Decompressive (cooling rarefaction) shock in optically thin radiative plasma

D.Kh. Morozov

Instituto de Ciencias Nucleares, UNAM, 70-543, CU, 04510, Mexico, D.F., Mexico

M. Pekker

Institute for Fusion Studies, The University of Texas, Austin, TX 78712, USA

Abstract

It is shown that the decompressive shock, i.e. a shock where the pressure behind the front is smaller than the pressure ahead of it, is possible in a radiative plasma. This is in contrast to the situation in classic gas dynamics. An example of a steady-state decompressive shock wave for a simple, but realistic model for radiative losses, is presented. It is shown that it satisfies the Landau stability criteria.

52.35.Nx,52.35.Tc, 47.70.Mc

I. INTRODUCTION

As it is shown in many articles (see for example [1] and [2]) the presence of radiative impurities in a plasma not only influence the speed of waves and front propagation in optically thin radiative plasma, but also leads to phenomena that do not exist in pure plasmas.

It is well known that decompressive shock waves, in which the pressure behind the front (P_1) is smaller than the pressure ahead of it (P_2), are forbidden by thermodynamics laws in classical gasdynamics [3]. In contrast, we show in this paper, that the existence of radiative impurities in a plasma can lead to the appearance of decompressive shocks (cooling rarefaction shock waves). This change is due to two distinct factors: First, a radiative plasma is essentially an open system, therefore the law of increasing entropy cannot be applied to it. Second, the impurities can change the speed of propagation and the damping of sound waves so much [4,5] that Landau stability criteria for shocks (the speed of the shock front has to be larger than the speed of sound in unperturbed media and lower than the speed of sound in perturbed one) can be satisfied for decompressive waves in radiative plasma.

In section II, the conditions are found under which impurities and thermal conductivity suppress the propagation of short wavelength oscillations in front of the decompressive shock wave, and cause long wavelength oscillations to propagate slower than the shock wave.

In section III, the equation for a stationary shock wave, taking into account the radiative processes, is given. It is shown that the problem for decompressive shock waves is an eigenvalue problem, the conditions for the existence of solutions are also derived.

In section IV, a numerical solution corresponding to a decompressive shock wave is found, and is shown to satisfy the Landau stability criteria.

It has to be noted, that the existence of decompressive thermal fronts has been predicted for collisionless plasma with a tail of fast particles [6,7]. The existence of solutions corresponding to expansion of thermal fronts has also been predicted in plasmas with strong heating by B. Ahlbom and W. Liese [8] however, the stability of such systems has not been examined.

II. STABILITY

Let us write down a one dimensional system of equations corresponds to the propagation of shock waves in a radiative plasma:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0, \tag{1}$$

$$m\,\frac{\partial}{\partial t}(nv) + \frac{\partial}{\partial x}\left(mnv^2 + P - \mu\frac{\partial v}{\partial x}\right) = 0, \tag{2}$$

$$\frac{\partial}{\partial t}\left(\frac{P}{\gamma-1} + \frac{mnv^2}{2}\right) + \frac{\partial}{\partial x}\left(\left(\frac{\gamma}{\gamma-1}P + \frac{mnv^2}{2}\right)v - \kappa\frac{\partial T}{\partial x}\right) = S - Q, \tag{3}$$

Where n — density, v — velocity, T — temperature, $P = 2nT$ — the plasma pressure (we suppose, that the ion and electron temperatures are equal $T_e = T_i$), m — main ion mass, μ — ion viscosity, γ — adiabatic index ($\gamma = 5/3$), κ— classical electron heat conductivity $\left(\kappa = \kappa_0 \left(T/T_0\right)^{5/2}\right)$, and S and Q — heating and cooling functions, respectively, related to the presence of impurities in plasma. Functions S and Q will be defined bellow. In Eqs. (1)-(3) the space coordinate X coincides with the direction of propagation of the shock.

Let us find out the criteria for stability of decompressive shock waves with the assumption that the time of shock passing is sufficiently long to neglect effects related to the final impurity relaxation time [9]. After a simple calculation, we get the dispersion equation corresponding to the system (1)-(3):

$$i\nu_{ph}\left(\left(\nu_{ph}^2 - \gamma\right) - \nu_\mu\left(\nu_\kappa + \nu_{RT}\right)\right) - \left(\nu_{Rn} + \nu_\mu\right) - \left(\nu_{ph}^2 - 1\right)\left(\nu_\kappa + \nu_{RT} + \nu_\mu\right) = 0, \tag{4}$$

where $\nu_{ph} = \omega/kv_T$ is the dimensionless frequency (ω — frequency, k — wave number of perturbations, $v_T = \sqrt{2T/m}$ — ion thermal speed), $\nu_{Rn} = \frac{\gamma-1}{2Tkv_T}\frac{\partial(Q-S)}{\partial n}$,$\nu_{RT} = \frac{\gamma-1}{2nkv_T}\frac{\partial(Q-S)}{\partial T}$, $\nu_\kappa = \frac{\gamma-1}{2}\frac{\kappa k}{nv_T}$ and $\nu_\mu = \frac{k}{mv_T n}\mu$.

Let us present the solutions to the dispersion equation (4) for a few limiting cases.

In the long wavelength limit: $\nu_{Rn}, \gg 1$, $|\nu_{RT}| \gg 1$, equation (4) has three roots [4,5]. The first:

$$\nu_{ph} = -i\left(\nu_{RT} + \nu_\kappa + \nu_\mu\right) \tag{5}$$

corresponds to aperiodic damping ($|\nu_{ph}| \gg 1$); the other two:

$$\nu_{ph} = \pm\left(1 - \frac{\nu_{Rn}}{\nu_{RT} + \nu_\kappa + \nu_\mu}\right)^{1/2} + \frac{i}{2\left(\nu_{RT} + \nu_k + \nu_\mu\right)}\left(1 - \gamma - \frac{\nu_{Rn}}{\nu_{RT} + \nu_\kappa + \nu_\mu} - \nu_\mu\left(\nu_\kappa + \nu_{RT}\right)\right) \tag{6}$$

correspond to modified sonic oscillations ($|\nu_{ph}| \leq 1$). Since ν_κ and ν_μ are always positive, $\nu_{RT} > 0$ is sufficient for damping of long wavelength sonic oscillations.

If ν_{RT} and ν_{Rn} are close to unity, then all modes are highly damped.

For shorter wavelengths, the sound wave dispersion tends to be dominated by viscosity and thermal conductivity rather than the radiative losses. If the wave vector k_R that makes ν_{RT} and ν_{Rn} close to unity, is much smaller than the wave vector k_κ, that brings ν_κ close to unity:

$$k_R = \frac{\gamma - 1}{2Tv_T}\frac{\partial(Q - S)}{\partial n} \ll k_\kappa = \frac{\gamma - 1}{2}\frac{\kappa}{nv_T} \tag{7}$$

then there exists a set of values of k:

$$k_R \ll k \ll k_\kappa \tag{8}$$

such that $\nu_{Rn} \ll 1$, $|\nu_{RT}| \ll 1$ and $\nu_\kappa \ll 1$ simultaneously. In this case the dispersion equation (4) has three roots: the first

$$\nu_{ph} = \frac{i}{\gamma}\left(\nu_{Rn} - \nu_{RT} - \nu_\kappa\right) \tag{9}$$

corresponds to radiative condensation mode ($\nu_{ph} \ll 1$) while the other two solutions:

$$\nu_{ph} = \pm\sqrt{\gamma} - \frac{i}{2\gamma}\left(\nu_{Rn} + \gamma\nu_\mu + (\gamma - 1)\left(\nu_{RT} + \nu_\kappa\right)\right) \tag{10}$$

corespond to slightly modified sound waves. The case of very short wave length

$$\frac{2nv_T}{\kappa(\gamma-1)} \ll k \tag{11}$$

corresponds to isothermic sonic oscillations:

$$\nu_{ph} = \pm\left(1 - \frac{\nu_{Rn}+\nu_\mu}{2\left(\nu_{RT}+\nu_\kappa+\nu_\mu\right)}\right) - i\,\frac{(\gamma-1)+\nu_\mu\left(\nu_\kappa+\nu_{RT}\right)}{2\left(\nu_{RT}+\nu_\kappa+\nu_\mu\right)} \simeq \pm 1 - i\,\frac{kv_T}{4\nu_{ii}}. \tag{12}$$

Expression (12) takes into account that ion-ion collisions are determined by $\mu = 0.96nTk/\nu_{ii}$ (where ν_{ii} is the frequency of ion-ion collisions). Obviously, the isothermic sound for very short waves

$$k_\mu \sim \frac{\nu_{ii}}{v_T} \tag{13}$$

can not propagate. If $kv_T \gg \nu_{ii}$, the MHD approximation fails. Under these conditions the ion sound is also damped by Landau damping. If

$$k_R \gg k_\mu, \tag{14}$$

then for all k, either the viscosity is high or the radiative losses are high, e.g. either $\nu_{Rn}, \gg 1$, $|\nu_{RT}| \gg 1$ or $\nu_\mu \gg 1$. In this case, only the modified sonic oscillations (6) corresponding to small values of k and $|\nu_{ph}| \gg 1$ may propagate in the plasma.

For the existence of decompressive shock waves it is necessary that sonic oscillations in front of the shock are either strongly damped, or they propagate slower than the shock wave, otherwise the shock wave will disperse.

It easy to see that if in a "warm" plasma, in front of the shock, conditions (13) and (14) are met in addition to:

$$\nu_{RT} + \nu_\kappa > \nu_{Rn}, \tag{15}$$

only modified sound (6) may propagate. Therefor, the speed of the shock wave, c, must be greater than the speed of the modified sound (6) in front of the shock wave:

$$c^2 > 1 - \frac{\nu_{Rn}}{\nu_{RT}}. \tag{16}$$

In deriving (16), we neglected the influence of viscosity and conductivity on the speed of the modified sound (6). It is worth noting that, even if conditions (13) and (14) are not fulfilled, short wavelength oscillations may still be highly damped due to anomalous viscosity caused by plasma turbulence.

Next, we will show that for acceptable parameters conditions when all types of sound, with the exception of modified sound (6), do not propagate through plasma do actually exist, even in a classical turbulence free plasma without anomalous viscosity. As an example, let us consider a plasma with temperature $T \approx 100\, eV$, coefficient of ionization $\xi = 0.2$, adiabatic index $\gamma = 5/3$, $Q \approx 10^{-19}\, erg \cdot sm^3/s$, and $d\ln(Q)/d\ln(T) \approx 2$ (see Sec. IV) then the ratio of $k_R = \frac{\gamma-1}{2nkv_T}\frac{\partial Q}{\partial T}$ and $k_\mu = \frac{\nu_{ii}}{v_T}$ is approximately 0.3, satisfying the desired creterion.

Behind the decompressive shock wave, the temperature and density are considerably lower than in front of it. Therefor, condition (7) may be met, so modified sound (10), behind the front of the shock wave, may propagate faster than the shock wave.

III. RADIATION LOSS MODEL

Let us choose the following model of radiative loss [5]:

$$Q = n^2 \xi L(T), \tag{17}$$

$$S = \text{const}, \tag{18}$$

where $L(T)$ is the temperature dependent function of plasma cooling due to the presence of impurities, ξ is the relative concentration of impurities in the plasma. Since, the gradients of the fluxes of particle momentum, and energy must be zero on both sides of the shock [3], the temperature and density of the plasma in Eqs. (1)-(3) at $x = \pm\infty$ must be such that $S - Q = 0$,

$$S - n^2_{x=\pm\infty} L\left(T_{x=\pm\infty}\right) = 0. \tag{19}$$

We will be considering a decompression shock wave moving from the right to the left, so $x = -\infty$ corresponds to an undisturbed plasma (with high temperature and density), and $x = +\infty$ corresponds to plasma behind the front (with lower temperature and density). In accord with this condition, the inequality (9) at $x = -\infty$ has the form:

$$\left.\frac{d\ln(L)}{d\ln(T)}\right|_{x-\infty} > 2. \tag{20}$$

In accord with (16), we get the following condition on the speed of the decompressive shock wave:

$$c^2 > 1 - \frac{2}{\left.\frac{d\ln(L)}{d\ln(T)}\right|_{x-\infty}}. \tag{21}$$

Combining inequalities (20) and (21) we get:

$$\frac{2}{1-c^2} > \left.\frac{d\ln(L)}{d\ln(T)}\right|_{x-\infty} > 2. \tag{22}$$

IV. STATIONARY FRONT EQUATIONS

In the coordinate system moving along with the shock wave, the shock wave is described by the following stationary system of equations:

$$nv = \Gamma_1 = \text{const}, \tag{23}$$

$$mv\Gamma_1 + P - \mu\frac{dv}{dx} = \Gamma_2 = \text{const}, \tag{24}$$

$$\frac{d}{dx}\left(\left(\frac{\gamma}{\gamma-1}P + \frac{mnv^2}{2}\right)v - \kappa\frac{dT}{dx}\right) = S - Q. \tag{25}$$

The system of equations (23)-(25) is reduced from the system of Eqs. (1)-(3) by discarding the time dependent terms, and a subsequent integration of equations (1) and (2). The constants Γ_1 and Γ_2 are determined through the plasma parameters at $x = -\infty$:

$$\Gamma_1 = \alpha n_0\sqrt{\frac{2_0}{m}}, \qquad \Gamma_2 = 2n_0T_0(1+\alpha^2), \tag{26}$$

where n_0, v_0, T_0 — density, speed, temperature of the plasma in front of the shock wave, and $\alpha = \sqrt{mv_0^2/\left(2T_0\right)}$ — speed of the decompressive shock wave. In terms of the dimensionless variables to be

$$\widetilde{n} = \frac{n}{n_0}, \quad \tau = \frac{\widetilde{T}}{T_0}, \quad \widetilde{v} = \frac{v}{v_0}, \quad z = -\frac{x}{l}, \tag{27}$$

where $l = \kappa/\Gamma_1$, we get the following system of equations:

$$\frac{d}{dz}\left(\frac{d\widetilde{T}^{7/2}}{dz} + \frac{2\gamma}{\gamma-1}\widetilde{T} + \alpha^2\widetilde{v}^2\right) = \widetilde{Q} - \widetilde{S}, \tag{28}$$

$$\alpha^2\widetilde{v}^2 - (1+\alpha^2)\widetilde{v} + \widetilde{T} - \frac{\mu v_0 \widetilde{v}}{2T_0 l}\frac{d\widetilde{v}}{dz} = 0. \tag{29}$$

If we neglect viscosity in equation (29), it becomes a quadratic with solution

$$\widetilde{v}(\widetilde{T}) = \frac{1+\alpha^2 \pm \sqrt{\left(1+\alpha^2\right)^2 - 4\alpha^2\widetilde{T}}}{2\alpha^2}. \tag{30}$$

The graph of the function $\widetilde{v}(\widetilde{T})$ is shown on fig. 1. Substituting (30) into equation (28) we get the equation which coincides completely with the equation for motion of a Newtonian particle in a given field with friction:

$$\frac{d^2y}{dz^2} + \lambda(y)\frac{dy}{dz} = -\frac{dU}{dz}, \tag{31}$$

$$y = \widetilde{T}^{7/2}, \quad \lambda(y) = \frac{4\gamma}{7(\gamma-1)}y^{-5/7} + 2\alpha^2\widetilde{v}(y)\frac{d\widetilde{v}}{dy}, \quad U(y) = \int(\widetilde{S} - \widetilde{Q})dy.$$

The $+$ sign in (30) corresponds to plasma in front of the shock wave, and the $-$ sign to plasma behind the shock wave. Equation (31) is useful for a qualitative analyses of possible solutions for different radiative impurity models. It is worth while to note that the coefficient of friction λ has a singularity at the point of maximum allowed temperature $\widetilde{T}_{\max} = \left(1+\alpha^2\right)^2/\left(4\alpha^2\right)$. However, it does not mean, that equation (28) should also have the same singularity. In the neighborhood of the critical point $\widetilde{T} = \widetilde{T}_{\max}$, like in the theory of classical shock waves, it is necessary to take viscosity into account. From equation (29) it is easy to see, that at this point $d\widetilde{v}/dz = 0$ and $d\widetilde{T}/dz = 0$. So, in the neighborhood of the critical point the temperature equation (28) becomes:

$$\frac{d^2\widetilde{T}^{7/2}}{dz^2} = \widetilde{Q} - \widetilde{S}. \tag{32}$$

At $\widetilde{T} = \widetilde{T}_{\max}$, the second derivative of temperature with respect to z is negative due to the sign of $\widetilde{Q} - \widetilde{S}$ (see below). The condition that $d\widetilde{T}^{7/2}/dz = 0$ at $\widetilde{T} = \widetilde{T}_{\max}$ allows us to connect together the solutions corresponding to the two branches of oscillations (see. 30). In our calculations we used the integrodifferential equation:

$$\frac{d\widetilde{T}^{7/2}}{dz} + \frac{2\gamma}{\gamma - 1}\widetilde{T} + \alpha^2\widetilde{v}^2 = \int_0^z (\widetilde{Q} - \widetilde{S})dx + \text{const}, \tag{33}$$

which is equivalent to equation (28). It is easy to see that equation (33) does not have a singularity solution at point $\widetilde{T} = \widetilde{T}_{\max}$ even if we use expression (30), instead of equation (29) for $\widetilde{v}(\widetilde{T})$. So $z < 0$ corresponds to the plasma in front of the shock wave, and $z > 0$ to the plasma behind it.

Let us consider what must be the form of the function $L(T)$ in expression (10), so that equation (33) would have a solution in the form of a decompressive shock wave and at the same time in the vicinity of $\widetilde{T} = 1$ would satisfy (22). Since at $\widetilde{T} = 1$ the derivative $d\ln(L)/d\ln(\widetilde{T})$ must be greater than 2 (see (20)), and at the same time $\widetilde{T} = 1$ must be a point of equilibrium (corresponding to the plasma in front of the shock wave) the function $L(\widetilde{T})$ must be double humped. The negative sign of the derivative of the function $Q(\widetilde{T})$ at $\widetilde{T} = 1$ is insured by a decrease in the plasma density with decreasing of temperature. The condition that $\widetilde{T} = 1$ is an equilibrium point has the form:

$$\frac{2}{1 - \alpha^2} > \left.\frac{d\ln(L)}{d\ln(\widetilde{T})}\right|_{\widetilde{T}_{\max}}. \tag{34}$$

It is easy to see that inequality (34) is equivalent to inequality (21). The form of the function $L(\widetilde{T})$ corresponds to the radiative losses of different impurities in the coronal Hydrogen plasma with Maxwellian electron distribution [11]. By addition of different impurities it is possible to obtain the required profile of the function $L(\widetilde{T})$ so that equation (33) has a solution in the form of a decompressive shock wave. From mathematical point of view the eigenvalues of equation (33) with fulfillment of condition (22) at $\widetilde{T} = 1$ correspond to a decompressive shock wave.

SHOCK SOLUTION

In this section we give an example of a decompressive shock wave found by numerically solving equation (32) for the function $\widetilde{L}(\widetilde{T})$ shown in fig. 2, and $\alpha = 0.5$. It should be noted that not for every double humped function $\widetilde{L}(\widetilde{T})$ that satisfies condition (22), there exists a decompressive shock wave solution.

As was noted above, $\widetilde{T} = 1$ is the equilibrium point, that corresponds to undisturbed plasma in front of the shock wave. This point is located on the growing branch in fig. 1 (solid line). As the distance from the shock decreases, the plasma speed grows with the growth of temperature in accord with the solid curve in fig. 1. After crossing the shock front the plasma speed must increase with decrease in temperature in accord with the dashed curve in fig. 1. If the solution of equation (33) does not switch to the other branch, one recovers the well recognized "cooling wave" (see for example [12]). For numerical solution of equation (33), we started from the point $z = 0$ corresponding to $\widetilde{T}_{\max} = \left(1+\alpha^2\right)^2 / (4\alpha^2)$. We used this to choose the integration constant in equation (33). Integration for positive and negative z was conducted independently.

In fig. 3 we display the temperature (solid line) and density (dashed line) profile from the numerical solution of equation (33). The shock wave propagates from the right to the left. The temperature profile, as is apparent from fig. 3, is very asymmetric. The temperature slowly falls for negative z and suddenly drops for positive z. It is easy to check that the speed of the sonic wave propagation behind the shock front is greater than α — the speed of this decompressive shock wave, which corresponds to the shock wave stability criteria of Landau.

V. CONCLUSION

It is shown that the decompressive (rarefaction) shock is possible in radiative plasmas in contrast to classical gasdynamics. It is possible for two reasons. First, a radiative plasma is an open system, and the entropy production may be arbitrary. Second, the usual sound may be damped in an unperturbed hot plasma, and only the slow modified sound may propagate. Simultaneously, the usual sound may propagate in a cool perturbed plasma. The shock front speed may be faster than the sound speed in unperturbed hot plasma and slower than the sound speed in the perturbed cool plasma. Thus, the Landau stability criterion may be satisfied. The eigenfunction and eigenvalue problem has been solved numerically for a simple but realistic radiation model. The temperature shock profile is presented.

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FIGURE CAPTIONS

Fig. 1. The dimensionless velocity versus the temperature. The solid line coresponds to undisturbed plasma (in front of the shock wave),and the dashed — to the disturbed plasma (behind the shock wave). Point corresponds to the equilibrium state.

Fig. 2. Radiative losses $\widetilde{Q}$ (solid line), and the heat source $\widetilde{S}$ (dashed line) versus the temperature.

Fig. 3. The temperature profile (solid line), and density profile (dashed line) in the shock wave. The wave runs from the right to the left.

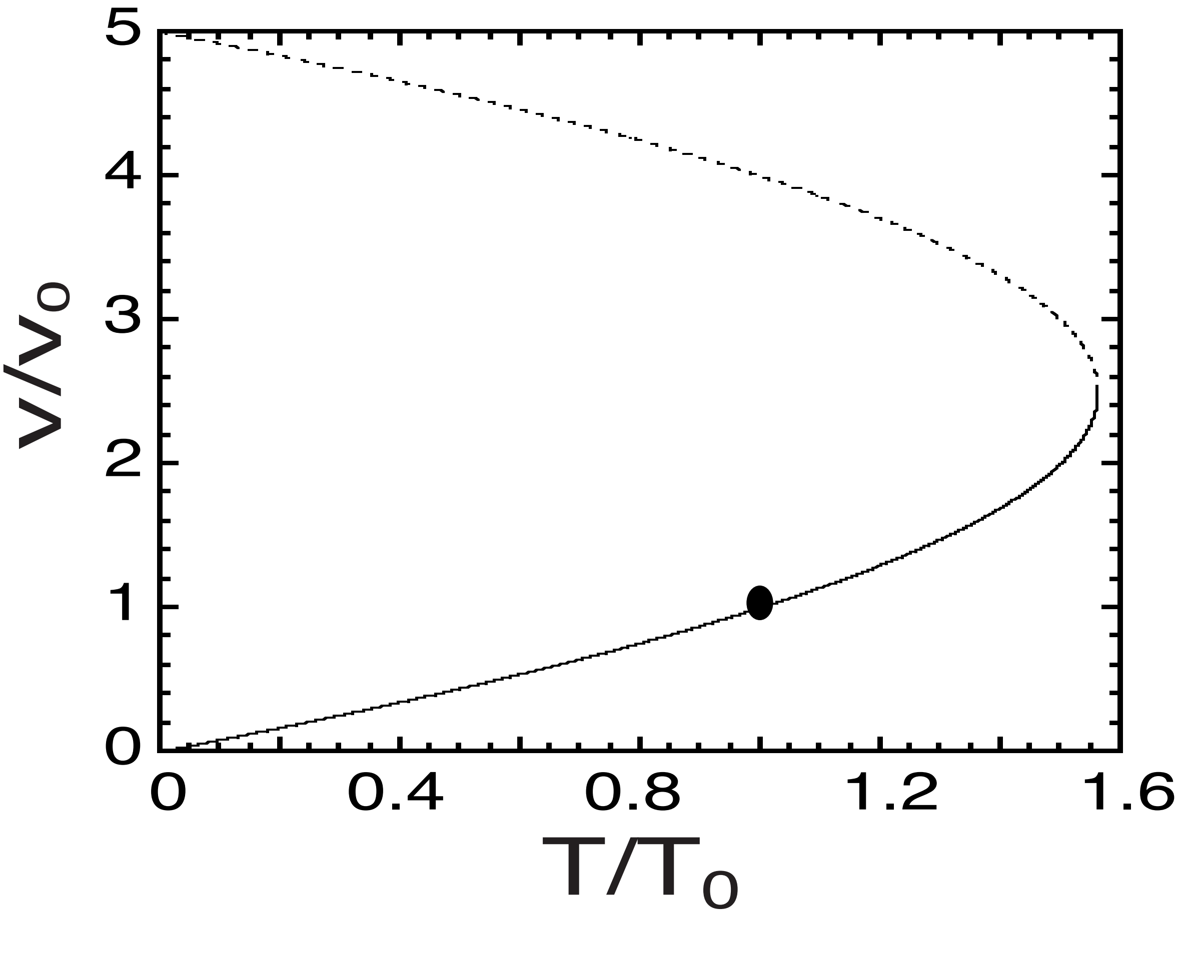

Fig.1

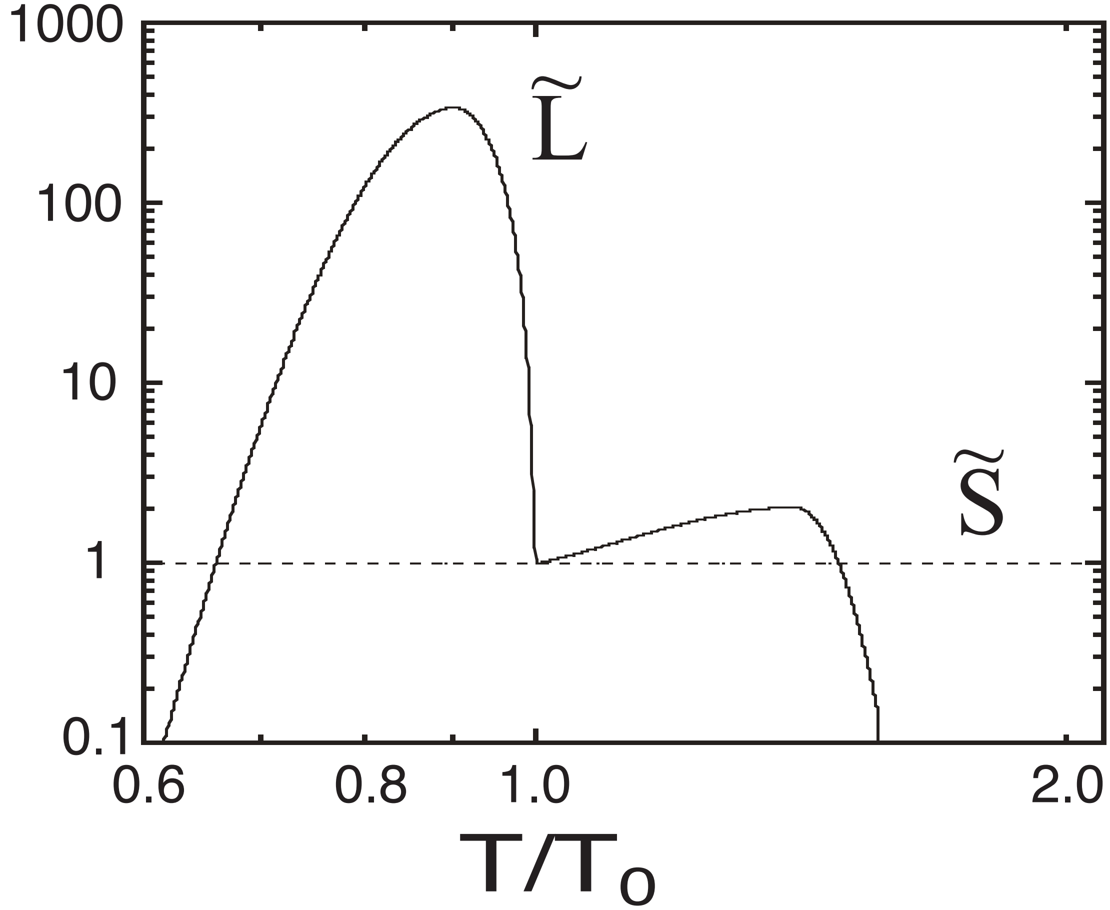

Fig.2

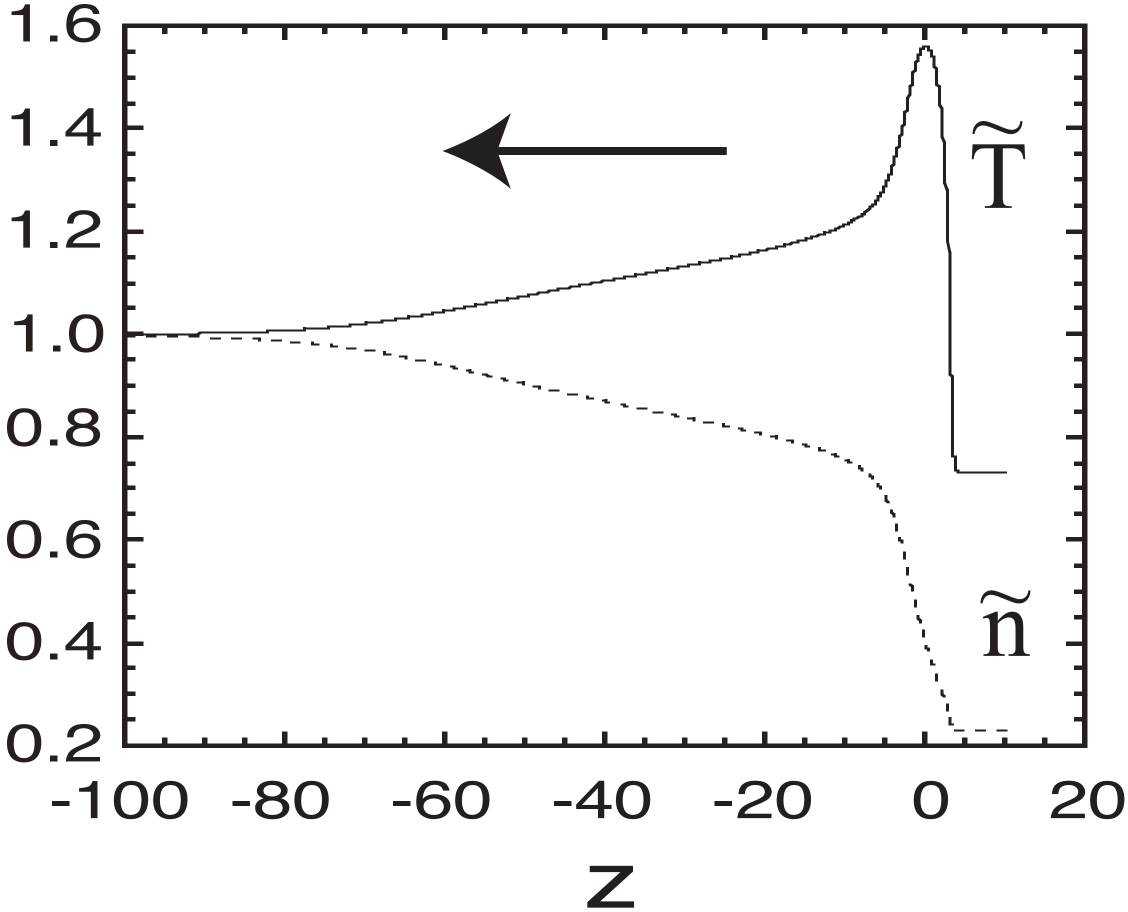

Fig.3