On the Robustness of the localized spatiotemporal structures in electron–positron–ion plasmas

S.M. Mahajan

Institute for Fusion Studies, The University of Texas at Austin, Texas 78712

V.I. Berezhiani

Institute for Fusion Studies, The University of Texas at Austin, Texas 78712

and

Department of Plasma Physics, Institute of Physics, Tbilisi, Georgia

R. Miklaszewski

Institute of Plasma Physics and Laser Microfusion,

00–908 Warsaw, str. Hery 23, P.O. Box 49, Poland

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Abstract

It is shown that, in an electron–positron plasma with a small fraction of ions, large–amplitude localized spatiotemporal structures (light bullets) can be readily generated and sustained. These light bullets are found to be exceptionally robust: they can emerge from a large variety of initial field distributions and are remarkably stable.

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I. INTRODUCTION

Electromagnetic wave dynamics in a relativistic electron–positron (e–p) plasma will, perhaps, be an essential determinant of the radiation properties of astrophysical objects like the pulsars, and the active galactic nuclei (AGN) [1,2]. The early universe in the MeV epoch is another system where this dynamics can play an important role [3]. Although the electron–positron pairs may be the dominant constituent of these objects, a minority population of heavy ions is also likely to be present. For instance, from the present baryon asymmetry, it can be estimated that the numbers of protons (and neutrons) was roughly 10^{-9} to 10^{-10} of the number of light particles (electrons, neutrinos, photons) in the MeV epoch that prevails up to times ~ 1 second after the Big Bang. This minority ion population, by creating a small number asymmetry, may end up imparting interesting new properties to the composite system. Thus a three–component electron–positron–ion (e–p–i) plasma, spanning physical systems like the pulsar magnetospheres and intergalactic Jets [4], has become an object of serious investigation. In several recent publications [5], it has been shown that an electron–positron plasma with a small concentration of ions (protons) can sustain stable large amplitude relativistic solitons with large density bunching. Such spatio–temporally localized stable solitons, to borrow a phrase from nonlinear optics, can be rightfully called "light bullets" [6]. These "light bullets," being a source of concentrated mass and energy, may lead to a gravitational instability sowing the seeds for a possible mechanism for the large–scale structure formation in the early universe. It is also obvious that formation of the light bullets could have a definite impact on the radiative properties of AGN.

If these localized solutions are to play an important part in the overall long–term dynamics of the systems in which they are found to be possible, the solutions have to possess a number of additional characteristics. The chief amongst these are:

1) Stability— Once formed, the bullets should be stable to a variety of perturbations, not only to small but to large amplitude perturbations as well. The solutions have to be nonlinearly stable;

2) Accessibility— Even if the bullets turn out to be stable, when formed, what is the guarantee that such exotic objects can be formed at all. Thus it should be demonstrated that, starting from a variety of disparate initial conditions (including those that are far from the eventual solution), it is possible to realize the desired solution in the times of interest. Note that accessibility and nonlinear stability are not entirely independent—if an arbitrary set of initial conditions can lead to the bullet formation, then it is natural that the bullet will be stable to arbitrary perturbations.

3) Survival in a Bullet–Bullet Collision— it will also be important to learn the consequences of the mutual interactions of the bullets. If the bullets disintegrated on a collision and the collision rates were sufficiently high, then the dynamics of the system may not be seriously affected by these localized solutions. Survival after a collision can be seen as a mark of stability in an extended sense.

In this paper, we have investigated whether the bullet solutions of an e–p–i plasma display the aforementioned characteristics. We show in Secs. II and III that the light bullets are exceptional robust; there exists a wide range of initial parameters of the fields that lead to the formation of light bullets. We have employed both analytical and numerical methods in our study; the analytical methods are used mainly to guide and interpret the numerical work. In Sec. IV, we give a summary of our results.

II. PULSE DYNAMICS: STABILITY AND ACCESSIBILITY

It was shown in Ref. [5] that the dynamics of a finite amplitude electromagnetic (EM) pulse propagating in an e–p–i relativistic plasma is governed, under appropriate circumstances, by the generalized nonlinear Schrödinger equation. This equation, in dimensionless form, can be written as (for details, see [5]):

$$i\,\frac{\partial A}{\partial t} + \frac{\partial^2 A}{\partial \xi^2} + \Delta_\perp A + \left(1 - \frac{1}{(1+|A|^2)^2}\right) A = 0, \tag{1}$$

where A is the slowly varying amplitude of the circularly polarized EM pulse, $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$, and $\xi = z - v_g t$ is the "comoving" (with group velocity v_g) coordinate. Equation (1)

is written in terms of the dimensionless quantities $A = |e|A/M_e c^2$, $r_\perp = (\epsilon\omega_e/8^{1/2}c)r_\perp$, $\xi = (\epsilon\omega_0/4c)\xi$, where ω_0 is carrier frequency of the pulse, $\omega_e = (4\pi e^2 n_{0e}/M_e)^{1/2}$ is the electron Langmuir frequency and $\epsilon = n_{0i}/n_{0e}(\ll 1)$, where n_{0i}, n_{0e} are the unperturbed number densities of ions and electrons, respectively. Notice that $M_e = m_e G(T/m_e c^2)$ (the function $G(T/m_e c^2)$, a combination of modified Bessel functions, is defined in Ref. [5])is the temperature ($T = T_e = T_p$) dependent effective mass of electrons. For nonrelativistic temperatures ($T \ll m_e c^2$), the effective mass reduces to $M_e = m_e + 5T/2c^2$, while for the ultrarelativistic high temperatures it becomes $M_e = 4T/c^2 \gg m_e$, the inertial mass. In deriving Eq. (1), we have assumed that the plasma is transparent (i.e. $\omega_0 \gg \omega_e$), and that the longitudinal extent of the pulse is much shorter than its transverse dimensions.

Equation (1) admits a "spherically" symmetric three dimensional solitary wave solution, i.e. the field depends on the variable $r = (r_\perp^2 + \xi^2)^{1/2}$ alone (Note however, that in the unnormalized coordinate space, the soliton has an ellipsoidal shape that is stretched across the propagation direction). For stationary solitons, the ansatz

$$A = A(r)\exp(i\Omega^2 t) \tag{2}$$

reduces Eq. (1) to

$$A_{rr} + \frac{2}{r}A_r - \Omega^2 A + \left(1 - \frac{1}{(1+A^2)^2}\right)A = 0 \tag{3}$$

where Ω is a constant corresponding to a nonlinear frequency shift. The comprehensive analysis of this eigenvalue problem for the ground state solution ($(dA/dr)_{r=0}$, $A(\infty) = 0$) can be found in Ref. [5]. Numerical solutions of Eq. (3) reveal that the soliton tends to become wider at both low and high intensities, attaining its smallest size at $A_m \approx 1.7$, where $A_m = A(0)$ is the amplitude of the soliton. Notice that for ultrarelativistic strong radiation (i.e. $A_m \gg 1$), the defining equation can be approximated by a linear equation, and a nearly analytical solution can be obtained. Indeed, for the region where $A_m \approx A \gg 1$, the solution of Eq. (3) is $A = A_m \sin(kr)/kr$ (where $k = (1-\Omega^2)^{1/2}$) and in the asymptotic region ($r \to \infty$), the solution decays as $A \sim \exp(-\Omega r)/\Omega r$.

To check the stability of these stationary solitonic solutions to small perturbations, we invoke the well–known stability criterion of Vakhitov and Kolokolov [7]: the solitons are stable if $\partial N/\partial\Omega^2 > 0$, where N

$$N = \int d\mathbf{r}_\perp d\xi |A|^2, \tag{4}$$

the "photon number," is a measure of the pulse energy. It can be readily seen that the photon number N is an integral of motion of Eq. (1). In the rest of the paper the words photon number and the energy invariant will be used interchangeably. For the solutions of Eq. (1), the variation of N with the soliton amplitude is presented in Fig. 1.

Since $\partial\Omega^2(A_m)/\partial A_m > 0$, the stability criterion implies that the solutions are stable provided the condition $\partial N/\partial A_m > 0$ is satisfied. We learn from Fig. 1 that for every $N > N_c = 69$ we have two ground state equilibria. However, it is only the solution with positive slope on the amplitude–energy curve that will correspond to a stable solution. Thus the pulses with a critical energy ($N > N_c$) and an amplitude $A > A_c = 0.7$ may emerge as light bullets in an e–p–i plasma.

The solutions have passed the first crucial test: as long as the pulse energy (N) is an increasing function of the "frequency," the pulse is stable to small perturbations. The next question to investigate is the nonlinear stability and accessibility: what happens if the initial shape and strength of the EM fields (or its fluctuations) is far from the shape and strength of the stable soliton solution? Naturally, this question must be answered before one can investigate their astrophysical and/or their cosmological consequences.

For this purpose, we must investigate the general dynamical properties of the nonstationary solutions of Eq. (1). The principal tool for the investigation of this rather complex problem will have to be computer simulation. However, theoretical analysis is essential in providing guidelines for computer simulations. Our small perturbation stability analysis has already helped us to derive a necessary condition for the emergence of light bullets from a given initially distributed field: the pulse energy must exceed the critical energy, i.e. $N > N_c$. A second important conclusion about the EM beam dynamics can be obtained by examining

the Hamiltonian structure of Eq. (1).

Indeed Eq. (1), along with the integral of motion (4), has the second integral of motion,

$$H = \int d\mathbf{r}_\perp d\xi \left[|\nabla_\perp A|^2 + |\partial_\xi A|^2 - \frac{|A|^4}{1+|A|^2} \right], \tag{5}$$

and H can be viewed as the Hamiltonian. For $H < 0$, a simple manipulation of Eqs. (4) and (5) leads to the following relation, first exhibited by Zakharov *et al.* [8],

$$\max |A|^2 > \frac{|H|}{N} \tag{6}$$

implying that if the characteristic function H is negative for an initial field distribution, then the field intensity has a time independent lower bound. Thus the field self–trapping occurs, the amplitude cannot go below a given value. It is also to be noted that for systems with saturation nonlinearity (like the one under investigation), the so called wave collapse does not take place.

The presence of integrals (4) and (5) permits important statements on the existence and stability of the stationary solutions of Eq. (1); the Vakhitov and Kolokolov criterion is one example. In fact, Eq. (3) can be reformulated in terms of the functional extremum problem $\delta(H + \Omega^2 N) = 0$. If the Hamiltonian H is bounded from below for fixed N then it follows that there is a stationary solution realizing the Hamiltonian minimum, and for the stable solution the second conditional variation of Hamiltonian ($\delta^2 H$) should be positive. This statement leads to the Vakhitov and Kolokolov criterion [9], although the original derivation was quite different.

Bearing in mind this fact one may develop the following picture concerning the fate of an initial field structure : on the positive–slope (negative–slope) side of the curve $N = N(A_m)$ in Fig. 1 (e–line), the field evolves in an "effective potential" (This "particle dynamics" analogy is actually constructed in Sec. III) which has a minimum (maximum). This implies that initial symmetric pulses of radiation with a given energy N and amplitude A_0, situated in the region enclosed by the e–curve, and near the positive–slope part of the curve, will "roll down" to stable equilibria. Emitting a radiation spectrum, that provides some kind

of an attenuation during this process, the fields make damped "oscillations" around the equilibrium minimum and eventually settle as a stable stationary solution—the light bullet. Similarly, any initial state in the region outside but on the right–hand side of the positive slope part of the e–curve, and with $N > N_c$ will roll down again to a stable equilibrium, and depending on the level of energy flowing away from the pulse and the initial pulse amplitude (A_0), it will be either trapped with eventual formation of the light bullet or it will overcome the potential barrier near the unstable equilibrium and will be diffracted away. Notice that this second alternative is forbidden when $H < 0$. In Fig. 1, the h–curve corresponds to zero Hamiltonian. Any initial state above the h–line ($H < 0$) is always trapped. For all other initial states (characterized by $N < N_c$ and/or the states lying to the left–hand side of the negative slope part of the e–curve) in Fig. 1, monotonic diffraction will occur.

It is obvious that the qualitative description presented above does not provide us with details of the time evolution of the light pulses; it rather serves as a guideline for understanding the results of numerical simulation. Numerical simulations of Eq. (1) confirm that if an initial profile of the pulse is close to the stable equilibrium one, the pulse quickly attains the profile of the ground state soliton and propagates for a long distance without distortion. We also find that if the initial pulse is in a domain of parameters corresponding to a profile far from the equilibrium one, it will either focus or defocus to the ground state equilibrium state exhibiting damped oscillations around it. The pulsations are damped due to the appearance of the radiation spectrum.

Since a light bullet has spherical symmetry, in our simulations, we first studied the spatio–temporal dynamics of symmetric, initially Gaussian shaped pulse $|A(r,0)| = A_0 \exp[-r^2/2a_0^2]$ (see Figs. 2–4). In all these cases we assumed that the initial pulses have plane fronts and have the same energy $N = (\pi)^{3/2} A_0^2 a_0^3 = 113$ (the horizontal line in Fig. 1). Although the initial amplitude of the beam is quite different from the amplitude of the corresponding equilibrium solution, in all these cases the light bullets appear after a period of damped pulsations about the equilibrium. In order to have a better view of the initial

focusing (defocusing) before reaching the equilibrium, we have plotted in Fig. 5 the time evolution of the central field amplitude $|A(0,t)|$ for different initial amplitudes $A_0 = 0.5$, 0.8, 1.5 corresponding to the points B, C, and D. All these states eventually reach the equilibrium state. Since N in the body of the pulse is not conserved due to radiation losses, different initial points go to different but nearby equilibria. We have, thus, demonstrated that spherically symmetric pulses of EM radiation always emerge as light bullets even when their initial parameters are quite far from the equilibrium ones. During their evolution towards equilibrium, the pulses maintain spherical symmetry and do not decay spontaneously into filaments.

III. ASYMMETRIC PULSES—POSSIBILITY OF PULSE TRAINS

One would expect that the dynamics of initially asymmetric pulses (i.e., the pulses with different effective lengths in different directions) will exhibit more complicated behavior. We remind the reader that Vakhitov and Kolokolov criterion was derived for general but small perturbations of the ground state solution. Thus the initial pulses of arbitrary symmetry which are very close to the equilibrium, and which satisfy the criterion, will form the light bullets. The next order of business, then, is to study the dynamics of initially asymmetric pulses which are far from the stable equilibrium.

We carried out numerical simulations of Eq. (1) for cylindrically symmetric pulses ($\Delta_\perp = \partial^2/\partial r_\perp^2 + r_\perp^{-1}\partial/\partial r_\perp$, where $r_\perp = (x^2+y^2)^{1/2}$). We consider initially Gaussian–shaped pulses $|A(r_\perp,\xi,0)| = A_0 \exp[-(r_\perp^2/2a_{0\perp}^2 + \xi^2/2a_{0\xi}^2)]$ with plane fronts. The simulations show that the pulses above the critical energy are trapped. The pulse–shapes undergo complicated but damped oscillations around the equilibrium which is closest to the corresponding equilibrium energy of the pulse $N(\sim a_{0\perp}^2 a_{0\xi} A_0^2)$. In spite of the initial asymmetry, in the final stages of evolution, the effective widths of the pulses ($a_\perp^{ef}$ and a_ξ^{ef}) tend to become equal ($a_\perp^{ef}$, $a_\xi^{ef} \to a_{eq}$) corresponding to the symmetric stable pulse, i.e., the light bullet formation does take place. However, formation of the light bullet may take a longer time compared to the

symmetric pulses with the same input energy. In Fig. 6 we display the time history of a pulse (in the transverse and propagation directions) with asymmetric initial conditions $a_{0\perp} = 4$ and $a_{0\xi} = 3$,and with energy $N = 115$. One can clearly see the formation of the light bullet.

The results presented, so far, are based on numerical simulations as well as on the Vakhitov and Kolokolov criterion. It is possible, however, to obtain general dynamical properties of nonstationary solutions for pulses, initially far from the parameters of equilibrium solutions, by using approximate analytical methods like the Variational Principle. This approach gives qualitatively good results provided that the pulses do not undergo structural changes as they evolve. The first standard step is to construct the Lagrangian

$$L = |\nabla_\perp A|^2 + |\partial_\xi A|^2 + \frac{i}{2}(A\partial_t A^* - A^*\partial_t A) - F(|A|^2), \tag{7}$$

where the asterisk denotes complex conjugation, and $F = |A|^4/(1+|A|^2)$. Appropriate variation of the Lagrangian ($\delta L/\delta A^* = 0$) yields Eq. (1) as the Euler–Lagrange equation. In the optimization procedure, the first variation of the variational functional must vanish on a set of suitably chosen trial function. As a trial function, we will use the Gaussian–shaped pulse

$$A = A_0(t)\exp\left[-\frac{x^2}{2a_x^2(t)} - \frac{y^2}{2a_y^2(t)} - \frac{\xi^2}{2a_\xi^2(t)} + i\psi\right] \tag{8}$$

where $\psi = x^2 b_x(t) + y^2 b_y(t) + b_\xi(t)\xi^2 + \phi(t)$. The self–similar evolution of the pulse is parameterized by the t–dependent amplitude A_0, the spatial widths $a_{x,y,\xi}$ and the phase ϕ. The parameters $b_{x,y,\xi}$ are the wave front curvatures. Substituting the expression (8) into Eq. (7) and demanding that the variation of the spatially–averaged Lagrangian with respect to each of these parameters be zero, we get the corresponding set of spatially–averaged Euler–Lagrange equations:

$$\frac{d^2\mathbf{R}}{dt^2} = -\frac{\partial}{\partial\mathbf{R}}V(\mathbf{R}) \tag{9}$$

where $\mathbf{R} = (a_x, a_y, a_\xi)$ is the width–vector, and the effective potential V has the form

$$V = \frac{2}{a_x^2} + \frac{2}{a_y^2} + \frac{2}{a_\xi^2} - \frac{K(A_0^2)}{A_0^2} \tag{10}$$

with the nonlinearity function

$$K(u) = 4\pi^{-1/2} \int_0^\infty dp p^2 F(ue^{(-p^2)}). \tag{11}$$

During the pulse evolution, the photon number (i.e., the energy) is conserved, $A_0^2(t)a_x(t)a_y(t)a_\xi(t) = \text{const}$. Thus, Eqs. (9)–(10) are equivalent to those describing the dynamics of a particle in a three–dimensional potential. Straightforward analysis shows that if the energy of the pulse is less than a critical value ($N < N_c = 65$), $\partial V/\partial \mathbf{R} < 0$ and consequently the force pushes an effective particle in the direction $\mathbf{R}$. In other words, the effective widths of the pulse increase continuously leading to its spreading in all directions. However for ($N > N_c$), the potential attains the shape of a three–dimensional well with two absolute extrema. If the particle is trapped in the well (i.e. the initial widths of the pulse are inside the well) than it will oscillate inside reflecting from the walls of the well. With increasing N, the potential well deepens and the trapping area becomes wider and wider. The equilibrium parameters of the pulse (obtained by using the relation $\partial V/\partial \mathbf{R} = 0$) are $a_{eq} = a_{x,y,\xi} = A_{eq}/K(A_{eq}^2)^{1/2}$, where a_{eq} and A_{eq} are, respectively, the equilibrium width and amplitude of the pulse. Stable equilibrium pertains if $\partial^2 V/\partial^2 R_{eq} > 0$, which is satisfied when the pulse amplitude is larger the than the critical $A_c = 0.6$. This stable and spherically symmetric solution actually represents the light bullet. Dependence of the photon number on the equilibrium amplitude is essentially the same as was obtained by exact numerical computation and presented in Fig. 1. We would like to state that even when the variation solution deviates markedly from the exact one (as it does for larger amplitude cases) the variational results do provide us with a simple understanding of the pulse dynamics. If we artificially introduce a weak attenuation in Eq. (9) (to simulate the radiation loss), then this force will settle the trapped particle in the bottom of the potential well. In other words, the light bullet formation will take place.

IV. SUMMARY AND CONCLUSIONS

Thus using numerical simulation as well as the variational approach, we demonstrated that in e–p–i plasmas, EM pulses with energies above a critical value evolve towards light bullets. Light bullets are exceptionally robust objects that can be generated in a large range of parameters—even far from the stable equilibrium. It is interesting to note that the light bullets can be generated using the mechanism of the modulation instability of self–trapped beams. This type of instability of the lowest–order self–trapped beam was first considered by Zakharov and Rubenchik [10]. Recently, Akhmediev and Soto–Crespo have shown that a longitudinally modulated cylindrical light beam, propagating in an optical medium (with saturating nonlinearity $f = |A|^2/(1 + |A|^2)$), can spontaneously break into a train of light bullets—the optical "machine gun" [11]. It was shown that the irreversible convergence of the solution to the regular train of optical bullets takes place. This convergence shows that light bullets may serve as attractors. Since general dynamical properties of the light bullet formation should not be sensitive to the type of saturating nonlinearity we expect similar behavior of the EM beams in e–p–i plasma as well. Indeed we show,in Figs. [7]–[8] the contour and the three–dimensional plots of the dynamics of a longitudinally (weakly) modulated beam (the beam is a self–trapped two–dimensional stationery solution of the system). As a result of the modulation instability the beam breaks up into a periodic train of light bullets.

The light bullets are already shown to be robust in the sense that they are stable and can be generated from a large variety of initial conditions. To complete the story of their robustness, one must examine their survivability when two of them collide [12]. Initial investigations show that the e–p–i bullets, depending on the initial relative velocities, energy and phase difference, either survive in tact or may pass through each other in a quasi–solitonic fashion, forming a third stationary bullet. For lower velocities, after collision, soliton fusion takes place that leads to formation of light bullets with energy higher than each of colliding components. Similarly, for non central collisions, the light bullets form

rotating structures and depending on initial velocities, bullets will either fuse or diverge from each other.

The simple message is that the class of light bullets is a very stable, accessible and rugged class and their presence will, almost necessarily, play a fundamental part in the long term dynamics of the systems which can sustain such solutions. The e–p–i system seems to be one of those systems which can provide a comfortable substrate for these highly interesting long–lived objects, the carriers of large amounts of mass and energy. It is clear that the bullets and the periodic trains of bullets will impart considerable spatiotemporal inhomogeneity to the medium in which they are born and in which they propagate. One may not be remiss in speculating that these "concentrations" of inhomogeneity could explain the 'clumpiness' observed in the jets which are conjectured to be emanating from black holes or the AGN. In any case, it is time to explore the astrophysical and cosmological significance of these fascinating objects. Such investigations are under progress and will be presented elsewhere.

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FIGURE CAPTIONS

FIG. 1. Energy N as a function of the soliton amplitude A_m (e–line). The h–curve corresponds to zero Hamiltonian. The points B, C, and D indicate, respectively, the initial values for A (for $N = 113$) used to simulate the results shown in Figs. 2-4.

FIG. 2. Field intensity I versus r and t for $A_0 = 0.5$, $N = 113$. Generation of light bullet takes place with initial focusing.

FIG. 3. Light–bullet generation for $A_0 = 0.8$ and $N = 113$.

FIG. 4. Generation of a light bullet with initial defocusing; $A_0 = 1.5$ and $N = 113$.

FIG. 5. Evolution of the peak amplitude of pulses $|A(0,t)|$ as a function of time t, for three different initial intensities (B, C, and D). After a few oscillations, all of these settle down to nearby equilibrium states.

FIG. 6. Intensity contour plot for time evolution of initially asymmetric pulse with $a_{0\perp} = 4$, $a_{0\xi} = 3$ and $N = 115$.

FIG. 7. Intensity contour plot for weakly modulated beam. As a result of modulation instability the beam breaks up into a periodic train of light bullets.

FIG. 8. The same phenomenon as in Fig. 7 displayed in a three dimensional plot.