Collisionless electron heating in inductively coupled discharges

K.C. Shaing and A.Y. Aydemir

Institute for Fusion Studies, The University of Texas at Austin

Austin, Texas 78712

Abstract

A kinetic theory of collisionless electron heating is developed for inductively coupled discharges with a finite height L. The novel effect associated with the finite-length system is the resonance between the bounce motion of the electrons and the wave frequency, leading to enhanced heating. The theory is in agreement with results of particle simulations.

PACS Nos.: 52.80.pi, 52.50.-b, 52.50.dg, 52.75.-d

Collisionless electron heating is the dominant heating mechanism in low-density inductively coupled discharges.[1–4] It can be described quantitatively in terms of the surface impedance. The standard theoretical model is to assume that the height of the system L is infinite.[1,5,6] When compared with results of particle simulations, this model is found to be valid for $\delta_a \ll L$.[1] Here, δ_a is the anomalous skin depth defined as $\delta_a = [v_t c^2/(\sqrt{\pi}\omega\omega_{pe}^2)]^{1/3}$ with v_t the electron thermal speed, c the speed of light, ω_{pe} the electron plasma frequency, and ω the wave frequency. In particle simulations, the surface impedance deviates and decreases significantly from the standard theoretical model when $\delta_a \lesssim L$.[1] We develop a kinetic theory to allow L to be finite. The novel effect associated with the finite height L is the bounce resonance between the bounce motion and the wave frequency. The results of the theory are in agreement with those from particle simulations. Because we adopt an optimum ordering, i.e. we assume that the electron collision frequency ν is of the order of the wave frequency ω, the theory is applicable to both collisional $\nu \gg \omega$, and collisionless $\nu \ll \omega$ plasmas. Thus, it can be useful in modeling inductively coupled discharges in searching for an optimum operation regime[7,8]. Note that the physics associated with a finite height L we describe here is not included in Ref. 9, in which a fluid theory is developed to simulate the kinetic effects in a half-infinite (i.e. $L \to \infty$) system. Our work, on the other hand, is similar to that of Ref. 10. With proper modifications of the electron equilibrium distribution, the theory is also applicable to conductors and semi-conductors.[5]

We employ a simplified theoretical model to describe the source. At $z = 0$, there is a spiral, planar antenna coil in which an oscillating current is driven with frequency ω. Plasma is confined in the region $0 \leq z \leq L$. The side wall in the radial direction is assumed to be at infinity. This assumption is valid if the collisional mean-free-path ℓ is less than the radial dimension of the source.

The coupled system of kinetic-Maxwell equations is solved as follows. We first solve the linearized kinetic equation for the perturbed electron distribution in response to the azimuthal wave electric field driven by the current in the coil. The azimuthal plasma current density is calculated by taking the velocity moment of the electron distribution. Inserting the plasma current in the Maxwell equations, we solve for the self-consistent azimuthal wave electric field and calculate the surface impedance.

The linearized kinetic equation away from the sheath regions located at $z \simeq 0$ and $z \simeq L$ is[5,6]

$$\frac{\partial \tilde{f}_1}{\partial t} + v_z \frac{\partial \tilde{f}_1}{\partial z} + \frac{e}{M} \tilde{\mathbf{E}}_1 \cdot \frac{\partial f_M}{\partial \mathbf{v}} = C(\tilde{f}_1), \tag{1}$$

where $\tilde{f}_1$ is the perturbed electron distribution function, v_z is the electron speed in the z-direction, e is the electron charge, M is the electron mass, $\mathbf{v}$ is the electron velocity, and $f_M = [N/(\pi^{3/2}v_t^3)]\exp(-v^2/v_t^2)$ is the Maxwellian distribution with N the plasma density, and $v = |\mathbf{v}|$. The collision operator $C(\tilde{f}_1)$ in Eq. (1) can be approximated by a Krook model, i.e. $C(\tilde{f}_1) = -\nu \tilde{f}_1$. Here, ν is the electron collision frequency which includes electron-electron Coulomb collisions, electron-ion Coulomb collisions, and electron-molecule collisions. In general, ν is a function of v. We assume that the width of the plasma sheath at $z \simeq 0$ and $z \simeq L$ is much smaller than the skin depth so that we can neglect the structure of the sheath. Assuming $\widetilde{\mathbf{E}_1} = \mathbf{E}_1(z)e^{i\omega t}$ and $\tilde{f}_1 = f_1(z)e^{i\omega t}$, we simplify Eq. (1) to

$$i\omega f_1 + v_z \frac{\partial f_1}{\partial z} + \frac{e}{m} \mathbf{E}_1 \cdot \frac{\partial f_0}{\partial \mathbf{v}} = -\nu f_1. \tag{2}$$

The solution to Eq. (2) is

$$f_1 = \int_L^z dz' \frac{D}{v_z} e^{\int_z^{z'} dz'' \frac{\nu+i\omega}{v_z} dz''} + g e^{-\int^z dz' \frac{\nu+i\omega}{v_z}}, \tag{3}$$

where g is an integration constant with $\partial g/\partial z = 0$, and $D = -(e\mathbf{E}_1/M) \cdot \partial f_M/\partial \mathbf{v}$.

The constant g and thus f_1 are determined by the boundary conditions at $z = 0$ and $z = L$, where the electrons are assumed to be specularly reflected by the plasma sheath potentials. To be specific, the boundary coditions are

$$\begin{aligned} f_1(v_z > 0,\ z = 0) &= f_1(v_z < 0,\ z = 0), \\ f_1(v_z > 0,\ z = L) &= f_1(v_z < 0,\ z = L). \end{aligned} \tag{4}$$

With Eq. (4), f_1 is found to be

$$f_1^+ = \int_L^z dz' \frac{D}{v_z} e^{\frac{\nu+i\omega}{v_z}(z'-z)} - \left(1 - e^{-2\frac{\nu+i\omega}{v_z}L}\right)^{-1}$$

$$\times \int_L^0 dz' \frac{D}{v_z} \left[e^{-\frac{\nu+i\omega}{v_z}(z'+z)} + e^{\frac{\nu+i\omega}{v_z}(z'-z)} \right], \tag{5}$$

where the superscript "+" in f_1 indicates that $v_z > 0$. For $v_z < 0$, f_1^- has the same form as f_1^+, i.e.

$$f_1^- = \int_L^z dz' \frac{D}{v_z} e^{\frac{\nu+i\omega}{v_z}(z'-z)} - \left(1 - e^{-2\frac{\nu+i\omega}{v_z}L}\right)^{-1}$$

$$\times \int_L^0 dz' \frac{D}{v_z} \left[e^{-\frac{\nu+i\omega}{v_z}(z'+z)} + e^{\frac{\nu+i\omega}{v_z}(z'-z)} \right], \tag{6}$$

The azimuthal current density J_θ can be calculated by taking the v_θ moment of Eqs. (5) and (6). Here v_θ is the azimuthal electron speed. Extending the domain of interest from $0 \le z \le L$ to $-L \le z \le L$, and employing the relation for the azimuthal wave electric field $E_\theta(-z) = E_\theta(z)$, we can cast J_θ in a symmetric form

$$J_\theta = \frac{2}{\sqrt{\pi}} \frac{Ne^2}{Mv_t} \int_{-L}^{L} dz' E_\theta(z') K_a(|z-z'|), \tag{7}$$

where

$$K_a(|z-z'|) = \int_0^\infty dx\, x^3 e^{-x^2} \int_0^{\pi/2} d\Theta \frac{\sin^3\Theta}{\cos\Theta} \left[e^{-\frac{a|z-z'|}{x\cos\Theta}} \right.$$

$$\left. + \left(e^{\frac{a|z-z'|}{x\cos\Theta}} + e^{-\frac{a|z-z'|}{x\cos\Theta}} \right) \frac{e^{-2aL/(x\cos\Theta)}}{1 - e^{-2aL/(x\cos\Theta)}} \right]. \tag{8}$$

In Eq. (8), $x \equiv v/v_t$, $a = (v + i\omega)/v_t$, and Θ is the angle in the spherical coordinates (v, Θ, ϕ) in the velocity space such that $v_z = v\cos\Theta$, and $v_\theta = v\sin\Theta\cos\phi$.

Combining Faraday's law and Ampére's law, we obtain

$$\frac{d^2E_\theta}{dz^2} + \frac{\omega^2}{c^2} E_\theta = \frac{4\pi i\omega}{c^2} J_\theta. \tag{9}$$

With Eq. (7), Eq. (9) can be expressed as

$$\frac{d^2E_\theta}{dz^2} + \frac{\omega^2}{c^2} E_\theta = i\alpha \int_{-L}^{L} dz'\, E_\theta(z') K_a(|z-z'|), \tag{10}$$

where $\alpha = 2\omega\omega_{pe}^2/(\sqrt{\pi} v_t c^2)$.

Equation (11) can be solved by expanding in Fourier series with the boundary condition $E_\theta(L) = E_\theta(-L) = 0$. Note also that dE_θ/dz at $z = 0$ is not continuous because of the current in the coil, and we have $dE_\theta/dz|_{z\to 0^+} = -dE_\theta/dz|_{z\to 0^-} = \mu$. The solution for $E_\theta(z)$ is then

$$E_\theta(z) = -\frac{2\mu}{L}\sum_{n=0}^{\infty}\frac{\cos[(2n+1)\pi z/2L]}{[\frac{(2n+1)\pi}{2L}]^2 - \frac{\omega^2}{c^2} + i\alpha L k_a^n}, \tag{11}$$

where $k_a^n = L^{-1}\int_{-L}^{L} dz \cos[(2n+1)\pi z/2L] K_a(|z|)$. The surface impedance Z is defined as $Z = -(4\pi i\omega/c^2)E_\theta(0)/\mu$. From Eq. (11), we find

$$Z = \frac{4\pi i\omega}{c^2}\frac{2}{L}\sum_{n=0}^{\infty}\frac{1}{[\frac{(2n+1)\pi}{2L}]^2 - \frac{\omega^2}{c^2} + i\alpha L k_a^n}. \tag{12}$$

The explicit expression for k_a^n is

$$k_a^n = \frac{1}{L}\int_0^\infty dx\, x^3 e^{-x^2}\int_0^{\pi/2} d\Theta\,\frac{\sin^3\Theta}{\cos\Theta}\left[\frac{2a}{x\cos\Theta} + \frac{(-1)^n(2n+1)\pi}{L\sinh(\frac{aL}{x\cos\Theta})}\right]\times$$
$$\left\{\left(\frac{a}{x\cos\Theta}\right)^2 + \left[\frac{(2n+1)\pi}{2L}\right]^2\right\}^{-1}. \tag{13}$$

If ν is not a function of v, k_a^n can be simplified to

$$k_a^n = \int_0^\infty dy e^{-y^2}\left[aL + \frac{(-1)^n(n+\frac{1}{2})\pi y}{\sinh(aL/y)}\right]\Big/\left\{(aL)^2 + \left[(n+\tfrac{1}{2})\pi y\right]^2\right\}. \tag{14}$$

The first term in the square brackets of Eq. (13) and (14) describes the wave-particle resonance in a half-infinite system, whereas the second term describes the bounce resonance between the bounce motion and the wave frequency in a system with a finite height L in the collisionless, i.e. $\nu \ll \omega$ case. Because the $\omega < \omega_{pe}$, ω^2/c^2 term in Eq. (12) can be neglected. If the collisional mean-free-path $\ell = v_t/\nu$ is much less than L, the bounce resonance term can be neglected, $k_a^n \approx \sqrt{\pi}/(2aL)$, and the collisional surface impedance is

$$Z \cong \frac{4\pi i\omega}{c^2 k}\tanh(kL), \tag{15}$$

where $k = (\omega_{pe}/c)\left(\sqrt{\phi(1+\phi)/2} + i\sqrt{\phi(1-\phi)/2}\right)$, and $\phi = (1+\nu^2/\omega^2)^{-1/2}$. This result is in agreement with that obtained by solving the Maxwell equations with a collisional Ohm's law.[1,10] If $\nu \ll \omega$, both resonance terms contribute to Z. However, if $\delta_a \ll L$, the wave-particle resonance effect dominates $k_a^n \simeq 1/(2n+1)$, and

$$Z \simeq \frac{4\pi\omega}{c^2}\frac{2\delta_a}{3}\left(i + \frac{1}{\sqrt{3}}\right). \tag{16}$$

This result agrees with that obtained in the $L \to \infty$ limit.[1,6,11,12] If $\delta_a \lesssim L$, the bounce resonance term becomes dominant. The individual contributions to the real part of the surface impedance ζ from each of these two terms, and the real part of the total impedance as a function of δ_a/L are shown in Fig. 1. The bounce resonance term ζ_{BR} is seen to be dominant for most of the relevant range of the parameter δ_a/L. Note that the total impedance shown here is not equal to the sum of the individual parts because of a cancellation that occurs when both terms are considered simultaneously, and the nonlinear dependence of ζ on its individual parts. The dashed line in Fig. 1 indicates the result of the $L \to \infty$ limit ζ_∞ shown in Eq. (16). It is seen that ζ deviates and decreases significantly from ζ_∞ when $\delta_a/L \gtrsim 0.1$. The theory can be used to find an accurate low-density operation limit at which ζ reaches its maximum, i.e. $d\zeta/d(\delta_a/L) = 0$. Because we adopt an optimum ordering, i.e. $\nu \sim \omega$ in solving the kinetic-Maxwell equations, the theory is applicable to both collisional and collisionless plasmas. This is demonstrated in Fig. 2 where ζ as a function of ν/ω is shown. When $\nu/\omega < 0.1$, ζ becomes independent of ν/ω indicating the onset of collisionless heating. The results in Figs. 1 and 2 are in agreement with the particle simulations presented in Refs. 1 and 9.

In summary, a kinetic theory of the collisionless electron heating is developed for inductively coupled discharges with a finite height L. The novel effect associated with the finite-size of the system is the bounce resonance between the bounce motion of the electrons and the wave frequency. The theory is in good agreement with results of particle simulations. The surface impedance obtained in the $L \to \infty$ limit Z_∞ is valid if $\delta_a \ll L$. However, if $\delta_a \lesssim L$, the value of the surface impedance decreases significantly from Z_∞. The value of δ_a/L at which the real part of the surface impedance ζ reaches maximum i.e. $d\zeta/d(\delta_a/L) = 0$ can be employed to define an accurate low density operation limit. Because we adopt an optimum ordering, the theory can be used in searching for the optimum operation regime. It can be tested experimentally by continuously varying the ratio ν/ω. With proper modification of the equilibrium distribution function, it can also be applicable to conductors and semi-conductors.

Acknowledgments

This work was supported by the U.S. Dept. of Energy Contract No. DE-FG03-96ER-54346. The authors would like to thank Dr. M. Turner for a useful discussion and providing us with his simulation data.

References

1. M.M. Turner, Phys. Rev. Lett. **71**, 1844 (1993).

2. J. Hopwood, C.R. Guarnieri, S.J. Whitehair, and J.J. Cuomo, J. Vac. Sci. Technol. A **11**, 147 (1993).

3. J. Hopwood, C.R. Guarneiri, S.J. Whitehair, and J.J. Cuomo, J. Vac. Sci. Technol. A **11**, 152 (1993).

4. M.S. Barnes, J.C. Forster, and J.H. Keller, Appl. Phys. Lett. **62**, 2622 (1993).

5. G.E.H. Reuter and E.H. Sondheimer, Prog. Roy. Soc. A (London) **195**, 336 (1948).

6. K.C. Shaing, Phys. Plasmas (to be published).

7. T.J. Sommerer and M.J. Kushner, J. Appl. Phys. **71**, 1654 (1992).

8. P.L.G. Ventzek, R.J. Hoekstra, T.J. Sommerer, and M.J. Kushner, Appl. Phys. Lett. **63**, 605 (1993).

9. M.M. Turner, Plasma Sources Sci. Technol. **5**, 159 (1996).

10. J.A. Reynolds, H.A. Blevin, and P.C. Thonemann, Phys. Rev. Lett. **22**, 762 (1969).

11. S. Ichimaru, "Basic Principles of Plasma Physics: A Statistical Approach," *Frontiers in Physics*, Vol. 41 (W.A. Benjamin, Inc., Reading, MA, 1973) p. 84.

12. E. S. Weibel, Phys. Fluids **10**, 741 (1967).

FIGURE CAPTIONS

FIG. 1. The real part of the surface impedance as a function of δ_a/L. ζ_{BR}-line has only the bounce resonance contribution from Eq. 14, whereas the ζ_{WP}-line has the contribution from the wave-particle resonance term. The plasma parameters are: $\nu/\omega = 0.01$, $L = 4$cm, $\omega = 2\pi \times 13.56\,$MHz, and $T_e = 5\,$eV. The dashed line is the $L \to \infty$ limit from Eq. 16.

FIG. 2. Real part of the impedance ζ as a function of ν/ω. The plasma parameters are the same as in Fig. 1, and $N = 10^{11}\mathrm{cm}^{-3}$.