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On the Quasihydrostatic Flows of Radiatively Cooling Self-Gravitating Gas Clouds

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On the quasihydrostatic flows of radiatively cooling self-gravitating gas clouds

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Abstract

Two model problems are considered, illustrating the dynamics of quasihydrostatic flows of radiatively cooling, optically thin self-gravitating gas clouds. In the first problem, spherically symmetric flows in an unmagnetized plasma are considered. For a power-law dependence of the radiative loss function on the temperature, a one-parameter family of self-similar solutions is found. We concentrate on a constant-mass cloud, one of the cases, when the self-similarity indices are uniquely selected. In this case, the self-similar flow problem can be formally reduced to the classical Lane-Emden equation and therefore solved analytically. The cloud is shown to undergo radiative condensation, if the gas specific heat ratio $\gamma > 4/3$. The condensation proceeds either gradually, or in the form of (quasihydrostatic) collapse. For $\gamma < 4/3$, the cloud is shown to expand. The second problem addresses a magnetized plasma slab that undergoes quasihydrostatic radiative cooling and condensation. The problem is solved analytically, employing the Lagrangian mass coordinate.

I. INTRODUCTION

At the initial stage of star formation in self-gravitating interstellar gas clouds, the clouds, or cloud fragments are too hot and rarefied to become gravitationally unstable (Shu *et al.* 1987, Lada *et al.* 1992, Bodenheimer 1992). Under these conditions, the cloud is normally in (approximate) hydrostatic equilibrium. In many cases, a cloud *reaches* such an equilibrium through evolution. Indeed, if the cloud mass initially exceeds the Jeans mass, the cloud will contract rapidly, on the free-fall time scale. If the cloud is magnetized, the magnetic buoyancy (or Parker) instability (*e.g.* Shu 1992) can expel the magnetic field from the cloud and lead to a rapid reconstruction of the density distribution and possibly to gravitational contraction. However, if the heat removal from the system is not efficient enough, the (thermal plus magnetic) pressure buildup can arrest the contraction, so that, after some oscillations, the system will reach a hydrostatic equilibrium. In the opposite case, when the cloud mass is smaller than the Jeans mass, rapid expansion starts, which, after dynamic saturation, can also be followed by a hydrostatic equilibrium. In the both cases, after a few dynamic times, the gas cloud *becomes* marginally stable from the viewpoint of the (simplified) Jeans instability criterion: the cloud mass is *equal* to the Jeans mass. Similar physical problems arise in the galaxy formation (Larson 1990, Schweizer 1990, Efstathiou 1990). It is important that such marginally stable regimes are in fact *quasi-* hydrostatic, as they can evolve significantly on a longer time scale. For example, relatively slow radiative cooling of the cloud is always accompanied by the gas flow, necessary to maintain the hydrostatics. One such a flow is inflow, when cooler and denser clouds (or cloud cores) develop. Similar radiative condensation flows have been encountered in many applications (probably, the most famous of them are cooling flows in the intra- cluster medium of the galactic clusters, see, *e.g.*, Fabian *et al.* (1991)). If the cooling is slow on the dynamic time scale, this flow proceeds quasihydrostatically. At a later stage, a contracting' cloud can undergo much faster gravitational collapse. However, as we shall show in this paper, there exists another possibility: the cloud remains quasihydrostatic, but still develops density collapse. Such a "slow" collapse can provide an alternative (nonJeans) scenario for star formation. (Another example of a slow, nonJeans condensation involves the ambipolar diffusion in a self-gravitating magnetized cloud, and it was suggested by Shu *et al.* (1987) as a mechanism for the low-mass star formation.) An alternative radiative cooling-induced flow in a self-gravitating cloud represents outflow, or expansion, and we shall address it as well. To the best of our knowledge, no analytic models of "marginally stable" quasihydrostatic radiatively cooling flows of self-gravitating gas clouds are available. It is our aim to develop such a model by looking at two simple problems, which can be studied analytically.

In the first problem, we investigate spherically symmetric quasihydrostatic flows in self-gravitating gas clouds without magnetic fields. We formulate this problem in Sec. II. Assuming a power-law dependence of the radiative loss function on the temperature, we find a one-parameter family of self-similar solutions, describing different flow regimes. Among them, there are quasihydrostatic flows developing singularities in a finite time. These are collapse and "explosive expansion," when the gas density goes to infinity or to zero in a finite time. In Sec. III we concentrate on the dynamics of a radiatively cooling cloud with fixed mass. We show that the dynamics crucially depends on the specific heat ratio of the gas γ. For $\gamma > 4/3$, the cloud undergoes condensation (either gradual, or collapse-like, depending on the exponent of the power-law radiative loss function). For $\gamma < 4/3$, the cloud always expands. We show that these time-dependent problems can formally be reduced to the classical Lane-Emden equation and solved analytically in terms of the Lane-Emden functions.

The second model problem, presented in Sec. IV, addresses a quasihydrostatically contracting plasma slab, with or without magnetic field. This time we account for the cutoff in the radiative loss function at low temperatures and show that the condensation process involves two stages. At the first stage, there is a (nonuniform) "volume" cooling of the slab, accompanied by plasma inflow (significant for small and insignificant for large magnetic fields). As the denser central regions cool faster, they reach the radiation cutoff temperature first. Then a traveling cooling front develops, which propagates from the mid plane outward, until all the slab cools down to the cutoff temperature, plasma flow terminates, and true hydrostatic equilibrium is achieved. Transforming to the Lagrangian mass variable, we shall be able to solve the problem analytically. Sec. V contains a brief summary and discussion of the results.

II. SPHERICALLY SYMMETRIC HYDROSTATIC FLOWS AND SELF-SIMILAR SOLUTIONS

We start with simple hydrodynamic equations for spherically symmetric self-gravity flows:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho v\right) = 0, \tag{1}$$

$$\rho \frac{dv}{dt} = -\frac{\partial p}{\partial r} - \rho \frac{Gm}{r^2}, \tag{2}$$

$$\frac{\partial m}{\partial r} = 4\pi \rho r^2, \tag{3}$$

where ρ, p and v are the gas density, pressure and velocity, respectively, m is the mass inside radius r, G is the gravitational constant, and $d/dt = \partial/\partial t + v\partial/\partial r$. To close the set, we need an equation of state and energy equation. Consider a perfect gas with constant specific heats:

$$p = \frac{R_g}{\mu_g}\rho T, \tag{4}$$

where R_g is the gas constant and μ_g is the effective molar mass. Neglecting any external heating processes, we assume that the (optically thin) gas is cooling by its own radiation, and we approximate the radiative loss function $L(\rho, T)$ by a power law $L = F_\nu \rho^2 T^\nu$, where, for a selected interval of the temperature T, the parameters ν and F_ν are constants. The quadratic dependence of L on the density results from the binary character of radiation collisions, while the temperature dependence is determined by many types of radiation collisions (see, *e.g.* Spitzer 1978, Kaplan and Pikel'ner 1979). Note, that the index ν can be both positive, and negative.

The energy equation takes the following form:

$$\frac{1}{\gamma-1}\frac{dp}{dt}+\frac{\gamma}{\gamma-1}p\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v\right)+F_\nu \rho^2 T^\nu=0. \tag{5}$$

We shall be interested in flows with a negligible inertia. This implies, in particular, that the characteristic time scale of the flow (determined by the radiative cooling) is much longer than the dynamical time scale. Neglecting the inertial term in the left-hand side of Eq. (2) (the corresponding criteria will be checked *a posteriori*), we arrive at the approximate quasi-hydrostatic relation

$$\frac{\partial p}{\partial r}+\rho\frac{Gm}{r^2}\simeq 0, \tag{6}$$

which replaces the complete Euler equation (2). [A different quasihydrostatic flow was considered by Shu (1983) in the context of ambipolar diffusion in a magnetized self-gravitating plasma slab.]

Let us introduce the characteristic time, length and mass scales of the problem,

$$\tau_r=\frac{p_0}{(\gamma-1)F_\nu\rho_0^2 T_0^\nu},\quad l_J=\left(\frac{p_0}{4\pi G\rho_0^2}\right)^{1/2},\quad M_J=4\pi\rho_0 l_J^3,$$

and rescale the independent and dependent variables:

$$\tilde{r}=r/l_J\,,\quad \tilde{t}=t/\tau_r\,,\quad \tilde{\rho}=\rho/\rho_0\,,\quad \tilde{p}=p/p_0\,,\quad \tilde{T}=T/T_0\,,\quad \tilde{m}=m/M_J\,,\quad \tilde{v}=\tau_r v/l_J.$$

Note, that the characteristic length l_J and mass M_J represent characteristic (initial) Jeans length and mass of the problem, while τ_0 is the characteristic radiative cooling time. In the scaled variables, the continuity equation (1) does not change, while the rest of equations can be rewritten as

$$\frac{\partial p}{\partial r}+\frac{\rho m}{r^2}=0, \tag{7}$$

$$\frac{\partial m}{\partial r} - \rho r^2 = 0, \tag{8}$$

$$\frac{\partial m}{\partial t} + v\frac{\partial p}{\partial r} + \gamma\frac{p}{r^2}\frac{\partial}{\partial r}(r^2 v) + \rho^{2-\nu}p^{\nu} = 0, \tag{9}$$

where we have omitted the tildes.

It can be easily checked that Eqs. (1) and (7)–(9) admit a one-parameter family of self-similar solutions:

$$\rho = (t_0 - t)^{\nu_1} R(\mu)\,, \quad v = (t_0 - t)^{\nu_2} V(\mu)\,, \quad p = (t_0 - t)^{\nu_3} P(\mu)\,, \quad m = (t_0 - t)^{\nu_4} M(\mu), \tag{10}$$

where the similarity variable is $\mu = r/(t_0 - t)^{\sigma}$, $t < t_0$, and indices ν_i are the following:

$$\nu_1 = \frac{2\sigma(1-\nu)-1}{\nu}\,, \quad \nu_2 = \sigma - 1\,, \quad \nu_3 = \frac{2\sigma(2-\nu)-2}{\nu}\,, \quad \nu_4 = \frac{\sigma(2+\nu)-1}{\nu}\,. \tag{11}$$

The form of the self-similar substitution (10) assumes that the flow develops a singularity in a finite time t_0. We shall call such flows singular. Alternatively, the same equations admit nonsingular self-similar substitution, which is obtained if we replace $t_0 - t$ by $t - t_0$ and take $t > t_0$ in Eq. (10) and in the similarity variable μ. The corresponding self-similar solution for the temperature is given by the ratio of the solutions for the pressure and density.

As it stands, the self-similarity index σ is an arbitrary parameter, as the governing equations alone are not sufficient to select it. Therefore, some additional information, such as an initial or boundary condition or conservation law (Barenblatt, 1979), should be invoked to select σ (see below).

In the similarity variable, Eqs. (1) and (7)–(9) become a set of ordinary differential equations:

$$\pm\,[1 + 2\sigma(1-\nu)]\,R \pm \nu\sigma\mu\frac{dR}{d\mu} + \frac{\nu}{\mu^2}\frac{d}{d\mu}(\mu^2 R V) = 0, \tag{12}$$

$$\frac{1}{\mu^2}\frac{d}{d\mu}(M) = R, \tag{13}$$

$$\frac{dP}{d\mu} + \frac{RM}{\mu^2} = 0, \tag{14}$$

$$\pm\frac{[2-2\sigma(2-\nu)]}{\nu}P+(\pm\sigma\mu+V)\frac{dP}{d\mu}+\frac{\gamma}{\mu^2}P\frac{d}{d\mu}(\mu^2V)+R^{2-\nu}P^{\nu}=0, \tag{15}$$

where the upper and lower signs stand for the singular and nonsingular flows, respectively. Eq. (15) is formally inapplicable in the case of $\nu = 0$ (the radiative loss function independent of the temperature). However, a separate analysis shows that this case is in fact describable by the general formulae, obtained below, therefore we shall not rule out this case.

Using Eq. (13), we can immediately integrate Eq. (12) to obtain

$$\mu^2RV\pm\sigma\mu^3R\mp\frac{\sigma(\nu+2)-1}{\nu}M=\text{const}, \tag{16}$$

with the same sign rule as before. The constant should be taken zero, if we require a well-behaved density (see below) and zero velocity at the cloud center $\mu = 0$.

Further analysis requires selection of the parameter σ. One can think about two alternative additional constraints, which would select it. The first is a constant, prescribed value of the gas pressure at infinity, while the second is mass conservation of the cloud. The first case corresponds to the cloud core being surrounded by a very large amount of "passive" ambient gas, which dictates the constant external pressure. Obviously, in this case, the mass of the core is not preserved, as inflow/outflow of the gas is allowed. The second case corresponds to the cooling of an isolated massive cloud, confined only by its self-gravity.

In the case of a constant external pressure, $p(r \to \infty) = p_\infty = \text{const}$, we must require $\nu_3 = 0$, which gives $\sigma = (2-\nu)^{-1}$. Then, the remaining ν_i-indices are immediately selected: $\nu_1 = (\nu-2)^{-1}, \nu_2 = (\nu-1)/(2-\nu)$ and $\nu_4 = 2(2-\nu)^{-1}$.

Alternatively, the constraint of a constant total mass can be written as

$$(t_0-t)^{[\sigma(2+\nu)-1]/\nu}\int_0^\infty \mu^2R(\mu)d\mu = M_c = \text{const}, \tag{17}$$

where M_c is the cloud mass, expressed in the units of the initial Jeans mass M_J. Eq. (17) immediately yields $\sigma = (2+\nu)^{-1}$. In this case the ν_i-indices are $\nu_1 = -3(\nu+2)^{-1}, \nu_2 =$

$-(\nu+1)/(\nu+2)$, $\nu_3 = -4(\nu+2)^{-1}$, and $\nu_4 = 0$. The same indices are obtained also for the mass-conserving nonsingular flows.

III. CLOUD WITH A CONSTANT MASS

Once σ is selected, one should solve the set of three ordinary differential equations (13)–(15) and the integral (16). In the following we shall concentrate on the constant mass case, as we found this case to be integrable analytically. Before we solve these equations, let us give a preliminary classification of possible flow regimes. We have already assumed that the density at the center is finite (until the time moment of singularity, if any). This implies that the similarity function $R(\mu)$ must be finite at $\mu = 0$. At large μ, the function R must go to zero (faster than μ^{-3}, as we require normalization, see Eq. (17)). Consider the singular solutions with the selected ν_i. The form of solutions with $\nu > -2$ (see Eq. (10)) describes collapse: the gas density goes to infinity at a finite time t_0, while the cloud shrinks into the center $r = 0$. For $\nu < -2$ Eq. (10) describes "explosive expansion": the density goes to zero everywhere at a finite time t_0. On the contrary, for the nonsingular mass-preserving flows we can have expansion at $\nu > -2$, and condensation at $\nu < -2$, but no singularities develop in a finite time.

As we see later, not all of these solutions actually exist. However, we can already notice that the character of the flow behavior can significantly depend on the form of the radiative loss function (that is, on the dominating mechanism of radiative cooling).

Putting $\sigma = (2+\nu)^{-1}$ in the integral (16), we immediately solve it for $V(\mu)$:

$$V(\mu) = \mp(2+\nu)^{-1}\mu \tag{18}$$

with the same sign rule as before. Returning to Eqs. (10) and (11), we obtain the full solution for the gas velocity:

$$v(r,t) = \frac{r}{(2+\nu)(t-t_0)}. \tag{19}$$

We see again that in the case of $\nu < -2$ there is a gas inflow from the periphery towards the center (for the singular flows), and outflow from the center (for the nonsingular flows), and vice versa for $\nu < -2$.

Substituting Eq. (18) in Eq. (15), we arrive at the following algebraic relation between the similarity functions of the pressure and density:

$$P(\mu) = \left[\frac{\pm(2+\nu)}{3\gamma-4}\right]^{\frac{1}{1-\nu}} [R(\mu)]^{\frac{2-\nu}{1-\nu}} \tag{20}$$

(the same sign rule). Equation (20) reminds the usual gas polytrope. However, this equation holds only for the similarity functions P and R. Going back to Eqs. (4) and (10) with $\sigma = (2+\nu)^{-1}$, we see that the full pressure p and density ρ of the gas are in fact not related by any polytrope.

The right-hand side of the relation (20) is not always well-defined. For the singular flows, it is well-defined either for $\gamma > 4/3$ and $\nu > -2$ (collapse), or for $\gamma < 4/3$ and $\nu < -2$ (explosive expansion). For the nonsingular flows, it is well-defined either for $\gamma < 4/3$ and $\nu > -2$ (gradual condensation), or for $\gamma > 4/3$ and $\nu < -2$ (gradual expansion). Also, Eq. (20) is not defined for $\nu = 1$. Going back to Eq. (15) and putting $\sigma = (2+\nu)^{-1} = 1/3$, we would arrive at either a nonphysical, or a trivial solution. Similarly, we have only a trivial solution in the special case of $\gamma = 4/3$. The case of $\nu = 2$ Eq. (20) must be also ruled out, as it predicts a constant pressure, which, in view of Eq. (14), implies a zero density. Therefore, we arrive at the following list of "bad" values of parameters: $\nu = 1$ and 2, and $\gamma = 4/3$.

Substituting Eq. (20) into Eqs. (13) and (14), we arrive at a single, second-order differential equation for the density:

$$\frac{c_\nu}{\mu^2}\frac{d}{d\mu}\left(\mu^2 R^{\frac{\nu}{1-\nu}}\frac{dR}{d\mu}\right) = -R(\mu), \quad \text{where} \quad c_\nu = \left(\frac{2-\nu}{1-\nu}\right)\left[\frac{\pm(2+\nu)}{3\gamma-4}\right]^{\frac{1}{1-\nu}} \tag{21}$$

(the same sign rule). Introduce a new independent variable,

$$\xi = \mu \mid n+1 \mid^{-1/2} \left[\frac{3\gamma-4}{\pm(3-n)}\right]^{\frac{1}{2n}}, \tag{22}$$

and dependent variable

$$\Theta(\xi) = \{R[\mu(\xi)]\}^{1/n}, \tag{23}$$

where $n = 1 - \nu$. Then Eq. (21) becomes

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\Theta}{d\xi}\right) = -\text{sign}(n+1)\Theta^n. \tag{24}$$

For $n > -1$ (that is, $\nu < 2$), this equation represents the celebrated Lane-Emden equation of index n (Chandrasekhar, 1939), while for $n < -1$ (that is, $\nu > 2$) we have an "anomalous" version of that equation. The Lane-Emden equation appears in the simplest formal model of stellar structure (reproduced in almost every textbook on stellar astrophysics), which considers *equilibrium* of a polytropic self-gravitating gas cloud. In particular, polytropic relations arise, if one adopts a nonrelativistic or ultra-relativistic completely degenerate electron gas as a star material (Landau and Lifshitz 1987; Shu 1992). The "normal" Lane-Emden equation (the minus sign at the right hand side of (24)) has been extensively studied and tabulated (Chandrasekhar, 1939). In our case, Eq. (24) arises in the problem of a *time-dependent* self-similar flow.

We have to solve Eq. (24) subject to two constraints. The first is the usual boundary condition

$$\frac{d\Theta}{d\xi}(\xi = 0) = 0, \tag{25}$$

that immediately follows from the assumed boundedness of the gas density at the center. This condition defines a one-parametric family of the so called "E-solutions" to the Lane-Emden equation (Chandrasekhar, 1939). The admissible self-similar flows , that we are looking for, must belong to this family. The second constraint is the already assumed mass conservation, which gives a normalization condition,

$$\alpha^3 \int_0^\infty \Theta^n(\xi)\xi^2 d\xi = M_c, \tag{26}$$

where

$$\alpha = \mid n+1 \mid^{1/2} \left[\frac{\pm(3-n)}{3\gamma-4}\right]^{\frac{1}{2n}} = \mid (2-\nu) \mid^{1/2} \left[\frac{\pm(2+\nu)}{3\gamma-4}\right]^{\frac{1}{2(1-\nu)}} . \tag{27}$$

In general, a normalization condition is much less convenient for a nonlinear equation, than, say, an additional boundary condition at $\xi = 0$. (In the special case $n = 1$, that is $\nu = 0$, the Lane-Emden equation becomes linear and its solution elementary, see below.) However, we can employ the well-known fact that the Lane-Emden equation remains invariant under a homology transformation. Indeed, if $\Theta_n(\xi)$ is a solution of the Lane-Emden equation for a fixed $n \neq 1$, and a is an arbitrary constant, then $a^{2/(n-1)}\Theta_n(a\xi)$ is also a solution. This invariance enables us to find the solution, subject to the boundary condition (25), and another boundary condition, say $\Theta(\xi = 0) = 1$, and then rescale it, using the normalization constraint (26). Specifically, we do the following. We find the solution to the Cauchy problem with $\Theta(\xi = 0) = 1$ and $(d\Theta/d\xi)(\xi = 0) = 0$ (let us call this solution $\Theta_n(\xi; 1)$). According to the homology theorem, the function $a^{2/(n-1)}\Theta_n(a\xi; 1)$ is also a solution, where a is arbitrary constant. Using Eq. (26), we determine this constant:

$$a = \left[\frac{M_c}{\alpha^3 I(1)}\right]^{\frac{n-1}{3-n}} , \tag{28}$$

where

$$I(1) = \int_0^\infty \Theta^n(\xi; 1) d\xi.$$

Therefore, for any admissible n (correspondingly, for any admissible ν), we can express the solution of our problem in terms of the Lane-Emden functions, satisfying the "standard" boundary conditions $\Theta = 1$ and $d\Theta/d\xi = 0$ at $\xi = 0$.

It is well known that the "normal" Lane-Emden equation has E-solutions, normalizable according to Eq. (26), only for $0 \leq n \leq 5$ (Chandrasekhar, 1939), that is $-4 \leq \nu \leq 1$. E-solutions with $-1 < n < 0$ (that is, $1 < \nu < 2$) have received much less attention, because in the equilibrium problem of they correspond to an "exotic" polytrope $p \sim \rho^k$ with $k < 1$. They are unacceptable in our problem too, because the gas density would go to infinity at

$\xi \to \infty$. Finally, the "anomalous" Lane-Emden equation with $n < -1$ (that is, $\nu > 2$) also has E-solutions, but we checked that they must be ruled out as nonnormalizable.

Let us summarize our results on the possible character of the spherically symmetric radiatively cooling flows. If the specific heat ratio of the gas $\gamma > 4/3$, the mass-preserving self-similar flow is possible for $-4 \leq \nu < 1$, except for $\nu = -2$. This flows represents an inflow (condensation). For $-4 \leq \nu < -2$, the condensation is gradual. For $-2 < \nu < 1$ the flow develops singularity (collapse): the gas density goes to infinity in a finite time.

If $\gamma < 4/3$, the mass-preserving self-similar flow are possible for the same values of ν. However, the character of the flow is now entirely different. Indeed, this flows always represents an outflow (expansion). When $-2 < \nu < 1$, the expansion is gradual. When $-4 \leq \nu < -2$, the flow develops singularity (explosive expansion), when the gas density goes to zero in a finite time. Therefore, radiative cooling is unable to compress a constant mass cloud quasihydrostatically, if $\gamma < 4/3$ (at least, in the regime of a self-similar flow).

The "normal" Lane-Emden equation has well-known E-type solutions in elementary functions for $n = 0, 1$ and 5 (Chandrasekhar, 1939). Among these, only $n = 1$ (that is, $\nu = 0$) and $n = 5$ (that is, $\nu = -4$) are permitted in our problem. In the case of $n = 5$ we have

$$\Theta_5(\xi; 1) = \left(1 + \frac{1}{3}\xi^2\right)^{-1/2}.$$

This solution is nonlocalized (extends to infinity), but normalizable. For $n < 5$ (that is, in all other cases) the solutions are localized, that is defined on a finite interval $0 < \xi < \xi_{\mathrm{max}}$, and vanish at $\xi = \xi_{\mathrm{max}}$. ξ_{max} increases with n, but remains finite for $n < 5$. This means that, at every time moment, the cloud radius r_c is uniquely determined, and it changes with time like

$$r_c = \alpha(t_0 - t)^{1/(3-\nu)}\xi_{\mathrm{max}} = \alpha(t_0 - t)^{1/(\nu+2)}\xi_{\mathrm{max}}$$

for the singular flows, and according to the same expression, but with $t_0 - t$ replaced by $t - t_0$ for the nonsingular flows. For $n = 5$ the cloud radius goes to infinity.

In the special case $n = 1$, the Lane-Emden equation becomes linear, and its solution $\Theta_1(\xi; 1)$ is localized on the interval $(0, \pi)$:

$$\Theta_1(\xi; 1) = (\sin\xi)/\xi \text{ if } 0 < \xi < \pi, \text{ and } 0 \text{ otherwise}.$$

In this case, directly calculating the normalization integral (26), we obtain the normalization coefficient $a = M_c/(\pi\alpha^3)$, so that the function $a\Theta_1(\xi; 1)$ yields the required solution.

Figures 1-3 show the spatial profiles (similarity functions) for the density, pressure and temperature for different values of the index n, that is for different exponents ν of the power-like radiative loss function. All the profiles can be easily calculated, once the corresponding Lane-Emden function is known. Recall that the velocity profile is linear, see Eq. (18). Figure 4 shows an example of the self-similar time evolution of the density for $\gamma = 5/3$ and $n = 1.5$ (that is, $\nu = -0.5$) in the "physical" coordinates r and t. For the chosen parameters, this flow develops collapse. The gas density at the center ultimately goes to infinity, and the cloud shrinks to the center.

Now let us discuss the validity of the quasihydrostatic approximation. Technically, one arrives at the hydrostatic relation (11) neglecting the inertial term $\rho(dv/dt)$ in the Euler equation (2). Therefore, the necessary criteria for the validity of this approximation represent smallness of the terms $\rho\partial v/\partial t$ and $\rho v\partial v/\partial r$, evaluated on the quasihydrostatic solution that we have found, compared to any of the two terms entering Eq. (11). Let us first check these criteria for the singular flows. The inequality

$$\rho\frac{\partial v}{\partial t} \ll \frac{\partial p}{\partial r}$$

can be rewritten as

$$(\nu+2)^{-1}(t_0-t)^{-\frac{2\nu+1}{\nu+2}}R(\mu)\left[(\nu+1)V+\mu\frac{dV}{d\mu}\right] \ll \frac{dP}{d\mu}.$$

We assumed that this inequality holds at $t = 0$. It is seen that if $-2 < \nu < -1/2$, the inequality will persist until the time moment of singularity. Otherwise, the inequality becomes weaker with time, and finally is violated. One can check that the second inequality,

$$\rho v \frac{\partial v}{\partial r} \ll \frac{\partial p}{\partial r},$$

leads to exactly the same criterion $-2 < \nu < -1/2$, if we want the quasihydrostatic solution to be applicable up to the time moment of singularity.

For the nonsingular solutions, the corresponding criterion is the opposite: $\nu < -2$ or $\nu > -1/2$ for the "eternal" validity of the quasihydrostatic solution.

Combining these criteria with the intervals of existence of different types of solutions, we can summarize the results in the following way. Start with $\gamma > 4/3$. We have found that the flow develops collapse if $-2 < \nu < 1$. If $-2 < \nu < -1/2$, the inertial term remains small until the time moment of the collapse. In the case of $-1/2 < \nu < 1$, the inertial term will grow and ultimately become large, so that the character of the flow will change. Gradual condensation occurs for $-4 \leq \nu < -2$. In this case the inertial term will remain small "forever."

Now let us go to the case of $\gamma < 4/3$. We have seen that explosive expansion occurs for $-4 \leq \nu < -2$. The aforementioned estimates of the inertial term show that this regime will finally cease to exist, and the character of the flow will change. Gradual expansion occurs for $-2 < \nu < 1$. When $-1/2 < \nu < 1$, the flow remains quasihydrostatic "forever." When $-2 < \nu < -1/2$, the inertial term grows, and the character of the flow will finally change.

We clearly see that the character of the flow depends crucially on the details of the temperature dependence of the radiative loss function.

IV. RADIATIVE COOLING OF A MAGNETIZED PLASMA SLAB

If the plasma cloud is magnetized, the radiative cooling flow can be modified significantly. In order to solve the corresponding problem analytically, we consider the slab geometry. Let the magnetic field be in the x direction, and all physical quantities be functions of z only. Now, the magnetohydrodynamic equations can be written as

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z}(\rho v) = 0, \tag{29}$$

$$\frac{\partial g}{\partial z} = -4\pi G\rho, \tag{30}$$

$$\frac{\partial}{\partial z}\left(p + \frac{B^2}{4\pi}\right) - \rho g = 0, \tag{31}$$

$$\frac{\partial B}{\partial t} + \frac{\partial}{\partial z}(vB) = 0, \tag{32}$$

$$\frac{1}{\gamma - 1}\frac{dp}{dt} + \frac{\gamma}{\gamma - 1}p\frac{\partial v}{\partial z} + L(\rho, T) = 0, \tag{33}$$

where g is the self-gravity acceleration. We have already assumed that the process is quasi-magnetohydrostatic and neglected the inertial term in Eq. (31). The resulting equation describes an (approximate) quasistatic balance between the total (thermal + magnetic) pressure gradient and the self-gravity force. Also, we have neglected the ambipolar magnetic diffusion in the induction equation (32), which is legitimate if the characteristic time of the radiative cooling is much shorter than the characteristic ambipolar diffusion time.

Using the continuity equation (29) and the induction equation (32), we arrive at the well-known equation

$$\frac{\partial}{\partial t}\left(\frac{B}{\rho}\right) + \frac{\partial}{\partial z}\left(v\frac{B}{\rho}\right) = 0, \tag{34}$$

implying that the magnetic field lines are "frozen" into the plasma, and that during the cloud condensation, the field lines get "compressed," so that the magnetic field increases.

Equations (29)–(33), supplemented by the perfect gas equation of state (4), represent a closed set. Let us transform to the Lagrangian mass coordinate (*e.g.* Shu, 1992). It is defined as

$$m = \int_0^z \rho(z', t)dz', \tag{35}$$

that is, m is the mass of a column of a unit area and height z. In the planar problem we are interested in, this quantity has the dimension of the surface mass density. Also, introduce the specific volume of the plasma, $u(m,t) = \rho^{-1}(m,t)$. In the new variables, Eqs. (29)–(33) become

$$\frac{\partial u}{\partial t} = \frac{\partial v}{\partial m} \tag{36}$$

$$\frac{\partial g}{\partial m} = -4\pi G \tag{37}$$

$$\frac{\partial}{\partial m}\left(\frac{p + B^2}{8\pi}\right) - g = 0 \tag{38}$$

$$\frac{\partial}{\partial t}(Bu) = 0 \tag{39}$$

$$\frac{1}{\gamma - 1}\frac{\partial p}{\partial t} + \frac{\gamma}{\gamma - 1}\frac{p}{u}\frac{\partial v}{\partial m} + L'(u,T) = 0 \tag{40}$$

where $L'(u,T) \equiv L(u^{-1},T)$. The Poisson equation (37) is immediately integrated:

$$g = -4\pi G m \tag{41}$$

(the integration constant must be taken zero because of the symmetry of the problem with respect to the mid plane). The frozen-in magnetic field can be expressed, in view of Eq. (39), as a function of the specific volume and initial conditions:

$$B(m,t) = f(m)u^{-1}(m,t)\,, \quad f(m) = B(m,t=0)u(m,t=0). \tag{42}$$

Using Eqs. (41) and (42), we integrate Eq. (38) and arrive at the following force balance relation:

$$p + \frac{1}{8\pi}f^2(m)u^{-2} + 2\pi G m^2 = F(t), \tag{43}$$

where $F(t)$ is an arbitrary function of time. Let us assume, like in Sec. III, that the cloud mass is preserved, and that B, p and $\rho = u^{-1}$ vanish at $z \to \infty$. Then we find

$$F(t) = 2\pi G M^2 = \text{const},$$

where M is the total mass (more precisely, the half-sided surface density) of the plasma slab. Therefore, the total (thermal + magnetic) plasma pressure is

$$p + \frac{1}{8\pi} f^2(m) u^{-2} = 2\pi G(M^2 - m^2), \tag{44}$$

which simply means that the total pressure at any "level" m is equal to the weight of the material "above" (that is, outside) this level. Substituting Eqs. (36) and (44) into Eq. (40), we obtain a simple first order partial differential equation for the plasma density:

$$\frac{1}{\gamma - 1}\left[(2-\gamma)\frac{f^2(m)}{8\pi u^2} + 2\pi\gamma G(M^2 - m^2)\right]\left(\frac{\partial u}{\partial t}\right) = -uL'[u, T(u,m)]. \tag{45}$$

$T(u,m)$ in the second argument of the radiative loss function is given by the following expression:

$$T(u,m) = \frac{\mu_g}{R_g}\left[2\pi G u(M^2 - m^2) - \frac{f^2(m)}{8\pi u}\right],$$

obtained from the Eqs. (4) and (44). Since the Lagrangian coordinate m enters Eq. (45) only as a parameter, this equation is essentially an ordinary differential equation. Moreover, it is separable and can be integrated in quadrature for any prescribed radiative loss function. Having found the specific volume $u(m,t)$, we can easily determine the rest of variables. Indeed, the pressure, temperature and magnetic field can be found from algebraic relations (44), (4), and (42), respectively. Using Eq. (36), we find the plasma velocity in the Lagrangian mass variable:

$$v(m,t) = \int_0^m \frac{\partial u}{\partial t} dm' = \frac{\partial}{\partial t}\int_0^m u dm'.$$

Finally, the relationship between Lagrangian and Eulerian coordinates, necessary for a transformation back to Eulerian coordinates z and t, is given by

$$z(m,t) = \int_0^m u(m',t) dm'. \tag{46}$$

Therefore, we can concentrate on Eq. (45). Consider the simplest example, when the radiative loss function is zero at $T < T_1$ and independent of the temperature for $T > T_1$:

$$L(u,T) = \frac{F_0}{u^2}\theta(T - T_1), \tag{47}$$

where $\theta(w)$ is the Heaviside step-function. We can say that the simple function (47) is an alternative to the power law used in the previous section, as it takes into account the radiation cutoff, but ignores any "smooth" temperature dependence. [Stress again that Eq. (45) admits, in principle, analytical solution for any $L'(u,T)$.] Let the initial temperature of the cloud be constant and equal to $T_0 > T_1$. Transform to the scaled time $\tau = t/t_0$ and Lagrangian coordinate $\zeta = m/MM$, and introduce scaled variables $\tilde{T} = T/T_0, \tilde{u} = u/u_0, \tilde{v} = v/v_0$ and $\tilde{B} = B/B_0$, where $v_0 = M/(\rho_0 t_0)$ and

$$t_0 = \frac{\gamma T_0 R_g}{(\gamma-1)\mu_g \beta_G \rho_0 F_0}.$$

Also, define $\tilde{f} = f/f_0$, where $f_0 = B_0 u_0$, and introduce two dimensionless parameters

$$\beta_G = \frac{R_g T_0}{2\pi \mu_g G M^2 u_0} \quad , \quad \beta_B = \frac{B_0^2}{16\pi^2 G M^2}.$$

Parameters β_G and β_B represent the ratio of the thermal and magnetic pressure, respectively, to the total (thermal + magnetic) pressure. The magnetic field can be neglected (at least, at the initial stage of the condensation), if $\beta = \beta_B/\beta_G \ll 1$. In the scaled variables, Eq. (45) becomes

$$\left(\frac{2-\gamma}{\gamma\beta_B \frac{f^2(\zeta)}{u^2} + 1 - \zeta^2}\right)\frac{\partial u}{\partial \tau} = -\frac{1}{u}, \tag{48}$$

where we have omitted tildes. The initial condition for the specific volume is uniquely determined by the assumed constant temperature at $\tau = 0$:

$$u_0(\zeta) \equiv u(\zeta, 0) = \frac{\beta_G}{1 - \zeta^2 - \beta_B B^2(\zeta, 0)}.$$

Integrating Eq. (48), we obtain the following implicit expression for the time evolution of the specific density :

$$\frac{2-\gamma}{\gamma}\beta_B f^2(\zeta) \ln\left[\frac{u(\zeta,\tau)}{u_0(\zeta)}\right] + \frac{1}{2}(1-\zeta^2)\cdot\left[u^2(\zeta,\tau) - u_0^2(\zeta)\right] = -\tau. \tag{49}$$

At fixed ζ, this relation holds until the temperature drops to the cutoff value T_1 (that is, to the value T_1/T_0 in the scaled variables). After that moment, the solution does not change in time:

$$u(\zeta) = \frac{T_1\beta_G}{2T_0(1-\zeta^2)}\left[1+\left(1+4f^2(m)\frac{T_0^2}{T_1^2}\frac{\beta_B}{\beta_G^2}(1-\zeta^2)\right)^{1/2}\right]. \tag{50}$$

Using Eq. (50) and (44) and equation of state, we can easily find the corresponding temperature.

The solution becomes especially simple, if we can neglect the magnetic field. In this case,

$$u(\zeta,\tau) = \frac{[\beta_G^2 - 2(1-\zeta^2)\tau]^{1/2}}{1-\zeta^2} = u(\zeta,0)\left[1-\frac{2(1-\zeta^2)}{\beta_G^2}\tau\right]^{1/2}. \tag{51}$$

Correspondingly, the temperature, density and velocity are the following:

$$T(\zeta,\tau) = \left[1-\frac{2(1-\zeta^2)}{\beta_G^2}\tau\right]^{1/2}, \tag{52}$$

$$\rho(\zeta,\tau) = \frac{1}{u} = \frac{1-\zeta^2}{\beta_G}\left[1-\frac{2(1-\zeta^2)}{\beta_G^2\tau}\right]^{-1/2}, \tag{53}$$

$$v(\zeta,\tau) = \left(\frac{1}{2\tau}\right)^{1/2}\ln\left[\left(\frac{2\tau}{\beta_G^2-2\tau}\right)^{1/2}\zeta+\left(\frac{2\tau}{\beta_G^2-2\tau\zeta^2+1}\right)^{1/2}\right]. \tag{54}$$

Equations (51)–(54) describe quasihydrostatic condensation (contraction) of the radiatively cooling plasma. These equations are valid on the whole interval $0<\zeta<1$ up to the time moment, when the temperature drops to the cutoff temperature $T=T_1$ (that is, to the temperature T_1/T_0 in the scaled variables). For the chosen initial condition, the cutoff temperature T_1 is reached first at the mid plane, so that further cooling and condensation there are arrested, while the flow velocity becomes zero. At the next time moment, T_1 is reached at the adjacent planes, etc. Therefore, a traveling cooling front develops, starting from the mid plane and moving outwards. From this time on, Eqs. (51)–(53) describe the pre-front solutions, valid for $|\zeta|>\zeta_f(\tau)$. The pre-front gas velocity is the following:

$$v(\zeta,\tau) = \left(\frac{1}{2\tau}\right)^{1/2}\ln\left[\frac{\zeta+\left(\frac{\beta_G^2-2\tau}{2\tau}+\zeta^2\right)^{1/2}}{\zeta_f+\left(\frac{\beta_G^2-2\tau}{2\tau}+\zeta_f^2\right)^{1/2}}\right]. \tag{55}$$

The position $\zeta_f(\tau)$ and speed v_{fL} of the cooling front in the Lagrangian coordinate are the following:

$$\zeta_f(\tau) = \left(1 - \frac{\tau}{\tau_f}\right)^{1/2}, \quad v_{fL}(\tau) = \frac{d\zeta_f}{d\tau} = \frac{\tau_f}{2}\tau^{-3/2}\left(\tau - \tau_f\right)^{-1/2}, \quad \tau \geq \tau_f,$$

where $\tau_f = (1/2)\beta_G^2\,(1 - T_1^2/T_0^2)$ is the (scaled) time, when the temperature at the mid plane first reaches the cutoff value T_1 (that is, T_1/T_0 in the scaled variables). One can see that the cooling front is decelerating with time (its speed is formally infinitely large at $\tau = \tau_f$ and falls like τ^{-2} for $\tau \gg \tau_f$). The post-front density, temperature and velocity are the following:

$$u(\zeta,\tau) = \frac{\beta_G T_1}{T_0(1-\zeta^2)}, \qquad T(\zeta,\tau) = \frac{T_1}{T_0} = \mathrm{const}, \quad v(\zeta,\tau) = 0, \qquad |\zeta| < \zeta_f.$$

Behind the front, where the temperature is constant, we can easily transform back to the Eulerian coordinate. We integrate equation (46) and obtain

$$z = z_0 \,\mathrm{arctanh}(\zeta) \quad , \qquad z_0 = \frac{\beta_G T_1 M}{T_0 \rho_0}.$$

Now we express the scaled Lagrangian coordinate ζ through the Eulerian coordinate z,

$$\zeta = \tanh(z/z_0),$$

and rewrite the density profile in the dimensional variables as

$$\rho(z) = \frac{M}{z_0}\cosh^{-2}\left(\frac{z}{z_0}\right).$$

This post-front solution in the Eulerian coordinate represents, of course, the classical solution for the isothermal self-gravitating gas slab in equilibrium (Spitzer 1942). Now we can easily calculate the position $z_f(t)$ and speed $v_{fE}(t)$ of the cooling front in the Eulerian coordinate:

$$z_f(\tau) = \frac{1}{2}\, z_0 \ln \frac{1+\zeta_f}{1-\zeta_f}\,, v_{fE}(\tau) = dz_f \overline{d}\tau = \frac{dz_f}{d\zeta_f}\frac{d\zeta_f}{d\tau} = \frac{1}{2}\, z_0 \tau_f^2 \tau^{-5/2}(\tau - \tau_f)^{-1/2}.$$

so that the front speed goes down like τ^{-3} at large times.

Figures 5-7 show the time evolution of the density, temperature and velocity profiles in the Lagrangian coordinate.

If the magnetic field is significant, the solution (see Eqs. (49) and (50)) looks different. The condensation time becomes longer, and the condensation process is hindered because of the magnetic pressure buildup. An example of the density dynamics in this case is shown in Figures 8 and 9 for $\beta_B = 0.1$ and 0.5, respectively. In this example, the initial condition for the plasma density corresponds to a finite-width plasma slab in the Eulerian coordinates. One can see that the larger the magnetic field, the less pronounced is the plasma condensation.

V. DISCUSSION AND CONCLUSIONS

The main aim of this work was to illustrate the general principle of marginal stability in the dynamics of self-gravitating gas clouds, relevant to the problem of star formation. We noticed that "fast" hydrodynamic or magnetohydrodynamic instabilities (like Jeans instability and Parker instability), or any other loss of hydrostatic equilibrium often resolve themselves, after several dynamic times, into marginally stable, quasihydrostatic configurations. These configurations can then evolve significantly on a longer time scale. We have solved analytically two simple model problems, describing "marginally stable" quasihydrostatic radiatively cooling flows of constant-mass self-gravitating clouds.

The first problem dealt with spherically-symmetric flows. Assuming a power-law dependence of the radiative loss function on the temperature with the power ν (which, of course, is quite a restrictive assumption, if one uses it for a wide temperature range), we have found a self-similar solution to the problem. We have seen that, depending on the specific heat ratio of the gas and on the exponent of the temperature dependence of the radiative loss function, entirely different cooling flow regimes are possible.

For $\gamma > 4/3$ and $-4 \leq \nu < 1$ ($\nu \neq -2$) the constant mass-cloud undergoes contraction.

This contraction is either gradual (for $-4 \leq \nu < -2$), or collapse-like (for $-2 < \nu < 1$). In the latter case, the character of collapse is not changed by the flow inertia if $2 < \nu < -1/2$.

For $\gamma < 4/3$, the constant-mass cloud is shown to expand. The expansion proceeds gradually for $-2 < \nu < 1$. If $-1/2 < \nu < 1$, the flow inertia remains insignificant up to the end of the expansion. For $-4 \leq \nu < -2$ the quasihydrostatic model predicts explosive expansion, when the density goes to zero in a finite time. However, we have found that the flow inertia grows with time in this case, and can finally change the character of the flow.

Therefore, this simple model gives very definite predictions of the character of the flow, depending on γ and ν. Interesting enough, it does not permit any self-similar solution for $\nu > 1$. It means that the character of the flow in the case of $\nu > 1$ must be very different. We can speculate that it is the constant external pressure regime (or, more generally, any regime, violating mass conservation) that will set it in this case. In a constant-mass cloud, this would mean development of an isolated core, which could exchange mass with the rest of the cloud, and where a quasihydrostatic self-similar flow with different self-similarity indices would be possible. Another unresolved issue concerns the dynamics of the same system under a realistic (nonpower-like) dependence of the radiative loss function on the temperature. Obviously, the simplicity of self-similarity will be lost, and the governing equations (1) and (7)–(9) will require numerical solution. Still, our self-similar solutions will be helpful both in search for "traces" of similarity in the numerical solution ("intermediate asymptotics," see Barenblatt 1979), and in checking the numerical code in a wide range of parameters.

The second problem dealt with a quasihydrostatic self-gravitating plasma slab, either magnetized or not. We considered radiative cooling of the slab, accompanied by the plasma inflow and contraction. Employing Lagrangian mass coordinate, we have been able to solve the problem analytically. We accounted for the low-temperature "cutoff" in the radiative loss function. This feature manifests itself in the existence of two distinct stages in the slab dynamics. At the first stage, there is a (nonuniform) "volume" cooling of the slab, accompanied by plasma inflow and condensation (significant for small and insignificant for large magnetic fields). As the denser central regions cool faster, they reach the radiation cutoff temperature first. Then the second stage starts, when a traveling cooling front develops at the center and propagates outward. The process proceeds until all the slab cools down to the cutoff temperature, plasma flow terminates, and true hydrostatic equilibrium, described by Spitzer (1942), is achieved.

Considering the second problem, we disregarded the ambipolar diffusion. This is justified, if the characteristic cooling time is less than the characteristic time of the ambipolar diffusion (Shu 1983, 1992). If this criterion is violated, one should include the ambipolar diffusion term in the induction equation, still using the quasihydrostatic approximation.

In this work, we concentrated on mass-preserving flows. Alternative constraints, like constant pressure density at infinity, must be considered separately, as well as a realistic, nonpower-like radiative loss function in the first problem. Other possible extensions include the question of stability. In the first problem, one should check, whether, starting from some "reasonable" initial conditions, the solution will finally approach the self-similar solution that we have found. Also, one should perturb the one-dimensional flows in a generic way (that is, three-dimensionally in the first problem, and two-dimensionally in the second one) and investigate their stability. In general, we should address the three following well-known instabilities (modified by the radiative-cooling induced flows). First, in the cases of inflow, the quasihydrostatically contracting cloud can ultimately become gravitationally unstable. The frequently used simplistic approach to the Jeans instability deals with an inequality between the mass of the cloud and Jeans mass, as an instability criterion. The marginal stability requires, in this approach, that the self-gravity force be equal to the pressure gradient force. Our work shows that this condition can be in fact satisfied all the time, without any significant flow acceleration. A proper (three-dimensional) analysis of the Jeans instability of a slowly cooling flow should address, as the background state, the quasihydrostatic density and pressure profiles that we have found. Second, possibility of the convective instability and its nonlinear consequences should be considered. Third, in the case of a magnetized slab, the magnetic buoyancy instability can develop. All these problems seem relevant in view of possible applications of the quasihydrostatic theory to the star formation. Now, that the basic physical insight into the nature of quasihydrostatic radiatively cooling flows has been achieved, these problems are worth investigating.

ACKNOWLEDGMENTS

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FIGURE CAPTIONS

FIG. 1. Similarity profiles for the spherically symmetric quasihydrostatic flows. Shown are the normalized gas density profiles $R(\mu)$ for different values of $n = 1 - \nu$. The similarity variable $\mu = r/(t_0 - t)^{1/(2+\nu)}$ for the singular flows, and $r/(t - t_0)^{1/(2+\nu)}$ for the nonsingular flows.

FIG. 2. Similarity profiles for the spherically symmetric quasihydrostatic flows. Shown are the normalized gas pressure profiles $P(\mu)$ for different values of $n = 1 - \nu$.

FIG. 3. Similarity profiles for the spherically symmetric quasihydrostatic flows. Shown are the normalized gas temperature profiles $T(\mu)$ for different values of $n = 1 - \nu$.

FIG. 4. An example of self-similar spherically symmetric quasihydrostatic flow, developing collapse. Shown is the time evolution of the gas density profile for $\gamma = 5/3$ and $n = 1.5$. The initial density at the center $\rho_0 = 10^{-20} g/cm^3$, the initial temperature $T_0 = 10^4 K$. For these parameters the characteristic time of the collapse is $t_0 = 2 \cdot 10^6$ years.

FIG. 5. Time evolution of a planar flow (no magnetic field). Shown are the gas density profiles $\rho(\zeta, t)$ in the scaled Lagrangian mass coordinate ζ at different time moments for $\beta_G = 1$ and scaled radiation cutoff temperature 0.1.

FIG. 6. Same as in Fig. 5, but shown are the gas temperature profiles $T(\zeta, t)$.

FIG. 7. Same as in Fig. 5, but shown are the gas velocity profiles $v(\zeta, t)$.

FIG. 8. Planar flow of a magnetized plasma. Shown are the gas density (a) and temperature (b) profiles in the scaled Lagrangian mass coordinate ζ at different time moments.

The scaled radiation cutoff temperature is 0.1, $\beta_G = 1$, and $\beta_B = 0.1$. The initial magnetic field is equal to unity for $\zeta < 0.94$ and zero elsewhere.

FIG. 9. Same as in Fig. 8, but $\beta_B = 0.5$, and the initial magnetic field is equal to unity for $\zeta < 0.7$ and zero elsewhere.

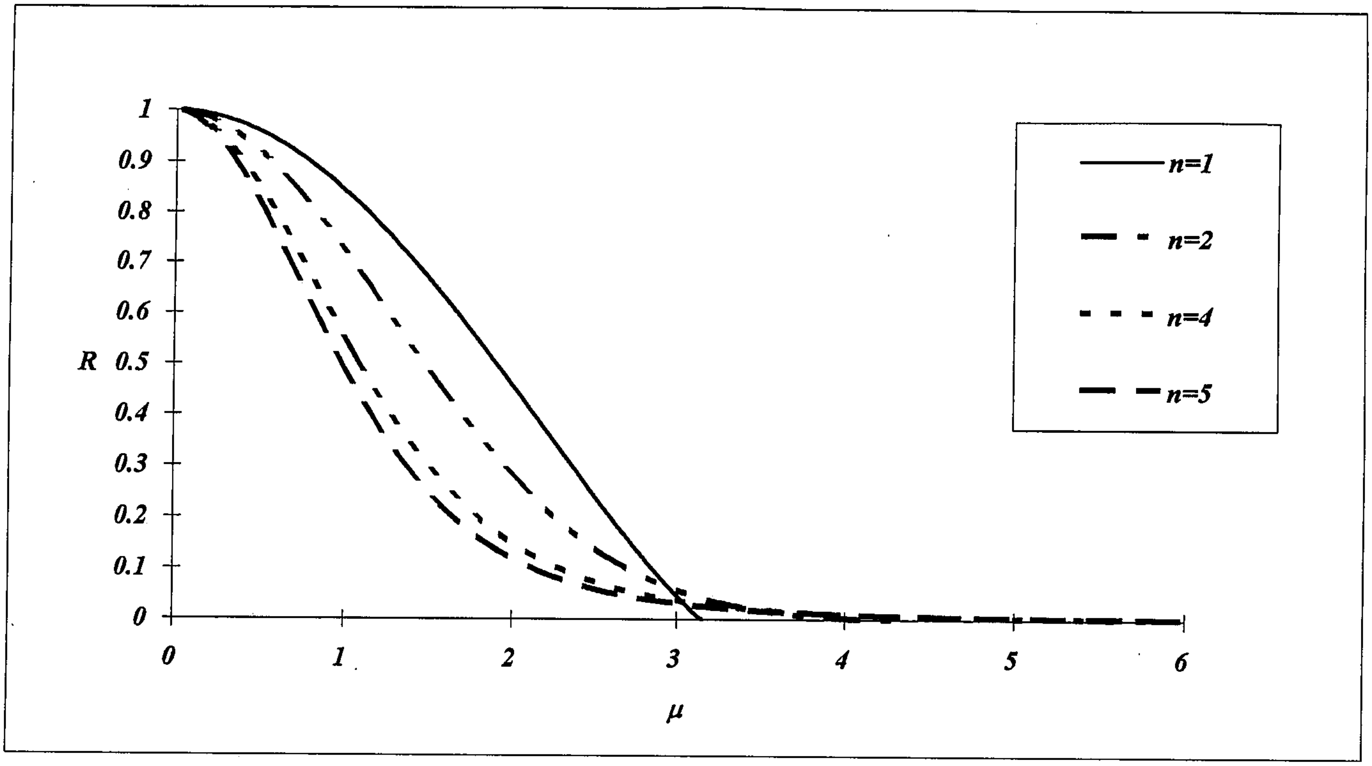


FIG 7

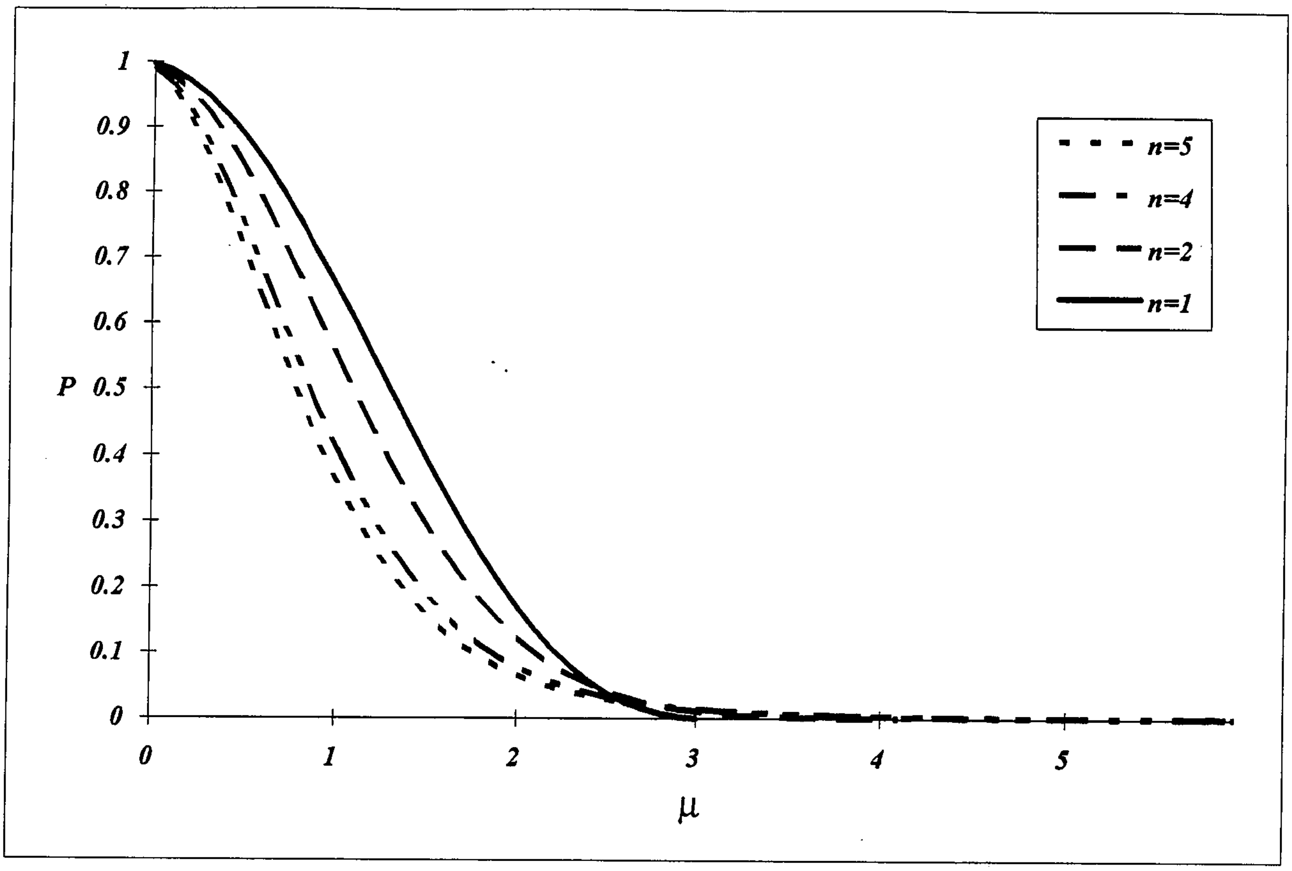


Fig. 2

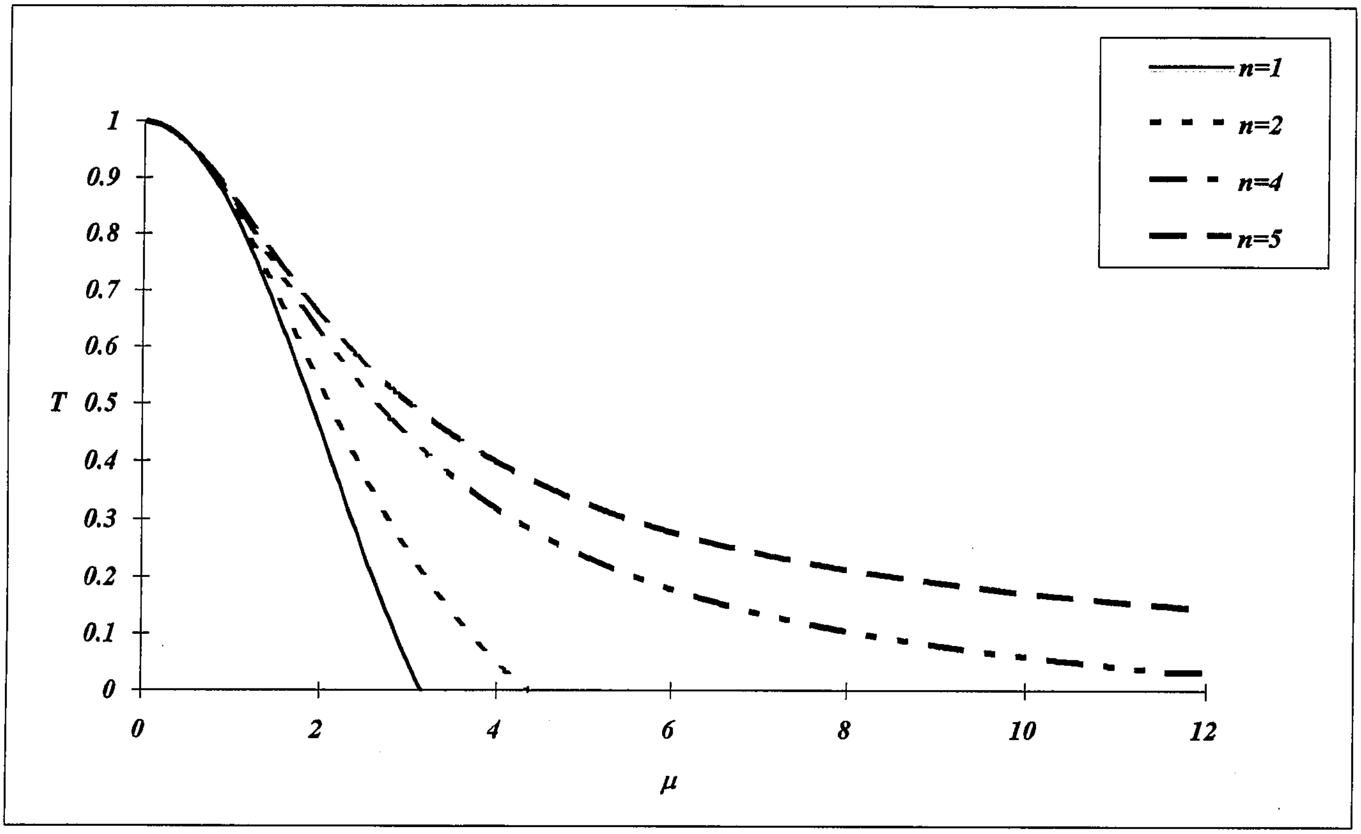


FIG 3

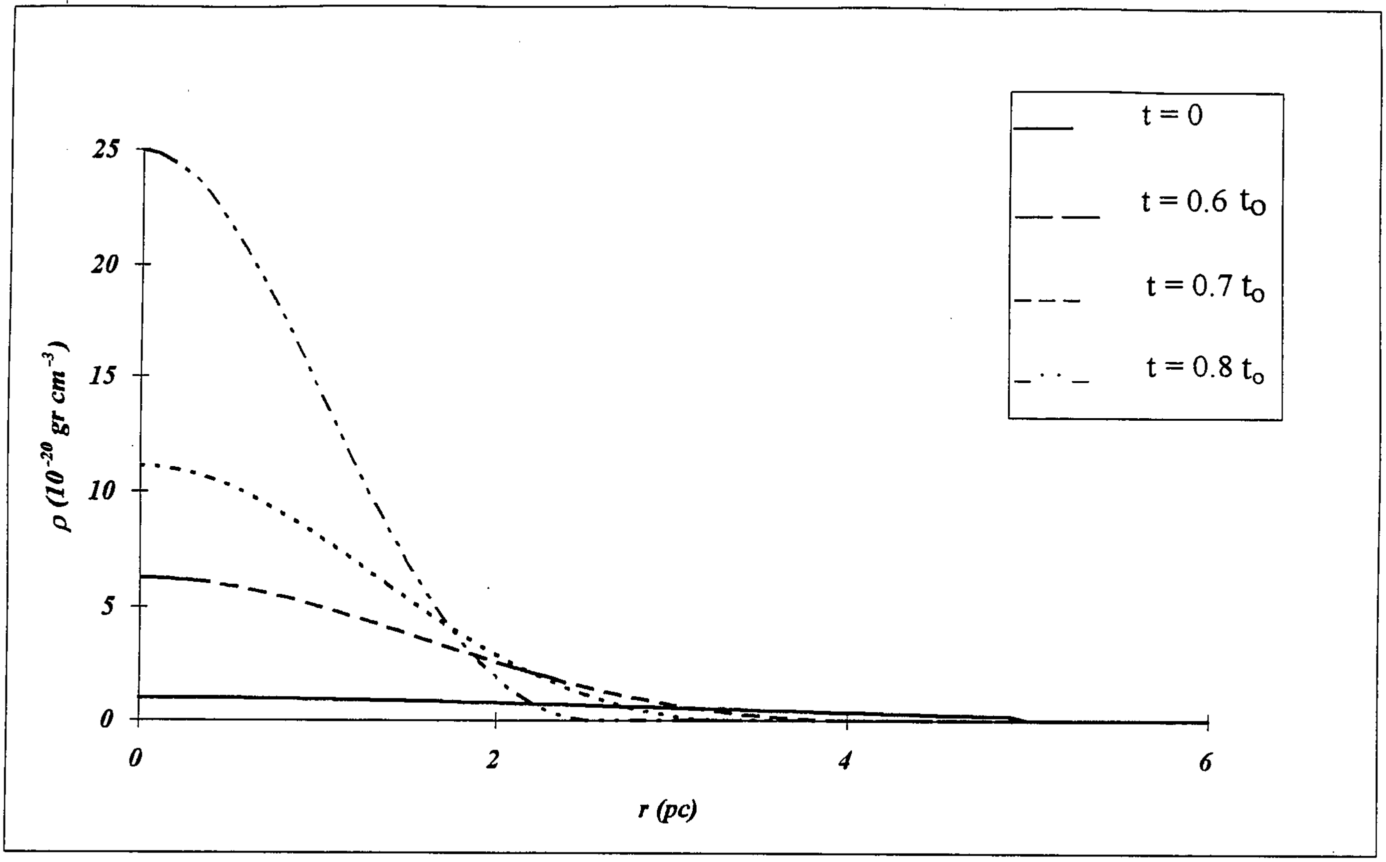


Fig. 4

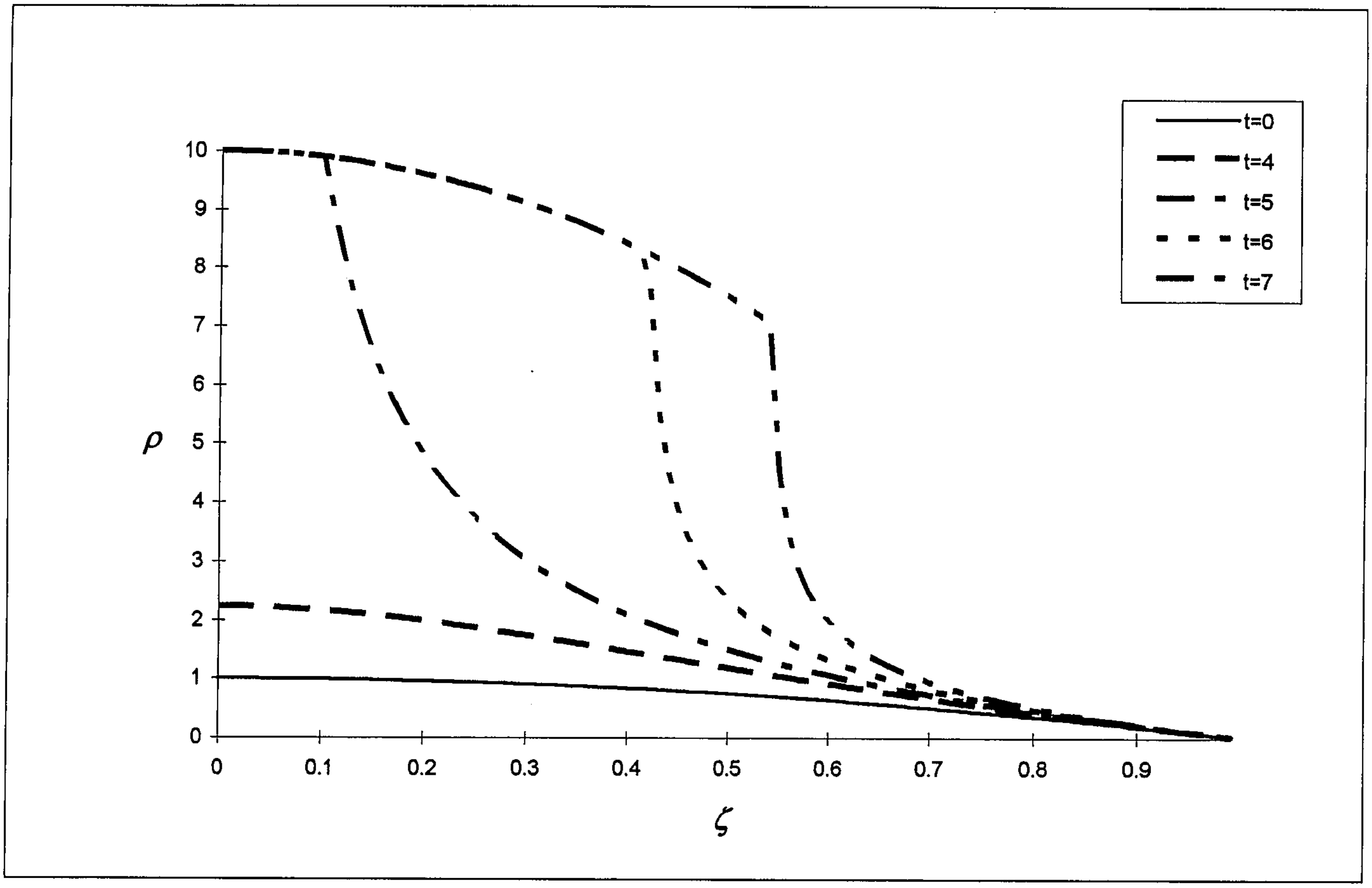
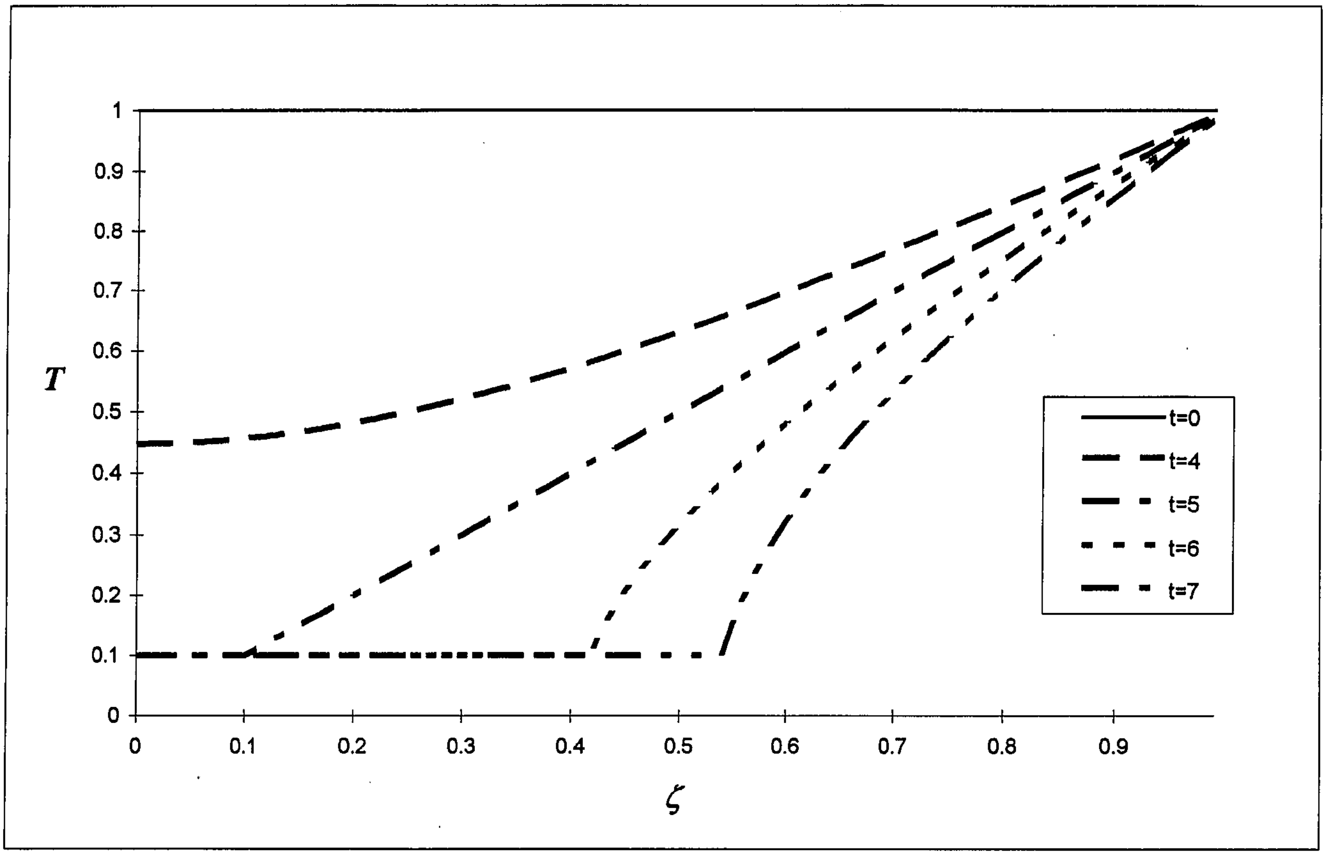
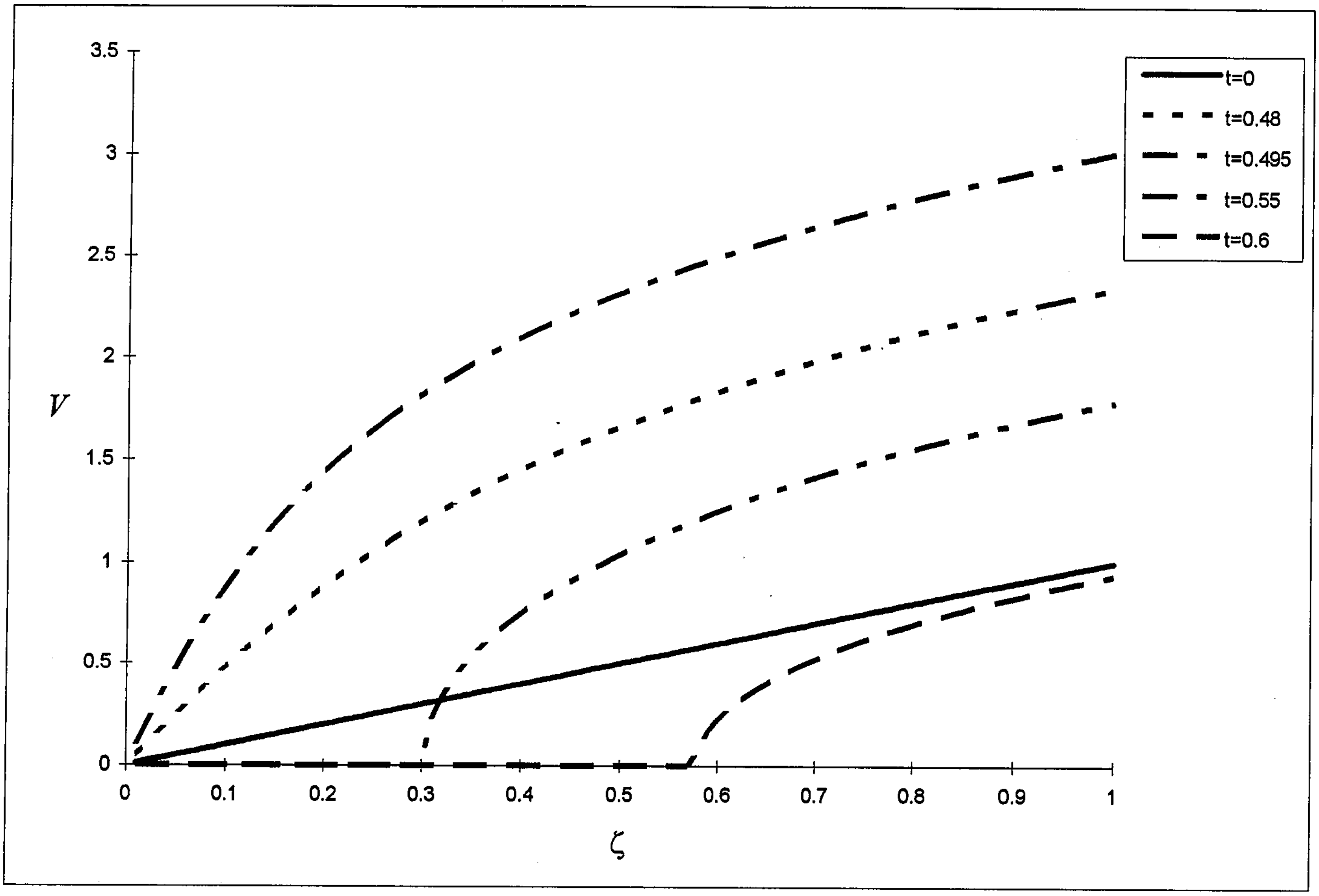


FIG 5



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
T
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
ζ
t=0
t=4
t=5
t=6
t=7

Fig 6



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3
2.5
2
1.5
1
0.5
0
V
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
ζ
t=0
t=0.48
t=0.495
t=0.55
t=0.6

FIG 7

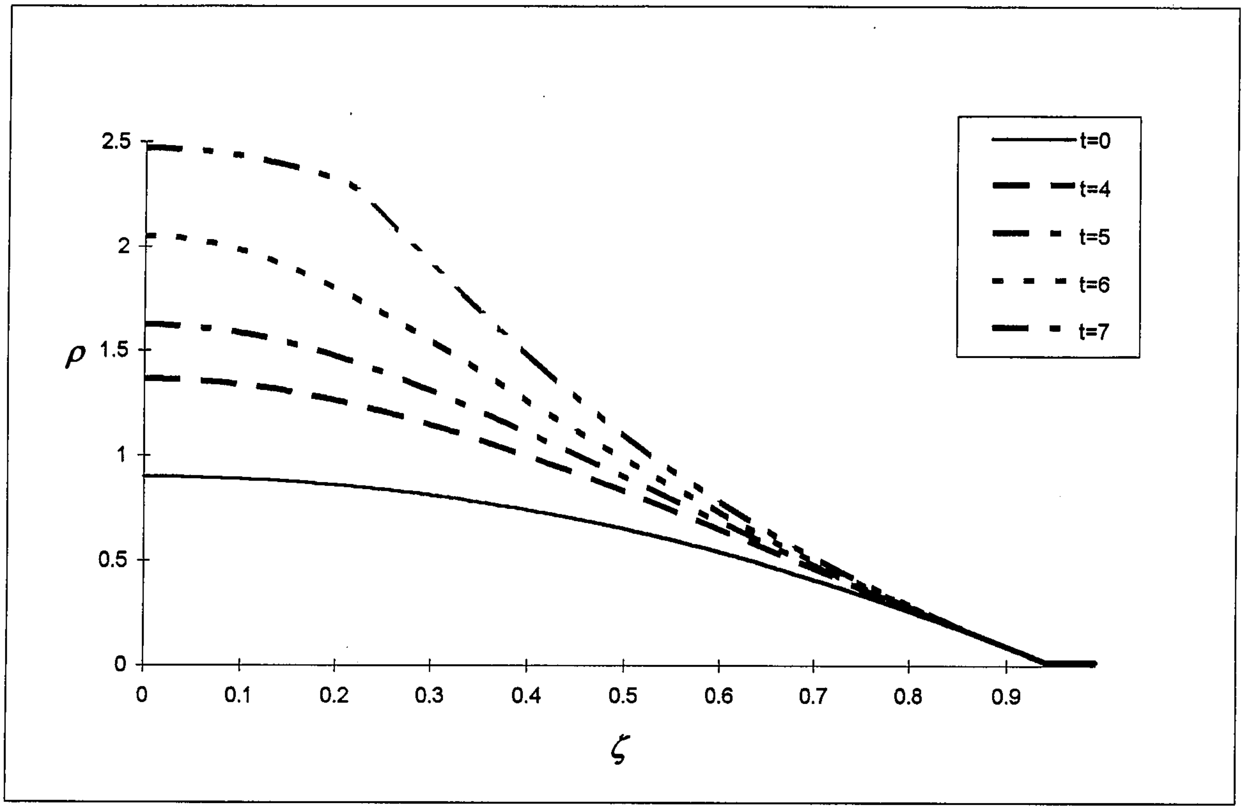
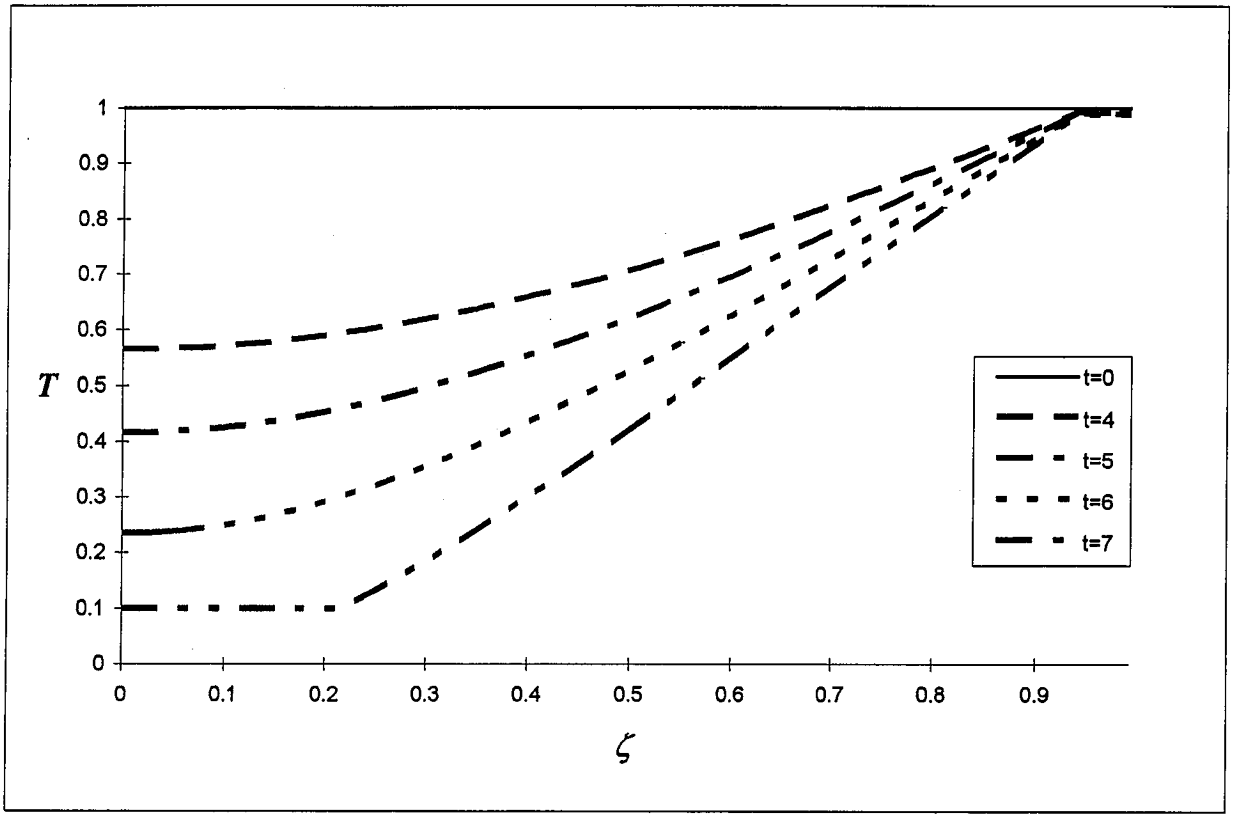


Fig. 8a



T
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0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
ζ
t=0
t=4
t=5
t=6
t=7

Fig. 8B

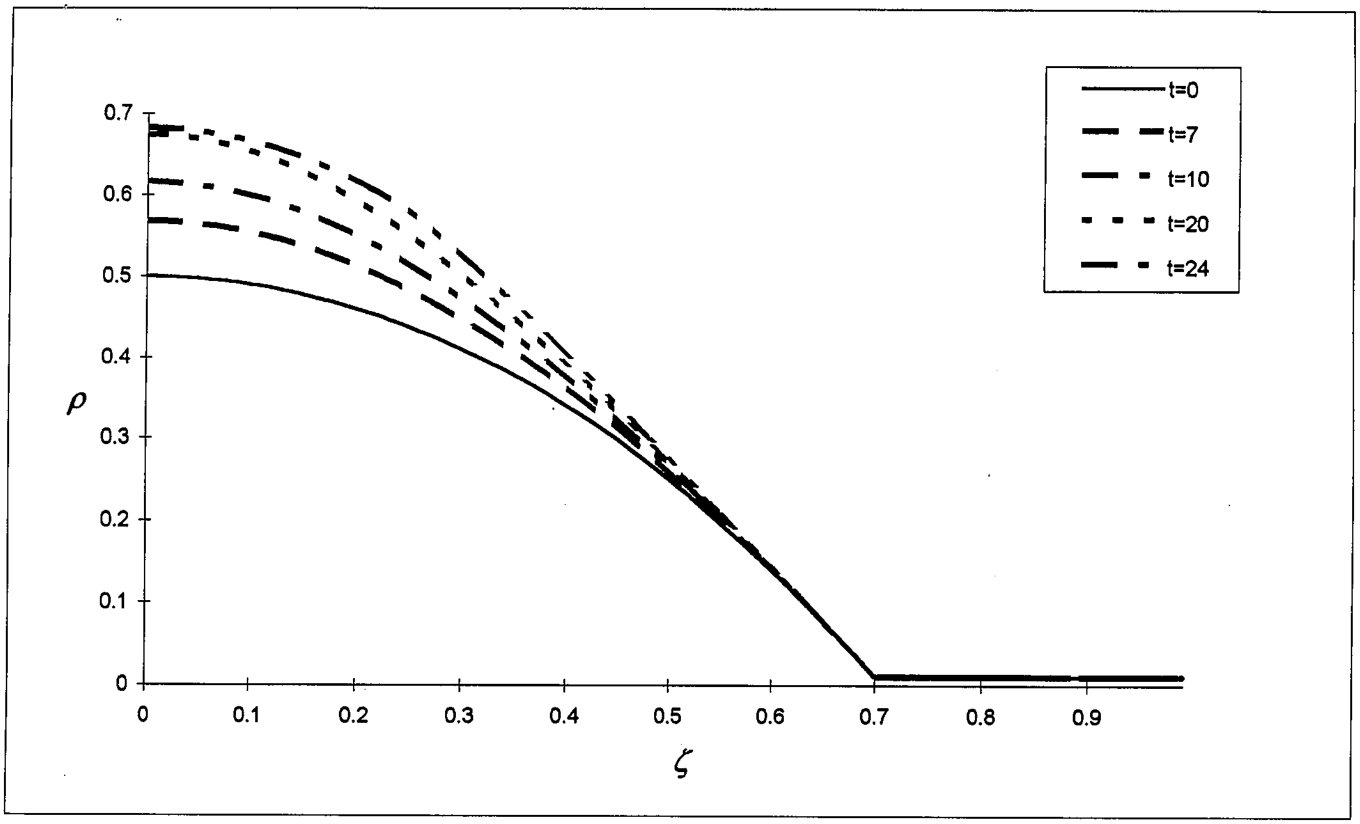
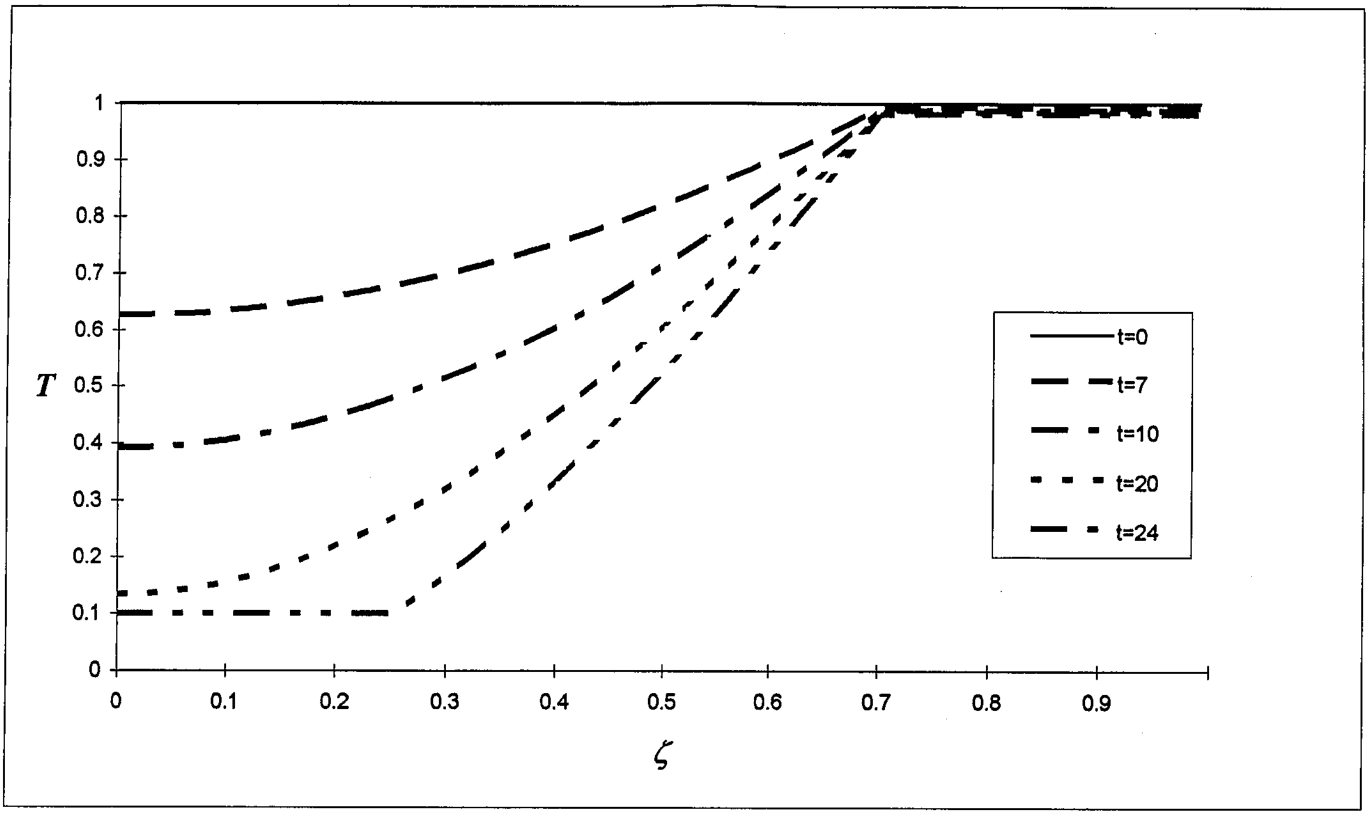


Fig. 9a



1
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0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
T
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
ζ
t=0
t=7
t=10
t=20
t=24

Fig. 9[illegible]