Plasma Near the Horizon of a Schwarzschild Black Hole

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Abstract

Very close to the horizon of a black hole, the gravitational acceleration becomes so large that vacuum can begin to radiate (the Hawking radiation). The temperature of this radiation can exceed (twice of) the rest mass of electrons and positrons at the distance to the horizon on the order of the Compton wavelength. In this vicinity, even within $3R_s$ (R_s is the Schwarzschild radius), an electron-positron plasma is realized and self-sustained. Using the 3+1 paradigm of general relativistic hydrodynamics, we find a steady equilibrium solution and that there is an opaque layer around the horizon so that the apparent temperature of a black hole may be lower than the Hawking temperature. We find that this plasma (in the "QED sea") is hydrodynamically marginally stable. Away from this vicinity, we also find several nontrivial hydrodynamic equilibrium solutions of plasma on the equatorial plane. The plasma above the "QED sea" may be unstable under certain conditions giving rise to such salient phenomena as (general and special) relativistic jets. These equilibrium solutions provide a good starting point to study the dynamics of plasma around a black hole.

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I. INTRODUCTION

The physics of a black hole has been developed mainly on its spacetime properties and little attention has been paid on matter associated with a black hole. This may be in part due to an assertion that no matter may be around it stably. On the other hand little development in general relativistic plasma physics has been carried out and its little impact on astrophysics is found, perhaps due to the esoterics of the subject. The enormous gravitational field around a black hole greatly affects the surrounding plasma medium, so that plasma physics in the vicinity of a black hole should be a subject of interest in astrophysics. The presence of a plasma will also alter observational signatures via its radiative and opacity properties. Based on the 3+1 paradigm [1] [2], a more intuitive mathematical description of general relativistic hydrodynamics of a plasma around a black hole could help ease the study and such has been introduced by Tarkenton [3] and Daniel & Tajima [4] [5]. We apply this method to investigate the properties of the plasma around a black hole. (As we shall see, plasma is generated and suspended around the black hole horizon).

Hawking discovered in 1974 [6] [7] that a Schwarzschild black hole can radiate electromagnetic waves due to its gravitational acceleration. The radiation is characterized as a black body radiation with temperature (the Hawking temperature) $T_{\rm H}$

$$k_{\rm B}T_{\rm H} = \frac{\hbar g_{\rm H}}{2\pi c}, \tag{1}$$

where $k_{\rm B}$ is Boltzmann's constant, $\hbar$ is Planck's constant, $g_{\rm H} = c^2/(2R_s)$ is the surface gravity of a black hole, and $R_s = 2GM/c^2$ is the Schwarzschild radius. This temperature ($T_{\rm H}$) is observed at an infinite distance. Verifying them was not straightforward because at that time there was not any agreed-upon formalism for quantum field theory in curved spacetime. But by 1975 each of the experts had repeated Hawking's calculation in his own way and had obtained the same results: thermal emission (see historical reviews in, e.g., [8] [9]). A key insight into this thermal radiation came from Unruh [10], who further discovered that an accelerating particle detector in a flat, empty spacetime should behave as though it were bathed in a thermal radiation with temperature

$$k_{\mathrm{B}}T = \frac{\hbar a}{2\pi c}, \tag{2}$$

where a is the detector's acceleration. This temperature T is related to the Hawking temperature T_{H} but these two temperatures are different. In our study the temperature T in (2) plays the most important role.

A static observer (who is a FIDucial Observer, hereafter, a FIDO) just above the horizon can be viewed as an accelerated observer in a flat spacetime with an acceleration

$$a = g = g_{\mathrm{H}}/\alpha, \tag{3}$$

where g is the local gravitational acceleration and α is the gravitational redshift function,

$$\alpha = \sqrt{1 - \frac{R_s}{r}}. \tag{4}$$

The reason why α is a gravitational redshift function is clear in the Schwarzschild metric as we shall come to it again later. The locally measured temperature of the radiation observed by a FIDO is divergently large as $\alpha \to 0$ at $r \to R_s$, and thus depends on the precise location of the FIDO. The redshifted temperature (i.e. the Hawking temperature) is related to the temperature in (2) by

$$T_H = \alpha T, \tag{5}$$

and hence T_H measured by a distant observer is finite. The redshifted temperature T_H is also called the surface temperature of the hole. Therefore a remote observer will see the black hole radiates with the temperature T_H, which is Hawking's result. In a local FIDO frame, the temperature T climbs towards the horizon and the energy of the corresponding thermal radiation can become higher than the rest mass of electron. Adopting this Hawking (or Unruh) mechanism, we are thus led to conclude that there is an electron-positron plasma generated by such a thermal radiation at a distance very close to the horizon. The apparent paradox that a FIDO above the horizon sees a thermal atmosphere but a freely falling observer sees pure vacuum was explained by Unruh and Wald [11]: two "vacua", one in the FIDO and the other in the free-fall frame, are related by the Bogoliubov transformation.

We shall show in the following how this plasma is realized and what the equilibrium it is in. This is one of a few reasons for the presence of a plasma around a black hole. Additional mechanisms for it include (i) the advection of a plasma from a surrounding object such as an accretion disk [12]; (ii) a plasma generation by the presence of strong fields (magnetic [13] or accelerating fields) and that of seed matter (perhaps from (i)). We will show some of the properties of these plasmas as well.

II. FORMALISM

A. The Metric and Coordinates System

The Schwarzschild metric arises from the solution of the vacuum Einstein equation that describes the gravitational field outside a neutral, non-rotating point mass. In Schwarzschild coordinates (a modification of flat spherical coordinates) the metric takes the form [14]

$$ds^2 = -\left(1 - \frac{R_s}{r}\right) c^2 dt^2 + \frac{dr^2}{1 - R_s/r} + r^2 d\Omega^2. \tag{6}$$

where $R_s = 2GM/c^2$ is the Schwarzschild radius associated with an object of mass M and $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the elemental solid angle. From this form we can identify the quantity $\sqrt{1 - R_s/r}$ is α defined in Eq.(4) and may be called the lapse function because it is the coefficient of cdt in the metric and thus describes the amount of time that measured by a FIDO elapses during the passage of a unit amount of universal time. This function was called the gravitational redshift function in the previous section since a photon emitted with frequency f_0 by a FIDO at radius r_0 will be measured to have the redshifted frequency $f = \alpha(r_0) f_0$ by an observer far from the hole. The third meaning of α is that it relates with the locally measured gravitational acceleration by

$$g = -c^2 \nabla \ln \alpha = -c^2 \frac{d\alpha}{dr}. \tag{7}$$

Note that the gradient operator ∇ in the above equation is defined in a curved space metric and hence different from its usual meaning in flat space.

The development of the 3+1 formulation of general relativity by Thorne et al. [1] provides a method in which the electrodynamics and plasma physics may be treated similarly to the formulation in flat spacetime while taking accurate account of general relativistic effects such as curvature. Taking this $3 + 1$ point of view [15], the general relativistic spacetime (t, r, θ, ϕ) can be split up into an absolute three dimensional space (r, θ, ϕ) plus a universal time t. The spatial metric of a non-rotating (Schwarzschild) black hole can then be written as

$$\begin{aligned} ds^2 &= \frac{dr^2}{1-(R_s/r)} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \\ &= \alpha^{-2} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2, \end{aligned} \tag{8}$$

where R_s is the Schwarzschild radius described earlier. A caution should be raised when manipulating differential equations in such a metric since the Schwarzschild space (and the corresponding 3+1 splitting) is not flat. In order to avoid such a confusion and gain a more intuitive insight, we should employ a locally flat space, which is Rindler's coordinates. A coordinates transformation

$$\begin{cases} x &= R_s \cdot (\phi_0 - \phi) \\ y &= 2R_s\alpha(r) \\ z &= R_s \cdot (\theta_0 - \theta)\alpha \end{cases} \quad \text{or} \begin{cases} \tilde{r} &= 2R_s\alpha(r) \\ \tilde{\theta} &= \theta \\ \tilde{\phi} &= \phi \end{cases} \tag{9}$$

converts the Schwarzschild line element into a locally flat one,

$$\begin{aligned} ds^2 &\approx dx^2 + dy^2 + dz^2 \\ &\approx d\tilde{r}^2 + R_s^2 d\tilde{\theta}^2 + R_s^2 \sin^2\tilde{\theta} d\tilde{\phi}^2 \end{aligned} \tag{10}$$

in Rindler's coordinates. The (Rindler) Cartesian coordinates will be used later when we discuss hydrodynamics in a co-moving frame. Note that Rindler's spherical coordinates is slightly different from the usual spherical one. In this coordinates the lapse function is simply

$$\alpha = \tilde{r}/(2R_s), \tag{11}$$

and the local gravitational acceleration can be written as

$$g = -c^2 \frac{d\alpha}{dr} = \frac{c^2 R_s}{2\alpha r^2} = \frac{c^2(1-\alpha^2)^2}{2\alpha R_s} \approx \frac{c^2}{\tilde{r}}. \tag{12}$$

B. Hawking's Effect in Rindler's Coordinates

Unruh's effect implies that the temperature T measured by a FIDO is [Eq.(2)]

$$\begin{aligned} k_{\rm B}T &= \frac{\hbar g}{2\pi c} = \frac{\hbar c(1-\alpha^2)^2}{4\pi\alpha R_s} = 5\times 10^{-18}\frac{(1-\alpha^2)^2}{\alpha}\frac{M_\odot}{M}\text{MeV} \\ &\approx \frac{\hbar c}{2\pi\tilde{r}} = \frac{3.1\times 10^{-12}}{\tilde{r}}\text{MeV}, \end{aligned} \tag{13}$$

where $\tilde{r}$ is the distance to the horizon in Rindler's coordinates. For a FIDO very close to the horizon the temperature may climb higher than the rest mass of an electron. When the distance to the horizon becomes

$$\tilde{r}_0 < \hbar/(4\pi m_e c) = \lambda_{\text{Compton}}/(4\pi) = 3.1\times 10^{-12}\text{cm}, \tag{14}$$

or

$$\alpha < 5\times 10^{-18}\frac{M_\odot}{M}, \tag{15}$$

we expect the temperature of a local vacuum exceeds the rest masses of an electron and a positron. This condition can be cast in T as

$$\frac{k_B T(\tilde{r})}{2m_e c^2} = \frac{\hbar}{4\pi m_e c\tilde{r}} > 1. \tag{16}$$

Where Eq.(16) [or (14)] is fulfilled, vacuum radiates photons whose energy is high enough to generate electrons and positrons by pair creation:

$$\gamma\ +\gamma \longleftrightarrow e^+\ +e^-.$$

This will give rise to a plasma with electrons, positrons, and photons. In a thermal equilibrium the density of each ingredient can be calculated by statistical mechanics [16] [17]

[18]. When we consider the case of pure radiation in vacuum, which corresponds to the surroundings of an isolated black hole where no matter accrets from the outside world, the density of electrons (and positrons) can be written as

$$n = \frac{m_e^3 c^3}{\pi^2 \hbar^3} \int_0^\infty \frac{s^2 ds}{\exp[(u+1)\Phi] + 1}, \tag{17}$$

where $u = (1+s^2)^{1/2} - 1$, $\Phi = m_e c^2 / k_B T$. In the limit $\Phi << 1 (k_B T / m_e c^2 >> 1)$, this integral is evaluated as

$$n = 1.803 \frac{m_e^3 c^3}{\pi^2 \hbar^3} \left(\frac{k_B T}{mc^2} \right)^3 = 2 \times 10^{31} T_{\rm MeV}^3 ({\rm cm}^{-3}) = 6 \times 10^{-4} \tilde{r}^{-3}, \tag{18}$$

where we have used the correlation of temperature and position, Eq.(13). Such a plasma may be regarded as an atmosphere of a black hole. Heckler [19] [20] suggested a "QED photosphere" for a small size black hole by QED (and QCD) processes.

On the other hand, when $k_B T << mc^2$, we have the density of electrons as

$$n = \frac{\sqrt{\pi} m^3 c^3}{4\pi^2 \hbar^3} \left(\frac{2}{\Phi} \right)^{3/2} e^{-\Phi} = 2.21 \times 10^{30} \Phi^{-3/2} e^{-\Phi}, \tag{19}$$

where $\Phi = mc^2/T >> 1$. The density of electrons and positrons decays rapidly as the distance to the horizon becomes greater than λ_C. Therefore, if there is sufficient matter outside the atmosphere, [i.e. if the density is greater than n in Eq.(19)], it is not generated by the Hawking radiation but from the matter accreted from outside the black hole or the one generated by the assistance of strong fields with (a small amount of) seed mater. This is the usual region considered by the previous works on relativistic plasmas near the horizon [21] [22] [23]. Though they also considered electron-positron plasmas, they did not resort to the Hawking (or Unruh) radiation as the ee^+ source. Hence they ([21] [22] [23]) had no such simple relations among the density, temperature and position in equations (13) and (18). We discuss the equilibrium and the dynamics of plasmas inside such a QED sea around the (isolated) black hole, adopting a (magneto-)hydrodynamical (MHD) point of view. We will also discuss briefly several possible hydrodynamical equilibria outside the QED atmosphere where accreting matter dominates.

III. HORIZON PLASMA

A. Governing Equations

Adopting the above mechanism, plasma is generated around the horizon inside the Compton wavelength and we thus have a QED atmosphere. The basic equations that govern the horizon plasma in 3+1 membrane paradigm were first introduced by MacDonald and Thorne [1] [2] (see also Thorne et al. [15]):

- Particle Number:

$$\frac{1}{\alpha}\frac{\partial(\Gamma n)}{\partial t}+\frac{1}{\alpha}\nabla\cdot(\alpha\Gamma n\mathbf{V})=0, \tag{20}$$

- Momentum:

$$\begin{aligned}&\frac{w}{c^2}\left(\frac{1}{\alpha}\frac{\partial\mathbf{V}}{\partial t}+(\mathbf{V}\cdot\nabla)\mathbf{V}\right)=\\&-\nabla P+\frac{w}{c^2}\mathbf{g}+\rho_e\mathbf{E}+\frac{1}{c}\mathbf{J}\times\mathbf{B}-\frac{\mathbf{V}}{c^4}\left(w\mathbf{V}\cdot\mathbf{g}+\mathbf{J}\cdot\mathbf{E}+\frac{1}{\alpha}\frac{\partial P}{\partial t}\right),\end{aligned} \tag{21}$$

- Energy:

$$\frac{1}{\alpha}\frac{\partial w}{\partial t}+\frac{1}{\alpha}\nabla\cdot(\alpha w\mathbf{V})=\frac{w}{c^2}\mathbf{V}\cdot\mathbf{g}+\mathbf{J}\cdot\mathbf{E}+\frac{1}{\alpha}\frac{\partial P}{\partial t}, \tag{22}$$

- Maxwell's Equations:

$$\nabla\cdot\mathbf{E}=4\pi\rho_e, \tag{23}$$

$$\nabla\cdot\mathbf{B}=0, \tag{24}$$

$$\frac{1}{\alpha c}\frac{\partial\mathbf{E}}{\partial t}=\frac{1}{\alpha}\nabla\times(\alpha\mathbf{B})-\frac{4\pi}{c}\mathbf{J}, \tag{25}$$

$$\frac{1}{\alpha}\frac{\partial\mathbf{B}}{\partial t}=-\frac{1}{\alpha}\nabla\times(\alpha\mathbf{E}), \tag{26}$$

where $\mathbf{g} = -g\hat{r}$ is the local gravitational acceleration as defined in equation (12), $w = \Gamma^2(\epsilon + P)$ is the enthalpy density, ϵ is the energy density including the rest mass energy, P is pressure, $\mathbf{V}$ is the fluid velocity, $\Gamma = (1 - V^2/c^2)^{-1/2}$ is the Lorentz factor and other symbols stand for their usual meanings. In these equations we assumed that the net of sinks and sources is balanced in a thermal equilibrium. Although we do not dwell on the problems with electric and magnetic fields in the present work, we keep those EM terms here for completeness and references. Note that the particle number density n and the pressure P are measured in the frame co-moving with the fluid, while the other physical quantities are measured by a local FIDO. These equations were solved successfully by the technique developed for solving the Grad-Shafranov equation for laboratory plasma configurations [24] [25] [3]. Here in this paper, we investigate the hydrodynamical aspect of a plasma without dealing with the electric and magnetic field. Also, we are interested in the region whose gravitational field is so strong that the temperature of the plasma is determined by the Unruh effect, and such a temperature is much higher than the rest mass of an electron so that the electron-positron density is determined by equation (18). We thus obtain the density profile without solving the hydrodynamic equations (20),(21), (22). This procedure much simplifies the problem.

B. QED Atmospheric Conditions

We have already had the density distribution from the argument of statistical mechanics (18):

$$n = 6 \times 10^{-4}\tilde{r}^{-3}. \tag{27}$$

We assume that the ee^+ plasma is an ideal gas. The gas pressure is then simply

$$P_g(\tilde{r}) = n(\tilde{r})k_{\mathrm{B}}T(\tilde{r}) = 1.8 \times 10^{-15}T^4_{(K)} = 3.0 \times 10^{-21}\tilde{r}^{-4}\mathrm{erg}\cdot\mathrm{cm}^{-3}. \tag{28}$$

The photon pressure is

$$P_\gamma(\tilde{r}) = \frac{a}{3}T^4 = 4.2 \times 10^{-21}\tilde{r}^{-4}\mathrm{erg}\cdot\mathrm{cm}^{-3}, \tag{29}$$

where $a = 7.56 \times 10^{-15}$erg cm^{-3}K^{-4}. This formula is valid only in the region where it is opaque to photons. We will check the validity later in the discussion. The total pressure P is thus

$$P = P_g + P_\gamma = 7.2 \times 10^{-21}\tilde{r}^{-4}\mathrm{erg}\cdot\mathrm{cm}^{-3}. \tag{30}$$

We further assume the electron-positron gas in the plasma has relativistic adiabatic gas constant $\gamma_g = 4/3$ and its temperature is much higher than $m_e c^2$. Then its enthalpy density can be written as

$$w = \Gamma^2\left(P\frac{\gamma_g}{\gamma_g - 1} + m_e n c^2\right) = 4\Gamma^2 P\left(1 + \frac{m_e c^2}{4T}\right) \approx 4P. \tag{31}$$

IV. THE QED SEA

A. Opaqueness

Beneath the QED atmosphere there lies a layer of plasma where it is opaque to photon. This layer may be called a QED sea. At the distance $\tilde{r}$ to the horizon it is opaque to photon if it satisfies

$$\int_{\tilde{r}}^{\lambda_C} d\tilde{r}\sigma n >> 1, \tag{32}$$

where λ_C is the Compton wavelength inside which electron-positron plasmas are realized, and σ is the cross section of the photon-electron (photon-positron) interaction. The left hand side of the inequality is the number of collisions of a photon with an electron on its way to escape from the QED sea. A layer beneath a certain depth of the sea is opaque if the integral is much greater than unity. We use the Klein-Nishina formula to estimate the cross-section

$$\sigma = \sigma_{\mathrm{KN}} = \frac{3}{8}\frac{mc^2}{k_B T}\sigma_{\mathrm{T}}, \tag{33}$$

where $\sigma_{\rm T} = (8\pi/3)(e^2/mc^2)^2 = 6.65 \times 10^{-25}{\rm cm}^2$ is the Thompson cross section. Using Eqs.(13) and (18) for T and n, the integral Eq.(32) gives a criterion of opaqueness as

$$\tilde{r} << 2 \times 10^{-18}{\rm cm}. \tag{34}$$

It means that the QED sea with the depth of $\sim 10^{-18}$cm around the black hole horizon traps the radiation therein and re-radiates it. In this region the plasma is opaque and photons can strongly interact with electrons and positrons. Note that this distance is much smaller than the length suggested by Heckler's photospheric surface [19] [20].

For a remote observer far away from the black hole, he does not see the Hawking radiation coming directly from the bare horizon, which would have a temperature of $k_{\rm B}T_{\rm H} = \hbar c/(4\pi R_s)$ after gravitationally redshifted. Instead, he observes radiation from the surface of $\tilde{r}_o = 2 \times 10^{-18}$cm, whose temperature is $k_{\rm B}T_\infty = \hbar c(1-\alpha^2)^2/(4\pi R_s)$, cooler than $k_{\rm B}T_{\rm H}$. We find that the difference in these two temperatures

$$\frac{T_\infty - T_{\rm H}}{T_{\rm H}} \approx -2\alpha^2. \tag{35}$$

This is small for an astrophysical black hole, since $R_s > 10^6$cm and $\alpha = \tilde{r}/2R_s < 10^{-24}$. However, for a primordial black hole of a size on the order of $M = 10^{-23}M_\odot$ whose Schwarzschild radius R_s is on the order of 10^{-18}cm, the deviation (35) becomes significantly large. In this case we need to replace the criterion of opaqueness (32) by

$$\int_{r_o}^{\infty}(dr/\alpha)\sigma n = \int_{\alpha_o}^{1} d\alpha \frac{2R_s}{(1-\alpha^2)^2} \cdot \frac{1 \times 10^{-26}}{T_\infty} \cdot 2 \times 10^{31} T_\infty^3 >> 1, \tag{36}$$

because Rindler's approximation is no longer valid. This integral yields a condition

$$\frac{1}{\alpha_o} + 2\alpha_o - \frac{\alpha_o^3}{3} - \frac{8}{3} >> \frac{R_s({\rm cm})}{10^{-18}}. \tag{37}$$

For example, if R_s is equal to 10^{-18}cm (a black hole of mass $M = 10^{10}$g), the opaque layer extends to $\alpha_o = 0.33$, and the deviation (35) of the apparent temperature T_∞ from the Hawking temperature $T_{\rm H}$ is 22%.

The reduction of the apparent temperature of a primordial black hole may have much significant meaning in astrophysics. Primordial black holes radiating with temperature higher than 1MeV have been estimated to have a lifetime comparable to the age of the universe [28]. Some argued that primordial black holes may be considered as candidates for very high energy γ-ray sources [29]. If there were no opaque layer, the lifetime of a black hole τ_H would be estimated as

$$\tau_H = \frac{Mc^2}{4\pi R_s^2 \cdot \sigma_{SB} T_{\rm H}^4}, \tag{38}$$

where σ_{SB} is the Stefan-Boltzmann constant. Since we have found that there is an opaque QED sea surrounding the black hole, the apparent temperature T_∞ is lower than the Hawking temperature $T_{\rm H}$ and hence the evaporation rate is reduced from that computed by Hawking. The lifetime τ can be estimated as

$$\tau = \frac{Mc^2}{4\pi r_o^2 \cdot \sigma_{SB} T_\infty^4}, \tag{39}$$

where r_o is the radius of the surface of the opaque layer. The lifetime is thus lengthen by

$$\frac{\tau - \tau_H}{\tau_H} = \left(\frac{R_s}{r_o}\right)^2 \left(\frac{T_H}{T_\infty}\right)^4 - 1 = \frac{1}{(1-\alpha_o^2)^3} - 1. \tag{40}$$

This effect is pronounced for small black holes. For example, the lifetime of a black hole with mass $M = 10^{10}$g is longer than that has been estimated without an opaque layer by 41%. According to Heckler's calculation [19] [20], however, his QED photosphere does not change the lifetime of a black hole significantly.

B. Isotropy

The strong gravitational field around a black hole may introduce spatial anisotropy. However, as we shall show, this is not the case in the opaque layer (QED sea). In order to examine the anisotropy of the plasma in the QED atmosphere, we compare two scale lengths, the mean free path and the pressure scale height. The mean free path $\lambda_{\rm mfp}$ of the plasma at a distance $\tilde{r}$ from the horizon is estimated to be

$$\lambda_{\rm mfp} = \frac{1}{n(\tilde{r})\sigma_{\rm KN}}, \tag{41}$$

where we use the Klein-Nishina formula again to estimate the cross-section of photon-electron interaction. From Eqs.(13) and (18) in cgs units, we obtain the mean free path of plasma at $\tilde{r}$ away from the horizon as

$$\lambda_{\rm mfp} = 4.5 \times 10^{16} \tilde{r}^2. \tag{42}$$

On the other hand the pressure scale height H at position $\tilde{r}$ is

$$H = \left| \frac{P(\tilde{r})}{\frac{dP(\tilde{r})}{d\tilde{r}}} \right| = \frac{\tilde{r}}{4}. \tag{43}$$

When $\lambda_{\rm mfp} << H$, the plasma is isotropic. Otherwise the strong gravitational field causes anisotropy in the plasma. This criterion for isotropy is

$$\tilde{r} << 5 \times 10^{-18} (\rm cm). \tag{44}$$

Since the opaque layer discussed above (34) is already inside this region (44), the plasma in the QED sea is in fact isotropic. As for the QED atmosphere layer between 10^{-18}(cm) and 10^{-12}(cm), the electron-positron plasma is anisotropic. Its equilibrium configuration is beyond the scope of the present paper.

C. Hydrostatic Equilibrium and Stability of the Sea

The plasma above the horizon is stratified as shown in Sec.III B. This plasma is supported by the stratified distribution of the Hawking radiation. We investigate the properties of this plasma, such as its equilibrium and stability.

From the QED atmospheric conditions [Eqs.(27),(30), and (31)], we find that the pressure gradient term happens to balance the gravitational force term in the momentum equation (21),

$$-\nabla P + \frac{w}{c^2} \cdot (-c^2/\tilde{r}) = \frac{4P}{\tilde{r}} - \frac{4P}{\tilde{r}} = 0. \tag{45}$$

Thus we establish that the steady equilibrium with no flow ($V = 0$) is a solution to the hydrostatic equations (20),(21),(22). This means that the plasma generated by the Hawking radiation is "self-suspended" in a steady hydrostatic equilibrium. To the best of our knowledge, such a suspended plasma equilibrium near the black horizon has never been mentioned.

As we shall show in the following, this equilibrium is marginally stable against hydrodynamic convection. The Schwarzschild criterion [26] [27] for convective stability is

$$\nabla < \nabla_{\rm ad}, \tag{46}$$

where $\nabla = d\ln T/d\ln P$ is the ambient temperature gradient, $\nabla_{\rm ad} = (d\ln T/d\ln P)_{\rm ad}$ is adiabatic gradient and is equal to $(\gamma_{\rm g} - 1)/\gamma_{\rm g}$ for an ideal gas. In the case of the black hole atmosphere the adiabatic gas constant $\gamma_{\rm g} = 4/3$ because the gas is fully relativistic; and because the relation between the pressure and temperature is $P \propto T^4$ as in Eqs.(28) and (29), we obtain

$$\nabla = \nabla_{\rm ad} = 1/4. \tag{47}$$

This means that the plasma is convectively marginally stable. It seems astonishing to us that the plasma generated by the (local) Hawking radiation is exactly in equilibrium and exactly in marginal stability. We expect that in the presence of magnetic field, this equilibrium may be destabilized, which will be discussed in a future publication.

V. PLASMA OUTSIDE THE QED ATMOSPHERE

A. Freely Falling Plasma

Far away from the QED sea (but still $\tilde{r} << R_s$), one may expect that matter falls toward the horizon since there is not enough force to support matter against the gravity. When the pressure term is absent, the momentum equation (21) can be written in spherical Rindler's coordinates $(\tilde{r}, \theta, \phi)$ as

$$\left(V_{\tilde{r}}\frac{\partial}{\partial\tilde{r}}+V_{\theta}\frac{\partial}{R_s\partial\theta}+V_{\phi}\frac{1}{R_s\sin\theta}\frac{\partial}{\partial\phi}\right)(V_{\tilde{r}}\hat{\tilde{r}}+V_{\theta}\hat{\theta}+V_{\phi}\hat{\phi})$$
$$=-\frac{c^2}{\tilde{r}}\hat{\tilde{r}}+\frac{V_{\tilde{r}}}{\tilde{r}}(V_{\tilde{r}}\hat{\tilde{r}}+V_{\theta}\hat{\theta}+V_{\phi}\hat{\phi}). \tag{48}$$

Here Eq.(48) is (special) relativistic though only velocity, instead of momentum, appears. This is because w's are cancelled out from both hand sides of Eq.(48). Note that the gradient operator here in Rindler's coordinates is different from that in the usual spherical coordinates. Noticing that

$$\frac{\partial\hat{\tilde{r}}}{\partial\theta}=\hat{\theta},\ \frac{\partial\hat{\theta}}{\partial\theta}=-\hat{\tilde{r}},\ \frac{\partial\hat{\phi}}{\partial\theta}=0,$$
$$\frac{\partial\hat{\tilde{r}}}{\partial\phi}=\sin\theta\hat{\phi},\ \frac{\partial\hat{\theta}}{\partial\phi}=\cos\theta\hat{\phi},\ \frac{\partial\hat{\phi}}{\partial\phi}=-\sin\theta\hat{\tilde{r}}-\cos\theta\hat{\theta},$$

we can decompose the momentum equation in the $\hat{\tilde{r}},\hat{\theta}$ and $\hat{\phi}$ directions:

$$\hat{\tilde{r}}: V_{\tilde{r}}\frac{\partial V_{\tilde{r}}}{\partial\tilde{r}}+V_{\theta}\frac{\partial V_{\tilde{r}}}{R_s\partial\theta}-\frac{V_{\theta}^2}{R_s}-\frac{V_{\phi}^2}{R_s}=\frac{-c^2}{\tilde{r}}+\frac{V_{\tilde{r}}^2}{\tilde{r}}, \tag{49}$$

$$\hat{\theta}: V_{\tilde{r}}\frac{\partial V_{\theta}}{\partial\tilde{r}}+V_{\theta}\frac{\partial V_{\theta}}{R_s\partial\theta}+\frac{V_{\tilde{r}}V_{\theta}}{R_s}-\frac{V_{\phi}^2}{R_s}\frac{\cos\theta}{\sin\theta}=\frac{V_{\tilde{r}}V_{\theta}}{\tilde{r}}, \tag{50}$$

$$\hat{\phi}: V_{\tilde{r}}\frac{\partial V_{\phi}}{\partial\tilde{r}}+V_{\theta}\frac{\partial V_{\phi}}{R_s\partial\theta}+\frac{V_{\tilde{r}}V_{\phi}}{R_s}+\frac{V_{\phi}V_{\theta}}{R_s}\frac{\cos\theta}{\sin\theta}=\frac{V_{\tilde{r}}V_{\phi}}{\tilde{r}}. \tag{51}$$

We make a further assumption that the gas moves only on the equatorial plane. This is most like the case when the gas accreting to the black hole is from an accompany binary star. In this case we can easily solve the radial and azimuthal components of velocity. It is interesting to note from the $\hat{\tilde{r}}$ component equation that the circular motion with $V_{\tilde{r}}=0$ is not allowed since the azimuthal velocity would become greater than the speed of light c. This is consistent to the well-known fact that a circularly corpuscular motion does not exist with $3R_s$. Having a non-zero radial velocity $V_{\tilde{r}}$, the azimuthal velocity is solved to be

$$V_{\phi}=c\frac{\tilde{r}}{a}\exp(-\tilde{r}/R_s)\approx c\frac{\tilde{r}}{a}, \tag{52}$$

where the approximation $\tilde{r}<<R_s$ is adopted and the constant a depends on the angular momentum. Note that this solution is a "rigid" rotation. The radial velocity is

$$V_{\tilde{r}}^2 = c^2(1 + \frac{2\tilde{r}^3}{a^2 R_s} - \frac{\tilde{r}^2}{b^2}), \tag{53}$$

where the constant b is determined by the boundary condition, and $2\tilde{r}/R_s < a^2/b^2$. The corresponding relativistic Lorentz factor Γ relates with the position $\tilde{r}$ as

$$\Gamma \approx \frac{ab}{\tilde{r}\sqrt{a^2 - b^2}}. \tag{54}$$

The number density can be determined directly from the continuity equation. From Eq. (20) one can deduce that

$$\alpha \Gamma n V_{\tilde{r}} = \text{constant},$$

we then obtain

$$n(\tilde{r}) = \frac{n_0}{\sqrt{1 + \frac{2\tilde{r}^3}{a^2 R_s} - \frac{\tilde{r}^2}{b^2}}}. \tag{55}$$

The density increases slightly as the distance $\tilde{r}$ increases, because the third term in the radical is greater than the second.

B. Solutions in a Co-moving Frame

A typical belief is that the vicinity of the horizon is pure vacuum. The reason behind it has been that a corpuscular body does not have a stable equilibrium orbit at a radius within $3R_s$ [14]. Hence the particle that crosses $3R_s$ will quickly fall toward the horizon. However, the gas (or plasma) dynamics is different from the corpuscular dynamics, because the fluid is under the gaseous (or plasma) pressure influence in addition to the gravitational force and the centrifugal force. When the distance to the horizon is small enough, the pressure term in the momentum equation (21) can not be neglected. It has been demonstrated, in fact, that there exist equilibria of plasma with or without magnetic field around the event horizon [24] [25] [3]. Here we take a local point of view to study this vicinity.

The metric for local Cartesian coordinates has been given in Eq.(9). The lapse function in this Cartesian Rindler coordinates is

$$\alpha(y) = \frac{y}{2R_s}, \tag{56}$$

and the gravitational acceleration measured in this frame is

$$|g| = \frac{c^2}{y}. \tag{57}$$

(Note that $y = \tilde{r}$ to relate to the previous notation).

With careful manipulations, the basic general relativistic MHD equations in a local co-moving Rindler's frame can be derived from equations (20) through (26) as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\alpha \rho \mathbf{V}) = 0, \tag{58}$$

$$\frac{1}{\alpha}\frac{\partial(\rho \mathbf{V})}{\partial t} + \nabla \cdot \left(\rho \mathbf{V}\mathbf{V} + p\mathbf{I} - \frac{\mathbf{B}\mathbf{B}}{4\pi} + \frac{\mathbf{B}^2}{8\pi}\mathbf{I}\right) - \rho \mathbf{g} + 2\rho \mathbf{\Omega} \times \mathbf{V} = 0, \tag{59}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\alpha \mathbf{V} \times \mathbf{B}), \tag{60}$$

$$\begin{aligned} &\frac{1}{\alpha}\frac{\partial}{\partial t}\left(\rho U + \frac{1}{2}\rho \mathbf{V}^2 + \frac{\mathbf{B}^2}{8\pi}\right) \\ &+ \frac{1}{\alpha}\nabla \cdot \left[\alpha\left(\rho U + p + \frac{1}{2}\rho \mathbf{V}^2\right)\mathbf{V} + \frac{\alpha c}{4\pi}\mathbf{E} \times \mathbf{B}\right] - \rho \mathbf{g} \cdot \mathbf{V} = 0, \end{aligned} \tag{61}$$

where

$$U = \frac{1}{\gamma - 1}\frac{p}{\rho}, \tag{62}$$

$$\mathbf{E} = -\frac{1}{c}\mathbf{V} \times \mathbf{B}. \tag{63}$$

The Lorentz factor disappears since we adopt a co-moving frame and assume that the fluid velocity with respect to this frame is not relativistic. The $\mathbf{\Omega} \times \mathbf{V}$ term is the relativistic Coriolis force term. In this co-moving Rindler's coordinates the x-axis points to the negative azimuthal direction, the y-axis to the radial direction and the z-axis to the vertical direction. The gravitational acceleration $\mathbf{g}$ is measured in this co-moving frame. Its main component is in the negative z-direction since the radial component of the gravitational force is mainly cancelled by the centrifugal force. It is assumed that g_y, which is the radial gravitational acceleration minus the centrifugal acceleration, is proportional to g_z. Both g_y and g_z are assumed to be independent of z.

We find that the hydrostatic solution to the force equilibrium in this co-moving frame is:

$$P(y, z) = P_0 \exp(-y/L) \exp(-z/H), \tag{64}$$

$$\rho(y, z) = \frac{\gamma P_0}{T(y)} \exp(-y/L) \exp(-z/H), \tag{65}$$

where

$$T(y) = \gamma H g_z(y), \tag{66}$$

where H and L are the scale heights in the z and y direction, respectively.

We study the stability of this equilibrium by adopting the Schwarzschild criterion of convection [26] [27]. Consider an imaginary bubble at position (y, z) moves toward a new position $(y + \delta y, z + \delta z)$. The changes in the pressure P and density ρ are

$$P(y + \delta y, z + \delta z) \approx P(y, z) \left(1 - \frac{\delta y}{L}\right) \left(1 - \frac{\delta z}{H}\right), \tag{67}$$

$$\rho(y + \delta y, z + \delta z) \approx \rho(y, z) \left[1 + (\frac{1}{y} - \frac{1}{L})\delta y - \frac{1}{H}\delta z\right]. \tag{68}$$

The ambient change of the density is denoted by $\delta\rho_{\rm amb} = \rho(y + \delta y, z + \delta z) - \rho(y, z)$. We have

$$\frac{\delta\rho_{\rm amb}}{\rho} \approx (\frac{1}{y} - \frac{1}{L})\delta y - \frac{\delta z}{H}. \tag{69}$$

On the other hand, the adiabatic change of density is

$$\frac{\delta\rho_{\rm ad}}{\rho} = \frac{\delta P}{\gamma P} = -\frac{\delta y}{\gamma L} - \frac{\delta z}{\gamma H}, \tag{70}$$

where γ is the adiabatic gas constant and is equal to 4/3 in relativistic case. The Schwarzschild criterion for stability has been given in Eq.(46), or equivalently, $\delta\rho_{\rm amb} < \delta\rho_{\rm ad}$.

Thus, the perturbation in the present equilibrium is unstable if it moves in the direction of $(\delta y, \delta z)$ which lies in the section (in the first quadrant) defined by

$$\frac{\delta z}{\delta y} < \frac{H}{y}\frac{\gamma}{\gamma - 1} - \frac{H}{L}. \tag{71}$$

The scale heights H and L are functions of the distance to the horizon (y), thus the condition (71) is sensitive to y (i.e., $\tilde{r}$). This instability may be called the general relativistic hydrodynamic convection. It is interesting to note that the general relativistic effect manifests through the fact that the gravity changes rapidly as a function of the lapse function α and thus y, even though the region we consider is so small that the y dependence of other physical quantities such as shearing is negligible. In the non-general-relativistic limit y and L approach to infinity and there is no instability.

The overall picture (on the equatorial plane) of the plasma in the atmosphere around a Schwarzschild black hole is shown in Fig. 1. The atmosphere may be categorized into six different regions. In region I, where the distance to the horizon is smaller than 10^{-18}cm, the atmosphere (or the sea) is plasma composed of electrons and positrons generated by the Hawking radiation and it is opaque to photons. This plasma is found to be in a stable static equilibrium (Sec.IV C). In region II ($10^{-18}\text{cm} < \tilde{r} < 10^{-12}$cm) the atmosphere is filled with plasma, but it is not opaque and is spatially anisotropic. In region III the atmosphere is composed of matter due to accreted gas from outside. The combination of the centrifugal force and pressure gradient balances the gravitational force. The plasma in this region may be in a stratified rotating equilibrium as shown in Eqs.(64)(65). We find that this equilibrium is against the general relativistic convection instability. In regions IV and V the atmosphere is not self suspended but the gas is spirally falling toward the hole. The difference of region V from region IV is that in region V the pressure may be neglected and the plasma is in a dynamical equilibrium given in Eqs.(52), (53) and (55). Region VI is outside $3R_s$ and it is the usual region on which many studies of accretion disks (starting from [30]) have been carried out.

VI. SUMMARY

In summary, we have found the presence of plasmas even in the vicinity (within $3R_s$) of the horizon of a Schwarzschild black hole. The origin and property of this plasma vary depending upon the distance away from the horizon. Within the Compton wavelength away from the horizon of an isolated black hole an electron-positron plasma is generated by thermal radiation due to the Hawking effect (Unruh effect). This yields the creation of the QED atmosphere. The particle number density, temperature, pressure, and enthalpy density of the plasma are locally determined by the condition of Unruh radiation and are simple power of $\tilde{r}$, where $\tilde{r}$ is the distance to the horizon. Deep in such a QED atmosphere, the plasma is opaque to photons (the "QED sea"). The presence of this "sea" leads to a number of important conclusions. A remote observer cannot see the radiation coming directly from the bare horizon. Therefore the apparent temperature T_∞ observed at infinity (such as us) is lower than the Hawking temperature T_{H}. This apparent temperature T_∞ can be significantly less than T_{H} for a small black hole. The lifetime of Hawking evaporation is considerably lengthened for such a black hole. This "sea" is shown to be isotropic. This solution satisfies the hydrodynamic equations with a steady equilibrium with no flow $V = 0$, where the gravitational force is balanced by the (radiation) pressure gradient. It is shown that this equilibrium is convectively marginally stable. This plasma is "suspended in vacuum near the horizon". (As is for the e^-e^+ pair creation where $T > 2mc^2$, there should be $\nu\bar{\nu}$ creation when the local Hawking temperature becomes sufficiently high. Thus it is expected such neutrinos are emitted. The emergence and energy of such neutrinos depend on the neutrino mass). In the QED atmosphere above the "sea", it is a layer of an optically thin, anisotropic electron-positron plasma generated by Hawking radiation.

Just above the QED atmosphere level, the gas may pile up instead of falling into the horizon. We have shown an equilibrium solution in a local co-moving frame in this region and found that it is unstable against a convective motion. Such instabilities may manifest as a relativistic jet (both general relativistic and special relativistic). Beyond this region, the plasma is spirally falling inward. This solution may be consistent with the advective accretion [12].

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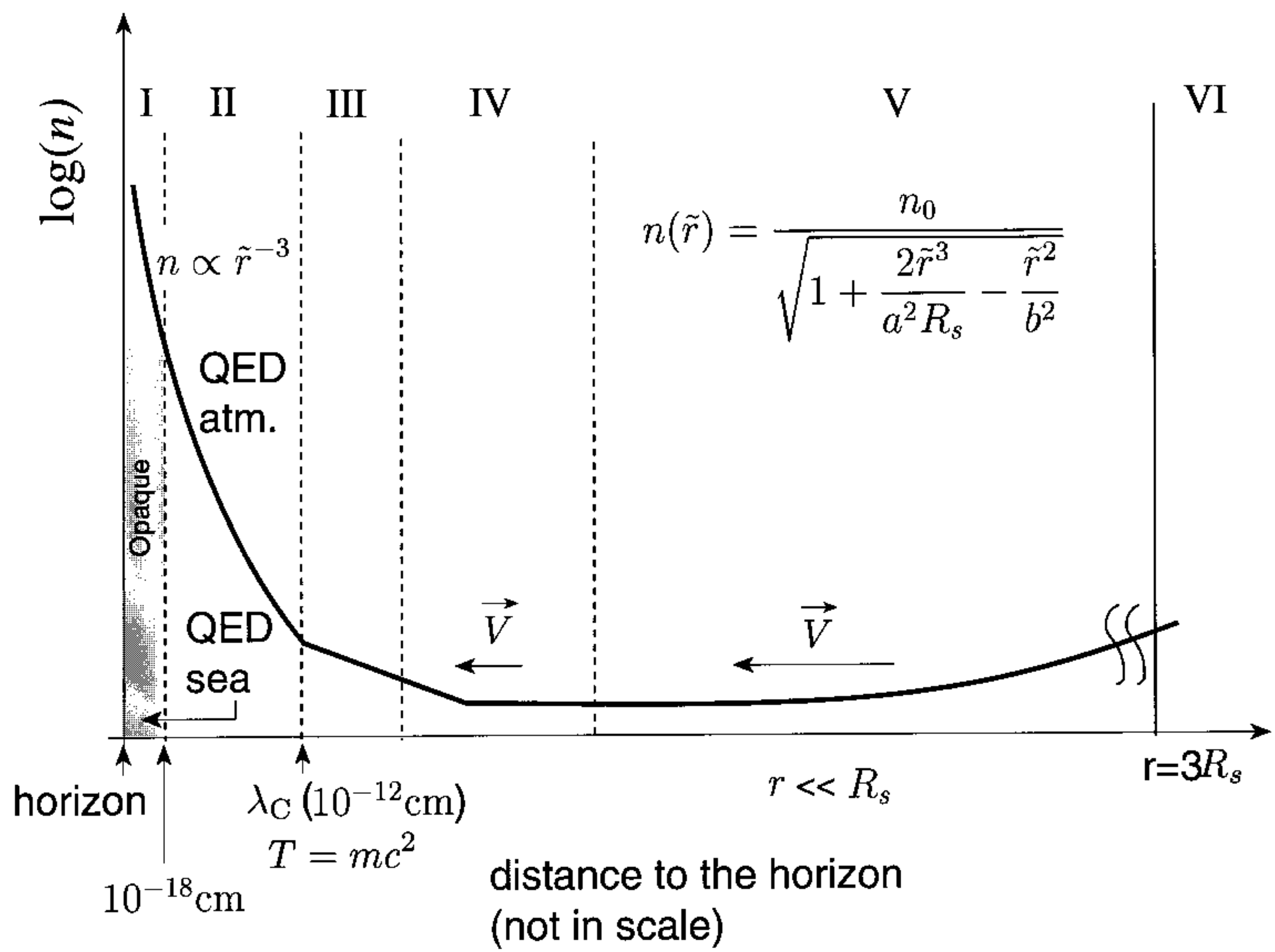
FIGURE CAPTION

Fig. 1 Schematic picture showing the density profile of the black hole atmosphere. Region I is an opaque, spatially isotropic layer of electron-positron plasma (QED sea) generated by the Hawking effect. Region II is an optically thin, spatially anisotropic layer (QED atmosphere). Region III is composed of circularly rotating gases where the gravitational force is balanced by the pressure gradient and the centrifugal force. Region IV and V contain spirally falling gases but the pressure term may be neglected in region V. Region VI is the place where the standard accretion disk is considered.



log(n)
I
II
III
IV
V
VI
$n \propto \tilde{r}^{-3}$
QED atm.
Opaque
QED sea
$n(\tilde{r}) = \frac{n_0}{\sqrt{1 + \frac{2\tilde{r}^3}{a^2 R_s} - \frac{\tilde{r}^2}{b^2}}}$
$\vec{V}$
$\vec{V}$
horizon
10^{-18}cm
$\lambda_C\,(10^{-12}$cm)
$T = mc^2$
$r << R_s$
r=3R_s
distance to the horizon
(not in scale)