Interchange Trigger for Substorms in a Nonlinear Dynamics Model

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Abstract

Here we describe the generalization of the Lorenz model of Rayleigh–Benard convection to the finite pressure plasma interchange dynamics. We include the usual three ODE's of the Lorenz model and two new modes describing the coupling of the plasma convection to the shear Alfvén waves in the system. Thus, we have a $d = 5$ phase space with an attractor or chaotic attractor depending on the system parameters. The system describes the creation of field line currents driven by the onset of the convective interchange which we interpret as the symmetry breaking in the ambient region 1 current system to form the initial phase of the substorm current wedge.

I. INTRODUCTION

The flux of momentum from the solar wind incident on the planetary magnetic dipole produces a stressed magnetotail with a trapped high pressure plasma confined in the night-side plasma sheet. During periods of enhanced solar wind dynamic pressure and enhanced erosion of dayside magnetic flux from an oppositely directed IMF magnetic field, the stress in the nightside magnetosphere exceeds the threshold for the onset of large scale (MHD-like) convection and magnetic reconnection. From general arguments [*Chang*, 1992] it is thought that the nonlinear dynamics of the highly stressed magnetotail system can be described with a few degrees of freedom. For example, the six dimensional WINDMI model [*Horton and Doxas*, 1996, 1998a] provides a global solar wind driven–ionospheric damped model capable of explaining the complex, correlated changes in six energy components resolved between the magnetotail and the ionosphere. The global correlation of the magnetospheric and ionospheric events during magnetic substorms gives strong evidence that this complex dynamical system evolves with a large scale coherence. Thus, we continue our development of low dimensional models (LDM) by deriving here the obvious generalization of the 3-dimensional Lorenz model [*Lorenz*, 1963] of a neutral fluid to $d = 5$ magnetohydrodynamic model of convection in the reduced MHD equations. The two additional degrees of freedom (ψ_1, ψ_2) describe the magnetic fluxes generated by the frozen in magnetic flux constraint of $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$ during the generation of the plasma convection. The resulting low order dynamical model appears to share most of the bifurcation structure leading from steady convection to chaos of the Lorenz dynamics on the three dimensional submanifold.

We describe the quasi–two–dimensional dynamics in the x–y coordinates in the equatorial plane ($z = 0$) in nightside magnetosphere. Stability analysis shows that the transition region where the normalized plasma pressure $\beta = 2\mu_0 p/B_n^2 \sim 1$ is the first region to go unstable. Locally the unstable dispersion relation is

$$\omega^2 = -\frac{k_y^2}{k_\perp^2}\frac{T_i \kappa_c}{m_i L_p} + \frac{k_\parallel^2 B_n^2}{\mu_0 \rho_m}$$

where $\kappa_c = \max(\widehat{\mathbf{b}} \cdot \nabla \widehat{\mathbf{b}}) = 1/R_c \sim B_x'/B_z = \mu_0 j_y/B_z$, and $L_p^{-1} = d_x \ell n p(x, 0, 0)$. Here T_i is the CPS ion temperature $\sim 4\,\mathrm{kev}$, $k_{\|} \sim 1/L_{\|}$ is the effective length of the ballooning mode, and ρ_m the mass density.

For the full dynamical equations we start with the reduced MHD equations for the electrostatic potential $\phi(x,y,t)$, the parallel vector potential $\psi(x,y,t)$ and the plasma pressure $p(x,y,t)$. The dynamical equations are as follows. The divergence of the current yields:

$$\frac{\rho_0}{B_n^2}\nabla^2\frac{\partial\phi}{\partial t} = \frac{\kappa}{B_n}\frac{\partial p}{\partial y} + \nabla_{\|}j_z + \frac{\mu\rho_0}{B_n^2}\nabla^4\phi. \tag{1}$$

The resistive Ohm's law yields:

$$\frac{\partial\psi}{\partial t} + \mathbf{v}\cdot\nabla\psi = \nabla_{\|}\phi + \eta\nabla^2\psi - E_o. \tag{2}$$

$$\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla p + \Gamma p\nabla\cdot\mathbf{v} = \chi\nabla^2 p. \tag{3}$$

Here the convective derivative is given by

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla = \frac{\partial}{\partial t} + \frac{1}{B_n}\left(\frac{\partial\phi}{\partial x}\frac{\partial}{\partial y} - \frac{\partial\phi}{\partial y}\frac{\partial}{\partial x}\right) \tag{4}$$

and the dissipation from viscosity μ, resistivity η and thermal conductivity χ is added to represent the transfer of energy to unresolved scales in the collisionless system. Equation (1) is the $\widehat{\mathbf{z}}$–component of the vorticity produced by $\mathbf{E}\times\mathbf{B}$ convection in the equatorial plane $(z=0)$ at which $\mathbf{B} = B_n\widehat{\mathbf{z}}$ and $\boldsymbol{\kappa} = (\mathbf{b}\cdot\nabla\mathbf{b}) = \widehat{\mathbf{x}}/R_c$.

When the growth of the magnetotail stretching from the increase of j_y/B_n becomes sufficiently large, the interchange destabilizing term exceeds the Alfvén wave term for $\beta > NL_pR_c/L_{\|}^2$ as given by *Hurricane* [1997] and MHD convection sets in. The deep tail where the plasma pressure $\beta \sim B_{xo}^2/B_n^2 > 100$ the compressibility of the plasma stabilizes the local interchange motions.

We will show that the pressure gradient driven convection creates a sheared $B_y(x,t)$ field by convection $\mathbf{v}$ of the perturbed flux ψ_1. Thus, it is useful to start the calculation with a small constant B_y field so that the gradient along the magnetic field is

$$\nabla_{\|} = \frac{B_y}{B_n}\frac{\partial}{\partial y} + \frac{\widehat{\mathbf{z}}\times\nabla\psi\cdot\nabla}{B_n} \tag{5}$$

where $b_y = B_y/B_n$ is taken as small. The B_y field may be a remnant from the IMF field that has soaked into the magnetosphere. It can be an important "seed" field in the process considered here.

In the equilibrium state $p = p_0(x) = p_0(1 + x/L_p)$, $\eta\nabla^2\psi = E_0$ and $\mathbf{v} = 0$. We now consider the projection of the full dynamics of Eqs. (1)–(3) onto the low–order description of the fields. The fields are chosen to describe the linear instability and the dominant nonlinear interactions on the large scales and the scale of $2k_x$ as in the classical Lorenz model.

Our procedure follows the Lorenz reduction of the Rayleigh–Benard convection problem and includes the electromagnetic dynamics important for the plasma substorm.

A. Low order representation of fields

The low order representation in the midnight sector relevant for the onset of current diversion and convection is given by

$$\phi = \phi_1(t)\sin(k_x x)\sin(k_y y) \tag{6}$$

$$\psi = \psi_1(t)\sin(k_x x)\cos(k_y y) + \psi_2(t)\sin(2k_x x) \tag{7}$$

$$p = p_1(t)\sin(k_x x)\cos(k_y y) + p_2(t)\sin(2k_x x). \tag{8}$$

The ϕ_1, p_1, p_2 amplitudes are the x, y, z variables of the Lorenz model. The magnetic disturbances are given by the flux amplitudes ψ_1 and ψ_2. The (linear) Alfvén wave coupled to the convection is given by ψ_1 and the self-generated sheared B_y–field by ψ_2.

Now we show some of the nonlinear calculations:

$$\mathbf{v}\cdot\nabla\psi_1 = \phi_1\psi_1 k_x k_y[-\cos\theta_x\sin\theta_y\sin\theta_x\sin\theta_y - \tag{9}$$

$$\sin\theta_x\cos\theta_y\cos\theta_x\cos\theta_y] = \frac{1}{2}\,k_x k_y\phi_1\psi_1\sin(2\theta_x)$$

where $\theta_x = k_x x$ and $\theta_y = k_y y$. The same calculation applies to the convection of the pressure fluctuation:

$$\mathbf{v}\cdot\nabla p_1 = -\frac{1}{2}\,k_x k_y \phi_1 p_1 \sin(2\theta_x). \tag{10}$$

Now the convection of the ψ_2 and p_2 terms give

$$\mathbf{v}\cdot\nabla\psi_2 = -2k_x k_y \phi_1\psi_2 \sin\theta_x \cos\theta_y \cos(2\theta_x) \tag{11}$$

$$= k_x k_y \phi_1 \psi_2 \cos\theta_y \left[\sin\theta_x - \sin 3\theta_x\right]$$

and likewise for $\mathbf{v}\cdot\nabla p_2$.

Finally, we evaluate the term

$$\widehat{\mathbf{z}}\cdot\nabla\psi\times\nabla(\nabla^2\psi)$$

$$= \widehat{\mathbf{z}}\cdot\nabla\psi_1\times\nabla\nabla^2\psi_2 + \widehat{\mathbf{z}}\cdot\nabla\psi_2\times\nabla\nabla^2\psi_1$$

$$= \psi_1\psi_2\left(k_y^2 - 3k_x^2\right)\sin\theta_y\left[\sin\theta_x - \sin 3\theta_x\right] \tag{12}$$

Thus, Eqns. (11) and (12) produce new terms $\sin\theta_y \sin 3\theta_x$ outside of the representation in Eqs. (6)–(8). The problems associated with truncation not including these terms are discussed in *Thiffeault and Horton* [1997] for the Rayleigh–Benard problem. We return to this issue later.

Substituting these fields and nonlinear terms in Eqs. (1)–(3) and equating the linearly independent variations yields the following five ordinary differential equations:

$$\begin{aligned}\frac{\rho_0 k_\perp^2}{B_n^2}\,\frac{d\phi_1}{dt} &= \frac{k_y\kappa}{B_n}\,p_1 - \frac{k_y B_y}{B_n}\,\frac{k_\perp^2\psi_1}{\mu_0}\\ &-\frac{k_x k_y}{\mu_0 B_n}\,(4k_x^2 - k_\perp^2)\psi_1\psi_2 - \frac{\rho_0\mu}{B_n^2}\,k_\perp^4\phi_1\end{aligned} \tag{13}$$

$$\frac{d\psi_1}{dt} = \frac{k_y B_y}{B_n}\,\phi_1 - \frac{k_x k_y}{B_n}\,\phi_1\psi_2 - \eta k_\perp^2\psi_1 \tag{14}$$

$$\frac{d\psi_2}{dt} = \frac{k_x k_y}{2B_n}\,\phi_1\psi_1 - 4k_x^2\eta\psi_2 \tag{15}$$

$$\frac{dp_1}{dt} = \frac{k_y p_0'}{B_n}\,\phi_1 - \frac{k_x k_y}{B_n}\,\phi_1 p_2 - \chi k_\perp^2 p_1 \tag{16}$$

$$\frac{dp_2}{dt} = \frac{k_x k_y}{2B_n}\,\phi_1 p_1 - 4\chi k_x^2 p_2 \tag{17}$$

II. ENERGIES AND ENERGY CONSERVATION

The full field equations have the following energy components. The kinetic energy $K(t)$ in the $\mathbf{E} \times \mathbf{B}$ convection is

$$K(t) = \frac{1}{2} \frac{\rho_0}{B_n^2} \int (\nabla \phi)^2 dx dy. \tag{18}$$

The magnetic energy $W_{B_{R1}}(t)$ associated with the field aligned currents is given by

$$W_{B_{R1}} = \frac{1}{2\mu_0} \int (\nabla \psi)^2 dx dy. \tag{19}$$

It is important to note that the magnetic energy $W_{B_{R1}}$ in Eq. (19) is that associated with the field aligned currents $j_{||}$ and the corresponding parallel vector potential $\nabla^2 \psi = \mu_o j_{||}$. The much larger magnetic energy associated with the geomagnetic tail lobe fields, which is the principal energy reservoir in the global WINDMI model (*Horton et al.*, 1998b), is derived fom a dawn–to–dusk vector potential $A_y(x,z)$ with $\nabla^2 A_y = -\mu_o j_y$.

The potential energy $U(t)$ that drives the convection is from the expansion of the flux tubes filled with high pressure plasma. The local measure of this potential energy is

$$U = \int \frac{x}{R_c} p \frac{dx dy}{L_x L_y} \tag{20}$$

where $1/R_c = \kappa$ is the curvature of the magnetic field lines. The rate of change of U is given by using Eq. (3) and the condition $\int_{\partial\Omega} d\mathbf{a} \cdot \mathbf{v} p = 0$, which annihilates the compression term, so that

$$\frac{dU}{dt} = \kappa (q + q_0) \tag{21}$$

where $q_0 = -\chi_0 \frac{dp}{dx} \to 0$. In the LDM we solve below there are special cases where $U(t \to \infty) \to 0$ releasing all the stored thermal energy. In general, there are only incomplete, episodic releases of the thermal energy U.

The conservation of energy between $U(t)$ and $K(t)$ is the same as in the Rayleigh–Benard–Taylor problem. The analysis is somewhat complicated, but is developed in detail in *Thiffeault and Horton* [1997]. Thus, we look here at the transfer between W_B and K in more detail.

From Eqs. (1)–(2) it is straightforward to show that

$$\frac{dW_B}{dt} = T_1 + T_2 - \eta \int (\nabla^2\psi)^2 dx dy \tag{22}$$

$$\frac{dK}{dt} = -\kappa q - T_1 - T_2 - \frac{\mu\rho_0}{B_n^2} \int (\nabla^2\phi)^2 dx dy \tag{23}$$

and

$$\frac{dU}{dt} = \kappa q - \kappa\chi \int \frac{dp}{dx}\, dx dy \tag{24}$$

where q is the thermal flux given by $q = \langle v_x p \rangle$.

The two energy transfer terms T_1 and T_2 give the power transfer from

$$j_z\, E_{||}^{\mathrm{es}} = -j_z \left(\frac{B_y}{B_n}\right)\left(\frac{\partial\phi}{\partial y}\right)$$

and

$$j_z(\mathbf{v}\cdot\nabla\psi) = j_z\, E_{||}^{\mathrm{conv}}$$

where $E_{||}^{\mathrm{es}}$ is the electrostatic parallel electric field and $E_{||}^{\mathrm{conv}}$ is the parallel inductive electric field from convection of the magnetic flux, ψ. The MHD constraint requiring that the total parallel electric field vanish relates to the power transfer since

$$E_{||} = -\frac{B_y}{B_n}\frac{\partial\phi}{\partial y} + \frac{\partial\psi}{\partial t} + \mathbf{v}\cdot\boldsymbol{\nabla}\psi = 0 \tag{25}$$

yields, after multiplying by $j_{||} = \frac{1}{\mu_0}\nabla^2\psi$ and integrating over the volume Ω,

$$\frac{d}{dt}\frac{1}{2\mu_0}\int_\Omega (\nabla\psi)^2 d\Omega + \int_{\partial\Omega} \frac{1}{\mu_0}\frac{\partial\psi}{\partial t}\nabla\psi\cdot d\mathbf{a} = T_1 + T_2 \tag{26}$$

where electrostatic parallel field due to $B_y \neq B_n \neq 0$ gives

$$T_1 = -\int_\Omega \frac{B_y}{B}\frac{\partial\phi}{\partial y}\, j_z d\Omega = \int \phi\, \frac{B_y}{B}\frac{\partial j_z}{\partial y}\, d\Omega \tag{27}$$

$$= \frac{B_y}{B}\frac{k_y k_\perp^2}{\mu_0}\,\phi_1\psi_1$$

and the convection of the ψ–flux gives

$$T_2 = \int \mathbf{v} \cdot \nabla \psi j_z dx dy = \frac{k_x k_y k_\perp^2}{B} \psi_1 \psi_2 \phi_1. \tag{28}$$

Thus, T_1 and T_2 are two components of the power flow between the region 1 currents and the $\mathbf{E} \times \mathbf{B}$ kinetic energy in the plasma flow driven by the pressure gradient–interchange. In the limit of $b_y = 0$ the MHD constraint is $E_{||} = \partial_t \psi + \mathbf{v} \cdot \nabla \psi = 0$ so that ψ is convected by $\mathbf{v} = \hat{\mathbf{z}} \times \nabla \phi / B_n$. The MHD statement that $\int E_{||} j_{||} d^3x = 0$ becomes $dW_B/dt + \int \mathbf{S} \cdot d\mathbf{a} = T_2$ where T_2 is driven by ϕ_1 and $\mathbf{S}$ is the Poynting flux.

Thus, in the absence of forcing and damping the low order description conserves the total energy

$$\frac{d}{dt}(K + W_B + U) = 0. \tag{29}$$

For the low order representation in Eqs. (6)–(8) these energies reduce to

$$K = \frac{\rho_0}{8B_n^2} k_\perp^2 \phi_1^2(t) \tag{30}$$

$$W_B = \frac{1}{8\mu_0} k_\perp^2 \psi_1^2 + \frac{1}{\mu_0} k_x^2 \psi_2^2 \tag{31}$$

and

$$U = -\frac{\kappa}{2} \frac{p_2}{k_x}. \tag{32}$$

Thus, we conclude that the low dimensional model (LDM) contains the essential physics of the full pde systems with regard to energy transfer.

III. TRIGGER MECHANISM

Consider the dynamics of a pressure fluctuation δp from the convection in the equatorial plane where $p = p_0(x) + \delta p(x, y, t)$. The dynamics is given by

$$\frac{\partial}{\partial t} \delta p + \mathbf{v} \cdot \nabla \delta p - \frac{1}{B_n} \frac{\partial \phi}{\partial y} \frac{\partial p_o}{\partial x} + \Gamma p_0 \left(\nabla_\perp \cdot \mathbf{v}_\perp + \nabla_{||} v_{||} \right) = 0. \tag{33}$$

There are two important cases: (i) fast motions in which the plasma is compressed so that $\Gamma p_0 \nabla \cdot \mathbf{v} \neq 0$ and Γ enters the stability equation and (ii) slow motions that are to a high degree incompressible $\nabla_{||} v_{||} \cong -\nabla_{\perp} \cdot \mathbf{v}_{\perp}$ and thus the motion is *independent* of Γ. The stability limit obtained by Hurricane, that is $\beta > N\, L_{||}^2 / L_p R_c$ is independent of Γ, and thus applies to slow motions where kinetic effects are important. Since he enforces the constraint of incompressible displacement $\boldsymbol{\xi}$ on the trial functions to obtain the lowest possible value of the MHD δW.

Now we show that for fast motions the parallel $v_{||}$ cannot cancel the cross–field compression and there is a stabilizing threshold proportional to the adiabatic gas constant Γ. We have also determined from collisionless kinetic theory the effective value of $\Gamma_{\mathbf{kin}}$. This fact follows from solving the linearized parallel flow equation

$$-i\omega\rho_0 \widetilde{v}_{||} = -ik_{||}\left(\delta p + \frac{1}{\mu_0}\, B_0 \delta B_{||}\right)$$

and estimating $\nabla_{||} v_{||} / \nabla_{\perp} \cdot \mathbf{v}_{\perp}$ using $\delta p/p \sim \delta n/n \sim \nabla \cdot \mathbf{v}_{\perp} / i\omega$. The analysis gives

$$\nabla_{||} v_{||} \sim \frac{k_{||}^2 p_0}{\omega^2 \rho_0}\, \nabla \cdot \mathbf{v}_{\perp} \tag{34}$$

showing that for $\omega^2 \gg k_{||}^2 v_{T_i}^2$ the parallel compression is too small to balance the cross–field compression. Thus, for fast modes (and especially small $k_{||}$ or flute modes) the compression

$$\boldsymbol{\nabla} \cdot \mathbf{v} = -2\boldsymbol{\kappa} \cdot \mathbf{v} = -\frac{2v_x}{R_c} \tag{35}$$

is not eliminated by the parallel mass flows. Only near marginal stability where $\omega^2 \ll k_{||}^2 v_i^2$ do the incompressible criteria apply.

We can find an important thermodynamic relation by multiplying Eq. (33) by δp and averaging over the region of the $\Delta\Omega$ of the disturbance. The averaging annihilates the convection $\mathbf{v} \cdot \nabla \delta p$ of the pressure and yields

$$\frac{\partial}{\partial t} \frac{\langle \delta p^2 \rangle}{2} + \left(\frac{\partial p_0}{\partial x} - \Gamma p_0 \kappa\right) q = -\chi_0 \Big\langle (\nabla \delta p)^2 \Big\rangle \tag{36}$$

where q is the heat flux

$$q = \langle v_x \delta p \rangle = -\frac{1}{B_n} \left\langle \frac{\partial \phi}{\partial y} \delta p \right\rangle . \tag{37}$$

The heat flux q is down the pressure gradient so that $q = -\chi(dp_0/dx)$ and the pressure fluctuations $\langle \delta p^2 \rangle$ grow. The pressure fluctuation power spectrum $\langle \delta p^2 \rangle = \int P(k)dk$ develops high k components for which the arbitrarily small χ_0 eventually absorbs the power transmitted through the $P(k)$ spectrum. In certain cases, with continuous driving of the system, the system reaches a steady state with the entropy production by turbulence

$$\dot{S}_a = -q \frac{\partial p_0}{\partial x} = \chi \left(\frac{\partial p_0}{\partial x} \right)^2 = \chi_0 \left\langle (\nabla \delta p)^2 \right\rangle \tag{38}$$

balanced by microscopic absorption of the shortest scale δp–fluctuations. Equation (38) applies when there is a turbulent steady state.

IV. PHYSICS OF LOCAL SUBSTORM CURRENT WEDGE TRIGGER

A localized ion pressure fluctuation in the region of $\beta = 2\mu_0 p_0 / B_z^2 \lesssim 1/\Gamma$ creates a vortex flow of current as shown in Fig. 1. In a uniform field the flow has $\nabla \cdot \delta \mathbf{j}_\perp = 0$ but due to the gradient of B_z the flow is slower on the Earthward side of the vortex and faster on the tailward side. The result is a charge accumulation proportional to

$$\partial_t \rho_q = -\nabla \cdot \delta \mathbf{j} = -\left(\frac{B'_x}{B_n^2} \right) \left(\frac{\partial \delta p}{\partial y} \right) .$$

The polarization of the plasma takes place with a return current $\mathbf{j}_p = (\rho_0/B_n^2)(d\mathbf{E}_\perp/dt)$ such that $\nabla \cdot \mathbf{j}_p = +(\rho_0/B_n^2) k_\perp^2 (d\phi_1/dt) = -(\kappa/B_n)\partial_y \delta p_1$ producing the exponential growth when the energy released from $\partial p_0/\partial x$ exceeds that required to compress the plasma. Beyond $x = -10\, R_E$ the compressional displacements on the fast interchange time scale $\tau_{\rm int} = (L_p R_c)^{1/2}/v_i$ are suppressed. Only the slower modes that have the complicated parallel structure with compensating parallel compression may be residually unstable.

A simple estimate of the dimensionless plasma pressure required for fast compressible interchange modes follows Eq. (36) where $\nabla_\perp \cdot \mathbf{v}_\perp = -2v_x/R_c$ is used to compute the cross–field compression. In the tail region the magnetic curvature $\kappa_{\max} = B'_x/B_n$ equilibrium pressure balance allow us to reduce

$$\frac{\partial p_0}{\partial x} - \Gamma p_0 \kappa = j_y B_n - \frac{\Gamma p_0}{B_n}\frac{\partial B_x}{\partial z} = j_y B_n \left(1 - \frac{\mu_0 p_0}{B_n^2}\Gamma\right) \tag{39}$$

using $\mu_0 j_y = \frac{\partial B_x}{\partial z} - \frac{\partial B_n}{\partial x} \simeq \frac{\partial B_x}{\partial z}$ in the region beyond $(5-10)R_E$. Thus, we obtain

$$\beta = \frac{2\mu_0 p_0}{B_n^2} < \frac{2}{\Gamma} \tag{40}$$

as a necessary condition for instability from interchange.

For the sufficient condition we must consider the Alfvén wave stabilization. The point is that the charge accumulated from the $k_y^2\,\kappa(\partial p_0/\partial x)$–drive is partially drained off by parallel currents $\mu_0 j_z = \nabla_\perp^2 \psi$ with $\nabla_{||} j_z = -\nabla \cdot \delta \mathbf{j}_{\rm int}$. This is the physics of both the linear and nonlinear terms in $\nabla_{||}\nabla_\perp^2\psi/\mu_0$ that give the T_1 and T_2 energy transfers. The stability condition is obtained with $(k_y B)^2 \to k_{||}^2 B^2$ from Eqs. (13), (14) and (16) as

$$k_\perp^2 \frac{k_{||}^2 B^2}{\mu_0} > \frac{k_y^2 \kappa}{\rho_0}\frac{\partial p}{\partial x} \tag{41}$$

which require $\beta > L_p R_c / L_{||}^2$ where $L_{||}$ is roughly the length of the magnetic field line to the ionosphere. Thus, the instability producing the onset of the seed of the substorm current wedge can only occur in a narrow range of plasma β that is satisfied near $X \simeq -10\,R_E$. In this region the onset will first appear in a small Δy sector corresponding to a ± 1 hr of MLT. However, the growth of the SCW current shown in the nonlinear model will cause an expansion of that same current loop.

The expansion in MLT of the current loop show in in Fig. 1 increases the mutual inductance of this loop with the crosstail current loop which gives a rapid nonlinear increase in the inductive electric field driving the SCW current loop. This produces the rapid increase in precipitating electrons and the formation of the northward bulge in the auroral ionosphere.

V. CONCLUSIONS

Motivated by the observation [*Baker and Pulkkinen*, 1991; *Pulkkinen et al.*, 1991] that substorm dynamics is a correlated series of events in the nightside magnetosphere that appear to initiate from electrodynamic activity at $X = -10 R_E$ to $-15 R_E$ in the near to midnight

MLT sector during periods of highly stretched magnetotail field lines (large j_y/B_n), we argue that low dimensional models derived from projections of the partial differential equations (1)–(3) on to suitable basis functions capture the essential physical processes of symmetry breaking, the unloading of the plasma pressure and the generation of the substorm current wedge.

Here we focus our attention on the local interchange instability as a candidate for the onset of the substorm dynamics at the end of the growth phase when the current sheet is well stressed. The global cycling of the geotail is best described by our earlier WINDMI model. Here we focus on the smaller time scales of minutes associated with the expansion phase.

We have derived a new $d = 5$ dimensional model that generalizes the $d_L = 3$ Lorenz model by taking into account the inductive electric field $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$ driven by the onset of convection from the interchange of flux tubes in maximum growth region. The convection is driven by the product of the field line curvature $\kappa = B_x'/B_n \simeq \mu_o j_y/B_n$ since $\partial B_x/\partial z \gg \partial B_z/\partial x$ and the Earthward pressure gradient $dp/dx = j_y B_n$. The convection generates $\mathbf{E}$ and the field aligned currents $j_{||}$ with the symmetry required to drive the substorm current wedge. In periods of order minutes the system goes into the nonlinear dynamics limited by the unloading of the local pressure gradient and the generation of Alfvén waves. Here we derive the model and trace the transfer of energy through the mode coupling terms in the truncated model. We do not attempt to explore the bifurcation sequences of the system leaving this for future studies. We note that the nonlinearities in the flow Eqns. (13) - (17) are volume conserving (due to their origin from the convection $\mathbf{v} \cdot \nabla$ and line bending $\delta\mathbf{B} \cdot \nabla$) so that there is a uniform volume contraction in the $d = 5$ phase space. Thus, in the strongly unstable system we may expect to find a $4 + \epsilon$ chaotic attractor in analogy with the 2.06 dimensional Lorenz attractor. In the dissipation limit we have briefly examined the large amplitude pump depletion oscillations of the model and verified that these nonlinear oscillations lie on a $d = 4$ energy surface in the phase space.

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FIGURE CAPTION

FIG. 1. Geometry of the unstable growth of the localized pressure fluctuation δp due to the interchange instability. The magnetic field is dragged with the convection due to the frozen in condition, which in turn generates the symmetry breaking of the region 1 currents to form the substorm current wedge.