Test Particle Simulations for Neoclassical Transport in a Reversed Shear Plasma

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Abstract

The authors present results of test particle simulations for neocalssical transport in the core region of a reversed shear plasma by solving the guiding center equations of motion in toroidal geometry together with the Monte Carlo Coulomb collisional pitch angle scattering. The radial transport is not diffusive in the core plasma where the orbit topology is much different from what is assumed in standard neoclassical theory. The results indicate that the unusual orbit topology and steep gradients in density and q profiles are not sufficient to explain the very low ion thermal conductivity observed in the ERS experiments.

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Greatly improved particle and ion thermal transport in the reversed shear region was demonstrated by the recent experiments on TFTR(Tokamak Fusion Test Reactor). [1] According to the time dependent transport analysis in work using the TRANSP code, the ion thermal diffusivity drops substantially to a level that is much less than the estimated neoclassical value, which has widely been believed to be the irreducible minimum plasma transport rate. Several analyses to resolve this apparent contradiction have been performed. Lin *et al.* [2] relaxed the basic neoclassical assumption that the ion's orbital excursions are much smaller than the local toroidal minor radius and equilibrium scale lengths of the system and established a revised theory where the ion poloidal gyroradius can be of the same order of magnitude as the local minor radius or the equilibrium pressure gradient scale length. Shaing *et al.* [3] investigated ion transport in the region close to the magnetic axis by solving the drift kinetic equation and found that the ion thermal conductivity remains finite in that region. Chang [4] obtained a finite banana width correction to the neoclassical ion thermal conductivity under the conventional assumption that the particle flow parallel to magnetic field lines dominates the trapped particle's orbital dynamics, and showed that negative radial gradients in plasma density or safety factor can reduce the neoclassical ion thermal conductivity when the banana width is a significant fraction of the gradient scale length.

In this work, we investigate the neocalssical transport in a reversed shear plasma by test particle simulations that solve numerically the guiding center equations of motion in toroidal coordinates for ensembles of trapped ions including the effect of Coulomb collisional pitch angle scatterings. For the region $r \gtrsim 0.3a$ for which our results indicate that the neoclassical transport by collisions is diffusive, we calculate the positive definite microscopic diffusion coefficient, defined by

$$\mathcal{D}(\mathcal{E}) = \lim_{t\to\infty} \frac{1}{N} \sum_{i=1}^{N} \frac{[r_i(t) - r_i(0)]^2}{2t}. \tag{1}$$

For the number of sample particles $N \gtrsim 256$ we have reproduced the Chang-Hinton diffusivity [5] for the case of normal shear plasma in Ref. [6]. In the region $r \lesssim 0.3a$, however, the diffusion coefficient does not converge to a well defined value, at least in a calculation time practically possible ($\omega_{bi} t \lesssim 10^4$ where ω_{bi} is the ion bounce frequency.) Thus we measure the transition time [7] during which the ions traverse from the core initial position to the reference position taken to be the minimum location of the safety factor, $r = 0.35\,a$, and from the result infer the effective diffusion coefficient taking the transport as if it were diffusive.

We solve the guiding center equations in toroidal coordinates, [6]

$$\dot{\epsilon} = -G_1 F_1, \tag{2}$$

$$\dot{\theta} = G_2 F_2 + G_1 F_3, \tag{3}$$

$$\dot{\phi} = G_3 F_2 + G_4 F_3, \tag{4}$$

$$\dot{\rho}_{\|} = -G_2 F_1, \tag{5}$$

where the functions $F_i(\epsilon, \theta, \rho_{\|})$ and $G_i(\epsilon, \theta, \rho_{\|})$ are given in Ref. [6] in detail. In the above equations, $\epsilon (= r/R_0)$, θ, and ϕ are the toroidal coordinates, $\rho_{\|} (\equiv v_{\|}/\Omega_c)$ is the parallel gyroradius, $v_{\|} = \mathbf{v} \cdot \widehat{b}$, $\widehat{b} = \mathbf{B}/B$, and the gyrofrequency $\Omega_c = eB/(mc)$. In the derivation of Eqs. (2)–(5), we used the standard magnetic field model in a large aspect ratio tokamak with circular cross section

$$\mathbf{B} = \frac{1}{1 + \epsilon \cos\theta} \left(\widehat{\phi} + \frac{\epsilon}{q} \widehat{\theta} \right), \tag{6}$$

where q is the safety factor.

We also consider the Coulomb collisional pitch angle scattering operator derived in Ref. [6]. This collision induces the following changes of the velocity variables $(\rho_{\|}, \mu)$:

$$(\rho_{\|})_f = (\rho_{\|})_i \cos\gamma + \sqrt{\frac{2\mu_i}{B_i}} \sin\gamma \cos\alpha, \tag{7}$$

$$\frac{2\mu_f}{B_f} = \left[(\rho_{\|})_i^2 + \frac{2\mu_i}{B_i} \right] \sin^2\gamma \sin^2\alpha + \left[\sqrt{\frac{2\mu_i}{B_i}} \cos\gamma - (\rho_{\|})_i \sin\gamma \cos\alpha \right]^2, \tag{8}$$

where subscripts $_i$ and $_f$ refer to the initial and final values, respectively, and the normalized magnetic moment $\mu = m^2 c^2 v_\perp^2 / (2e^2 B)$. In Eqs. (7) and (8), two angles of α and γ are determined as

$$\alpha = 2\pi\eta_1, \qquad \gamma - \left[-\nu\delta t \ln(1-\eta_2)\right]^{1/2}, \tag{9}$$

where ν is collision rate, η_1 and η_2 are two random numbers on [0,1]. We calculate the collision rate from local density and temperature shown in Fig. 1 which are chosen to represent the experiment in Ref. [1].

We investigate numerically the transport properties of ions by integrating the equations of motion in toroidal coordinates of Eqs. (2)–(5) and calculating the microscopic diffusion coefficients. We use the system parameters of TFTR, i.e., major radius $R_0 = 260cm$, minor radius $a = 94cm$, centerline field strength $B_0 = 4.6T$, and the assumed theoretical fit of the experimentally measured q, ion density and temperature profiles in the TFTR ERS (Enhanced Reversed Shear) experiment [1]. The q profile, the corresponding shear function, $s(r) \equiv \frac{r}{q}\frac{dq}{dr}$, the ion density and temperature profiles are shown in Fig. 1.

We consider the motion of deuterium ions (D^+) in the circular toroidal equilibrium specified above. Figure 2 shows the radial excursion of the ions with various initial radial position and pitch($v_\parallel/v$), and kinetic energy equal to the ion temperature at the position. We launched the particles at $\theta = \phi = 0$. From the results of the calculation we find that for $r \lesssim 0.05a$, all the co-going particles with $v_\parallel > 0$ are trapped and for $r \lesssim 0.17a$, the banana widths of co-going ions are restricted by the local initial minor radius, which were described qualitatively as important reasons of enhanced confinement in reversed shear plasma in Ref. [2]. We also see that the radial excursions of counter-going particles with $v_\parallel < 0$ constantly increase as $r \to 0$ and largest banana width of an ion starting on magnetic axis reaches to $0.24\,a$ near the minimum safety factor surface. Trapped fraction of counter-going particles decreases as $r \to 0$. [2] The largeness of the orbit width and the smallness of the trapped fraction are the reason that the transport in the core region is not diffusive, as described below.

We consider an ensemble of D^+ ions composed of 512 counter-going trapped particles that initially have same radial position and random θ's with the pitch variable $\lambda = 1$ and kinetic energy equal to the ion temperature at the position to investigate the transport. Change of initial λ does not make any noticeable difference in the result of transport calculation since the particles spread out in a short time in (λ, θ) space. [6] In Fig. 3 we show the time dependencies of the running diffusion coefficients,

$$\mathcal{D}(t) = \frac{1}{2t}\frac{1}{N}\sum_{j=1}^{N}\left[r_j(t) - r_j(0)\right]^2 , \tag{10}$$

corrected for the effect of the radially bounded motion of trapped particles [6] for the ensemble described above. The total integration time, T, is $1.5 \times 10^8 \Omega_0^{-1}$. From Fig. 3(a) we can see that the transport at $r = 0.1a$ is not diffusive. At least for a computation time during which we can practically follow the motion of particles there is no sign of convergence for the running diffusion coefficient. As can be seen in Fig. 3(b), the running diffusion coefficient for the ensemble of particles starting at $r = 0.5\,a$ shows a curve convergent to well defined constant value indicating that the radial transport there is a diffusion process. For the case, when $\mathcal{D}(t)$ appears convergent, we regard that $\mathcal{D}(t)$ is ergodic and replace the ensemble average by its time average. We obtain the diffusion coefficient from time series of $\mathcal{D}(t)$ as

$$\overline{\mathcal{D}} = \frac{1}{T - T_0}\int_{T_0}^{T}\mathcal{D}(t)dt \tag{11}$$

and estimate the standard deviation by

$$\delta\mathcal{D} = \left[\frac{1}{T - T_0}\int_{T_0}^{T}(\mathcal{D}(t) - \overline{\mathcal{D}})^2 dt\right]^{1/2} , \tag{12}$$

where T_0 is the time when convergence is observed to set in. For the case of Fig. 3(b) the total integration time is 4.5×10^3 ion bounce times.

The results of our running diffusion coefficient calculation indicate that the region where the transport is diffusive is $r \gtrsim 0.3a$. Thus we measure the transport in the core region by the transition time method, [7] that is, we calculate the average time elapsed while particles traverse from the core to the minimum q surface at $r = 0.35\,a$. For comparison with diffusivity in other works we estimate 'diffusion' coefficients in the region assuming as if transport is diffusive there, though a diffusion coefficient can not be defined there. We estimate the equivalent 'diffusion' coefficient as

$$\mathcal{D} = \frac{d^2}{T_t}, \tag{13}$$

where d is the distance between initial position and minimum q position and T_t is the calculated transition time. Equation (13) gives for a diffusion process the usual diffusion coefficient calculated by Eq. (11). In Fig. 4 we present the result of transport calculation. We can see from the result that neoclassical transport in reversed shear plasma is lower than that calculated from standard neoclassical formula [5] in whole plasma region. Our previous work [6] already showed that the diffusivity calculated with same tool as used here for normal shear plasma almost exactly matches with the standard neoclassical formula. The discrepancy becomes large as $r \to 0$. Our results show the same tendency with those in Ref. [4] in which the steep gradients in density and q induce several tens of percent change in diffusivity. Since in the analytic work only the steep gradient effect was considered, it is natural that our calculation, which automatically includes all the orbit topology effects such as restriction of radial excursion by the local minor radius, gives lower transport in core region. However, Fig. 4 does not agree with the result in Ref. [1] where diffusivity is argued to be very small in core region, based on the thermal flux for power balance divided by the large temperature gradient. One possibile reason for the difference is the large radial electric field or its shear, that may be present in the region.

In summary, we solved the guiding center equations of motion in toroidal geometry together with the Monte Carlo Coulomb collisional pitch angle collision operator to investigate the core neoclassical transport in a reversed shear plasma. We have attempted to simulate the situation of the ERS experiment in Ref. [1] and [4]. Results of test particle simulation of the radial transport indicate that for $r \lesssim 0.3a$ where banana widths of counter-going trapped particles are very large, the radial transport is not diffusive. We calculated the microscopic diffusion coefficient for the region in which the transport is diffusive, and transition time otherwise, from which we estimate the 'diffusivity' for comparison with results produced by other methods. Our results qualitatively agree with those in Ref. [4] but do not agree with experimental results in Ref. [1]. Thus we conjecture that what makes the core neoclassical transport in the ERS experiments almost zero is not the unusual orbit topology or steep gradients in density or q included in our calculation, but the large radial electric field present in the region.

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FIGURE CAPTIONS

FIG. 1. Radial profiles of (a) safety factor $q(r)$, (b) shear $s(r)$, (c) ion density $n(r)$, and (d) temperature $T(r)$.

FIG. 2. Radial excursion of D^+ ions in the plasma depicted in Fig.1. Particles initially have $\theta = \phi = 0$, kinetic energy equal to the temperature at local radial position, and various radial position and pitch.

FIG. 3. Time dependences of the running diffusion coefficients for ensembles of particles initially located at (a) $r = 0.1a$ and (b) $r = 0.5a$.

FIG. 4. Comparison of the transport calculation results with the standard neoclassical diffusion coefficient. Solid curve is Chang and Hinton's formula, lowermost shortdashed line is the numerical result from microscopic diffusion coefficient calculation, and the dotted line which spans only the core region is from the transition time calculation.