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NON-MAXWELLIAN ION DISTRIBUTIONS CAUSED BY NEOCLASSICAL HEAT CONDUTION

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Abstract

It is shown that in tokamaks with neoclassical banana regime heat conduction and ion self collisions dominant for energy scattering, the distribution tail diffuses outwards radially and downwards in energy maintaining a constant effective temperature $\left(-\partial \ell n f_i / \partial \varepsilon\right)^{-1}$, as observed in some experiments.

Measurements of the ion distribution function (f_i) in tokamaks by analyzing the charge exchange neutrals emerging along a line of sight have often produced $\ell n f_i$ versus energy (ε) plots with two straight lines.[1] The original explanation by Russian workers[2] has generally been accepted. The lower energy particles, which exhibit a low effective temperature $T_i = \left(-\,\partial \ell n f_i / \partial \varepsilon\right)^{-1}$, are assumed to come from the outer regions of the discharge where neutrals are plentiful and T_i is low; the higher energy particles with higher T_i are assumed to come from the hot core. However, recent charge exchange measurements[3] on PDX using a modulated diagnostic neutral beam as the neutral source, with discharge conditions $I = 495\text{kA}$, $B_T = 22.5\text{kG}$, $\bar{n}_e = 2.9\times 10^{13}\text{cm}^{-3}$, $Z_{eff} = 2.5$, $T_{io} = 5\text{keV}$ in hydrogen with deuterium neutral beam injection, have shown that f_i, even for a single minor radius (away from the magnetic axis), exhibits the two straight lines on a $\ell n f_i$ plot. The steeper slope at lower energies gives an effective temperature decreasing with radius as expected, but the higher energy part retains an approximately constant temperature close to the central ion temperature T_{io}.

An earlier charge exchange measurement showing strong evidence for a non-Maxwellian f_i was made by Goldston[4] on ATC. Analyzing charge exchange particles emerging along lines of sight which were tangential to magnetic surfaces in a toroidal sense, he found the temperature of ions moving antiparallel to the current, designated by $T_{\nparallel}$, decreased with r with an approximately parabolic dependence, whereas the temperature for ions moving parallel to the current remained constant out to near the wall. This was for $I = 65\text{kA}$, $B_T = 15\text{kG}$, $\bar{n}_e = 2\times 10^{13}\text{cm}^{-3}$, $Z_{eff} = 4$, $T_{io} = 220\text{eV}$ with only ohmic heating in deuterium. Changing to $I = 85kA$ and hydrogen, some fall off in $T_{\parallel}$ was observed but it was only by 22% at $r/a = 0.75$. A similar toroidally tangential measurement made by Goldston et al.[5] on ohmically heated discharges in PDX ($I = 150kA$, $B_T = 20kG$, $\bar{n}_e = 2\times 10^{13} cm^{-3}$, $Z_{eff} = 1$, $T_{io} = 600eV$) showed only a small difference between $T_{\parallel}$ and $T_{\perp}$, the difference being within the experimental error.

A basic assumption of neoclassical theory is that the ion Larmor radius in the poloidal magnetic field is small compared with the radial gradient scale lengths. This is not satisfied for ions in the tail of f_i in the above and most tokamak discharges. Hence, a breakdown of standard neoclassical theory is to be expected and, in particular, of the argument that in lowest order f_i must be Maxwellian. Neoclassical ion heat conduction has, therefore, been reconsidered without assuming f_{io} Maxwellian. It has been found that if ion self collisions are dominant for energy scattering so that heating or cooling of the tail particles by electrons or beam ions can be neglected, neoclassical theory requires the tail of f_i, which is diffusing outwards radially and downwards in energy, to maintain a constant effective temperature, leading to the observed two component f_i in outer radii. This and other considerations lead one to the conclusion that the discontinuity in the slope of $\ell n f_i$ is the equivalent in the two-dimensional phase space r,v, of a contact discontinuity in gas dynamics.[6] The flow of the "fluid" f_i is outwards in r and downwards in v on the higher energy, larger r, side of the discontinuity and inwards in r and upwards in v on the reverse side, the flows being tangential to the discontinuity.

The modifying effect of energy scattering collisions with electrons or beam particles will be considered at the end of this letter. The former are invoked to explain the fall off of $T_\parallel$ with hydrogen in ATC and the null result with ohmic heating in PDX as well as to explain the increase in effective temperature of the inward diffusing low-energy part of f_i. An explanation of how the two component f_i leads to the observed difference in $T_\parallel$ and $T_\perp$ is also deferred until the end of this letter. A reference frame is chosen in which the mean toroidal velocity of the ions ($\simeq V_\parallel$) is zero. This modifies the radial electric field E_r but centrifugal force terms are assumed higher order. Separating f_i into parts f^+, f^-, which are, respectively, even and odd in $v_\parallel$, the part of f^+ which is independent of the poloidal angle θ is expanded in the even Legendre polynomials $P_\ell(v_\parallel/v)$ and only the first term is retained, i.e., $f_0 \equiv \bar{f}_0^+(r,v) = a_0 P_0$. From consideration of the collision operator the coefficient in the next term $a_2 P_2$ should satisfy $a_2 \sim 2a_0/Z^i_{eff}\ell(\ell+1) = a_0/3Z^i_{eff}$, where $Z^i_{eff} = (n_i + \Sigma n_z Z^2)/n_i$. The other parts of f_i, namely $\tilde{f}^+(\theta)$ and f^- are derived in the usual neoclassical manner as functions of f_0. Although the parameter ρ_0/L is not assumed small compared with unity, its magnitude is assumed limited such that $(2/3)(\rho_\theta/L)^2(r/R) \ll 1$; then in f^-, only the term first order in both (ρ_θ/L) and $(r/R)^{1/2}$ need be retained.

Averaging the drift-kinetic equation with Fokker-Planck collision operator over both pitch angle and poloidal angle θ, one obtains to first order in $(r/R)^{1/2}$

$$\frac{1}{r}\frac{\partial r\Gamma_v}{\partial r} + \frac{eE_r^*}{mv^2}\frac{\partial v\Gamma_v}{\partial v} = -\frac{\gamma}{v^2}\frac{\partial}{\partial v}v^2\left(f_0\frac{\partial H}{\partial v} - \frac{1}{2}\frac{\partial^2 G}{\partial v^2}\frac{\partial f_0}{\partial v}\right) \tag{1}$$

where $E_r^* = E_r - V_\parallel B_\theta$, $\gamma = 4\pi e^4 \ell n\Lambda/m_i^2$, H and G are the Rosenbluth potentials as defined in Ref. 7, and $4\pi v^2\Gamma_v dv$ is the radial diffusive flow per unit area for ions in the velocity element dv . For the banana regime, Γ_v is given by

$$\Gamma_v = -0.49\left(\frac{r}{R}\right)^{1/2}\frac{m_i^2 v^2 \nu_{PA}}{e^2 B_\theta^2}\left(\frac{\partial f_0}{\partial r} + \frac{eE_r^*}{mv}\frac{\partial f_0}{\partial v}\right) \tag{2}$$

and $\nu_{PA} = \sum_{i,z}(\gamma/v^2)\partial G_j/\partial v$. Energy scattering by collisions with impurity ions will be small and has been omitted, and that by electrons and beam ions has been assumed small.

Here, Eq. (1) will be solved only for the energetic tail of f_0 , it being assumed that the low-energy part of f_0 is given; namely, for $0 \leqslant v < V$, f_0 is the Maxwellian f_c , with temperature $T_c(r)$ and $(V/v_{T_c})^2 \gtrsim 3.5$ so that $\exp(-V^2/v_{T_c}^2) \ll 1$ and $\hat{n}_c \equiv \int_0^V 4\pi v^2 f_{0c} dv \simeq n_i$. With these assumptions, the Rosenbluth potentials can be approximated by considering only the f_c contributions in the case of f_i . The approximations are

$$\frac{\partial H}{\partial v} = -\frac{\hat{n}_c}{v^3} \quad , \quad 0.5\frac{\partial^2 G}{\partial v^2} = \frac{\hat{n}_c T_c}{m_i v^3}$$

$$\text{and} \quad \nu_{PA} = \gamma Z_{eff}^i \frac{\hat{n}_c}{v^3} \; . \tag{3}$$

Substituting Eqs. (2) and (3) into Eq. (1) gives

$$\frac{1}{r\hat{n}_c}\frac{\partial}{\partial r} r\alpha^2\hat{n}_c\left(\frac{\partial f_0}{\partial r} + \frac{eE_r^*}{m}\frac{\partial f_0}{\partial w}\right) + \alpha^2 \frac{eE_r^*}{m}\left(\frac{\partial^2 f_0}{\partial w \partial r} + \frac{eE_r^*}{m}\frac{\partial^2 f_0}{\partial w^2}\right)$$

$$= -\frac{\partial}{\partial w}\left(f_0 + \frac{T_c}{m}\frac{\partial f_0}{\partial w}\right) \tag{4}$$

where $w = (1/2)v^2$ and $\alpha^2 = 0.49\ Z_{eff}^i(r/R)^{1/2}(m/eB_\theta)^2$.

Equation (4) will be solved for the radial range from some inner radius r_0 , which could, for example, be the edge of the sawtooth region, to an outer radius, which is either the wall or a smaller radius within which charge exchange collisions can be neglected. The boundary condition assumed at $r = r_0$ is for $0 \leqslant v < V$, $f_0 = f_c$ with $T_c = T_{co}$ and for $V < v < \infty$, $f_0 = f_H \sim \exp(-mw/T_H)$ with T_H somewhat larger than T_{co} . The justification for assuming a small temperature difference already in existence at r_0 is that the part f_c has arrived at r_0 by diffusion from an outer lower temperature region and f_H has come from the higher temperature inner region. Also, many of the trapped particle constituents of f_H at r_0 will have orbits passing near the magnetic axis where T_i is a maximum.

Neglecting the weak radial dependence of $r\alpha^2\hat{n}_c$ over the radial range of interest, Eq. (4) is solved by a perturbation method initially neglecting the weak radial dependence of T_c . Changing coordinates from r,w to r , $\hat{\varepsilon}$ where

$$\hat{\varepsilon} = w - \frac{e}{m}\int_{r_0}^{r} E_r^*\, dr \quad , \tag{5}$$

Eq. (4) becomes

$$\alpha^2 \frac{\partial^2 f_0}{\partial r^2} = -\frac{\partial}{\partial \hat{\varepsilon}} \left(f_0 + \frac{\overline{T}_c}{m} \frac{\partial f_0}{\partial \hat{\varepsilon}} \right) , \tag{6}$$

which can be solved by the separable coordinates method. Taking k^2 as an example separation constant, the example solution, which is small at large r , is

$$f_0 = \exp\left(-\frac{kr}{\alpha}\right)\left[C_1 \exp(-\beta_1 \hat{\varepsilon}) + C_2 \exp(-\beta_2 \hat{\varepsilon}) \right] \tag{7}$$

where $\beta_{1,2}$ are the solutions of

$$\frac{\overline{T}_c}{m} \beta^2 - \beta + k^2 = 0 . \tag{8}$$

However, the boundary condition at r_0 requires $\beta = m/T_H$ and hence, Eq. (8) becomes an equation for a unique value of k^2 . Finally, a more accurate solution allowing for the radial dependence of T_c can be obtained assuming the same dependence of f_0 on $\hat{\varepsilon}$ and using the WKBJ method; one obtains

$$f_0 = C\left(\frac{T_H - T_{co}}{T_H - T_c} \right)^{1/4} \exp\left(-\int_{r_0}^{r} \frac{dr}{\lambda} \right) \exp\left(-\frac{mw}{T_H} \right) \tag{9}$$

where

$$\frac{1}{\lambda} = -\frac{e(E_r - V_{\parallel} B_\theta)}{T_H} + \frac{1}{\rho_{\theta H}} \left\{ \frac{2[1 - (T_c/T_H)]}{0.49 Z^i_{eff}} \right\}^{1/2} \left(\frac{R}{r}\right)^{1/4}$$

and $\rho_{\theta H} = (2mT_H)^{1/2}/e\overline{B}_\theta$.

It should be emphasized that the solutions in Eqs. (7) and (9), giving a constant effective temperature T_H for the tail of f_i , only occur because of the velocity dependence of Γ_v in the banana regime and the velocity dependence of the coefficients in C_{ii} for the tail particles. Under other conditions, the constant T_H will not occur and, of particular interest, is the effect of electron collisions. The term $C_{ie}(f_0)$ will produce an extra contribution to the right-hand side of Eq. (6) equal to

$$-\frac{\nu_{ei}}{\gamma \hat{n}_c} \left(\frac{m_e}{m_i}\right) \frac{\partial}{\partial \hat{\varepsilon}} v^3 \left(f_0 + \frac{T_e}{m_i} \frac{\partial f_0}{\partial \hat{\varepsilon}}\right) \quad .$$

Comparing this term to the C_{ii} term with $T_e \approx T_c$, it is smaller by the factor $(m_e/m_i)^{1/2}$, larger by the factors $(m_i v^2/2T_e)^{3/2}$ and n_e/n_i ; it is typically only a fraction of the C_{ii} term. However, the presence of the extra factor v^3 means that the ion deceleration increases with energy which will have the effect of decreasing T_H as f_H diffuses outwards. The change in T_H as f_H flows a radial distance Δr will be proportional to

$$\Delta t C_{ie} \approx \left[\frac{\Delta r}{(\Gamma_v/f_0)}\right] C_{ie} \sim T_i^{1/2}\, m_i^{3/2}\, T_e^{-3/2}\, I^2 (Z^i_{eff})^{-1} \quad .$$

In the change from the deuterium to the hydrogen experiment in ATC, the factor $m_i^{-3/2}\, I^2$ is increased by $2\sqrt{2}(85/65)^2 = 4.8$, so that the change in $T_\parallel$ (identified with T_H below) would be 4.8 times larger in the hydrogen experiment. In the PDX ohmic heating experiment which gave $T_\parallel \approx T_\perp$, the factor $T_i^{1/2}\, I^2 (Z^i_{eff})^{-1}\, T_e^{-3/2}$ is increased by 7.6 over the ATC deuterium case; hence a fall off of $T_\parallel$ with radius is expected which is larger than observed for hydrogen in ATC. In the PDX neutral beam heating experiment[3], the cooling of the f_H particles due to C_{ie} should again be substantial, but it is suspected that this is balanced approximately by the heating due to beam particles.

Turning to the case of f_c , the inward-flowing lower energy part of f_i , the magnitude of the C_{ii} term is much smaller--it must be calculated more accurately than in Eq. (4) to get a non-zero value--and the mean radial velocity for the f_c particles is smaller by the factor $\hat{n}_H/\hat{n}_c$ (~5), where $\hat{n}_H = \int_V^\infty 4\pi v^2 f_H dv$, since detailed ambipolarity[8] requires

$$\int_0^\infty 4\pi v^2 \Gamma_v dv = \Gamma_{enc} \simeq 0 . \qquad (10)$$

The C_{ie} term will, therefore, be much more important for f_c and, in ohmic discharges, can be given the main credit for the increase of T_c with decreasing radius.

Turning to the observations in the two toroidal directions, particles are being detected satisfying $\xi \equiv (v_\parallel/v) \simeq \pm 1$ and hence, $f^+ + f^-$ is being measured. In the banana regime, f is constant on a banana orbit apart from a small collisional correction: hence, $f[r, w, \xi = \pm(2r/R)^{1/2}]$ will be the same as $f(r \mp \delta r_V,\ w \mp \delta r_V e E_r^*/m,$

$\xi = 0$) where $\delta r_V = (2r/R)^{1/2} mv/eB_\theta$. Also, if pitch-angle scattering is strong compared with energy scattering $\left(Z^i_{eff} \gg 1\right)$, the change in f from $\xi = \pm(2r/R)^{1/2}$ to $\xi = \pm 1$ will be small. Hence, from standard neoclassical theory[7]

$$f(r,w,\xi = \pm 1) \simeq f_0 + f_1^- = f_0 \mp \delta r_V \frac{\partial f_0}{\partial r} \mp \frac{\delta r_V e E_r^*}{m} \frac{\partial f_0}{\partial w} \quad . \qquad (11)$$

If the energy for the discontinuity is denoted by $W(r) = 1/2[V(r)]^2$, it should be observed for $\xi = \pm 1$ at $\hat{W} \equiv 1/2\hat{V}^2 = W(r \mp \delta r_V) \pm \delta r_V e E_r^*/m$. Hence, using Eq. (11), the effective temperate for $\xi = +1$ will be

$$\left(-\frac{\partial \ell n f}{m \partial w}\right)^{-1} = T_H \left[1 + \frac{1}{2}\left(\frac{2r}{R}\right)^{1/2} \rho_H \left(\frac{v_{TH}}{v}\right) \left(\frac{1}{\lambda} + \frac{eE_r^*}{T_H}\right)\right] \qquad \text{for} \quad w > \hat{w}$$

$$= T_c(r - \delta r_V) \left\{1 - \frac{1}{2}\left(\frac{2r}{R}\right)^{1/2} \rho_c \frac{v_{Tc}}{v} \left[\frac{eE_r}{T_c} + \frac{n_c'}{n_c} + \frac{T_c'}{T_c}\left(\frac{mw}{T_c} - \frac{3}{2}\right)\right] + \left(\frac{2r}{R}\right)^{1/2} \frac{m(v - \hat{V})}{eB_\theta} \frac{T_c'}{T_c}\right\} \qquad \text{for} \quad w < \hat{w} \quad .$$

$$(12)$$

For the ATC parameters, the factors in the square brackets are found to involve corrections in the range 5-15%, so that parallel observation should exhibit the two temperatures T_H , $T_c(r - \delta r_V)$. Similarly, for anti-parallel observations, the two temperatures should be T_H , $T_c(r + \delta r_V)$. However, in work to be published it has been found that these two component distributions require a substantially larger negative E_r to satisfy Eq. (10) than in standard neoclassical theory. Thus, for parallel observations $\hat{W}$ will be smaller than W and the energy range over which f_H is detected is increased and could produce the dominant slope determining $T_\parallel$. For anti-parallel observations, $\hat{W}$ is increased and the slope of f_c could dominate. (The one ℓnf <u>versus</u> w plot illustrated in Ref. 4 does in fact show evidence for a discontinuity at $mw = 700$ eV, the four lowest energy data points falling on a straight line with the expected lower effective temperature.)

In conclusion, the non-Maxwellian ion distributions observed in ATC and PDX and which are expected to exist in many other tokamak experiments, can be explained as an outcome of neoclassical heat conduction under conditions where ion self collisions are dominant for energy scattering. One is led to the conclusion that the discontinuity between the two parts of f_i is a contact discontinuity in the phase space r,v between the outward and inward flowing parts of f_i. The presence of the non-Maxwellian tail will modify substantially the predictions for ion heat conduction, transfer of energy from ions to electrons and the ion energy per unit volume in the outer regions, the ion bootstrap current J_{iz} and the parameter $E_r - V_\parallel B_\theta$. The measurement of T_i from the Doppler broadening of impurity spectral lines will give values close to T_c which is not a measure of the local ion energy per unit volume when the tail f_H exists.

Footnotes and References

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