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KINETIC THEORY OF RF WAVE IN A PLASMA IN AN INHOMOGENEOUS MAGNETIC FIELD

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ABSTRACT

Kinetic effects on propagation and absorption of radio frequency (rf) waves in inhomogeneous and dispersive plasmas are studied, where effects of finite gyroradius and wave-particle interactions are included. The generalized linear propagator in the presence of the inhomogeneity of magnetic field strength along the field line is calculated for obtaining the nonlocal conductivity tensor. Instead of a plasma dispersion function a new function is introduced. The influence of the inhomogeneity to the rf wave-energy deposition scheme is found to be appreciable in high temperature plasmas. Parameter dependence of this new function is studied. We derive a kinetic wave equation taking the corrections due to the plasma inhomogeneity and dispersion. Nonlocal effects on the wave energy flux and on the power deposition to each plasma species are examined. This approach is applied to toroidal plasmas.

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1. Introduction

Problems of radio frequency waves in plasmas have attracted attention for many years from the viewpoint of the physics interests and applications to the thermonuclear fusion research. Recently, the investigations on the heating, the current-drive and the flux-control by use of rf waves have been intensively studied both theoretically and experimentally. These analyses are the critical issues for the development of the thermonuclear fusion program. In addition to these applications, various concepts to utilize the rf wave have been proposed, such as rf end plugging or the rf stabilization of the MHD instabilities. Experiments have been performed to examine the availability of these schemes. The plasmas of experimental concerns are confined in complex geometries and hence there still exist discrepancies between the theoretical prediction and the experimental observations, which have revealed the necessity of the comprehensive studies of the rf-wave accessibility, propagation and absorption in inhomogeneous and dispersive plasmas.

Previous theoretical analyses on this subject[1] have been done mainly by use of the magnetohydrodynamic (MHD) wave-equation studies and the ray-tracing method. The problem is that the key issues, to determine the wave propagation structures in the inhomogeneous dispersive plasma and to obtain the power and momentum absorption rates to each plasma species, have not been resolved by these analyses. From the MHD analyses, we may have the rough estimation of the mode structure and can estimate the power absorption by introducing an artificial damping rate.[2] In high temperature plasmas, the Coulomb collision becomes rare and the characteristic problem of the MHD analyses apparently appears: The singular nature of the MHD equation gives rise to the divergence behavior of the wave form and the mode conversion to the hot mode is excluded. By the ray-tracing method, the correct wave form and the energy deposition rates in the plasma are not obtained when the wave length becomes of the order of the scale length of the inhomogeneity. In addition to it, the usual ray-tracing analyses have been done by use of the conductivity tensor which was derived for homogeneous plasmas.[3]

On the local damping rate of the launched wave, the effect of the nonuniformity of the magnetic field strength along the field line have been studied.[4-8] However the effect on the global propagation form of the wave field has not been studied.

In this article, we study the kinetic effects on the propagation and absorption of rf waves in the inhomogeneous and dispersive plasma immersed in an inhomogeneous magnetic field. We include the finite-gyroradius effect and wave-particle interactions to derive the wave propagation equation. The modification of the linear propagator due to the inhomogeneity of the magnetic field strength along the magnetic field line is also included. The case where the magnetic field strength is inhomogeneous along the guiding center orbit of particles is often found in magnetic confinement systems, such as the toroidal devices and open mirrors. Because of this effect the damping mechanism of the wave becomes insensitive to the resonance condition $|\omega - n\Omega| \sim |k_\parallel v_{T\parallel}|$ (Ω: cyclotron frequency, $k_\parallel$: parallel wave number and $v_{T\parallel}$: the parallel thermal velocity). All particles can interact with the wave and the dissipation remains to be finite even if $k_\parallel \to 0$.

For the consistent analyses of the wave propagation and absorption in a dispersive nonuniform plasma, we also analyze the effect of the nonlocality on the perturbed current. The nonlocal effects of the mobility tensor and the wave are simultaneously included in order for the wave energy density to be conserved.

The constitution of this paper is as follows. In Section 2, we show the general formalism of the wave propagation equation and discuss the effect of the magnetic field strength inhomogeneity along the field line of force. A dispersion function Φ_n is introduced instead of the plasma dispersion function. The nonlocal effect on the perturbed current and the propagation equation are presented by means of the nonlocal mobility tensor. In Section 3 we apply the general wave equation to plasmas in toroidal magnetic confinement devices. The dissipation rate as an effective collision is estimated for the typical parameters of the plasma, which is shown to be an appreciable contribution. In the final section, summary and discussion are presented.

2. Wave Equation in Inhomogeneous Magnetic Field

In this section, we construct the basic formalism of the rf wave equation in a dispersive plasma immersed in an inhomogeneous magnetic field. The model magnetic field is given by $B_s = B_0(1+s/\ell_1 + s^2/\ell_2^2)$, where s is the coordinate along the main magnetic field, ℓ_1 and ℓ_2 are the typical scale lengths of the field inhomogeneity and the condition $|s| < \ell_1, \ \ell_2$ is used. Assuming that the inhomogeneity is weak, $(\rho/\ell_{1,2})^2 \ll 1$, ($\rho$ being the gyroradius), the particle orbit is approximately given by the sum of the guiding center motion along the field line and the perpendicular gyromotion. The cyclotron frequency varies along the field line and hence the modification appears in the phase of the wave, which particles feel, $\phi(\tau) = \omega(t-t') - \vec{k} \cdot (\vec{x}-\vec{x}')$, $\tau \equiv t' - t$. The parallel velocity indeed is not constant of the motion. However, the temporal change of $v_\parallel$ and the associated change of $k_\parallel v_\parallel$ is much smaller than the modification of $\Omega = eB/m$ for the waves in the range of the cyclotron frequency except for the magnetically trapped particles which are close to the turning point. We approximate $v_\parallel$ to be constant for the simplicity. We take the x-coordinate in the direction of the perpendicular wave number, $\hat{x} = \vec{k}_\perp/|k_\perp|$, $\vec{k}_\perp = (\vec{k} \times \hat{s}) \times \hat{s}$ and $\vec{k}$ is the wave vector. The position of the particle at t′ is given as ($\hat{y} = -\hat{x} \times \hat{s}$ and $s' = 0$ at $\tau = 0$)

$$x' - x = \rho \cos[-\Omega_0\tau + \frac{\Omega_0 v_\parallel}{2\ell_1}\tau^2 - \frac{\Omega_0 v_\parallel^2}{3\ell_2^2}\tau^3 + \alpha] - \rho \cos\alpha \tag{1-1}$$

$$y' - y = \rho \sin[-\Omega_0\tau + \frac{\Omega_0 v_\parallel}{2\ell_1}\tau^2 - \frac{\Omega_0 v_\parallel^2}{3\ell_2^2}\tau^3 + \alpha] - \rho \sin\alpha \tag{1-2}$$

$$s' = v_{\parallel}\tau \tag{1-3}$$

where (x,y,s) is the position at $\tau = 0$, α is the gyrophase at $\tau = 0$ and terms of the order of $(v_{\parallel}/\Omega\ell_{1,2})^2$ are neglected. The phase $\phi(\tau)$ is then given as

$$\phi(\tau) = (k_{\parallel}v_{\parallel} - \omega)\tau - k_{\perp}\rho\{\cos(\alpha+\phi) - \cos\alpha\} \tag{2}$$

where

$$\phi = \Omega_0\tau - \frac{\Omega_0 v_{\parallel}}{2\ell_1}\tau^2 + \frac{\Omega_0 v_{\parallel}^2}{3\ell_2^2}\tau^3 \tag{3}$$

$\Omega_0 = e_s B_0/m_s$ and the suffix s stands for the particle species.

The perturbed distribution function $\tilde{f}$ is given by integrating the Vlasov equation as

$$\tilde{f}_k(v) = \frac{-ie_s}{m_s\omega}\{\frac{\partial f_0}{\partial \vec{v}} - \int_0^\infty d\tau\ [\frac{\partial}{\partial\tau}(\frac{\partial f_0(\vec{v}')}{\partial \vec{v}'})$$

$$- i\vec{k}\cdot\frac{\partial f_0(\vec{v}')}{\partial\vec{v}'}\vec{v}']\exp(-i\phi - \eta\tau)\}\tilde{\vec{E}}_k \tag{4}$$

where the infinitesimal positive parameter η indicates the analytic continuation to satisfy the causality. The equation (3) shows that the gyrophase ϕ is not a linear function of τ. Writing the flow $\tilde{\vec{\Gamma}}$ to be $N\overleftrightarrow{\mu}$ $\tilde{\vec{E}}$, (N being the number density of the plasma species), the path integral along the unperturbed orbit, Eq. (1), gives the mobility tensor $\overleftrightarrow{\mu}$ as

$$\overleftrightarrow{\mu}\tilde{\vec{E}} = \int d^3\vec{v}\, \frac{ie_s}{m_s\omega}\, \vec{v} \left\{ -\begin{pmatrix} \cos\alpha \dfrac{\partial f_0}{\partial v_\perp} \\ \sin\alpha \dfrac{\partial f_0}{\partial v_\perp} \\ \dfrac{\partial f_0}{\partial v_\parallel} \end{pmatrix} + \sum_{n=-\infty}^{\infty} \sum_{n'=-\infty}^{\infty} e^{-i(n-n')\alpha}\, J_{n'}(\zeta) G_n \right. \tag{5}$$

$$\left. \times \left(\frac{n\Omega}{v_\perp}\frac{\partial f_0}{\partial v_\perp} + k_\parallel \frac{\partial f_0}{\partial v_\parallel}\right) \begin{pmatrix} \dfrac{n\Omega}{k_\perp} J_n(\zeta) \\ iv_\perp J_n'(\zeta) \\ v_\parallel J_n(\zeta) \end{pmatrix} \right\} \cdot \tilde{\vec{E}}$$

where $\zeta = k_\perp \pi$, J_n is the n-th order Bessel function and G_n is the linear propagator defined by

$$G_n = i \int_0^\infty d\tau \, \exp i\{(\omega - k_\parallel v_\parallel + i\eta)\tau - n\phi\} \, . \tag{6}$$

We take the equilibrium distribution function as a local Maxwellian with the temperature anisotrophy $T_\perp$ and $T_\parallel$ to be

$$f_0(\vec{v}) = N\left(\frac{m}{2\pi}\right)^{3/2} \frac{1}{T_\perp \sqrt{T_\parallel}} \exp\left[\frac{-mv_\perp^2}{2T_\perp} - \frac{mv_\parallel^2}{2T_\parallel}\right] \tag{7}$$

for electrons and ions. Substituting Eq. (7) into Eq. (5), we have

$$\overleftrightarrow{\mu}_s = \frac{-ie_s}{m_s\omega}\,\frac{T_{\perp s}}{T_{\parallel s}}\left[-\frac{T_{\parallel s}}{T_{\perp s}}\overleftrightarrow{I} + \sum_n \overleftrightarrow{M}_n\right] , \tag{8}$$

where

$$\overleftrightarrow{M}_n = \begin{bmatrix} \frac{n^2\Lambda_n}{\lambda}\Phi_n^{(0)}, & in\Lambda_n'\,\Phi_n^{(0)}, & \left(\frac{\omega-n\Omega}{\Omega}\,\frac{k_\perp}{k_\parallel}\right)\frac{n\Lambda_n}{\lambda}\Phi_n^{(1)} \\ -in\Lambda_n'\,\Phi_n^{(0)}, & \left(\frac{n^2\Lambda_n}{\lambda} - 2\lambda\Lambda_n'\right)\Phi_n^{(0)}, & -i\left(\frac{\omega-n\Omega}{\Omega}\,\frac{k_\perp}{k_\parallel}\right)\Lambda_n'\,\Phi_n^{(1)} \\ \left(\frac{\omega-n\Omega}{\Omega}\,\frac{k_\perp}{k_\parallel}\right)\frac{n\Lambda_n}{\lambda}\Phi_n^{(1)}, & i\left(\frac{\omega-n\Omega}{\Omega}\,\frac{k_\perp}{k_\parallel}\right)\Lambda_n'\Phi_n^{(1)}, & \left(\frac{\omega-n\Omega}{\Omega}\,\frac{k_\perp}{k_\parallel}\right)\frac{\Lambda_n}{\lambda}\Phi_n^{(2)} \end{bmatrix} \tag{9}$$

$\Lambda_n = I_n(\lambda)\cdot\exp(-\lambda)$, $\lambda = k_\perp^2\pi^2$, $\Lambda_n' = d\Lambda_n/d\lambda$, $\Omega = \Omega_0(1+s/\ell_1 + s^2/\ell_2^2)$,

and

$$\Phi_n^{(\ell)} = \frac{1}{\sqrt{2\pi T_\parallel/m}}\int_{-\infty}^{\infty} dv_\parallel \left(\frac{k_\parallel v_\parallel}{\omega-n\Omega}\right)^{\ell}\left(k_\parallel v_\parallel + \frac{n\Omega T_\parallel}{T_\perp}\right)\exp\left(-\frac{mv_\parallel^2}{2T_\parallel}\right) G_n . \tag{10}$$

The presence of the magnetic field inhomogeneity changes the usual plasma dispersion function $Z(\xi) \equiv \int_{-\infty}^{\infty} dt\, \exp(-t^2)/(t-\xi)\sqrt{\pi}$ and its derivatives to the new function $\Phi_n^{(\ell)}$ given by Eq. (10). This field inhomogeneity contributes to the broadening of the resonance condition of the wave particle interaction from $v_\parallel = \hat{\omega}/k_\parallel$ to $|v_\parallel - \hat{\omega}/k_\parallel| \leq \Delta(\omega/k_\parallel)$, $(\hat{\omega} \equiv \omega - n\Omega)$. The width $\Delta(\omega/k_\parallel)$ is given as follows. The propagator for G_n in $\Phi_n^{(\ell)}$ is rewritten

$$G_n = i \int_0^\infty \exp\, i(\alpha\tau + \beta\tau^2 - \gamma\tau^3) d\tau \qquad (11)$$

where

$$\alpha = (\omega - k_\parallel v_\parallel + i\eta) \qquad (12\text{-}1)$$

$$\beta = \frac{n\Omega v_\parallel}{2\ell_1} \qquad (12\text{-}2)$$

$$\gamma = \frac{n\Omega v_\parallel^2}{3\ell_2^2} \ . \qquad (12\text{-}3)$$

The second and third terms in the bracket of Eq. (11) cause the decorrelation and the resonance width $\Delta(\omega/k_\parallel)$. We consider the function Φ defined as

$$\Phi = \hat{\omega} \sqrt{\frac{m}{2\pi T_\parallel}} \int_{-\infty}^{\infty} dv_\parallel \exp\left(-\frac{mv_\parallel^2}{2T_\parallel}\right) G_n \ . \qquad (13)$$

In the limit of $\ell_{1,2} \to \infty$, the values β and γ vanish and hence the resonances of particles and waves occur for $\alpha = 0$. Namely

$$\lim_{\ell_{1,2}\to\infty} \Phi(\alpha,\beta,\gamma) \to \xi_n Z(\xi_n) \qquad (14)$$

where Z is the plasma dispersion function of the argument $\xi_n = (\omega - n\Omega)/\sqrt{2}\ |k_\parallel|v_{T\parallel}$.

Figure 1 shows the real and imaginary parts of Φ as compared with the values of $\xi Z(\xi)$. The parameter ζ

$$\zeta \equiv \frac{n\Omega v_{T\parallel}}{2\ell_1(\omega - n\Omega)^2} \tag{15}$$

is taken to be ±0.1, ±0.3, ±1, ±3 and ±10 for the case of $1/\ell_2 = 0$. As is clearly shown in Fig. 1, the inhomogeneity of the cyclotron frequency along the field line not only broadens the resonance condition but also causes an effective collision in the off-resonant limit $|\omega/k_\parallel| \gg v_{T\parallel}$.[4] The non-resonant dissipation can be interpreted as follows. The propagator G_n is equivalent to that of the particles responding to the wave with frequency modulation, which means that the ω-spectrum has the finite width. The width of the spectrum is estimated to be $\sqrt{\beta}$. When the wave has the finite spectrum, the decorrelation between particle and the wave gives rise to the effective collision, which contributes to the absorption/emission of the wave. We show the typical parameter dependence of this collision term. In the limit $1/\ell_{1,2} \to 0$ or the resonant limit ($\alpha \to 0$), the asymptotic form is given as

$$G_n \simeq \begin{cases} \frac{1}{\alpha}\left[-1 + \frac{2i\beta}{\alpha^2} - \frac{6\gamma}{\alpha^3} + \ldots\right] & \alpha \to \infty \\ \frac{1}{2}\sqrt{\frac{\pi}{2}}\frac{1}{\sqrt{\beta}}(-1+i) & \beta > 0 \\ \frac{i}{(i\gamma)^{1/3}}\frac{\pi}{\sqrt{3}}\frac{1}{\Gamma(2/3)} & \beta \simeq 0 \quad \alpha \to 0 \\ \frac{1}{2}\sqrt{\frac{\pi}{2}}\frac{1}{\sqrt{-\beta}}(1+i) & \beta < 0 \end{cases} \tag{16}$$

When the magnetic field inhomogeneity is small, $G \sim -1/\alpha$ holds and the wave-particle interaction takes place mainly by usual Landau and cyclotron resonance. This resonance vanishes when $k_\parallel = 0$. If the decorrelation exists, $\beta \neq 0$, the damping appears even for $k_\parallel = 0$. Substituting $k_\parallel = 0$ into Eq. (13) one finds

$$\mathrm{Im}\Phi = \mathrm{Im} \lim_{\eta \to 0} i \int_0^\infty e^{(it - \eta t - \frac{\zeta^2}{4} t^4)} dt \ . \tag{17}$$

The equation (17) can be evaluated by use of the method of the steepest descent for smaller values of ζ. We approximately have

$$\mathrm{Im}\Phi \sim \begin{cases} \sqrt{\frac{2\pi}{3}}\,\zeta^{-1/3}\exp\left(-\frac{3}{8}\zeta^{-2/3}\right)\cos\left(-\frac{\pi}{6} + \frac{3\sqrt{3}}{8}\zeta^{-2/3}\right) \quad (\zeta \to 0) & \text{(18-1)} \\ \Gamma\left(\frac{5}{4}\right)\sqrt{\frac{2}{\zeta}} \quad (\zeta \to \infty) & \text{(18-2)} \end{cases}$$

Figure 2 illustrates the real and imaginary parts of Φ function as a function of ζ for the case of $|\xi| = \infty$. The result implies that when the plasma temperature is high enough for the classical collision to be rare, this effective collision plays an important role for the wave energy deposition scheme.

This dissipation also becomes important when one considers the concepts utilizing the ponderomotive force generated by the rf waves. The ponderomotive force in the MHD limit is written as

$$\vec{F}_s = -\nabla \frac{N_s e_s^2}{m_s} \{\frac{\tilde{E}_\perp^2}{\omega^2 - \Omega_s^2} + \frac{\tilde{E}_\parallel^2}{\omega^2}\} . \tag{19}$$

One origin of the ponderomotive force is the nonuniformity of the rf wave amplitude. The other is the inhomogeneity of the cyclotron frequency. When $\nabla_\parallel |B|$ and $\tilde{E}_\perp$ exist, the absorption of the wave occurs even if the condition $|\omega^2 - \Omega^2| \gg k_\parallel^2 v_{T\parallel}^2$ is satisfied: It should be noted that the energy absorption by particles is always associated with the generation of the ponderomotive force in an inhomogeneous magnetic field. Therefore in such a case the adiabaticity of a particle no longer holds.

In order to obtain the dispersion equations of the wave-propagation and the energy deposition, we next investigate the nonlocal effects on the conductivity tensor. In the inhomogeneous dispersive plasma, the local conductivity tensor cannot be used. When we consider the stationary state, the wave energy should satisfy the conservation law, i.e., the power deposition rate should be balanced with the divergence of the Poynting vector. Namely the condition

$$\frac{1}{\mu_0} \nabla \cdot (\tilde{\vec{E}} \times \tilde{\vec{B}}) + \tilde{\vec{J}} \cdot \tilde{\vec{E}} = 0 \tag{20}$$

should be satisfied. The quantity $\tilde{\vec{J}}$ is the perturbed current induced by the rf fields. The nonlocal effect on the conductivity tensor has been calculated in Refs. [9-10]. When the inhomogeneity of the equilibrium is weak, the conductivity tensor $\hat{\vec{\sigma}}$ is defined as,[11]

$$\vec{J}(\vec{r},t) \simeq \int d\vec{r}'dt' \, \hat{\vec{\sigma}}\left(\vec{r}-\vec{r}', \, t-t' \, ; \, \frac{\vec{r}+\vec{r}'}{2} \, , \, \frac{t+t'}{2}\right) \vec{E}(\vec{r}',t') \tag{21}$$

where the two former arguments of $\vec{\vec{\sigma}}$ stand for the dispersion of the medium and the latter two indicate the equilibrium inhomogeneity. The Fourier representation has the relation

$$\vec{\vec{\sigma}}(\vec{k},\omega; \, \vec{r},t) = \int d\vec{r}_1,dt_1, \, \hat{\vec{\sigma}}(\vec{r}_1,t_1; \, \vec{r},t)e^{-i(\vec{k}\cdot\vec{r}_1-\omega t_1)} \; . \tag{22}$$

The mobility tensor $\vec{\vec{\mu}}$ derived in Eq. (8) gives $\vec{\vec{\sigma}}$ as

$$\vec{\vec{\sigma}}(\vec{k},\omega; \, \vec{r},t) = \sum_s \vec{\vec{\sigma}}_s = \sum_s N_s e_s \vec{\vec{\mu}}_s \tag{23}$$

where N_s and e_s are the number density and the charge of s species. The $\hat{x}$-direction is taken in the direction of inhomogeneity. Expanding $\vec{\vec{\sigma}}$ with respect to k_x as

$$\vec{\vec{\sigma}}(\vec{k},\omega;x) = \vec{\vec{\sigma}}(\omega;x) + \vec{\vec{\sigma}}'(\omega;x)k_x + \frac{1}{2}\vec{\vec{\sigma}}''(\omega;x)k_x^2 \ldots \tag{24}$$

where the prime denotes $\partial/\partial k_x$, we have

$$\vec{J}(\vec{r}) = \sum_{n=0} \sum_{m=0} \frac{i^{n-m}}{n!m!2^{n+1}} \left[\left(\frac{\partial}{\partial x}\right)^m \left\{ \frac{\partial^{2n+m} \overleftrightarrow{\sigma}(\vec{k},\omega;x)}{\partial x^n \partial k_x^{n+m}} \cdot \vec{E}(\vec{r}) \right\} \right.$$

$$\left. + (-1)^n \frac{\partial^{2n+m} \overleftrightarrow{\sigma}(\vec{k},\omega;x)}{\partial x^n \partial k_x^{n+m}} \cdot \frac{\partial^m \vec{E}(\vec{r})}{\partial x^m} \right] . \qquad (25)$$

Up to the second order corrections, Eq. (25) reduces to

$$J_i(x) = \sum_j \left[\!\left[\sigma_{ij}(x)E_j(x) - i\sqrt{\sigma'_{ij}}\frac{\partial}{\partial x}\left(\sqrt{\sigma'_{ij}}\,E_j\right) \right.\right.$$

$$\left.\left. - \frac{1}{4}\left[\frac{\partial}{\partial x}\left\{\sqrt{\sigma''_{ij}}\frac{\partial}{\partial x}\left(\sqrt{\sigma''_{ij}}\,E_j\right)\right\} + \sqrt{\sigma''_{ij}}\frac{\partial}{\partial x}\left\{\sqrt{\sigma''_{ij}}\frac{\partial E_j}{\partial x}\right\}\right] \right]\!\right] . \qquad (26)$$

where i,j stand for x,y and z. Substituting Eq. (8), (23) and (25) into the Maxwell's equation

$$\nabla \times \tilde{\vec{E}} = -\frac{\partial \tilde{\vec{B}}}{\partial t} , \qquad (27\text{-}1)$$

$$\nabla \times \tilde{\vec{B}} = \mu_0\left(\tilde{\vec{J}} + \varepsilon_0 \frac{\partial \tilde{\vec{E}}}{\partial t}\right) , \qquad (27\text{-}2)$$

we obtain the basic wave equations.

From Eq. (26) one can calculate the energy deposition. For s-th species of particles, the absorbed energy P_s is calculated as

$$P_s = \sum_{ij} \frac{1}{4}\left[2\sigma_{sHij}\, E_i^* E_j - \sqrt{-1}\,\sigma'_{sHij}\left(E_i^* \frac{\partial E_j}{\partial x} - \frac{\partial E_i^*}{\partial x} E_j\right)\right.$$

$$- \frac{1}{4} \frac{\partial^2 \sigma''_{sHij}}{\partial x^2} E_i^* E_j - \frac{1}{2} \frac{\partial}{\partial x} \{\sigma''_{sHij} \frac{\partial}{\partial x} (E_i^* E_j)\} + \sigma''_{sHij} \frac{\partial E_i^*}{\partial x} \frac{\partial E_j}{\partial x}] \quad , \qquad (28)$$

where the suffix H stands for the Hermitian part of $\overleftrightarrow{\sigma}$. We also have the Poynting vector S_x as

$$S_x = \frac{1}{4\mu_0} \frac{2}{\omega} \mathrm{Im}\{ E_y^* (\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}) - E_z^* (\frac{\partial Ex}{\partial z} - \frac{\partial Ez}{\partial x})\}$$

$$- \sum_s \sum_{ij} \frac{1}{4} [\sqrt{-1}\, \sigma'_{sAij} E_i^* E_j + \frac{1}{2} \sigma''_{sAij} (E_i^* \frac{\partial E_j}{\partial x} - \frac{\partial E_i^*}{\partial x} E_j)] \qquad (29)$$

where the suffix A denotes the anti-Hermitian part of the tensor. The equations (28) and (29) satisfies $\frac{\partial S_x}{\partial x} + \sum_s P_s = 0$ which is the energy conservation law.

3. Application to Toroidal Plasmas

In a low-β toroidal plasma, the magnetic field strength changes approximately proportional to $B_0/(1+r\cos\Theta/R)$ where r and R are minor and major radii and Θ is the poloidal angle. Owing to the rotational transform of the magnetic field line, the cyclotron frequency varies as particles move along the magnetic field line. We study the response function in a low-β tokamak with circular cross section. We use (x,y,z) coordinates, $x = r\cos\Theta$, $y = r\sin\Theta$ and z is taken along the axis of the torus. For the simplicity of the analysis we keep only the $1/(1+x/R)$ dependence of the toroidal magnetic field and assume that the pitch $d\Theta/dz$, is constant along the magnetic field line.

The particle velocity $\vec{v}$ is represented to be $\vec{V} + \vec{v}_g$ where $\vec{V}$ is the guiding center motion and $\vec{v}_g$ is the gyromotion. The ∇B drift of particles appears in the guiding center motion. However, this drift velocity is small compared to the thermal velocity. In addition to it, the ∇B drift velocity is perpendicular to ∇B, so that this motion does not give rise to the change of Ω in the lowest order correction. We then simply write

$$\vec{v} = (0, r/qR, 1)\, v_\parallel + \vec{v}_g \, , \tag{30}$$

where q is the safety factor and the parallel velocity $v_\parallel$ is also treated to be constant. The phase $\phi(\tau)$ is given as

$$\phi(\tau) = (k_\parallel v_\parallel - \omega)\tau - k_\perp \rho\{\cos(\psi-\phi-\alpha) - \cos(\psi-\alpha)\} \tag{31}$$

and

$$\phi = \Omega_0\tau + \frac{r\Omega_0}{R\nu}\left\{\sin(\Theta_0-\nu\tau) - \sin\Theta_0\right\} \tag{32}$$

where $k_x = k_\perp \cos\psi$, $k_y = k_\perp \sin\psi$, $\nu = v_\parallel/qR$, $\Omega_{0s} = e_s B_0/m_s$ and Θ_0 is the poloidal angle at $\tau = 0$. Expanding ϕ given by Eq. (32) with respect to $\nu\tau$ and retaining terms up to $(\nu\tau)^2$, we obtain

$$\phi \simeq \Omega\tau + \frac{nr\Omega v_\parallel}{2qR^2}\sin\Theta_0\tau^2 - \frac{nr\Omega v_\parallel{}^2}{6q^2R^3}\cos\Theta_0\tau^3 \ . \tag{33}$$

From Eqs. (31) and (33), the mobility tensor is calculated. Substituting $\ell_1 = qR^2/r \sin\Theta_0$ and $\ell_2^2 = 2q^2R^3/r \cos\Theta_0$, the mobility tensor in the toroidal geometry, $\vec{\mu}_t$, is given as

$$\overleftrightarrow{\mu}_t = \overleftrightarrow{U}^{-1}\,\overleftrightarrow{\mu}\,\overleftrightarrow{U} \tag{34}$$

and

$$\overleftrightarrow{U} = \begin{bmatrix} \cos\psi\ , & -\sin\psi\ , & \frac{-r}{qR}\sin\Theta_0 \\ \sin\psi\ , & \cos\psi\ , & \frac{r}{qR}\cos\Theta_0 \\ \frac{r}{qR}\sin(\Theta_0-\psi), & \frac{-r}{qR}\cos(\Theta_0-\psi)\ , & 1 \end{bmatrix} \tag{35}$$

where $\overleftrightarrow{\mu}$ is given by Eqs. (8) and (9). In deriving Eq. (34) terms of order $(r/qR)^2$ are neglected.

In the toroidal geometry, the cyclotron frequency Ω varies as a function of x, and the cyclotron resonance condition $n\Omega = \omega$ is satisfied on a surface $x = x_r$ where $\Omega(x_r) = \Omega_0/(1+x_r/R) = \omega/n$ holds $(n \geqslant 1)$. Since the cyclotron resonance damping rate is proportional to $\exp\{-(n\Omega-\omega)^2/2k_\parallel^2 v_{T\parallel}^2\}$ $(\ell_{1,2} \to \infty)$, the cyclotron interaction takes place in the region satisfying $(n\Omega-\omega)^2/2k_\parallel^2 v_T^2 \lesssim 1$. When the spatial change of $k_\parallel v_{T\parallel}$ is weak, the cyclotron damping appears in a thin layer around the surface x_r, which is given by

$$|x-x_r| \lesssim \delta_n \equiv \frac{k_\parallel \rho}{n} R \quad \ldots \quad (n \geqslant 1) \tag{36}$$

Out of this layer, the cyclotron resonance decreases exponentially, and actually vanishes. The decorrelation due to the magnetic field inhomogeneity causes an additional energy exchange. For $|x-x_r| > \delta_n$, $k_\parallel v_{T\parallel}$ is small and Eq. (18) holds. This damping exponentially decreases for $\zeta^{-2/3}/2 > 1$. A strong damping is expected in a region

$$|x-x_r| \lesssim \Delta_n \equiv \frac{r|\sin\Theta|\rho}{nqR^2} R \ldots (n \geqslant 1) \tag{37}$$

This interaction region is much wider than that for the cyclotron resonance, given by Eq. (36).

This damping mechanism works effectively for the energy absorption by ions when one applies the rf waves. In the case of the rf wave heating of toroidal plasmas in the range of the ion cyclotron frequency, the rf waves are easily absorbed by electrons in a wide region of the plasma column unless $|k_\parallel|$ is not extremely small. On the contrary, the linear contribution of the ion cyclotron damping given by the conductivity tensor in a homogeneous B field is localized in the region $|x-x_r| < \delta_n$. To study the heating by the perpendicular electric field $\tilde{E}_x$, let us consider the contribution by $\langle \tilde{E}_x \sigma_{sxx} \tilde{E}_x \rangle$ (The brackets $\langle\ \rangle$ stand for the phase average). If we use the conductivity tensor for the homogeneous magnetic field, the energy input to the s species is evaluated by integrating $(Ne^2/m\omega)|\tilde{E}_x|^2 \mathrm{Im}Z(\xi_n)\Omega/\sqrt{2}\ |k|v_{T\parallel}|_s$ over x. The order estimate is given in the $\lambda \to 0$ limit as

$$\int dx\ \langle \tilde{E}_x^* \sigma_{sxx} \tilde{E}_x \rangle \sim \frac{Ne^2}{m\omega} \int_{x_r-\delta_n}^{x_r+\delta_n} |\tilde{E}_x|^2 \frac{\Omega}{\sqrt{2}|k_\parallel|v_T} \sim \frac{Ne^2}{m\omega} |\tilde{E}_x|^2 R \quad (n=1). \tag{38}$$

The amplitude $|\tilde{E}_x|^2$ is evaluated at the resonance surface. The result does not explicitly depend on the thickness of the layer nor on $k_\parallel$[5] so long as the rf field is uniform in the resonance region. On the contrary, if we correctly use the conductivity tensor which includes the magnetic field inhomogeneity, the integrand $\langle \tilde{E}_x^* \sigma_{sxx} \tilde{E}_x \rangle$ remains finite in a much wider region of x. For instance, when $|k_\parallel|v_{T\parallel}$ is small, the damping exists in the region $|x-x_r| \lesssim \Delta_n$ and the integral is estimated to be

$$\int \langle \tilde{E}_x^* \sigma_{xx} \tilde{E}_x \rangle \sim \frac{Ne^2}{m\omega} \overline{|\tilde{E}_x^2|}\ R \quad (n=1) \tag{39}$$

where the average of $\overline{|\tilde{E}_x^2|}$ is taken in the region $|x-x_r| \lesssim \Delta_n$. Comparing Eq. (38) and (39), one sees that the energy input is the same for these two calculations, provided that the amplitude of the wave is homogeneous. When the amplitude of the rf field is not homogeneous, it is necessary to employ the correct conductivity tensor which includes the effect of the magnetic field inhomogeneity. For instance, if one utilizes the two-ion hybrid resonance (heavy majority ions and light minority ions), the mode conversion from the fast wave to the ion Bernstein wave is expected. The large amplitude wave is excited associated with this mode conversion. This wave does not propagate to the cyclotron resonance layer of the minority ions. The cyclotron damping by the majority ions does not take place except for the large values of $k_{\parallel}$. The damping caused by the magnetic field inhomogeneity works as the mechanism by which ions can absorb the wave energy, and contributes appreciably to the ion heating.

The equation (18) shows that the typical decorrelation time can be evaluated by

$$\tau_c \sim \left(\sqrt{\frac{r\sin\Theta\Omega v_T}{qR^2}}\right)^{-1}$$

for $|x-x_r| < \Delta_n$. This effective collision frequency is to be compared with the cyclotron frequency Ω. One has

$$\frac{1}{\tau_c\Omega} \sim \sqrt{\frac{r\rho|\sin\Theta_0|}{qR^2}} \sim 10^{-2}\sqrt{\frac{r|\sin\Theta_0|}{qR^2}\frac{\sqrt{T_i}}{B}}$$

where T_i is in eV and r,R and B are in MKSA unit. This value can be of order 10^{-3} to 10^{-2} in the central column of the plasma of the present tokamak experiments. We see that $1/\tau_c$ is much greater than the classical collision frequency except in the periphery of the plasma column.

4. Summary and Discussions

In summary, we derived the kinetic rf wave equation in an inhomogeneous and dispersive plasma. The conductivity tensor which governs the propagation is obtained in a nonuniform magnetic field. Nonuniformity of the magnetic field strength along the line of force is included, and a new function Φ_n instead of the plasma dispersion function is introduced. The function Φ_n represents wave-particle interactions and contains an off-resonant type dissipation.

In formulating the wave equation, we include the nonlocal effects of the conductivity tensor. When we consider the dispersion of the wave field, the neglect of the nonlocality of $\overleftrightarrow{\sigma}$ yields the discontinuity of the Poynting flux in a plasma, i.e., the correct energy deposition rate and profile for each species can not be obtained. The basic equations for toroidal plasmas are derived in Section 3.

The dissipation caused by the magnetic field inhomogeneity can be interpreted as an effective collision between a wave-packet and particles, and gives an appreciable contribution to the rf-wave propagation and absorption especially in high temperature plasmas. This dissipation enlarges the resonant layer of waves, and the absorption region becomes large such as $|\omega - n\Omega| \lesssim \sqrt{n\Omega v_{T\parallel}/2\ell_1}$. The modulation of the cyclotron frequency makes the particles resonate with waves even when $|k_\parallel v_T| \ll |\omega - n\Omega|$.[4] This implies that the ion-Bernstein mode of $k_\parallel = 0$ can be absorbed by ion species. The absorbed power in ICRF heating for toroidal plasma is also evaluated. The evaluation of the total absorption Eq. (39) is same for the constant B approximation Eq. (38), assuming that the rf field is <u>constant</u>; the total absorption does not depend explicitly on $k_{\parallel}$ as is shown in Refs. [5,7,8]. However, the wave field is affected by the plasma profile, the magnetic field inhomogeneity and the wave number $k_{\parallel}$. The results on the power absorption clearly shows the importance to know the global wave form in the nonuniform and dispersive plasma.

The gradient of the field strength along the magnetic field line, $|\nabla B_{\parallel}| \neq 0$, is also found in a mirror or a bumpy machine. Therefore, when we launch ICRF wave to such a plasma, an absorption and an emission of the wave as well as the averaged force (ponderomotive force) by the wave due to this effect are to be seen.[7] The fact implies that one must be careful to assume the adiabaticity of particles when the ICRF waves are used for the end plugging of the mirror machine. Also implied is that the pitch angle scatterings are enhanced. When a spontaneous field fluctuation or an instability of this frequency range like loss cone mode[12] is considered, this effective collision may be responsible to the enhancement of the scattering.

The equation (25) is symmetrized with respect to the field and plasma inhomogeneities.[13] Substituting Eqs. (8), (9) and (23) into Eq. (25), one sees that the representation Eq. (25) is self-adjoint.[13,14,15] The coefficient 2^{-n} in Eq. (25) is originated from the formulation Eq. (21). The requirement of the self-adjointness does not uniquely determine the representation of $\vec{J}(\vec{r})$. The consistent calculation of $\hat{\hat{\sigma}}$ is necessary[14].

In order to construct firm bases of the ICRF wave study we have done one-dimensional kinetic calculations of the wave propagation and absorption, having clarified the mode characters and the heating mechanisms. The results are to be reported in a separate paper.[16]

We here neglected the trapped particle contribution in deriving the conductivity tensor. When the trapped particles have coherences with the considered waves, the effect should be examined, allowing for future analyses.

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Figure Captions

Figure 1

The real part (a,c) and the imaginary part (b,d) of Φ vs. ξ for various values of ζ. In the limit $\zeta \rightarrow 0$, Φ reduces to $\xi Z(\xi)$ which is shown by the dashed line.

Figure 2

Φ is shown in the off-resonant limit, $k_{\parallel} = 0$. The dashed lines are the analytic evaluation, Eq. (18).

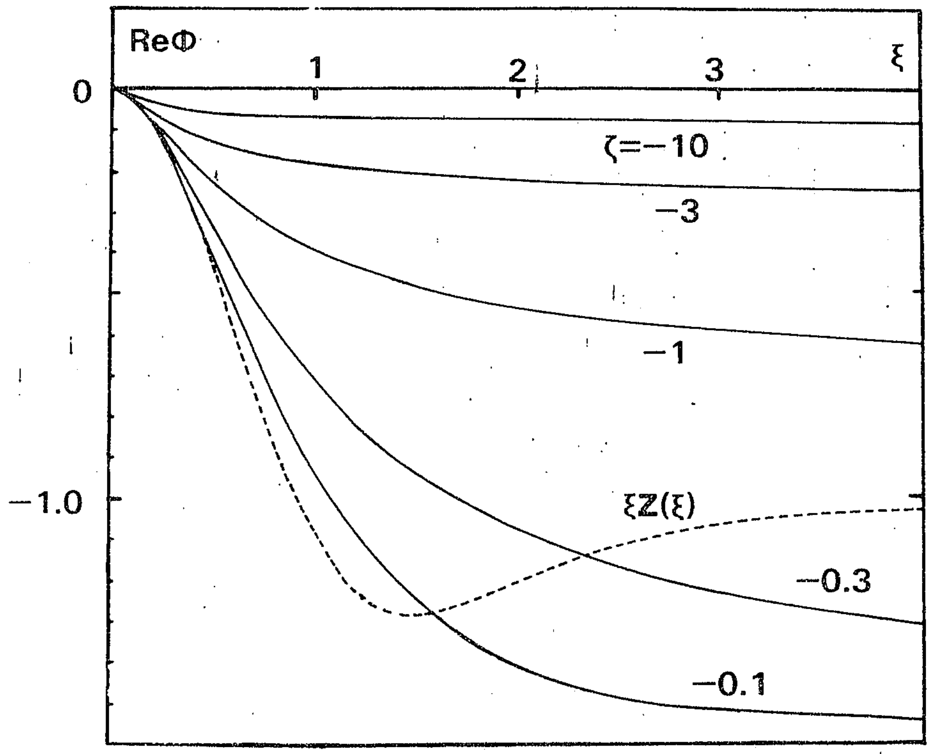

FIG. 1(a)

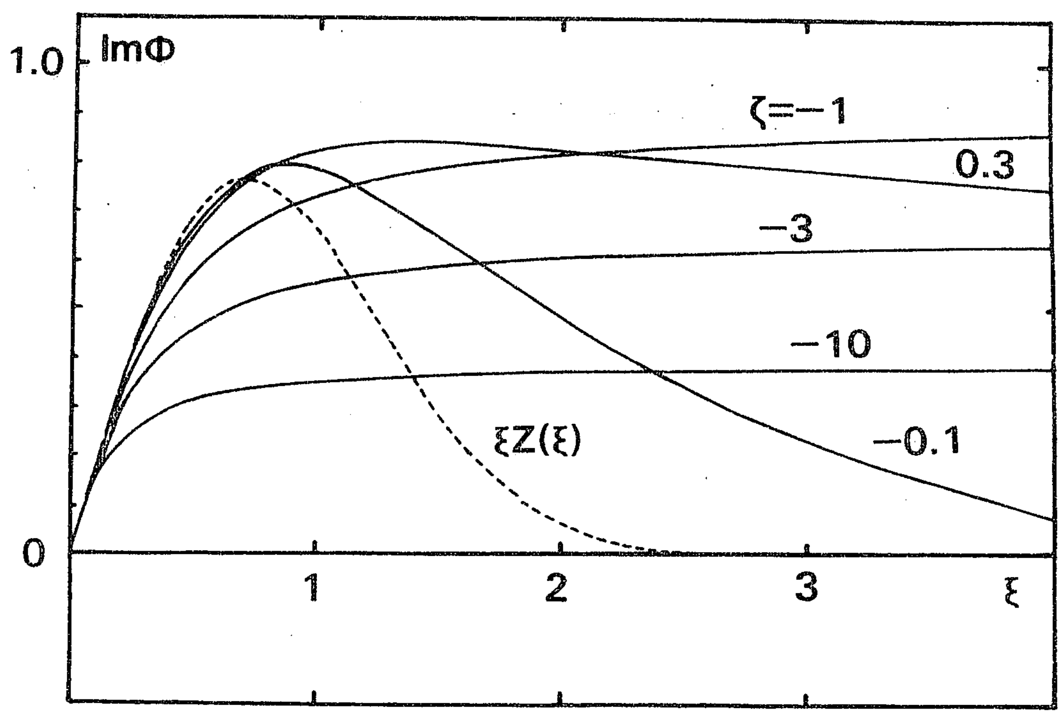

FIG. 1(b)

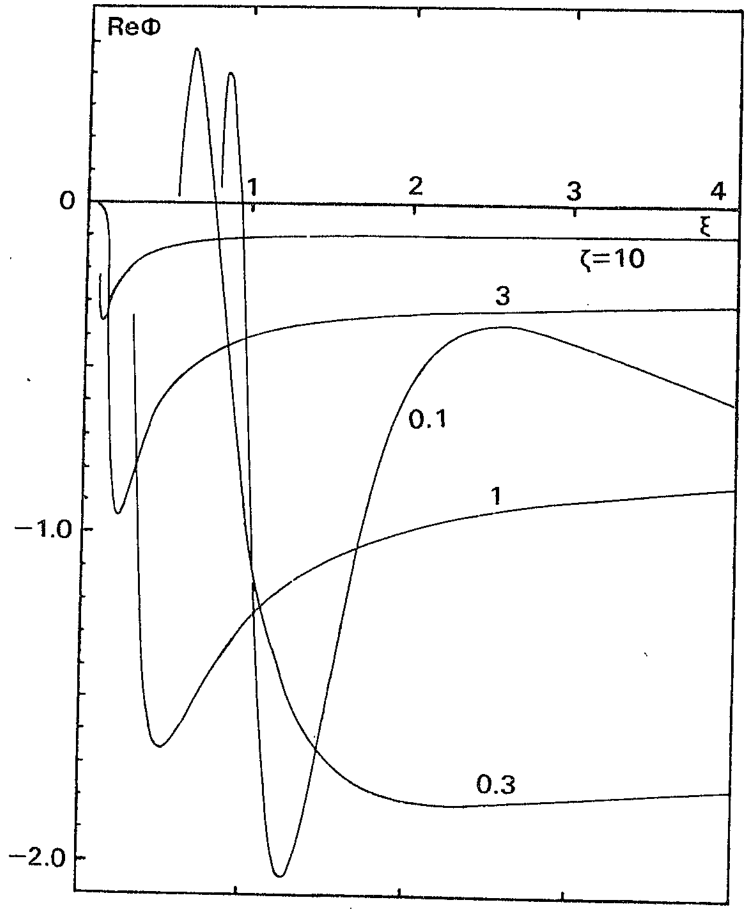

FIG. 1(c)

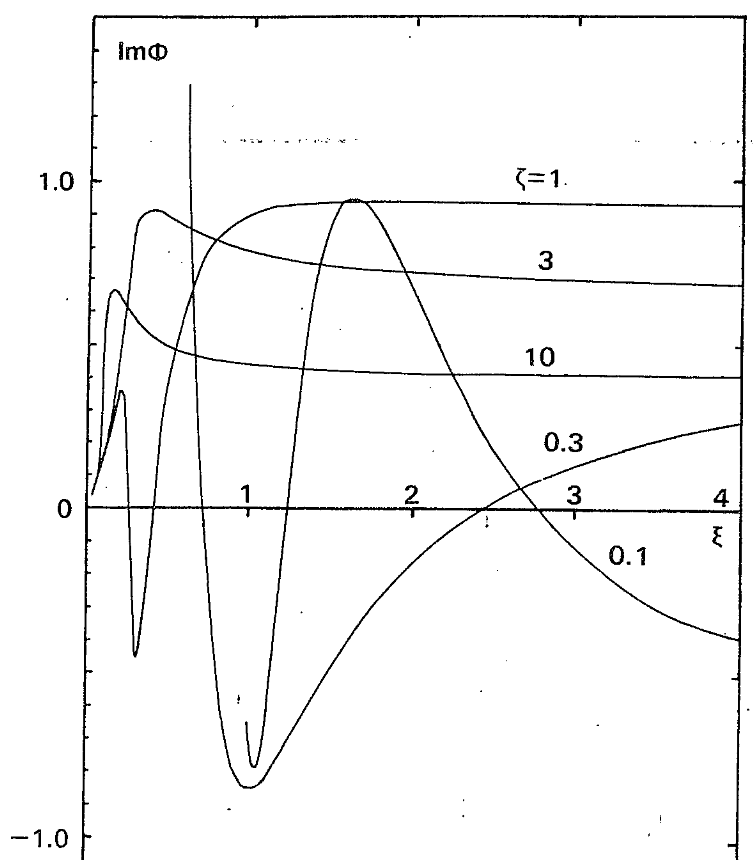

FIG. 1(d)

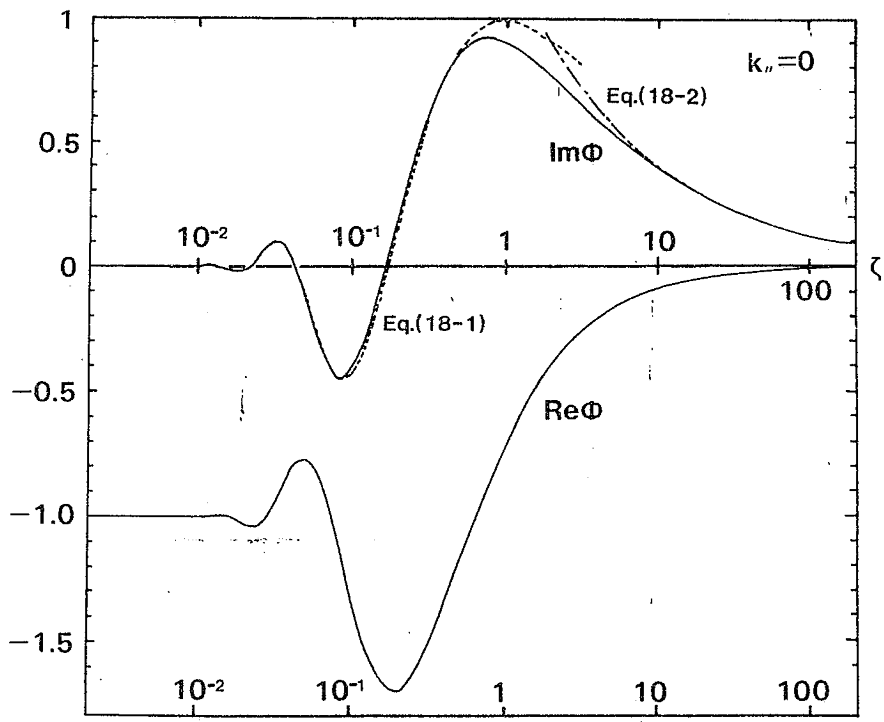

FIG. 2

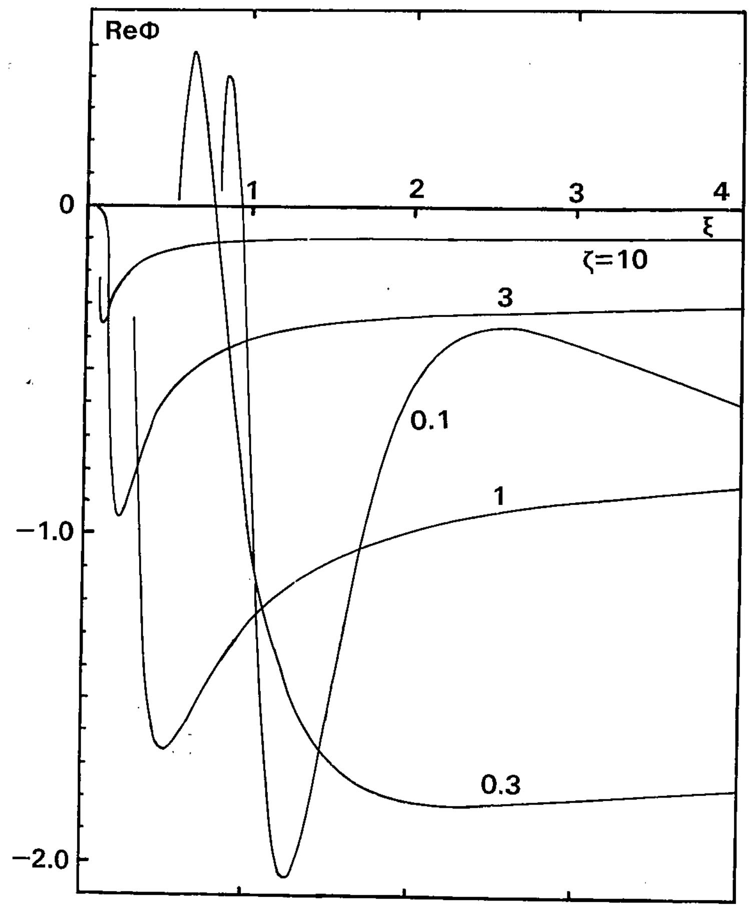

FIG. 1(c)

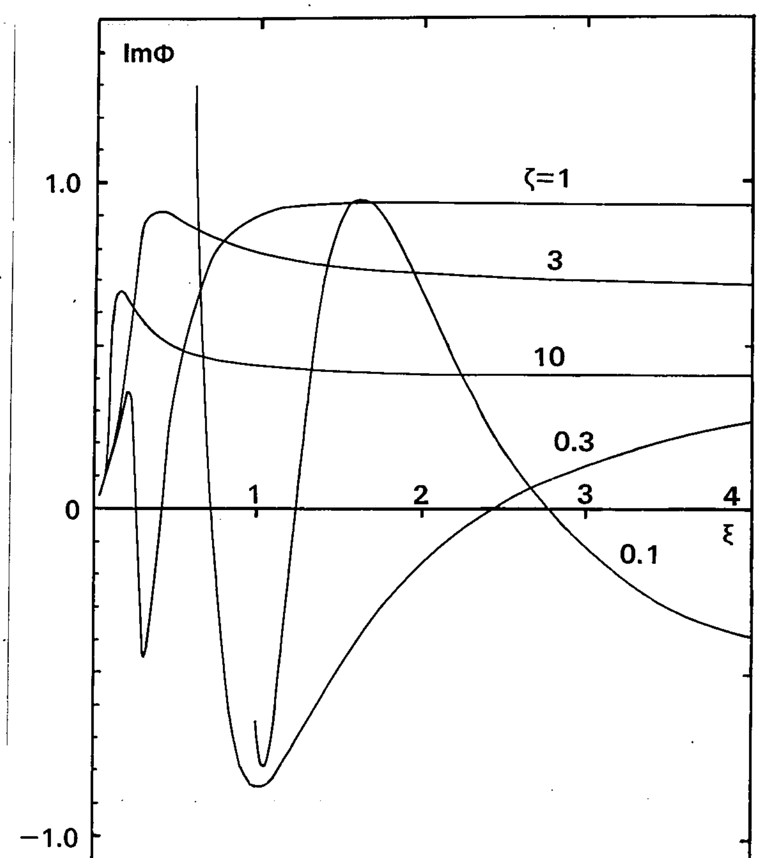

FIG. 1(d)

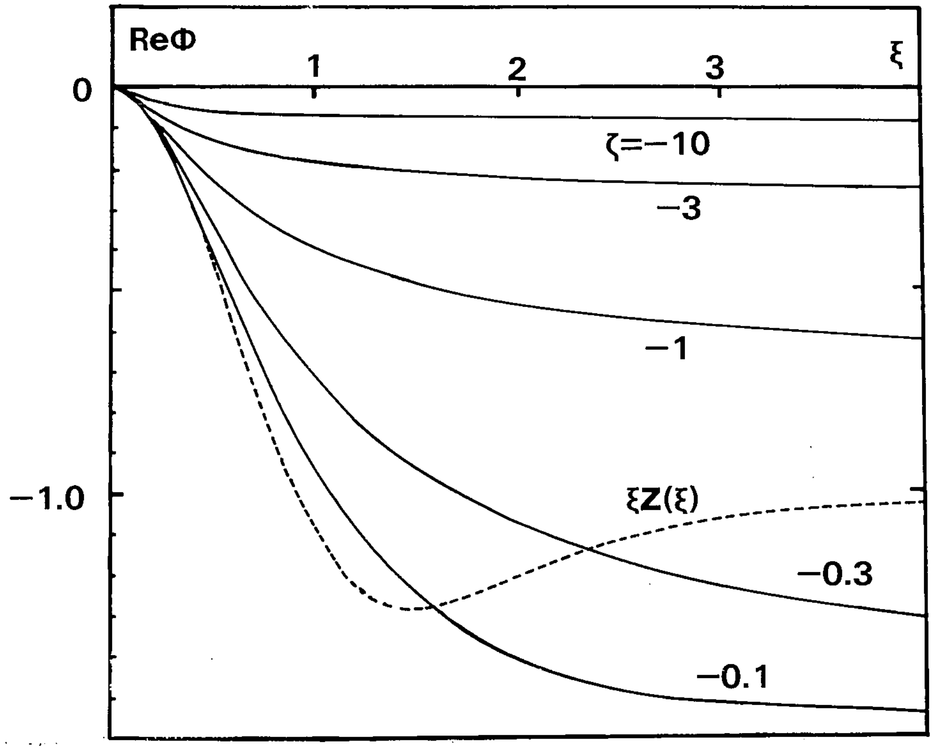

FIG. 1(a)

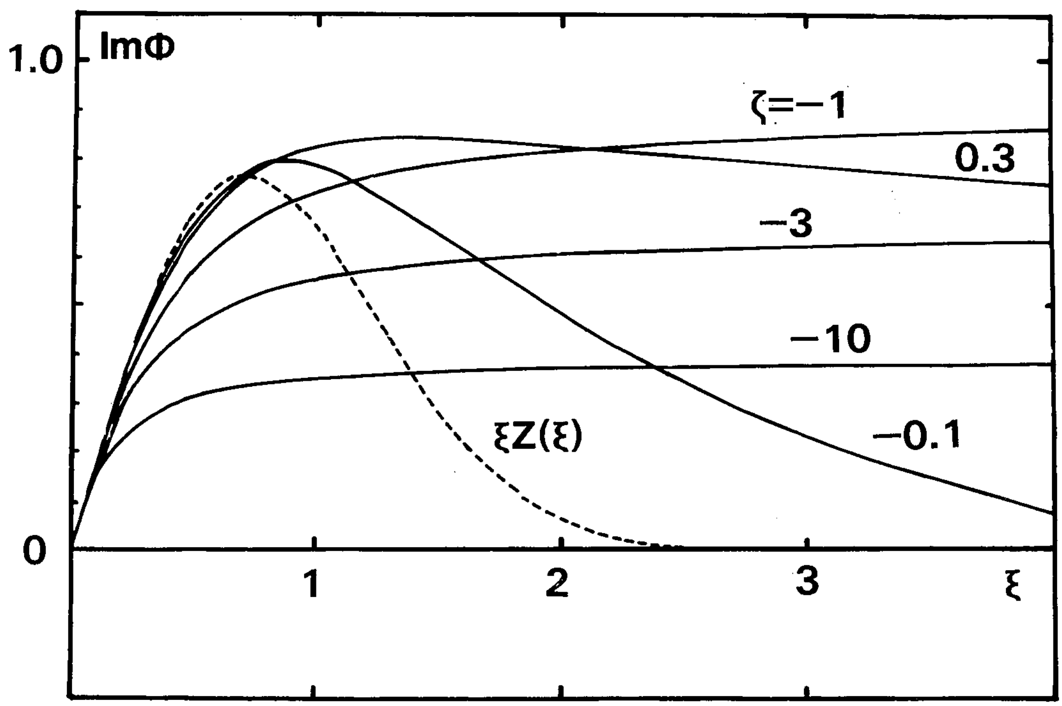

FIG. 1(b)

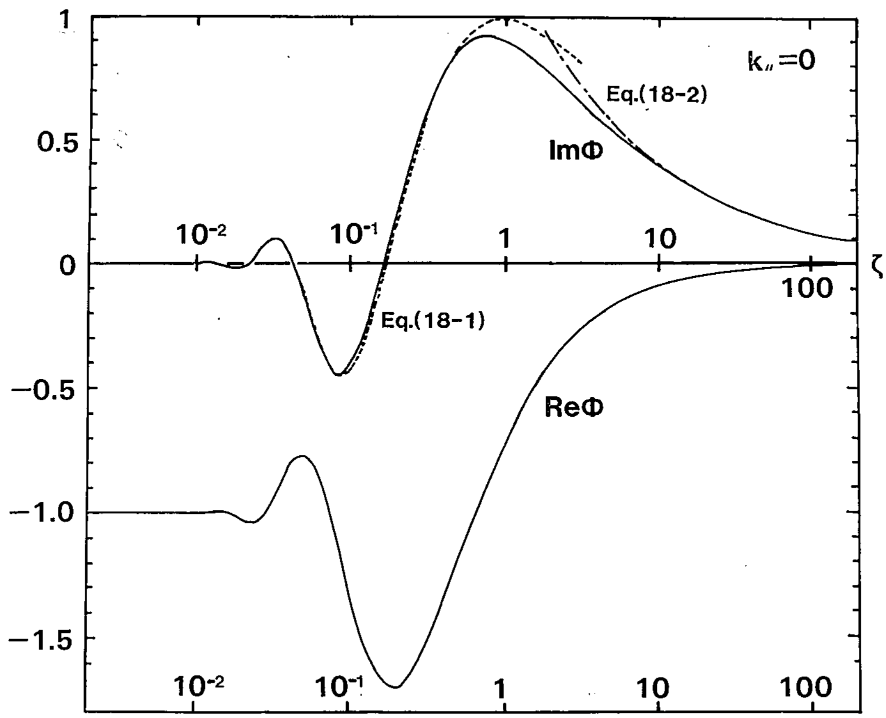

FIG. 2