp\sqrt{G_\infty/2}$, Eq. (43) can be written as:

$$i\frac{\partial A}{\partial z} + \nabla_\perp^2 A + \Psi A = 0 \quad (44)$$

where $\Psi = 1 - n_0(T_0)$ represents the generalized nonlinearity. The companion equation (40) can be viewed as a transcendental algebraic relation between T_0 and $|A|^2$. Thus we conclude that Ψ is a function of $|A|^2$ ($\Psi = \Psi(|A|^2)$). We note that Eq. (44) can be written in the Hamiltonian form $iA_z = \delta H/\delta A^*$, where $H = \int d\mathbf{r}_\perp [|\nabla_\perp A|^2 - F(|A|^2)]$ and $F(t) = \int_0^t \Psi(t')dt'$. The Hamiltonian structure implies that Eq. (44) conserves the Hamiltonian H in addition to the power (“photon number”) $N = \int d\mathbf{r}_\perp |A|^2$.

Unfortunately, it is not possible, in general, to derive an explicit analytic relation $\Psi = \Psi(|A|^2)$ for arbitrary value of T_∞ . Some qualitative deductions readily follow. Equation (40) shows that the presence of EM radiation reduces the temperature T_0 . Since $df(T_0)/dT_0 > 0$, from Eq. (42) we conclude that the plasma density is also reduced in the region of the EM field localization which is in accordance with adiabatic motion of the plasma. For higher strength of the EM field, a complete expulsion of plasma i.e. plasma cavitation can take place ($n_0 \rightarrow 0$); this has been predicted in Ref. [23]. Thus the nonlinearity function Ψ shows a saturating character with the increase of EM field strength (note that present model is valid provided $|A|^2/(G_\infty^2 - 1) \leq 1$). To illustrate, we exhibit in Fig.1 a plot of Ψ versus $|A|^2$ for $T_\infty = 0.1$. One can see that the nonlinearity function indeed saturates at high intensity. For small temperatures, we can even obtain an analytic expression for the function Ψ . Remembering $T_0 \leq T_\infty$, assuming $T_\infty \ll 1$, and by using Eq. (42) along with

the asymptotic expansions $G_0(\approx 1 + 5T_0/2)$ and $f(\approx T_0^3)$, we derive for the nonlinearity function:

$$\Psi = 1 - \left(1 - \frac{|A|^2}{5T_\infty}\right)^{3/2}. \quad (45)$$

Equations (44)-(45) admit a stationary, nondiffracting axially symmetric solution of the form $A/\sqrt{5T_\infty} = U(r)\exp(i\lambda z)$ where $r = (x^2 + y^2)^{1/2}$ and λ is the nonlinear wave-vector shift. The radially dependent envelope $U(r)$ obeys an ordinary nonlinear differential equation:

$$\frac{d^2U}{dr^2} + \frac{1}{r}\frac{dU}{dr} - \lambda U + \Psi(U^2)U = 0 \quad (46)$$

where $\Psi = 1 - (1 - U^2)^{3/2}$. This equation corresponds to a boundary value problem with the boundary conditions: U has its maximum U_m at $r = 0$, and $U \rightarrow 0$ as $r \rightarrow \infty$. We remind the reader that it has been shown in a seminal paper of Vakhitov and Kolokolov [24] that such solutions exist for arbitrary saturating nonlinearity functions Ψ , provided that the eigenvalue λ satisfies $0 < \lambda < \Psi_m$, where Ψ_m is a maximal value of the nonlinearity function. Equation (46) admits an infinity of discrete bound states characterized by $j = 0, 1, 2, \dots$ zeros at finite r . We consider only the lowest-order nodeless solution of Eq. (46), i. e. “ground state” that is positive and monotonically decreasing with increasing r . In the asymptotic region the solution must decay as $U_{r \rightarrow \infty} \sim \exp(-\sqrt{\lambda}r)/\sqrt{\lambda r}$. Our nonlinearity function Ψ has a maximum $\Psi_m = 1$ found at $U_m (= 1)$, i. e. at the maximally allowed strength of the field. As a consequence the upper bound of the propagation constant λ_c must satisfy $\lambda_c < \Psi_m$. Numerical simulations show that the amplitude of the ground state solution $U_m = U(r = 0, \lambda)$ is a growing function of λ (see Fig. 2) and it acquires its maximum value ($= 1$) at $\lambda = \lambda_c \approx 0.29$. The solution represents a trapped, localized EM solitary beam. The beam becomes wider at low amplitudes.

The stability of the solitonic solutions can be investigated using the criterion of Vakhitov and Kolokolov [24] — the soliton is stable against small, arbitrary perturbations if $dN/d\lambda > 0$, where N is the power of the trapped mode:

$$N(\lambda) = 2\pi \int_0^\infty dr r U^2(r, \lambda). \quad (47)$$

In Fig. 3 we plot the numerically obtained solutions of N for various λ . Since the curve has positive slope everywhere, the corresponding ground state solution is stable for $0 < \lambda < \lambda_c$. Notice that the power of the solitary beam always exceeds a certain critical value $N > N_c \approx 7.8$. We also know that N must be bounded from above ($N \leq N_m \approx 10.5$).

For arbitrary temperatures, explicit form of $\Psi = \Psi(|A|^2)$ can not be found. However, due to its saturating character, solutions with properties similar to the small temperature case (which can be explicitly solved) could be expected. Using relations (34), (40) and (42), we numerically find a stationary solution of Eq. (44) for arbitrary T_∞ . For convenience we use following representation of vector potential $A/A_c = U(r)\exp(i\lambda z)$, where $A_c = (G_\infty^2 - 1)^{1/2}$. Though the maximum value of U is still restricted by the condition $0 < U_m \leq 1$, the amplitude of vector potential A_m can reach a considerable value. For ultrarelativistic temperatures, $T_\infty \gg 1$, we have $A_c = \sqrt{15T_\infty} \gg 1$ and since $0 < A_m \leq A_c$ the soliton solution with ultrarelativistic strength of EM field is possible. Here we present results of simulations for $T_\infty = 1$ (i.e. $T_\infty \approx 0.5 \text{ MeV}$). In Fig. 4 we plot the amplitude of the ground state solution U_m versus the propagation constant λ . The solution exists provided $0 < \lambda < \lambda_c (\approx 0.22)$. The profiles of the field $U(r)$ the plasma density $n_0(r)$ and the temperature $T_0(r)$ are exhibited in Fig. 5 for $\lambda = 0.1$. One can see that in the region of field localization, the plasma temperature and density is reduced. Similar plots could be obtained for all allowed values of λ . When $\lambda \rightarrow \lambda_c$, plasma cavitation takes place, i.e. at $r = 0$ the plasma density and temperature tends down to zero. Appearance of zero temperature is not surprising since the corresponding region is the plasma vacuum; all particles are gone away.

The dependence of N on λ is presented in Fig. 6. One can see that the curve $N = N(\lambda)$ has a positive slope and according to Vakhitov and Kolokolov criterion, the corresponding solitary solutions are stable against small perturbations.

The detailed dynamics of arbitrary field distribution must be studied by direct simulations of Eq. (44). We can learn much, however, from the recent extensive elucidations of the dynamical properties of the solutions of NSE with saturating nonlinearity. It seems that the general features of evolution are not sensitive to the details of the saturating nonlinearity

(see for instance [15,25] and references therein). For all such systems the beam will monotonically diffract if the beam power is below a critical value ($N < N_c$), and it will be trapped if $N > N_c$ and the Hamiltonian $H < 0$ is negative. In the latter case, the beam parameters will oscillate near the equilibrium, ground state values. These oscillations monotonically decrease with increase z due to the appearance of the radiation spectrum. For larger z , the oscillations are damped out, and the formation of the soliton in its ground state takes place. If the initial profile of the beam is close to the equilibrium one, the beam quickly reaches the ground-state equilibrium, and propagates for a long distance without distortion of its shape. The initial beam, even when its parameters (i.e. amplitude, effective width and phase) are quite far from equilibrium, will either focus or defocus to the ground state, exhibiting damped oscillations around it. As a consequence the beam reaches an equilibrium with its final power slightly smaller than the initial. Such an evolutionary scenario may not hold for very intense beams with $N \gg N_c$; the beam may then break up into filaments due to a modulation instability. However, each filament, will tend to evolve towards its own equilibrium state corresponding to the power it carries. Thus, the ground-state equilibrium seems to be an attractor.

Our own studies indicate that Eq. (44), with the nonlinearity particular to the problem at hand, reproduces the general expected behavior described above. However we find that the soliton formation requires the initial beam power to be in the range $N_c < N < N_m$. For $N > N_m$, the multistream motion of the plasma prevents the system from settling in a steady state.

V. CONCLUSIONS

We have investigated the nonlinear propagation of strong EM radiation in a relativistic, unmagnetized two-fluid plasma. The treatment is fully relativistic — in the coherent or directed motion as well as in the random or thermal motion of the plasma particles. The assumption that prior to the switching of the field-plasma interaction, the generalized vor-

ticity is zero, greatly simplifies the system of relativistic fluid equations. In particular, in the electron-ion dominated plasma, under well defined conditions the system of Maxwell-fluid equation [Eqs. (25)-(26)] turns out to be structurally similar to the one obtained for a cold plasma. Consequently we would expect that results already established in cold plasma limit can find appropriate analogy in the hot plasma case.

We presented a somewhat detailed study of EM beam propagation in transparent $e-p$ plasmas. Applying a reductive perturbation technique, the system of relativistic Maxwell-fluid equations is reduced to a 2D nonlinear Schrödinger equation with a saturating nonlinearity. We found that if the strength of the EM field amplitude is below the wave breaking limit, the beam can enter the self-trapped regime resulting in the formation of stable, self-guided 2D solitonic structures. The beam-trapping owes its origin to the thermal pressure (which opposes the ponderomotive pressure) — naturally such trapping regimes are not accessible in the relativistic but cold plasma limit. In the region of beam trapping, the plasma density as well as its temperature is reduced and under certain conditions these parameters can be reduced considerably (i.e. plasma cavitation takes place).

The fact that relativistically hot $e-p$ plasmas are capable of sustaining high amplitude localized structures of high amplitude electromagnetic fields should be a result of considerable importance to an understanding of the complex radiative properties of different astrophysical objects where such plasmas are known to exist.

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Figure Captions

FIG. 1 The nonlinearity function Ψ versus $|A|^2$ for $T_\infty = 0.1$.

FIG. 2 Nonlinear dispersion relation: the amplitude U_m as a function of λ , the eigenvalue.

The plasma temperature $T_\infty \ll 1$.

FIG. 3 The beam power N versus λ ($T_\infty \ll 1$).

FIG. 4 The nonlinear dispersion relation, U_m versus λ for $T_\infty = 1$.

FIG. 5 Normalized EM field U , plasma temperature T_0 and density n_0 versus r for $T_\infty = 1$.

FIG. 6 The beam power N versus λ ($T_\infty = 1$).

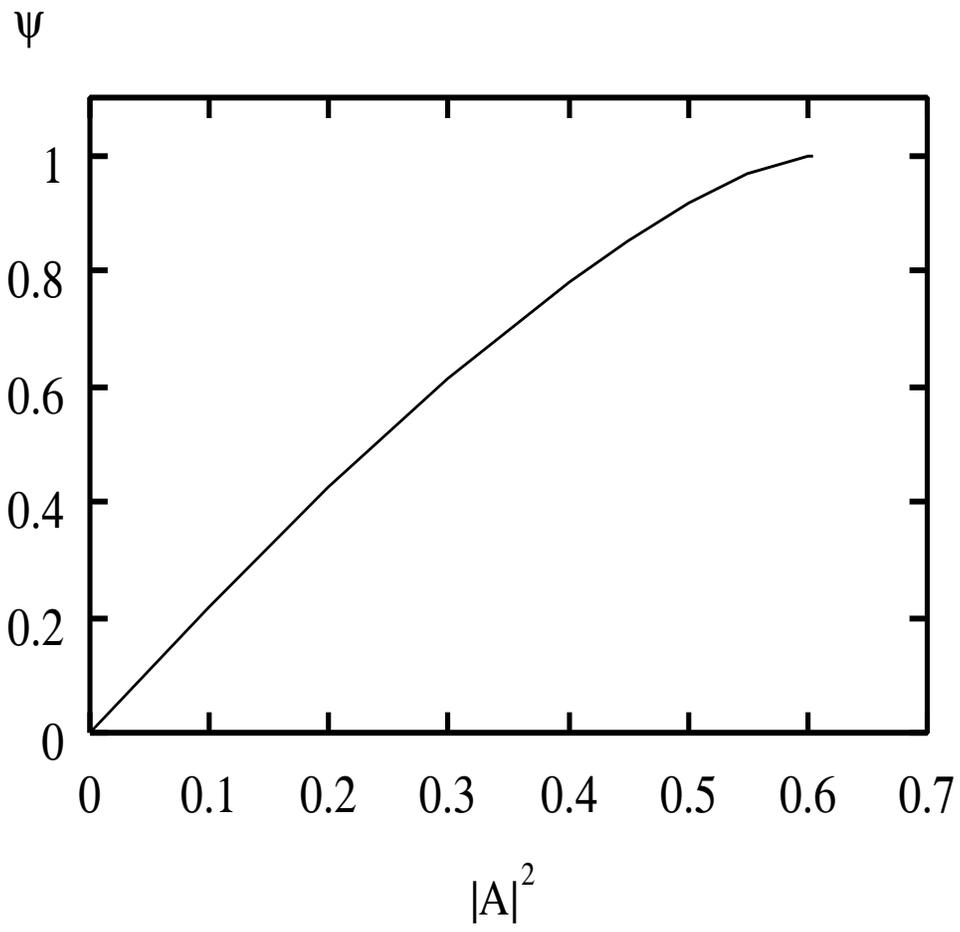


FIG. 1

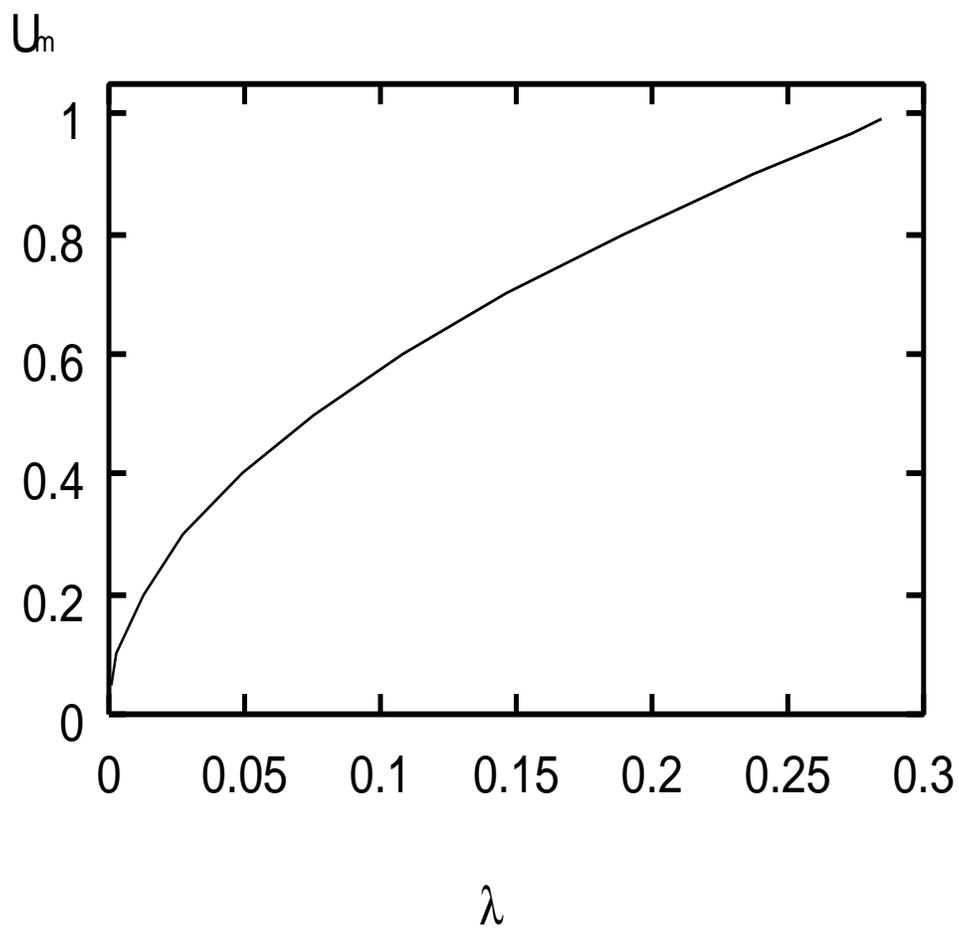


FIG. 2

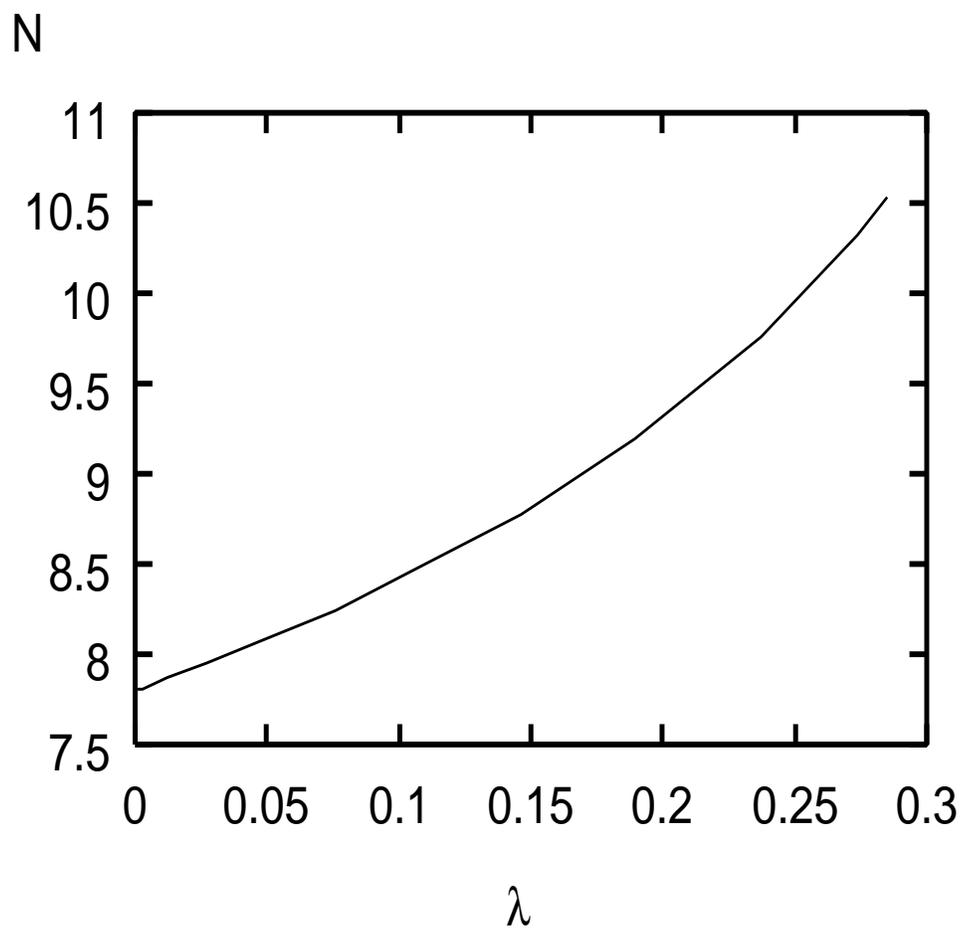


FIG. 3

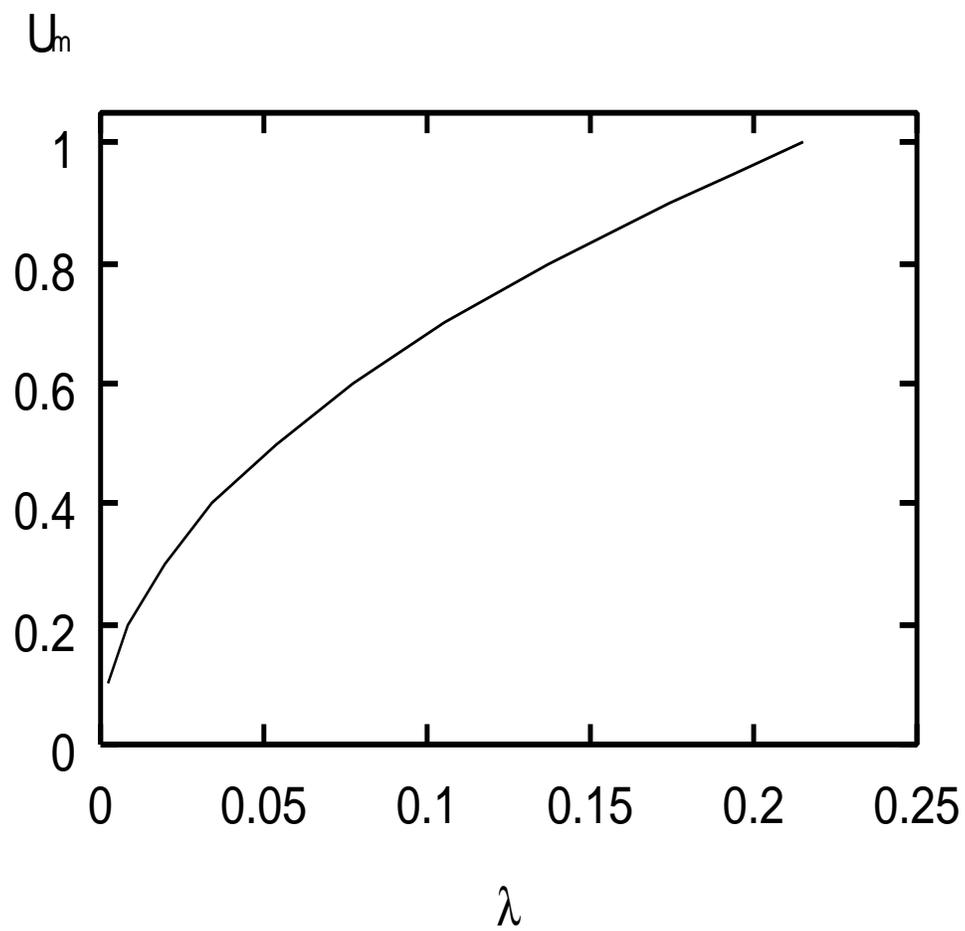


FIG. 4

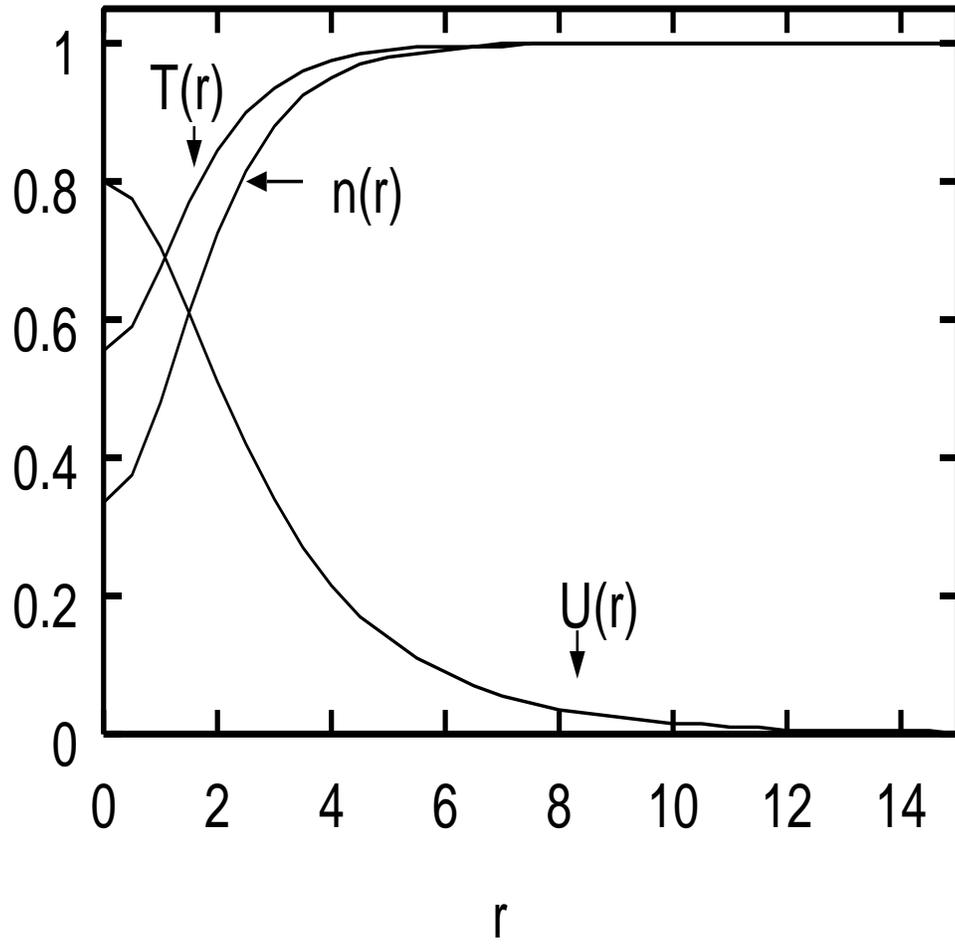


FIG. 5

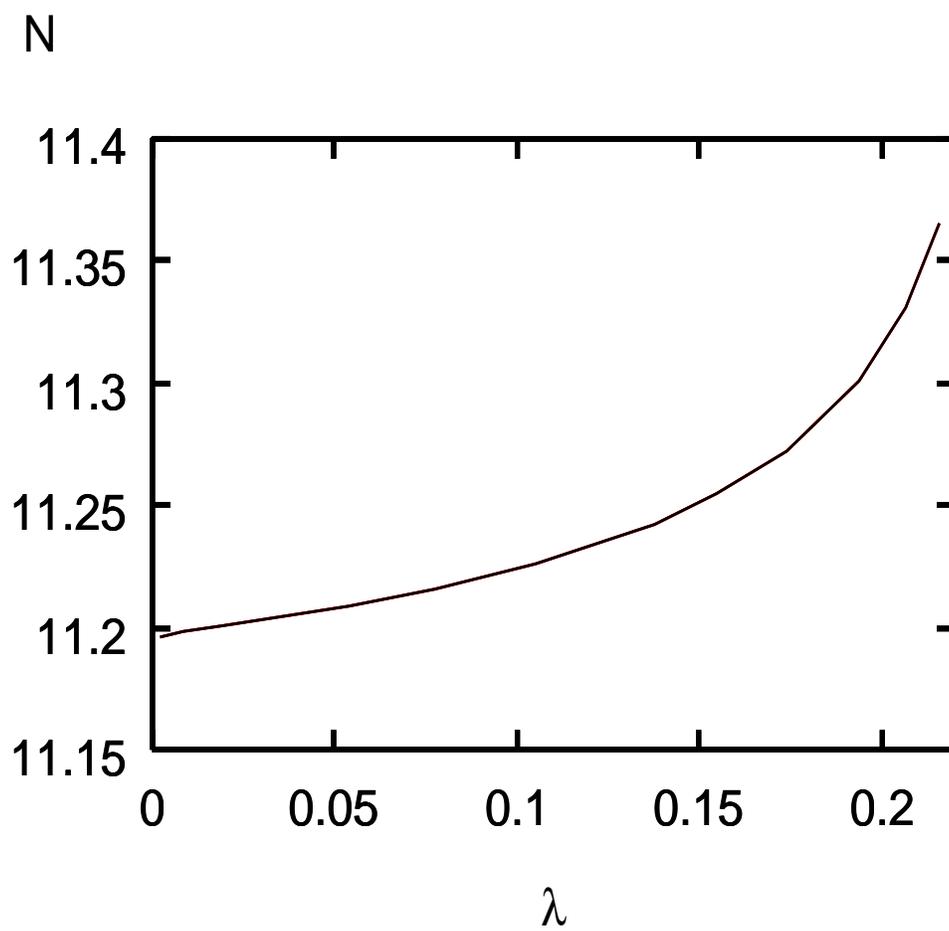


FIG. 6