Ion plateau transport near the tokamak magnetic axis

K.C. Shaing and R.D. Hazeltine

Institute for Fusion Studies, The University of Texas at Austin, Austin, Texas 78712

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Abstract

Conventional neoclassical transport theory does not pertain near the magnetic axis, where orbital variation of the minor radius and the poloidal field markedly change the nature of guiding-center trajectories. Instead of the conventional tokamak banana-shaped trajectories, near-axis orbits, called potato orbits, are radially wider and lead to distinctive kinetic considerations. Here it is shown that there is a plateau regime for the near-axis case; the corresponding potato-plateau ion thermal conductivity is computed.

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I. INTRODUCTION

Tokamak transport at small collision frequency ν depends upon the structure of the collisionless orbits. Conventional neoclassical theory [1] takes this structure into account, but computes the orbits in a thin-banana approximation, in which the poloidal magnetic field B_P and minor radius r vary only weakly over the orbit. It has been pointed out [2,3] that this approximation is invalid near the tokamak magnetic axis, where B_P and r are very small and subject to large fractional change. When the near-axis orbits are studied allowing for such change, they are found to have a distinctive shape—more similar in cross-section to a potato than to a banana—and a thickness that scales differently with tokamak parameters. The result is low-collisionality transport coefficients that significantly differ from the conventional neoclassical formulae.

Previous work [4] considered the extreme low-collisionality regime, in which the potato frequency—the frequency at which a charged particle executes its poloidally closed orbit—is large compared to the effective collision frequency. Our purpose here is to consider an intermediate collision-frequency regime, in which the only untrapped-particle orbits survive Coulomb scattering. This regime is closely analogous to the plateau regime of conventional neoclassical theory; we will call it the "potato plateau," or simply plateau regime.

Collisionality is conventionally denoted by ν_* and measured by the ratio of the effective collision frequency, ν/f_t^2, where f_t is the fraction of trapped particles, to the trapped-particle bounce frequency, $f_t\omega_t$:

$$\nu_* = \frac{\nu}{f_t\omega_t^3}. \tag{1}$$

Here we have introduced the transit frequency

$$\omega_t \equiv \frac{v_t}{qR},$$

where $v_t = (2T/m)^{1/2}$ (with T the temperature and m a particle mass) is the thermal speed,

q is the safety factor and R the major radius. Plasma dynamics will be dominated by collisions unless the collision frequency is smaller than the transit frequency:

$$\frac{\nu}{\omega_t} = f_t^3 \nu_* \ll 1.$$

When the trapped-particle fraction is small,

$$f_t \ll 1, \tag{2}$$

one can distinguish two low-collisionality regimes, potato and plateau; if

$$\nu_* \ll 1 \tag{3}$$

trapped particles complete their orbits, and potato-transport applies. This case has been studied previously. The present work assumes

$$f_t^{-3} \gg \nu_* \gg 1 \tag{4}$$

so that collisions prevent the completion of trapped orbits, while the faster motion of transiting particles is only mildly perturbed.

The analogy between these formula and their banana versions is evidently very close. Since the nature of plateau transport for the banana case is well understood, it is reasonable to ask whether the present calculation is necessary. In fact the potato analysis, even when ν_* is large, is very different from the conventional one—and not only because of the different shape of the underlying orbits. Kinetic analysis close to the magnetic axis requires different orderings and different variables, essentially because the radial excursions of particles from flux surfaces can no longer be considered a perturbative effect. For the same reason our results for potato-plateau transport cannot be inferred from comparison to the banana-plateau transport coefficients.

To get a sense of when our plateau analysis applies, we recall [4] the potato formula

$$f_t^3 = \frac{2q^2\omega_t}{\Omega}, \tag{5}$$

where $\Omega = eB/mc$ is the gyrofrequency. After substitution into (1) it is not hard to show that ν_* can be written as

$$\nu_* = \frac{1}{2}\frac{\nu}{\omega_*}\frac{R^2}{a^2}$$

where

$$\omega_* = \frac{v_t^2}{\Omega a^2}$$

is a nominal measure of the drift frequency; a is the tokamak minor radius. Since $\omega_* \sim \nu$ in typical experiments, we see that ν_* can easily exceed unity. On the other hand, since

$$\omega_t \ll \Omega$$

it is not hard to also satisfy

$$\frac{\nu}{\omega_t} = \frac{\nu}{\omega_*}\frac{q^2R^2}{a^2}\frac{\omega_t}{\Omega} \ll 1.$$

In other words the requirements for potato-plateau physics will often pertain near the magnetic axis.

Finally we consider *where*—in what fraction of the tokamak cross-section—potato orbits and the new analysis are pertinent. Conventional tokamak transport theory assumes that the change in minor radius Δr associated with a trapped-particle orbit is small compared to r, the radius itself. Under this assumption the radial excursion is found to be the banana width

$$\Delta r_b \equiv \left(\frac{r}{R}\right)^{1/2}\rho_P$$

where ρ_P is the gyroradius in the poloidal field. Hence the conventional orbit analysis is consistent only if

$$r \gg \frac{\rho_P^2}{R}.$$

While the right-hand side of this inequality is very small, it does not specify the radius at which potato effects become important. The latter is a radius at least as large as the potato width, Δr_p. From an orbit analysis [4] that allows $\Delta r \sim r$ one finds that

$$\Delta r_p \sim (2q)^{2/3} \rho^{2/3} R^{1/3}. \tag{6}$$

For ion parameters in most tokamak experiments this radius is in the centimeter range. Thus ion potato transport applies to a core region, several centimeters in radius, surrounding the tokamak magnetic axis. From here on we focus on ion transport, omitting species-subscripts.

II. KINETIC EQUATION

In this section we solve the ion drift-kinetic equation that has been linearized using the small parameter

$$\Delta \equiv \frac{\Delta r_p}{a}. \tag{7}$$

Thus we write the ion distribution as

$$f = f_M + f_1,$$

where f_M denotes a Maxwellian distribution,

$$f_M = \frac{n}{\pi^{3/2}\, v_t^3}\, e^{-v^2/v_t^2}$$

and the correction f_1 is measured by Δ:

$$f_1 \sim \Delta f_M. \tag{8}$$

The linearized equation has the form [4]

$$(v_\parallel \mathbf{b} + \mathbf{v}_d) \cdot \nabla f_1 - C(f_1) = -\mathbf{v}_d \cdot \nabla f_M. \tag{9}$$

Here the gradients are performed at fixed magnetic moment $\mu = v_\perp^2/2B$ and kinetic energy $E = v^2/2$; $\mathbf{b} = \mathbf{B}/B$ is a unit vector in the direction of the magnetic field $\mathbf{B}$, $v_\parallel = \mathbf{b}\cdot\mathbf{v} = \sqrt{2(E-\mu B)}$ is the parallel velocity,

$$\mathbf{v}_d = v_\parallel \mathbf{b} \times \nabla \left(\frac{v_\parallel}{\Omega}\right)$$

is the guiding center drift velocity and ψ denotes the poloidal flux, which depends only on the radial variable, r:

$$\psi = \psi(r).$$

Finally C denotes the linearized Coulomb collision operator.

An important feature of (9) is that it advances f_1 according to the full guiding-center velocity, including the radial drift $\mathbf{v}_d \cdot \nabla\psi$, rather than only the lowest order velocity, $v_\parallel \mathbf{b}$. This distinctive feature is required near the magnetic axis because f_1 varies more rapidly in minor radius than f_M. Indeed we will find that, despite (8), the derivatives of the two functions are comparable:

$$\frac{\partial f_1}{\partial \psi} \sim \frac{\partial f_M}{\partial \psi}. \tag{10}$$

Here—unlike the case of conventional theory—it is important that the derivatives are expressed in terms of ψ, rather than r. The point is that

$$\frac{\partial}{\partial r} = \frac{d\psi}{dr}\frac{\partial}{\partial \psi}$$

where

$$\frac{d\psi}{dr} \propto r$$

is a rapidly varying function, whose relative change over an orbit can be comparable to the function itself. On the other hand, for small Δ it is consistent to assume

$$\frac{\partial f_M}{\partial \psi} = \text{constant} \tag{11}$$

on any ion guiding-center orbit.

Thus the radial derivative of f_1 at fixed (E, μ, θ), where θ is the poloidal angle, is large. Recall, however, that tokamak axisymmetry implies that the canonical angular momentum

$$\psi_* = \psi - \frac{I v_\parallel}{\Omega} \tag{12}$$

where $I(\psi) = RB_T$ and B_T is the toroidal field, is constant along the orbit. Hence the coordinate transformation

$$(E, \mu, \psi, \theta) \to (E, \mu, \psi_*, \theta)$$

removes the radial derivative from (9) and yields the kinetic equation

$$(v_\parallel \mathbf{b} + \mathbf{v}_d) \cdot \nabla\theta \, \frac{\partial f_1}{\partial \theta} - C(f_1) = -\mathbf{v}_d \cdot \nabla f_M. \tag{13}$$

Here only the poloidal drift survives; we will find that even this drift has significant, zeroth-order effect.

We next simplify the kinetic equation by transforming the distribution function, $f_1 \to g$, where

$$f_1 = -\frac{I v_\parallel}{\Omega} f_M \left(\frac{p'}{p} + \frac{e\Phi'}{T} + y \frac{T'}{T} \right) + g. \tag{14}$$

Here the primes denote ψ-derivatives and y is a constant parameter to be chosen presently. In terms of g our kinetic equation becomes

$$\omega_P \frac{\partial g}{\partial \theta} - C(g) = -\mathbf{v}_d \cdot \nabla\psi f_M \left(\frac{E}{T} - y - \frac{5}{2} \right). \tag{15}$$

where

$$\omega_P \equiv v_\parallel \mathbf{b} \cdot \nabla\theta + \mathbf{v}_d \cdot \nabla\theta \tag{16}$$

is the poloidal speed. The potato orbit equations show that [4]

$$\omega_P = \frac{3}{4} \frac{\Omega_0}{I q R_0} \left(\psi + \frac{2}{3} \frac{I v_{\parallel 0}}{\Omega_0} \right) \tag{17}$$

where the 0-subscripts denote evaluation at $(\theta = 0, \psi = 0)$. By evaluating (12) at $\psi = 0$ we see that

$$\psi_* = -\frac{Iv_{\|0}}{\Omega_0}$$

whence

$$\omega_P = \frac{3}{4}\frac{\Omega_0}{IqR_0}\left(\psi - \frac{2}{3}\psi_*\right). \tag{18}$$

Thus the turning point for an orbit labelled by ψ_* occurs at $\psi = (2/3)\psi_*$, rather than at the point $\psi = \psi_*$, where $v_\| = 0$. The difference reflects poloidal drifts; that this drift contributes in lowest order—changing the coefficient by $\mathcal{O}(1)$—is a peculiarity of potato motion.

Plateau transport is dominated by particles near the turning point, $\omega_P \cong 0$, and therefore in the trapped region of velocity space. We recall that this region consists of the range

$$0 \leq \kappa \leq 1. \tag{19}$$

where

$$\kappa = \frac{4}{27}\frac{\Omega_0 R_0}{q}\frac{\left|v_{\|0}\right|^3}{(v_{\|0}^2 + \mu B_0)^2}.$$

The turning points occur on the inside of the poloidal cross-section, where

$$\cos\theta < 0. \tag{20}$$

It can be shown [4] that

$$\omega_P \sim f_t \omega_t \ll \omega_t$$

in the trapped region. Notice also that for small ion gyroradius

$$\rho \equiv \frac{v_t}{\Omega_0} \ll R, \tag{21}$$

the parallel flow must be relatively small in the trapped region, allowing the approximation

$$\kappa \cong \frac{4}{27}\frac{\Omega_0 R_0}{q}\frac{\left|v_{\|0}\right|^3}{(\mu B_0)^2}. \tag{22}$$

This is a convenient point to verify the ordering (10). The definition (12) of ψ_* allows us to express (14) as

$$f_1 = (\psi_* - \psi)\left(\frac{p'}{p} + \frac{e\Phi'}{T} + y\frac{T'}{T}\right) + g,$$

whence, assuming g to be comparable to the first term in f_1, we infer,

$$f_1 \sim \Delta\psi \, \frac{\partial f_M}{\partial \psi}$$

where $\Delta\psi$ measures the change in ψ over the orbit. We have already noted that in the potato case, $\Delta\psi \sim \psi$; the ordering (10) immediately follows.

For the plateau regime the most convenient variables are E, ω_P, ψ_* and θ. Therefore we write

$$\frac{\partial g}{\partial \theta}|_{E,\mu,\psi_*} \rightarrow \frac{\partial g}{\partial \theta}|_{E,\omega_P,\psi_*} + \frac{\partial \omega_P}{\partial \theta}|_{E,\mu,\psi_*} \, \frac{\partial g}{\partial \omega_P}|_{E,\omega_P,\psi_*},$$

and our kinetic equation assumes the form

$$\omega_P\left(\frac{\partial g}{\partial \theta} + \frac{\partial \omega_P}{\partial \theta}\,\frac{\partial g}{\partial \omega_P}\right) - C(g) = -\mathbf{v}_d \cdot \nabla\psi f_M \left(\frac{E}{T} - y - \frac{5}{2}\right). \tag{23}$$

This equation has a familiar analog in the banana regime, but with a significant difference. In the banana-regime version, the second term on the left (involving the parallel-velocity derivative) corresponds to the mirror force, and is typically smaller than the first term by a power of the inverse aspect ratio. In other words the magnetic curvature acts on banana particles as a perturbation, keeping bananas relatively thin. Here the second term can be seen—for typical particles—to be fully comparable to the first: both are measured by ω_t. This difference reflects that fact that the potato width is finite in an orbit-perturbation sense. Moreover it implies that using ω_P as a velocity coordinate complicates the kinetic equation in a serious way; indeed our new coordinates are useful only for a small class of poloidally "resonant" particles. However it is precisely these resonant particles that determine plateau transport.

To understand the resonance, and to confirm the collisionality orderings discussed previously, we first note that the orbital frequency ω_P, which nominally dominates the collision frequency in any low-collisionality regime, vanishes at the trapped-particle turning points. Thus, if the collision operator were neglected the solution would be singular at $\omega_P = 0$. As collisions resolve this singularity, a boundary-layer structure occurs in the solution g: a narrow range of ω_P in which sharp ω_P-variation defeats the nominal ordering, resolving the singularity. We denote the width of the boundary layer by $\Delta\omega_P$, and find that the two terms on the left-hand side of (23) are estimated by $\Delta\omega_P$ and

$$\frac{\omega_P^2}{\Delta\omega_P} \sim \frac{(f_t\omega_t)^2}{\Delta\omega_P}$$

respectively. Here we suppress a common factor of g and note that $\partial\omega_P/\partial\theta \propto (\Delta\omega_P)^{-1}$ becomes large near the turning points, so that the product

$$\omega_P \frac{\partial\omega_P}{\partial\theta} \sim \omega_P^2 \sim f_t^2\omega_t^2$$

is finite. Finally the collision term in (23) is measured by

$$C(g) \sim \nu \frac{\omega_t^2}{\Delta\omega_P^2},$$

reflecting pitch-angle diffusion.

Potato analysis in the low-collisionality limit assumes that, even near the turning points, the first two terms in(23) are comparable and dominate the collisional term. Here we assume instead that collisions resolve the $\omega_P = 0$ singularity; this potato-plateau balance is characterized by

$$\Delta\omega_P \sim \nu \frac{\omega_t^2}{\Delta\omega_P^2}$$

or

$$\Delta\omega_P \sim \nu^{1/3}\omega_t^{2/3}. \tag{24}$$

We use this result to estimate the size of the second term on the left,

$$\frac{(f_t\omega_t)^2}{\Delta\omega_P} \sim f_t\omega_t\nu_*^{-1/3},$$

and conclude that it is smaller than the first term by a factor of

$$\frac{f_t\omega_t\nu_*^{1/3}}{\Delta\omega_P} \sim \nu_*^{-2/3}.$$

It follows that when (4) holds the drift-kinetic equation has the lowest order form

$$\omega_P \frac{\partial g}{\partial \theta} - C(g) = -\mathbf{v}_d \cdot \nabla\psi f_M \left(\frac{E}{T} - y - \frac{5}{2} \right), \tag{25}$$

which is to be solved for $g(E, \omega_P, \psi_*, \theta)$ in the limit $\nu \ll \omega_P$.

III. SOLUTION OF THE KINETIC EQUATION

The radial drift has the familiar [1] expression

$$\mathbf{v}_d \cdot \nabla\psi = -\frac{1}{2\Omega} I\mathbf{b} \cdot \nabla\theta (v^2 + v_\parallel^2) \frac{\partial \log B}{\partial \theta}. \tag{26}$$

As usual we assume that the θ-dependence of B comes mainly from the $1/R$-dependence of the toroidal field; then

$$\frac{\partial \log B}{\partial \theta} \cong \frac{r}{R_0} \sin\theta.$$

Note that this expression involves *two* factors that vary significantly over the orbit: θ and

$$\frac{r}{R_0} = \alpha\sqrt{\psi},$$

where

$$\alpha \equiv \sqrt{\frac{2q}{IR_0}} \tag{27}$$

can be assumed constant. Now the right-hand side of (25) can be expressed as

$$-\mathbf{v}_d \cdot \nabla\psi f_M \left(\frac{E}{T} - y - \frac{5}{2} \right) = Q \sin\theta$$

with

$$Q \equiv \frac{1}{2\Omega} I\mathbf{b}\cdot\nabla\theta(v^2+v_\parallel^2)\alpha\sqrt{\psi}f_M\left(\frac{E}{T}-y-\frac{5}{2}\right)\frac{T'}{T}.$$

With regard to the left-hand side of (25), we recall that the limit

$$g_0 = \lim_{\nu/\omega_P\to 0} g$$

is independent of the detailed form of the collision operator, and correctly computed from a Krook model,

$$C(g) = -\nu g.$$

Thus our drift-kinetic equation has become

$$\omega_P\frac{\partial g}{\partial\theta} + \nu g = Q\sin\theta, \tag{28}$$

which is easily solved. The $\nu/\omega_P \to 0$ limit of the resonant solution is

$$g_0 = \pi\delta(\omega_P)Q(E, \omega_P = 0, \psi_*)\sin\theta. \tag{29}$$

The definition (16) shows that

$$v_\parallel \ll v,$$

at $\omega_P = 0$. Hence we can neglect $v_\parallel$ compared to v in Q, and obtain the resonant distribution

$$g_0 = \frac{\pi I^2}{qB_0R_0^2\Omega_0}\alpha\sqrt{\psi}E\left(\frac{E}{T}-y-\frac{5}{2}\right)\frac{T'}{T}f_M\delta(\omega_P)\sin\theta. \tag{30}$$

where δ represents the Dirac delta-function.

The form of g_0 resembles that of the conventional plateau distribution, but in fact it is rather different. In the conventional case, all turning points (for given values of E and ψ_*) lie on the same flux surface; but here the delta-function factor, localizing g_0 to the orbital turning points, marks a contour in (ψ, θ) that intersects a range of flux surfaces. This fact is clear from (17), since $v_{\|0}$ can vary over the entire trapped region of phase space. We recall (19) and(22) to infer the range

$$0 < v_{\|0} < \frac{3}{2^{4/3}} \left(\frac{q}{\Omega_0 R_0} \right)^{1/3} v^{4/3}.$$

The corresponding range of ψ,

$$\Delta\psi = -\frac{I}{\Omega_0} \left(\frac{q}{2\Omega_0 R_0} \right)^{1/3} v^{4/3} \tag{31}$$

corresponds to the potato width anticipated in (6).

Because of this radial spread of turning points, potato transport coefficients are less radially localized than their banana counterparts. Essentially for this reason [5] a consistent transport theory requires radially averaging the particle and heat fluxes, in this case over a flux tube of radius $\Delta\psi$ enclosing the magnetic axis. In other words, if F represents a conventional, flux-surface-averaged particle or heat flow, then the potato transport description is obtained from

$$F \rightarrow \frac{1}{\Delta\psi} \int_o^{\Delta\psi} d\psi F. \tag{32}$$

IV. ION THERMAL CONDUCTIVITY

Collisional momentum conservation implies ambipolarity of the radial particle flux, thus requiring the ion radial particle flux, Γ_i, to be proportional to the square of the electron gyroradius, ρ_e. Since both the ion radial drift and our distribution function g_0 are proportional to $\rho \gg \rho_e$, the particle flux obtained from our kinetic theory will be unphysically large unless the parameter y is chosen appropriately. In other words, ambipolarity is satisfied by requiring $\Gamma_i(y) = 0$, where

$$\Gamma_i(y) = \left\langle \int d^3v \, \frac{1}{\Delta\psi} \int_0^{\Delta\psi} d\psi \mathbf{v}_d \cdot \nabla\psi g_0(y) \right\rangle. \tag{33}$$

Here the angle-brackets denote a flux-surface average, formally defined by

$$\langle F\rangle = \left(\oint \frac{d\theta}{\mathbf{B}\cdot\nabla\theta}\right)^{-1} \oint \frac{d\theta F}{\mathbf{B}\cdot\nabla\theta}.$$

To verify that the flux surface average has the simple lowest-order form,

$$\langle F\rangle \cong \frac{1}{2\pi}\oint d\theta F, \tag{34}$$

we note that $\mathbf{B}\cdot\nabla\theta = B_T/qR$, which does not vary strongly over the orbit.

After substituting (26) and (30) into (33) we obtain a product of integrals for the ion particle flux. Only two of the integrals require comment: first, the poloidal integral is

$$\oint \frac{d\theta}{2\pi}\sin^2\theta\ \delta(\omega_P) = \frac{1}{4}\delta(\omega_P)$$

because all turning points occur on the inside of the torus, as noted in (20). Second, the ψ-integral is

$$\frac{1}{\Delta\psi}\int_0^{\Delta\psi} d\psi\psi = \frac{(\Delta\psi)^2}{2}$$

since the drift velocity and the distribution each contribute a factor of $\sqrt{\psi}$. Here the right-hand side is specified by (31). With these remarks, the particle flux is straightforward to evaluate; we find that

$$\Gamma_i = \frac{\sqrt{\pi}}{2^{1/3}16}\left(\frac{v_t}{\Omega_0 R_0}\right)^{10/3} \frac{q^{1/3}I^3}{B}\, nv_t\, \frac{T'}{T}\,\mathcal{I}$$

where $\mathcal{I}$ is expressed in terms of the gamma-function as

$$\mathcal{I} \equiv \Gamma\left(\frac{14}{3}\right) - \frac{5}{2}\Gamma\left(\frac{11}{3}\right) - y\Gamma\left(\frac{11}{3}\right)$$

Requiring $\mathcal{I}(y) = 0$ we find that

$$y = \frac{7}{6}.$$

It is now a simple matter to compute the radial heat flux:

$$\langle \mathbf{q}\cdot\nabla\psi\rangle = \left\langle \int d^3v\, \frac{1}{\Delta\psi}\int_0^{\Delta\psi} d\psi \mathbf{v}_d\cdot\nabla\psi\left(E - \frac{5}{2}T\right) g_0\right\rangle.$$

We find that

$$\langle \mathbf{q} \cdot \nabla\psi \rangle = -1.3 \left(\frac{v_t}{\Omega_0 R_0} \right)^{10/3} \frac{q^{1/3} I^3}{B} n v_t T'. \qquad (35)$$

V. DISCUSSION

The formula (35) corresponds to a thermal diffusion coefficient given by

$$\chi_\psi = 1.3 \left(\frac{v_t}{\Omega_0 R_0} \right)^{10/3} \frac{q^{1/3} I^3}{B} v_t \qquad (36)$$

where the subscript reminds us that radial distance is measured with respect to poloidal flux ψ. We observe that χ_ψ is independent of collision frequency, as expected for a plateau transport process. It also displays a surprisingly strong dependence on ion gyroradius, essentially because the fraction of trapped potato orbits depends on gyroradius—unlike the banana case, where trapping is determined by the aspect ratio.

We can understand the form of (36) in terms of the random-walk (RW) formula for ion thermal diffusion,

$$\chi_{RW} \sim f_t \nu_{\text{eff}} S(\psi)^2 \qquad (37)$$

where f_t is the fraction of particles that take part in the particular random walk being considered,

$$\nu_{\text{eff}} \sim \frac{\nu}{f_t^2}$$

is their effective collision frequency, and $S(\psi)$ measures their random step-size, in units of poloidal flux. It is clear from the previous section that the fraction relevant to plateau transport is

$$f_t = \frac{\Delta\omega_P}{\omega_t}.$$

The plateau step-size is given by

$$S(\psi) \sim \frac{\mathbf{v}_d \cdot \nabla\psi}{\nu_{\text{eff}}}$$

since the radial step in the plateau regime is a collisional artifact: collisions prevent the completion of otherwise closed orbits. Combining these estimates into (37), we obtain the thermal diffusivity

$$\chi_\psi \sim \left(\frac{\Delta\omega_P}{\omega_t}\right)^3 \frac{(\mathbf{v}_d \cdot \nabla\psi)^2}{\nu}$$

But we have found, in (24),

$$\left(\frac{\Delta\omega_P}{\omega_t}\right) = \left(\frac{\nu}{\omega_t}\right)^{1/3}$$

whence

$$\chi_\psi \sim \frac{(\mathbf{v}_d \cdot \nabla\psi)^2}{\omega_t}. \tag{38}$$

To verify that (38) agrees with our result, (36), we note that the latter is estimated by

$$\chi_\psi \sim f_t^2 \omega_t (\Delta\psi)^2, \tag{39}$$

where $\Delta\psi$ is defined by (31). Since

$$\Delta\psi \sim \frac{\mathbf{v}_d \cdot \nabla\psi}{\omega_P}.$$

we can write

$$\chi_\psi \sim f_t^2 \omega_t \frac{(\mathbf{v}_d \cdot \nabla\psi)^2}{\omega_P^2},$$

which, after we recall $\omega_P = f_t\omega_t$, reproduces (38).

Of course the formula (39) has a simple random-walk interpretation of its own: each trapped particle (the number of which is measured by f_t) executes a random radial step by $\Delta\psi$ on each poloidal circuit—that is, at the rate $\omega_P = f_t\omega_t$. This point of view has obvious mnemonic advantages but leaves out the subtle role of collisions in plateau transport.

To get a sense of the quantitative significance of our result, we compare it to conventional plateau transport. First we express the familiar plateau coefficient for thermal diffusion in radius, $\chi_r \sim q\rho^2 v_t/R$ in terms of the radial variable ψ, using

$$|\nabla\psi| \cong B_P R \nabla r \cong B_P R.$$

Then the conventional plateau formula [1] yields the estimate

$$\chi_{\psi\,\mathrm{conv}} \sim \rho^2 \omega_t r^2 B_T^2$$

which is to be compared with (36). The ratio is

$$\frac{\chi_\psi}{\chi_\psi\,\mathrm{conv}} \sim \frac{q^2 \rho^{4/3} R^{2/3}}{r^2}. \tag{40}$$

Equation (6) shows that this quantity is close to unity at the edge of the potato region, $r \sim \Delta r_P$. At smaller radii, conventional plateau theory underestimates ion thermal diffusion.

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