“Shear-Langmuir Vortexes:” New elementary mode of plasma collective behavior

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Abstract

Linear evolution of electrostatic perturbations in an unmagnetized, zero temperature, two-component plasma *shear* flow is studied. It is shown that the velocity shear induces, due to the non-normality of linear dynamics, a new elementary mode of plasma *non-periodic* collective behavior—“Shear-Langmuir Vortexes”—with vortical motion of plasma species, characterized by intense energy exchange with the mean flow.

1 Introduction

The subject of the stabilty of shear flows remains an enthralling problem of fluid mechanics and plasma physics. While Classical stability theory (*normal mode approach*) has been prosperous in dealing with different kinds of shear flows, yet in some, quite basic and important cases, the approach has serious problems, evoked by the non-self-adjoint character of the governing equations.[1−3].

The core of the problems lies in shortcomings of the normal mode approach, which only allows to examine asimptotic stability of the flow. Whereas non-self-adjoint differential equations own eigenfunctions that are not mutually orthogonal and therefore the eigenfunctions may strongly interfere, ensuring algebraic behavior for early time[4]. Actually, recent investigations[1−3,5,6] (with basic origins ascribed to Lord Kelvin[7] and W. Orr[8,9]) revealed that a superposition of decaying normal modes may grow initially and that this *transient* growth can be siginificant even for subcritical values of Reynolds numbers.

In physical terms it means that in parallel shear flows, where above mentioned mathematical nuances are influential, one should look attentively for new modes of disturbance dynamics (new modes of particle collective behavior), which might easily been overlooked in the framework of traditional analysis. A convenient tool of such survey is so-called *nonmodal approach* (due to Kelvin[7]), which implicates a change of independent variables from a laboratory to a moving frame and a study of temporal evolution of *spatial Fourier harmonics* (SFH's) of disturbances. This approach is well-adjusted for tracing of a time history of non-exponentially evolving disturbances. Nonmodal analysis has already been applied to several important kinds of hydrodynamical[5,6,10−14], hydromagnetic[15−21] and plasma[22−23] shear flows and helped to espy some unexpected and basic phenomena associated with *linear dynamics* of disturbances.

In particular, transiently (algebraically) growing solutions, characterised by vortical motion of particles, are common for a wide range of shear flows: accretion shear flows[10], MHD shear flows of standard[16] and electron-positron[19] plasmas. In[23] it was shown that velocity shear induces excitation of a completely *new class* of non-periodic, electrostatic perturbations with vortical motion of the plasma *ion* component. These perturbations with their notable ability for effective energy exchange with a mean flow may play a *dominant* role in the linear dynamics of perturbations.

In the present article we shall demonstrate the existence of yet another mode of plasma transient behavior for the most *foundational* kind of arbitrary two-component, nonrelativistic plasma flow with zero thermal temperature of species. The paper is arranged in the following fashion: Next section deals with generalized foramlism of the phenomena. We write down linearized electrostatic equations for the shear flow and implementing the nonmodal approach derive general system of ODE's, describing interplay of conventional plasma features of such simple system with velocity shear induced effects. The third section is dedicated to the study of a simplified, *zero mutual streaming*, case and disclosure a new class of nonperiodic solutions, describing shear-induced vortical motion of plasma species. We call these solutions "Shear-Langmuir Vortextes" and discuss basic features of this new, elementary mode of plasma shear-induced *non-periodic* collective behavior in the final section.

2 General Formalism

Consider a two-component plasma in which each specie individually behaves like cold fluids and are completely characterized by proper number densities n_s, lab-frame velocities $\mathbf{V}_s$, and temperatures T_s, satisfying $kT_s \ll m_s c^2$ (so that thermal motions are negligible). Here and henceforth subscript s refers to a number of specie $[s = 1, 2]$. We assume that in equilibrium plasma is quasineutral (electric charges q_s and mean number densities N_s obey the expression $q_1 N_1 + q_2 N_2 = 0$), and the background mean velocities of the species may be different but are equally and linerly sheared:

$$\vec{\mathcal{V}}_s = \{\mathcal{V}_s + Ay,\ 0\}, \tag{1}$$

further, without loss of generality, we can take $\mathcal{V}_2 > \mathcal{V}_1$.

Furthermore, adopting the standard and the simplest layout[24], we assume that there is no magnetic field in the unperturbed state. We consider only nonrelativistic velocities and neglect the effects of the magnetic field, produced by the particle streams. Taking account of the electrostatic nature of oscillations we can write for electric potential:

$$\Delta\varphi = -4\pi(q_1 n_1 + q_2 n_2), \tag{2}$$

while linearized equations of motion for the species give:

$$\mathcal{D}_s V_{sx} + AV_{sy} = -(q_s/m_s)\partial_x\varphi, \tag{3}$$

$$\mathcal{D}_s V_{sy} = -(q_s/m_s)\partial_y\varphi, \tag{4}$$

$$\mathcal{D}_s V_{sz} = -(q_s/m_s)\partial_z\varphi, \tag{5}$$

where $\mathcal{D}_s \equiv \partial_t + (\mathcal{V}_s + Ay)\partial_x$.

The system (2–5) is completed by linearized forms of continuity equation for both species:

$$\mathcal{D}_s n_s + N_s(\partial_x V_{sx} + \partial_y V_{sy} + \partial_z V_{sz}) = 0. \tag{6}$$

Note that $\mathcal{D}_s$'s obey the following commutation relations:

$$[\mathcal{D}_s^n,\ \partial_y] = -An\mathcal{D}_s^{n-1}\partial_x, \tag{7a}$$

$$[\mathcal{D}_s^n,\ \Delta] = -2An\mathcal{D}_s^{n-1}\partial_x\partial_y - A^2n(n-1)\mathcal{D}_s^{n-2}\partial_x^2, \tag{7b}$$

Introducing auxiliary notation $\delta \equiv q_2m_1/q_1m_2$, $\omega_s \equiv (4\pi q_s^2 N_s/m_s)^{1/2}$ (the latter are plasma frequencies of the species) and applying $\mathcal{D}_1$ and $\mathcal{D}_2$ to both components of (6), respectively, we get:

$$\mathcal{D}_1^2 n_1 + \omega_1^2 n_1 = (m_2/m_1)\omega_2^2 n_2 + 2AN_1\partial_x V_{1y}, \tag{8a}$$

$$\mathcal{D}_2^2 n_2 + \omega_2^2 n_2 = (m_1/m_2)\omega_1^2 n_1 + 2AN_2\partial_x V_{2y}. \tag{8b}$$

Note that $\mathcal{D}_s$ operators possess the following conserved quantities:

$$\mathcal{D}_s\{N_s[(\partial_x^2 + \partial_z^2)V_{sy} - \partial_y(\partial_x V_{sx} + \partial_z V_{sz})] + A\partial_x n_s\} = 0, \tag{9}$$

which, after taking of x-th derivative and rearranging of terms, leads to

$$\mathcal{D}_s\{N_s\Delta V_{sy} + \mathcal{D}_s\partial_y n_s + 2A\partial_x n_s\} = 0. \tag{10}$$

The latter expressions, by taking into account (7a) and (7b), help to derive from (8a) and (8b):

$$\Delta\mathcal{D}_1^3 m_1 n_1 + \mathcal{D}_1\Delta\{m_1\omega_1^2 n_1 - m_2\omega_2^2 n_2\} = 0, \tag{11a}$$

$$\Delta\mathcal{D}_2^3 m_2 n_2 + \mathcal{D}_2\Delta\{m_2\omega_2^2 n_2 - m_1\omega_1^2 n_1\} = 0, \tag{11b}$$

Note that $m_1\omega_1^2 n_1 - m_2\omega_2^2 n_2 = 4\pi q_1 N_1(q_1n_1 + q_2n_2) = -q_1N_1\Delta\varphi$. Hence above equations may be rewritten in a remarkably simple way:

$$\Delta\mathcal{D}_1^3(q_1n_1) + \omega_1^2\mathcal{D}_1\Delta(q_1n_1 + q_2n_2) = 0, \tag{12a}$$

$$\Delta\mathcal{D}_2^3(q_2n_2) + \omega_2^2\mathcal{D}_2\Delta(q_1n_1 + q_2n_2) = 0, \tag{12b}$$

This pair of *exact* partial differential equations defines the mathematical background of the problem.

In order to initiate standard nonmodal analysis[11,18] let us make the substitution of variables $x' = x - (\mathcal{V}_1 + Ay)t$, $y' = y$, $z' = z$, $t' = t$, that leads to the corresponding change of operators $[\Delta\mathcal{V} \equiv \mathcal{V}_2 - \mathcal{V}_1]$: $\mathcal{D}_1 = \partial_{t'}$, $\mathcal{D}_2 = \partial_{t'} + \Delta\mathcal{V}\partial_{x'}$, $\partial_x = \partial_{x'}$, $\partial_y = \partial_{y'} - At\partial_{x'}$, and $\partial_z = \partial_{z'}$. The Fourier transform in the new spatial variables, $F = \int dk_{x'}dk_{y'}dk_{z'}\hat{F}(k_{x'}, k_{y'}, k_{z'}, t)exp[i(k_{x'}x' + k_{y'}y' + k_{z'}z')]$, now converts (2–6) to a set of first order, ordinary differential equations (ODE's) for the evolution of the spatial Fourier harmonics (SFH). Let us introduce dimensionless notations: $D_1 \equiv i\hat{n}_1/N_1$, $D_2 \equiv i\hat{n}_2/N_2$, $\beta_0 \equiv k_{y'}/k_{x'}$, $\gamma \equiv k_{z'}/k_{x'}$, $R \equiv A/ck_{x'}$, $\tau \equiv ck_{x'}t'$, $\beta(\tau) \equiv \beta_0 - R\tau$, $\vec{U}_s \equiv \hat{\mathbf{V}}_s/c$, $\varepsilon \equiv \Delta\mathcal{V}/c$, $\Phi \equiv i|q_1|\hat{\varphi}/m_1c^2$, $W_s \equiv \omega_s/ck_{x'}$. We get:

$$[1+\beta^2+\gamma^2]\Phi = W_1^2[D_1 - D_2], \tag{13}$$

$$\partial_\tau U_{1x} + RU_{1y} = -\Phi, \tag{14a}$$

$$\partial_\tau U_{1y} = -\beta\Phi, \tag{14b}$$

$$\partial_\tau U_{1z} = -\gamma\Phi, \tag{14c}$$

$$(\partial_\tau + i\varepsilon)U_{2x} + RU_{2y} = -\delta\Phi, \tag{15a}$$

$$(\partial_\tau + i\varepsilon)U_{2y} = -\delta\beta\Phi, \tag{15b}$$

$$(\partial_\tau + i\varepsilon)U_{2z} = -\delta\gamma\Phi, \tag{15c}$$

$$\partial_\tau D_1 = U_{1x} + \beta U_{1y} + \gamma U_{1z}, \tag{16}$$

$$(\partial_\tau + i\varepsilon)D_2 = U_{2x} + \beta U_{2y} + \gamma U_{2z}. \tag{17}$$

In the framework of this representation the integrals of $\mathcal{D}_s$ operators, given by (9), reduce to the following pair of algebraic relations:

$$(1+\gamma^2)U_{1y} - \beta(U_{1x} + \gamma U_{1z}) = RD_1 + C_1, \tag{18a}$$

$$(1+\gamma^2)U_{2y} - \beta(U_{2x} + \gamma U_{2z}) = RD_2 + C_2 e^{-i\varepsilon\tau}, \tag{18b}$$

where C_1 and C_2 are some constants.

Next, we can reduce (8) to the following second order ODE's:

$$\partial_\tau^2 D_1 + W_1^2 D_1 = W_1^2 D_2 - 2RU_{1y}, \tag{19a}$$

$$(\partial_\tau + i\varepsilon)^2 D_2 + W_2^2 D_2 = W_2^2 D_1 - 2RU_{2y}, \tag{19b}$$

and combining (13-19) derive the following pair of explicit equations:

$$\partial_\tau^2 P + \left[W_1^2 + \frac{3R^2}{\mathcal{K}^4}\right]P = W_1^2 e^{-i\varepsilon\tau}Q - \frac{2RC_1}{\mathcal{K}^3}, \tag{20a}$$

$$\partial_\tau^2 Q + \left[W_2^2 + \frac{3R^2}{\mathcal{K}^4}\right]Q = W_2^2 e^{i\varepsilon\tau}P - \frac{2RC_2}{\mathcal{K}^3}. \tag{20b}$$

where $\mathcal{K}(\tau) \equiv \sqrt{1+\beta^2(\tau)+\gamma^2}$, and $P \equiv D_1\mathcal{K}^{-1}$, $Q \equiv D_2 e^{i\varepsilon\tau}\mathcal{K}^{-1}$.

These equations constitute a basic set of ODE's describing temporal evolution of SFH in the two-component plasma shear flow with zero temperature of the species. Evidently in the shearless ($R=0$, $C_s=0$) limit the equations encompass traditional plasma effects associated with this simple kind of electrostatic plasma system: plasma oscillations and (under certain, well-known, conditions) two stream instabiity. One can easily surmise that the insertion of velocity shear makes the system much complex and its behavior will be determined by the interplay of those standard plasma effects with specific velocity shear induced effects: (a) variation of a wave number of each SFH in time (due to the effect of the shearing background on the wave crest); and (b) appearance of algebraic (transient) solutions.

Mathematically the first effect is exposed by the temporal variation of the function $\mathcal{K}(\tau)$. As regards transient, vortical solutions (the main subject of the present paper) their appearance is connected with the existence of nonhomogeneity (terms proportional to C_1 and C_2) in (20).

3 “Shear-Langmuir” Vortexes

In order to examine the latter effect *per se* let us simplify further the layout and consider zero mutual streaming ($\mathcal{V}_1 = \mathcal{V}_2 = \mathcal{V}$, $\varepsilon = 0$) case. The initial system (2–6) may be elegantly rewritten through one-fluid variables: perturbed charge density $\varrho \equiv q_1 n_1 + q_2 n_2$ and current density $\mathbf{J} \equiv q_1 N_1 \mathbf{V}_1 + q_2 N_2 \mathbf{V}_2$. The result is:

$$\Delta\varphi = -4\pi\varrho \tag{21}$$

$$\mathcal{D}J_x + AJ_y = -(\omega_p^2/4\pi)\partial_x\varphi, \tag{22a}$$

$$\mathcal{D}J_y = -(\omega_p^2/4\pi)\partial_y\varphi, \tag{22b}$$

$$\mathcal{D}J_z = -(\omega_p^2/4\pi)\partial_z\varphi, \tag{22c}$$

$$\mathcal{D}\varrho + \partial_x J_x + \partial_y J_y + \partial_z J_z = 0, \tag{23}$$

where $\omega_p^2 \equiv \omega_1^2 + \omega_2^2$.

Using, again, nonmodal approach and its conventional dimensionless notations, complemented by $W \equiv \omega_p / k_{x_1} c$, $D \equiv i\hat{\varrho}/c|q_1|N_1$, $\mathbf{j} \equiv \hat{\mathbf{J}}/c|q_1|\mathcal{N}_1$, for SFH of perturbations we get:

$$j_x^{(1)} = -Rj_y - (W/\mathcal{K})^2 D, \tag{24a}$$

$$j_y^{(1)} = -(W/\mathcal{K})^2 \beta D. \tag{24b}$$

$$j_z^{(1)} = -(W/\mathcal{K})^2 \gamma D, \tag{24c}$$

$$D^{(1)} = j_x + \beta j_y + \gamma j_z. \tag{25}$$

Note that, in accordance with general algebraic relations (18), the above one-fluid variables are bounded by the following relation [$\mathcal{C} \equiv C_1 - C_2$]:

$$(1+\gamma^2) j_y - \beta(j_x + \gamma j_z) = RD + \mathcal{C}. \tag{26}$$

As regards energy of SFH it may be written as:

$$\mathcal{E} \equiv \frac{1}{2}\left[j_x^2 + j_y^2 + j_z^2 + \frac{W^2}{\mathcal{K}^2} D^2 \right], \tag{27}$$

which satisfies the following differential equation

$$\mathcal{E}^{(1)} = -R\left(j_x j_y - \frac{W^2 \beta}{\mathcal{K}^4} D^2 \right), \tag{28}$$

implying that in the shearless ($R = 0$) limit energy of each SFH is a conserved quantity.

¿From (24–25) it easily follows that $D^{(2)} + W^2 D + 2Rj_y = 0$, and taking into account (26) for $Y \equiv D/\mathcal{K}$ we get:

$$Y^{(2)} + \left[W^2 + \frac{3R^2(1+\gamma^2)}{\mathcal{K}^4} \right] Y = -\frac{2R\mathcal{C}}{\mathcal{K}^3}, \tag{29}$$

which, certainly, may also be derived from (20) implying $Y \equiv P - Q$. Further analysis will be dedicated to the solution of this equation. Note that all physical variables characterizing the perturbations are readily expressed through $Y(\tau)$ and $Y^{(1)}(\tau)$. Here we display, for further convenience, the general expressions of this kind:

$$D(\tau) = \mathcal{K}Y, \tag{30a}$$

$$j_x + \gamma j_z = \mathcal{K}^{-3}\Big[(1+\gamma^2)\mathcal{K}^2 Y^{(1)} - R\beta(\mathcal{K}^2+\gamma^2)Y - \mathcal{C}\beta\mathcal{K}\Big], \tag{30b}$$

$$j_y = \mathcal{K}^{-3}\Big[\beta\mathcal{K}^2 Y^{(1)} + R(1+\gamma^2)Y + \mathcal{C}\mathcal{K}\Big]. \tag{30c}$$

It should be noted that (29) is somewhat analogous with equations which govern shear-induced evolution of two-dimensional[11] and three-dimensional[14] acoustic perturbations in hydrodynamical parallel shear flows and with similar equation, which describes evolution of electrostatic ion perturbations in plasma shear flow[23]. Below we shall exploit the analogy, bearing in mind mathematical methods used in Refs. 11, 14, and 23.

Clearly in the shearless limit (29) describes elementary plasma oscillations. In the presence of a nonzero shear the oscillations become modified: they acquire shear induced dispersion. Besides, and this is more important, velocity shear ensures appearance of *new class* of solutions, corresponding to the inhomogeneous term in (29). General solution of the equation is the sum of the *particular* solution of this equation and the *general* solution of the corresponding homogeneous equation (with $\mathcal{C}=0$). In other words, it may be said that Eq. (29) describes two different modes of plasma collective behavior:
(a) Plasma (Langmuir) oscillations, modified by the presence of the velocity shear—which correspond to the case $\mathcal{C}=0$; and
(b) Aperiodic vortex perturbations—which correspond to the case $\mathcal{C}\neq 0$.

This classification is strongly justified for flows with $R\ll 1$, while when $R\simeq 1$ it becomes ill-defined. In the former case a particular solution of (29), proportional to the nonhomogeneity parameter $\mathcal{C}$, may be readily found. Introducing a pair of auxiliary notations: $\mathcal{Y}\equiv W^2Y/2R\mathcal{C}$, $\nu\equiv R/W$ we can reduce (29) to:

$$\nu^2\frac{\partial^2\mathcal{Y}}{\partial\beta^2} + \left[1+\frac{3\nu^2}{(1+\beta^2)^2}\right]\mathcal{Y} + \frac{1}{(1+\beta^2)^{3/2}} = 0. \tag{31}$$

The value of ν parameter $\sim A/\omega_p$ is small in almost all cases of practical (astrophysical) importance. For example, for electrons, $\omega_e = 2\pi 10^{3.95} n_e^{1/2}$, and n_e is of the order of $10^4 cm^{-3}$ in Earth's megnetosphere, $10^8 cm^{-3}$ in Solar corona, $10 cm^{-3}$ in Solar wind and $10^{12} cm^{-3}$ in a neutron star atmosphere[24]. At the same time a shear parameter A in all these cases, estimated as a ratio of characteristic velocity to characteristic length scale, appears to be less by at least one order of magnitude. Thus ν appears to be the natural small parameter of the problem and its smallness allows us to display particular solution of (29) by the following series[25,11,14,23]:

$$\mathcal{Y} = \sum_{n=0}^{\infty}\nu^{2n}\mathcal{Y}_n, \tag{32a}$$

$$\mathcal{Y}_0(\beta) = -(1+\beta^2)^{-3/2}, \tag{32b}$$

$$\mathcal{Y}_n(\beta) = -\left[\frac{\partial^2\mathcal{Y}_{n-1}}{\partial\beta^2} + \frac{3\mathcal{Y}_{n-1}}{(1+\beta^2)^2}\right], \tag{32c}$$

Furthermore, it is easy to show that Eq.(29), rewritten in properly choosed variables, reduces to Hill's nonhomogeneous equation. Let us introduce a couple of new (angular) variables: θ, which measures an angle between the $\mathbf{k}(\tau)$ vector and the Y axis; and ϕ which measures an angle between the X axis and $\mathbf{k}$ vector's projection onto $X0Z$ plane. Clearly $k_x = |\mathbf{k}|sin\theta cos\phi$, $k_y = |\mathbf{k}|cos\theta$, and $k_z = |\mathbf{k}|sin\theta sin\phi$. Shear-induced drift of wave vectors, as we have seen above, ensures temporal variation of k_y $[k_y(\tau) = k_{x_1}\beta(\tau)]$, so that θ also varies with time, while ϕ remains constant. In particular, $tan\phi = \gamma = const$ and $cos\theta(\tau) = \beta(\tau)/\mathcal{K}(\tau)$. It is easy to find that $\partial_\tau = Rcos\phi sin^2\theta\partial_\theta$, while $\partial_\tau^2 = R^2cos^2\phi sin^4\theta(\partial_\theta^2 + 2cot\theta\partial_\theta)$. So that if we introduce new dimensionless parameter $\alpha \equiv Rcos\phi/W = \nu cos\phi$ and new function $\Psi \equiv (W/2\alpha\mathcal{C})sec^2\phi sin\theta Y$ we can derive from (29) the following equation:

$$\alpha^2\frac{\partial^2\Psi}{\partial\theta^2} + \Big[4\alpha^2 + sin^{-4}\theta\Big]\Psi + 1 = 0, \tag{33}$$

which, evidently, is a special case of *Hill's nonhomogeneous differential equation*[26].

Obviously, α (like ν in the initial expansion) may be considered as a natural small parameter of the problem and approximate particular solution of (33) is:

$$\Psi = \sum_{n=0}^{\infty}\alpha^{2n}\Psi_n, \tag{34a}$$

$$\Psi_0 = -sin^4\theta, \tag{34b}$$

$$\Psi_n = -sin^4\theta\left[\frac{\partial^2\Psi_{n-1}}{\partial\theta^2} + 4\Psi_{n-1}\right], \tag{34c}$$

For sufficiently small α's good approximation is to take $\Psi \simeq \Psi_0$ (or, in initial expansion, $\mathcal{Y} \simeq \mathcal{Y}_0$). In this, *zeroth order approximation*, the solution of our problem is readily found and the result is:

$$RD = -2\alpha^2\mathcal{C}sin^2\theta, \tag{35a}$$

$$j_x = -\mathcal{C}cos^3\phi[tan^2\phi\times\theta + sin\theta cos\theta], \tag{35b}$$

$$j_y = \mathcal{C}cos^2\phi sin^2\theta, \tag{35c}$$

$$j_z = \mathcal{C}sin\phi cos^2\phi[\theta - sin\theta cos\theta]. \tag{35d}$$

As for the energy of SFH we get the following simple result:

$$\mathcal{E} \simeq \frac{\mathcal{C}^2}{2}cos^4\phi\Big[sin^2\theta + tan^2\phi\times\theta^2\Big]. \tag{36}$$

Note that in this expression a contribution of the last term in the energy expression (27) is neglected, because direct calculation shows that D^2 term it is at least α^2 times less than other terms.

Certainly the same solutions also may be exposed in initial notations, or derived from the expansion (32). Below we write these expressions in their explicit form:

$$D = -\Big(\frac{2R\mathcal{C}}{W^2}\Big)\frac{1}{1 + \gamma^2 + \beta^2}, \tag{37a}$$

$$j_x = -\left(\frac{\mathcal{C}}{(1+\gamma^2)^{3/2}}\right)\left[\gamma^2 acot\left(\frac{\beta}{\sqrt{1+\gamma^2}}\right) + \frac{\beta\sqrt{1+\gamma^2}}{1+\gamma^2+\beta^2}\right], \tag{37b}$$

$$j_y = \frac{\mathcal{C}}{1+\gamma^2+\beta^2}, \tag{37c}$$

$$j_z = -\left(\frac{\mathcal{C}\gamma}{(1+\gamma^2)^{3/2}}\right)\left[acot\left(\frac{\beta}{\sqrt{1+\gamma^2}}\right) - \frac{\beta\sqrt{1+\gamma^2}}{1+\gamma^2+\beta^2}\right], \tag{37d}$$

while the expression (36) for the energy of SFH takes the form:

$$\mathcal{E} \simeq \frac{\mathcal{C}^2}{2(1+\gamma^2)^2}\left[\frac{1+\gamma^2}{1+\beta^2+\gamma^2} + \gamma^2 acot^2\left(\frac{\beta}{\sqrt{1+\gamma^2}}\right)\right]. \tag{38}$$

4 Discussion

Let us discuss briefly main features of above found analytic vortical solutions, which are referred as *Shear-Langmuir Vortexes* (SLV) throughout this paper. It is convenient to consider two-dimensional (2D) and three-dimensional (3D) cases separately.

- **2D case:** i.e., the case when γ=0 ($\phi = 0$)—the pertyrbations are “locked” in the XOY plane. In this case general SLV solutions reduce to the following simple expressions:

$$D = -\left(\frac{2R\mathcal{C}}{W^2}\right)\frac{1}{1+\beta^2}, \tag{39a}$$

$$j_x = -\frac{\mathcal{C}\beta}{1+\beta^2}, \tag{39b}$$

$$j_y = \frac{\mathcal{C}}{1+\beta^2}, \tag{39c}$$

$$\mathcal{E} \simeq \frac{\mathcal{C}^2}{2(1+\beta^2)}. \tag{40}$$

These solutions closely resemble well-known expression, describing the “transient” growth of the energy of SFH for 2D[11] acoustic perturbations in parallel shear flows of neutral fluids and with similar equation, which describes evolution of electrostatic ion perturbations in plasma shear flow[23]. In particular, transient increase of the energy takes place if initially $k_{y_1}/k_{x_1} > 0$ ($\beta_0 > 0$) and occurs nearby the $\tau_* \equiv \beta_0/R$ moment of time, when $\beta(\tau)$ tends to zero and $\mathcal{K}(\tau)$, which is equal to $(1+\beta^2)^{1/2}$ in 2D case, attains its minimum value.

The rate of the transient energy increase depends on initial orientation of the perturbation wave vector in space. Namely, $\mathcal{E}_{max}/\mathcal{E}_0 = (1+\beta_0^2)$, so that only those SLV's undergo substantial transient increase, which initially possess large values of β_0 parameter. Geometrically, these are the perturbations which initially have wave-vectors with small θ_0 angle between their wave vectors and the Y axis.

- **3D case**: is considerably more complex and interesting. Here trigonometric representation of solutions is more convenient for analysing of the LSV lineaments.

  Shear-induced drift of wave number vectors happens in such a way that even when initially θ_0 is very small (let us speak, for clarity, about a SFH with positive initial values of k_x, k_y, and k_z) in the course of the evolution θ traverses monotonously through the range $[\theta_0,\ \pi]$, matching $\theta_{\tau_*} = \pi/2$ value at $\tau = \tau_* \equiv \beta_0/R$ moment, and attaining in $\tau \to \infty$ asymptotics $\theta_\infty = \pi$ value.

  This circumstance ensures appearance of the analogous evolutionary features for the components of the current density, which are normal to "shear" (Y) axis. In fact, Eqs.(35–36) imply that the behavior of charge density and curreent density y component remains still "transient" and is not much affected by the admittance of the extra dimension. However, the siuation with current density x-th and z-th components is crucially different. Namely, they acquire nontransient, monotonously increasing terms, proportional to the angle θ.

  As regards the energy of LSV, from (36) it is easy to write the ratio of its asymptotic and initial values:

$$\frac{\mathcal{E}_\infty}{\mathcal{E}_0} = \frac{\pi^2 tan^2\phi}{sin^2\theta_0 + tan^2\phi \times \theta_0^2}, \tag{41}$$

  which further reduces to $\mathcal{E}_\infty/\mathcal{E}_0 \simeq sin^2\phi(\pi/\theta_0)^2$ when $\theta_0 \ll 1$. These estimations imply that the asymptotic energy of LSV (which is equal to zero for 2D perturbations) may be either less or greater than its initial energy. Namely, for quasi-2D LSV's, with very small values of ϕ, $\mathcal{E}_\infty < \mathcal{E}_0$ even when $\theta_0 \ll \pi$. However, for $\phi > \phi_t$, where

$$\phi_t \equiv atan\left(\frac{sin\theta_0}{\sqrt{\pi^2 + \theta_0^2}}\right), \tag{42}$$

  stands as some threshold value of ϕ, the asymptotic energy of LSV becomes greater than its initial energy. In other words, the LSV's, which satisfy condition (42) imposed on their initial orientation in space, are able to extract energy from the mean flow not only transiently but also asymptotically.

  It is not easy to envisage at once the potential importance of LSV's in different plasma situations. However, it seems reasonable to expect that these and similar structures should play important role in a wide class of plasma flows including various laboratory, fusion, geophysical and astrophysical examples. Besides, apart from possible technical and theoretical areas of their manifestation, LSV's seem to be worthy of interest as yet another, rather simple but principally new mode of nonperiodic collective behavior evoked by the presense of the kinematic shear in plasma flows.

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