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ELECTROMAGNETIC DRIFT MODES DRIVEN BY ION PRESSURE GRADIENTS IN TOKAMAKS

Wendell Horton
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712
and
Duk-In Choi and Bong-Guen Hong
Korea Advanced Institute
of Science and Technology
Chongyangi, Seoul
Korea

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Wendell Horton
Institute for Fusion Studies
and
Department of Physics
The University of Texas at Austin
Austin, Texas 78712
United States of America

Duk-In Choi and Bong-Guen Hong
Department of Physics
Korea Advanced Institute of Science and Technology
Chongyangni, Seoul,
Korea

Abstract

A hybrid of hydrodynamics and kinetics is used to study the effect of finite plasma pressure on the ion pressure-gradient driven toroidal drift modes. The linear drift modes of the system are given by a fifth-order polynomial describing the coupling of the electron drift, the ion acoustic, and the shear Alfvén oscillations. The characteristic frequencies, growth rates, and polarization of the electromagnetic modes are investigated as a function of the parameters of toroidicity, plasma gradients, and plasma pressure.

I. INTRODUCTION

The ion pressure-gradient driven modes in toroidal plasma have been the subject of numerous recent linear[1-4] and nonlinear[5-7] studies. The relevance of the modes to the beam-heated tokamaks with high ion temperature gradients is well recognized[1-7], and the modes may be relevant to the fluctuations observed during high-power heating conditions.[8,9] In addition, a thermonuclear reacting plasma with an ion temperature well above the electron temperature may be subject to a form of the ion pressure gradient instability.

The previous studies[1-7] use the electrostatic approximation due to the fact that the maximum growth rate is well below the shear Alfvén wave transit frequency for low beta plasmas. To be applicable to the higher plasma pressure achieved with auxiliary or alpha particle heating the theory must take into account the electromagnetic components of the instability.

General formulations of the electromagnetic drift mode problem derived from the linearized Vlasov-Maxwell system of self-consistent field equations are given by Antonsen and Lane[10], Tang *et al.*[11], Hastie and Hesketh[12], Rewoldt *et al.*[13], Cheng[14], and Itoh *et al.*[15] with different emphases. When the linear mode equations are reduced by using the ballooning mode theory for the structure of the toroidal eigenmodes, the problem becomes solvable including many, if not all, of the kinetic theory effects even in the actual finite pressure toroidal equilibrium. Due to the large number of physical phenomena contained in the kinetic equations and the toroidal equilibrium, the mode equations are complicated and difficult to solve. Some important aspects of the stability problems do not depend sensitively on the inclusion of the kinetic effects. An example of the degree of dependence on kinetic effects is found in the study of Terry et al.[4] which compares kinetic growth rates and frequencies with hydrodynamic growth rates and frequencies in the electrostatic dispersion relation.

In this work we formulate the problem of electromagnetic drift modes using the hybrid description based on hydrodynamic and kinetic equations. We thus restrict ourselves to the main features of the electromagnetic drift modes. The formulation presented here neglects the wave-particle resonances, the difference between trapped and circulating particles, and does not correctly describe the frequency dispersion for perpendicular wavelengths comparable with the ion gyroradius. The advantages of the hybrid approach, however, are the clear physical interpretations of the features of the dynamics contained in the moment equations and the possibility of extending the study into the nonlinear regime with nonlinear partial differential equations.

The organization of the paper is as follows. In Section II the electromagnetic fields are defined and the reduced formulas for the plasma currents required for Maxwell's equations are determined. Using the momentum balance across the magnetic field, the divergence and the rotational parts of the perpendicular plasma currents are computed. Total pressure balance across the magnetic field is solved for the parallel component of the magnetic oscillation. In Section III the gyrokinetic equations are linearized, and the parallel components of the plasma currents are computed. With these expressions for the plasma currents, two alternative forms of the reduced electromagnetic drift mode equations are derived in Section IV. In Section V, the dispersion relation for the coupled low frequency oscillations is derived and the appropriate dimensionless parameters are defined within the local approximation. The transformation of the frequencies, growth rates and polarizations are investigated with variation of the plasma pressure for the five low-frequency modes. Section VI contains the summary and conclusions.

II. FORMULATION

We use a representation[16] of the electric field of the form

$$\underset{\sim}{E} = -\nabla\phi - \nabla \times \left(a\hat{b}\right) - \frac{1}{c}\left(\frac{\partial A_{\parallel}}{\partial t}\right)\hat{b} \tag{1}$$

with $\hat{b} = \underset{\sim}{B}/B$. Fourier analyzing in time as $\exp(-i\omega t)$ and operating with $\hat{b}\cdot$, $\nabla_{\perp}\cdot$, and $\hat{b}\cdot\nabla\times$ on Ampere's law we have

$$\nabla_{\perp}^2 \frac{\partial\psi}{\partial s} = -i\left(\frac{4\pi\omega}{c^2}\right)\delta J_{\parallel} \tag{2}$$

$$\frac{\partial}{\partial s}\nabla_{\perp}^2 \frac{\partial\psi}{\partial s} = i\left(\frac{4\pi\omega}{c^2}\right)\nabla_{\perp}\cdot\delta\underset{\sim}{J} \tag{3}$$

and

$$\nabla_\perp^2 \, \delta B_\parallel = -\frac{4\pi}{c} \hat{b} \cdot \nabla \times \delta \underset{\sim}{J}_\perp \quad . \tag{4}$$

In Eqs. (2) through (4) the field ψ is related to the parallel component of the vector potential $A_\parallel$ by

$$\frac{\partial \psi}{\partial s} = i \frac{\omega}{c} A_\parallel = \hat{b} \cdot \nabla \psi \tag{5}$$

with $E_\parallel = -\hat{b} \cdot \nabla (\phi - \psi)$. The parallel component of the magnetic perturbation $\delta B_\parallel$ is given by

$$i \frac{\omega}{c} \, \delta B_\parallel = \nabla_\perp^2 a \quad . \tag{6}$$

The coupled mode equations are obtained when we express the perturbed plasma currents on the right-hand sides of Eqs. (2) through (4) in terms of the field quantities $(\phi, a, A_\parallel)$.

We first compute the cross-field current from the momentum balance equation solved for $\underset{\sim}{J}_\perp$,

$$\underset{\sim}{J}_\perp = \frac{c\hat{b}}{B} \times \left[\nabla p + m_i n_i \left(\frac{\partial}{\partial t} + \underset{\sim}{u} \cdot \nabla \right) \underset{\sim}{v}_E \right] \tag{7}$$

where the $\underset{\sim}{u}$ is given by the sum of the $\underset{\sim}{E} \times \underset{\sim}{B}$ velocity $\underset{\sim}{v}_E$ and the diamagnetic velocity $\underset{\sim}{v}_d$,

$$\underset{\sim}{u} = \underset{\sim}{v}_E + \underset{\sim}{v}_d = \frac{c\underset{\sim}{E}\times\hat{b}}{B} + \frac{c\hat{b}\times\nabla p}{eBn_i} \quad . \tag{8}$$

The finite Larmor radius (FLR) part of the stress tensor is properly taken into account in this reduction as shown by Hinton and Horton.[17] The linearization of the perpendicular current gives

$$\delta \underset{\sim}{J}_\perp = \frac{c}{B_0}\hat{b}_0 \times \left[\nabla\delta p - \frac{\delta B_\parallel}{B_0}\nabla p_0 + m_i n_0 \left(\frac{\partial}{\partial t} + \underset{\sim}{v}_{d0}\cdot\nabla\right)\underset{\sim}{v}_E\right] \quad . \tag{9}$$

The zero suffix denotes the unperturbed quantities which are dropped in the following unless necessary to avoid confusion. The prefix δ denotes the perturbed quantities. We operate with $\nabla\cdot$, and $\hat{b}\cdot\nabla\times$ on Eq. (9) to obtain

$$\nabla\cdot\delta\underset{\sim}{J}_\perp = \frac{c}{B}\nabla\delta p\cdot\left(\hat{b}\times\frac{\nabla B}{B} + \nabla\times\hat{b}\right) - c\nabla\times\left(\frac{\delta B_\parallel\hat{b}}{B^2}\right)\cdot\nabla p$$

$$\frac{-i\omega m_i nc^2}{B^2}\left(1 - \frac{\omega_{*pi}}{\omega}\right)\nabla_\perp\cdot\underset{\sim}{E}_\perp \tag{10}$$

and

$$\hat{b}\cdot\nabla\times\delta\underset{\sim}{J}_\perp = \nabla\cdot\left(\frac{c}{B}\nabla_\perp\delta p_\perp\right) - c\nabla_\perp\cdot\left(\frac{\delta B_\parallel}{B^2}\nabla_\perp p\right)$$

$$-i\omega m_i\left(1 - \frac{\omega_{*pi}}{\omega}\right)\nabla_\perp\cdot\left(\frac{cn\underset{\sim}{v}_E}{B}\right) \quad . \tag{11}$$

Retaining the contribution from the first term in Eq. (11) and substituting it into the right-hand side of Eq. (4), we obtain the pressure balance relationship between $\delta p_\perp$ and $\delta B_\parallel$,

$$\nabla_\perp^2 \left(B\delta B_\parallel + 4\pi\delta p_\perp \right) = 0$$

or

$$B\delta B_\parallel + 4\pi\delta p_\perp = \text{const.} \tag{12}$$

The pressure balance equation (12) replaces the third component Eq. (4) of the original three coupled-mode equations (2 - 4). The compressional magnetic field fluctuation $\delta B_\parallel$ is expressed in terms of the perpendicular pressure fluctuation $\delta p_\perp$. In order to obtain closure for the coupled-mode equations, we need expressions for $\delta J_\parallel$ and $\delta p_\perp$ in terms of the fields ϕ and ψ .

III. PARALLEL CURRENT AND PRESSURE FLUCTUATIONS FROM KINETIC THEORY

In order to compute the parallel current fluctuation for Eq. (2), we solve the linearized drift kinetic equation obtained from

$$\frac{\partial f}{\partial t} + \left(\underset{\sim}{v}_\parallel + \underset{\sim}{v}_E + \underset{\sim}{v}_D \right) \cdot \nabla f + \left[\frac{e}{m} \underset{\sim}{E} \cdot \left(\underset{\sim}{v}_\parallel + \underset{\sim}{v}_D \right) + \mu \frac{\partial B}{\partial t} \right] \frac{\partial f}{\partial \varepsilon} = 0 ,$$

with

$$\underset{\sim}{v}_D = \frac{\hat{b}}{\Omega} \times \left[\mu \nabla B + v_\parallel^2 (\hat{b} \cdot \nabla) \hat{b} \right] \tag{13}$$

and $\varepsilon = v^2/2$ and $\mu = v_\perp^2/2B$.

Specifying the unperturbed equilibrium distribution function as the local Maxwellian,

$$F_0 = F_M = n_0(m/2\pi T)^{3/2} \exp(-m\varepsilon/T) ,$$

we obtain the solution of the linearized equation

$$\delta f = -\frac{e}{T}\phi F_0 + F_0 \frac{(\omega - \omega_{*t})}{(\omega - k_\parallel v_\parallel - \omega_D)} \left[\frac{e}{T}\left(\phi - \frac{v_\parallel}{c} A_\parallel\right) + \frac{m}{T}\mu\delta B_\parallel\right]$$

where we defined

$$\omega_{*t} = \omega_* \left(1 - \frac{3}{2}\eta + \frac{m\varepsilon}{T}\eta\right)$$

$$\omega_D = \underset{\sim}{k} \cdot \underset{\sim}{v}_D$$

$$\omega_* = \underset{\sim}{k}_\perp \cdot \hat{b} \times \nabla \ln n_0 \left(\frac{cT}{eB}\right)$$

$$\eta = \frac{d\ln T}{d\ln n_0} \tag{14}$$

and the species index j is suppressed. We define $\tau = T_e/T_i$.

Noting that the mode frequencies of interest lie between the electron and ion transit frequencies ($\omega_{ti} < \omega < \omega_{te}$ with $\omega_{tj} = v_j/L_c$), we expand the denominators in the appropriate smallness parameter for each species. We obtain for electrons

$$\delta f_e = \phi F_{0e} - F_{0e}\left(1 - \frac{\omega_{*te}}{\omega}\right)\left\{\psi - \left[\phi - \frac{m_e v_\perp^2}{2T_e}\left(\frac{\delta B_\parallel}{B}\right)\right]\frac{\omega}{k_\parallel v_\parallel}\right\}$$

$$\cdot\left[1 + \left(\frac{\omega - \omega_{De}}{k_\parallel v_\parallel}\right) + (\quad)^2 + \ldots\ldots\right] , \tag{15a}$$

and for ions

$$\delta f_i = -\tau\phi F_{0i} + F_{0i}\left(1 - \frac{\omega_{*ti}}{\omega}\right)\left[\tau\left(\phi - \frac{k_\parallel v_\parallel}{\omega}\psi\right) + \frac{m_i v_\perp^2}{2T_i}\left(\frac{\delta B_\parallel}{B}\right)\right]$$

$$\times\left(1 - \frac{\omega_{Di}}{\omega}\right)^{-1} \times \left\{1 + \left[\frac{k_\parallel v_\parallel}{\omega}\left(1 - \frac{\omega_{Di}}{\omega}\right)^{-1}\right] + (\quad)^2 + \ldots\right\} \tag{15b}$$

The potential fields ϕ and ψ in this expression and hereafter are normalized to T_e/e. As noted in the introduction, the solutions in the form of Eq. (15) preclude the full kinetic effects such as the ion drift resonance, the finite Larmor radius effects and the trapped particles. We restrict ourselves here to the main features of the modes without these effects. We now compute the parallel current fluctuation $\delta J_\parallel$ and the perpendicular pressure fluctuation $\delta p_\perp$ from these solutions. We obtain

$$\delta J_\parallel = n_0 e\frac{\omega}{k_\parallel}\left[\left(1 - \frac{\omega_{*e}}{\omega}\right)(\psi - \phi) + \left(1 - \frac{\omega_{*pe}}{\omega}\right)\left(\frac{\delta B_\parallel}{B} - \frac{\omega_{\nabla B}^e + \omega_\kappa^e}{\omega}\psi\right)\right.$$

$$+ \left(\frac{k_\parallel c_s}{\omega}\right)^2 \left(1 - \frac{\omega_{*pi}}{\omega}\right)(\phi - \psi)\Bigg] \tag{16}$$

and

$$\delta p_\perp = p_e\left[\left(1 - \frac{\omega_{*pi}}{\omega}\right)\phi - \left(1 - \frac{\omega_{*pe}}{\omega}\right)\psi\right] . \tag{17}$$

A more general form of this equation is given in Eq. (18) of Ref. 16. The various drift frequencies of the pressure gradient, the curvature, and grad B are given by

$$\omega_{*pj} = \omega_{*j}(1 + \eta_j)$$

$$\omega_\kappa^j = \frac{cT_j}{e_j B} \hat{b} \times \hat{b}\cdot\nabla\hat{b} \cdot \underset{\sim}{k}_\perp$$

$$\omega_{\nabla B}^j = \frac{cT_j}{e_j B} \hat{b} \times \frac{\nabla B}{B} \cdot \underset{\sim}{k}_\perp \quad .$$

We can now express $\delta B_\parallel / B$ in terms of ϕ and ψ by substituting Eq. (17) into Eq. (12)

$$\frac{\delta B_\parallel}{B} = -\frac{\beta_e}{2}\left[\left(1 - \frac{\omega_{*pi}}{\omega}\right)\phi - \left(1 - \frac{\omega_{*pe}}{\omega}\right)\psi\right] \tag{18}$$

with $\beta_j = 8\pi p_j/B^2$. Making use of Eqs. (17) and (18) in Eqs. (10) and (16), we compute $\nabla \cdot \delta \underset{\sim}{J}_\perp$ and $\delta J_\parallel$ in terms of ϕ and ψ . We have

$$\nabla \cdot \delta\underset{\sim}{J}_\perp = i n_0 e 2\omega_\kappa^e\left[\left(1 - \frac{\omega_{*pe}}{\omega}\right)\psi - \left(1 - \frac{\omega_{*pi}}{\omega}\right)\phi\right]$$

$$+ \frac{i\omega n_0 T_e m_i c^2}{eB^2}\left(1 - \frac{\omega_{*pi}}{\omega}\right)\nabla_\perp^2\phi \tag{19}$$

and

$$\delta J_\parallel = \frac{n_0 e\omega}{k_\parallel}\left\{1 - \frac{\omega_{*e}}{\omega} + \left(1 - \frac{\omega_{*pi}}{\omega}\right)\left[\frac{\beta_e}{2}\left(1 - \frac{\omega_{*pe}}{\omega}\right) - \frac{k_\parallel^2 c_s^2}{\omega^2}\right](\psi - \phi)\right.$$

$$\left. - \frac{2\omega_\kappa^e}{\omega}\left(1 - \frac{\omega_{*pe}}{\omega}\right)\psi\right\}. \tag{20}$$

In the computation of Eq. (19) for the contribution from the diamagnetic current, we make use of the equilibrium force balance relationship,

$$\nabla\left(\frac{B^2}{8\pi} + p\right) = \frac{1}{4\pi}\,\underset{\sim}{B}\cdot\nabla\underset{\sim}{B} \quad , \tag{21}$$

to combine the first three terms of Eq. (10) to express

$$\nabla_\perp \cdot \delta\underset{\sim}{J}^d = \frac{c}{B}\nabla\delta p_\perp \cdot \left[\hat{b} \times \frac{1}{B^2}(\underset{\sim}{B}\cdot\nabla)\underset{\sim}{B} + \nabla\times\hat{b}\right]$$

$$= \frac{2c}{B}\nabla(\delta p_\perp)\cdot\hat{b} \times (\hat{b}\cdot\nabla)\hat{b}$$

$$= -2i\,\omega_\kappa^e\left(\frac{e\delta p_\perp}{T_e}\right). \tag{22}$$

Based on Eq. (21) it is also easily demonstrated that the following identities hold:

$$\frac{\beta_e}{2}\left(\omega_{*pe} - \omega_{*pi}\right) + \omega^e_{\nabla B} = \omega^e_\kappa$$

and

$$\frac{\beta_e}{2}\left(\omega_{*pe} - \omega_{*pi}\right) - \tau\omega^i_{\nabla B} = -\tau\omega^i_\kappa \tag{23}$$

with $\tau = T_e/T_i$.

We also used Eq. (23) in the reduction of Eq. (20).

IV. THE COUPLED MODE EQUATIONS

We now substitute Eqs. (20) and (19) into the right-hand sides of Eqs. (2) and (3) respectively. We obtain

$$\rho^2\frac{\partial}{\partial s}\nabla^2_\perp\frac{\partial\psi}{\partial s} + \frac{2\omega^e_\kappa\omega}{v_A^2}\left(1 - \frac{\omega_{*pe}}{\omega}\right)\psi$$

$$+ \frac{\omega^2}{v_A^2}\left\{\left(1 - \frac{\omega_{*e}}{\omega}\right) + \left(1 - \frac{\omega_{*pi}}{\omega}\right)\left[\frac{\beta_e}{2}\left(1 - \frac{\omega_{*pe}}{\omega}\right) - \frac{k_\parallel^2 c_s^2}{\omega^2}\right]\right\}(\phi - \psi) = 0 , \tag{24}$$

and

$$\rho^2\frac{\partial}{\partial s}\nabla^2_\perp\frac{\partial\psi}{\partial s} + \frac{2\omega^e_\kappa\omega}{v_A^2}\left[\left(1 - \frac{\omega_{*pe}}{\omega}\right)\psi - \left(1 - \frac{\omega_{*pi}}{\omega}\right)\phi\right]$$

$$+ \frac{\omega^2}{v_A^2}\left(1 - \frac{\omega_{*pi}}{\omega}\right)\rho^2\nabla_\perp^2\phi = 0 \,. \tag{25}$$

We define the Alfvén velocity $v_A = B/(4\pi m_i n_0)^{1/2}$ and the cross-field wavelength unit $\rho = c(m_i T_e)^{1/2}/eB$. The coupled Eqs. (24) and (25) are the basic equations describing the electromagnetic fields ϕ and ψ . However, an alternative presentation of the equations is useful. Choosing the quasi-neutrality constraint

$$\frac{\partial}{\partial s}\delta J_\parallel + \nabla_\perp \cdot \delta \underset{\sim}{J}_\perp = 0 \tag{26}$$

that follows from $\partial/\partial s$ of Eq. (2) subtracted from Eq. (3), a convenient alternate pair of equation results.

By combining Eqs. (24) and (25), making use of Eq. (26), we obtain the quasi-neutrality relationship of

$$\left\{1 - \frac{\omega_{*e}}{\omega} + \left(1 - \frac{\omega_{*pi}}{\omega}\right)\left[\frac{\beta_e}{2}\left(1 - \frac{\omega_{*pe}}{\omega}\right) - \frac{k_\parallel^2 c_s^2}{\omega^2}\right]\right\}(\phi - \psi)$$

$$- \left(1 - \frac{\omega_{*pi}}{\omega}\right)\left(\rho^2\nabla_\perp^2\phi - \frac{2\omega_\kappa^e}{\omega}\,\phi\right) = 0 \,. \tag{27}$$

Now the parallel component equation of the Ampere's law of Eq. (24) can be simplified by Eq. (27) to obtain

$$\frac{\partial}{\partial s}\left(\frac{v_A^2\rho^2}{\omega^2}\nabla_\perp^2\frac{\partial\psi}{\partial s}\right)+\frac{2\omega_\kappa^e}{\omega}\left(1-\frac{\omega_{*pe}}{\omega}\right)\psi+\left(1-\frac{\omega_{*pi}}{\omega}\right)\left(\rho^2\nabla_\perp^2\phi-\frac{2\omega_\kappa^e}{\omega}\phi\right)=0\ . \tag{28}$$

We may now take the quasi-neutrality condition Eq. (27),and the parallel component of Ampere's law, Eq. (28),as the fundamental coupled equations describing the electromagnetic fields ϕ and ψ . This system of equations is equivalent to that of Eqs. (24) and (25). However, we can easily see that by letting $\psi = 0$ and $\beta_e = 0$ in Eq. (27) that we recover the electrostatic mode equation.[1] Thus, the system of Eqs. (27) and (28) is more convenient in certain cases.

V. ELECTROMAGNETIC DRIFT WAVE DISPERSION RELATION

In this section we consider that the axial eigenvalue problem has been solved to obtain modes localized to the regions of the bad curvature drifts. We do not consider in detail the conditions required for ballooning of the wave functions in the regime of the bad curvature. The ballooning problem is considered in Refs. 1-3 for the electrostatic equation and in Refs. 11-15 for the electromagnetic equations. Physically, there is appreciable ballooning when the local growth rate exceeds the frequency for the parallel group velocity to propagate wave energy from the outside to the inside of the torus. The dispersion relation for a ballooning mode is given approximately by the local dispersion relation evaluated on the outside of the torus where ω_κ obtains its maximum unfavorable value.

To obtain the local dispersion relationship, we put $\partial/\partial s \to ik_\parallel$ and $\nabla_\perp^2 \to -k_\perp^2$ in the modal equations. From Eqs. (27) and (28) we obtain

$$\left\{1 - \frac{\omega_{*e}}{\omega} + \left(1 - \frac{\omega_{*pi}}{\omega}\right)\left[\frac{\beta_e}{2}\left(1 - \frac{\omega_{*pe}}{\omega}\right) - \frac{\omega_s^2}{\omega^2}\right]\right\}(\phi - \psi)$$

$$+ \left(1 - \frac{\omega_{*pi}}{\omega}\right)\left(k_\perp^2\rho^2 + \frac{2\omega_\kappa^e}{\omega}\right)\phi = 0 \tag{29}$$

and

$$\left[k_\perp^2\rho^2\left(\frac{\omega_A^2}{\omega^2}\right) + \frac{2\omega_\kappa^e}{\omega}\left(1 - \frac{\omega_{*pe}}{\omega}\right)\right]\psi = \left(1 - \frac{\omega_{*pi}}{\omega}\right)\left(k_\perp^2\rho^2 + \frac{2\omega_\kappa^e}{\omega}\right)\phi \tag{30}$$

with

$$\omega_A = k_\parallel v_A \quad \text{and} \quad \omega_s = k_\parallel c_s \,.$$

Rewriting the quasi-neutrality Eq. (29) as

$$A(\omega)\phi = B(\omega)\psi \tag{31}$$

and the parallel component of Ampere's law of Eq. (30) as

$$C(\omega)\phi = D(\omega)\psi \;, \tag{32}$$

the dispersion relation, a fifth order polynomial in ω, is given by

$$AD - BC = 0 \tag{33}$$

and the polarization relation is given by

$$\alpha(\omega) = \frac{\psi}{\phi} = \frac{A(\omega)}{B(\omega)} = \frac{C(\omega)}{D(\omega)} . \tag{34}$$

The quantities A through D are

$$A = \omega^2(\omega - \omega_{*e}) + (\omega - \omega_{*pi}) \left[k_\perp^2\rho^2\omega^2 + 2\omega_\kappa^e\omega + \frac{\beta_e}{2}\omega(\omega - \omega_{*pe}) - \omega_s^2\right]$$

$$B = \omega^2(\omega - \omega_{*e}) + (\omega - \omega_{*pi}) \left[\left(\frac{\beta_e}{2}\right)\omega(\omega - \omega_{*pe}) - \omega_s^2\right]$$

$$C = (\omega - \omega_{*pi})\,(k_\perp^2\rho^2\omega + 2\omega_\kappa^e)$$

$$D = k_\perp^2\rho^2\omega_A^2 + 2\omega_\kappa^e(\omega - \omega_{*pe}) .$$

An alternative form of the dispersion relation to that in Eq. (33) is obtained by combining Eqs. (29) and (30) as

$$\left[\frac{\omega-\omega_{*e}}{\omega-\omega_{*pi}} - \frac{\omega_s^2}{\omega^2} + \frac{\beta_e}{2}\left(1 - \frac{\omega_{*pe}}{\omega}\right)\right] \times \left[1 - \frac{(\omega-\omega_{*pi})(\omega k_\perp^2\rho^2+2\omega_\kappa^e)}{k_\perp^2\rho^2\omega_A^2+2\omega_\kappa^e(\omega-\omega_{*pe})}\right]$$

$$= -\left(k_\perp^2\rho^2 + 2\,\frac{\omega_\kappa^e}{\omega}\right) . \tag{35}$$

When the curvature drift is neglected, i.e. $\omega_\kappa^e \simeq 0$, Eq. (35) reduces to the well known (e.g. Eq. (8) of Ref. 18) dispersion relation for the coupled drift waves and shear Alfven waves by the finite ion gyroradius effect. The generalized version of that dispersion relation is Eq. (35) which describes the coupling of β corrected drift waves and curvature modified shear Alfven waves by the two effects of finite ion gyroradius and curvature.

The basic dimensionless parameters of the system are $\varepsilon_n = r_n/R$, $\beta = 8\pi p/B^2$ and the order unity parameters $\tau = T_e/T_i$, η_i and η_e . All frequencies are measured in units of c_s/r_n and the cross-field wavenumber $\underset{\sim}{k}_\perp$ in units of $\rho = c(m_i T_e)^{1/2}/eB$. The dimensionless frequencies in Eq. (35) are

$$\omega_{*e} = k, \qquad \omega_{*pe} = k(1 + \eta_e), \qquad \omega_\kappa^e = k\varepsilon_n$$

$$\omega_{*pi} = -\frac{k(1 + \eta_i)}{\tau}, \qquad \omega_s = \frac{\varepsilon_n}{q}, \qquad \omega_A = \left(\frac{2}{\beta_e}\right)^{1/2}\left(\frac{\varepsilon_n}{q}\right) \tag{36}$$

Following Ref. 16 we define three plasma pressure regimes (A) $\beta \sim \varepsilon_n^2$ where $\omega_A \sim 1$, (B) $\beta \sim \varepsilon_n$ where $\omega_A \sim \varepsilon_n^{1/2}$ and (C) $\beta \sim 1$ where $\omega_A \sim \omega_s \sim \varepsilon_n$.

In the limit $k \to 0$ the dispersion relation reduces to

$$(k_\perp\rho)^2\omega\left[\omega^2\left(1 + \tfrac{1}{2}\beta_e\right) - \omega_s^2\right]\left(\omega^2 - \omega_A^2\right) = 0$$

with the five roots $\omega = 0$, $\omega = \pm\omega_s/[1 + \beta_e/2]^{1/2}$ and $\omega = \pm\omega_A$. For general k we solve the fifth-order polynomial numerically. For each root the polarization $\alpha = \psi/\phi$ defined in Eq. (34) is calculated. The polarization of the mode is defined as (1) electrostatic, ES, for $|\alpha| < 1/2$, as (2), magnetohydrodynamic, MHD, if $|\alpha - 1| < 1/2$

and (3) electromagnetic, EM, if $|\alpha| > 3/2$. In this manner the five roots $\omega = \omega_\ell(k)$ and polarizations $\alpha_\ell(\omega)$ are calculated as shown in Fig. 1.

A. Low β Regime

In the low plasma pressure regime $\beta \sim \varepsilon_n^2$ the dispersion relation decouples into an electrostatic branch with $|\alpha| \ll 1$ given by

$$\omega^2\left(1 + k^2\right) - \omega k\left[1 - 2\varepsilon_n - \frac{k^2}{\tau}\left(1 + \eta_i\right)\right] + k^2(2\varepsilon_n))\frac{\left(1 + \eta_i\right)}{\tau} = 0 \qquad (37)$$

which yields the fluid approximation to the electrostatic pressure gradient driven drift wave instability derived from Vlasov theory.[2-4] In this plasma regime the MHD branch with $\alpha \simeq 1$ is given by

$$\omega\left(\omega - \omega_{*pi}\right) - \omega_A^2 = 0 \ . \qquad (38)$$

with stable oscillation at

$$\omega(k) = \frac{1}{2}\,\omega_{*pi} \pm \left(\omega_A^2 + \frac{1}{4}\,\omega_{*pi}^2\right)^{1/2} .$$

We define[1,4] the electron drift wave phase velocity

$$u_k = \frac{\left[1 - 2\varepsilon_n - \frac{k^2}{\tau}\left(1 + \eta_i\right)\right]}{1 + k^2} \qquad (39)$$

from Eq. (37) and the characteristic electrostatic growth parameter $\gamma_0 = \left[\frac{2\varepsilon_n(1+\eta_i)}{\tau}\right]^{1/2}$. The electrostatic drift mode is unstable for slow phase velocity where $u_k^2 < 4\gamma_0^2/(1+k^2)$ occurring for k centered around $k_0 = [\tau(1-2\varepsilon_n)]^{1/2}/(1+\eta_i)^{1/2}$. The electrostatic frequency and growth rate $\omega = \omega_k + i\gamma_k$ are given by

$$\omega(k) = \frac{1}{2} k u_k \qquad \gamma(k) = \pm k\left(\frac{\gamma_0^2}{1+k^2} - \frac{1}{4} u_k^2\right)^{1/2} . \qquad (40)$$

The maximum growth rate is $\gamma_m = (2\varepsilon_n)^{1/2}$ and occurs at $k_\theta\rho = k_0 \simeq [\tau/(1+\eta_i)]^{1/2}$.

B. Moderate β Regime

With increasing plasma pressure the Alfvén frequency $\omega_A = (2/\beta_e)^{1/2}(\varepsilon_n/q)$ decreases to couple with the drift mode frequencies. In particular, for $\beta \sim \varepsilon_n/q^2$ the Alfvén frequency drops to the maximum electrostatic growth rate $\gamma_m = (2\varepsilon_n)^{1/2} \sim \omega_A$.

With the plasma pressure in regime (B) where $\beta \sim \varepsilon_n$, the dispersion relation remains complicated. Collecting the terms which define the drift wave phase velocity u_k, as given by Eq. (39) for low β, we obtain

$$(1+k^2)u_k = 1 - 2\varepsilon_n - \frac{k^2}{\tau}(1+\eta_i) - \frac{(2\varepsilon_n)\ [1+\eta_e+\frac{1}{\tau}(1+\eta_i)]}{\omega_A^2} \qquad (41)$$

which shows that increasing plasma pressure lowers the phase velocity of the drift wave-like oscillation.

The critical wavenumber k_0 for vanishing phase velocity is

$$k_0 = \left[\frac{1 - 2\varepsilon_n[1 + \eta_e + \frac{1}{\tau}(1+\eta_i)]/\omega_A^2}{(1 + \eta_i)/\tau} \right]^{1/2} . \tag{42}$$

As the plasma pressure increases, the critical wavelength $2\pi/k_0$ goes to infinity. In the present model, in which the FLR parameter ρ/r_n is vanishingly small, this transition means that the mode transforms from wavelengths that scale with ρ to low-order azimuthal modes with finite $k_\theta r \simeq m \simeq \ell q$. From Eq. (42) the critical beta β_c for the onset of instability in the low order azimuthal modes is

$$\beta_c = \frac{\varepsilon_n(1+\frac{1}{\tau})}{[1+\eta_e+\frac{1}{\tau}(1+\eta_i)]q^2} . \tag{43}$$

where we use $\beta = \beta_e(1+1/\tau)$. As β approaches β_c from below, the fastest-growing mode moves from the microscopic wavelength $1/k_0 \sim \rho[(1 + \eta_i)/\tau]^{1/2}$ to the macroscopic wavelength of order r_n. During the transition, the growth rate remains of order $\varepsilon_n^{1/2}$.

For $\beta > \beta_c$ given by Eq. (43) the dispersion relation contains the approximate magnetohydrodynamic modes $(\phi = \psi)$ given by

$$\omega\left[\omega + \frac{k}{\tau}(1 + \eta_i)\right] + (2\varepsilon_n)\left[1 + \eta_e + \frac{1}{\tau}(1 + \eta_i)\right] - \omega_A^2 = 0 . \qquad (44)$$

The unstable oscillations $\omega = \omega(k) + i\gamma(k)$ occur at

$$\omega(k) = -\frac{k}{2\tau}(1 + \eta_i)$$

$$\gamma(k) = \pm\left[(2\varepsilon_n)\left[1 + \eta_e + \frac{1}{\tau}(1 + \eta_i)\right] - \omega_A^2 - \frac{k^2}{4\tau^2}(1 + \eta_i)^2\right]^{1/2} \qquad (45)$$

in the wavenumber range $0 \leqslant k \leqslant k_m$ where

$$k_m = 2\left[\frac{(2\varepsilon_n)\left[1 + \eta_e + \frac{1}{\tau}(1 + \eta_i)\right] - \omega_A^2}{(1 + \eta_i)/\tau}\right]^{1/2} . \qquad (46)$$

The maximum growth rate occurs at $k = 0$ with $\gamma = \gamma_m$ where

$$\gamma_m = \left\{2\varepsilon_n\left[1 + \eta_e + \frac{1}{\tau}(1 + \eta_i)\right]\right\}^{1/2}(1 - \beta_c/\beta)^{1/2} . \qquad (47)$$

For $\beta \gg \beta_c$ this maximum growth rate γ_m exceeds the low plasma pressure growth rate by the factor $\left[1 + \eta_e + \frac{1}{\tau}(1+\eta_i)\right]^{1/2}$.

In the moderate plasma pressure regime $\beta \sim \varepsilon_n$ there remains an essentially electrostatic instability connected with ion-acoustic waves with $\omega_s < \omega_{*pi}$ and a stable electromagnetic electron drift wave. Both these modes are characterized by small $\delta j_{\parallel}(k,\omega)$ and $E_{\parallel} \neq 0$. For the electrostatic oscillations $|\phi| \gg |\psi|$ and $\delta j_{\parallel}(k,\omega) \simeq 0$. The mode has $\omega \simeq \omega^{i}_{\nabla B}$ and $\gamma(k) = \pm\omega_s(1+\eta_i)^{1/2}$ and should be treated with kinetic theory except in the regime $\tau(1+\eta_i) \gg 1$.

The remaining mode is an electromagnetic or inductively polarized drift wave with $|\psi| \gg |\phi|$. Taking $\alpha \gg 1$ in the second factor of the left hand side from Eq. (35) we have

$$\omega^2 - (\omega_{*e} + 2\omega^e_\kappa)\omega + 2\omega^e_\kappa\omega_{*pe} - k^2_\perp\rho^2\omega^2_A = 0$$

from which we obtain the stable electromagnetic drift wave oscillation

$$\omega(k) = \omega_{*e} + 2\omega^e_\kappa + \frac{k^2_\perp\rho^2\omega^2_A}{\omega_{*e}} \qquad \text{and} \qquad \gamma(k) = 0.$$

In the dimensionless variables the mode is $\omega(k) \cong k(1+2\varepsilon^2_n/\beta_e q^2)$.

The transitional region near $\beta \sim \beta_c$ is the region where the five modes are strongly coupled through the fifth-order equation and the polarizations are of mixed character. We proceed to examine this regime numerically. For $\varepsilon_n = 0.25$, $q = 2$, $rq'/q = 1$, and $\eta_i = \eta_e = \tau = 1$ we solve for $\omega_\ell(k)$, $\gamma_\ell(k)$, and $\alpha_\ell(k)$ for $\ell = 1,2,3,4$ and 5 as beta varies from below β_c to above β_c. From Eq. (43), $\beta_c = 0.032$.

Figure 1 shows the transition of the modes, given by the fifth-order polynomial derived from the hybrid model, occurs as the local plasma β varies from below the MHD beta critical to above the MHD beta critical. For reference, we note that often-used measure of danger $\gamma(k)/k^2$ from an instability increases from approximately $0.3/(.5)^2 = 1.2[D_0]$ at $\beta = 0.02$ to $0.4/(.1)^2 = 40[D_0]$ at $\beta = 0.03$. Here, $D_0 = (\rho/r_n)(cT_e/eB)$, the drift wave diffusion scale factor of anomalous transport theory. The divergence of γ/k^2 in the case (c) where $\beta = 0.05$ can be interpreted as meaning that $k^2 \to \rho/r_n$ and thus $\gamma/k^2 \to cT_e/eB$, or a qualitatively faster plasma loss rate prevails.

VI. SUMMARY AND CONCLUSIONS

The low frequency stability of the tokamak system is investigated on the basis of hybrid dynamical equations. The cross-field plasma currents are derived from the perpendicular component of the momentum balance equations in terms of oscillating fields $\underset{\sim}{E}$, $\delta\underset{\sim}{B}$ and pressure fluctuation δp_j . The oscillating densities δn_j and pressures δp_j required for quasi-neutrality and the cross-field currents are derived from the linearized drift-kinetic equation.

In the non-resonant, fluid limit we show that the self-consistent field equations reduce to a fifth-order polynomial describing the low-frequency modes of the confined plasma. At low plasma pressure $\beta \sim \varepsilon_n^2$ and high plasma pressure $\beta \sim 1$, the modes separate into a cubic equation describing three electrostatic oscillations and a quadratic equation for the MHD modes. For intermediate plasma pressure $\beta \sim \varepsilon_n$ the roots of the fifth-order polynomial are strongly interacting.

For plasma pressure approaching the MHD critical beta β_c the wavelength of the fastest-growing mode increases and the polarization of the unstable mode becomes mixed. In this transitional regime it is essential to retain the interaction of the roots of the fifth-order system. In the transitional regime the mode frequencies are of the same order and the system <u>cannot</u> be factored into the quadratic equation of FLR-MHD theory and a cubic equation.

For plasma pressure above the MHD critical beta β_c by a factor of approximately two $(\beta \gtrsim 2\beta_c)$, the fifth-order system again factors into the $E_\parallel = 0$ MHD equations and a cubic equation. The cubic equation describes the inductive $(\phi = 0)$ electron drift wave and the electrostatic $(\psi = 0)$ ion acoustic waves. The ion acoustic-drift waves are unstable for $\eta_i \gg 1$ with growth rates of order $\gamma_s \simeq (\varepsilon_n/q)[\frac{(1+\eta_i)}{\tau}]^{1/2}[c_s/r_n]$ which is smaller by $\varepsilon_n^{1/2}$ than the FLR-MHD instability with $\gamma_m \sim (2\varepsilon_n)^{1/2}[1+\eta_e+\frac{1}{\tau}(1+\eta_i)]^{1/21/2}[c_s/r_n]$ and $E_\parallel = 0$.

We conclude that, the hybrid model contains more of the physical processes occurring in the low-frequency plasma dynamics than the idealized FLR-MHD theory. The hybrid dynamics predicts the same critical value of plasma pressure β_c, Eq. (43), as MHD theory, but its meaning is different from that of MHD theory. In the hybrid model the system is <u>unstable</u> both above and below the critical beta. The MHD beta critical in the hybrid model marks the point at which the growth rate $\gamma(k)$ maximized over k moves from a wavelength determined by the ion inertial scale length (finite $k\rho$) to a wavelength independent of the ion inertial scale length ($k\rho = 0$ and finite $kr = m$).

The change in the physical meaning of the critical plasma pressure may be useful for the experimental interpretation of the low-frequency fluctuations and anomalous transport in high pressure plasmas.

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Figure Caption

Fig. 1 The frequencies and growth rates as a function of $k = k_\theta \rho$ for ballooning modes with average $k_\parallel r_n = \varepsilon_n / q$ for the transitional values of the plasma pressure β. The local plasma parameters are $\tau = \eta_i = \eta_e = 1$, $q = 2$ and $\varepsilon_n = 0.25$.

ELECTROMAGNETIC DRIFT MODES
FIFTH ORDER POLYNOMIAL