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Impurity Transport Studies in TEXT

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Abstract

The results of impurity transport in TEXT are compared with the predictions of a turbulence-based transport model. In the experiments, Sc was injected into the plasma using laser ablation and the time resolved profiles of critical Sc ionization stages were measured along with the potential fluctuation profile. The experiment was simulated using a 1-D radial transport code with the standard transport flux $\Gamma = -D\frac{\partial n}{\partial r} + nV$. The diffusion coefficient D and convective velocity V parameters were varied until the time dependent 1-D simulations reproduce the data. This representation for the empirical impurity transport is compared with the $\mathbf{E}\times\mathbf{B}$ turbulent diffusivities and mobility based on the fluctuation data and the measured radial electric field. The agreement is best with the $\mathbf{E}\times\mathbf{B}$ diffusivity taken in the strong turbulence regime where $D = c_D(\widetilde{\phi}/B_T)$, while the comparison with the weak turbulence diffusivity and the collisional (no fluctuations) diffusivity results in qualitative disagreements. The percolation theory diffusivity $(\widetilde{\phi}/B_T)^{7/10}(\Delta\omega/k_\perp^2)^{3/10}$ is also briefly discussed.

I Introduction

Predicting the transport of impurities in tokamak confinement systems is of key importance for projections of the operation of fusion systems. In addition, the analysis of the transport of impurity ions injected at sufficiently low levels of concentration that the collective modes responsible for the fluctuations are not materially altered provides important information for testing basic transport theory. Here we report the results of studies of the impurity transport experiments in which trace amounts of scandium are injected by laser ablation and the observed transport is compared with the $\mathbf{E} \times \mathbf{B}$ turbulent diffusivities for test particles. For this purpose experiments were performed to provide both impurity profile evolution data and fluctuation data in the same, and in similar, discharges. We restrict attention to those injection experiments where $n_z Z^2 \ll n_e$ so that the effect on the collective modes is predicted to be negligible. Thus, we may consider that the impurity scandium ions are being convected by the $\mathbf{E} \times \mathbf{B}$ fluctuation velocity as test particles.

The study of test particle (ion) transport by the turbulent $\mathbf{E} \times \mathbf{B}$ drifts in electric fields that model drift wave-type fluctuations has been an active area of theoretical research.[1–7] Rather well understood test particle diffusivities from the $\mathbf{E} \times \mathbf{B}$ drifts are reported in the extensive computer experiments in which the fully-nonlinear dynamics is taken into account.[1–4] The results show that once the fluctuation level exceeds a relatively low level, the diffusion leaves the weak turbulence regime in which $D_\perp = \widetilde{E}_\perp^2 \tau_c / B^2$ and enters the strong turbulence, or renormalized turbulence regime, in which $D_\perp \propto \widetilde{E}_\perp \lambda_\perp / B \sim \widetilde{\phi}/B$. At still higher fluctuation amplitudes, the diffusion increases even more slowly with the percolation theory limit[4,6,7] predicting $D_\perp \propto (\widetilde{\phi}/B)^{7/10} (\Delta\omega / k_\perp^2)^{3/10}$ where $\Delta\omega, k_\perp$ are the characteristic frequency band and mean wavenumber of the fluctuations. These predictions are tested numerically in Isichenko *et al.*[4] and compared with earlier works.[2,3,5,6] The three regimes of turbulent diffusivity are summarized in Table I along with the neoclassical Pfirsch-Schlüter regime collisional transport formula.

In addition to the diffusivity the inward radial electric field E_r produces a mobility μ component to the impurity flux that is also proportional to the fluctuation level. In collisional transport theory the mobility μ_z and diffusivity D_z are strictly related by the Einstein relationship $D_z = \mu_z T_z$. Due to the radial electric field $E_r = -d\Phi/dr < 0$, this value of the mobility produces a strong inward impurity flux such that the Boltzman relationship $n_z(r) = n_0 \exp(-Ze\Phi(r)/T_z)$ is an equilibrium state at zero impurity flux $\Gamma_z = 0$. In the turbulent $\mathbf{E} \times \mathbf{B}$ transport the mobility-to-diffusivity ratio can be much less than the value given by the Einstein relation. Since the theoretical studies for the effective mobility are not as complete as those for the diffusivity we report here the effective coefficient c_E (with E for Einstein relation) in the mobility relationship $\mu_z = c_E(D_z/T_z)$ that gives the best agreement with the evolution of the impurity density profiles.

In brief we report here that in the test particle regime the scandium injection experiment can be explained within a factor of 2 by the formulas

$$\Gamma_z = -D_z \frac{dn_z}{dr} + \mu_z n_z Ze E_r$$

with $D_z = c_D(\tilde{\phi}/B_T)$ and $\mu_z = c_\mu(\frac{\tilde{\phi}}{B_T T_z})$ with $c_D = 0.28 \pm 0.09$ and $c_\mu = 0.043 \pm 0.007$ for hydrogen as a working gas. For these values of the coefficients the deviation from the Einstein formula is given by $c_E = \mu T_z/D_z = c_\mu/c_D \simeq 0.2$ compared with unity. While for He, $c_D = 0.22 \pm 0.09$, $c_\mu = 0.16 \pm 0.07$, $c_E \simeq 0.7$. While the domains of applicability of the transport formulas are still an active area of theoretical research, these formulas are consistent with the present data set and with the extensive theoretical work and computer experiments performed for the nonlinear $\mathbf{E} \times \mathbf{B}$ drift motion of test particles in drift wave-type fluctuating electric fields.

Earlier analysis[7] of the fluctuation data in TEXT with respect to the regime of the

$\mathbf{E} \times \mathbf{B}$ transport has indicated that the strong nonlinearity condition $R_E = \Omega_E/\omega_k > 1$ is well satisfied. The actual value of $R_E \simeq 1.5$ to 2.5 where Ω_E is the rotation rate around the maximum and minimum of $\tilde{\phi}$ which for a fluctuation with $\mathbf{k} = (k_x, k_y)$ is approximately given by $\Omega_E = k_x k_y \tilde{\phi}/B \sim k_\perp^2 D$. In the regime $R_E > 1$, the nonlinearity determines the decorrelation time. Thus, the structure of the transport of trace amounts of impurity elements is providing an important comparison with strongly nonlinear regime of $\mathbf{E} \times \mathbf{B}$ turbulent transport. Transport experiments in a Q-machine that are also explained by the strongly nonlinear turbulent formula are reported by McWilliams *et al.*[8]

In contrast to neoclassical theory where the numerical coefficients such as c_D can be evaluated precisely by performing certain velocity space integral over Maxwellian distribution, the coefficients in the turbulent transport formulas can only be estimated. Even estimation of the numerical values for a given fluctuation spectrum requires extensive integrations of the chaotic $\mathbf{E} \times \mathbf{B}$ trajectories as given in Refs. 1–5. The value of c_D determined from the regression fits to the ensemble averaged diffusivity is a functional of the fluctuation spectrum. Some results for the sensitivity of c_D to changes in modeling of the fluctuation spectrum are shown in Horton *et al.*[9] and Kim *et al.*[10] In Kim *et al.*[10] the effect of Coulomb pitch angle scattering along with the turbulent $\mathbf{E} \times \mathbf{B}$ flow is reported and the neoclassical diffusivity is recovered at a sufficiently low fluctuation level.

II Injection and Thermalization of the Impurity Element

The laser ablation method creates a burst of scandium ions in a localized radial region. The scandium ions stream over the magnetic surface in time $qR/v_{zi} \simeq 10^{-4}\,\mathrm{s}$ and scatter in velocity space by Coulomb collisions with the working gas ions n_i, m_i, Z_i, T_i. This initial relaxation is given by the kinetic equation

$$\frac{\partial f_z}{\partial t} + \frac{v_{\parallel}}{qR}\frac{\partial f_z}{\partial \theta} = C_{zi}(f_z) = \frac{\partial}{\partial \mathbf{v}} \cdot \left(\frac{\mathbf{v} - \mathbf{u}_i}{\tau_s} f_z + D_v \frac{\partial f_z}{\partial \mathbf{v}} \right) \tag{1}$$

where the slowing-down time τ_s from the collisions with the thermal ions is given by

$$\frac{1}{\tau_s} = \frac{16\pi^{1/2}}{3} \frac{n_i e^4 Z^2 Z_i^2}{m_z m_i v_i^3} \ell n \Lambda \ . \tag{2}$$

In Braginskii Eq. (2) occurs as the impurity-ion collision frequency $\nu_{zi} = 1/\tau_s$ with

$$\nu_{zi} = \frac{4(2\pi)^{1/2}}{3} \frac{n_i e^4 Z^2 Z_i^2 \ell n \Lambda}{m_z m_i^{-1/2} T_i^{3/2}} \tag{3}$$

where Eq. (3) follows (2) with $v_i = (2T_i/m_i)^{1/2}$. For the hydrogen discharge at $n_i = 3 \times 10^{13}\,\mathrm{cm}^{-3}$, $T_i = 300\,\mathrm{ev}$, this time is $\tau_s \simeq 10^{-4}\,\mathrm{s}$ and for the helium discharge the time is approximately twice as long. The velocity diffusion coefficient in Eq. (1) is given by $D_v = T_i/(m_z \tau_s)$ so that in the absence of driving forces the impurity-ion collisions force $f_z(\mathrm{x}, \mathrm{v}, t)$ to $f_z = n_z(r,t) \exp[-m_z(\mathrm{v} - \mathrm{u}_i)^2/2T_i]$ for times long compared with τ_s. Here $\mathrm{u}_i(\mathrm{x}, t)$ and $T_i(\mathrm{x}, t)$ are the center of mass flow velocity and temperature of the working gas.

We note that collisional impurity transport formulas are often presented in terms of the ion-impurity collision frequency $\nu_{iz} = (n_z m_z/n_i m_i)\nu_{zi}$ and the thermal ion gyroradius $\rho_i = (2m_i T_i)^{1/2}/eZ_i B_T$ (rather than the gyroradius ρ_z of the impurity ions) which changes the appearance of the formulas. The relationships are given in the Appendix.

In the regimes of these experiments the impurity-ion collision frequency is high enough that the impurity mean free path $v_z \tau_s$ is less than qR so the collisional transport is in the Pfirsch-Schlüter regime with radial diffusion given in Table I. Typical values for these experiments with Sc are $\rho_z(Z = 18) \simeq 0.025\,\mathrm{cm}$, $\nu_{zi} = 1/\tau_s = 10^4\,/\mathrm{s}$ and $v_z \tau_s = 3 \times 10^6\,\mathrm{m/s} \cdot 10^{-4}\,\mathrm{s} = 300\,\mathrm{cm}$. For $q = 3$ the P-S diffusivity is then

$$\begin{aligned} D_z^{\mathrm{PS}} &= (1 + 2q^2)\nu_{zi}\rho_z^2 \\ &= (1 + 18)(0.025\,\mathrm{cm})^2 10^4\, s^{-1} \cong 120\,\mathrm{cm}^2/\mathrm{s} \end{aligned} \tag{4}$$

which is about one order of magnitude smaller than the observed rate of diffusion. Previous experiments[11,12] have attempted to use collisional transport theory[13] to explain impurity transport and have met with a similar difficulty. We note that $\nu_{zi}\,\rho_z^2$ is the same for all charge states Z and independent of m_z. These features of the collisional impurity transport[13] and the origin of the $1+2q^2$ factor from the equilibrium flows are given in the Appendix.

Once the fast transport processes (submillisecond) described by Eq. (1) have established the local impurity density $n_z(r,t=0) = n_z^0 \exp\left(-(r-r_0)^2/2\Delta r^2\right)$ localized to given radial region $(r_0 \pm \Delta r)$ and uniformly spread over the magnetic surface, the slower $\mathbf{E}\times\mathbf{B}$ convection process becomes effective in transporting the impurity ions. Taking the density moment of the impurity kinetic equation eliminates the thermal velocity spread of the impurity ions and gives the $\mathbf{E}\times\mathbf{B}$ transport problem. This reduction that eliminates all velocity space considerations depends on $\Delta\omega \gg k_{\|}\,v_z$. For the 10^4 to 10^5 Hz fluctuations under consideration this condition is satisfied due to high mass of the Sc ions. With $T_z \leq 300$ ev and $m_z = 45$ amu we have $v_z \lesssim 3\times10^6$ cm/s and using $|k_{\|}| \leq 1/qR$ gives $|k_{\|}|v_{zi} \lesssim 10^4$. Thus, this neglect of the parallel dynamics of the Sc ions may be the first limiting condition on the applicability of the 2D-$\mathbf{E}\times\mathbf{B}$ transport model. Parallel motion effects were considered in a related problem for electrons by Horton *et al.* and from this experience we believe the neglect of $k_{\|}\,v_z/\Delta\omega$ is justified here.

For a general $\mathbf{E}_{\perp}(\mathbf{x},t)$ field the exact motion of the particles has been studied extensively in theoretical works. The studies show that the initial distribution

$$n_z(\mathbf{x},t) = \sum_{j=1}^{N} \delta(\mathbf{x}-\mathbf{r}_j(t)) \tag{5}$$

with

$$\frac{d\mathbf{r}_j}{dt} = \frac{\mathbf{E}(\mathbf{r}_j,t)\times\mathbf{B}}{B^2} \tag{6}$$

will be well described by an effective diffusion coefficient D_x after the convection over several spatial correlation lengths $\ell_c = 1/\Delta k_{\perp}$. After this transit the impurity density spreads, in the absence of ionization and recombination, according to

$$\frac{\partial n_z}{\partial t} = \frac{-1}{r}\frac{\partial}{\partial r}(r\,\Gamma_z) \tag{7}$$

where

$$\Gamma_z = -D\,\frac{dn_z}{dr} + Ze\,n_z\,\mu\,E_r\ . \tag{8}$$

Now the precise determination of the effective diffusion coefficient D and mobility μ are difficult problems in nonlinear dynamics and depend on the spectral features of the fluctuations.[4–7]

Here we will use some of the results from the studies of Horton,[1,3] Kleva-Drake,[2] Misguich,[5] and in particular the recent results of Isichenko *et al.*[4] for the nonlinear $\mathbf{E}\times\mathbf{B}$ transport. We do not attempt to discuss the nonlinear dynamics theories here. The principle results for D are given in Table I. The main point is that for the measured fluctuation spectrum in TEXT the impurity dynamics is highly nonlinear with the decorrelation rate ν determined by the turbulent $\mathbf{E}\times\mathbf{B}$ motion itself rather than parallel transit motions or other effects. We express the associated mobility as

$$\mu = c_E\,\frac{D}{T_z} \tag{9}$$

where c_E is a constant between zero and unity. For $c_E = 1$ the mobility is given by the Einstein relation so that the $\Gamma_z = 0$ state corresponds to the Boltzmann impurity density profile $n_z = n_0\exp(-Ze\,\Phi(r)/T_z)$ when $E_r = -d\Phi/dr$. Some $\mathbf{E}\times\mathbf{B}$ nonlinear dynamics studies[3] indicate that the mobility may fall well below this value so that $c_E \ll 1$. Since c_E has not been accurately determined in the literature we leave the coefficient $c_E = c_\mu/c_D$ to be determined by the impurity transport data.

III Analysis of the Transport Experiment

The transport of Sc was measured in H and He discharges with common external control parameters, $B_T = 2.8\,\mathrm{T}$, $I_p = 250\,\mathrm{kA}$, $\overline{n}_e = 2\times10^{19}\,\mathrm{m}^{-3}$. In the experiment, a small quantity of Sc was injected into the plasma using laser ablation. The electron density perturbation was less than 1%. The observed temporal behavior of the Sc ionization stages is shown in Fig. 1. This figure illustrates that Sc is confined to the plasma for only a finite interval. After injection, the impurity trace is transported through the plasma and then out of the plasma through the last closed flux surface. On encountering a surface outside of the plasma, it sticks and is not recycled in significant quantity. For measurement of transport, the impurity spatial distributions are as important as the temporal data. Because Sc ($m = 45, Z = 21$) must be very close in its transport properties to Ti ($m = 48, Z = 22$), an ambient impurity, the spatial distributions for Ti can be combined with the temporal data for Sc in an appropriate analysis to provide the transport coefficients for either species. The radial distributions for Ti^{+14} and Ti^{+16} are shown in Fig. 2. Because the two ions together have significant concentration over the entire plasma and since they peak in different locations, they will adequately describe the spatial structure of Sc or Ti impurities. The analysis that is described below properly accounts for the small (atomic) differences in the two elements, Sc and Ti. In the analysis, the impurity transport of both Sc and Ti was simulated with a fixed background plasma model consisting of measured profiles of T_e and n_e. As described in Sec. II, a theoretically based turbulence model will be tested. The mean electrostatic potential $\overline{\phi}(r)$, and the root-mean-square fluctuating potential $\widetilde{\phi}(r)$ were measured with a heavy ion beam probe to evaluate the model for these discharges.

The impurity flux, Γ, is represented with the standard empirical formula

$$\Gamma = -D\frac{\partial n_z}{\partial r} - n_z V \tag{10}$$

where both D and V may be dependent on radius, but are taken independent of the ion species or ion charge. The impurity data is then simulated using the set of impurity transport equations

$$\frac{\partial n_z}{\partial t} + \nabla \cdot \Gamma = -n_e n_z (S_z + \alpha_z) + n_e n_{z-1} S_{z-1} + n_e n_{z+1} \alpha_{z+1} \tag{11}$$

where the S_z is the ionization rate coefficient for the z^{th} ionization stage of Sc and α_z is the recombination rate coefficient. The functions S_z and α_z depend principally on the $T_e(r)$ profile which is well-known from Thomson scattering and ECE emission in TEXT.

In the simulation there are two sets of equations, one for Ti and one for Sc. $D(r)$ and $V(r)$ are common to the two. The simulation of the temporal transient for Sc is shown in Fig. 3. The agreement between experiment and simulation is excellent for these two as it is for the intervening stages of ionization which are omitted for clarity of presentation. The comparison of measurement with the simulated radial distributions is equally good as shown in Fig. 2. Finally, the diffusion and the convection parameter are shown in Fig. 4 for the He discharge.

The predicted flux is

$$\Gamma_j = -D_j \left(\frac{\partial n_j}{\partial r} + c_E \frac{n_j e_j}{T_j} \frac{\partial \phi}{\partial r} \right) . \tag{12}$$

In terms of the components of the model flux used in the simulation

$$D = c_D \frac{\tilde{\phi}}{B_T} \tag{13}$$

$$V = c_\mu \frac{\tilde{\phi}}{B_T} \frac{e_j}{T_j} E_r \tag{14}$$

and the dimensionless ratio in Eq. (12) is

$$c_E = c_\mu / c_D . \tag{15}$$

The radial profile for $\tilde{\phi}$ was measured with a HIBP diagnostic. An example of the results is shown in Fig. 5. A comparison of the diffusivity from the simulation with theory is in Fig. 6. The shaded region in 6a represents the evaluation of D_{ps} for discharges near those studied here.

The dimensionless ratio c_D is in Fig. 7 for both H and He. Note that the parameter c_D varies over the outer 6 cm of the profile. The variation is about 30% for H and 40% for He.

Perhaps the most important feature here is that the magnitude of c_D is approximately the same for the two discharges. This lends credence to the use of Eq. (13) to represent the diffusion.

The convective velocity requires the radial electric field as well. This data was also taken with the HIBP. An example for H working gas is shown in Fig. 5b. In Fig. 8 is the comparison of theory with experiment for the convective velocity V. Note that the evaluation of Eq. (14) with $c_\mu = 1$ is shown.

The curve labeled "Eq. (16)" in Fig. 8, was calculated using

$$V = c_E \left(c_D \frac{\tilde{\phi}}{B} \right) \frac{e_j}{T_j} E_r \tag{16}$$

where c_D is in Fig. 7 and $c_E = 1$. Both of the theoretical representations have the shape found in the simulation. The magnitudes indicate that the mobility is well below the value given by the Einstein relation with $c_E = 1$. The factor c_μ is calculated for each of the two discharges with $c_D = 1$ and displayed in Fig. 9a. Employing the c_D of Fig. 7, c_μ is shown in Fig. 9b. In both cases, the value for H and He are disparate. Using the data of Figs. 7 and 9a, for H, $c_E = 0.2$ and for He, $c_E = 0.7$. On the basis of the limited data shown here, the mobility representation provides only a qualitative parameterization of the empirical V. Perhaps something might be learned from a more broadly based study.

Finally, we express the quality of the theoretical parameterization of the spectroscopically inferred value of D and V by showing the variation that is required in the constants c_D and c_μ introduced in Eqs. (13) and (14). The variations in c_D and c_μ required to obtain the radial profile for both hydrogen and helium as working gases is shown in Figs. 7 and 9. For the diffusivity essentially the same value of c_D works for hydrogen and helium, but there is systematic radial variation of c_D decreasing from 0.3 to 0.1 over the radial range of $r = 16$ cm to 24 cm.

IV Summary and Conclusions

In the two cases presented here, we conclude that the predictions of $\mathbf{E} \times \mathbf{B}$ turbulent transport of the impurities explains the measured impurity transport. In this work, extensive temporally and spatially resolved impurity measurements were first collapsed to just two parameters, $D(r)$ and $V(r)$. These were then compared to a turbulence-based transport model consisting of a strong turbulence diffusivity via the Einstein relation.

The empirical diffusivity is consistent with well-known $\mathbf{E} \times \mathbf{B}$ turbulent diffusivity.[1–4] This diffusivity is linearly proportional to fluctuating electrostatic potential and is determined to within a constant factor that depends on the details of the turbulent spectrum. In each of two discharges studied here, essentially the same spatially dependent factor was found. For H, the spatially averaged coefficient of the turbulent diffusivity is $c_D = 0.22 \pm 0.09$ and for He, $c_D = 0.28 \pm 0.08$. Previous efforts[11,12] to explain the impurity transport using neoclassical formulas have led to severe difficulties. Because of this initial success, it is worthwhile to extend these comparisons to a wider range of experimental conditions.

Appendix – Equilibrium Flows of the Impurities

Once the impurities are released, they flow on the magnetic surface to set upon equilibrium poloidal flow pattern that gives rise to the Pfirsch-Schlüter enhancement factor of $1 + 2q^2$. Since the flows are fast and of fundamental importance we review the key equations for the establishment of these flows.

Designating the working gas by 1 and the impurity ions by 2 we have

$$e_1 n_1 \left(\mathbf{E} + \frac{\mathbf{v}_1}{c} \times \mathbf{B}\right) - \nabla p_1 - m_1 n_1 \nu_{12}(\mathbf{u}_1 - \mathbf{u}_2) = 0 \tag{A1}$$

$$e_2 n_2 \left(\mathbf{E} + \frac{\mathbf{v}_2}{c} \times \mathbf{B}\right) - \nabla p_2 - m_2 n_2 \nu_{21}(\mathbf{u}_2 - \mathbf{u}_1) = 0 \tag{A2}$$

with $m_1 n_1 \nu_{12} = m_2 n_2 \nu_{21}$. Taking the perpendicular and parallel components we have

$$n_1 \mathbf{u}_1 = \frac{n_1 \mathbf{B}}{B^2} \times \left[-\mathbf{E} + \frac{\nabla p_1}{e_1 n_1} + \frac{m_1 n_1 \nu_{12}}{e_1 n_2}(\mathbf{u}_1 - \mathbf{u}_2)\right] \tag{A3}$$

and

$$e_1 n_1 E_{\|} - \nabla_{\|} p_1 = m_1 n_1 \nu_{12}(u_{1\|} - u_{2\|}) \tag{A4}$$

with the appropriate interchange of 1 and 2 for the second ion species.

Now the continuity equation in the toroidal magnetic field forces the parallel flow to be established on the time scale qR/v_T such that

$$\frac{B_\theta}{rB}\frac{\partial}{\partial\theta}(n_1 u_{\|1}) = -\nabla \cdot \left[\frac{n_1 \mathbf{B}}{B^2}\left(-\mathbf{E} + \frac{\nabla p_1}{e_1 n_1}\right)\right] = \frac{2\sin\theta}{BR}\left(n_1 E_r - \frac{1}{e_1}\frac{\partial p_1}{\partial r}\right)$$

giving

$$n_1 u_{\|1} = -\frac{2r\cos\theta}{B_\theta R}\left(n_1 E_r - \frac{1}{e_1}\frac{\partial p_1}{\partial r}\right) . \tag{A5}$$

Computing $u_{\|1} - u_{\|2}$ from Eq. (A5) and substituting into (A4) gives the variation on the

$$\frac{1}{Br}\frac{\partial}{\partial\theta}(e_1 n_1 \Phi + p_1) = \frac{m_1 n_1 \nu_{12}}{B_\theta^2 R}(2r\cos\theta)\left(\frac{p_1'}{e_1 n_1} - \frac{p_2'}{e_2 n_2}\right) \tag{A6}$$

and

$$\frac{1}{Br}\frac{\partial}{\partial\theta}\left(e_2 n_2 \Phi + p_2\right) = \frac{m_2 n_2 \nu_{21}}{B_\theta^2 R}\left(2r\cos\theta\right)\left(\frac{p_1'}{e_1 n_1} - \frac{p_2'}{e_2 n_2}\right) . \tag{A7}$$

Now the poloidal E_θ and $\partial p_{1,2}/\partial\theta$ forces generate the radial convective fluxes given by the radial component of Eq. (A3). From Eqs. (A3) and (A6)–(A7) we obtain

$$(n_1 \mathbf{u}_1)_r = -\frac{B_\phi}{B^2}\left(1 + \frac{2r\,B^2\cos\theta}{R\,B_\theta^2}\right)\left(\frac{m_1 n_1 \nu_{12}}{e_1 B}\right)\left(\frac{p_1'}{e_1 n_1} - \frac{p_2'}{e_2 n_2}\right) \tag{A8}$$

$$(n_2 \mathbf{u}_2)_r = -\frac{B_\phi}{B^2}\left(1 + \frac{2r\,B^2\cos\theta}{R\,B_\theta^2}\right)\left(\frac{m_1 n_1 \nu_{12}}{e_2 B}\right)\left(\frac{p_2'}{e_2 n_2} - \frac{p_1'}{e_1 n_1}\right) . \tag{A9}$$

These formulas show that there is ambipolarity in the ion-impurity flux before making the surface average. The formulas also give the full r-θ pattern of convection from E_θ and $\partial p_{1,2}/\partial\theta$ before making the flux surface average. Once the surface average $dS = r d\theta (R_0 + r\cos\theta) d\phi$ is carried out by integrating over dS, the direct collisional random walk flux and the toroidal flow enhancement combine as

$$D_z^{ps} = (1 + 2q^2)\nu_{zi}\,\rho_z^2 \tag{A10}$$

where D_z^{ps} is defined such that

$$\Gamma_z = -D_z^{ps}\frac{1}{T_z}\left(\frac{\partial p_z}{\partial r} - \frac{n_z Z}{n_i Z_i}\frac{\partial p_i}{\partial r}\right) .$$

Formula (A10) is derived by Rutherford[12] with a generalization to include the thermo-electric effect in the frictional force in Eq. (A1) which he takes as $\mathbf{R}_{iz} = -c_1\left(\frac{m_i n_i}{\tau_{iz}}\right)(\mathbf{v}_i - \mathbf{v}_z) - c_2 n_i \nabla_\parallel T_i$ with the values of c_1, c_2 first order rational polynomials in $\alpha = n_z Z^2/n_i$. The Rutherford formula for the impurity particle flux is

$$\Gamma_z = -\frac{n_i \rho_i^2 q^2}{T_i Z^2 n_z \tau_{iz}}\left[(c_1 + c_2^2/c_3)\left(\frac{\partial p_z}{\partial r} - \frac{n_z Z}{n_i}\frac{\partial p_i}{\partial r}\right) - \frac{5}{2}\frac{c_2}{c_3}(n_z Z)\frac{\partial T_i}{\partial r}\right] \tag{A11}$$

where c_3 arises from the parallel ion thermal conductivity $q_{i\parallel} = c_2 n_i T_i u_\parallel - c_3(n_i T_i \tau_{iz}/m_i)\nabla_\parallel T_i$. In the trace impurity experiments of interest here the terms proportional to n_z/n_i in Eq. (A11) can be dropped and the Rutherford formula reduces to that used in the text.

Now the question arises as to how this impurity flux depends on the charge state and the mass of the impurity ions. The $\mathbf{E} \times \mathbf{B}$ turbulent flux is independent of Z and m_z provided $k_\perp \rho_z \ll 1$ and $k_\| v_z/\omega \ll 1$. Now using Eq. (3) we find that

$$\rho_z^2 \nu_{zi} = \frac{m_z T_z}{e^2 Z^2 B^2} \cdot \frac{n_i e^4 Z_i^2 Z^2 \ell n \Lambda}{m_z m_i^{-1/2} T_i^{3/2}} \tag{A12}$$

so that both m_z and Z^2 drop out. Thus, all charge states of Sc and Ti etc. will be transported at the same rate from the collisions with the working gas ions.

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Figure Captions

1. Temporal evolution of the chord-averaged emission for several charge states of Sc after laser injection into the He plasma at 0.3s.

2. Radial distributions for Ti^{+14} (open circles) and Ti^{+16} (filled circles) in a He plasma. The solid lines represent the results of the simulation.

3. Comparison of injection measurement (points with error bars) to injection simulation for Sc^{+12} and Sc^{+18} in a He plasma. The results of the simulations are shown by solid and dashed lines.

4. The empirical diffusivity and the convection parameter inferred from the impurity data in the H.

5. The radial profile of $\tilde{\phi}$, (a) the fluctuating part and (b) E_r derived from the mean value $\overline{\phi}$ of the electrostatic potential, for the H working gas. The data was taken with a heavy ion beam probe.

6. A comparison of Eq. (13) ($c_D = 1$) with experimental D. The working gas is H (a) and for He (b). For comparison, an evaluation of D_{ps} is represented by the shaded region in (a).

7. The radial variation of the diffusivity parameter c_D defined in Eq. (13). The mean and standard deviation over the radial range shown are given in the box.

8. Comparison of the experimental convection parameter to the mobility prediction bsed on the turbulent diffusivity defined in Eq. (14) ($c_\mu = 1$) and to the mobility prediction with $c_\mu = 1$ and using the c_D in Fig. 7.

9. The comparison parameter c_μ versus minor radius. This quantity is the ratio of the empirical convection to that predicted from the turbulent diffusivity prediction (a) and to that predicted from the empirical diffusivity (b) i.e. includes c_D of Fig. 7. The mean and standard deviation over the radial range shown are given in the box.

Table I

Summary of Theoretical Diffusivities

Quasilinear Theory

$$D_x = \sum_{\mathbf{k}} \frac{E_y^2}{B_T^2} \tau_c(\mathbf{k}) \qquad k_\perp \tilde{v}_E \ll \Delta\omega$$

Renormalized Turbulence Theory

$$D_x = \frac{\tilde{E}_y}{k_\perp B} = c_D \frac{\tilde{\phi}}{B_T} \qquad k_\perp \tilde{v}_E \gtrsim \Delta\omega$$

Percolation Theory

$$D_x = \left(\frac{\tilde{\phi}}{B_T}\right)^{7/10} \left(\frac{\Delta\omega}{k_\perp^2}\right)^{3/10} \qquad k_\perp \tilde{v}_E \gg \Delta\omega$$

Collisional Diffusion (Pfirsch-Schlüter Regime)

$$D_x = (1 + 2q^2)\rho_z^2 \nu_{zi}$$

Note $\rho_z^2 \nu_{zi}$ is often rewritten as $\nu_{ii} \rho_i^2$ using $\nu_{zi} = (Z^2 m_i / Z_i^2 m_z)\nu_{ii}$ and $T_z = T_i$.

Table II

Parameters for Scandium in Hydrogen and Helium Plasmas

$$m_z = 45\,\mathrm{amu}$$

$$Z = 12, 16, 17, 18$$

$$n_z \leq 2 \times 10^7\,\mathrm{cm}^{-3}$$

For $Z = 18$ and $m_z = 45\,\mathrm{mH}$ we have

$$\nu_{zi} = 10^4/\mathrm{s}$$

$$\rho_z = 0.03\,\mathrm{cm}$$

$$v_z = 3 \times 10^6\,\mathrm{cm/s}$$

Note the working gas is taken as hydrogen at $T_i = 300\,\mathrm{eV}$, and impurities are ${}_{45}\mathrm{Sc}^{21}$ ($3d4s^2$) and 48 Ti^{22} ($3d^24s^2$).