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ELECTRON TEMPERATURE LIMIT FOR POLOIDAL EQUILIBRIUM AND TOKAMAK CONTAINMENT SCALING

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Abstract

The experimentally observed scaling laws for the electron temperature (T_{eo}) and energy containment time (τ_{Ee}) with Ohmic heating are derived from the two conditions: (a) the predicted critical T_{eo} for loss of poloidal equilibrium and (b) the proportionality of the ion heat flux to the Poynting vector. Condition (b) is inferred from the large fraction of Ohmic power shown to be transferred to the ions.

Several authors[1] have used the experimental results from the many tokamak experiments throughout the world to produce empirical scaling laws for the energy confinement. The most extensive study appears to be that of Pfeiffer and Waltz[2], who found that the central electron temperature T_{eo} in Ohmically heated discharges is given

$$T_{eo}=e^{-12\pm1.6}\, a^{-1.24\pm0.25}\, R^{0.97\pm0.23}\, I^{0.81\pm0.10}\, Z_{eff}^{\;0.37\pm0.09} \langle n_e\rangle^0 B_T^0 A_i^0 \equiv \Pi_1 \qquad (1)$$

where the units are mks except that T_{eo} is in keV. The various symbols have their usual meaning. The constant has been put in exponential form, e being the base for Naperian logarithms. They found no statistically significant dependence of T_{eo} on the volume-averaged electron density $\langle n_e \rangle$, the toroidal magnetic field B_T or the hydrogen ion atomic mass number A_i. These three parameters have been included in Eq. (1) with zero exponents to complete the observed empirical scaling. For the average temperature per electron over the discharge cross section, $\langle T_e \rangle$, a similar scaling law was obtained which will be denoted here by Π_2. The parametric dependences in Π_2 differ only by small amounts from Π_1.

For the electron energy containment time in such discharge (determined from the energy replacement time) Pfeiffer and Waltz obtained

$$\tau_{Ee} = e^{-43.8} \langle n_e \rangle^{0.90} a^{0.98} R^{1.63} Z_{eff}^{\;0.23} \, (T_{eo}/\Pi_2)^{\gamma} \qquad (2)$$

As discussed by these authors, the exponent γ is not determined by the steady-state Ohmic heating results. In this statistical analysis they found a broad minima in the variation of the best-fit parameter with γ, reasonably good fits occurring over the range $1.0 \leqslant \gamma \leqslant 2.5$.

It is first pointed out that the scaling laws for T_{eo} and τ_{Ee} are not independent. Thus, assuming that q is unity on the magnetic axis, the energy replacement time is

$$\tau_{Ee} \equiv 3\pi a^2 \langle n_e \rangle \langle T_e \rangle / I E_T = 3\pi^2 \sigma_{\|0} a^4 \langle n_e \rangle \langle T_e \rangle T_{eo}^{3/2} / q_a I^2 Z_{eff} \quad , \tag{3}$$

where E_T is the toroidal electric field and $\sigma_{\|0}$ is the approximate constant satisfying $\sigma = \sigma_{\|0} T_e^{3/2} Z_{eff}^{-1}$. Assuming $\langle T_e \rangle$ is proportional to T_{eo}, which was done by Pfeiffer and Waltz for some experimental cases, and for consistency neglecting the variation of q_a, Eq. (1) and (3) yield

$$\tau_{Ee} \sim \langle n_e \rangle \; a^{0.90} \; R^{2.43} \; Z_{eff}^{-0.07} \; I^{0.03} \; (T_{eo}/\Pi_1)^{2.5} \tag{4}$$

which is very close to the parametric dependence of Eq. (2). Thus a theoretical explanation of the scaling law $T_{eo} = \Pi_1$ would automatically lead to a close approximation for the observed parametric dependence of τ_{Ee}. Such a theoretical explanation is reported here based on the critical value of

T_{eo} for loss of poloidal equilibrium in the central saw-tooth region. A second condition required, which will be justified here, is that the ion heat flux is proportional to the incoming energy flux, the Poynting vector. The formula for the critical T_{eo} is valid only for $Z_{eff} \gtrsim 3$; the fact that the scaling laws seem to apply for lower Z_{eff} also is considered at the end of this letter.

The details of the analysis leading to the critical electron temperature for loss of poloidal equilibrium are given elsewhere[3]; a summary of the results is given here. Considering the central saw-tooth region of the plasma and taking the density and temperature gradients of the various particle species to be small in this region[3], there still remains one significant force tending to cause poloidal rotation, namely the electron neoclassical viscous force $\overline{(P_{e\|}-P_{e\perp})\sin\theta/R}$ driven by the electron current $J_{\|}$. (Here the over bar is used to denote the simple θ-average, with θ being the poloidal angle.) The overall response of the positive ions resisting poloidal rotation is highly non-linear in the poloidal velocity $\overline{V}_{\theta}$. This nonlinearity, which leads to the possibility of multiple equilibria, arises partly from convective inertia terms and partly from the electrostatic potential variation on a magnetic surface, $\Phi(\theta)$. In impure plasmas the main cause of this potential variation is the nonuniformity of the impurity ions on a magnetic surface which can be particularly pronounced as resonance with a very slow compressional magnetosonic wave is approached[3].

The multiple equilibria which are found differ in the magnitudes of $\overline{V}_\theta$ and the nonuniformity of the plasma. The analysis shows that the normal equilibrium with low $\overline{V}_\theta$ is lost when the electron temperature reaches the critical value T_{c1}. Poloidal acceleration towards a new equilibrium with higher $\overline{V}_\theta$ must occur, an m=0, n=0 instability. The new equilibrium can only exist for a short time due to both increased electron cooling and local decay of current density necessitated by the increased E_T required to maintain the current with increased electrostatic trapping. At a second lower critical temperature, a second instability occurs involving deceleration towards the original low $\overline{V}_\theta$ equilibrium. This relaxation oscillation has been identified with the observed saw-tooth oscillations. In particular, the critical temperature T_{c1}, which is the value of T_{eo} satisfying

$$(T_{eo}/T_{io})^{\frac{1}{2}} \sigma_{\|} E_T = C(m_H/m_e)^{\frac{1}{2}} A_i^{\frac{1}{2}} n_{io} e \, v_{Tz} \quad , \qquad (5)$$

predicts accurately the observed values of T_{eo} for those experimental cases where all the parameters needed in Eq. (5) are known. Here m_H is the proton mass and $v_{Tz} \equiv (2T_{io}/m_z)^{\frac{1}{2}}$.

The parameter C is an approximate constant depending very weakly on such quantities as (T_{eo}/T_{io}), Z, Z_{eff}, and (m_z/m_i). For Ohmically heated discharges with $1 \leq T_e/T_i \leq 4$ and with carbon or oxygen impurity a good approximation for $Cm_H^{\frac{1}{2}}/m_e^{\frac{1}{2}}$ is 20. (The analysis leading to Eq. (5) takes the electrons and hydrogen ions in the plateau regime and the impurity ions in the Pfirsch-Schluter regime, this being valid for most Ohmically heated discharges.) Hence, for such discharges using the approximation $\sigma_{\|0}=2.5n_e e^2 Z_{eff}/m_e \nu_e T_e^{3/2}$, where the factor 2.5 is an approximate mean for the range $1<Z_{eff}<16$, Eq. (5) can be written in the form

$$T_{eo}^2 = (20e\sigma_{\|0}^{-1}) A_i^{\frac{1}{2}} n_{io} T_{io}^{\frac{1}{2}} v_{TH} Z_{eff} E_T^{-1} A_z^{-\frac{1}{2}} . \qquad (6)$$

To compare Eq. (6) with the empirical scaling law a further relationship is needed and attention is turned to the energy balance for the positive ions. There is recent experimental evidence[4] that substantially more energy is transferred from the electrons to the ions than given by the collisional term proportional to (T_e-T_i). The case is made here that there are in fact a variety of processes by which Ohmic heat is transferred or diverted from the electrons to the ions and that, although the fraction of Ohmic power involved in each case is modest, their sum involves a substantial fraction. These processes are listed in Table I and where available, the second column gives the formula for the rate of energy transfer per unit volume. The third column gives an approximate range for the amount of power involved as a percentage of the total Ohmic power (EJ) for impure Ohmically heated discharges under saw-tooth conditions. The first process is the well known collisional energy transfer term. The second process is the frictional heating of the hydrogen ions due to their bootstrap current relative to the impurity ions, this current being driven predominantly by the radial ion heat flux q_i. The formula for Q_Ω was first obtained by Hinton and Moore[5]; the formula for J_{iz} comes form a universal Ohm's law which has been derived for the ion current[6]. The third process is neoclassical energy transfer and is treated in detail in reference 7. The part of the formula proportional to Γ_{NC} , the neoclassical diffusion, was first given by Rosenbluth et al.,[8] the part proportional to Γ_E, the electrostatic component of diffusion, by Hazeltine and Ware[9]. The percentage power indicated assumes that the neoclassical pinch effect is dominant in $(\Gamma_{NC} + \Gamma_E)$ with the inward diffusion being balanced by a positive anomolous component of diffusion as suggested by Tokamak results[10].

When instabilities are present in the plasma, all modes which are driven by the electron temperature or density gradients, by the electron current or by the toroidal current gradient and which experience ion Landau damping or other ion viscous damping will involve transfer of Ohmic power from the electrons to the ions. Process 4 is the energy transfer due to electrostatic drift waves[7], with the percentage power assuming $\sum_k \Gamma_k \sim -(\Gamma_{NC}+\Gamma_E)$. Saw-tooth oscillations involve energy transfer by the same process as in item 3, but during the disruption phase the rate is much larger, firstly because $-(\Gamma_{NC} + \Gamma_E)$ is larger due to both increased electron trapping and locally larger E_T and secondly, because $[\bar{V}_{\|} - (E_r/B_\theta)] \simeq (B/B_\theta)\bar{V}_\theta$ is much larger[3]. No formula is available for the mean energy transfer rate involved in this process or for the case of the Mirnov oscillations.

From Table I it is concluded that a substantial fraction ($\gtrsim 50\%$) of the Ohmic power is transferred to the positive ions. Because of the largeness of this fraction the total heat transfer will be assumed proportional to EJ and because of the limited accuracy of the scaling laws [see the factor exp(±1.6) in Eq.(1)] the proportionality factor will be taken as unity. Thus the ion heat flux q_i can be equated to the Poynting vector and taking the plateau formula for q_i with the approximations $n_i \triangleq n_{io}$, $T_i \simeq T_{io}$, $T_i' \simeq T_{io}/a$, $r=a$, $B_\theta \simeq 2I/ac$ gives

$$3\pi^{\frac{1}{2}} c^3 n_{io} T_{io}^2 A_i^{\frac{1}{2}} m_H v_{TH}\, a/4e^2 B_T I R^2 = cE_T B_\theta/4\pi \tag{7}$$

Eliminating the factor $n_{io} v_{TH} A_i^{\frac{1}{2}}/E_T$ between Eq. (6) and (7) and converting from gaussian units to mks and keV yields

$$T_{eo}(\mathrm{keV}) = 7.5\times10^{-7} a^{-5/4} RI\, Z_{eff}^{\frac{1}{2}} \left(B_T^{\frac{1}{2}} a^{\frac{1}{4}} T_{io}^{-3/4}\right) A_z^{-\frac{1}{4}} \tag{8}$$

The factor $(B_T^{1/2}a^{1/4}/T_{io}^{3/4})$, which satisfies the Connor and Taylor constraints[11], is an approximate constantfor Ohmically heated discharges. Thus in the sets of experimental data listed by Pfeiffer and Waltz[2], for the 69 cases which list T_{io} the average value of $(B_T^{1/2}a^{1/4}/T_{io}^{3/4})$ is 1.97, the mean-square deviation is 0.5 and the maximum and minimum values are 3.9 and 1.1. The reason for this approximate constant value follows from Eq. (7) which together with the relationship $\sigma_{\parallel 0}T_{eo}^{3/2}Z_{eff}^{-1}E_T \simeq J_o = 2B_T/R$ gives

$$\frac{B_T^{1/2}a^{1/4}}{T_{io}^{3/4}} \sim \left(\frac{a}{R}\right)^{1/4}\left(\frac{n_{io}T_{eo}}{B_{\theta a}^{2}}\right)^{1/4}\left(\frac{T_{eo}}{T_{io}}\right)^{1/8}\frac{A_i^{1/8}}{Z_{eff}^{1/4}} \tag{9}$$

showing very weak dependence on the various parameters indicated. Since for the typical impurites oxygen and carbon $A_z^{-1/4} \simeq 0.5$, the quantity $B_T^{1/2}a^{1/4}/T_{io}^{3/4}\ A_z^{1/4}$ has the approximate value of unity and Eq. (8) reduces to

$$T_{eo} \simeq e^{-14.1}\ a^{-5/4}\ R\ I\ Z_{eff}^{1/2} n_e^{\circ}\ B_T^{\circ}\ A_i^{\circ}\ , \tag{10}$$

the independence of T_{eo} on n_e, B_T and A_i being included for completeness.

Comparing Eq. (10) with the empirical scaling obtained by Pfeiffer and Waltz, Eq. (1), shows that the exponents of five of the seven parameters fall within the statistical range of Eq. (1) with the other two exponents (for I and Z_{eff}) being only 10% too large. The difference in the constant is accounted for almost completely by the small difference in the exponents for I which, in the mks and keV units, is the only parameter introducing a large number in the equation. The experimental range for I was 4 to 590 kA, which in amperes is exp(10.8±2.5) and the range for $I^{0.19}$ is exp(2.05±0.47).

Turning to the question of plasmas with low Z_{eff} such as in Alcator, the theory leading to Eq. (6) is inapplicable not only because $n_z Z^2 \gg n_i$ is invalid but since the radial limit r_s of the saw-tooth region in such discharges is small, (the poloidal Larmor radius at r_s is approximately equal to r_s) normal neo-classical theory is inapplicable in general[3]. Qualitative mechanisms have been suggested for the poloidal rotation instability under these conditions[3] but no quantitative predictions are available. It is somewhat surprising that the Alcator results satisfy the same scaling laws since Eq. (6) predicts values of ${T_{eo}}^2$ an order of magnitude too large. This discrepency is cancelled by a corresponding factor in Eq. (7) where either $n_{io} v_{TH} A_i / E_T$ or ${T_{io}}^{5/2}$ is predicted an order of magnitude too small. The Alcator results taken by Pfeiffer and Waltz are for comparatively low currents in deuterium where the energetic ions which make the dominant contributions to $\overline{(P_{i\parallel} - P_{i\perp})\sin\theta}$ and q_i have poloidal Larmor radii which are large fractions of the minor radius. Substantial deficits of these particles are expected and this could help explain both discrepancies.

In conclusion, it has been shown for impure plasmas that the predicted critical electron temperature for loss of poloidal equilibrium in the central saw-tooth region, together with the assumption that the net transfer of energy from electrons to ions is proportional to the Ohmic power input, leads to the scaling law for T_{eo} given by Eq. (8) and approximately by Eq. (10). For all seven parameters a, R, I, Z_{eff}, n_e, B_T, A_i, there is very close agreement with the empirical scaling law found by Pfeiffer and Waltz. As shown at the beginning of this letter, the scaling law for T_{eo} is all that is needed to deduce a good approximation for the empirical scaling law for the containment time τ_{Ee}. Hence the theory presented here explains the observed parametric dependence of τ_{Ee} and, in particular, its proportionality to $\langle n_e \rangle$.

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TABLE I

PROCESSES TRANSFERRING OHMIC POWER FROM ELECTRONS TO IONS.

PROCESS	FORMULA	PERCENTAGE OF EI
1. Collisional Transfer	$Q_{ie}=\sum_j 3n_e m_e \nu_{ej}(T_e-T_i)/m_j$	10-25%
2. Direct Ion Ohmic Heating	$Q_\Omega=J_{iz}E_T[1-(1/Z_{eff})]$, where $J_{iz}=(\sigma_\parallel)_i(2q_iB_\theta/5p_i)$	10-20%
3. Neoclassical Heat Transfer	$Q_{NC}=-(\Gamma_{NC}+\Gamma_E)[\bar{V}_\parallel-(E_r/B_\theta)]eB_\theta$	10-20%
4. Drift Waves	$Q_{DW}=\sum_k \Gamma_k\ \omega_k e\ B/k_\perp$	5-15%
5. Saw-tooth Disruptions	—	?
6. Mirnov Oscillations	—	?