DOE/ET-53088-635 IFSR #635

Superluminous Laser Pulse in an Active Medium

D.L. Fisher and T. Tajima
Department of Physics and
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

December 1993

Superluminous Laser Pulse in an Active Medium

D.L. FISHER AND T. TAJIMA
Department of Physics and
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

Abstract

Physical conditions are obtained to make the propagation velocity of a laser pulse and thus the phase velocity of the excited wake be at any desired value, including that equal to or greater than the speed of light. The provision of an active-plasma laser medium with an appropriately shaped pulse allows not only replenishment of laser energy loss to the wakefield but also acceleration of the group velocity of photons. A stationary solitary solution in the accelerated frame is obtained from our model equations and simulations thereof for the laser, plasma and atoms.

PACS numbers: 52.40.Db, 42.50.Rh

We propose the use of an active laser medium to control the envelope propagation velocity of a laser pulse in a plasma to any design velocity v_{env}, including the speed of light or greater. Such a pulse is said to be superluminous. It is possible with appropriate construction of the laser pulse profile in the direction of propagation to "accelerate" the group velocity of that pulse. This is related to the process of self induced transparency[1] and to the triple soliton solutions already obtained for a (nonactive) plasma system.[2, 3] Since the pulse can travel at the design speed with no change in its structure, the phase velocity of the accelerating field will also be at the design speed in an active-plasma medium. The pre-excited active medium plays the role of a nonlinear amplifier, amplifying the front of the laser pulse and absorbing energy at the rear of the pulse in such a manner as to maintain its shape but at the same time increase the overall pulse speed. This apparent acceleration of the group velocity even beyond the speed of light does not violate the special theory of relativity as energy and information flow in fact does not exceed c. The leading edge of the pulse which is necessary for acceleration already contains information about the pulse and this information is extracted through the nonlinear amplification process. If the pulse is of finite length, the peak will travel only to edge of the pulse (the edge travels with the pulse group velocity) with the design velocity and with the group velocity thereafter.

The process of effective acceleration of the photon group velocity and the recovery of laser energy loss by the active medium has applications in photonics and telecommunications as well as to wakefield accelerators. With application to acceleration[4] the increase of the phase velocity of the wakefield could help overcome slippage between the particle bunch and accelerating field as the particle bunch outruns the accelerating field due to the difference between the phase velocity of the field and the particle velocity $v \approx c$. Secondly, since the energy used to induce the wake comes from the active medium not from the laser pulse, pump depletion[5] may be eliminated. The energy to induce the wakefield originally comes from the active medium, leaving the structure of the pulse unchanged with the final energy density of the material reduced by the amount necessary to induce the wakefield. Thirdly, using a properly shaped active medium channel in the transverse direction, we should be able to replenish energy loss to diffraction and refraction, thereby optically guiding and effectively focusing the laser pulse.

Three conditions must be met to accelerate the pulse envelope to the design speed v_{env} and retain a stationary structure. First is the resonance condition: the laser photon energy should approximately match that of the transition energy in the active medium. Here we consider the case of exact resonance, but small detuning might be both possible and desirable to minimize unwanted nonlinear optical effects. Secondly the laser pulse duration must match the time scale of energy exchange between the active medium and laser pulse (reciprocal of the average Rabi frequency). Thirdly a certain energy density in the active medium is necessary for the acceleration of the pulse envelope.

Starting with the wave equation

$$\left(\nabla^2 - \frac{\eta^2}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A} + c\nabla\frac{\partial \phi}{\partial t} = -\frac{4\pi}{c}\mathbf{J} - \frac{4\pi}{c}\frac{\partial \mathbf{P}}{\partial t}, \tag{1}$$

where $\mathbf{A}$ is the vector potential of the laser pulse, ϕ is the potential for the plasma electric field, and the transverse component of polarization vector $\mathbf{P}$ contains only the active material dependent (atomic) response. None of the plasma effects are included in $\mathbf{P}$, they enter through ϕ and J. The index of refraction η is for the nonactive portion of the material response (atomic). The form of the fields in one dimension for a laser of frequency ω_0 and wavenumber k_0 are

$$P = [U(z,t) + iV(z,t)]e^{i[k_0 z - \omega_o t + \psi(z,t)]}, \tag{2}$$

$$A = a(z,t)ie^{i[k_0 z - \omega_o t + \psi(z,t)]}, \tag{3}$$

$$J = -\frac{e^2 n A}{\gamma m c}, \tag{4}$$

where $\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$, $U(z,t)$ and $V(z,t)$ are the slowly varying real (electric dipole dispersion) and imaginary (absorption) components of the material polarization vector,[1] $\psi(z,t)$ and $a(z,t)$ are the slowly varying phase and envelope of the laser pulse. The transverse current J is obtained from the conservation of canonical momentum and initially the plasma is stationary. With the assumption that the laser oscillation time and length scales are much shorter than those for $a(z,t)$, $\psi(z,t)$ and the plasma (ω_p), we multiply equation (1) by $\mathbf{A}^*$ and average over the fast laser scales.[1]

Our purpose here is to determine the form of a laser pulse envelope which travels in the medium with a design velocity v_{env} and retain its shape without pump depletion. In order to avoid phase slippage for the purpose of an application to an accelerator, we hereafter set $v_{\mathrm{env}} = c$. We make a change of variables to $\lambda = z - ct$, $\tau = t$ and since the purpose is to find stationary structures the time derivative is set to zero ($\frac{\partial}{\partial \tau} = 0$) for the present Letter. Assuming the real part of the dispersion $U(z,t) = 0$ (which can be included in η) and substituting (2)–(4) into (1) yields from the real and imaginary parts

$$(\eta^2 - 1)\frac{\partial^2 a}{\partial \lambda^2} = 2a\left(\frac{\eta^2\omega_0}{c} - k_0\right)\frac{\partial \psi}{\partial \lambda} + (\eta^2 - 1)a\left(\frac{\partial \psi}{\partial \lambda}\right)^2 - k_p^2\frac{na}{n_0\gamma} - 4\pi\frac{\partial V}{\partial \lambda}\,, \tag{5}$$

$$(\eta^2 - 1)\frac{\partial}{\partial \lambda}\left(a^2\frac{\partial \psi}{\partial \lambda}\right) = -\left(\frac{\eta^2\omega_0}{c} - k_0\right)\frac{\partial a^2}{\partial \lambda} - \frac{4\pi}{c}V\left(\omega_0 + c\frac{\partial \psi}{\partial \lambda}\right)a\,, \tag{6}$$

where $k_p = \frac{\omega_p}{c}$. Laser medium equations for a two state system with dipole transitions at exact resonance ($\Delta\mathcal{E} = \hbar\omega_0$) without relaxation gives[1]

$$\frac{\partial V}{\partial \lambda} = -\frac{\kappa^2}{c^2}aW\,, \tag{7}$$

$$\frac{\partial W}{\partial \lambda} = \frac{\omega_0^2}{c^2}aV\,, \tag{8}$$

where $\Delta\mathcal{E}$ is the atomic transition energy between the two states under consideration, $e\hbar\kappa$ is the specific dipole moment for the active state and the time constants for incoherent damping effects and energy damping of the active system are long compared to the interaction time.

The energy density of the active medium W is defined with respect to the ground state $W_0 = -N_a \frac{\hbar\omega_0}{2}$ where N_a is the density with all possible states active.

The longitudinal laser-plasma interaction is modeled through the ponderomotive force in the electron momentum equation and the ions are assumed immobile. The form for Poisson's equation and the plasma term in previous equations becomes[6]

$$\frac{\partial^2\phi}{\partial\lambda^2}\frac{k_p^2}{2}\frac{mc^2}{e}\left(\frac{1+\left(\frac{ea}{mc^2}\right)^2}{\left(1+\frac{e\phi}{mc^2}\right)^2}-1\right) , \tag{9}$$

$$-\frac{4\pi}{c}\mathbf{J} = k_p^2\frac{amc^2}{mc^2+e\phi} . \tag{10}$$

The third condition states that there be enough energy Γ stored in the active medium prior to the pulse arrival to amplify the leading edge of the pulse to the peak value in the distance L_v ($L_v = l_{\text{pulse}}\, v_{\text{env}}/(v_{\text{env}} - v_g)$) for the accelerated pulse envelope to outrun the otherwise nonaccelerated pulse over the pulse length l_{pulse}. That is, there must be energy equal to that of the pulse stored in the length L_v of the active medium. When the atomic effect dominates $\frac{1}{\eta} \ll \sqrt{1-\left(\frac{\omega_p}{\omega_0}\right)^2}$ with $\eta \approx 1$ and $\frac{\omega_0}{\omega_p} \gg 1$ (underdense plasma), this yields

$$W(-\infty)+\mid W_0 \mid = \Gamma_v \approx \frac{l_{\text{pulse}}}{L_v}\frac{E^2}{4\pi} = \left(1-\frac{c}{\eta v_{\text{env}}}\right)\frac{E^2}{4\pi} , \tag{11}$$

where $W(-\infty)$ is the value of W prior to the arrival of the pulse, Γ_v energy density necessary to accelerate the stationary structure to the design velocity v_{env} and E is the electric field of the laser.

The interaction between the laser pulse and active medium is characterized by two time scales, the average inverse Rabi frequency[7] Ω_R^{-1} and the laser pulse scale Ω_l^{-1}. The frequency of energy exchange between the laser and active medium (Ω_R) is derived using Eqs. (7) and (8) along with the approximate average laser intensity $\frac{a^2}{2}$ to yield

$$\frac{\partial^2 V}{\partial\lambda^2} = -\frac{\kappa^2\omega_0^2a^2}{2c^4}V . \tag{12}$$

This gives $\Omega_R \approx \frac{\kappa\omega_0 a}{\sqrt{2}c^2}$. Additionally the laser time scale comes from Eq. (6), keeping only leading order terms yields

$$\frac{\partial^2 a}{\partial\lambda^2} = \frac{4\pi\kappa^2 W a}{(\eta^2-1)c^2} . \tag{13}$$

Using Eq. (11) ($v_{\text{env}} = c$) for the energy density gives $\Omega_l \approx \frac{\kappa\omega_0 a}{\sqrt{2}c^2}$. The second necessary condition $\Omega_R = \Omega_l$ for a stationary superluminous structure is in fact demonstrated: these two time scales much match. This match is necessary since the energy must return to the active medium on the average Rabi time scale. If the Rabi time scale does not match the duration of the laser pulse, the energy return will be incomplete if the pulse is short and will again extract energy if the pulse is too long.

When the plasma effect dominates over the atomic effect $\frac{1}{\eta} \gg \left(1 - \left(\frac{\omega_p}{\omega_0}\right)^2\right)$, the stored energy for acceleration to the design velocity becomes,

$$W(-\infty) + \mid W_0 \mid = \Gamma_v \approx \left(1 - \frac{c}{v_{\text{env}}}\left(1 - \frac{\omega_p^2}{2\omega_0^2}\right)\right)\frac{E^2}{4\pi} . \tag{14}$$

Setting the velocity of the laser pulse envelope $v_{\text{env}} = c$ yields

$$W(-\infty) + \mid W_0 \mid \approx \frac{\omega_p^2}{2\omega_0^2}\frac{E^2}{4\pi} . \tag{15}$$

The induced wakefield is approximately $E_{\text{acc}} \approx \frac{I}{I_c}\frac{m\omega_p c}{e}$ where $\frac{I}{I_c} = \left(\frac{eE}{m\omega_0 c}\right)^2$. This gives the energy gain U of particles per unit length by the wakefield (the negative of the stopping power)

$$\left(\frac{\partial U}{\partial z}\right)_{\text{acc}} = 4\pi\frac{n_a}{n}\frac{\mathcal{E}_a}{\lambda_p} . \tag{16}$$

The energy for the wakefield excitation must be supplied by the active medium for the pulse to be stationary. For a laser pulse $l_{\text{pulse}} \approx \lambda_p$ the energy density necessary to overcome pump depletion is $\Gamma_{\text{pump}}(-\infty) \approx \left(\frac{\omega_p}{\omega_0}\right)^2\left(\frac{eE}{m\omega_p c}\right)^2\Gamma_c(-\infty)$. Since $\left(\frac{\omega_p}{\omega_0}\right)^2\left(\frac{eE}{m\omega_p c}\right)^2 \ll 1$ the energy density necessary for group velocity acceleration to c is much larger than that to overcome pump depletion.

Equations (5), (6), (7), (8), and (9) were numerically integrated with initial conditions at $\lambda = 0$, yielding a stationary structure in the moving reference frame with velocity $v_{\text{env}} = c$. The initial conditions include material constant κ, polarization V_0, energy density $W(0)$, $\frac{\omega_0}{\omega_p}$ and the index of refraction η. If the structure is to be stationary for all time, the pulse must be of infinite extent. At $\lambda = 0$ we give initial condition $a_0 \neq 0$ since the pulse must extend to infinity. The $\frac{1}{a^2}$ singularity in equation (6) has been handled by the method of analytic continuation. To determine the initial condition $V(0)$, the conservation of action is used,[7] which gives

$$V = \pm N_a \hbar \kappa \sqrt{1 - \left(\frac{n_a}{N_a}\right)^2} = \frac{2W_N\kappa}{\omega_0}\sqrt{1 - \left(\frac{W}{W_N}\right)^2}, \tag{17}$$

where $W = \frac{n_a \hbar \omega_0}{2}$, W_N is the value of W with all states active, n_a is the density of particles in the active states and $n_a(-\infty)$ is the density prior to the pulse arrival. The simulations were initialized with all possible states active prior to the pulse arrival ($n_a(-\infty) = N_a$). At $\lambda = 0$ the intensity is not equal to zero (must be of infinite extent) but very small compared to its peak value, therefore $1 - \left(\frac{n_a}{N_a}\right) \approx 0$. The value of $V(0)$ can not be determined exactly because the exact value of $W(-\infty)$ can not be determined beforehand without knowledge of the solution. Therefore trial values for $V(0)$ are attempted until energy conservation is achieved.

Figure 1 shows the stationary structure which is a short laser pulse ($\approx \frac{c}{\omega_p}$) with a peak laser field $a = 0.045\,\frac{mc^2}{e}$ for an initial atomic energy density of $W(0) = 0.001\ \mathrm{n}_0 mc^2$ inducing a wakefield of $E_{\text{peak}} = 0.0014\,\frac{m\omega_p c}{e}$. Initial condition are arbitrarily chosen with no material in mind are $\frac{\omega_0}{\omega_p} = 10$, $\eta = 1.0045$, $\kappa = 3.5\,\frac{e}{mc}$ and $V_0 = 2 \times 10^{-5}\frac{n_0 ec}{\omega_p}$. During the first half cycle the energy is absorbed by the laser pulse, reducing the energy density stored in the material. The second half completes the cycle, returning laser energy back to the active material. The difference of the energy density contained in the laser material initially $W(0)$ and the final is the energy used to produce the wakefield. This leaves the energy of the laser unchanged and therefore eliminates pump depletion.

Figure 2 is a special case of the general example of Fig. 1. In this case the parameters were adjusted so that the laser pulse produced and then absorbed the plasma oscillations leaving no wake. We see the peak laser pulse amplitude is $a = 0.046\,\frac{mc^2}{e}$ with the initial atomic energy density $W(0) = 0.001\,n_0mc^2$. Initial conditions are $\frac{\omega_0}{\omega_p} = 10$, $\eta = 1.0045$, $\kappa = 0.5\,\frac{e}{mc}$ and $V_0 = 7 \times 10^{-6}\frac{n_0ec}{\omega_p}$. Since there is no wakefield produced after the pulse passes, the atomic energy density returns to its original value at the tail of the laser pulse. Wakeless solutions have been discussed before,[2, 3] but without allowance for pump depletion effects and for the active medium.

The results of these two examples are slightly in the plasma regime $\left(\left(1 - \left(\frac{\omega_p}{\omega_0}\right)^2\right) < \frac{1}{\eta}\right)$, however, neither dominates. These examples demonstrate two of the necessary conditions previously described. Firstly the independence of the peak amplitude of the laser pulse on the pulse length, which is dependent on κ, but on the energy density $W(-\infty)$ as shown in Eqs. (11) and (14). Also Eqs. (11) and (14) (contributions from both plasma and atomic effects) correctly predicts the energy density $W(-\infty) \approx 0.001\,\frac{1}{n_0mc^2}$ for the laser peak amplitude of $a \approx 0.045\,\frac{mc^2}{e}$. Secondly the relation of the pulse length Eq. (13) to that of the average Rabi frequency Eq. (12) is shown to hold, predicting a pulse length of approximately $2.8\,\frac{c}{\omega_p}$ for the first case (Fig. 1) and approximately $20\,\frac{c}{\omega_p}$ for the second case (Fig. 2).

The question of stability arises for pulse shapes that are only approximately stationary. The initial value problem is investigated with a newly developed PIC-fluid code using a more realistic gaussian shaped laser pulse. Results for the gaussian show acceleration and long term pulse shape stability. More details of our solutions will be reported in future publications.

A number of conditions must be met for a material to be used as the active medium-plasma source. The basic concept we suggest is to use the outer shell electrons to form a plasma and the inner shell electron resonant transitions as the active source, as in the basic scheme used in x-ray lasers.[8] For applications using very high intensity laser pulses, the problems of ionization,[9, 10, 11] power broadening of the resonance and locating resonances in an intense electric field are to be addressed in future publications with more detail. The work was supported by the U.S. Department of Energy. We thank C.W. Siders and M.C. Downer for their illuminating discussions.

References

[1] S. McCall and E. Hahn, Phys. Rev. **183**, 457 (1969).

[2] K. Mima et al., Phys. Rev. Lett. **57**, 1421 (1986).

[3] C. McKinstrie and D. Dubois, Phys. Rev. Lett **57**, 2022 (1986).

[4] T. Tajima and J. Dawson, Phys. Rev. Lett. **43**, 267 (1979).

[5] W. Horton and T. Tajima, Phys. Rev. A **34**, 4110 (1986).

[6] P. E. Esarey, and A. Ting, Phys. Rev. A **41**(8), 4463 (1990).

[7] L. Allen and J.H. Eberly, *Optical Resonance and Two-level Atoms*, John Wiley and Sons, 1975.

[8] R. Elton, *X-Ray Lasers*, Academic Press, Inc., 1990.

[9] B.M. Penetrante and J.N. Bardsley, Phys. Rev. A **43**(6), 3100 (1991).

[10] S. D. D. Strickland, and S. L. Chin, J. Opt. Soc. Am. B **8**, 858 (1991).

[11] R.R. Freeman, P.H. Bucksbaum, H. Milchberg, S. Darack, D. Schumarcher, and M.E. Geusic, Phys. Rev. Lett. **59**(10), 1092 (1987).

Figure Captions

1. Laser pulse profile (a^2) (thick solid line), longitudinal wakefield (E) (thin line) and energy density of the active medium (W) (dashed line) for a general case with envelope speed $v_{\mathrm{env}} = c$.

2. Laser pulse profile (a^2) (thick solid line), longitudinal electric field (E) (thick dashed line) and energy density of the active medium (W) (dashed line) with initial conditions set for solitary structure with no wake. The wakefield is induced and absorbed within the laser pulse.

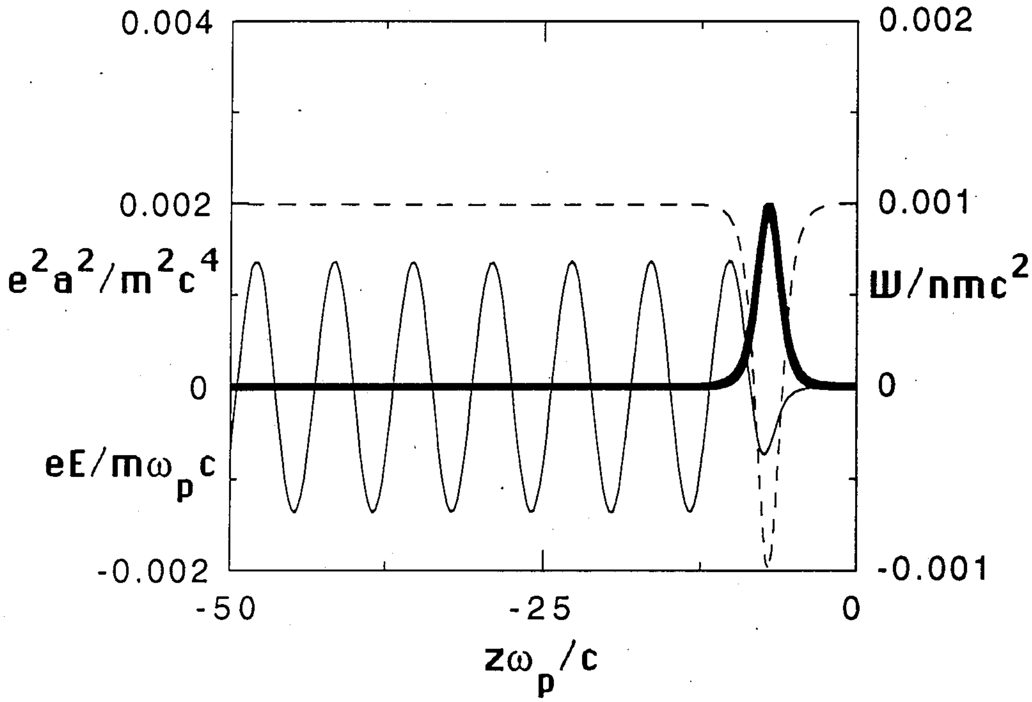

0.004
0.002
e^2a^2/m^2c^4
0
$eE/m\omega_p c$
-0.002
0.002
0.001
W/nmc^2
0
-0.001
-50
-25
0
$z\omega_p/c$

Fig. 1

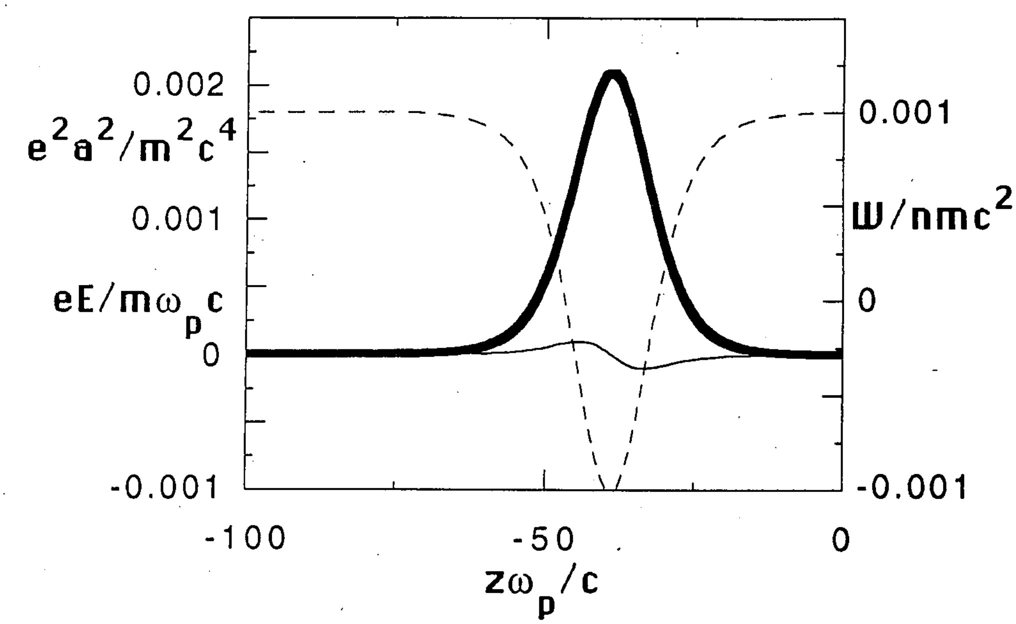
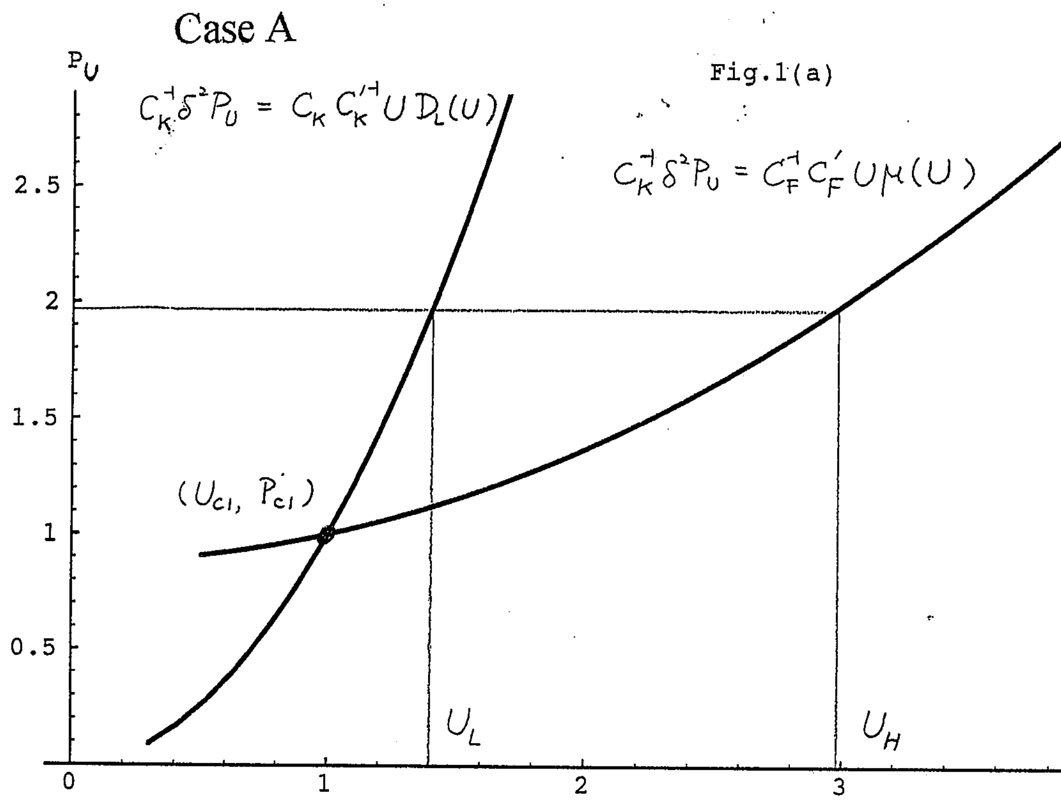
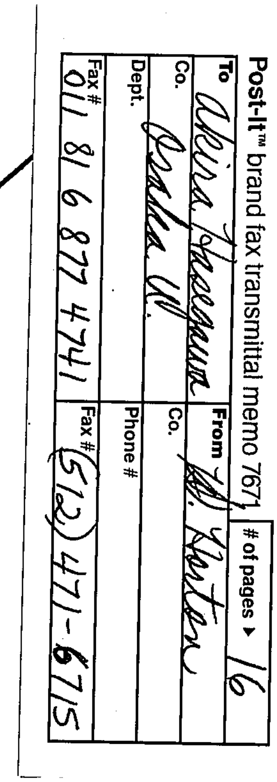
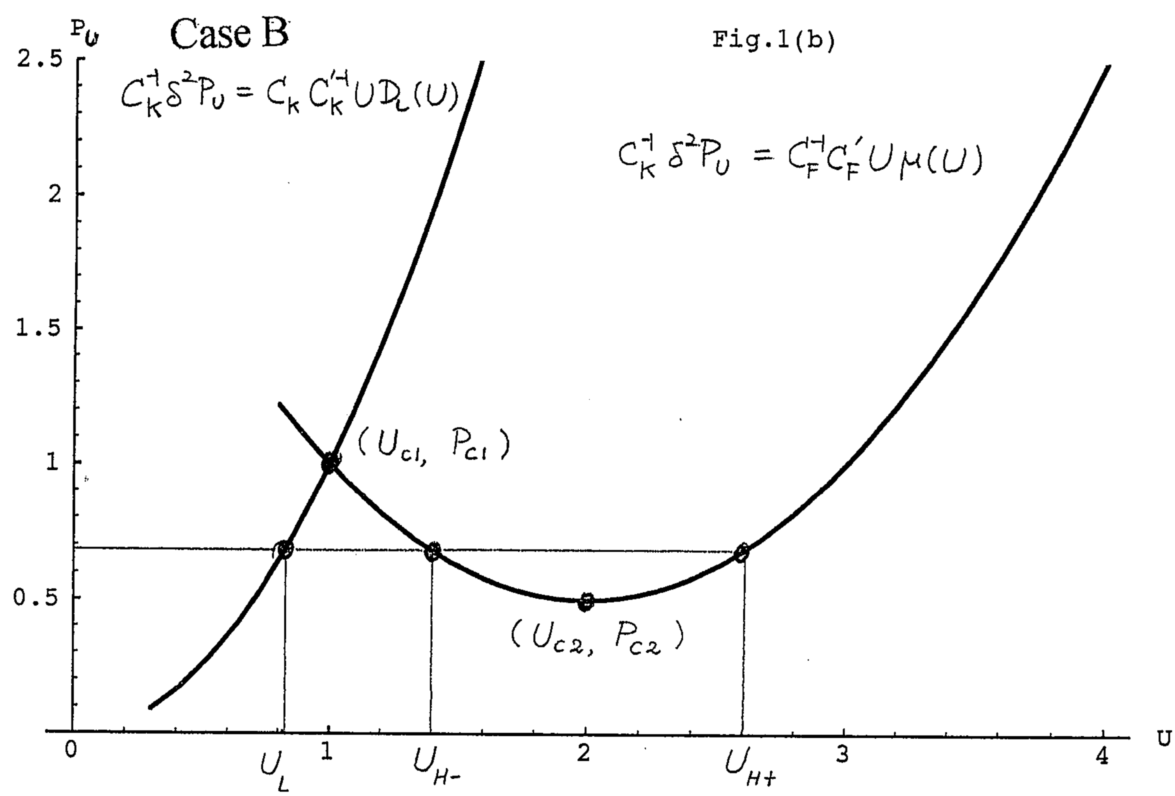

0.002
e^2a^2/m^2c^4
0.001
$eE/m\omega_p c$
0
-0.001
0.001
W/nmc^2
0
-0.001
-100
-50
0
$z\omega_p/c$

Fig. 2


Case A
Fig.1(a)
P_U
$C_K^{-1}\delta^2 P_U = C_K C_K'^{-1} U D_L(U)$
$C_K^{-1}\delta^2 P_U = C_F^{-1} C_F' U \mu(U)$
(U_{c1}, P_{c1})
U_L
U_H
U
0
1
2
3
4
0.5
1
1.5
2
2.5


Post-it™ brand fax transmittal memo 7671
# of pages ▶ 16
To
From
Co.
Co.
Dept.
Phone #
Fax # 011 81 6 877 4741
Fax # (512) 471-6715


Case B
Fig.1(b)
P_U
$C_K^{-1}\delta^2 P_U = C_K C_K'^{-1} U D_L(U)$
$C_K^{-1}\delta^2 P_U = C_F^{-1} C_F' U \mu(U)$
(U_{c1}, P_{c1})
(U_{c2}, P_{c2})
U_L
U_{H-}
U_{H+}
U
0
1
2
3
4
0.5
1
1.5
2
2.5

Figs. 1(a)-(b)

Fig.2(a)

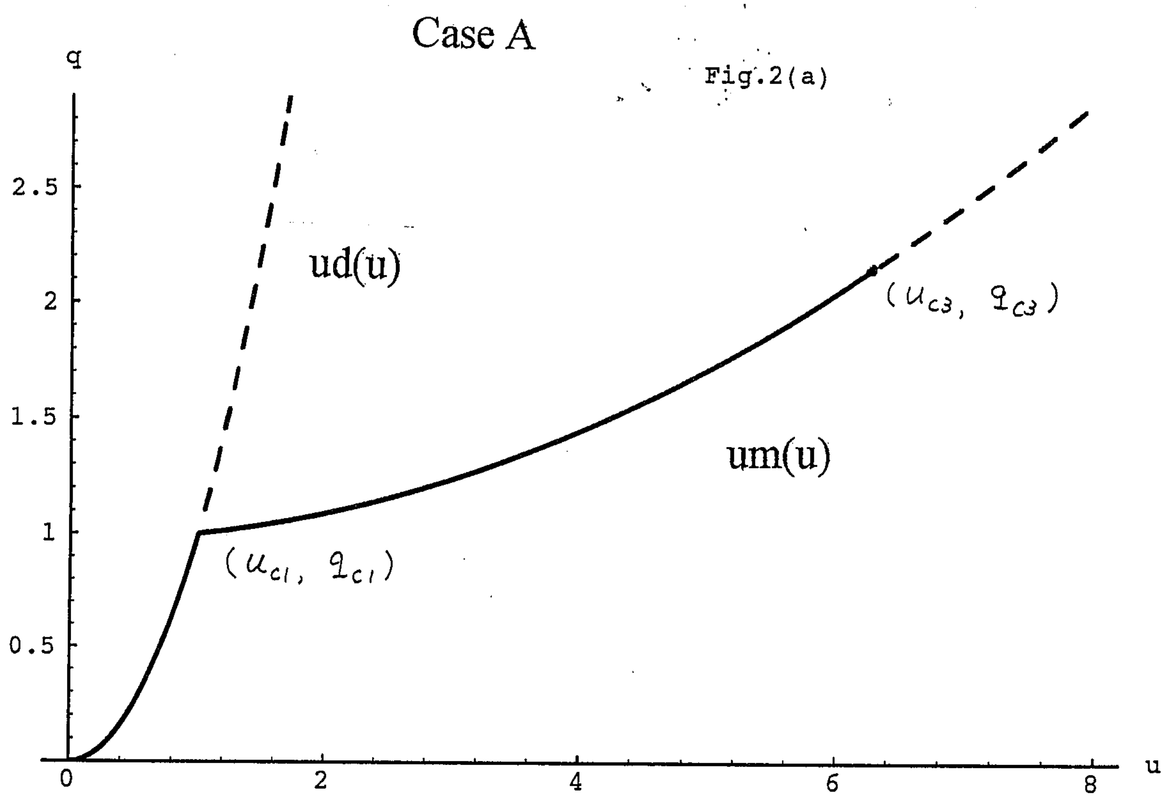
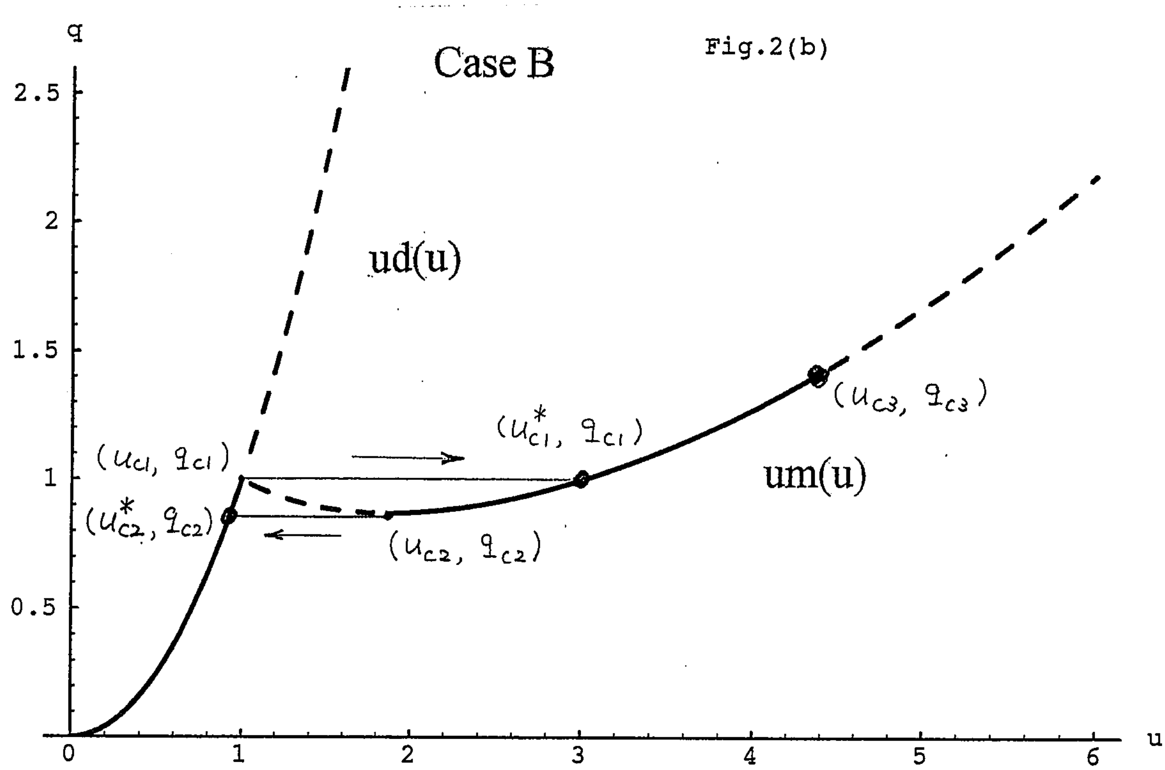


Fig.2(b)



Figs. 2(a)-(b)

Case A

$q = 0.8 < q_{c1}$

L state

Fig.3(a)

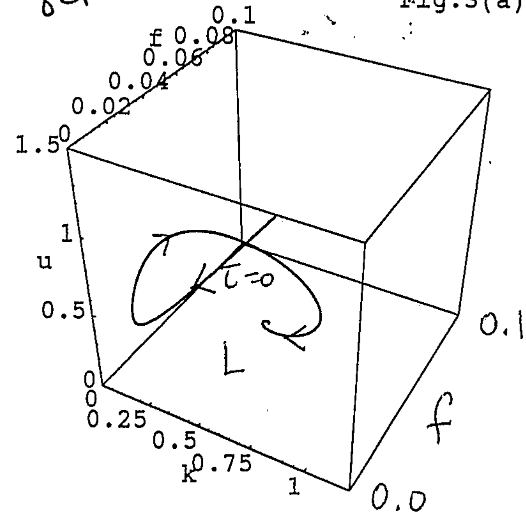


Fig.3(b)

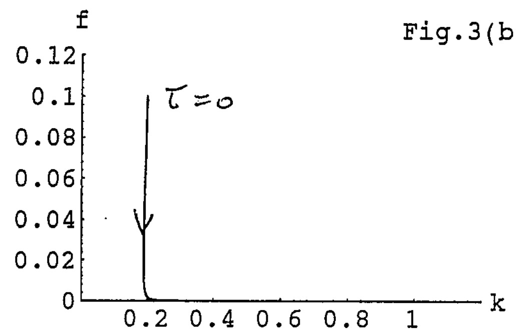
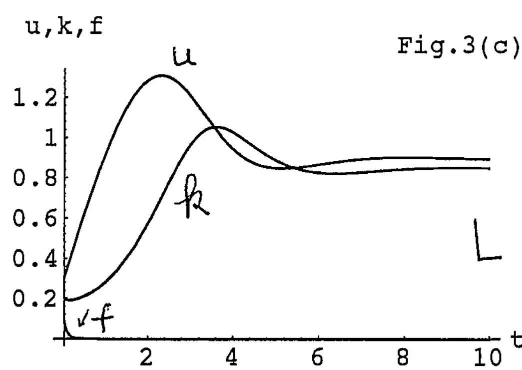


Fig.3(c)



L state

Case A

$q_{c1} < q = 1.05 < q_{c3}$

Fig.4(a)

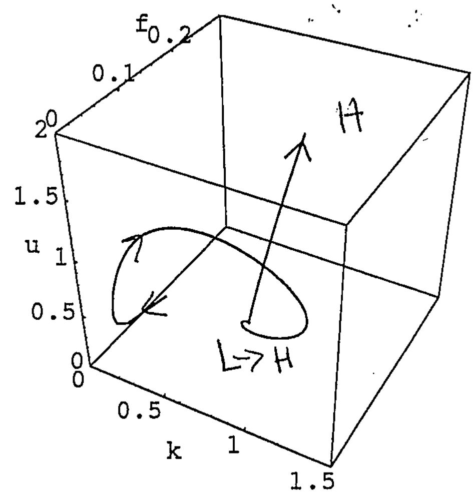


Fig.4(b)

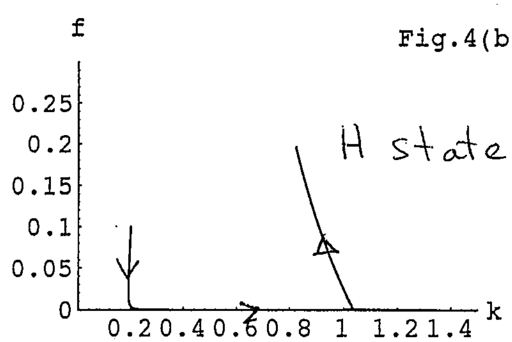
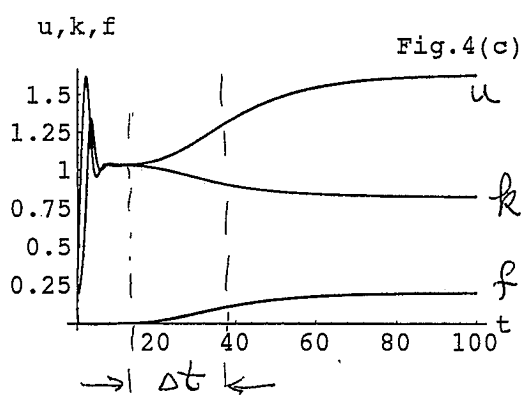


Fig.4(c)

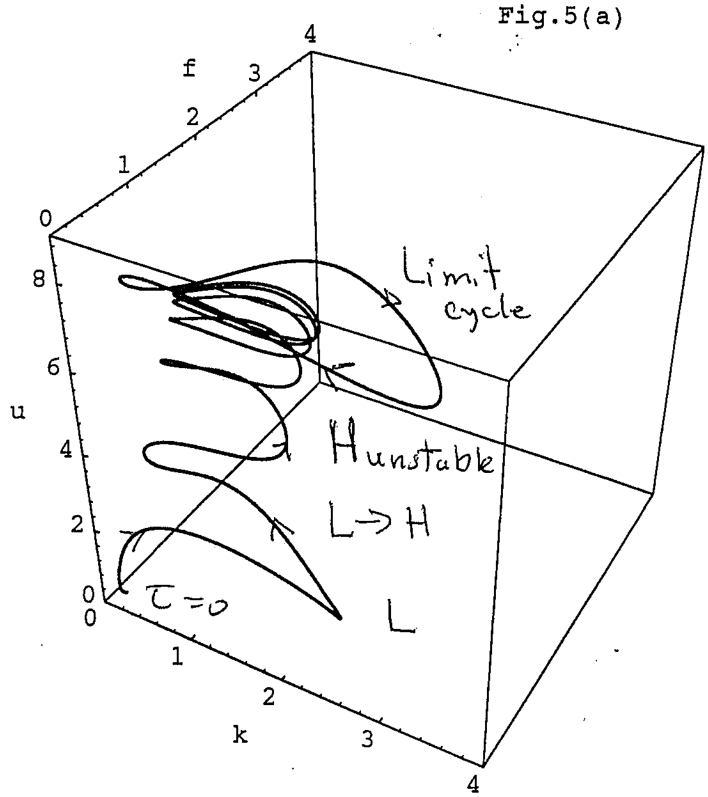


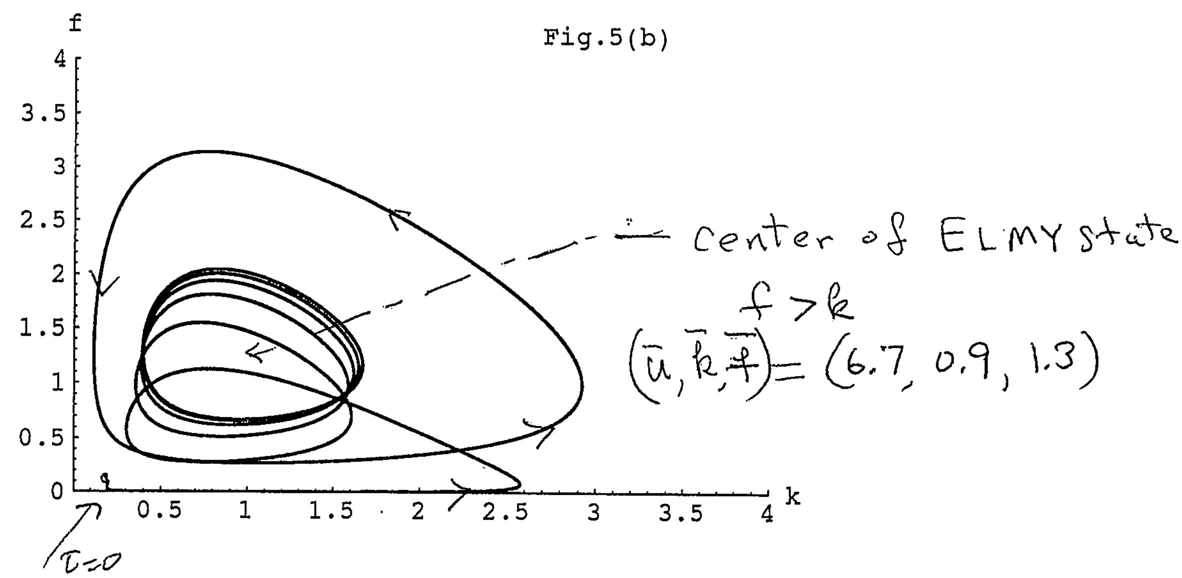
L → H transition

Figs. 4(a)-(c)

Case A

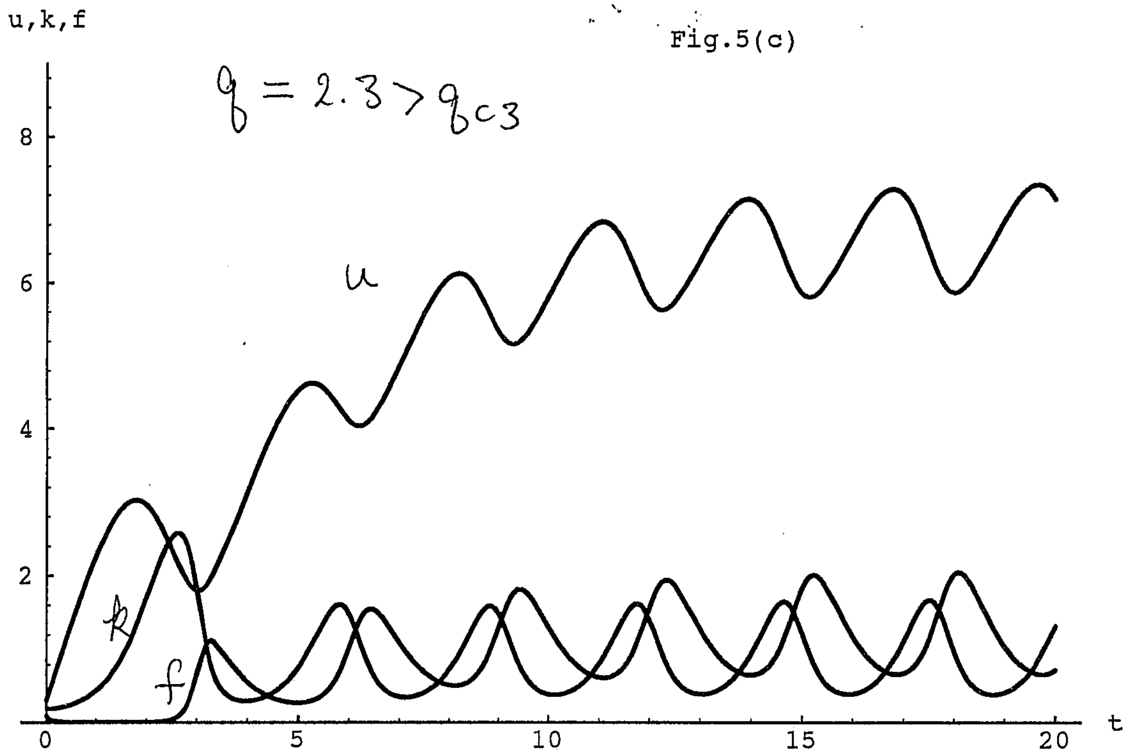
ELMy State

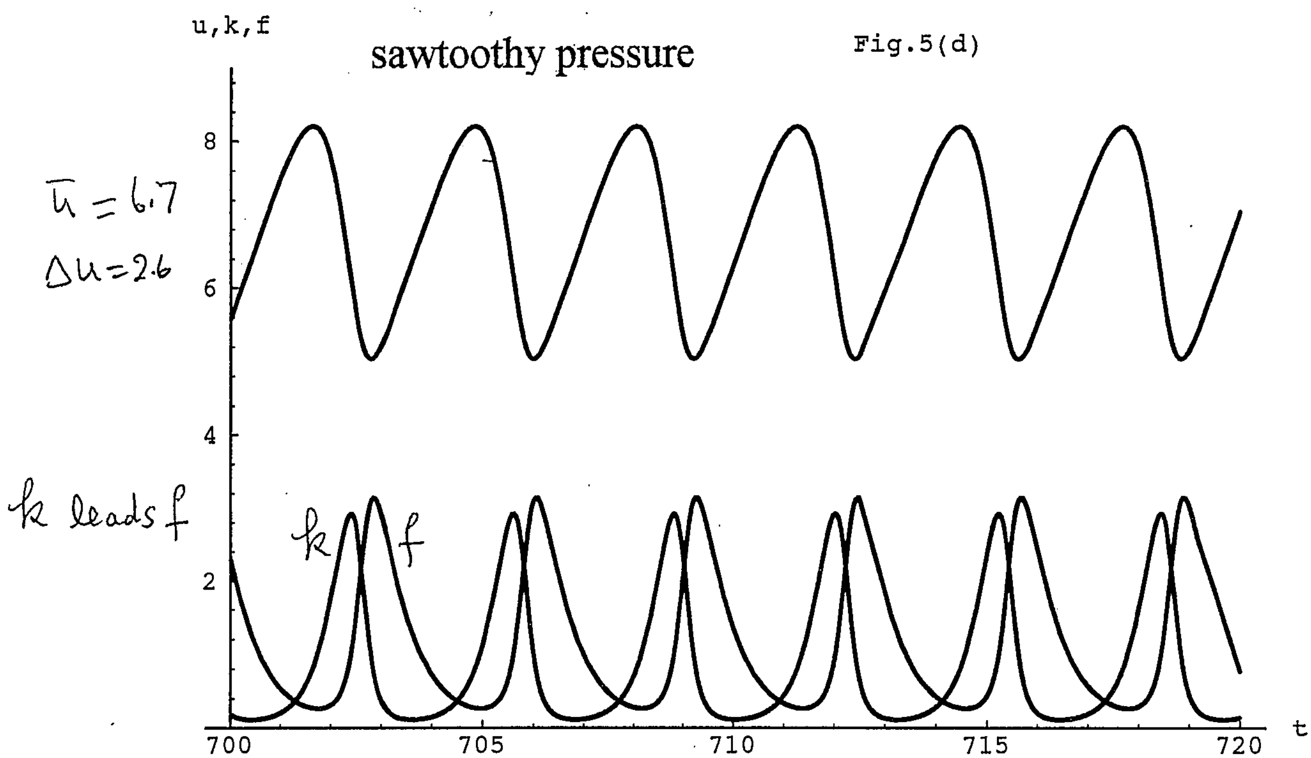




Figs. 5(a)-(b)

Case A

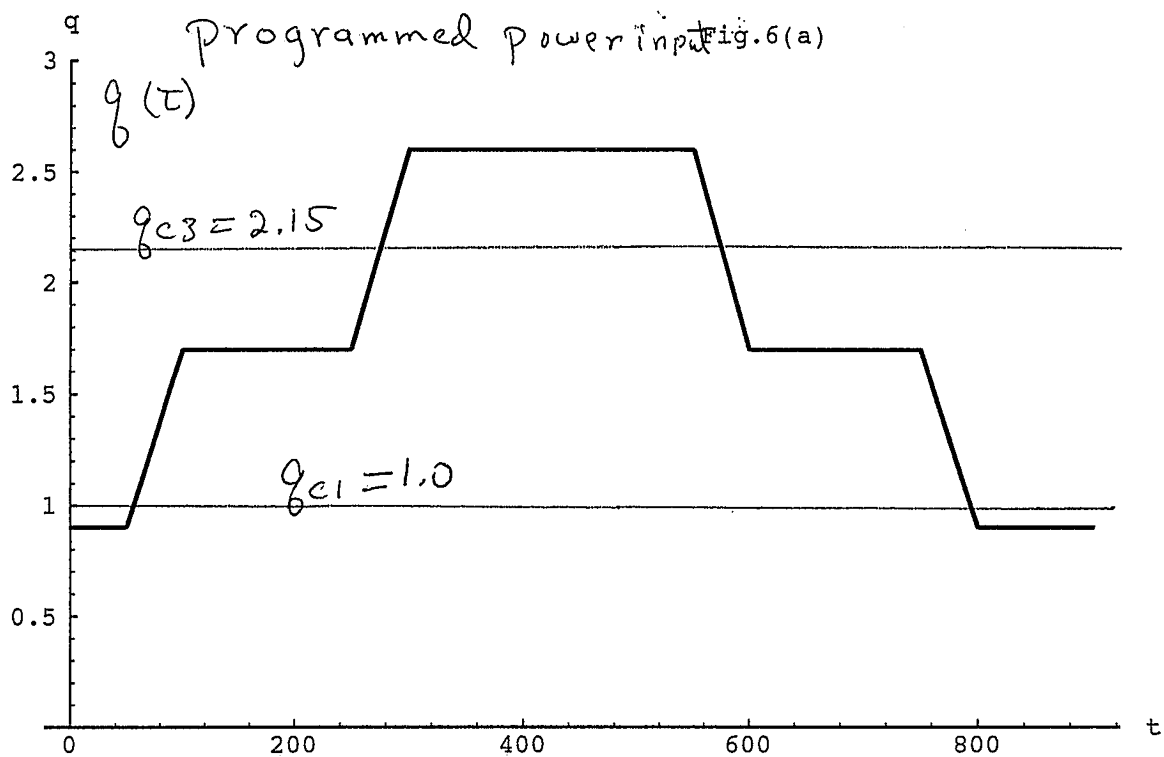


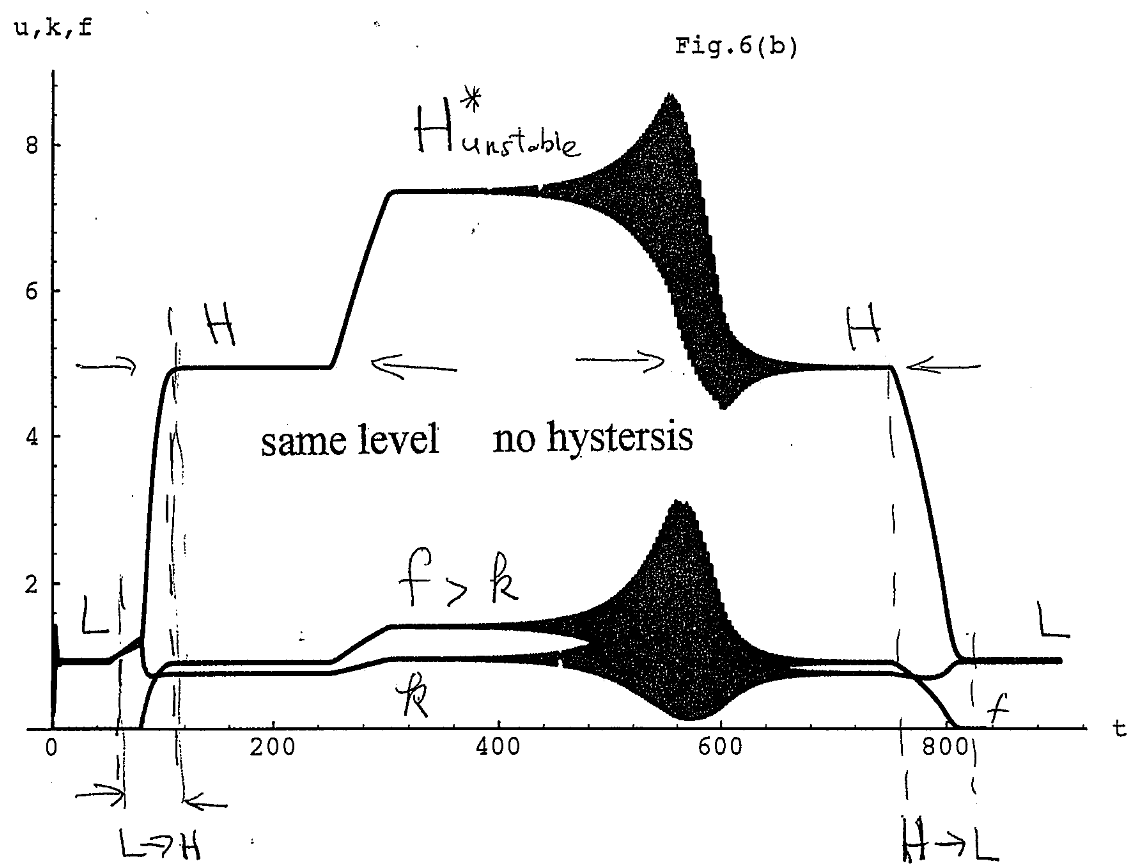


Figs. 5(c)-(d)

Fig.6(a)

Case A



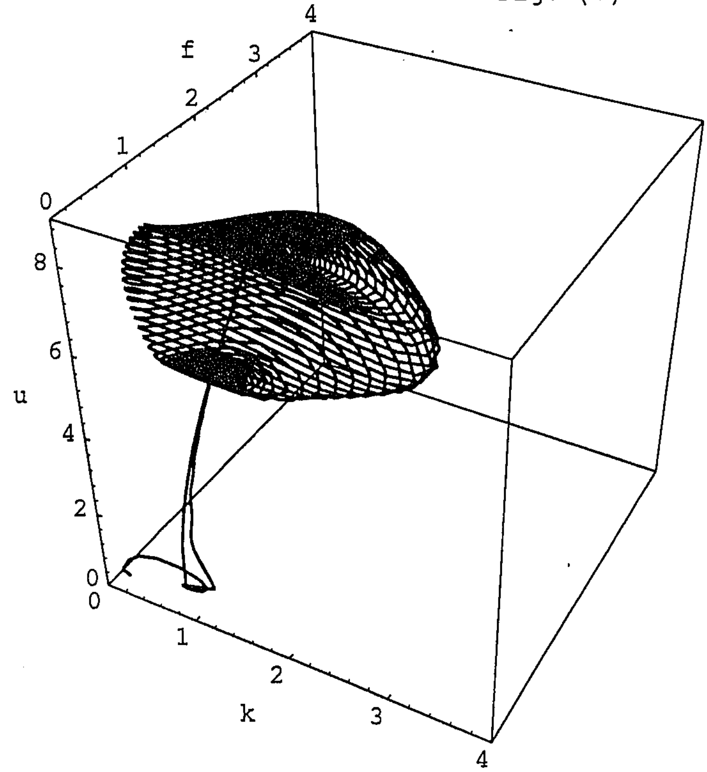


Figs. 6(a)-(b)

Case A

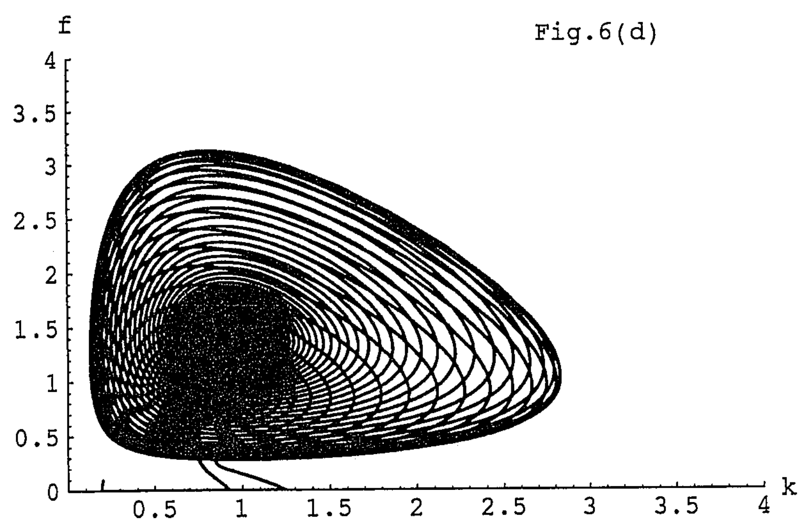
programmed g(τ)

Fig.6(c)



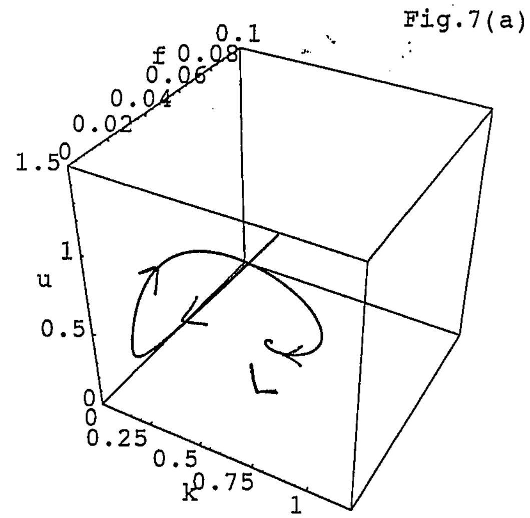
limit cycle

Fig.6(d)

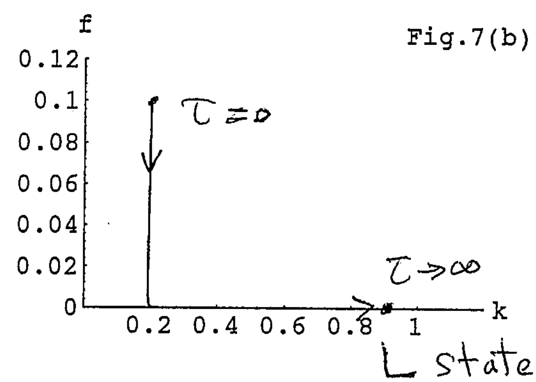


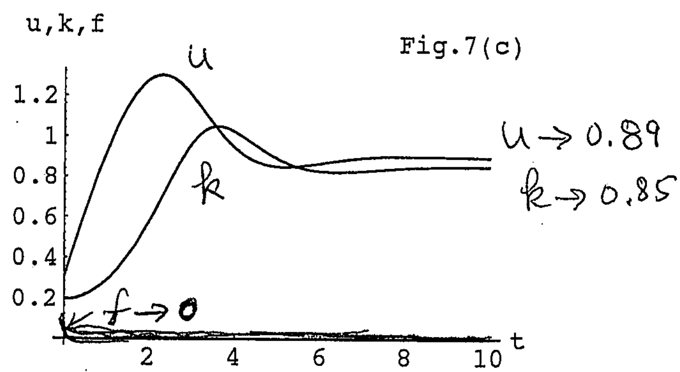
Case B

$q = 0.8 < q_{c2} = 0.87$



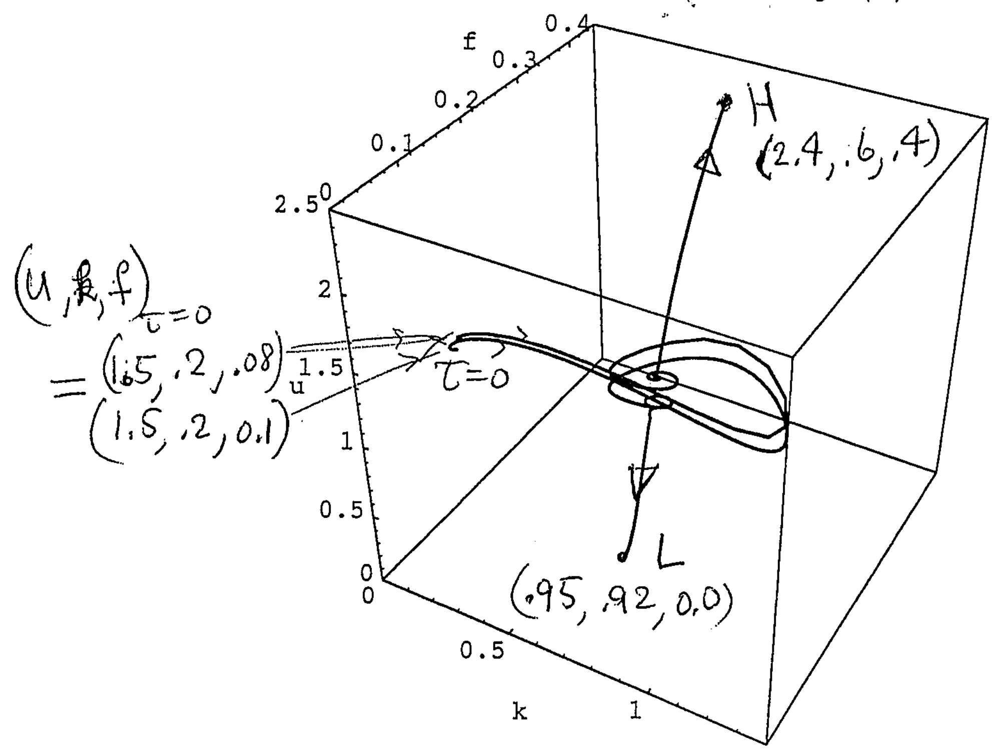
L state



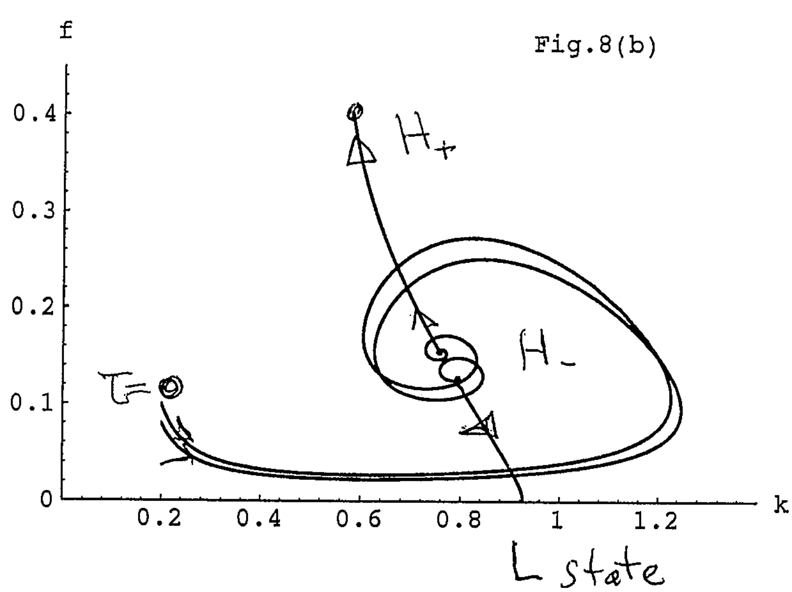


Figs. 7(a)-(c)

Case B

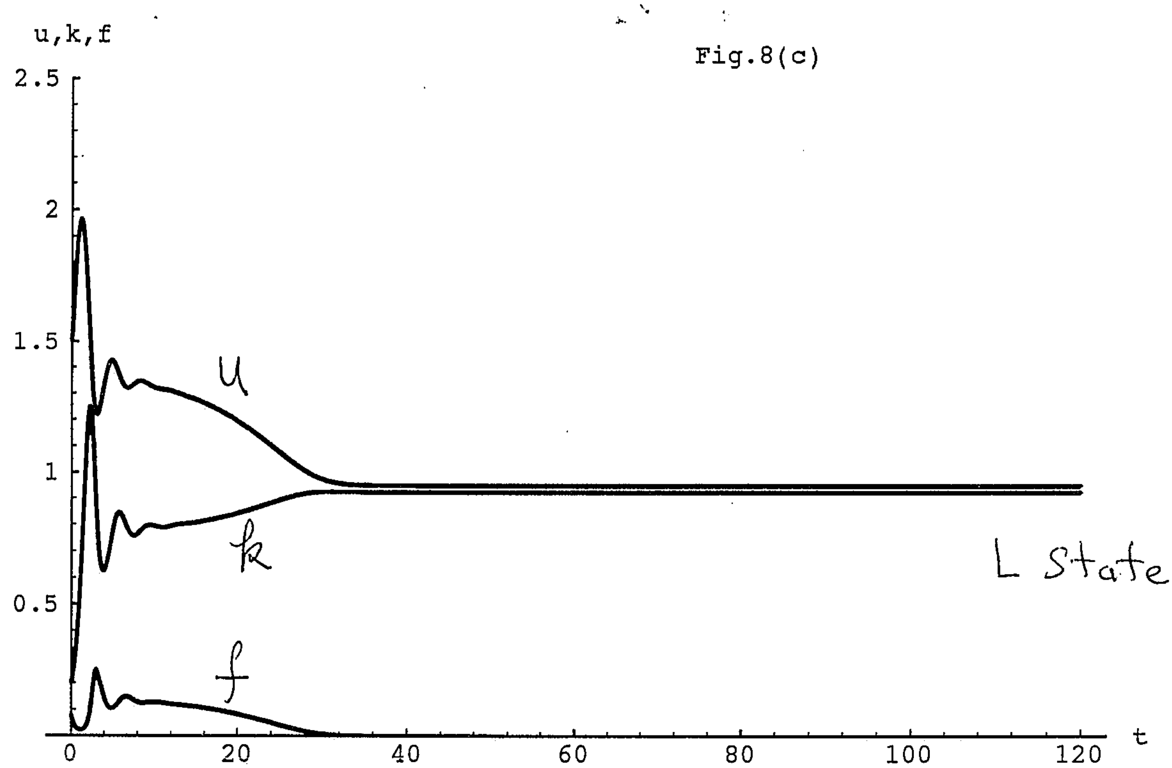


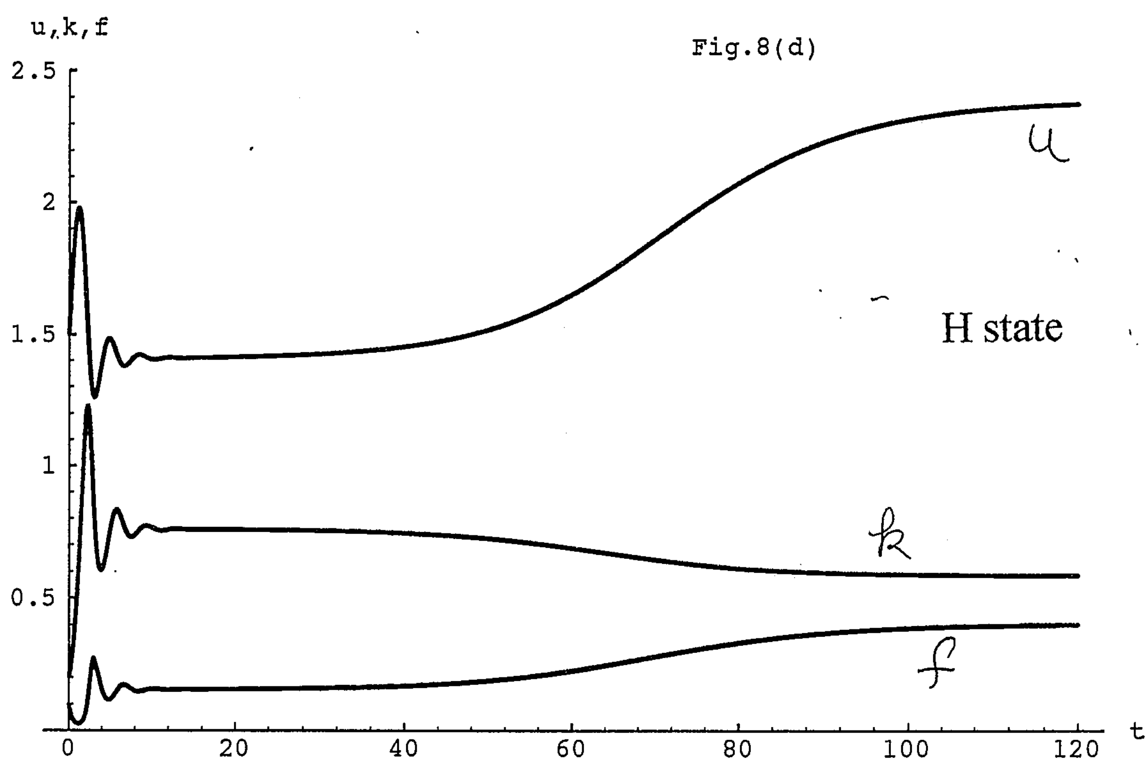
sensitivity to initial data



Figs. 8(a)-(b)

Case B

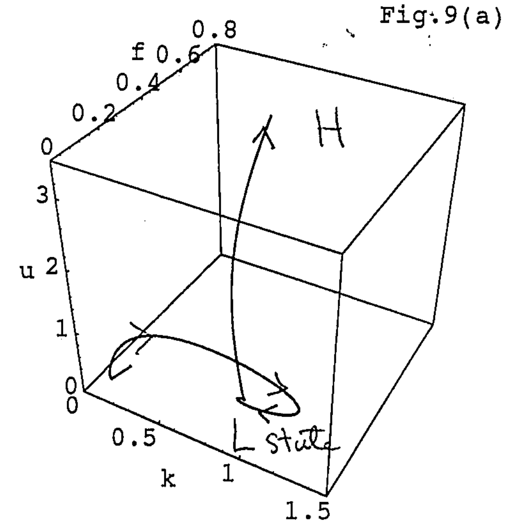




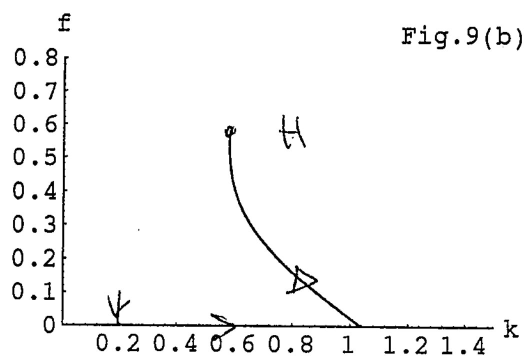
Figs. 8(c)-(d)

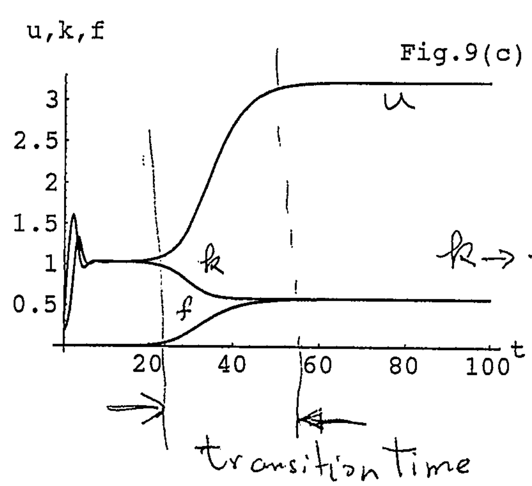
Case B $q = 1.05 > q_{c1}$

$q_{c1} < q < q_{c3}$



L → H transition





Figs. 9(a)-(c)

Case B

$q = 1.6 > q_{c3} = 1.4$

Fig.10(a)

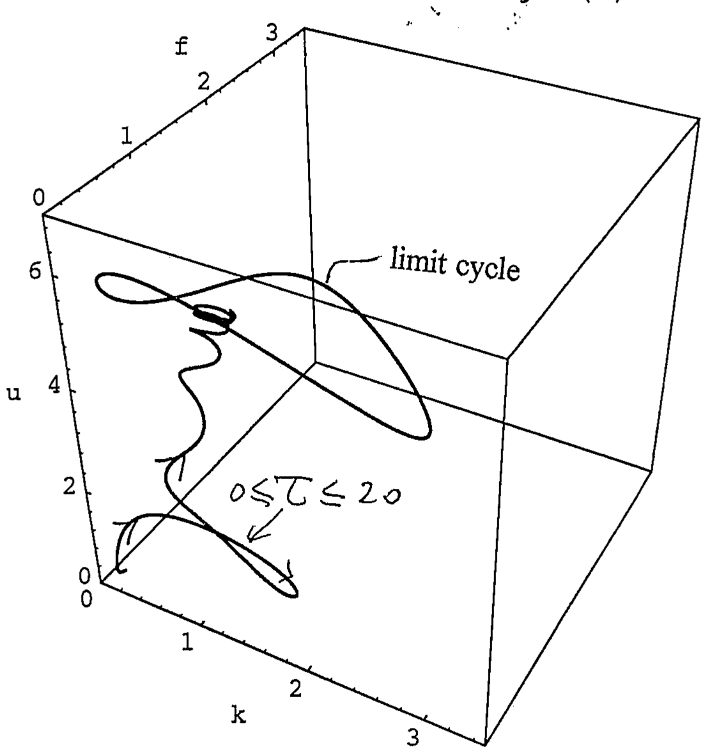
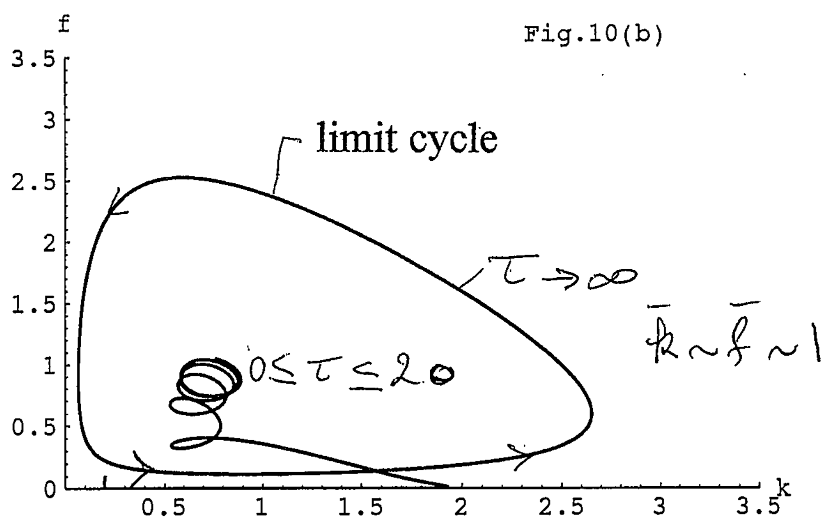
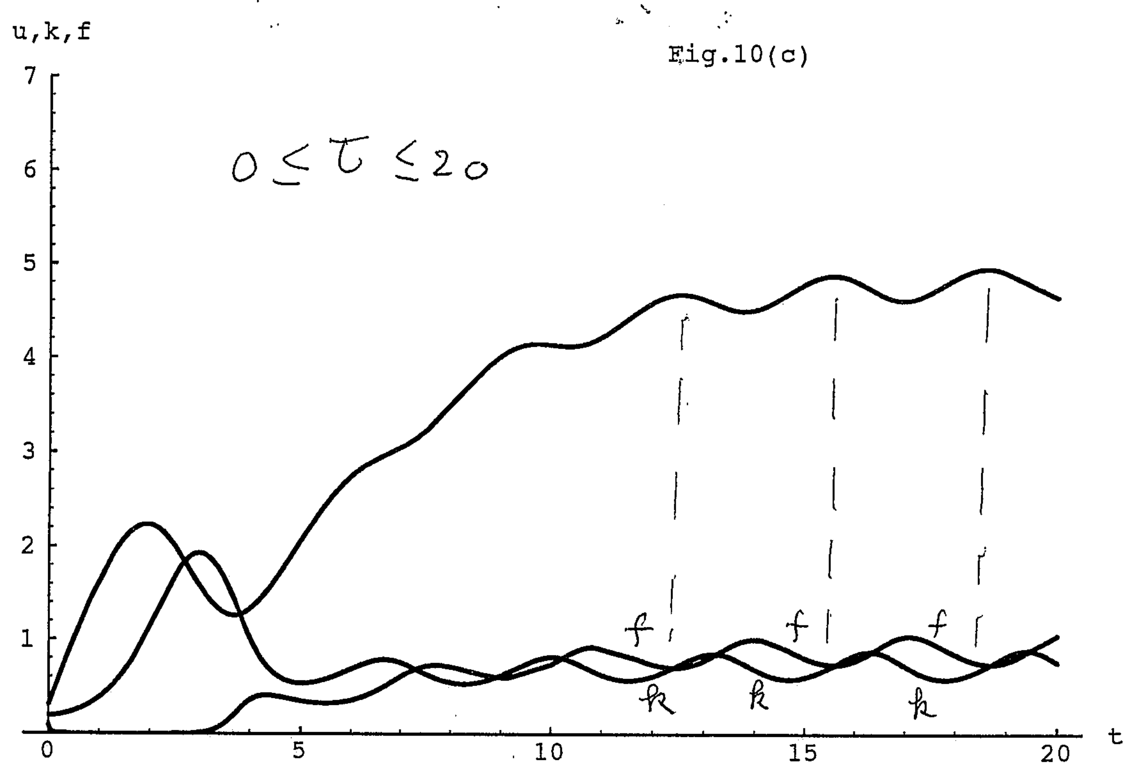


Fig.10(b)

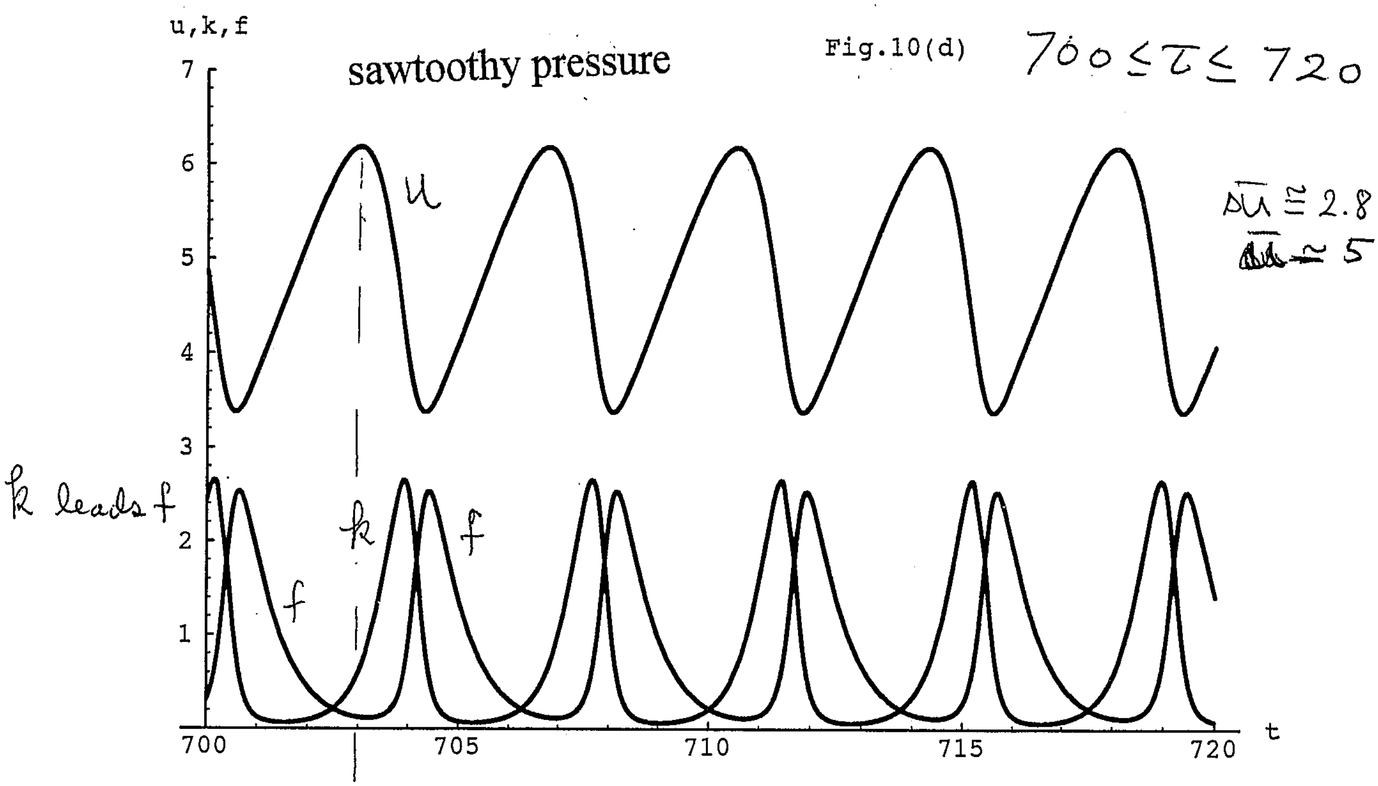


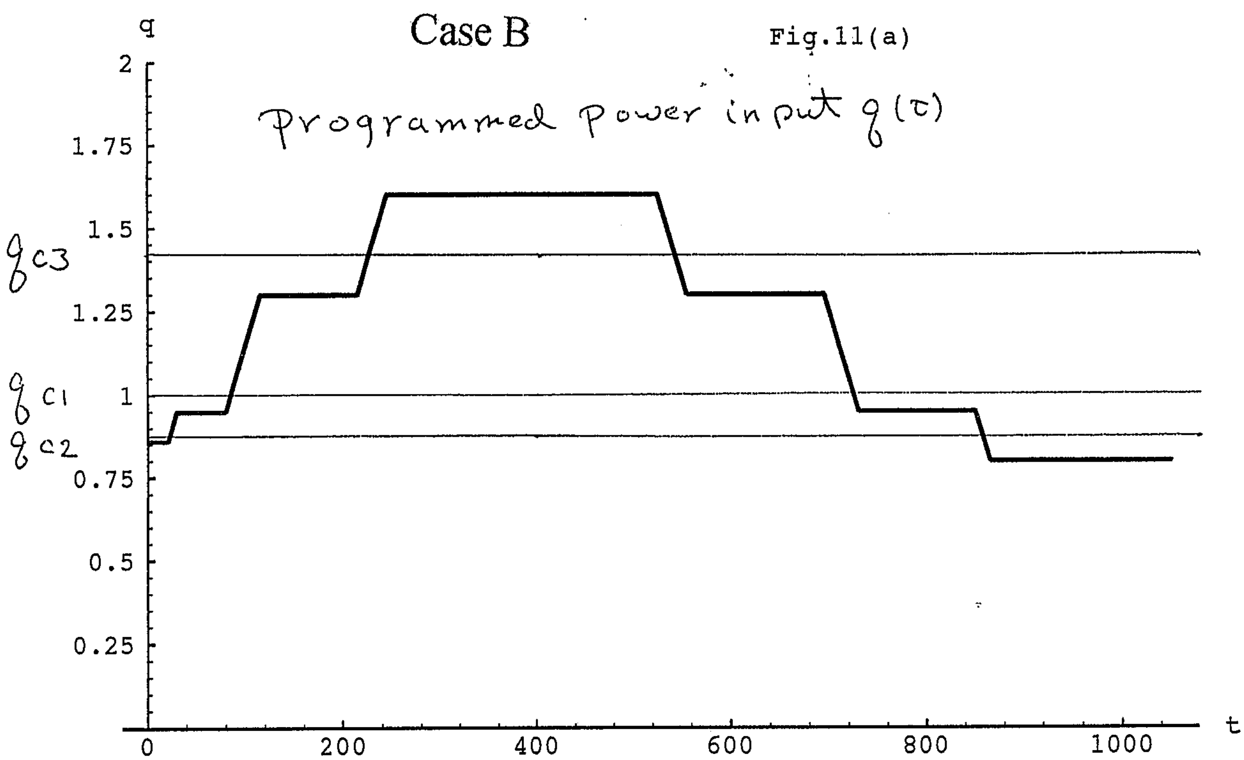
Figs. 10(a)-(b)

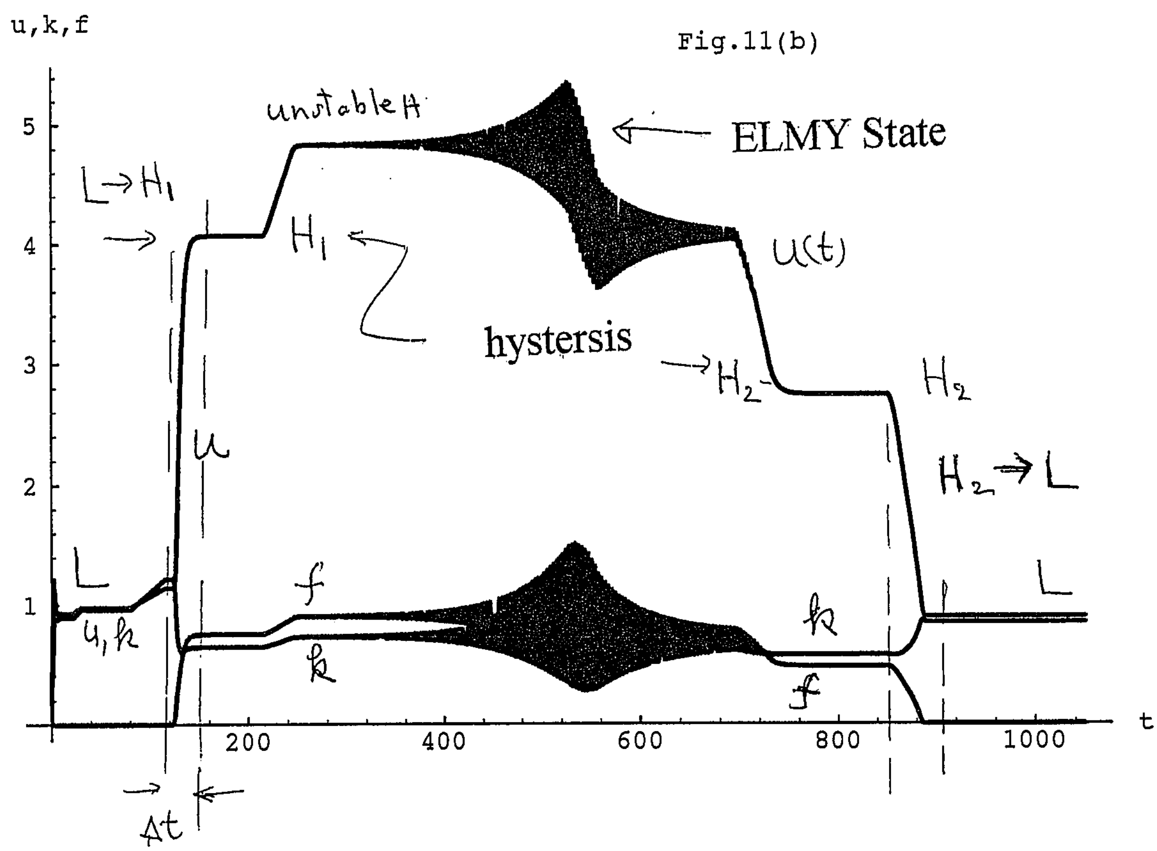
Case B $q = 1.6 > q_{c3}$



sawtoothy pressure



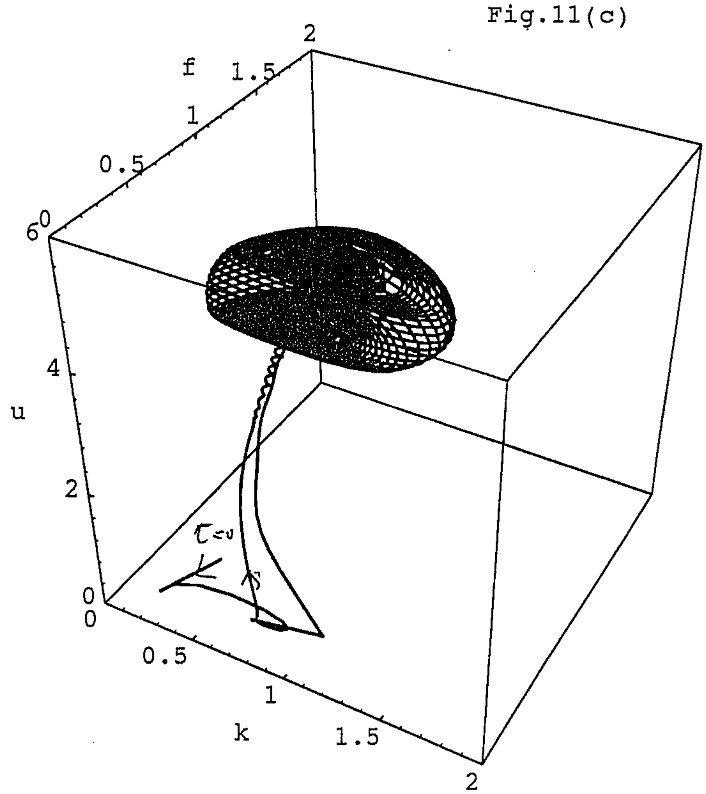

Case B
Fig.11(a)
Programmed power input q(τ)
q
t
q_{c3}
q_{c1}
q_{c2}
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
200
400
600
800
1000


u,k,f
Fig.11(b)
unstable H
ELMY State
L→H1
H1
u(t)
hystersis
→H2
H2
u
H2→L
L
f
k
u,k
Δt
t
5
4
3
2
1
200
400
600
800
1000

Figs. 11(a)-(b)

Case B

Programmed g(τ)



limit cycle