Analysis of the Bi-modal Nature of Solar Wind – Magnetosphere Coupling

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Abstract

It has been shown that the optimal linear prediction filter relating the solar wind electric field and the geomagnetic activity, as measured by the AL index, is both bi-modal and dependent on the level of activity in the magnetosphere. Further studies truncated the prediction filter to a five parameter model containing two low-pass filtered delta functions of arbitrary amplitude and delay time. The present study elaborates on the nature of the bi-modal response by using the five parameter model to quantify the effects of the level of geomagnetic activity on each of the modes of the filter individually. We find that at all levels of activity, the second mode, occurring at approximately one hour, is relatively unchanged. The first mode, however, has a necessary one parameter dependence on the level of activity in the magnetosphere. The amplitude of the first mode is shown to increase with respect to activity, and this dependence is sufficient in characterizing the changing properties of the magnetosphere with respect to activity.

I. INTRODUCTION

Shortly after the introduction of the method of linear prediction filtering [7] to the study of the solar wind - magnetosphere coupling, it was demonstrated that the response of the magnetosphere to the solar wind is probably bi-modal, and that this response is dependent on the level of activity in the magnetosphere [1,4]. The solar wind and magnetosphere were represented respectively by the incoming solar wind velocity times the southward component of the interplanetary magnetic field (as measured on the IMP-8 spacecraft and time shifted so as to be "simultaneous" with the ground based measurements) and the AL index (the lower envelope of the auroral latitude magnetic field at the earth's surface as measured by an array of magnetometer stations - indicative of the westward electrojet). The level of activity in this study was represented by the median value of the AL index. The 34 data sets were sorted into intervals of increasing activity and then concatenated in sets of five, to give 30 longer data sets [1].

The linear prediction filter for a given input and output signal is defined by the equation

$$\hat{O}(t) = \int_0^T h(\tau) I(t-\tau) d\tau, \tag{1}$$

where $\hat{O}(t)$ is the predicted output which in this case is the predicted AL index, $I(t)$ is the input data which in this case is $V \cdot B_s$. The filter, $h(t)$, is calculated so as to minimize the RMS error between the measured and predicted output. Thus the optimal filter is calculated using a least squares fit to the data, and is called a Wiener filter [12]. The resulting filters for the Bargatze data set [1] are shown in Figure 1. The filters were smoothed so that the large scale structure would be more apparent. The small scale structure is assumed to be noise or non-physical. The apparent double hump for low and medium activity and the single hump with a shoulder for high activity was attributed to the two part response of the magnetosphere to the solar wind: direct driving and loading-unloading. The first peak results from the direct driving of the magnetosphere by the solar wind caused by excessive dayside merging of magnetic field lines during the period of enhanced solar activity. The time delay is only that necessary for transport of the input energy to the auroral electrojet through direct coupling in the system. The second peak corresponds to the nightside loading-storage-unloading of energy by the tail lobes. The larger delay corresponds to the storage of energy before unloading. It can be seen that at high levels of activity the direct driving overwhelms the unloading response. Both 3D MHD simulations and low dimensional dynamical models [5,6,9,10] provide mechanisms for the direct coupling of the solar wind input to the auroral electrojet through normal modes of the magnetospheric cavity and field aligned currents.

Noting the double peak structure, a simple model was developed to investigate the effectiveness of an explicitly bi-modal filter on isolated substorms [11,2]. This model consisted of two delta functions of arbitrary time delay and amplitude, convolved with a low-pass filter to include the inductive effects of the magnetosphere. The time delays and amplitudes were highly variable, but the histogram of parameter occurrences showed that the two peaks were distinct: the first centered at about 30 minutes, the second at about 70 minutes, consistent with the results of the full linear filter. However, no variables were found to correlate with the variation of parameters in this case. One should note that this was a different data set from the previously mentioned analysis. This data set consists of only single isolated substorms.

This second model can be represented in several ways. Most analogous to the first, is

$$\hat{O}(t) = \int_0^T \int_0^T g_1(\tau_1) g_2(\tau_2 - \tau_1) I(t - \tau_2) d\tau_1 d\tau_2, \tag{2}$$

where $g_1(t)$ is the low pass filter, defined with the time constant τ:

$$g_1(t) = \frac{1}{\tau} exp(-t/\tau), \tag{3}$$

and $g_2(t)$ is the bi-modal response, with two time delays, t_1 and t_2, and two amplitudes, a_1 and a_2:

$$g_2(t) = a_1 \delta(t - t_1) + a_2 \delta(t - t_2), \tag{4}$$

for a total of five parameters.

In this paper will will make use of the five parameter model to investigate the bi-modal nature of the response of the magnetosphere to the solar wind. This simple model will allow us to differentiate the two modes, which cannot be done quantitatively with the full linear prediction filter. Extending the previous study [2] by making use of longer time records which contain multiple substorms, we are able to demonstrate a dependence of the model on the level of activity. In fact, we are able to localize this dependence to a single parameter.

The effectiveness of the fits will be presented by two measures: the Average Relative Variance (ARV), and the prediction efficiency (ρ). The average relative variance is defined [3] as

$$ARV = \frac{\sum_{j=1}^{M}(O_j - \hat{O}_j)^2}{\sum_{j=1}^{M}(O_j - \langle O \rangle)^2}, \tag{5}$$

and the prediction efficiency is defined as the correlation coefficient

$$\rho = \frac{\frac{1}{M}\sum_{j=1}^{M}(O_j - \langle O \rangle)(\hat{O}_j - \langle \hat{O} \rangle)}{\sigma_o \sigma_{\hat{o}}}. \tag{6}$$

A perfect match between the measured output, O, and the predicted output, $\hat{O}$, would yield $ARV = 0.0$ and $\rho = 1.0$.

II. ANALYSIS AND RESULTS

We consider first the results of the five parameter model on the original 34 short data sets compiled by Bargatze and co-workers [1]. The results are unremarkable. While the model does fit the data with reasonable ARV and ρ, the parameters (a_1, a_2, t_1, t_2, and τ) vary wildly from one fit to the next independent of any measure (such as average AL) we could find. Because each short record contains only a few substorms, it is not surprising that this is the same result one finds when fitting isolated substorms [2]. A short section of a typical fit to the AL index is shown in Figure 2 along with the full linear prediction filter result. (The main objective here is to investigate the effects of geomagnetic activity on solar wind - magnetosphere coupling in the context of a bi-modal interpretation, and not to develop a prediction scheme. The comparison with the Bargatze *et al.* results is provided as a benchmark for readers, many of whom are familiar with the results of Bargatze and co-workers. Also note that the Linear Prediction Filter results are those that would be obtained using the filters displayed in the Bargatze *et al.* publication, i.e. after smoothing.) The ARV for the bi-modal fit in Figure 2 was 0.43 and $\rho = 0.76$. The fit is remarkably close to that obtained using the full linear prediction filter for the same data. Even though the Wiener filter has roughly 100 free parameters and this model has only five, they both fit the data equally well. This suggests the relationship between the two time series is low dimensional in nature, and that the bi-modal model is a good approximation of that relationship. The fits shown are a typical comparison of the three.

Turning now to the concatenated data sets, we allow all five parameters to vary and get the best fit parameters as shown in Figure 3 for each data set. These are the parameters yielding the best ARV and ρ. The time constant appears to increase at high activity. The time delay of the first mode seems to stay roughly constant at about 10 minutes for low activity and drops some for high activity, while the time delay of the second mode has a sharp drop from nearly 1 hour at low activity to 14 minutes at high activity. The amplitudes of the modes experience similar changes, with the amplitude of the second mode slowly dropping, and the first having a sharp rise for high activity. Thus, a clear change occurs at high activity, especially in a_1, t_2, and τ. Note that both the ARV and prediction efficiency are reasonable for most of the fits (with the obvious exception of intervals one and two, whose levels of activity are low enough that the background noise can contribute more to the ARV and ρ).

At this point we should recall a common interpretation of the results of Bargatze *et al.*. It was that the vanishing bi-modal structure of the filters at high activity was caused by both an increase in the magnitude of the first mode and a decrease in the time offset of the second. The results shown in Figure 3 would seem to indicate that this is exactly what happens. But this is not the end of the story.

It is quite possible that the changes we are observing in the parameters are not independent of each other. Consider what would happen if we added a third mode that had the same time offset as the second. The amplitude of the second and third modes might then demonstrate some "dependence" on activity, but would always add to the same number. We want to eliminate this type of false dependence on activity, and find those parameter(s) which offer an independent and irreducible dependence on activity. Although the relationships among parameters in our five parameter model are not likely as simple as in the above example, we expect they are there nonetheless.

In order to resolve this interdependence, we look more closely at the results. Table I is a correlation table for the parameter occurrences. Two of our three strongly varying parameters, t_2 and τ, are very strongly anti-correlated, with a linear correlation coefficient of -0.9. A view of these two parameters on comparable scales is shown in Figure 4. This figure has error bars that were calculated in-sample as follows: A random sample of 80% of the data was used in the analysis, and the results recorded. This was repeated N times with different random samples each time. From these N fits, we calculated the median and MADM (median absolute deviation from the median). If the data were to posses Gaussian statistics, the MADM times 1.43 would be equivalent to the standard deviation, and median equivalent to the mean. Figure 4 shows the median, and the median $\pm 1.43 \times MADM$. The same analysis has been done for all the results of this work, but always looks similar to Figure 4, and would unnecessarily clutter the already complicated plots.

From Figure 4 and Table I, it seems likely that t_2 and τ do, in fact, possess some interdependence. We will attempt to remove this unnecessary degree of freedom by fixing one of those parameters with respect to activity. If the best fit for the other parameter then also remains fixed, and the quality of the fits to the data (as measured by ARV and ρ) is equivalent to that obtained with all of the parameters free, then we have successfully reduced the degrees of freedom in the dependence on activity of the model parameter space. We will do this in what follows.

Since t_2 and τ seem likely candidates for interdependence, we now reduce the fit-able parameter space by fixing the decay time for the low-pass filter. Given that the freely fit value varied between 0.4 and 1.7 hours, we consider only that range. What we find is that at most values of the decay time, the resulting best filters have wildly varying coefficients, similar to the fits obtained to the short data sets. At the lowest values of the decay time, however, a more smoothly varying structure is obtained for the parameters, as shown in Figure 5. The fit shown uses $\tau = 0.617$ hrs, the median value from medium and low activity of the 5-free parameter fits. We now see somewhat different behavior than in the five free parameter fits. First, notice that, as expected, t_2 is nearly fixed, with an average of 57.6 and standard deviation of 8.2 minutes. The time delay (first panel) of the first mode appears to be fixed at about 10.1 minutes (standard deviation of 3.6 minutes). The amplitude (second panel) of the first mode is what now seems to cleanly separate the two modes, with the first and second modes nearly equivalent at low and medium activity and the first mode dominant at high activity. The important feature here is that the amplitude of the second mode remains relatively unchanged and the amplitude of the first mode increases with respect to activity, while both have roughly constant time delays. So by reducing the free parameters by fixing τ, we have reduced the apparent dependence of the model on geomagnetic activity to only one parameter - a_1, and the radical changes of t_2 and τ have been shown to be offsetting effects. We will, of course, systematically investigate this reduction in what follows. We note that the ARV and prediction efficiency are remarkably similar to the fits obtained by allowing all five parameters to vary, suggesting there is no direct correlation between the decay constant and magnetospheric activity, as discussed above.

We continue the reduction of parameter space freedoms with respect to activity in the same manner as above, in an attempt to distill the essential dependence of solar wind - magnetosphere coupling on activity. We fix both time delays or both amplitudes and allow only the remaining two parameters to vary. We also consider the mixed sets, with (a_1, t_2) and (t_1, a_2) variable. These fits are all similar with one exception. All of the fits which allowed a_1 to vary with respect to activity had better ARV and ρ than all of the fits which did not, lending further support to there being a necessary dependence of a_1 on activity.

Using the previous results for the fixed values, we now allow only one parameter to vary.

We do this for each of the five parameters. The only one that stands out is the fit for a_1, which is better than the others at both high and low activity. (Medium activity is fit by all models equally well by definition - we are fixing the parameters at their median values.) This suggests that we need only one parameter to describe the dependence of this model on geomagnetic activity, and that parameter is the magnitude of the first mode. This is also what the 4 and 2 free parameter fits led us to expect. The results for the fits which allowed only a_1 and t_1 to vary are shown in Figures 6 and 7 respectively. We are now plotting the ARV and ρ as percent differences from the four parameter fits, since the changes from one model to the next are small.

As a final test to the freedom of only a_1 as being necessary and sufficient to describe the bimodal nature of solar wind - magnetosphere coupling, we fix a_1 at the values obtained in the 1-free parameter fits, and allow all four other parameters to vary. As we should have expected, t_1 and a_2 remain relatively constant while t_2 and τ demonstrate interdependence similar to that shown in Figure 4. We try again with τ fixed. The parameters that were free yielded values similar to those in Figure 5, but with the mode 2 amplitude varying less.

Finally we look more closely at the measure of "activity" of the magnetosphere. The Bargatze *et al.* data set is organized according to median activity. But instead of simply ordering the data with increasing activity as we have done, we can plot the a_1 coefficient with respect to an actual measure of activity, the average AL index. This is shown in Figure 8. We can now see that a definite increasing trend with respect to activity exists.

We also consider the results of trying again to fit the short data sets while holding four of the parameters fixed to try and make some relation to the previous results [2]. Allowing a_1 to be freely fit to these data sets, we still see a steady increase with respect to activity, but there is variation on the order of half the magnitude of a_1. This further validates our initial comment that there is too much noise in the short time series to find a good linear prediction filter. The same effect is seen regardless of whether one is using the many parameter Wiener filter, or the five parameter bi-modal filter.

III. SUMMARY AND CONCLUSION

In summary, we have shown that it is possible to ascertain a dependence of solar wind – magnetosphere coupling on geomagnetic activity within the context of a few parameter bi-modal model, and that dependence is in only a single parameter.

The original work on bi-modal filters by Blanchard and McPherron [2] was unable to resolve a consistent dependence on the level of activity in the magnetosphere. This was in contrast to the linear prediction filter work of Bargatze and co-workers [1]. We have found that the original short records compiled by Bargatze yield similar results to what was obtained by Blanchard and McPherron using isolated sub-storms. However, when these records are spliced together (as was done by Bargatze *et al.*), not only is a dependence recognizable, but it is the same dependence as observed when using the full linear prediction filters.

Recognizing the significance of reducing the dependence of the parameter space on activity to the essential one, we see that the only *necessary* dependence is in the a_1 parameter. Since the model having only a_1 depend on activity and that allowing all five parameters to depend on activity are essentially equivalent as evaluated using the ARV and prediction efficiency measures, the model having only a_1 free is therefore a *sufficient* description as well. We further see that while the preliminary analysis would indicate a strong dependence of the t_2 and τ parameters on activity, this is just a manifestation of their inter-dependence, and not necessarily physical.

Finally, we should note that it is likely that much of the random variation of the parameters is because of the quality of the data set used. It has small gaps which were interpolated, and the time offset between space and ground measurements depends on the position of the satellite and assumes a constant solar wind speed. We chose to use this data set because of its historical significance and the general familiarity of researchers in this field with this data set. Today one can acquire continuous measurements almost instantly from the many ground based stations and satellites, and this would be the preferred data to use. We will expand this study in future work showing the relationship to the low dimensional analogue models of Klimas et. al. [8,9] and Horton and Doxas [6].

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APPENDIX A: MODIFICATION OF THE DATA SET

The best fit parameters obtained for the full Bargatze data set are similar to those for the modified one, except that the t_1 parameter is negative between data sets 17 and 19. At the same time, t_2 is approximately 15 minutes, effectively replacing mode one for the "ten minute mode".

This region of anomalous fit parameters is worthy of some consideration. Since the fitting routine is a downhill simplex and the initial guesses were always positive numbers, the only way to arrive at a negative time offset is if the fit to the data got monotonically better while progressing in the negative direction. In other words, it actually is a better fit to the data than having both constants positive. In addition, when the method for obtaining error bars as described in the Analysis section is used here, the error bars for these questionable sets do not include positive numbers. Since negative time offsets violate causality this is a physically impossible response in the magnetosphere. Negative t_1 or t_2 could indicate that there is an error in the time offset added to synchronize the satellite observations with the ground based data, but this error is not likely to exceed five minutes. We could also take the unforgiving position of assuming that this model is not correct because it fails for this data set. We choose instead to state that this model is plausible for 94% of the data investigated, and merely eliminate the offending data from our analysis.

We consider the case in which the fewest possible short data sets are incompatible with this model. Given that the anomaly occurs only in concatenated intervals 17 through 19, and each interval is composed of five short time series, that implies that there are only two deviants, and that both of the faulty time series must be included before their properties overtake those of the correct time series. That makes the erroneous data the short intervals 19 and 21. Removing these from our database and concatenating the remaining data, we repeat the analysis, and get the results shown in Figure 5.

We chose not to further investigate the cause of the incompatibility of these data sets with our model because any conclusions we might reach would be uncertain and merely supposition. Instead we eliminated the data in question from the concatenations, so that, for example, the concatenated data set labeled #18 now contains the short Bargatze sets 18, 20, 22, 23, and 24, instead of 18 through 22.

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TABLES

TABLE I. Linear (Pearson) Correlation for 5-free parameter fits.

	a1	t1	a2	t2	tau
a1	1.000	-0.881	-0.649	-0.837	0.628
t1	-0.881	1.000	0.373	0.826	-0.699
a2	-0.649	0.373	1.000	0.560	-0.396
t2	-0.837	0.826	0.560	1.000	-0.901
tau	0.628	-0.699	-0.396	-0.901	1.000

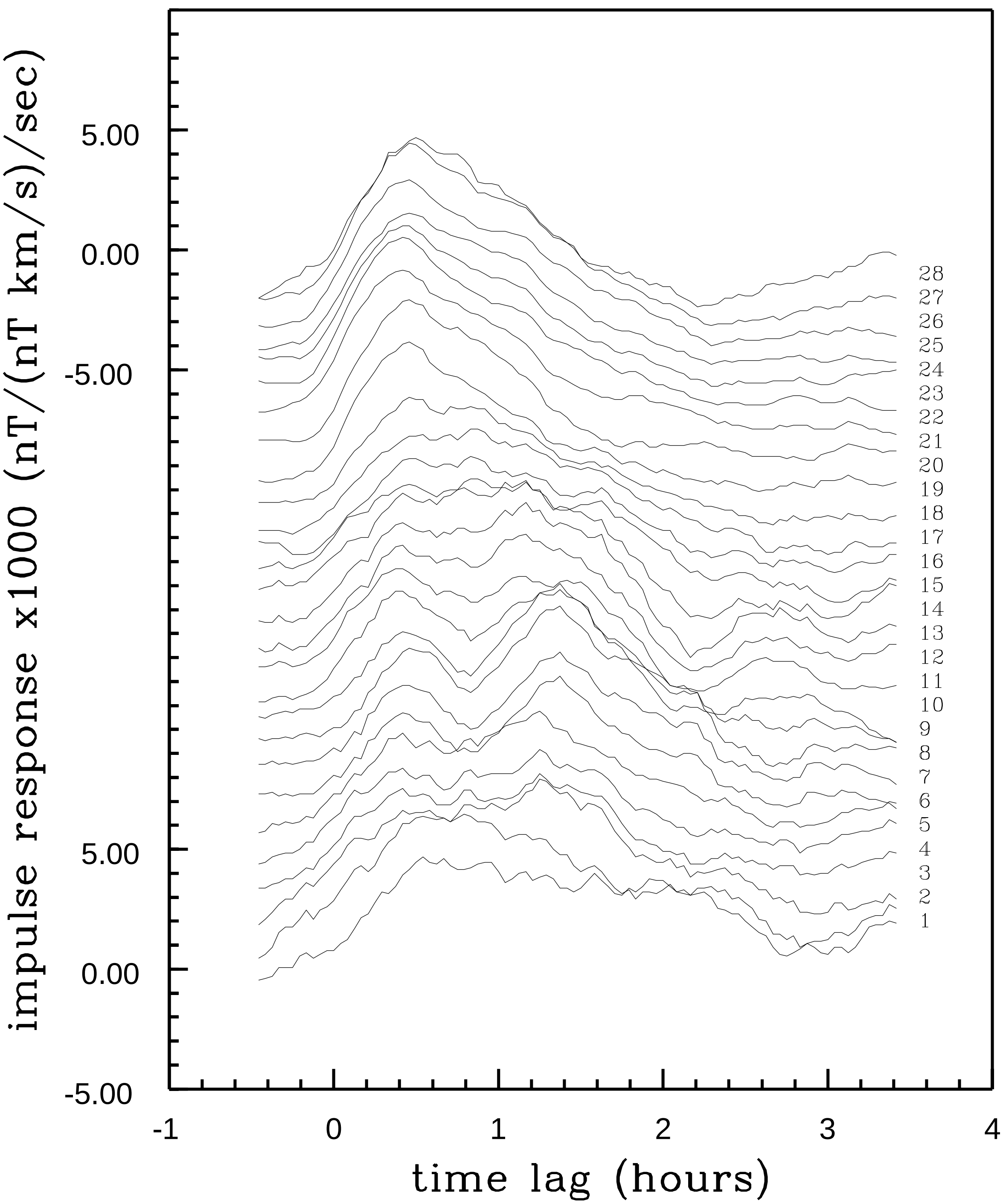

FIG. 1. Linear prediction filters for the Bargatze data set. Note the apparent double peak structure for low and medium activity and the single peak with a shoulder for high activity.

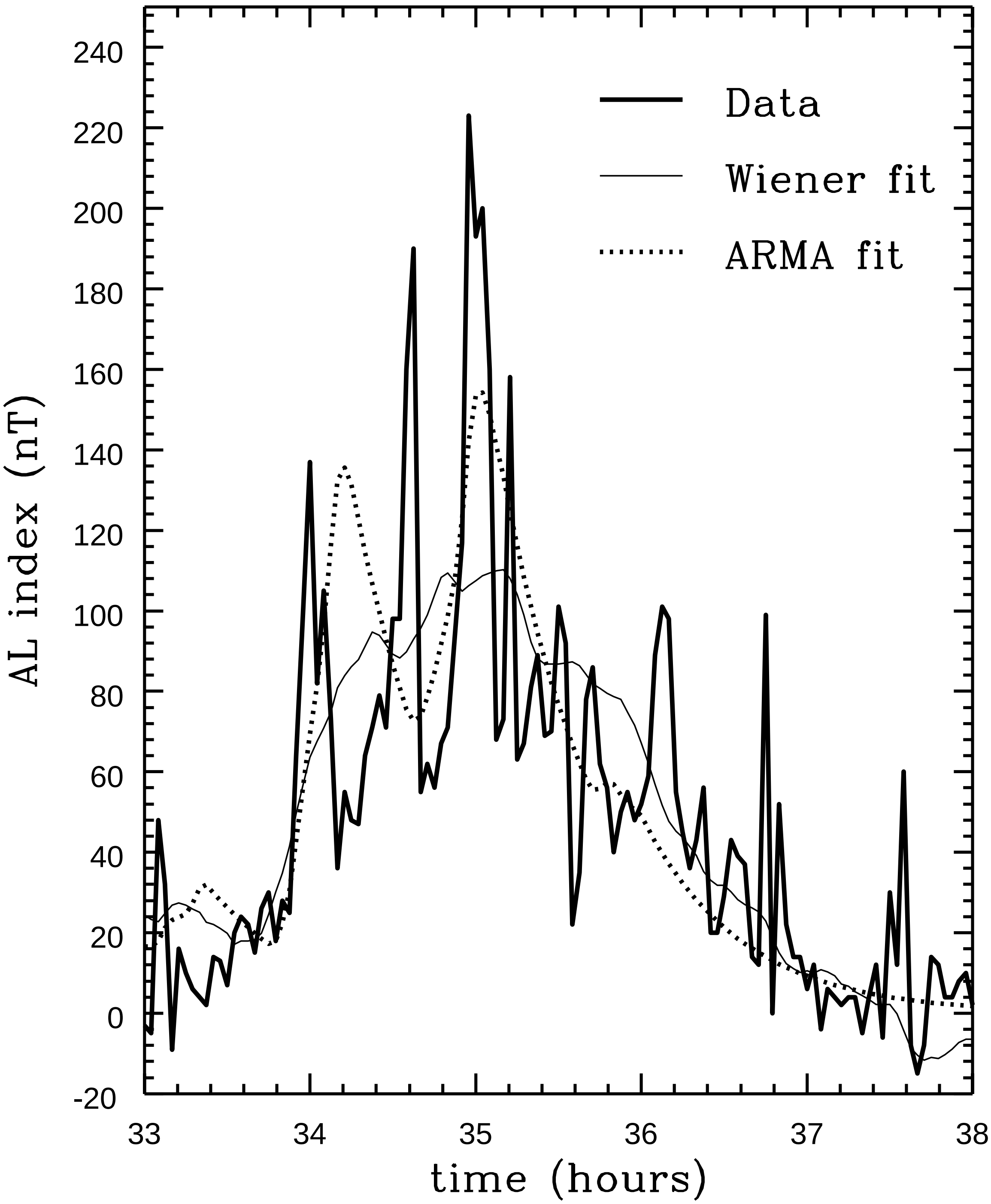

FIG. 2. Close-up of the fit to the AL index of Bargatze interval 20 using both the full linear prediction filter (Wiener filter), and the five parameter bi-modal model (an ARMA filter).

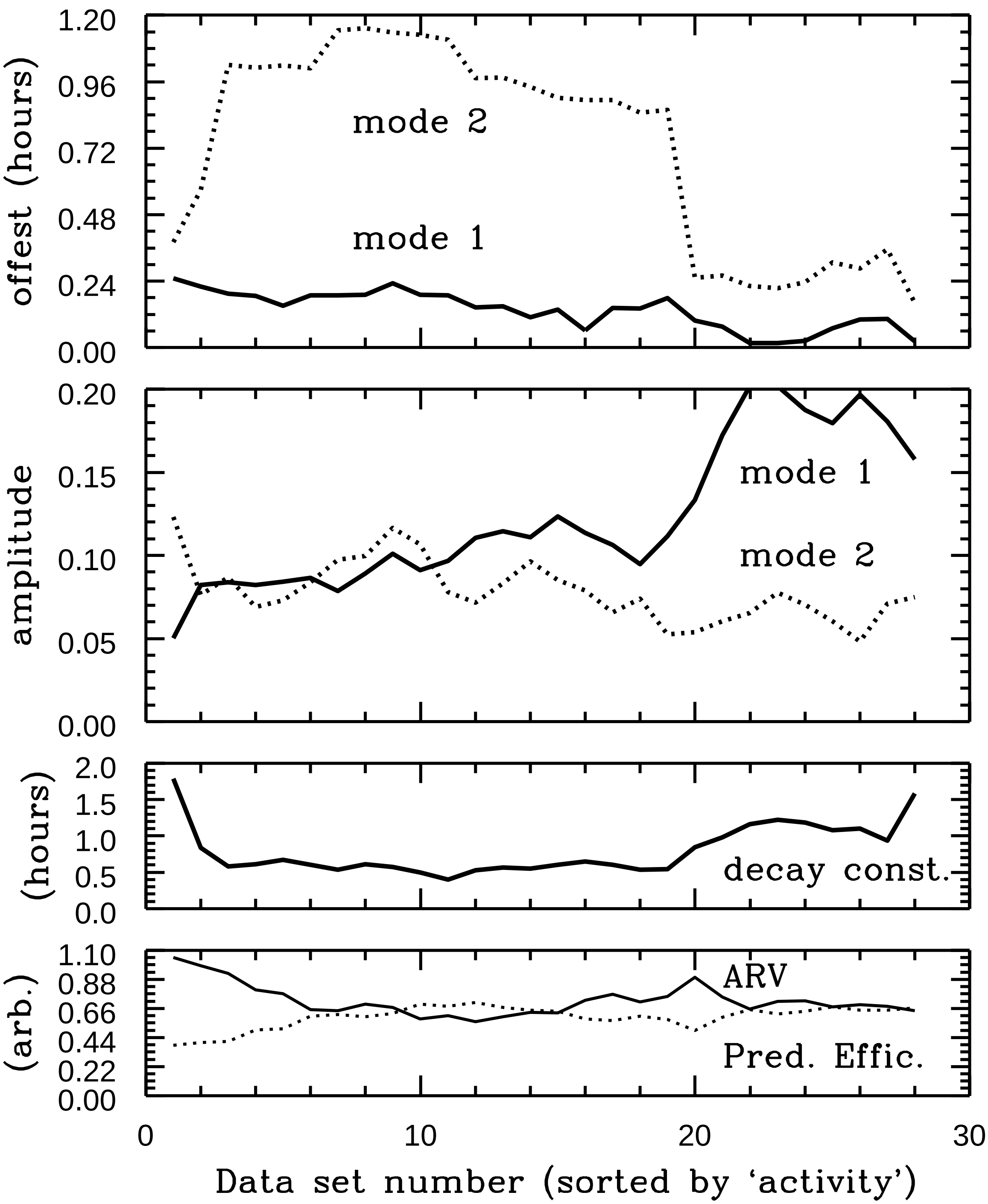

FIG. 3. Results of five parameter fit to the concatenated Bargatze data set with all five parameters, a_1, a_2, t_1, t_2, and τ, free to vary. (The amplitude is simply the output units over the input units, $nT/(nT \cdot km/s)$, or $1/km$)

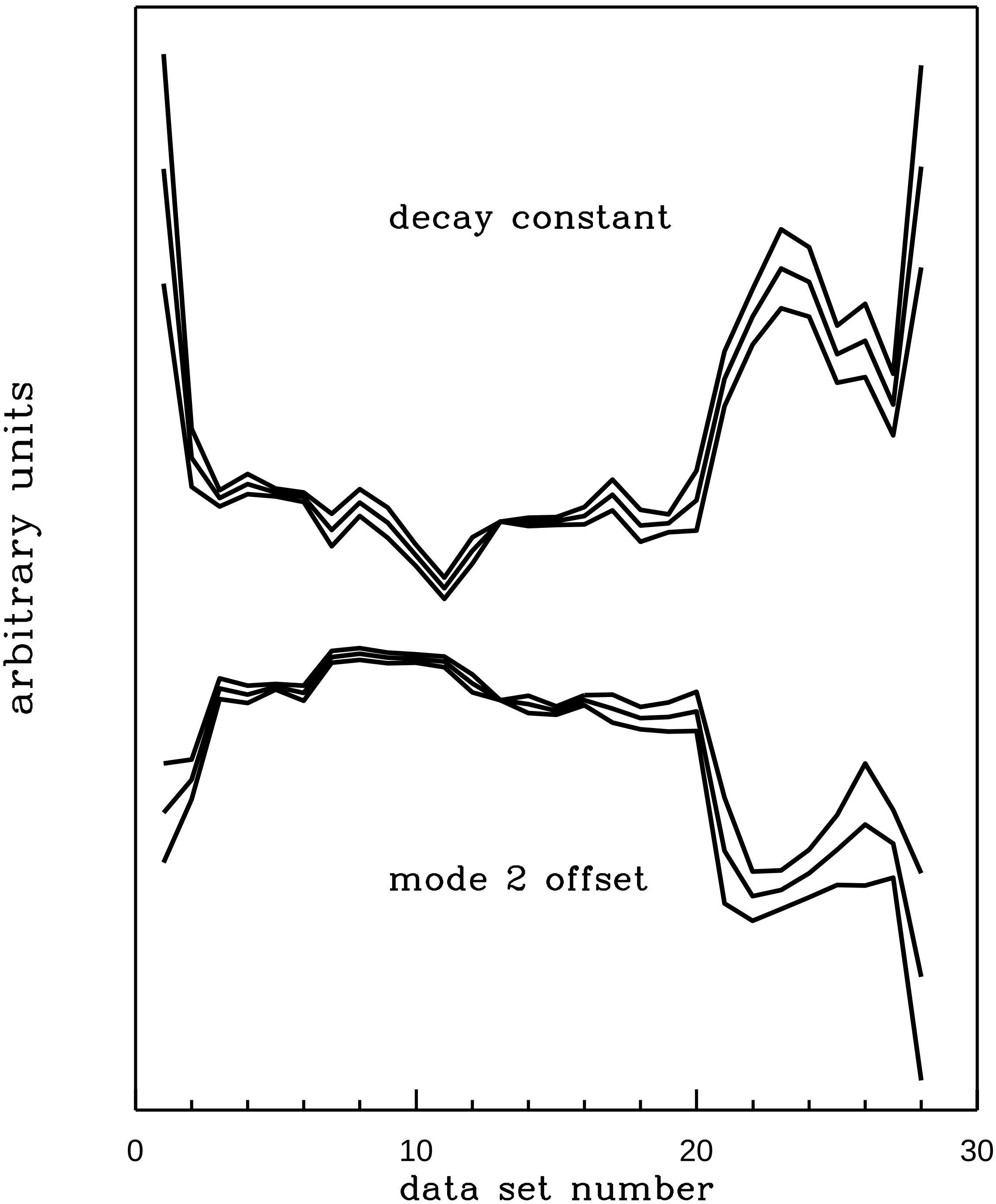

FIG. 4. Comparison of the parameter values of both t_2 and τ.

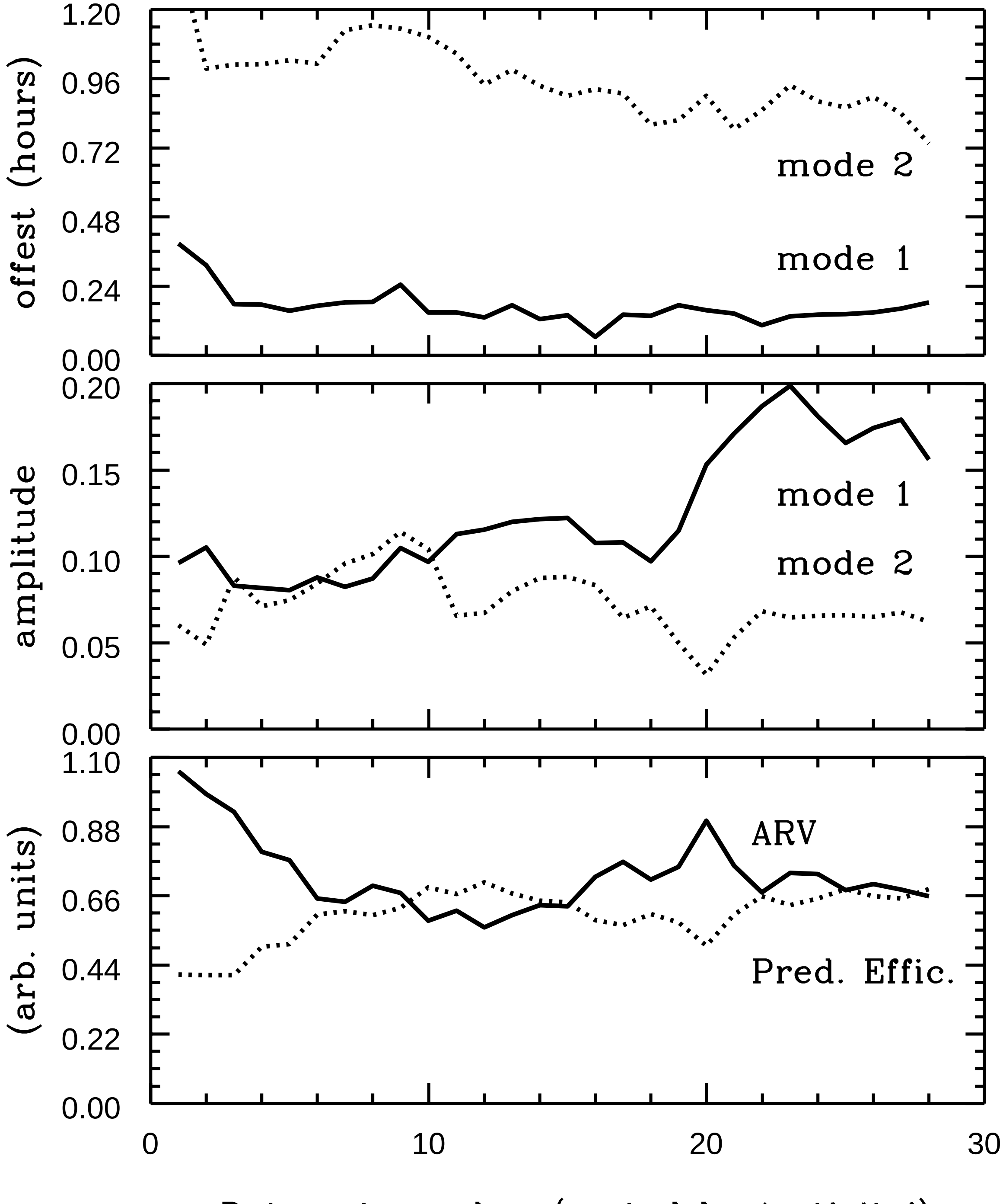

FIG. 5. Results of four parameter fit with the time constant for the low-pass filter fixed at $\tau = 0.617$ hours.

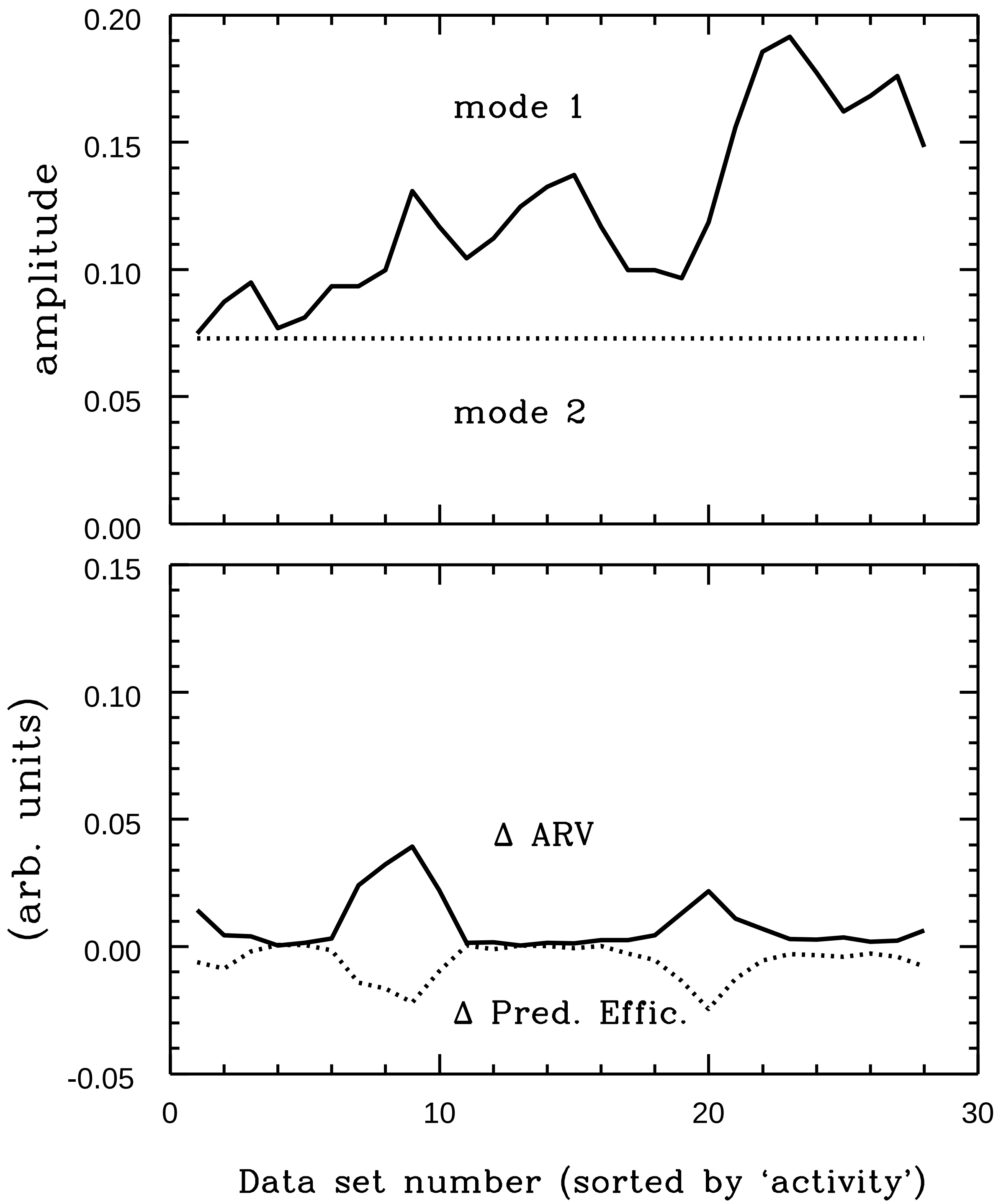

FIG. 6. Results for the fits with only the amplitude of the first mode variable. Of the four one parameter fits, this is noticeably the best, differing from the four parameter fits only slightly in sets 7 through 10 and 19 through 21.

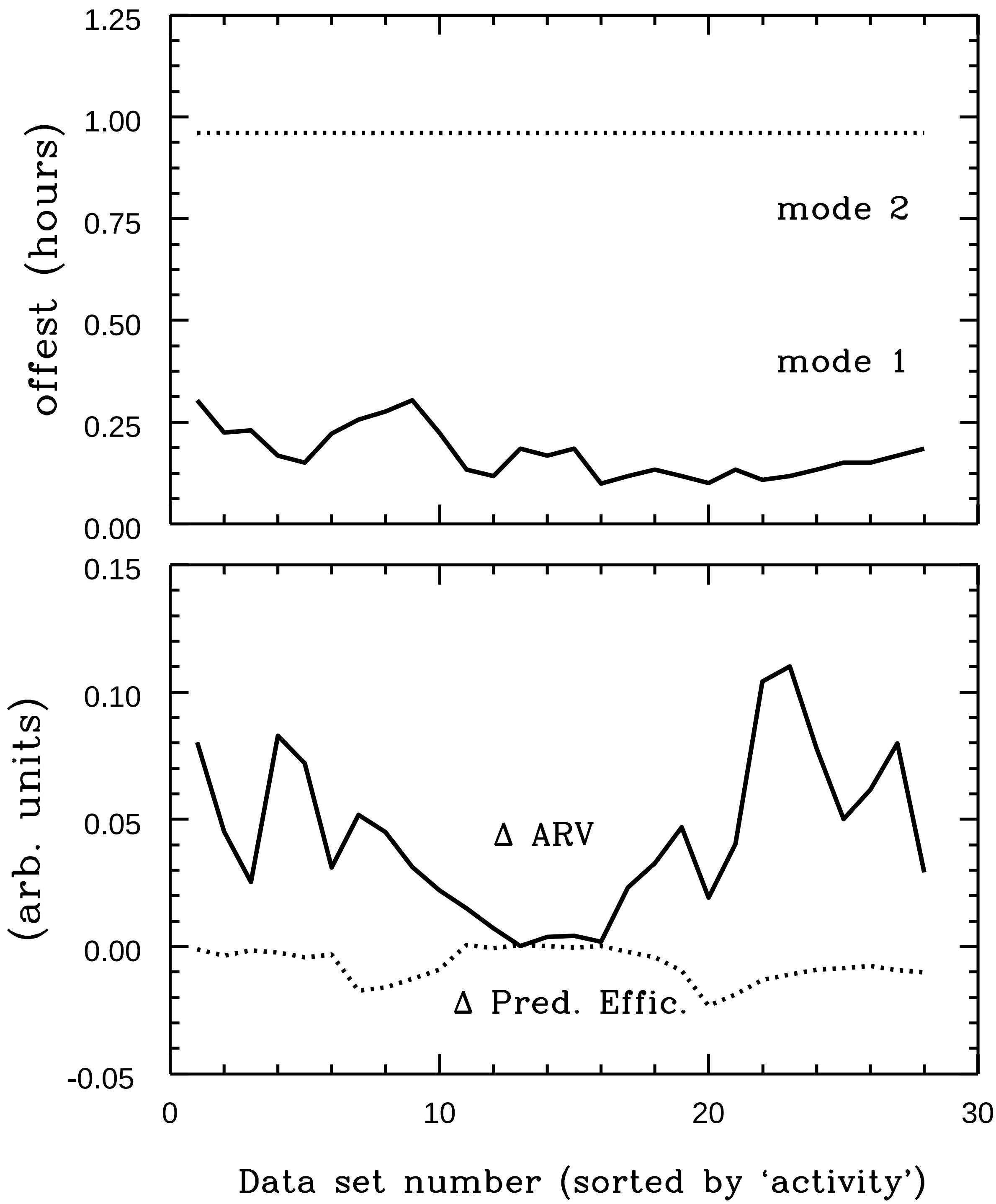

FIG. 7. Results for the fits with only the time delay of the first mode variable. This fit is only close to the four parameter fits at intervals 13 through 16 - the medium activity values to which it fits by definition.

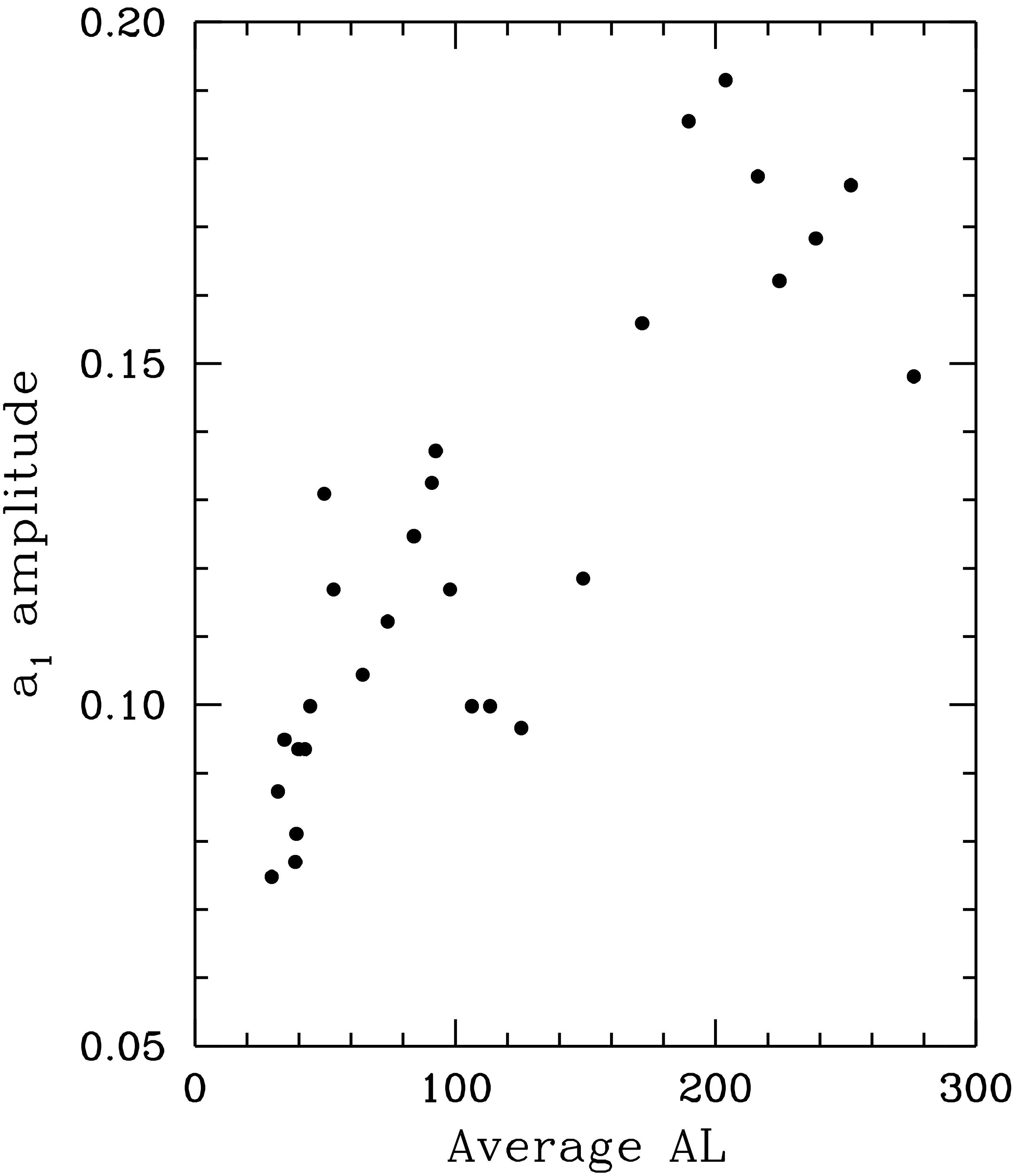

FIG. 8. Fit for a_1 only, same as in Figure 6, with a different ordinate.