SHEAR FLOW INDUCED WAVE COUPLINGS IN THE SOLAR WIND

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Abstract

A sheared background flow in a plasma induces coupling between different MHD wave modes, resulting in their mutual transformations with corresponding energy redistribution between the modes (Chagelishvili, Rogava & Tsiklauri (1996)). In this way, the energy can be transfered *from one wave mode to the other*, but energy can also be *added to or extracted from the background flow.* In the present paper it is investigated whether the wave coupling and energy transfer mechanisms can operate under solar wind conditions. It is shown that this is *indeed the case.* Hence, the long-period waves observed in the solar wind at $r > 0.3\,\mathrm{AU}$ *might be generated by much faster periodic oscillations in the photosphere of the Sun.* Other possible consequences for the observable peculiar beat phenomena in the wind and acceleration of the wind particles are also discussed.

Subject headings: Sun: solar wind – MHD waves – shear flow – wave coupling

1. Introduction

Long-period waves are observed in the solar wind at $r > 0.3\,\mathrm{AU}$ (Hollweg (1990)). The correlation between the magnetic field and the velocity fluctuations indicates that the observed waves are outward propagating Alfvén waves. It appears, thus, that the sun radiates Alfvén waves. These waves dissipate and heat heavy ions and protons. In the wind, the heavy ions (such as He^{2+}) are hotter as well as faster than the protons, $T_i/T_p \approx m_i/m_p$, and $V_i - V_p \approx v_A$. The latter condition strongly indicates that the waves could play a significant role in the heating and acceleration of the wind. For this mechanism to be taken seriously, we must find answers to the following questions:1) Since the observed waves have periods of several hours at 1 AU, what is the source of such low-frequency waves; it is hard to associate these waves with known processes in the photosphere, 2) Will there be enough power in the Alfvén waves to drive the high-speed solar wind streams?

In this paper we make an attempt to answer these questions. The coupling brought about by the velocity shear between the modes of a shear-less plasma, and the interactions of these modes with the velocity shear (with a possible energy exchange) will be the two central mechanisms invoked in this effort. The ability of an inhomogeneity like the velocity shear to couple various plasma modes has been common knowledge for a long time. What has not been generally known, however, is the fact that velocity shear, unlike other normal inhomogeneities, causes a profound change in the very nature of the eigenvalue problem associated with linear waves. The corresponding eigenvalue problem becomes non self- adjoint resulting in non-orthogonal eigenfunctions which do not have independent time evolution. Thus the asymptotic normal mode analysis (with exponential time dependence) cannot completely describe the time evolution of the system. In particular, all transient and algebraic processes, which may form an essential part of the dynamics, will be missing in the standard approach. Thus to know the rates of mode conversion or of energy exchange in a sheared plasma, we need a different approach. One, in fact, must go back to solving the initial value problem. Fortunately the mathematical frame work for posing and solving the initial value problem (in sheared flows) is now well developed, and a large number of relevant and interesting physics problems have also been worked out. For want of a better name, we call the new method (to be explained and used in this paper) 'non-modal analysis'.

In several papers, dealing with a wide variety of laboratory, geophysical and astrophysical shear flows, the techniques of the nonmodal analysis have been used to delineate most of the expected shear induced phenomena: the transient amplification and decay of perturbations, energy exchange between a given mode and the background, and the mutual conversion and energy exchange between different modes. A list of representative articles consists of: Chagelishvili, Rogava & Tsiklauri (1996); Chagelishvili, Rogava, & Segal (1994); Rogava, Mahajan,

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& Berezhiani (1996); Chagelishvili & Chkhetiani (1995); and Mahajan, Machabeli & Rogava (1997). In these papers, appropriate conditions for optimal exchange and the rates of exchange are also worked out.

Large empirical evidence (see the numerous Ulysses data in Winterhalter D. et al. 1996) implies that the solar wind is a quite essential example of an inhomogeneous plasma flow capable of sustaining a variety of modes. Are the conditions in the solar wind, then, favourable for the shear-mediated processes to effectively occur? In this paper we demonstrate that the answer to the preceding question is in the affirmative. This realization is likely to have far-reaching consequences: 1) the appearance of the long-period waves at $r > 0.3\,\mathrm{AU}$ may be attributed to the shear-induced transformation of much faster photospheric oscillations, e.g. fast magnetosonic modes, 2) The inter mode-flow energy exchange, jointly with wave transformations, may transfer a part of the wave energy to the flow resulting in the acceleration of the solar wind. Finally, the wave coupling can produce beats (Rogava & Mahajan (1997)), which may be detected in the solar wind. Due to its rather peculiar outward appearance, this effect may serve as a *bona fide* signature of the shear-induced effects.

Before plunging into a mathematical analysis of the solar wind problem, we should note that we consider two different kinds of MHD wave transformations. The first process ivolves the transformation of fast magnetosonic waves (FMW) into Alfvén waves (AW). It was recently shown (Rogava, Mahajan, & Berezhiani (1996), Chagelishvili et al. (1997)) that this process takes place in both electron-proton and electron-positron plasmas. The remarkable quality of this particular kind of velocity shear induced mode transformation is that its *efficiency is exclusively determined by the spectrum of initial perturbations*, i.e. the characteristics of the background flow do not impose any specific constraints.

The second process considered in the present paper is the velocity shear induced transformation of FMW into slow magnetosonic waves (SMW) (Chagelishvili, Rogava & Tsiklauri (1996)). This process is *efficient only when the Alfvén and sound speeds, V_A and V_s, are comparable*. In the solar wind, beyond $\sim 0.5\,\mathrm{AU}$, typically (see e.g. Sturrock (1994)), the ion and electron temperatures are approximately the same ($T_e \approx T_p \approx 10^5 K$), the proton number density is $n_p \approx 10\,\mathrm{cm}^{-3}$, and the background magnetic field is $B_0 \approx 10^{-4} - 10^{-5}\,\mathrm{G}$. For these values,

$$V_s \simeq 2.873 \times 10^6 \left(\frac{T}{10^5\mathrm{K}}\right)^{\frac{1}{2}} \mathrm{cm/s}, \quad (1)$$

$$V_A \simeq 3.449 \times 10^6 \left(\frac{B_0}{5\times 10^{-5}\mathrm{G}}\right) \left(\frac{n_p}{10\mathrm{cm}^{-3}}\right)^{-\frac{1}{2}} \mathrm{cm/s}, \quad (2)$$

yielding

$$\frac{V_A}{V_s} \simeq 1.201 \left(\frac{B_0}{5\times 10^{-5}}\mathrm{G}\right) \left(\frac{T}{10^5\mathrm{K}}\right)^{-\frac{1}{2}} \left(\frac{n_p}{10\mathrm{cm}^{-3}}\right)^{-\frac{1}{2}}, \quad (3)$$

which is of order unity.

This difference in efficiency conditions between the two processes implies that while FMW-AW transformations may take place in the whole extent of the solar wind the FMW-SMW transition may be of secondary importance since it may take place only at $r > 0.5\,\mathrm{AU}$ involving internally generated and/or remnant FMW. The FMW-AW transformations, on the other hand, may serve as a reliable source for the generation of the observed long-period Alfvén waves at $r > 0.3\,\mathrm{AU}$.

We consider a simple low-β fluid plasma to study both the FMW-AW and FMW-SMW processes. In both these transformations, the adiabatic character of the FMW energy evolution, induced by the presence of the velocity shear, may cause a partial transfer of the wave energy to the mean flow thus leading to the acceleration of wind particles. Finally, we show that, in either of these cases, *beat waves* are excited; the latter finding is similar to the recent results of Rogava & Mahajan (1997) where beat waves are seen in the parallel shear flow of a gravitationally stratified compressible neutral fluid.

2. General Formalism

In the present section we model the solar plasma as a sheared MHD flow, and derive the basic set of linearized equations, governing the evolution of small-scale perturbations in this flow. We shall follow the techniques of the *nonmodal analysis* (see for details, e.g. Marcus & Press (1977) and Criminale & Drazin (1990)): we apply a standard coordinate transformation to a comoving shearing reference frame, and convert the system to a set of coupled second order ordinary differential equations (ODEs) describing the temporal evolution of the MHD modes (AW, SMW, and FMW) sustained by the flow.

2.1. Physical model and equations

In order to investigate the essential features of the shear induced mode coupling in the solar wind, we consider a rather simple physical model. A uniform magnetized plane slab plasma is embedded in a constant magnetic field along the z-direction, i.e. $\mathbf{B}_0 = B_0\,\mathbf{e}_z$. The background flow is assumed to be directed along the magnetic field and to vary linearly in the x-direction: $\mathbf{V}_0 = V_0\,\mathbf{e}_z = Ax\,\mathbf{e}_z$, with a constant A (see Fig. 1).

The simplicity of this model flow, in particular, the assumption of the linear profile for the velocity shear, guarantees its applicability to a wide variety of terrestrial and astrophysical shear flows. The reason is simple: for small-scale perturbations with wavelengths much smaller than the length scale of the flow, an arbitrary piecewise linear 'shear profile' can be taken to be approximately linear on the length-scales of interest . The Goldreich-Lynden-Bell model (Goldreich, & Lynden-Bell (1965)), widely used since the sixties for astrophysical shear flows, is a well-known example of such an approximation.

To concentrate on the essentials of our stated physical problem, we consider a special case of the quite general magnetized plasma flow discussed in Chagelishvili, Rogava, & Tsiklauri (1997). By neglecting pressure anisotropy effects, the complications due to the firehose and mirror instabilities will be eliminated, and the resulting dynamics will be limited to an interplay of SMW, FMW, and AW. Small perturbations in the model shear flow obey the following linearized MHD equations:

$$(\partial_t + Ax\partial_z)\widehat{d} + \partial_x u_x + \partial_y u_y + \partial_z u_z = 0, \qquad (4)$$

$$(\partial_t + Ax\partial_z)u_x = -V_s^2\partial_x\widehat{d} + V_A^2\left[\partial_z\widehat{b}_x - \partial_x\widehat{b}_z\right], \qquad (5a)$$

$$(\partial_t + Ax\partial_z)u_y = -V_s^2\partial_y\widehat{d} + V_A^2\left[\partial_z\widehat{b}_y - \partial_y\widehat{b}_z\right], \qquad (5b)$$

$$(\partial_t + Ax\partial_z)u_z = -V_s^2\partial_z\widehat{d} - Au_x, \qquad (5c)$$

$$(\partial_t + Ax\partial_z)\widehat{b}_x = \partial_z u_x, \qquad (6a)$$

$$(\partial_t + Ax\partial_z)\widehat{b}_y = \partial_z u_y, \qquad (6b)$$

$$\partial_x\widehat{b}_x + \partial_y\widehat{b}_y + \partial_z\widehat{b}_z = 0, \qquad (7)$$

where the dimensionless perturbed density and magnetic field are defined by $\widehat{d} \equiv \rho'/\rho_0$ (ρ_0 =const) and $\widehat{\mathbf{b}} \equiv \mathbf{B}'/|\mathbf{B}_0|$ ($|\mathbf{B}_0|$ =const), respectively. The u's are the velocity perturbations, the operator ∂ denotes the partial derivatives, and the constants V_s and V_A refer to the sound and the Alfvén speeds, respectively.

2.2. Coupled oscillations

Following the standard procedure of the nonmodal approach (see e.g. Chagelishvili, Rogava, & Tsiklauri (1997)) we first affect the following change of variables:

$$x' = x, \quad y' = y, \quad z' = z - Axt; \qquad t' = t,$$

and then take a *spatial* Fourier transformation of the perturbed quantities F:

$$F = \int \widehat{F}(k_{x'}, k_{y'}, k_{z'}, t')\, e^{i(k_{x'}x' + k_{y'}y' + k_{z'}z')} dk_{x'} dk_{y'} dk_{z'}.$$

After eliminating b_z by means of Eq. (7), we may write, in dimensionless variables:

$$D^{(1)} = K_x(\tau)v_x + K_y v_y + v_z, \qquad (8)$$

$$v_x^{(1)} = -\varepsilon^2 K_x(\tau)D + [1 + K_x^2(\tau)]b_x + K_y K_x(\tau)b_y, \qquad (9a)$$

$$v_y^{(1)} = -\varepsilon^2 K_y D + [1 + K_y^2(\tau)]b_y + K_y K_x(\tau)b_x, \qquad (9b)$$

$$v_z^{(1)} = -\varepsilon^2 D - Rv_x, \qquad (9c)$$

$$b_x^{(1)} = -v_x, \qquad (10a)$$

$$b_y^{(1)} = -v_y, \qquad (10b)$$

where $F^{(n)}$ denotes the n-th order time derivative of $\widehat{F}$ and the dimensionless variables are defined as: $D \equiv i\widehat{d}$, $b_x \equiv i\widehat{b}_x$, $b_y \equiv i\widehat{b}_y$, $R \equiv A/(V_A k_{x'})$, $\varepsilon \equiv V_s/V_A$, $\tau \equiv V_A k_{x'} t_1$, $K_x(\tau) \equiv k_{x'}/k_{z'} - R\tau \equiv K_{x0} - R\tau$, $K_y \equiv k_{y'}/k_{z'}$, $v_i \equiv u_i/V_A$ $(i = x, y, z)$.

Note that, in these equations, R measures the normalised strength of the velocity shear, the speeds are normalized to the Alfvén speed, and the time is normalized to the Alfvén time. This normalization, different from the one used in Chagelishvili, Rogava, & Tsiklauri (1997) (velocities normalized to the sound speed), is dictated by convenience for studying the cold plasma limit.

By introducing a new variable, $\psi \equiv D + K_x(\tau)b_x + K_y b_y$, we can reduce the system (8–10) to three intercoupled second order ordinary differential equations :

$$\psi^{(2)} + \omega_1^2\psi = \mathcal{C}_1(\tau)b_x + \mathcal{C}_2 b_y, \qquad (11a)$$

$$b_x^{(2)} + \omega_2^2(\tau)b_x = \mathcal{C}_1(\tau)\psi + \mathcal{C}_3(\tau)b_y, \qquad (11b)$$

$$b_y^{(2)} + \omega_3^2 b_y = \mathcal{C}_2\psi + \mathcal{C}_3(\tau)b_x, \qquad (11c)$$

with the following auxiliary notation:

$$\omega_1 \equiv \varepsilon, \qquad (12a)$$

$$\omega_2(\tau) \equiv \sqrt{1 + (1 + \varepsilon^2)K_x^2(\tau)}, \qquad (12b)$$

$$\omega_3 \equiv \sqrt{1 + (1 + \varepsilon^2)K_y^2} \qquad (12c)$$

$$\mathcal{C}_1(\tau) \equiv \varepsilon^2 K_x(\tau), \qquad (13a)$$

$$\mathcal{C}_2 \equiv \varepsilon^2 K_y, \qquad (13b)$$

$$\mathcal{C}_3(\tau) \equiv -(1 + \varepsilon^2)K_y K_x(\tau). \qquad (13c)$$

Equations (11)–(13) describe coupled oscillations with three degrees of freedom. Uncoupled eigenfrequencies and coupling coefficients appearing in (11)–(13) are ω_i and $\mathcal{C}_i$, $(i = 1, 2, 3)$ respectively. The presence of shear in the flow ($R \neq 0$) ensures temporal variability of some of these quantities. However, their dependence on time may be considered as adiabatic when $R \ll 1$. Under certain circumstances, the coupling leads to energy exchange between the oscillators, and to the transformation of fundamental oscillations into each other. These equations are quite general and encompass all three linear MHD modes (AW, SMW, FMW). In the subsequent sections we examine two special cases of interest in the solar context, viz. FMW-AW and FMW-SMW transformations. We find that both processes are optimally favoured in the solar wind.

3. FMW–AW transformations

Since pressure effects are quite subsidiary to this transformation, we may simplify our basic setup further by neglecting the pressure. In this approximation, $\epsilon = 0$ with $\omega_1 = \mathcal{C}_1 = \mathcal{C}_2 = 0$, Eqs. (11)–(13) reduce to the following pair of coupled second order ODE's:

$$b_x^{(2)} + [1 + K_x^2(\tau)]b_x + K_y K_x(\tau) b_y = 0, \qquad (14a)$$

$$b_y^{(2)} + [1 + K_y^2]b_y + K_y K_x(\tau) b_x = 0, \qquad (14b)$$

describing the interaction of the compressional Alfvén (pressureless FMW) and the shear Alfvén waves (see Eq. (79) and Eq. (80) in Rogava, Mahajan, & Berezhiani (1996)). The eigenfrequencies are $\omega_2^2(\tau) \equiv [1 + K_x^2(\tau)]$ and $\omega_3^2 \equiv (1 + K_y^2)$, and the coupling coefficient $\mathcal{C}_3(\tau) \equiv K_y K_x(\tau)$. The system under investigation is mathematically equivalent to a pair of linear pendulums, connected by a spring with a varying stiffness coefficient. The length of one of these pendula also varies in time. Strictly speaking, due to this temporal variation, the canonical theory of coupled oscillations is no longer valid. However, when the system parameters vary slowly (adiabatically), as they do when $R \ll 1$, the standard theory of coupled oscillations may serve as a useful guide in understanding and interpreting the inherent physical processes. A brief overview of some useful facts from this well-known part of Classical Mechanics is presented in the Appendix.

In Eq. (14), the time dependence of the effective coupling coefficient $\mathcal{C}_3(\tau)$ is a direct consequence of the shear in the mean flow velocity. The coupling coefficient is also proportional to the wave vector component in the y-direction (K_y in our dimensionless notation) so that for $k_{y'} = 0$, the two waves decouple, even in the presence of a shear flow. Since the frequency ω_2 also varies in time, the presence of shear ($R \neq 0$) leads to a temporal variability of one of the uncoupled eigenfrequencies ($\omega_2(\tau)$) in addition to that of the coupling coefficient $C_3(\tau)$.

A remarkable feature of this particular kind of velocity shear induced wave coupling is that its effectiveness depends mainly on the wave characteristics (the values of K_{x0} and K_y, which determine the initial orientation of the wave vector) and *not explicitly* on the intrinsic physical characteristics of the flow. However, a dependence on some flow characteristics (average value of background flow velocity V_0 and the Alfvén velocity) is hidden in the shear parameter R. An estimate for R reads as $R \simeq (V_0/V_A)(l_z/L)$, with L signifying a characteristic length scale of the shear flow. Since we are considering only small-scale perturbations $l_z \equiv 1/k_{z'} < L$, it is clear that for both the *slow* (with $V_0 \simeq 3 \times 10^5\,\mathrm{m/sec}$), and the *fast* (with $V_0 \simeq 6 - 7 \times 10^5\,\mathrm{m/sec}$) solar wind flows, we have $R \ll 1$.

The 'normal frequencies' of these oscillations, calculated by the standard formula (see Appendix), are

$$\Omega_F^2(\tau) \equiv \Omega_+^2 = 1 + K_y^2 + K_x^2(\tau), \qquad (15a)$$

$$\Omega_A^2 \equiv \Omega_-^2 = 1, \qquad (15b)$$

and may easily be identified, respectively, as *compressional* and *shear Alfvén wave* (equivalently, FMW and AW) frequencies. The frequency of the FMW is *time dependent* and when $R \ll 1$, it varies adiabatically.

To demonstrate explicitly (for example, by a numerical solution of Eqs.(14)) the presence or absence of 'mode-transformation' it is essential to excite, initially, one of the 'pure' normal modes, and then observe the evolution of the entire system. This comprises a problem of the proper selection of *initial conditions*, which is readily resolved by means of a technique known in the mathematical theory of coupled mechanical oscillations (see Appendix).

We are now ready to present the results of the numerical solution of the initial value problem posed in Eq.(14). For Fig. 2, the relevant parameters are: $K_{x0} = 10$, $K_y = 0.1$, $R = 0.1$. From Fig. 2a, displaying the solution for $b_y(\tau)$, the conversion of an initially pure FMW into an AW around the time $\tau = \tau_* \equiv K_{x0}/R$, can be clearly seen. The energy history of this transformation is illustrated in Fig. 2b, which shows the temporal evolution of the perturbation energy

$$\mathcal{E}_{FA}(\tau) \equiv \frac{1}{2}\left[|v_x|^2 + |v_y|^2 + |v_z|^2 + |b_x|^2 + |b_y|^2 + |b_z|^2\right]. \qquad (16)$$

Note that in the shearless limit the energy is a conserved quantity (Rogava, Mahajan, & Berezhiani (1996)), while when $R \neq 0$, the temporal evolution of $\mathcal{E}_{FA}$ proceeds adiabatically. The 'adiabatic behavior' of the modes implies that they should normally follow the dispersion curves of their own: the spectral energy density of either the FMW or AW should be proportional to its corresponding normal frequency $\mathcal{E}_\pm \sim \Omega_\pm$ (Chagelishvili, Rogava & Tsiklauri (1996), Rogava, Mahajan, & Berezhiani

(1996), Chagelishvili, Rogava, & Tsiklauri (1997)). This mode of energy evolution, however, will not pertain in the 'degeneracy region' (DR, see appendix), where efficient transformation of one wave into the other may occur. Checking necessary conditions for efficient coupling (see Appendix), we learn that the difference $\Omega_+(\tau) - \Omega_-(\tau)$ attains its minimum value at $\tau = \tau_*$. It is, therefore, evident that the DR is in the neighborhood of τ_* (at times, when $0 < |K_x(\tau)| < 1$). In the vicinity of $\tau = \tau_*$, $K_y < 1$ leads to the most efficient mode coupling and, hence, to the possibility of mutual transformation of the modes. As regards the second ('slow passing') condition derived in the appendix, it readily holds in the DR when $R \ll 1$.

The FMW–AW transformation is not complete but only partial; this is shown in Fig. 2 (b). We see that the energy graph is not symmetric: the rate of adiabatic decrease in energy up to $\tau = \tau_*$ is greater than the corresponding rate of growth at $\tau > \tau_*$, indicating that the initial FMW was only partially converted into the AW, which contributed to the decrease of the efficiency with which the wave extracts energy from the mean flow at $\tau > \tau_*$. In other words, the FMW transfers energy to the mean flow until $\tau < \tau_*$ and cannot, afterwards, (at $\tau > \tau_*$) extract back the same amount of energy because the FMW has been partially transformed into an AW. The latter mode has a constant fundamental frequency $\Omega_- \equiv \Omega_A$ and, therefore, is not able to extract energy from the mean flow via the shear-induced process. This picture allows one to speculate that FMW–AW transitions may *contribute to the net acceleration of solar wind particles*.

Yet another impressive shear-induced phenomenon, which arises in this setup is the excitation of beat modes (Fig. 3). Similar kinds of shear induced beats among internal gravity and sound waves were originally reported in Rogava & Mahajan (1997).

In the MHD system, the subject of the present paper, the 'beat regime' is realized when $R \ll K_{x0} \ll 1$ ($\varepsilon \simeq 1$); the normal frequencies $\Omega_F(\tau)$ and Ω_A are, then, almost equal to each other. Hence, the coupling is inherently efficient and conditions are favourable for the excitation of 'beats.' A representative example of such a solution is presented in Fig. 3 for $K_{x0} = 0.1$, $K_y = 0.1$, and $R = 2 \times 10^{-4}$. Notice that the beat frequency $\Omega_b \equiv \Omega_F(\tau) - \Omega_A$ is variable, and varies in such a way that the beat period becomes smaller and smaller when τ exceeds τ_*.

4. FMW–SMW transformations

In the Introduction we have already shown that conditions in the solar wind environment are also favourable for FMW–SMW transformations. For an explicit demonstration, let us study another simple case, and consider 2D perturbations in the *presence of a thermal isotropic pressure*. In other words, let us investigate our basic set (11)-(13) with $K_y = 0 = b_y$. The remaining system turns out to be yet another interesting pair of intercoupled second order ODEs :

$$\psi^{(2)} + \varepsilon^2 \psi = \varepsilon^2 K_x(\tau) b_x, \tag{17a}$$

$$b_x^{(2)} + [1 + (1+\varepsilon^2) K_x^2(\tau)] b_x = \varepsilon^2 K_x(\tau) \psi. \tag{17b}$$

Chagelishvili, Rogava & Tsiklauri (1996), dealing with a similar system, have shown that the condition $\varepsilon \simeq 1$, which physically corresponds to the approximate equality of Alfvén and acoustic Mach numbers, ensures effective mutual transformations of SMW and FMW into one another with corresponding energy exchange between them. Interestingly enough, the solar wind parameters are precisely in the regime for optimal coupling (see introduction).

In the solar wind context we are primarily interested in establishing the possibility of FMW–SMW transitions. We performed a numerical analysis of Eq. (17) for the case $\varepsilon = 1.2$ ($K_{x0} = 10$, $R = 0.1$). Fig. 4a shows the temporal evolution of the function $v_z(\tau)$, which apparently reveals that the initially pure FMW is partially converted into a much slower oscillating SMW. Note that the transformation is not complete: the resulting wave is a mixture of a SMW and a FMW.

The situation becomes clearer on examining the evolution of the dimensionless total energy density of the perturbations (Chagelishvili, Rogava & Tsiklauri (1996)),

$$\mathcal{E}_{FS} \equiv (|v_x|^2 + |v_z|^2)/2 + \varepsilon^2 |D|^2/2 + (|b_x|^2 + |b_z|^2)/2. \tag{18}$$

In Fig. 4b the temporal evolution of $\mathcal{E}_{FS}(\tau)/\mathcal{E}_{FS}(0)$ is plotted for the same sample of parameters. As expected, the initially pure FMW follows its adiabatic ($\mathcal{E}_{FS} \sim \Omega_F$) route of evolution up to the moment $\tau = \tau_*$. However, after the wave passes through the DR, and is partially converted into SMW, the slope of its energy curve (energy increases at $\tau > \tau_*$) is noticeably smaller. This behaviour indicates that at $\tau > \tau_*$ we actually have some mixture of a FMW and a SMW. This means also that the initial perturbation, which is converted into a SMW, has transfered a part of its energy to the background flow contributing its fair share to the acceleration of the solar wind particles.

Beats are present in this case too. Figure 5 displays a vivid example of the excited beat modes (numerical solution is obtained for $K_{x0} = 10^{-2}$, $R = 10^{-4}$, and $\varepsilon = 1$).

5. Discussion

In this study, we had set out to find answers to a couple of questions: 1) what is the source of long-period Alfvén waves in the distant solar wind, and 2) could Alfvén-like modes accelerate the solar wind, and if yes, how? Our conjecture is that the short-period fast magnetosonic waves (amply produced in the photosphere), acting through the agency of the newly discovered velocity shear induced physical processes, could convert to long-period Alfvén waves as well as impart energy to the wind flow. To test this conjecture, we have examined if the shear mediated processes of mode-conversion (FMW to AW and FMW to SMW), and of energy exchange with the flow (FMW, SMW and the flow) could efficiently take place in the solar wind. The preliminary results are very encouraging. We found that the conditions for occurence of these processes are, in fact, very close to optimal. For reasons stated earlier, we considered the following two, rather simple, subclasses of velocity shear induced processes:

1. *FMW–AW transitions* (with thermal pressure effects ignored). We found that the velocity shear effectively couples FMW and AW modes, and for weakly 3D perturbations (i.e. those with $K_y \equiv k_{y'}/k_{z'} < 1$), ensures a partial transformation of FMWs into AWs. We argue that this process may be the source for the long-period Alfvén waves which are actually observed at $r > 0.3\,\mathrm{AU}$ (Hollweg (1990)). It was demonstrated that a small shear in the wind velocity can easily yield AWs in the corona with a period 10 times longer than the FMWs excited in the photosphere. The FMWs are converted into AWs around the moment when their energy density tends to its minimum value. This allows us to argue, also, that FMWs transfer a part of their energy to the background wind flow while they are converted into AWs. Thus the process may be a credible source of solar wind particles acceleration.

2. We found that FMWs may also be transformed into SMWs in the solar wind. Conditions for the effectiveness of this channel of wave transformations are more distinctive—it is necessary to have approximately equal values of the Alfvén and acoustic Mach numbers in the region of the wind. Fortunately, as simple estimates show, nature seems to favour this condition: such an equality ($V_A \simeq V_s$) is likely to exist for the known wind parameters at distances greater then $0.5\,\mathrm{AU}$ raising the possibility of an FMW–SMW transition. Like in the previous case FMWs manage to impart a considerable part of their energy to the mean flow while they are converted into SMWs, so that the process may provide yet another channel of wind acceleration.

In our investigations, we have also unearthed another remarkable 'shear' effect— the excitation of beat modes. It is shown to happen in both the FMW-AW and FMW-SMW regimes. This phenomenon, due to its spectacular appearance, may be gainfully employed as a reliable observational signature of velocity shear induced processes.

After these very promising indications of the importance of velocity shear induced effects in the solar wind we are eager to further extend the investigations. We intend to launch more profound and detailed studies based on more realistic plasma models. It is hoped that accurate quantitative statements regarding these transitions will follow. We would, then, be in a stronger position to judge if this mechanism can really account for the long-period Alfvén waves that are observed in the solar wind. As regards energy exchange processes, again, more detailed studies are required to find out to what extent this kind of energy transition takes place in the solar wind. In a forthcoming paper, based on the general equations (11)–(13) derived in the present paper, we will consider a general 3D setup and will look for mutual transformations of Alfvén and both magnetosonic modes, so that all transitions between linear MHD waves wil be investigated under typical solar wind conditions. Further generalizations of this model, including different T_i and T_e (present in the solar wind), pressure anisotropy, and finally the kinetic effects will also be studied.

Recent observational results Grall et al. (1996) indicate that the acceleration of the polar wind is almost completed by $10\,R_{\odot}$. This circumstance implies that the acceleration of the solar wind and and the heating of the solar corona occur in approximately the same region, the very region where velocity shear induced mutual transformations of MHD waves are anticipated. In this context it seems reasonable to beleive that the velocity shear induced mode conversion processes may be important also for the understanding of coronal expansion.

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A. Coupled Oscillations

The term ‘coupled oscillations’ refers to the case where two (or more) oscillators, *on equal footing*, are coupled tightly so that the motion of each one of them is affected by the other(s). Energy can flow from one oscillator to the other in contrast to the case of ‘forced oscillations’, where the feedback of energy from the driven system to the driver can be neglected.

The mathematical description of the motion of two coupled linear oscillators leads to the following pair of second order, ordinary differential equations:

$$F_1^{(2)} + \omega_1^2 F_1 + \mathcal{C} F_2 = 0, \tag{A1}$$
$$F_2^{(2)} + \omega_2^2 F_2 + \mathcal{C} F_1 = 0, \tag{A2}$$

where ω_1 and ω_2 are the oscillator *eigenfrequencies*, and $\mathcal{C}$ is the corresponding *coupling coefficient*. The above system describes general linear oscillations of coupled oscillators (Morse (1981), Magnus (1976)). Fixing one of the oscillators results in a simple harmonic oscillation of the other, but allowing both oscillators to move simultaneously results in a motion that is usually (with arbitrary initial conditions) *not periodic* (Morse (1981)).

However, for constant eigenfrequencies and coupling coefficient, it can be shown that the general solution of the system can always be represented as a combination of the *normal modes*:

$$F_1 = F_+ \cos(\Omega_+ t - \phi_+) + F_- \cos(\Omega_- t - \phi_-), \tag{A3}$$
$$F_2 = \sigma_+ F_+ \cos(\Omega_+ t - \phi_+) + \sigma_- F_- \cos(\Omega_- t - \phi_-), \tag{A4}$$

where the *fundamental* or *normal frequencies* of the coupled oscillations, $\Omega_\pm$, are determined by

$$\Omega_\pm^2 \equiv \frac{1}{2}\left[(\omega_1^2 + \omega_2^2) \pm \sqrt{(\omega_1^2 - \omega_2^2)^2 + 4\mathcal{C}^2}\right]. \tag{A5}$$

The auxiliary quantities $\sigma_\pm$ in Eq. (A4) relate the oscillation amplitudes of the two normal modes to each other and are defined as (see e.g. Magnus (1976)):

$$\sigma_\pm \equiv \frac{\Omega_\pm^2 - \omega_1^2}{\mathcal{C}} = \frac{\mathcal{C}}{\Omega_\pm^2 - \omega_2^2}, \tag{A6}$$

while the $\phi_\pm$ are the initial phases of the coupled oscillators.

In a coupled system described by Eqs. (A1)–(A2), it is always possible (with properly chosen initial conditions) to excite a simple harmonic motion in which *both* oscillators have *the same* frequency, viz. one of the fundamental frequencies Ω_+ or Ω_-. From Eqs. (A3)–(A4) it is easily seen that this regime is established when either F_+ or F_- is equal to zero. It immediately follows that (Magnus (1976)):

- $F_+ \neq 0$ and $F_- = 0$, when $F_2 = \sigma_+ F_1$, and $\partial_t F_2 = \sigma_+ \partial_t F_1$
- $F_- \neq 0$ and $F_+ = 0$, when $F_2 = \sigma_- F_1$, and $\partial_t F_2 = \sigma_- \partial_t F_1$.

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Note also that the value of Ω_- is smaller than either ω_1 or ω_2, while the value of Ω_+ is larger than both ω_1 and ω_2. In other words, *'coupling always spreads apart the natural frequencies'* (Morse (1981)).

When the eigenfrequencies and/or coupling coefficient $\mathcal{C}$ of the coupled oscillating system vary in time, and when the variation is slow or *adiabatic* (i.e. $|\Omega_\pm(\tau)^{(1)}| \ll \Omega_\pm^2(\tau)$), then the system exhibits notable mutual transformations of normal oscillations with corresponding energy transfer between them (Kotkin & Serbo (1971), Chagelishvili, Rogava & Tsiklauri (1996), Rogava, Mahajan, & Berezhiani (1996)). The *mechanical* example of the oscillatory system, governed by this kind of equations, is the system of two coupled pendulums with slowly (adiabatically) variable lengths (i.e. eigenfrequencies) and the interpendulum coupling coefficient. Kotkin & Serbo (1971), while considering the similar mechanical problem discovered two necessary conditions for the effectiveness of the energy exchange between the weakly coupled pendulums:

- (A) There should exist a so called "degeneration region," (DR) where $|\Omega_+^2 - \Omega_-^2| \leq |\mathcal{C}(\tau)|$. In other words, in the case of weak coupling this condition implies that $\Omega_- \approx \Omega_+$, which means that the maximum energy exchange between the pendulums occurs when they have approximately the same length.
- (B) The DR should be 'passed' slowly, i.e. in a sufficiently long time interval exceeding the 'beat period': $|\Omega_\pm^{(1)}(\tau)| \ll |\mathcal{C}(\tau)|$.

Certainly, these conditions are valid for arbitrary oscillatory systems, governed by the same kind of differential equations. So, they can be directly applied in the analyses of velocity shear induced intermode (or interwave) couplings.

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Figure Captions

Fig. A1.— Simple uniform slab model with linearly varying background flow.

Fig. A2.— Time dependence of $b_y(\tau)$ for $K_{x0} = 10$, $R = 0.1$, and $K_y = 0.1$ (Fig. 2a). The graph [numerical solution of Eqs. (14)] represents the partial transformation of a FMW, with fundamental frequency $\Omega_F(\tau)$, into a AW with frequency $\Omega_A = 1$ (more than 10 times lower!). Fig. 2b displays the time dependence of the energy $\mathcal{E}_{FA}(\tau)/\mathcal{E}_{FA}(0)$.

Fig. A3.— Beat waves [FMW–AW] displayed for $b_x(\tau)$ and $b_y(\tau)$ when $K_{x0} = 0.1$, $R = 2 \times 10^{-4}$, and $K_y = 0.1$.

Fig. A4.— The temporal evolution of $v_z(\tau)$ for an initially pure FMW which is partially transformed into a SMW. The graph represents results of numerical solution for $K_{x0} = 10$, $R = 0.1$, and $\varepsilon = 1.2$ (Fig. 4a). Fig. 4b shows time dependence of the energy $\mathcal{E}_{FS}(\tau)/\mathcal{E}_{FS}(0)$.

Fig. A5.— Beat waves [FMW-SMW] displayed for $D(\tau)$ and $b_x(\tau)$ when $K_{x0} = 10^{-2}$, $R = 10^{-4}$, and $\varepsilon = 1$.