Laboratory Laser Acceleration and High Energy Astrophysics: γ–ray Bursts and Cosmic Rays

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Abstract

Recent experimental progress in laser acceleration of charged particles (electrons) and its associated processes has shown that intense electromagnetic pulses can promptly accelerate charged particles to high energies and that their energy spectrum is quite hard. On the other hand some of the high energy astrophysical phenomena such as extremely high energy cosmic rays and energetic components of γ–ray bursts cry for new physical mechanisms for promptly accelerating particles to high energies. We suggest that the basic physics involved in laser acceleration experiments sheds light on some of the underlying mechanisms and their energy spectral characteristics of the promptly accelerated particles in these high energy astrophysical phenomena.

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1 Introduction

The advent of the laser technology for ultrashort pulses and the discovery of high fluence solid–state laser media have opened a new way to do science in a (relatively) compact laboratory setting (Perry and Mourou, 1994). For example, an intense short laser pulse is capable of driving a wake of waves in a plasma (wakefield) that is suitable for accelerating particles to high energies (Tajima and Dawson, 1979). The recent progress in short pulse laser technology allows short enough ($\lesssim 100$ fs) and intense enough ($\gtrsim 10^{18}$ W/cm^2) laser irradiation on a gas (or plasma) to create such wakefields appropriate for high energy acceleration (see. e.g. Chattopadhyay et $al.$, 1996). Since 1995 a series of laser acceleration experiments have been conducted and these experiments have been successful in creating bursts of accelerated electrons (Nakajima et $al.$, 1995). Although the main purpose of many experiments has been to investigate the laser wakefield's applicability to a future high energy accelerator (or its component), some of their observations that emerged are quite useful in understanding acceleration processes in astrophysical plasmas. High energy astrophysical phenomena such as extremely high energy cosmic rays (EHECR) and high energy components of γ–ray bursts (GRB) invariably involve some processes of acceleration of particles to high energies.

The most oft–resorted mechanism of high energy acceleration in astrophysics is the Fermi stochastic acceleration (Fermi, 1954). This and its associated shock acceleration (Blandford and Ostriker, 1977; Blandford and Eichler, 1981; Bell, 1978) have been believed to explain the well–known power–law spectrum of cosmic rays, perhaps up to 10^{15} eV. Beyond this energy the available magnetic field in the shock or galaxies may not be enough to account for such energies. Nonetheless, the Fermi mechanism has been spectacularly successful in cosmic ray acceleration. Seldom has any other credible alternative general mechanism emerged in astrophysical acceleration. The mechanism of pulsar field acceleration suggested by Gunn

and Ostriker (1969) is a notable exception.

However, several observational factors invite alternative mechanisms, as will be explained below. In order to clarify and explore such mechanisms, laboratory investigations of laser acceleration seem to provide important clues.

2   Laboratory Laser Acceleration

An intense laser pulse may be characterized by two parameters; one is the normalized vector potential of the electromagnetic field a_0

$$a_0 \equiv \frac{eE_0}{m\omega_0 c} \tag{1}$$

and the other is the ratio of the laser frequency ω_0 to the plasma frequency ω_p,

$$\gamma_0 = \frac{\omega_0}{\omega_p}. \tag{2}$$

For an underdense plasma, $\gamma_0 > 1$, while for an overdense plasma, $\gamma_0 < 1$. Since the laser propagation does not happen for $\gamma_0 < 1$ but for rare exceptions (Ashour–Abdalla et $al.$, 1981), we concentrate mostly on $\gamma_0 > 1$ cases in what follows.

In a regime of moderate laser intensity where a_0 is not tiny but still $a_0 \lesssim 1$, the excitation of laser wakefield behind the laser pulse is expected, where the amplitude of the wake electric field E_w is given as

$$E_w \approx E_0 a_0^2, \tag{3}$$

where $E_0 \equiv m\omega_p c/e$. This wakefield is a (primarily) longitudinal field with high phase velocity (Tajima and Dawson, 1979)

$$v_{ph} = c\sqrt{1 - \frac{1}{\gamma_0^2}}. \tag{4}$$

If the pulse length ℓ is sufficiently short ($\ell \lesssim \pi c/\omega_p$) such an excitation is expected to be efficient. However, even if the pulse length does not satisfy this condition, the Raman instability

can induce self–organized laser modulations that "automatically" satisfy the above condition (self–modulated laser wakefield excitation) and a similar wakefield excitation occurs. Further, when a_0 is sufficiently close to order unity, the excited wakefield has a sufficiently large amplitude, whose trapping width $p_{tr} \approx (m\,eE_w/\omega_p)$. As is easily seen, if the wakefield amplitude approaches $E_w \sim E_0$, the trapping width of electron momentum in such a wakefield becomes of the order of mc, i.e. relativistic. When this occurs, the wakefield with fast phase velocity can now trap electrons in the bulk plasma (i.e. originally stationary one) and begins to accelerate them to high energies. This is exactly what happened in some of the recent laser acceleration experiments (Joshi *et al.*, 1981; Modena *et al.*, 1995; Cowan *et al.*, 1997). Often in these experiments the energy spectrum of accelerated electrons is observed to have a power–law with a rather hard exponent. Figure 1 shows an example of the energy spectrum of electrons in laser acceleration. In this Nakajima *et al.* experiment (1995) the power exponent is approximately -1.8. Figure 2 shows the case of Cowan *et al.* (1997) yielding the hardest exponent of electron spectrum of -1.45, while the exponent ranges from -3.5 to -1.45. The experiment by Modena *et al.* (1996), on the other hand, shows the exponent of $-0.25 \sim -0.3$.

In a relativistic regime of strong laser $(a_0 > 1)$ the wakefield amplitude E_w saturates around E_0. This is because even a very strong ponderomotive force of laser can at best evacuate the entire electron population and thus the amount of available space charge of electrons is depleted, leading to the saturation of E_w. In addition, at this intensity of wakefield $E_w = E_0$, the wave breaking of the wakefield in two (or three) dimensions may limit the increase of E_w. In $a_0 > 1$ regime, however, the electromagnetic fields of the laser by themselves can begin to pick up electrons directly (Tajima, 1985). This is because electrons quickly acquire transverse momenta in the laser electric field, which in turn kick in the magnetic acceleration, which is pointed in the direction of laser propagation. This acceleration takes place within the laser pulse (not behind the pulse as the latter is the case

with the wakefield acceleration). This mechanism is called the snow–plow acceleration and the peak energy of electrons is proportional to a_0^2 and again a broad energy spectrum is expected (Tajima, 1985).

3   High Energy Astrophysical γ–ray Spectra

Many phenomena of high energy astrophysics exhibit γ–ray emission whose energy spectrum obeys a power–law. These include the events of γ–rays from the Crab nebulae (Sinitsyna, 1996). The most enigmatic of high energy astrophysics is the events of γ–ray bursts. These challenge the astrophysicist in many ways. The recent observation that γ–ray bursts are at a cosmological distance, as evidenced in GRB970508, along with their rapid variability demands that the energy release mechanism should be prompt, rather than stochastic cumulative events such as the Fermi stochastic acceleration that would result in too much time delay and diffuseness in time history of γ–ray burst events. A standard theory (Rees, 1997) for a γ–ray burst suggests a collision of neutron stars and its subsequent explosive sequence of energy conversion, generally termed as a "fireball theory." This theory is capable of explaining: (i) the energy level of our event at a cosmological burst $\sim 10^{51}$ erg/sec; (ii) spatial scale(s) consistent with the rapid temporal variability; (iii) the power–law light curve suggesting a compact explosion; (iv) a rough ballpark for a typical γ–ray energy ($\sim 10^2$ KeV). It is yet to explain a wide variety of burst patterns and time scales (ranging from μ sec to msec). In addition, there are, at least, two difficulties: one is that the energy spectrum of γ–ray bursts is rather far from thermal spectrum and often shows a power–law pattern, which sometimes becomes harder or higher in energy in earlier times; the other is associated with a puzzle whether the majority of energy of neutron star collision may be carried off by neutrinos and thus not enough energy left in γ photons.

In order to overcome these difficulties associated with original Rees' model, Takahashi and Tajima (1998) introduced a model in which two new theoretical ingredients confront

these problems. These are (i) higher temperature of collided matter and (ii) an acceleration mechanism that can detach a portion of matter from the thermal energy. This first ingredient accounts for higher entropy in high temperature quark–gluon plasmas (QGP) upon collisions, which thus is capable of generating much more fraction of energy into photons rather than neutrinos. We will not extend our discussion on this here any further. We shall dwell on the second point a little bit.

We believe that in order to account for the nonthermal (and often power–law) component of γ–rays and their temporal structure, a thermal fireball of hot plasma is not sufficient. Nor is its associated process of stochastic shock acceleration in front of the exploding fireball; energetically too little and temporally not broadly varied. On the other hand, the bulk shock acceleration cannot detach particles from the thermal energy, as the phase velocity of sound waves cannot be far different from the thermal speed.

As we discussed in Sec. II, a large amplitude (intense) electromagnetic wave ($a_0 > 1$) may be able to pick up particles to high energies. If there is a way to provide sufficient energy or channel it, a sufficient amount of energy can detach from the thermal component and form a high energy component. We (Takahashi and Tajima, 1998) suggest the following scenario: the collision (or repeated reverberations of it) of magnetized neutron stars can impart its collisional energy not only into a kinetic energy of compressible (i.e. ordinary) shock waves, but also into a kinetic energy of transverse (or shear) Alfvén shock waves (i.e. a large amplitude shear Alfvén wave). This transverse Alfvén shock wave has the phase velocity of $v_A = B/\sqrt{4\pi M n}$, where B is the magnetic field of the merged (or merging) neutron stars, M the mass of the constituent particles (such as neutrons, protons, or electrons/positrons), and n the density of that matter. If the plasma at this particular radius (or position) is already dominated by electrons and positrons (rather than baryonic particles), two branches of transverse electromagnetic waves (one is the Alfvén wave and the other is the usual electromagnetic wave) have a narrow band gap of evanescence (Daniel and Tajima,

1998). Thus when the strength of magnetic field is rapidly decaying as the Alfvén shock propagates outward, the originally lower–branched Alfvén wave can mode–convert itself into the upper–branch electromagnetic wave. Since a substantial fraction of collision energy is in kinetic motion (at least in its early stage before the kinetic energy thermalizes itself) and a substantial amount of such kinetic motion may be in the transverse Alfvén motion if there is strong enough magnetic field (a standard neutron star case), we expect that a substantial amount of collision energy is poured into the Alfvén shocks, which may be converted into the electromagnetic wave energy. One of the major differences of the EM wave from the Alfvén wave is its phase velocity: the EM wave is slightly greater than (or nearly equal to) the speed of light c, while v_A is less (or sometimes much less) than c. Thus once the slow phase velocity of the Alfvén wave which is capable of trapping a large population of electrons and positrons gets increased to a high value of c after the mode–conversion into the EM wave, many of those trapped particles are now being accelerated to high energies (the relativistic factor $\gg 1$) through the snow–plow mechanism (primarily) and the wakefield mechanism (in addition). This acceleration mechanism can detach a substantial amount of energy and particles from the thermal bulk and eventually accounts for the high energy component of γ–rays bursts. Note that the usual shock wave cannot do the same. The hard spectrum of power–law often observed in γ–ray bursts, thus, seems a natural consequence of this acceleration process.

Figure 3 shows the spectra of γ–ray bursts as a function of time. This shows that (i) the γ–ray spectrum tends to be harder at early stages of the burst; (ii) the spectrum can be characterized by a power–law with exponent -1 in the early stage, eventually tending to a softer (near) thermal spectrum. The temporal trend of the γ–ray spectra as characterized in Fig. 3 seems to be consistent with the above detachment theory of high energy particles. Harder and higher energy components of particles that have been accelerated by the snow-plow of EM waves that were mode–converted from the transversal (magnetic) shock waves

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should account for the early (or precursory) hard spectrum of γ–ray bursts. On the other hand, the bulk of hot quark–gluon plasma particles explodes and expands until the plasma cools down to several 10^2KeV. At this point the plasma ceases to be opaque and begins to emit thermal γ–rays, accounting for later arriving of the thermal component of γ–rays. In Fig. 4 we show additional examples that show the (near) power–law spectra of γ–rays from a variety of high energy astrophysical objects or events including that of the Crab nebula.

4 Extremely High Energy Cosmic Rays

The energy spectrum of cosmic rays is well characterized by a power–law of exponent negative 2.75 to the energy from 1 GeV through 10^{15} eV. (See Fig. 5). [Note: This exponent is measured one, while if one corrects with the leaky box propagation factor $a \sim -0.6$, the exponent at the source is ~ -2.15.] Beyond this energy there appears a knee and another power–law with exponent negative $3.0 \sim 3.2$ to the energy through 10^{19} eV. Beyond this it seems the exponent once again increases to ~ -2, though error bars are large (Hayashida, 1994). More interestingly, the recent AGASA observations (Hayashida, 1994) and others (Bird *et al.*, 1994) have shown that among these extremely high energy cosmic ray events (beyond several 10^{19} eV) some events seem to come from strongly correlated spatio–temporal positions in the sky. This observation caused both heightened excitement and bewilderment (Cronin, 1996). This is because high energy cosmic rays have been considered to be a product of stochastic Fermi acceleration by multiple shocks (perhaps created by supernovae) according to many astrophysicists (Blandford *et al.*, 1977). If some of extremely high energy cosmic ray events come from a compact object, the acceleration mechanism must be a prompt one. An additional difficulty is that charged particles (such as protons) suffer pionization (GZK process; Greisen, 1966; Zatsepin and Kuzmin, 1966) encountering microwave photons of the cosmic background and thus are supposed not to reach the Earth if they are higher than 5×10^{19} eV and their source is beyond 30 MPc distance.

In order to overcome the second difficulty, a number of scenarios have been suggested. Here, however, we will not elaborate on them. Rather, we will focus on the first problem of prompt acceleration as this is a physical problem common with the γ–ray burst mechanism.

We suggest that the snowplow/wakefield acceleration may be responsible for the highest energy gamma rays in GRB's. We contend that the snowplow mechanism in GRB's accelerates charged particles to very high energy in the following scenario and may account for the highest energy cosmic rays as well.

Substantial circumstellar materials are likely to exist in the space 100 km–1000 km away from the merging neutron stars, where an efficient photonic acceleration could take place. In the later stage of a GRB, or at a large distance from the source, the photon intensity would be somewhat diluted. Nonetheless, even at a far distance space (> 1000 km) from the source, the intensity is so high that the induced electric field is near the high–end of the sub–Schwinger Field ($< 10^{16}$ V/cm) (Schwinger, 1951). Continual gamma ray radiation with intensity modulation of 0.1–100 msec would keep producing a train of high electric fields over a long pass–length (> 1000 km) of the GRB photons. GRB's show many irregular trains of intensity variations. A typical GRB intensity at the presumed source region, $R = 30$ km, is 10^{23} times larger than the peta watt laser intensity. This extreme photon intensity $[N(R)]$ was seen for GRB970508 at a measured distance of $D \gtrsim 1.49 \times 10^{28}$ cm (Metzger *et al.*, 1997; Djorgovski *et al.*, 1997) and the observed intensity on earth, $N = (7 \times 10^{50}$ erg/sec$)/\langle E \rangle$, where the average energy $\langle E \rangle$ was ~ 700 KeV after the red-shift correction

$$N(R) = N \times D^2/R^2 \gtrsim 9.3 \times 10^{42}/\text{cm}^2 \sec . \tag{5}$$

Assuming the cumulative snowplow (or ponderomotive) lengths of L km, the prompt electric acceleration could bring charged particles up to the maximum energy of

$$E_{\max} = 10^{21} \left[\frac{L}{km} \right] \text{eV}. \tag{6}$$

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The above electric field $E_{\max}$ (per kilometer) can exceeds 10^{21} eV/km, if we extrapolate nonlinear snowplow field (quadratic). The actual maximum energy over 10^{22} eV is feasible, because L is not limited to 1 km.

Gamma rays of GRB can produce highly energetic electrons and protons. Bremsstrahlung photons, dN/dE (proportional to E^{-1}), will come out as secondaries from electrons. The energy loss of electrons is so quick that they do not propagate much distance and their acceleration may be practically limited to about ~ 100 TeV, much lower energies than those of protons. According to the Lattimer and Schramm (1976) calculation, about a few per cent of baryons in neutron stars become relativistic upon a merge of neutron stars. We can consider snowplow (and wakefield) acceleration from behind (and in front) of them for over a long distance.

Extremely high energy cosmic rays at 10^{21} eV can make interaction after acceleration and can produce in average a few secondary gamma rays and neutrinos of average energy, $E_g \sim 10^{20}$ eV. If we assume an efficiency (k) for the particle ratio (1%) and for acceleration (1%), k would be in the order of $\sim 10^{-4}$. A rough estimate with these conservative assumptions gives the integral gamma ray intensity at 10^{20} eV,

$$N_\gamma(> 10^{20}\,\mathrm{eV}) = 2k[7 \times 10^{51}\,\mathrm{erg}/1.6 \times 10^9\,\mathrm{erg}]/[4\pi R^2]/\mathrm{day} = 1/[100\,\mathrm{km}^2\,\mathrm{yr}\,\mathrm{sr}]. \quad (7)$$

This is comparable to the observed EHECR intensity, regardless of the primary particle species. It is a surprise that the GRB's at a cosmological distance can account for the energetics of the highest energy cosmic rays in spite of their tremendous distance. EHECR's above 10^{20} eV in cosmic ray spectrum already suggested that they could be a new extra-galactic component, turning up the intensity over an otherwise smooth extra-galactic component above $10^{18.5}$ eV.

As mentioned, AGASA (Hayashida *et al.*, 1994, 1996; Bird *et al.*, 1994; Uchihori *et al.*, 1996) has 100 km² detector and recorded 27 EHECR's above 4×10^{19} eV in five years. Among

them, the highest energy events ($E = 0.7 - 2 \times 10^{20}$ eV) had 3 different pairs. Each pair came from the same direction within 2.5–degree errors within 2 years separation. A chance coincidence can account for them only as a 2.9% probability. Worldwide data of pairs and tridents concluded random chance of less than 0.1% probability. EHECR protons cannot come from 30 Mpc or farther due to photo-pion production loss, i.e., Greisen-Zatsepin-Kuzmin (GZK) cut–off. Moreover, they should be dispersed for over 10,000–100,000 years by scattering in the intergalactic magnetic-field (Waxman, 1995; Waxman and Coppi, 1996) even if they originated at the same time from the same source within 30 Mpc. These multiplets clearly require point sources therefore, where extremely prompt acceleration occurs and the accelerated particles should not be attenuated or scattered during propagation.

EHECR neutrinos can propagate 40 Gpc of intergalactic space without interaction or energy loss, until they encounter high density targets. The hot dark matter in the vast halo of our Virgo cluster is at extreme intensity (10^{11} times that of baryons in the clusters), if they are thermal neutrinos with ~ 0.1 eV mass. Neutrino-neutrino interactions in the extended halo can also produce sufficiently high intensity of gamma-rays and protons above 10^{20} eV that can reach the Earth's atmosphere. The flux above 10^{20} eV estimated by this scenario is $\sim (0.1 - 1)$ particle/km^2/century, not far from the observed data, ~ 1 particle/km^2 century, and high occurrence rate of pairs ($\sim 30\%$ of all the events).

Although the details need more elaborate quantification, fundamental laser acceleration mechanisms seem more important and promising than ever in the understanding of the highest energy cosmic rays and high-energy astrophysics processes.

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FIGURE CAPTIONS

FIG. 1. Energy spectrum of accelerated electrons in the laser wakefield in the experiment by Nakajima *et al.* (1995). The exponent of the power–law spectrum is about -1.8.

FIG. 2. Energy spectra of accelerated electrons in the laser irradiated target experiment by Cowan *et al.* (1997). Two–dimensional distribution was also measured. The exponent is around -2 (ranging from -1.45 to -3.5, depending on experimental conditions).

FIG. 3. The γ–ray spectra observed by the BATSE satellite (Pendleton, 1994) of the γ–ray burst event. In early stage the spectrum is a power–law with experiment -1; in later stages it is a gaussian thermal spectrum.

FIG. 4. Various γ–ray spectra from high energy astrophysical events: (a) E^{-2} spectrum from Ophinchus (Hunter *et al.*, 1993); (b) E^{-2} spectrum from PKS 0528 + 134 (Collmar *et al.*, 1993); (c) $E^{-1.4}$ spectrum from Crab Nebula (Sinitsyna, 1996); (d) AGN spectra (Schönfelder, 1994).

FIG. 5. Energy spectrum of extremely high energy cosmic rays. It shows three broad ranges of power laws, the first with exponent of -2.75 ($10^{15} > E > 10^9$ eV), the second with -3 ($10^{19} > E > 10^{15}$ eV), and the third exponent -2 ($E >$ few 10^{19} eV).