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Impurity Effects on Linear and Nonlinear ITG Driven Modes

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Abstract

Linear and nonlinear stages in the development of the ion-temperature-gradient driven drift-wave instability are studied analytically in the presence of shear flows, magnetic shear, inhomogeneity, and curvature. In the linear regime, it is shown that the toroidal η_i mode can be destabilized by a small amount of impurities only if there exists an impurity build-up at the plasma edge. The slab η_i mode is destabilized by a small amount of inhomogeneities, and stabilized by a larger impurity content when the inhomogeneity and main ions density profiles are close to each other. In the nonlinear regime two types of coherent structures are found: generalized Hasegawa-Mima dipole vortex in the weak magnetic shear case, and a periodic, vortex-chain solution in the strong shear case, which corresponds to the saturated, large amplitude drift-tearing mode.

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I. INTRODUCTION

The presence of impurity ions in tokamak plasmas, often being heavy and highly charged, is an undesirable but unavoidable factor in all fusion experiments. Impurities may originate from the first wall and other structures inside the discharge vessel which are in contact with the plasma, such as limiters, antennas, and probes, or they can also be injected into the plasma as a part of the refueling pellet (e.g. as its coating). Even a small amount of impurities may drastically affect the tokamak discharge confinement as well known by the importance of low Z_{eff} for improved confinement regime. In the reactor regime the dilution of the fuel-hydrogenic ion density by the impurities directly degrades the fusion power yield.

It has been shown previously that the presence of impurities can both modify already existing modes, and give rise to new impurity driven modes,[1] which then may lead to an enhanced transport.[2] It is of particular interest to study the ion-temperature-gradient driven modes (also called ITG or η_i modes) in the presence of impurities, since these modes are widely recognized to provide the dominant mechanism for the transport of the ion thermal energy across the confining magnetic field.[3,4] These fast transport results were confirmed in the recent papers[5–9] devoted to a detailed study of the linear stability and the nonlinear stage of the η_i instability. The simulation aspect of these studies reveals that in the late stages of the nonlinear evolution large scale coherent structures were formed[6,9] in a background of turbulent waves even in the presence of magnetic shear. The role of sheared flows is shown in Hamaguchi and Horton[5] to have an important influence on the transport rate especially when the magnetic shear is relatively weak.

The first attempt to study the linear theory of the ion-temperature--gradient driven modes in the presence of impurities was reported by Fröjdh *et al.*[10] for the case of a toroidal η_i mode in the absence of magnetic shear. Authors of Ref. 10 studied stability properties using the fluid description, and calculated the quasilinear diffusion coefficients for the main and impurity ions. These results were improved in Ref. 11, to include the effects of the finite ion inertial length and higher order toroidal effects.

In this paper we present a nonlinear theory of the η_i mode in multi ion-species plasmas, accounting for all ion-acoustic, curvature, inhomogeneity, and compressibility effects to the leading order. Our calculations are based on the fluid model developed in Ref. 5, generalized to describe multiple ion species. We show that in the linear regime a small amount of impurities has a destabilizing influence on the slab η_i mode, if the density profiles of the main and impurity ions are close to each other. Conversely, a larger amount of impurities ($\geq 10\%$) has a stabilizing influence on the slab η_i mode. For the toroidal η_i mode, we show that it can be destabilized only in the relatively rare cases when the density profile of impurities is either flat, or oppositely oriented than that of the main ions, i.e. when there exists an impurity build-up at the plasma edge due to some sort of plasma decontamination.

The presence of the driving force for the linear instability yields a relatively high level of turbulent fluctuations associated with the unstable drift waves, and as a consequence one may expect that large scale coherent structures are formed, similar to those observed in the numerical simulations in the single-ion-species plasma.[6] Such self-organization results from the partial alignment of fluctuations due to the $\mathbf{E} \times \mathbf{B}$ plasma convection around the local maxima and minima of the electrostatic potential. These structures trap particles, and hence reduce the quasilinear transport, but they also give rise to a new strong turbulence transport mechanism, which is a result of the particle ejection from the locations where random inelastic collisions between such coherent structures take place.

Following the analytic procedure developed for the nonlinear η_i mode in single ion species plasmas,[9] we find two types of coherent structures. In the case of a weak shear we construct the usual dipole vortex, or modon, solution with a small amplitude monopole and quadrupole superimposed on the dipole. In a multispecies plasma there exists a family of modons, each of which is associated with the discontinuity of the pressure gradient of a different ion species. In the magnetic shear dominated case, however, we construct a solution in the form of a vortex chain, which can be identified as a saturated state of the linearly unstable drift-tearing mode.

The paper is organized as follows. Our model equations are derived in Sec. II, and the general travelling solution is studied in Sec. III. In the same section we derive the linear dispersion relation and discuss the destabilization of the η_i mode by a small amount of impurities. The two types of nonlinear solutions are constructed in Sec. V, and the conclusions are given in Sec. V.

II. MODEL EQUATIONS

We study the toroidal η_i mode in a collisionless plasma consisting of two, or more, ion species. The plasma is confined to a torus with the major radius R, and immersed in a sheared magnetic field $B_0(x)\mathbf{e}_\parallel$, whose shear length is equal to L_s. Keeping only the leading order inhomogeneity, curvature, and magnetic shear effects, the magnitude $B_0(x)$ and the direction $\mathbf{e}_\parallel$ of the magnetic field can be written as:

$$B_0(x) = B_0\left(1 - \frac{x}{R}\right)$$

$$\mathbf{e}_\parallel = \mathbf{e}_z + \frac{x - x_0}{L_s}\,\mathbf{e}_y - \frac{z - z_0}{R}\,\mathbf{e}_x \,. \tag{1}$$

The unperturbed inhomogeneous density $n_{0,j}(x)$ and temperature $T_j(x)$ of the ion species j are given by

$$n_{0,j}(x) = n_{0,j}\left(1 - \frac{x - x_0}{L_{n,j}}\right)$$

$$T_j(x) = T_j\left(1 - \frac{x - x_0}{L_{T,j}}\right) \,. \tag{2}$$

The ions are assumed to be relatively cold, $T_j \ll T_{0,e}$, but the corresponding temperature gradient may be strong

$$\frac{T_j}{T_{0,e}} \leq \frac{L_{T,j}}{L_{n,j}} < 1 \ . \tag{3}$$

There also exists a macroscopic sheared plasma flow, with the zero-order velocity of each ion species given by:

$$\mathbf{V}_{0,j} = c_{s,j}(x - x_0)\left(\frac{\mathbf{e}_z}{L_{\|,j}} + \frac{\mathbf{e}_y}{L_{\perp,j}}\right) \ , \tag{4}$$

where $c_{s,j} = (T_e/m_j)^{1/2}$ is the ion sound speed of the species j.

We study perturbations whose parallel phase velocity is much smaller than the electron thermal velocity and we also neglect all viscosity effects. Then we may describe electrons by the Boltzmann distribution

$$n_e = n_{0,e}(x)\left(1 + \frac{e\phi}{T_e}\right) \ . \tag{5}$$

Electron temperature gradient effects are also neglected, since they appear as higher-order corrections to Eq. (5).

In the regime studied in this paper the ions can be regarded as adiabatic, and thus a closed system of equations describing the plasma state is obtained from the hydrodynamic equations of continuity, parallel momentum (to the magnetic field), and thermal balance, for each ion species. These equations were derived and studied to some details in Refs. 5–8 and 13–15, assuming

$$\frac{\rho_{s,j}}{L} \ll \rho_{s,j}\nabla_\perp \ll 1 \ , \tag{6}$$

where $\rho_{s,j} = (m_j T_e)^{1/2}/(q_j B_0)$ is the ion inertial length, $q_j = Z_j e$ is the charge of the ion species j, and L is some typical scale length from Eqs. (1)–(4). To the leading order in the small parameter $\rho_{s,j}/L$, hydrodynamic equation for the ion species j can be written as,[5,9]

$$\left(\frac{\partial}{\partial t_j} + \mathbf{V}_{\perp,j}\cdot\nabla_j\right)\left(N_j - \nabla^2_{\perp,j}\Phi_j\right) + \partial_{\parallel,j}V_{\parallel,j}$$

$$= \mathbf{e}_z \times \nabla_j \cdot \{[(\mathbf{e}_z \times \nabla_j P_j)\cdot\nabla_j]\,\mathbf{V}_{\perp,j}\} + 2\epsilon_{n,j}\frac{\partial}{\partial y_j}(P_j + \Phi_j) \quad (7)$$

$$\left(\frac{\partial}{\partial t_j} + \mathbf{V}_{\perp,j}\cdot\nabla_j\right)V_{\parallel,j} = -\partial_{\parallel,j}(P_j + \Phi_j) \quad (8)$$

$$\left(\frac{\partial}{\partial t_j} + \mathbf{V}_{\perp,j}\cdot\nabla_j\right)P_j + \Gamma_j\partial_{\parallel,j}V_{\parallel,j}$$

$$= 2\Gamma_j\epsilon_{n,j}\frac{\partial}{\partial y_j}\left[\left(1 - \frac{T_{0,j}}{T_e} - \frac{K_j}{2\epsilon_{n,j}}\right)\Phi_j + 2P_j\right]. \quad (9)$$

In the above equations we used the standard dimensionless drift-wave variables

$$x_j = \frac{x - x_0}{\rho_{s,j}}\ ,\quad y_j = \frac{y}{\rho_{s,j}}\quad ,\quad z_j = \frac{z - z_0}{\rho_{s,j}}\quad ,\quad t_j = \frac{t c_{s,j}}{L_{n,j}}$$

$$\Phi_j = \frac{Z_j e\phi}{T_e}\frac{L_{n,j}}{\rho_{s,j}}\ ,\quad V_{\parallel,j} = \frac{v_{\parallel,j}}{c_{s,j}}\frac{L_{n,j}}{\rho_{s,j}}\ ,\quad P_j = \frac{p_j}{n_{0,j}T_e}\frac{L_{n,j}}{\rho_{s,j}}\ ,\quad N_j = \frac{n_j}{n_{0,j}}\frac{L_{n,j}}{\rho_{s,j}} \quad (10)$$

$\mathbf{V}_{\perp,j}$ is the perpendicular component of the ion hydrodynamic velocity, which in the dimensionless variables, Eq. (10), can be written as

$$\mathbf{V}_{\perp,j} = S_{\perp,j}x_j\mathbf{e}_y + \mathbf{e}_z \times \nabla_j\Phi_j\ , \quad (11)$$

$\partial_{\|,j}$ is the spatial derivative in the direction parallel to the magnetic field

$$\frac{\partial}{\partial_{\|,j}} = \frac{\partial}{\partial z_j} + S_{m,j} x_j \frac{\partial}{\partial y_j} , \tag{12}$$

and we used the following standard notation

$$S_{\perp,j} = \frac{L_{n,j}}{L_{\perp,j}} , \quad S_{\|,j} = \frac{L_{n,j}}{L_{\|,j}} , \quad S_{m,j} = \frac{L_{n,j}}{L_s} , \quad \epsilon_{n,j} = \frac{L_{n,j}}{R}$$

$$\eta_j = \frac{L_{n,j}}{L_{T,j}} , \quad K_j = \frac{T_{0,j}}{T_e}(1+\eta_j) , \quad \Gamma_j = \gamma_j \frac{T_{0,j}}{T_e} , \tag{13}$$

where $\gamma_j = C_{p,j}/C_{v,j}$ is the ratio of specific heats, which for a collisionless plasma has the numerical value $1 < \gamma_j < 3$, determined from the full Vlasov treatment. By introducing this species dependent scalings in Eq. (10) we are able to write the large system of partial differential equations compactly.

It is convenient also to express the dimensionless density N_j, parallel velocity $V_{\|,j}$, and pressure P_j of the ion species j as the sum of the corresponding zero-order and perturbed quantities

$$N_j = -x_j + \delta N_j$$

$$V_{\|,j} = S_{\|,j} x_j + \delta V_j$$

$$P_j = -K_j x_j + \delta P_j . \tag{14}$$

It should be emphasized here that the normalization introduced by Eqs. (10)–(13) is different for each ion species. Thus, in order to combine these equations and to close the system by the requirement of quasineutrality

$$\sum_j Z_j n_j = n_e , \tag{15}$$

it will be necessary to return to dimensional variables. The advantage of the new species-dependent normalization, however, is that for the theoretical, especially nonlinear, analysis, it allows an easy comparison with the existing literature on the ion-temperature-gradient driven modes in single ion species plasma.

Equations (7)–(9) account for all toroidicity, shear, and finite Larmor radius effects to the given order. Thus, two terms on the right-hand side of the continuity equation (7) describe the convection by the combined effects of the diamagnetic drift and the finite Larmor radius, and the convection by the toroidal drift, respectively. Similarly, the right-hand side in the energy balance equation (9) arises from the compressibility due to the cross-field transport.

III. NONLINEAR TRAVELLING SOLUTION AND THE LINEAR DISPERSION RELATION

In this section we proceed with the construction of the general nonlinear travelling solution of the equations (7)–(9), and (15). The linear mode, corresponding to the ion-temperature-gradient driven toroidal drift waves in a multi-ion species plasma, is then readily recovered from the asymptotic form of this general solution.

We seek a solution which is stationary and independent on z in the reference frame moving with the phase velocity $\mathbf{v}_{ph}$, given by

$$\mathbf{v}_{ph} = u\left(\mathbf{e}_y + \alpha^{-1}\mathbf{e}_z\right) \ . \tag{16}$$

In other words, we assume that the solution of equations (7)–(9), and (15) is dependent only on x and $y' = y + \alpha z - ut$. Note that in the linear case we have $u = \omega/k_y$, $\alpha = k_z/k_y$, and thus the phase velocity u becomes complex in the presence of a linear instability.

Using Eq. (16) and the species-dependent normalization introduced in Eq. (10) we can rewrite the perpendicular convective and parallel derivatives as

$$\begin{aligned} \frac{\partial}{\partial t_j} + \mathbf{V}_{\perp,j} \cdot \nabla_j &= \left[\mathbf{e}_z \times \nabla_j \left(\Phi_j - u_j x_j + S_{\perp,j}\,\frac{x_j^2}{2}\right)\right] \cdot \nabla_j \\ \partial_{\|,j} &= \left[\mathbf{e}_z \times \nabla_j \left(\alpha_j x_j + S_{m,j}\,\frac{x_j^2}{2}\right)\right] \cdot \nabla_j \ , \end{aligned} \tag{17}$$

where the dimensionless phase velocity u_j and the pitch angle α_j are given by

$$\begin{aligned} u_j &= L_{n,j}\,\frac{Z_j e B_0}{T_e}\,u \\ \alpha_j &= \frac{L_{n,j}}{\rho_{s,j}}\alpha \ . \end{aligned} \tag{18}$$

Hydrodynamic equations (7)–(9), together with Eq. (18), are very similar to those studied in Ref. 9 for a single ion species plasma, and we proceed with the solution following the method developed there. The compressibility terms in Eq. (9) are difficult to retain without approximation.

In plasmas with not too strong temperature gradients, $K_j < 1$, compressibility terms in Eq. (9) scale as T_j/T_e, and for $T_e > T_j$, these terms can be regarded as perturbations. In the leading order solution we set $\Gamma_j = 0$ and rewrite the energy balance as a complete vector product which can readily be integrated, yielding

$$\Phi_j - u_j x_j = F_j(P_j) \ , \tag{19}$$

where F_j is an arbitrary function, to be determined from boundary conditions. After the substitution of Eq. (19) into Eqs. (8), and (7), we can integrate them too in the same fashion

$$V_{\parallel,j} - \left(1 + \frac{1}{F_j'}\right)\left(\alpha_j x_j + S_{m,j}\,\frac{x_j^2}{2}\right) = G_j(P_j) \tag{20}$$

$$\left[N_j - \Phi_j - \nabla^2_{\perp,j} F_j + (2\epsilon_{n,j} + u_j)\, x_j - S_{\perp,j}\,\frac{x_j^2}{2}\right] F_j' + 2\epsilon_{n,j} x_j$$

$$+ \left(\alpha_j x_j + S_{m,j}\,\frac{x_j^2}{2}\right)\frac{F_j''}{2F_j'^2}$$

$$- \left(\alpha_j x_j + S_{m,j}\,\frac{x_j^2}{2}\right) G_j' - \nabla^2_{\perp,j} F_j + (\nabla_{\perp,j} P_j)\,\frac{F_j''}{2} = H_j(P_j) \ , \tag{21}$$

where G_j, H_j are also arbitrary functions of their respective arguments and $'$, $''$ denote their first and second derivatives.

When the characteristic wavelength of perturbations is small compared to the shear lengths

$$\max\left(S_{m,j}, S_{\perp,j}, S_{\parallel,j}\right) \ll 1 \ , \tag{22}$$

effects of small but finite compressibility can be, to the first order, taken into account by using Eqs. (19)–(21) to express the related terms in the energy balance equation. Proceeding with the same procedure as for $\Gamma_j = 0$, we find that with a finite value of Γ_j Eqs. (19)–(21) retain the same form, only with a small correction in the arguments of the functions F_j, G_j, and H_j

$$F_j = F_j\left(P_j - \delta K_j x_j\right)$$

$$G_j = G_j\left(P_j - \delta K_j x_j\right)$$

$$H_j = H_j\left(P_j - \frac{F_j'}{1+F_j'}\delta K_j x_j\right) , \tag{23}$$

where

$$\delta K_j = -2\Gamma_j \epsilon_{n,j}\left[1 - \frac{T_{0,j}}{T_e} + \frac{1}{F_j'}\left(2 - \frac{\alpha_j}{\epsilon_{n,j}}G_j' + \frac{K_j}{\epsilon_{n,j}}\frac{H_j'}{1+F_j'}\right)\right] . \tag{24}$$

To this end, functions F_j, G_j, H_j remain unknown, but their asymptotic behavior is determined by the requirement that the electrostatic potential and the perturbations of the ion density, velocity, and pressure defined in Eq. (14) remain finite for $x \to \pm\infty$.

First, we study the simple case when all shears are absent

$$S_{m,j} = S_{\perp,j} = S_{\parallel,j} = 0 , \tag{25}$$

when the functions F_j, G_j, H_j are asymptotically linear functions

$$F_j(P_j) = P_j F_{j,1}$$

$$G_j(P_j) = P_j G_{j,1}$$

$$H_j(P_j) = P_j H_{j,1} \ , \tag{26}$$

with the following slopes

$$F_{j,1} = \frac{u_j}{K_j + \delta K_j}$$

$$G_{j,1} = \frac{\alpha_j}{u_j}\left(1 + \frac{u_j}{K_j + \delta K_j}\right)$$

$$H_{j,1} = \frac{1}{K_j + \delta K_j}\left[\frac{u_j}{K_j + \delta K_j}\left(1 - u_j - 2\epsilon_{n,j}\right) - 2\epsilon_{n,j} \right.$$

$$\left. + \frac{\alpha_j^2}{u_j}\left(1 + \frac{u_j}{K_j + \delta K_j}\right)\right] . \tag{27}$$

The linear dispersion relation is now obtained requiring that functions F_j, G_j, H_j have their asymptotic forms, Eq. (26), on the whole x-y plane. Substituting these expressions into Eq. (21), using quasineutrality Eq. (15), and returning to the dimensional variables, we obtain after a somewhat lengthy but straightforward algebra the following dispersion relation

$$\sum_j n_{0,j} Z_j^2 \left(\frac{1}{Z_j} - 1 - \frac{H_{j,1}}{F_{j,1}^2} + \frac{1 + F_{j,1}}{F_{j,1}}\,\rho_{s,j}^2 k_\perp^2\right) = 0 \ , \tag{28}$$

which can be conveniently rewritten as

$$a_0 + a_1 \frac{\omega_{*,i}}{\omega} + a_2 \left(\frac{\omega_{*,i}}{\omega}\right)^2 + a_3 \left(\frac{\omega_{*,i}}{\omega}\right)^3 = 0 \ , \tag{29}$$

where $\omega_{*,i} = k_y v_{*,i}$, with $v_{*,i}$ being the electron diamagnetic drift velocity associated with the inhomogeneity of the ion species i, $v_{*,i} = T_e / \left(Z_i e B_0 L_{n,i}\right)$, and

$$a_0 = \sum_j n_{0,j} Z_j \left(1 + Z_j \rho_{s,j}^2 k_\perp^2\right)$$

$$a_1 = \sum_j n_{0,j} Z_j Z_i \frac{L_{n,i}}{L_{n,j}} \left[-1 + 2\epsilon_{n,j} + (K_j + \delta K_j)\rho_{s,j}^2 k_\perp^2\right]$$

$$a_2 = \sum_j n_{0,j} Z_j^2 \left[2\epsilon_{n,j}(K_j + \delta K_j)\left(\frac{Z_i L_{n,i}}{Z_j L_{n,j}}\right)^2 - \frac{m_i}{m_j}\left(\frac{c_{s,i} k_z}{\omega_{*,i}}\right)^2\right]$$

$$a_3 = -\sum_j n_{0,j} Z_j Z_i \frac{L_{n,i}}{L_{n,j}} \frac{m_i}{m_j} (K_j + \delta K_j) \left(\frac{c_{s,i} k_z}{\omega_{*,i}}\right)^2 . \tag{30}$$

It should be noted that in the presence of compressibility effects in the energy balance equation (9), coefficients a_n, $n = 0, 1, 2, 3$, in the linear dispersion relation, Eq. (29), are weakly ω, k_z dependent due to the presence of the small terms δK_j

$$\delta K_j = 2\Gamma_j \left[-\epsilon_{n,j}\left(1 - \frac{T_{0,j}}{T_e}\right) + K_j \left(1 - \frac{K_j + 1}{K_j + \dfrac{Z_j L_{n,j}}{Z_i L_{n,i}} \dfrac{\omega}{\omega_{*,i}}}\right) + \frac{Z_j L_{n,j}}{Z_i L_{n,i}} \frac{m_i}{m_j} \frac{c_{s,i}^2 k_z^2}{\omega_{*,i}\omega}\right] . \tag{31}$$

In a single species plasma our dispersion relation fully recovers earlier results for the η_i mode. Thus, in the limit $k_z = 0$, and with the use of $K \ll 1$, we obtain the usual dispersion relation for the toroidal η_i mode

$$1 + \rho_s^2 k_\perp^2 - \left(1 - 2\epsilon_n - K\rho_s^2 k_\perp^2\right) \frac{\omega_*}{\omega} + 2\epsilon_n K \left(\frac{\omega_*}{\omega}\right)^2$$

$$= -2\Gamma \left[-\epsilon_n \left(1 - \frac{T_e}{T_i}\right) + K \left(1 - \frac{K + 1}{K + \dfrac{\omega}{\omega_*}}\right)\right] \left(\rho_s^2 k_\perp^2 + 2\epsilon_n \frac{\omega_*}{\omega}\right) \frac{\omega_*}{\omega}, \tag{32}$$

which coincides with Eq. (8) from Ref. 6 for $\rho_s^2 k_\perp^2 \ll \min(1, \epsilon_n)$.

Likewise, in the limit $\epsilon_n = 0$ we obtain from Eq. (29) the slab η_i mode, which is destabilized by the coupling with ion acoustic waves, and described by

$$1 + \rho_s^2 k_\perp^2 - \left(1 - K\rho_s^2 k_\perp^2\right)\frac{\omega_*}{\omega} - \left(1 + K\frac{\omega_*}{\omega}\right)\left(\frac{c_s k_z}{\omega}\right)^2$$
$$= -2\Gamma\left(\frac{c_s k_z}{\omega}\right)^2\left[\rho_s^2 k_\perp^2 - \left(\frac{c_s k_z}{\omega}\right)^2\right] . \qquad (33)$$

For $K \ll 1$ and $\rho_s^2 k_\perp^2 \ll \min(1, c_s^2 k_z^2/\omega^2)$ this expression coincides with Eq. (86) from Ref. 5. The relation of these results with those obtained by the Swedish group from the Chalmers University are discussed in some detail in Ref. 16. Similarly, in multispecies plasmas our results are in agreement with Ref. 17.

In a plasma consisting only of main and impurity ions, denoted by i and I, respectively, it is convenient to rewrite Eq. (30) in terms of the following parameters

$$\beta = \frac{n_{0,I}}{n_{0,e}} , \ \sigma = \frac{L_{n,i}}{L_{n,I}} , \ \mu = \frac{m_I}{m_i} , \ p = \sigma\frac{K_I}{K_i} , \ \lambda_i = \frac{L_{n,i}}{L_{\parallel,i}} , \ \lambda_I = \frac{L_{n,i}}{L_{\parallel,I}} . \qquad (34)$$

Assuming that the main ion species is singly charged hydrogen, and using the quasineutrality of the unperturbed state, yielding

$$\frac{n_{0,i}}{n_{0,e}} = 1 - \beta Z_I$$
$$\frac{L_{n,i}}{L_{n,e}} = 1 + \beta Z_I(\sigma - 1) , \qquad (35)$$

and completely neglecting compressibility effects, $\Gamma_j = \delta K_j = 0$, we can readily write

$$a_0 = 1 + \rho_{s,i}^2 k_\perp^2 \left[1 + \beta(\mu - Z_I)\right]$$

$$a_1 = -1 + 2\epsilon_{n,i} + \rho_{s,i}^2 k_\perp^2 K_i + \beta Z_I \left[1 - \sigma + \rho_{s,i}^2 k_\perp^2 K_i \left(\frac{\mu p}{Z_I^2} - 1\right)\right]$$

$$a_2 = 2\epsilon_{n,i} K_i + \frac{c_{s,i} k_z}{\omega_{*,i}} \lambda_i - \left(\frac{c_{s,i} k_z}{\omega_{*,i}}\right)^2$$

$$+\beta Z_I \left[2\epsilon_{n,i} K_i \left(\frac{p}{Z_I} - 1\right) + \frac{c_{s,i} k_z}{\omega_{*,i}} \left(\frac{\lambda_I}{\mu^{1/2}} - \lambda_i\right) - \left(\frac{c_{s,i} k_z}{\omega_{*,i}}\right)^2 \left(\frac{Z_I}{\mu} - 1\right)\right]$$

$$a_3 = -\left(\frac{c_{s,i} k_z}{\omega_{*,i}}\right)^2 \left[1 + \beta Z_I \left(\frac{p}{\mu} - 1\right)\right] K_i \ . \tag{36}$$

As mentioned above, in the case of a negligible compressibility the linear dispersion relation, Eq. (30), simplifies to a cubic algebraic equation. It has a pair of complex-conjugate roots (corresponding to the presence of a linearly unstable mode) if

$$Q^3 - R^2 < 0 \ , \tag{37}$$

where

$$Q = -\frac{a_1}{3a_3} + \left(\frac{a_2}{3a_3}\right)^2$$

$$R = -\frac{a_0}{2a_3} + \frac{3}{2}\frac{a_1 a_2}{(3a_3)^2} - \left(\frac{a_2}{3a_3}\right)^3 \ . \tag{38}$$

These expressions allow us to study analytically the destabilizing influence of impurities on the linear η_i mode in the small β limit, when the instability condition Eq. (37) can be rewritten as

$$3\frac{\delta Q}{Q_0} < 2\frac{\delta R}{R_0} \, . \tag{39}$$

Here Q_0, R_0 are obtained from Eq. (38) in the limit $\beta = 0$ and at the marginal stability for the η_i mode, $Q_0^3 = R_0^2$, while δQ, δR are the corresponding perturbations arising from the small impurity component.

In the limit $k_z \to 0$, i.e. for a purely toroidal η_i mode described by Eq. (32), and for $\rho_{s,i}^2 k_\perp^2 \to 0$, we find from Eq. (39) that the destabilization by impurities occurs when

$$2\sigma \left[Z_I \left(1 - 2\epsilon_{n,i}\right) - 4\epsilon_{n,i} K_i \right] < Z_I \left(1 - 2\epsilon_{n,i}\right) \left(1 + 2\epsilon_{n,i}\right) \, . \tag{40}$$

For a typical value of the parameters

$$\epsilon_{n,i} < \frac{1}{2} \, , \quad K_I < Z_I \frac{1 - 2\epsilon_{n,i}}{4\epsilon_{n,i}} \, , \quad L_{n,i} > 0 \, ,$$

inequality (40) corresponds to

$$\frac{1}{L_{n,I}} < \frac{1 + 2\epsilon_{n,i}}{2L_{n,i}} \left[1 - \frac{4\epsilon_{n,i} K_I}{Z_I (1 - 2\epsilon_{n,i})} \right]^{-1} \, . \tag{41}$$

In other words, in the limit $c_s k_z / \omega_{*,i} \ll 1$, $\rho_{s,i}^2 k_\perp^2 \ll 1$, the destabilization of the toroidal η_i mode occurs if the density gradient of impurities is either smaller, or oppositely oriented from that of the main ions ($\sigma < 0$). However, in large tokamak machines the impurity density profile is typically close to the density profile of the main ion species, and consequently the presence of impurities will have a stabilizing effect on drift waves, as shown in Fig. 1.

Likewise, under the conditions when the η_i instability of the main ions arises solely from the coupling with sound waves, Eq. (33), i.e. in the slab limit $\epsilon_{n,i} = 0$, with $\rho_{s,i}^2 k_\perp^2 = 0$, and adopting for simplicity $c_s k_z / \omega_{*,i} \ll 1$, we find that the destabilization by impurities occurs when

$$\sigma + \frac{p - 2Z_I}{\mu} > 0 \ ,$$

which is equivalent to

$$\frac{L_{n,i}}{L_{n,I}} \left(\frac{m_I}{m_i} + \frac{T_{0,I}}{T_{0,i}} \frac{1+\eta_I}{1+\eta_i} \right) > 2Z_I \ . \tag{42}$$

Consequently, a small amount of heavy impurity ions contributes to the slab-type η_i instability even if the density profiles of the impurity and main ions are close, $L_{n,i} \sim L_{n,I}$. For example, titanium with $Z_I = 22$ and $m_I/m_i = 48$ gives $L_{n,i}/L_{n,I} > 2(22)/48$ from Eq. (42) for a destabilizing effect.

However, detailed numerical calculations, presented in Fig. 2, indicate that the destabilization by impurities is of a rather limited extend (note only a small shift of the boundary of the stable region in Fig. 2), and only for a small impurity content, $\beta \leq 0.1$. For a larger amount of impurities the slab η_i mode is stabilized, similarly to the toroidal case. As pointed out by Dong *et al.*[18] in the case of a pure "impurity plasma" where $\beta = 1/Z_I$ the ITG and η_i mode physics is exactly the same as the pure hydrogenic plasma.

IV. NONLINEAR VORTEX SOLUTIONS

It has been shown in Ref. 9 that in the nonlinear regime of ion-pressure-gradient driven toroidal drift-wave fluctuations two types of solitary vortices are possible, and the same is true for multiple ion species plasmas.

In the strong magnetic shear case, similarly to Ref. 9 we proceed analytically, adopting

$$S_{\perp,j} = S_{\parallel,j} = \Gamma_j = \alpha_j = 0 \ . \tag{43}$$

In this case, the functions F, G, H are asymptotically polynomials of the first, second, and third degree, respectively

$$\begin{aligned} F_j(P_j) &= P_j F_{j,1} \\ G_j(P_j) &= P_j G_{j,1} + P_j^2 G_{j,2} \\ H_j(P_j) &= P_j H_{j,1} + P_j^2 H_{j,2} + P_j^3 H_{j,3} \ , \end{aligned} \tag{44}$$

where

$$\begin{aligned} F_{j,1} &= \frac{u_j}{K_j} \\ G_{j,1} &= \frac{\alpha_j}{u_j}\left(1 + \frac{u_j}{K_j}\right) \\ G_{j,2} &= -\frac{S_{m,j}}{2u_j K_j}\left(1 + \frac{u_j}{K_j}\right) \\ H_{j,1} &= \frac{1}{K_j}\left[\frac{u_j}{K_j}(1 - u_j - 2\epsilon_{n,j}) - 2\epsilon_{n,j} + \alpha_j G_{j,1}\right] \end{aligned}$$

$$H_{j,2} = -\frac{S_{m,j} G_{j,1}}{2K_j^2} + \frac{2\alpha_j G_{j,2}}{K_j}$$

$$H_{j,3} = -\frac{S_{m,j} G_{j,2}}{K_j^2} \ . \tag{45}$$

Same as before, we require that the functions F, G, H have their asymptotic form, Eqs. (44) and (45), on the whole x-y plane, which permits us to rewrite the nonlinear wave equations (19)–(21) as

$$\left[\nabla_\perp^2 - \kappa^2 + f(\psi)\right] \psi = 0 \ , \tag{46}$$

where $\psi = (c_{s,i}/u)(e\phi/T_e)$, and

$$\kappa^2 = \frac{1}{D\rho_{s,i}^2} \left\{ 1 + \frac{\rho_{s,i} c_{s,i}}{L_{n,i} u} \sum_j \left[2\epsilon_{n,i} \left(1 + \frac{K_j}{u_j}\right) - \frac{L_{n,i}}{L_{n,j}} \right] \frac{n_{0,i}}{n_{0,e}} Z_j \right\}$$

$$f(\psi) = \frac{S_{m,\mathrm{eff}}}{2\rho_{s,i}^2} \frac{c_{s,i}^2}{u^2} \left(\psi^2 - 3\psi \frac{x}{\rho_{s,i}} + 2 \frac{x^2}{\rho_{s,i}^2} \right)$$

$$S_{m,\mathrm{eff}} = \frac{\rho_{s,i}^2}{DL_s^2} \sum_j \left(1 + \frac{K_j}{u_j}\right) \frac{m_i}{m_j} \frac{n_{0,j}}{n_{0,e}} Z_j^2$$

$$D = \sum_j \left(1 + \frac{K_j}{u_j}\right) \frac{m_j}{m_i} \frac{n_{0,j}}{n_{0,e}} \ . \tag{47}$$

Alternatively, we can rewrite Eq. (46) in the Hamiltonian form as

$$\left(\nabla_\perp^2 - \kappa^2\right) \psi = -\frac{\partial}{\partial \psi} U_{\mathrm{eff}}(\psi, x) \ , \tag{48}$$

where the effective potential U_{eff} is given by

$$U_{\mathrm{eff}}(\psi, x) = \frac{S_{m,\mathrm{eff}}}{2\rho_{s,i}^2} \frac{c_{s,i}^2}{u^2} \left(\frac{\psi^4}{4} - \psi^3 \frac{x}{\rho_{s,i}} + \psi \frac{x^2}{\rho_{s,i}^2} \right) - \frac{\kappa^2}{2} \psi^2 \ . \tag{49}$$

Our equation (48) is the same as Eq. (53) in Ref. 9, and consequently in our case of multi-ion-species plasma there exists an identical soliton-like solution as that described in Sec. IV of Ref. 9. This solution is an odd function of x, $\psi(x) = -\psi(-x)$, while in the y direction potential ψ is periodic. Physically, this corresponds to a nonlinear vortex chain. This odd parity vortex chain solution $\psi(x)$ is also associated with the finite amplitude drift-tearing mode which is an even parity parallel vector potential $A_{\|}(x)$ part of the solution driven by the parallel electron currents. The localization in the x-direction is possible if the potential ψ is large enough to change the sign of the effective square e-folding length in the neighborhood of $x = 0$

$$\kappa_{\text{eff}}^2 \equiv \kappa^2 - f(\psi) < 0 \quad , \quad |x| < x_c \qquad \text{and} \qquad \kappa_{\text{eff}}^2 > 0 \quad , \quad |x| > x_c \ .$$

Alternatively, the emergence of the nonlinear localization may be interpreted as the consequence of the trapping by the potential U_{eff}, which contains a positive definite term proportional to ψ^4. For more details on this soliton-chain solution we refer the reader to Refs. 9 and 13.

The second type vortex solution is obtained in the limit of a weak shear. This is essentially the usual Larichev-Reznik dipole, slightly perturbed by the shear.

In the weak shear limit, Eq. (22), we may allow for finite compressibility terms in the energy equations, and then the functions F, G, H are given by Eq. (23). Using the requirement that for $|x| \to \infty$ we have

$$\Phi_j \sim \delta N_j \sim \delta P_j \sim V_{\|,j} \to 0 \ , \tag{50}$$

we asymptotically have

$$F_j(\xi) = \xi F_{j,1} + \xi^2 F_{j,2}$$

$$G_j(\xi) = \xi G_{j,1} + \xi^2 G_{j,2}$$

$$H_j^{\rm out}(\xi) = \xi H_{j,1}^{\rm out} + \xi^2 H_{j,2}^{\rm out} \ , \qquad (51)$$

with the coefficients $F_{j,n}$, $G_{j,n}$, $H_{j,n}$ being given by

$$F_{j,1} = \frac{u_j}{K_j + \delta K_j}$$

$$F_{j,2} = \frac{S_{\perp,j}}{2K_j^2}$$

$$G_{j,1} = -\frac{S_{\parallel,j}}{K_j + \delta K_j} + \frac{\alpha_j}{u_j}\left(1 + \frac{u_j}{K_j + \delta K_j}\right)$$

$$G_{j,2}(\xi) = \frac{-1}{2K_j^2(F_{j,1} + 2\xi F_{j,2})}\left[S_{m,j}\left(1 + F_{j,1} + 2\xi F_{j,2}\right) + 2S_{\perp,j}\,\frac{\alpha_j}{u_j}\right]$$

$$H_{j,1}^{\rm out} = \frac{1}{K_j + \delta K_j}\left[-2\epsilon_{n,j}\left(1 + F_{j,1}\right) + F_{j,1}(1 - u_j) + G_{j,1}\alpha_j\right]$$

$$H_{j,2}^{\rm out}(\xi) = \frac{1}{2K_j^2}\Bigg[- F_{j,1}S_{\perp,j} - G_{j,1}S_{m,1} - 4K_jF_{j,2}\left(2\epsilon_{n,j} - 1 + u_j\right)$$

$$+4K_j\alpha_jG_{j,2}(\xi)\Bigg] + \frac{F_{j,2}}{K_j^2\left(F_{j,1} + 2\xi F_{j,2}\right)^2} \ . \qquad (52)$$

In the derivation of Eq. (52) we dropped all terms proportional to $S_j\Gamma_j$, S_j^2, and higher.

In the usual Larichev-Reznik scenario of vortex construction, functions

F_j, G_j, H_j are adopted in their asymptotic forms, Eq. (51), on the whole x-y plane except in a limited region called vortex core, in which the coefficients $F_{j,n}$, $G_{j,n}$, $H_{j,n}$ take different values. Naturally, functions F_j, G_j, H_j are required to be continuous on the whole x-y plane, and thus the edge of the modon core must be an equiline for all these functions. However, as they have different arguments, see Eq. (23), only one of these functions may be allowed to have discontinuous coefficients, while all the others will keep the same analytic form on the entire x-y plane.

We adopt the vortex core to be a circle with the radius r_0, centered at $x = 0$, $y = 0$, assuming the size of the core to be small compared with the shear lengths, $S_j r_0 \ll K_j$. All the functions F_j, G_j', H_j are adopted in their asymptotic form, Eq. (51), except one of them, e.g. that corresponding to the species j, which is within the core, $r < r_0$ adopted as

$$H_j^{\rm in}(\xi) = \xi H_{j,1}^{\rm in} + \xi^2 H_{j,2}^{\rm in} \ , \tag{53}$$

with $H_{j,0}^{\rm in}, H_{j,2}^{\rm in} \ll H_{j,1}^{\rm in}$.

An approximate vortex solution is obtained from Eqs. (19)–(21), with the use of Eqs. (23), and (51)–(53) after the expansion in P_j. At distances which are small compared to the shear length we may drop the terms of the third and higher orders. Returning to the dimensional variables, using the condition of quasineutrality, Eq. (15), and following the same procedure as that developed in Ref. 9, we obtain the following nonlinear wave equation

$$\left(\nabla_\perp^2 - \kappa^2\right)\left(\frac{e\phi}{T_e} - w\,\frac{x}{\rho_{s,i}}\right) = \mathcal{R}\ , \tag{54}$$

where

$$\kappa^2 = \frac{1}{D}\sum_j Z_j^2 n_{0,j}\left(\frac{1+F_{j,1}}{F_{j,1}}\,\kappa_j^2 + \frac{1}{Z_j} - 1\right)$$

$$w = \frac{1}{D\kappa^2}\,\frac{u}{c_{s,i}}\sum_j Z_j^2 n_{0,j}\left(\frac{1+F_{j,1}}{F_{j,1}}\,\kappa_j^2 + \frac{1}{Z_j} - 1\right) w_j$$

$$D = \sum_j Z_j^2 n_{0,j}\frac{1+F_{j,1}}{F_{j,1}}\rho_{s,j}^2\ , \tag{55}$$

with the species-related parameters κ_j, w_j having different values inside and outside the vortex core

$$\kappa_j^{\text{out}} = \frac{-H_j^{\text{out}}}{F_{j,1}\left(1+F_{j,1}\right)} = \frac{u_j - 1}{u_j + K_j + \delta K_j} + \frac{2\epsilon_{n,j}}{u_j} + \frac{\alpha_j S_{\parallel,j}}{u_j\left(u_j + K_j\right)} - \frac{\alpha_j^2}{u_j^2}$$

$$\kappa_j^{\text{in}} = \frac{-H_j^{\text{in}}}{F_{j,1}\left(1+F_{j,1}\right)}$$

$$w_j^{\text{out}} = 0$$

$$w_j^{\text{in}} = \left(1 - \frac{\delta K_j}{u_j + K_j}\right)\left(1 - \frac{\kappa_j^{\text{out}2}}{\kappa_j^{\text{in}2}}\right)\ . \tag{56}$$

The right-hand side term $\mathcal{R}$ in Eq. (54), related to the nonlinearities and shears, is extremely lengthy and we give it only in the Appendix.

We proceed with the construction of the vortex, treating the driving term $\mathcal{R}$ as a small perturbation. A spatially localized solution is obtained if the square e-folding length κ^2 has different signs inside and outside the core

$$\left(\kappa^{\mathrm{out}}\right)^2 > 0 \,, \qquad\qquad \left(\kappa^{\mathrm{in}}\right)^2 < 0 \,. \tag{57}$$

The leading order solution ϕ_1 is the usual Hasegawa-Mima dipole

$$\phi_1 = w^{\mathrm{in}} \cos\theta \frac{T_e}{e} \frac{r_0}{\rho_{s,i}} \frac{\kappa^{\mathrm{in}2}}{\kappa^{\mathrm{in}2} - \kappa^{\mathrm{out}2}} \begin{cases} \dfrac{r}{r_0}\left(1 - \dfrac{\kappa^{\mathrm{out}2}}{\kappa^{\mathrm{in}2}}\right) + \dfrac{\kappa^{\mathrm{out}2}}{\kappa^{\mathrm{in}2}} \dfrac{J_1(i\kappa^{\mathrm{in}}r}{J_1(i\kappa^{\mathrm{in}}r_0)} \,, & r<r_0 \\ \dfrac{K_1(\kappa^{\mathrm{out}}r)}{K_1(\kappa^{\mathrm{out}}r_0)} \,, & r>r_0 \end{cases} \tag{58}$$

where $r = (x^2 + y^2)^{1/2}$, $\theta = \arctan y/x$, and J_1 and K_1 are the Bessel functions of the first order. The wavenumber κ^{in} satisfies the following nonlinear dispersion relation

$$\kappa^{\mathrm{out}} \frac{K_1(\kappa^{\mathrm{out}}r_0)}{K_2(\kappa^{\mathrm{out}}r_0)} = -i\kappa^{\mathrm{in}} \frac{J_1(i\kappa^{\mathrm{in}}r_0)}{J_2\left(i\kappa^{\mathrm{in}}r_0\right)} \,. \tag{59}$$

Since the first-order solution ϕ_1 is a dipole, both the driving term $\mathcal{R}$ in Eqs. (54) and (a1), and also the second order solution $\delta\phi_1$, have only cylindrically symmetric (monopole) and quadrupole parts

$$\mathcal{R} = \mathcal{R}_0(r) + \mathcal{R}_2(r)\cos 2\theta$$

$$\delta\phi = \delta\phi_0(r) + \delta\phi_2(r)\cos 2\theta \,. \tag{60}$$

Perturbed equation (54) can be easily integrated using the Cauchy method of the variation of constants, yielding

$$\delta\phi_n = \frac{T_e}{e} \begin{cases} J_n(i\kappa^{\mathrm{in}}r)\left[a_n + \mathcal{Y}_n(r)\right] + Y_n\left(i\kappa^{\mathrm{in}}r\right)\mathcal{J}_n(r), & r < r_0 \\ K_n\left(\kappa^{\mathrm{out}}r\right)\left[b_n + \mathcal{I}_n(r)\right] + I_n\left(\kappa^{\mathrm{out}}r\right)\mathcal{K}_n(r) \,, & r > r_0 \end{cases} \qquad n = 0, 2 \tag{61}$$

where J_1, Y_1, and I_1, K_1 are respectively the ordinary and modified Bessel functions of the n-th order, a_n, b_n are constants of integration, and

$$\mathcal{Y}_n(r) = -\frac{\pi}{2}\int_{r_0}^{r} dr' r' Y_n\left(i\kappa^{\rm in} r'\right)\mathcal{R}_n(r')$$

$$\mathcal{J}_n(r) = \frac{\pi}{2}\int_{0}^{r} dr' r' J_n\left(i\kappa^{\rm in} r'\right)\mathcal{R}_n(r')$$

$$\mathcal{I}_n(r) = -\int_{r_0}^{r} dr' r' I_n\left(\kappa^{\rm out} r'\right)\mathcal{R}_n(r')$$

$$\mathcal{K}_n(r) = \int_{\infty}^{r} dr' r' K_n\left(\kappa^{\rm out} r'\right)\mathcal{R}_n(r') \quad , \quad n = 0, 2 \ . \qquad (62)$$

Obviously, the solution given by Eq. (61) is finite at $x = 0, \infty$, while the continuity at $r = r_0$ yields

$$a_n = \frac{1}{J_n\left(i\kappa^{\rm in} r_0\right)}\left[\frac{e}{T_e}\,\delta\phi_n(r_0) - Y_n\left(i\kappa^{\rm in} r_0\right)\mathcal{J}_n(r_0)\right]$$

$$b_n = \frac{1}{K_n\left(\kappa^{\rm out} r_0\right)}\left[\frac{e}{T_e}\,\delta\phi_n(r_0) - I_n\left(\kappa^{\rm out} r_0\right)\mathcal{K}_n(r_0)\right] \quad , \quad n = 0, 2 \ . \qquad (63)$$

As we have adopted the vortex core to be circular, with the radius r_0, centered at $x = 0$, $y = 0$, the continuity of the function H_j yields

$$\delta\phi_0(r_0) = \frac{H_{j,0}^{\rm in}}{H_{j,1}^{\rm out} - H_{j,1}^{\rm in}}\,\frac{\rho_{s,j}}{K_j}\,B_0 u$$

$$\delta\phi_2(r_0) = 0 \ . \qquad (64)$$

Finally, from the continuity of $\nabla\delta\phi_n$ at $r = r_0$, which for our choice of parameters corresponds to $(\partial/\partial r_0)\delta\phi_n(r_0)$, $n = 0, 2$, we obtain the following equations from which the remaining constants $H_{j,0}^{\rm in}$, $H_{j,2}^{\rm in}$ can be determined

$$\frac{\mathcal{K}_0(r_0)}{K_0\left(\kappa^{\rm out}r_0\right)}-\frac{\pi}{2}\frac{\mathcal{J}_0(r_0)}{J_0\left(i\kappa^{\rm in}r_0\right)}$$

$$=\frac{e}{T_e}\,\delta\phi_0(r_0)\left[\frac{\kappa^{\rm out}K_1\left(\kappa^{\rm out}r_0\right)}{K_0\left(\kappa^{\rm out}r_0\right)}-\frac{i\kappa^{\rm in}J_1\left(i\kappa^{\rm in}r_0\right)}{J_0\left(i\kappa^{\rm in}r_0\right)}\right] \qquad (65)$$

$$\frac{\mathcal{K}_2(r_0)}{K_2\left(\kappa^{\rm out}r_0\right)}-\frac{\pi}{2}\frac{\mathcal{J}_2(r_0)}{J_2\left(i\kappa^{\rm in}r_0\right)}=0\,. \qquad (66)$$

It is worth noting that in a multi-ion-species plasma the vortices of the Larichev-Reznik type are produced by the nonlinearities associated with only one ion species, since only one of the H functions is permitted to have different analytic expressions inside and outside the vortex core. This is a consequence of the compressibility terms in the energy balance equation which introduce small corrections $\delta K_j x_j$ in the argument of the function H_j, which are different for each ion species.

V. CONCLUSIONS

In this work we have investigated the influence of heavy, multiply charged impurity ions on both the linear and nonlinear ion-temperature-gradient driven drift modes. The model equations which were used include the effects of the ion temperature inhomogeneity, magnetic field shear and inhomogeneity, shear flow, finite ion Larmor radius, ion acoustic, and toroidal effects.

In the linear regime, it is shown that a small amount of impurities has a stabilizing effect on the toroidal η_i mode, except under relatively rare conditions of plasma self-decontamination (i.e. when the density profile of impurities is peaked at the plasma edge). For the slab η_i mode we found a weak destabilization by a small amount of impurities, but a larger impurity content ($\geq 10\%$) had a stabilizing influence.

In the nonlinear regime we found the same nonlinear structures as those existing in single ion species plasmas.[9] In the weak shear limit

$$1 \gg \left(\frac{S_\perp}{u} \, , \; \alpha \frac{S_m}{u^2 \rho_s^2} \right) \phi_{\max} \gg \left(\frac{S_m^2}{u^2 \rho_s} \right)^2 \phi_{\max}^2 \; ,$$

the usual Hasegawa-Mima dipole is found, with a small amplitude monopole and quadrupole superimposed on the modon. There exists a family of modons, each of them being associated with the nonlinearity in the energy balance equation of a different ion species.

In the strong magnetic shear limit

$$\frac{S_m^2}{u^2 \rho_s} \phi_{\max} \sim 1 \; ,$$

we demonstrated the existence of a periodic solution, arising from the self-trapping of the electrostatic potential in the nonlinear potential well. This solution is described by a real cubic nonlinear Schrödinger equation, with the leading order nonlinearity being determined by the magnetic shear. This odd parity electrostatic island change solution is the low plasma pressure limit of the finite amplitude.

VI. APPENDIX

The nonlinear term in Eq. (54) is obtained as the sum of nonlinear terms from the continuity equations for each ion species, and it can be written as

$$\mathcal{R} = \frac{1}{D}\sum_j Z_j n_{0,j}\frac{\rho_{s,j}}{L_{n,j}}\frac{1+F_{j,1}}{F_{j,1}}\mathcal{R}_j, \tag{A1}$$

where D is given by Eq. (55) and

$$\mathcal{R}_j = Z_j L_{n,j}^2\left(F_{j,1}\mathcal{P}_{j,1} + \frac{F_{j,2}}{F_{j,1}^2}\left\{\left(\nabla_\perp^2 - \frac{\kappa_j^2}{\rho_{s,i}^2}\right)\left[\left(\frac{e\phi}{T_e}\right)^2 - 2\frac{e\phi}{T_e}\frac{ux}{c_{s,i}\rho_{s,i}}\right]\right.\right.$$
$$\left.\left. - \frac{2F_{j,1}}{1+F_{j,1}}\left(\frac{e\phi}{T_e} - \frac{ux}{c_{s,i}\rho_{s,i}}\right)\left(\nabla_\perp^2 - \frac{\kappa_j^2}{\rho_{s,i}^2}\right)\left(\frac{e\phi}{T_e} - w_j\frac{ux}{c_{s,i}\rho_{s,i}}\right)\right\}\right). \tag{A2}$$

Here $\mathcal{P}_j$ is the usual nonlinear term in the continuity equation in single ion-species plasmas (see Ref. 9), but expressed in terms of the pressure P_j, rather than the potential ϕ

$$\mathcal{P}_j = \frac{-1}{F_{j,1}^3(1+F_{j,1})}\left\{\left(\frac{1}{2}+F_{j,1}\right)F_{j,2}\left[\nabla_\perp^2\left(\frac{e\phi}{T_e} - \frac{ux}{c_{s,i}\rho_{s,i}}\right)^2\right.\right.$$
$$\left. +2\left(\frac{e\phi}{T_e} - w_j\frac{ux}{c_{s,i}\rho_{s,i}}\right)\nabla_\perp^2\frac{e\phi}{T_e}\right] + \frac{H_{j,2}}{\rho_{s,j}^2}\left(\frac{e\phi}{T_e} - \frac{ux}{c_{s,i}\rho_{s,i}}\right)^2$$
$$\left. - \frac{g_{j,1}}{\rho_{s,j}^2 K_j}\frac{ux}{c_{s,i}\rho_{s,i}}\left(\frac{e\phi}{T_e} - \frac{ux}{c_{s,i}\,\rho_{s,i}}\right) - \frac{g_{j,2}}{\rho_{s,j}^2 K_j^2}\left(\frac{ux}{c_{s,i}\,\rho_{s,i}}\right)^2 + \frac{F_{j,1}^2 H_{j,0}}{Z_j^2\rho_{s,j}^2}\right\}$$

$$g_{j,1} = 2\left[F_{j,2}\left(2\epsilon_{n,j} - 1 + u_j\right) - \alpha_j G_{j,2}(0)\right]$$

$$g_{j,2} = \frac{1}{2}\left[F_{j,1}S_{\perp,j} + G_{j,1}S_{\perp,j} + 2\alpha_j^2\frac{F_{j,2}}{F_{j,1}^2}\right]. \tag{A3}$$

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FIGURE CAPTIONS

Fig. 1. Boundary between the stable and unstable regions for the purely toroidal η_i mode, with $s = 0$, in a hydrogen plasma with a small amount of multiply charged carbon ions. The main and impurity ions are assumed to have the same temperature and density profiles, which corresponds to $\sigma = 1$, $p = 1$, $K_i = K_I \equiv K$, $\epsilon_{n,i} = \epsilon_{n,I} \equiv \epsilon$, and the impurity content ranges from $0 - 20\%$. The parallel velocity shear is absent, $\lambda_i = \lambda_I = 0$, and the other plasma parameters are adopted as $r = 0.1$, $m = 12$, $Z = 5$.

Fig. 2. Boundary between the stable and unstable regions for the slab η_i mode, with $\epsilon = 0$. All the other parameters are the same as in Fig. 1.

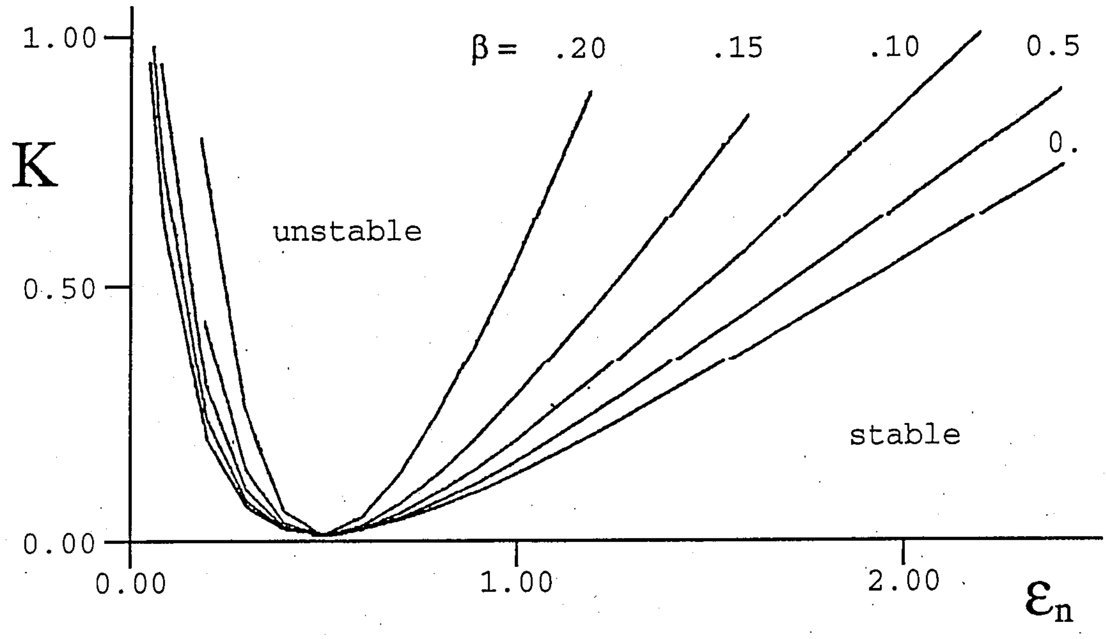

Figure 1

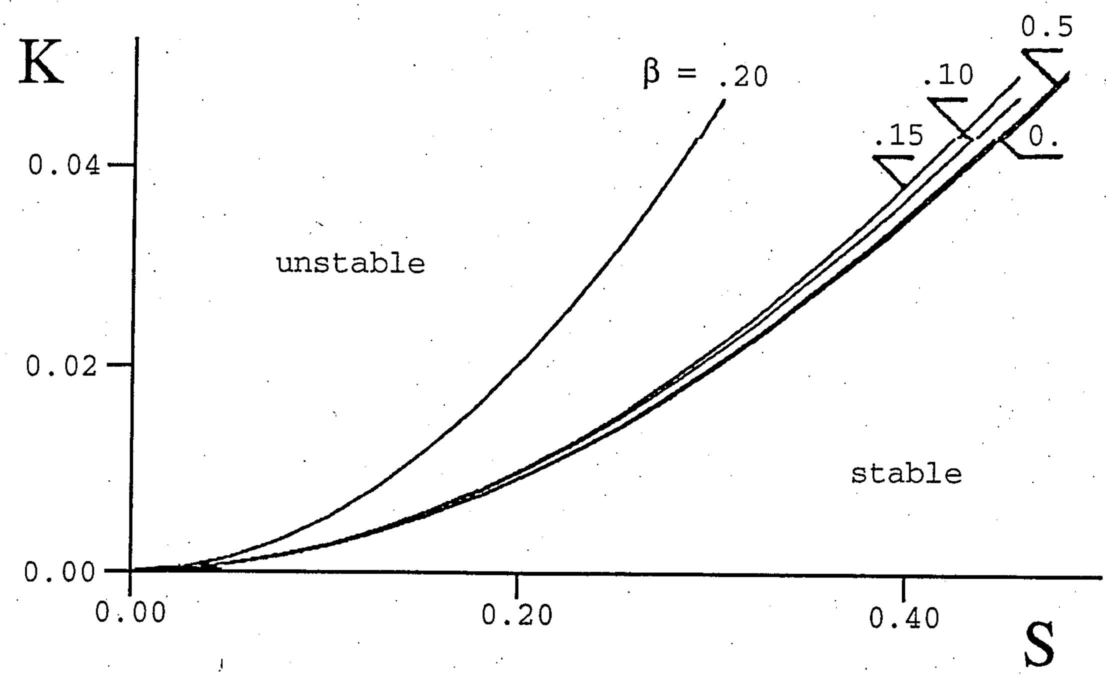

Figure 2

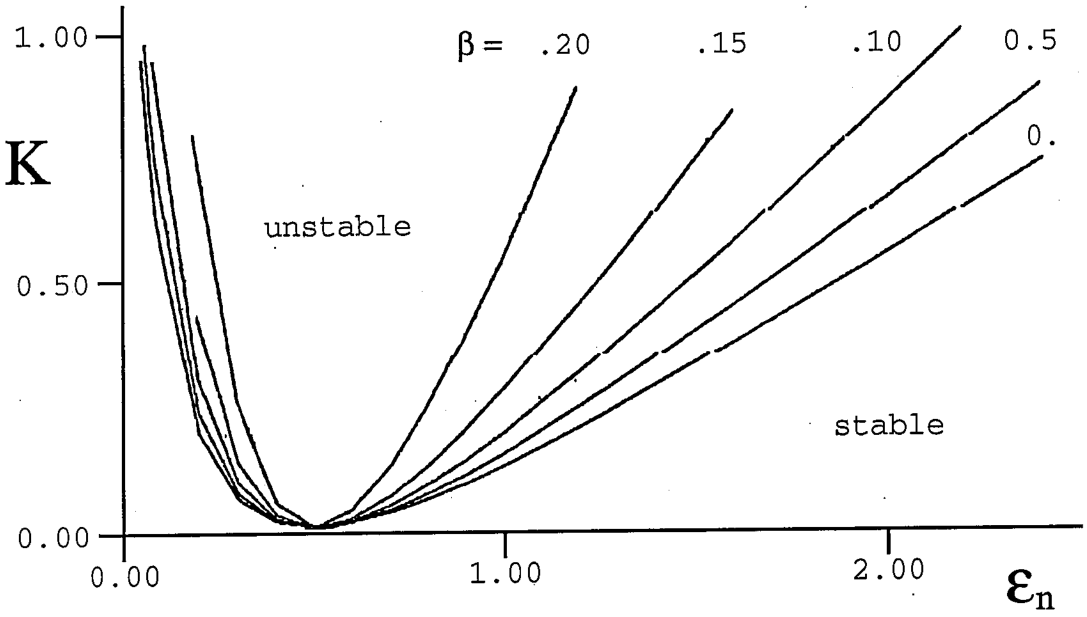

Figure 1

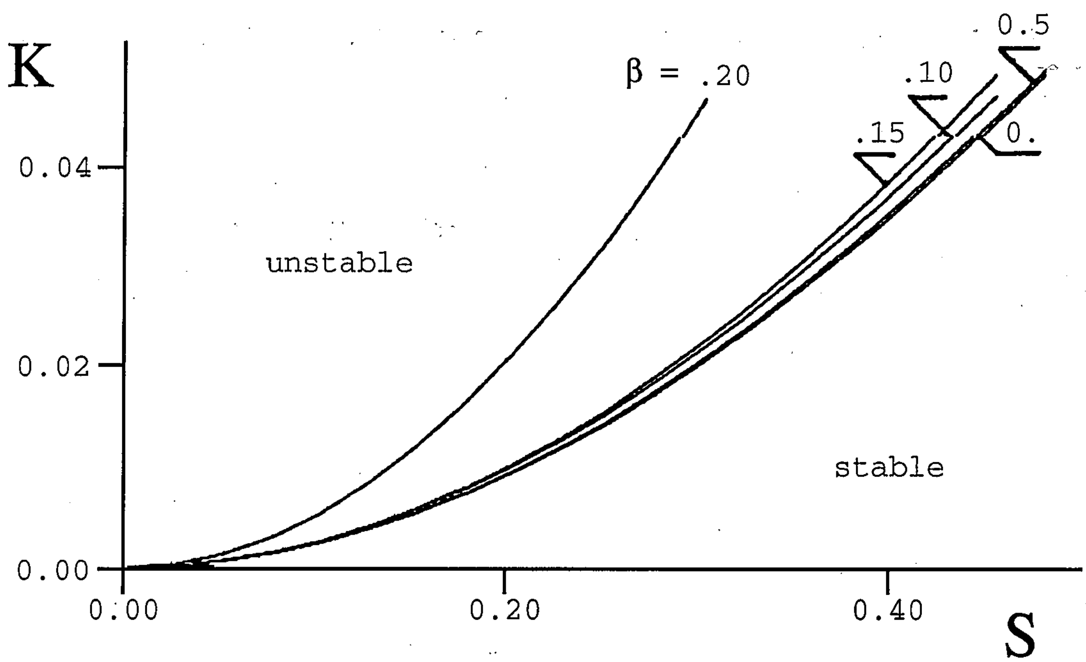

Figure 2

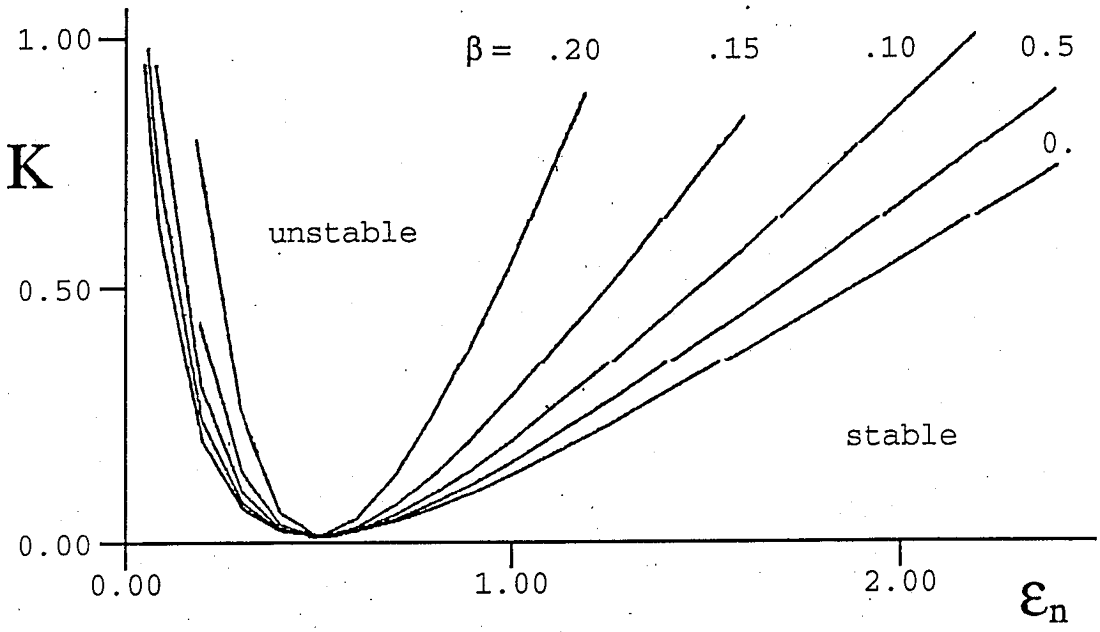

Figure 1

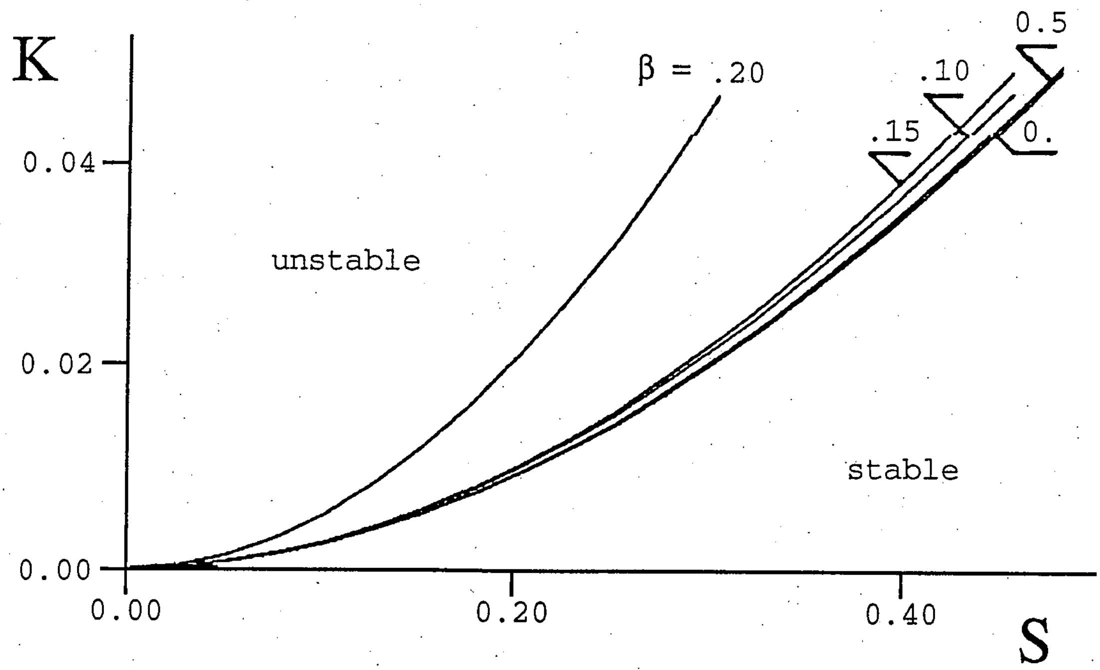

Figure 2

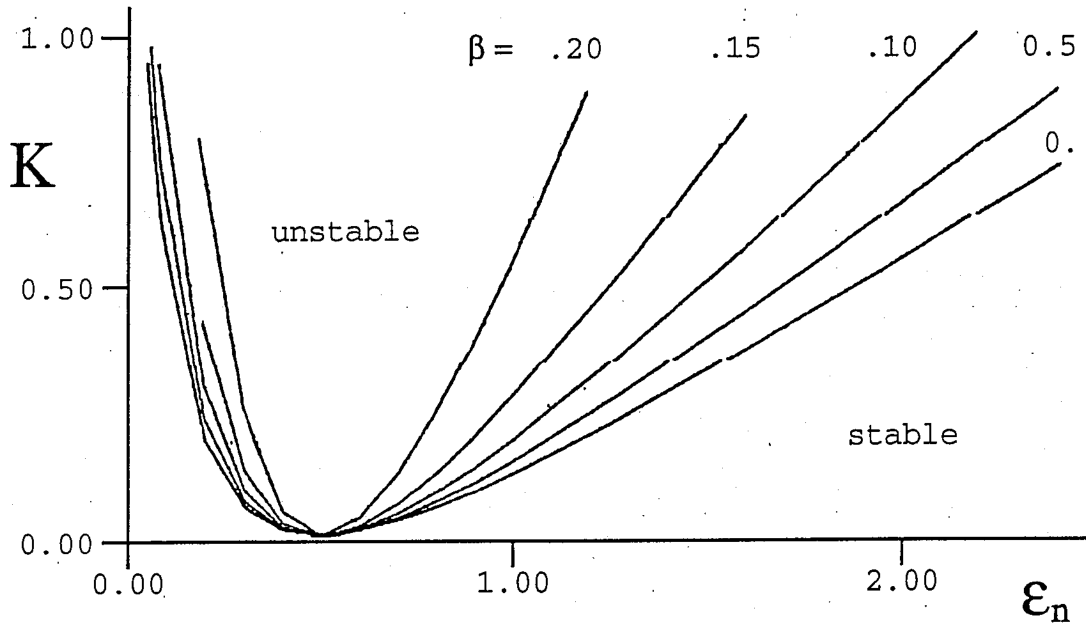

Figure 1

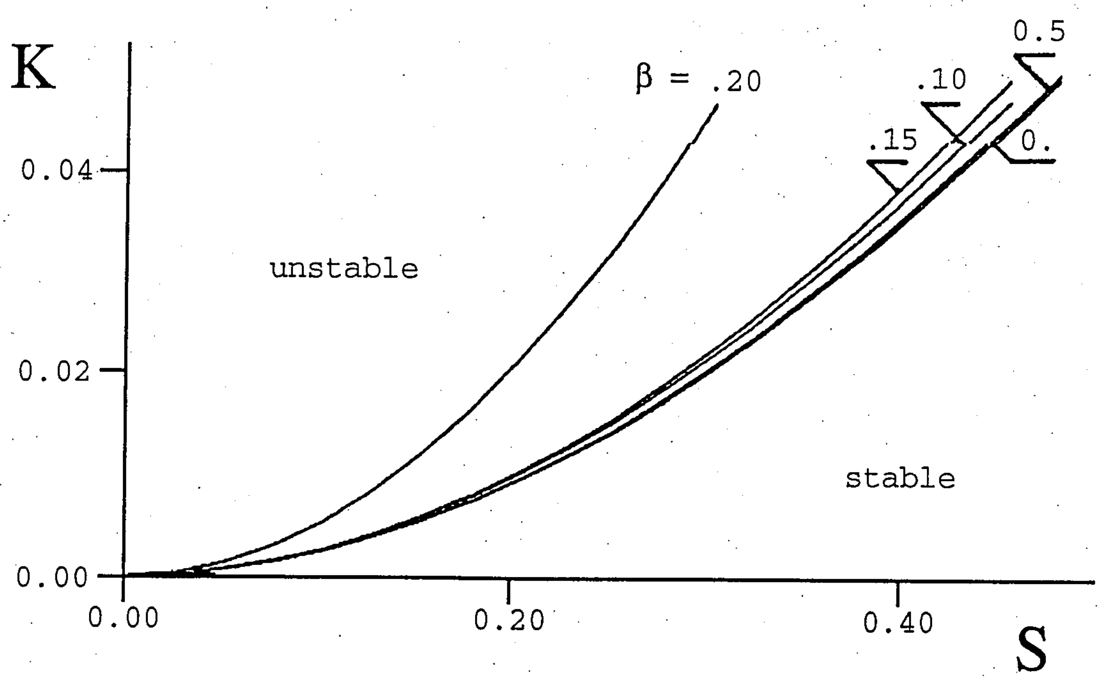

Figure 2