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Filamentation, Current Profiles and Transport in a Tokamak

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Abstract

A Tokamak with slightly imperfect magnetic surfaces should have a microscopically filamented current structure. If so, its equilibrium has an exact analog in the dynamics of interacting charged rods. Then there will be a natural current-profile, analogous to thermal equilibrium of the rods (and the natural profile can be calculated by conventional statistical mechanics). This would account for the phenomenon of profile consistency or resilience in Tokamaks. In addition to the natural profiles, this filamentary model also predicts an anomalous inward flux of both heat and particles in a Tokamak, as well as an anomalous diffusion. These 'inward-pinch' components are related to the current gradient.

I Introduction

In the theory of plasma relaxation,[1] a turbulent magnetized plasma reaches a state of minimum energy subject only to constant magnetic helicity. This theory has been successful in predicting or interpreting the magnetic field profiles, and other features, of many plasma confinement experiments — including the Toroidal Pinch, Spheromak and Multipinch. However, one configuration not described by the theory is the Tokamak — which in normal circumstances does not exhibit relaxation. In fact, relaxation in a Tokamak occurs only at a so-called disruption — when (in accord with theory) it leads to the well-known negative voltage spike. This is presumably because only at a disruption does the Tokamak plasma experience the strong turbulence which leads to the relaxed state.

Nevertheless, despite the fact that it does not fully relax, there is evidence that a normal Tokamak also has a preferred 'natural profile' (though the evidence is less conclusive than in the fully relaxed experiments mentioned above). This tendency towards a preferred profile is often called profile 'consistency' or 'resilience.'[2, 3, 4, 5, 6]

There have been several attempts[7, 8, 9, 10, 11] to account for preferred tokamak profiles in terms of some minimum principle similar to relaxation, but these have not gained general acceptance. The reason is that there has been no obvious mechanism which would lead to the proposed state. (In this regard it is important to recognize that relaxation is not a mathematical principle involving virtual displacements; it involves physical displacements which break lines of force, invalidate all MHD constraints except magnetic helicity, and dissipate energy.)

In this paper I show how one may arrive at a preferred, or natural, Tokamak profile *using only accepted physical principles.* These particular profiles are similar to ones proposed previously on the basis of ad hoc variational principles and the main purpose of the present paper is to provide a justification for them. However the present work also provides an interesting new interpretation of Tokamak fluctuations and leads to a new result — it predicts an inward particle and heat pinch related to the current profile.

The basic Tokamak model is described in Sec. 2. In Sec. 3 an analogy is developed between Tokamak profiles and the equilibrium of a realisable dynamical system. Then in Sections 4 and 5 natural Tokamak current profiles are obtained by applying conventional statistical mechanics to this analog. The implications for plasma transport are described in Sec. 6. Section 7 describes some extensions of the basic model.

II Tokamak Equilibria

For a plasma in equilibrium or moving slowly compared to Alfvén speed, the magnetic field satisfies

$$\nabla \times (\mathbf{J} \times \mathbf{B}) = 0 \tag{1}$$

and in the conventional Tokamak ordering (large aspect ratio, low-β) this reduces to

$$B_0 \frac{\partial J}{\partial z} + \mathbf{B}_\perp \cdot \nabla J = 0 \ , \tag{2}$$

where $J = J_z$ is the toroidal current. We also have

$$\mathbf{B}_\perp \simeq \widehat{\mathbf{z}} \times \boldsymbol{\nabla}\psi \tag{3}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \equiv \nabla_\perp^2 \psi = J \ . \tag{4}$$

We take Eqs. (2)–(4) as our basic model of Tokamak equilibria.

If the magnetic field possesses regular magnetic surfaces the model leads only to the standard equilibrium condition that J must be an arbitrary function of ψ. If, at the other extreme, the magnetic field is completely ergodic, then (since Eq. (2) implies that J is constant along $\mathbf{B}$) the current must be uniform throughout the torus. This is indeed the fully relaxed state of a large aspect-ratio Tokamak and accords with the concept that turbulence leads to breaking of all magnetic surfaces. However if the level of chaos in the magnetic field is small (so that a field line wanders only slowly from some ideal surface) then the conclusion that J is uniform throughout the torus is an unrealistic interpretation of Eq. (2). In reality small effects omitted from the model mean that the uniformity of J along $\mathbf{B}$ is valid only over a limited length of field line. (One might say that a field line 'forgets' its initial value of J before it gets far from its starting point.) Thus a more realistic interpretation of Eq. (2) is one in which J is locally uniform along $\mathbf{B}$ (i.e. on a microscopic scale) but varies on the macroscopic scale. In this event the current density must be highly irregular on the microscopic scale and in the form of filaments aligned with the local $\mathbf{B}$. (If the current were smooth but nonuniform at some toroidal cross-section, then mapping it along the chaotic field would produce an irregular filamented current elsewhere.) As one follows a filament along $\mathbf{B}$ its area is constant but its cross-section becomes more and more convoluted — until it diffuses or decays away. At the same time fresh filaments are generated by the action of the chaotic $\mathbf{B}$ on the macroscopic current gradient. (This is similar to the 'Clump' picture of MHD turbulence described and analyzed in detail by Tetreault[12])

III The Dynamical Model

From the picture described above we extract only the concept that the field is slightly chaotic and that as a result the current is microscopically filamented. For simplicity we assume for the present that all filaments have similar magnitude j_0. Then we make one more important conceptual step. We regard the coordinate z as a time, and $\mathbf{B}_\perp$ as a velocity. Then the configuration $\mathbf{r}_i(z) = (x_i(z), y_i(z))$ of the filaments is given by the Hamiltonian equations

$$j_0 \frac{dx_i}{dz} = \frac{1}{B_0}\frac{\partial H}{\partial y_i} \qquad j_0 \frac{dy_i}{dz} = \frac{-1}{B_0}\frac{\partial H}{\partial x_i} \tag{5}$$

$$H = \sum_{i<k} j_0^2\, U(\mathbf{r}_i, \mathbf{r}_k) \ , \tag{6}$$

where

$$\nabla_{\perp}^{2} U(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}') \tag{7}$$

i.e. $U(\mathbf{r}_i, \mathbf{r}_k)$ is the potential at $\mathbf{r}_i$ due to a unit strength filament at $\mathbf{r}_k$. (In an infinite region $U = \log|\mathbf{r}_i - \mathbf{r}_k|$ but we need to consider a bounded system so we leave U unspecified at present.) Apart from a scale factor the Hamiltonian H is the energy of the system — and energy is conserved. The current distribution

$$J = j_0 \sum_i \delta(\mathbf{r}-\mathbf{r}_i) \tag{8}$$

and the magnetic flux

$$\psi = j_0 \sum_i U(\mathbf{r},\mathbf{r}_i) \tag{9}$$

depend on z through the $\mathbf{r}_i(z)$, but their statistical averages are independent of z — corresponding to stationary equilibria if z were time (but see Sec. (7C) for an important exception).

Equations (5)–(7) have an easily visualized interpretation. Apart from a change of sign in the Hamiltonian they are identical to the equations of motion for a set of electrically charged rods moving with $\mathbf{E}\times\mathbf{B}$ velocities in a strong parallel magnetic field.[13, 14] (or the motion of parallel vortices in a 2-D Euler fluid.[15, 16, 17]). We could even restore the sign of the Hamiltonian if we envisaged that the rods also had a gravitational attraction which exceeded the electrostatic repulsion. Thus, the Tokamak equilibrium problem has an exact analog in the motion of a mechanical system. *We can therefore apply the methods of conventional statistical mechanics to it.* However it is important not to lose sight of the fact that Eqs. (5)–(7) do not describe the time evolution of the Tokamak filaments: they describe their configuration at a fixed time. It should also be noted that the constraint of periodicity in z (in order to represent a toroidal plasma) has been ignored. When there are many small filaments, only small displacememnts would be needed to make the current periodic in any configuration of statistical equilibrium. (It is not necessary to make individual filaments periodic if we regard them as indistinguishable.)

IV The Statistical Mechanics

Conventional statistical mechanics can be applied to our model in several ways according to ones preference in basic postulates. As energy is strictly conserved an appropriate description is the microcanonical ensemble, when the joint probability distribution for N filaments is given by

$$\rho\{\mathbf{r}_i\} \sim \delta\left[E - H\{\mathbf{r}_i\}\right] \ , \tag{10}$$

where $\{\mathbf{r}_i\} = (\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_N)$ denotes the positions of all the filaments. Then for a large number of filaments N, the entropy

$$S = -k \int n(\mathbf{r}) \log\, n(\mathbf{r}) d\mathbf{r} \tag{11}$$

is a logarithmic measure of the number of microscopic configurations corresponding to a macroscopic distribution $n(\mathbf{r})$. Statistical equilibrium is found by maximizing S subject to fixed energy

$$E = j_0 \int n(\mathbf{r})\, \psi(\mathbf{r}) d\mathbf{r} \ , \tag{12}$$

where

$$\psi(r) = j_0 \int U(\mathbf{r}, \mathbf{r}') n(\mathbf{r}') d\mathbf{r}' \ , \tag{13}$$

and fixed number of filaments

$$N = \int n(r) d\mathbf{r} \ . \tag{14}$$

This leads to an equilibrium current distribution

$$J(\mathbf{r}) = j_0 \langle n(\mathbf{r}) \rangle = K \exp\left(-\beta j_0 \psi(\mathbf{r})\right) \ , \tag{15}$$

where β arises as a Lagrange multiplier.

This result, of course, is just the 'most probable state' calculated by Montgomery, Turner and Vahala[18] in an early paper. However their work had a quite different motivation and concerned different basic models. (They considered axisymmetric equilibria $\mathbf{B} = (0, B_\theta(r), B_z(r))$, $\mathbf{j} = (0, j_\theta(r), j_z(r))$ in which B_z and j_z were somewhat arbitrarily discretized, and 2-D magneto-hydrodynamics $\mathbf{B} = (B_x, B_y, 0)$, $\mathbf{v} = (v_x, v_y, 0)$ in which the current j_z and the vorticity ω_z were discretized. In both models all quantities are independent of z. In contrast the present work concerns a quasi-2-D model, which need not be axisymmetric, and in which the third dimension z plays a crucial role. Also the discretization into filaments is considered to be a real effect, not merely a device for attaching statistical weights to a continuous function.)

An alternative approach would be to invoke a 'mean-field' approximation in which each filament moves in the average field of all the others. In this case the canonical distribution is appropriate and one obtains (15) immediately, with β the inverse 'temperature' of the filaments. Note that this 'temperature' is related to the interaction of the current filaments, i.e. to the energy of the magnetic field. It has nothing directly to do with usual plasma temperature and might be referred to as the 'magnetic temperature.'

V Equilibrium Profiles

From the previous sections we see that, starting from the Tokamak equilibrium Eqs. (2)–(4), and invoking only the concept that the current is microscopically filamented, we are led to natural Tokamak current profiles given by Eq. (15). It is convenient to set $\psi = 0$ on the magnetic axis of the Tokamak. Then these profiles form a one-parameter family:

$$J = J_0 \exp(-\lambda\psi) \tag{16}$$

Note that all specific characteristics of the current filaments have been absorbed into J_0 and λ so that the size and number of filaments does not affect the natural profiles.

The equilibria corresponding to the natural current profile are given by

$$\nabla_\perp^2 \psi = J_0 \exp(-\lambda\psi) \tag{17}$$

and for a circular cross-section Tokamak, with minor radius a,

$$\psi = \frac{2}{\lambda} \log\left(1 + \frac{\alpha r^2}{a^2}\right) \tag{18}$$

where $\alpha = J_0 \lambda a^2/8\pi$. This corresponds to a radial distribution of current

$$J(r) = \frac{J_0}{(1 + \alpha r^2/a^2)^2} \tag{19}$$

and a $q(r)$ profile

$$q(r) = q_0 \left(1 + \frac{\alpha r^2}{a^2}\right) . \tag{20}$$

Note that the parameter α determines the central-peaking of the current. Profiles such as these have often been proposed before. In addition to the work of Montgomery, Turner and Vahala,[18] they were obtained by Biskamp,[7] by Hsu and Chu,[8] and by Kadomtsev[9] — all essentially by minimizing the energy under virtual displacements of the magnetic flux with pressure and current being taken as invariant functions of ψ. Somewhat similar profiles were also proposed by Pfirsch and Pohl,[10] and by Minardi.[11] It is, therefore, appropriate to emphasize that the present interest lies in the underlying filamentary model. Accordingly we examine this further in the next section and derive some new results.

VI Fluctuations and Transport

We first consider the relation of the current distribution (19) to the underlying filamentary structure. The total current I is related to the number and strength of the filaments, $I = Nj_0$

and the current peaking coefficient α is related to their effective temperature $1/\beta$,

$$\beta N j_0^2 = \frac{8\pi\alpha}{(1+\alpha)} \ . \tag{21}$$

Note that $\alpha \to 0$ when the effective temperature $\to \infty$. Thus, the fully relaxed state (uniform current) occurs when the effective temperature of the fluctuations is large — as one would expect. The current channel contracts indefinitely, ($\alpha \to \infty$) when the effective temperature falls to $N j_0^2/8\pi \equiv T_c$. Because of the sign difference in the Hamiltonian, the range of positive effective temperatures ($T_c < T < \infty$) in the Tokamak corresponds to the range of negative temperatures investigated by Edwards and Taylor[13] and by Joyce and Montgomery[14] for the electrostatic charged-rod model. In the Tokamak itself, negative magnetic temperatures correspond to 'hollow' current profiles.

From the current profile alone one cannot determine separately the effective temperature or the individual filament strength, only the combinations $N j_0$ and βj_0. However, small scale fluctuations in the magnetic field of wavenumber $\mathbf{k}$ are given by (see Ref. 13)

$$\left\langle B_k^2 \right\rangle = \frac{N j_0^2}{\pi a^2} \cdot \frac{1}{k^2} \tag{22}$$

so that the magnetic temperature and the filament strength could in principle be inferred from these fluctuations. (Equation (22) can be obtained by noting that

$$\mathbf{B_k} = \mathbf{k}\psi_k = j_0 \left(\frac{\mathbf{k}}{k}\right) \sum_i \exp(i\mathbf{k}\cdot\mathbf{r}_i) \tag{23}$$

and that on a microscopic scale the $\mathbf{r}_i$ are randomly distributed so that $\langle \exp\, i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_k)\rangle = \delta_{ik}$.)

A more interesting deduction from the filamentary model concerns heat and particle transport in a Tokamak. To see this, consider a group of 'test-filaments' J_1 identified at some particular toroidal cross-section $z = z_0$. These filaments will 'evolve' along z according to a Fokker-Planck equation

$$\frac{\partial J_1}{\partial z} = \frac{-1}{r}\frac{\partial}{\partial r} r \left(D\frac{\partial J_1}{\partial r} + F J_1 \right) \ . \tag{24}$$

The test filaments will reach the z-independent distribution of the background only if

$$\frac{F}{D} = \frac{-1}{J}\frac{dJ}{dr} \,. \tag{25}$$

This can also be written, using (14), as $F/D = \beta j_0 \, d\psi/dr$. Recalling that in the electrostatic rod analog ψ is replaced by electric potential and j_0 by charge per rod, this is just the Einstein relation between mobility and diffusion of charged particles.

The transport described by Eq. (24) is a macroscopic effect. However the underlying process is the uniformity of J along the microscopic chaotic field $\mathbf{B}$, represented by $\mathbf{B}\cdot\nabla J = 0$. Now, if the parallel thermal conductivity is large, the (plasma) temperature satisfies a similar equation, $\mathbf{B}\cdot\nabla T \simeq 0$ and is also microscopically uniform along the same chaotic field. Therefore, there must be the same relation between the Fokker-Planck coefficients for radial heat transport as there is for that of current filaments — though the mean profiles will differ because there are internal sources of heat but not of current. (Also, heat is transported passively in the magnetic field, whereas diffusion of current affects the field.)

At this point we must recall that z is not the real time and that Eq. (24) describes the result of a stationary perturbation. However, if we associate some velocity, such as the electron thermal velocity, with transport of heat along the microscopic field, then the *ratio* of the Fokker-Planck coefficients for heat diffusion in real time will be the same as the *ratio* of the coefficients in Eq. (24). Therefore, in so far as it is anomalous and dominated by flow along a chaotic field, the real heat diffusion equation must be

$$\frac{\partial T}{\partial t} = \frac{-1}{r}\frac{\partial}{\partial r} r \left(\chi \frac{\partial T}{\partial r} + \nu T\right) + \text{ Sources} \tag{26}$$

where

$$\frac{\nu}{\chi} = \frac{-1}{J}\frac{dJ}{dr} \,. \tag{27}$$

Thus we are led to the interesting conclusion that *there must be an inward heat pinch in a Tokamak,* related to the current profile.

A similar argument applied to particle diffusion shows that there must also be an inward particle pinch velocity u, related to the current profile and the particle diffusion coefficient D,

$$\frac{u}{D} = \frac{-1}{J}\frac{dJ}{dr} . \tag{28}$$

For the natural current profiles (19) this is equivalent to

$$\frac{a^2 u}{r^2 D} = \frac{4\alpha}{(1 + \alpha r^2/a^2)} . \tag{29}$$

However, the natural current profile assumes a uniform effective magnetic temperature, whereas (27) and (28) require only a local magnetic temperature and may therefore have wider validity.

VII Extended Models

A. Variable strength filaments

So far we have considered the number and strength of the filaments to be fixed. It may be more appropriate to consider filaments which can break-up and merge with one another. A step in this direction is a model in which each filament carries an integral multiple of an elementary current j_0 but this multiple may change as filaments merge or break-up.

The entropy of a configuration with $n_s(r)$ filaments of strength sj_0 is

$$S = -k \sum_s \int n_s(\mathbf{r}) \log n_s(\mathbf{r}) d\mathbf{r} . \tag{30}$$

The interaction energy of the filaments is

$$E = j_0 \sum_s s \int n_s(\mathbf{r}) \psi(\mathbf{r}) d\mathbf{r} \tag{31}$$

and the total current is

$$I = j_0 \sum_s \int s\, n_s(\mathbf{r}) d\mathbf{r} . \tag{32}$$

Then the configuration of maximum entropy at fixed energy and current (which replaces fixed number of filaments) is given by

$$n_s = A \exp\left[-j_0(\beta\psi + \mu s)\right] \tag{33}$$

where β and μ are Lagrange multipliers. After summing over s this gives a current profile which can be written

$$J = J_0\, e^{-\lambda\psi} \Big/ \left(1 - ke^{-\lambda\psi}\right)^2 \qquad (k < 1)\,. \tag{34}$$

Hence this model yields a two-parameter family of natural current profiles. Unless $k \simeq 1$ they are not significantly different from those of the simpler model.

B. An additional constraint

In general we may regard energy as the only relevant isolating integral for the filaments. However, for a system bounded by a circular cylinder the Hamiltonian is invariant under rotations about the axis and there is an additional invariant of the equations (5)–(7),

$$P = \sum_i \left(x_i^2 + y_i^2\right) \;, \tag{35}$$

corresponding to conservation of angular momentum. When this invariant is introduced into the calculation of the maximum entropy configuration(for fixed strength filaments) one finds natural current profiles given by

$$J = J_0 \exp\left(-\lambda\psi - \omega r^2/2\right) \;. \tag{36}$$

Note that in this case current density is no longer specified as a function of the flux ψ alone; it is also a function of r. This implies a greater variety of natural current profiles in a circular cylinder than in a torus. An interesting aspect of this is discussed below.

C. Helical profiles

A striking feature of relaxation theory is that in some cases the relaxed state in a circular cylinder is not itself axisymmetric but helical. This is a form of spontaneous symmetry breaking. A similar phenomenon can occur in the present model for equilibria defined by the current distribution (36). In this case symmetry breaking leads to helical configurations which are the analog of the rotating equilibria found by Smith and O'Neil[19] for the electrostatic charged-rod model.

To see how these helical configurations arise we first note an important distinction between current profiles defined by Eq. (17) and those defined by (36). For profiles defined by (17), recalling that $\mathbf{B}_\perp = \widehat{\mathbf{z}} \times \nabla\psi$,

$$\mathbf{B} \cdot \nabla J = -B_0\, J\, \lambda\, \frac{\partial \psi}{\partial z} = 0 \qquad (37)$$

so that ψ is indeed independent of z as we have implicitly assumed. But for profiles defined by (36) we have,

$$\mathbf{B} \cdot \nabla J = -B_0\, J \left(\lambda\, \frac{\partial \psi}{\partial z} + \omega\, \frac{\partial \psi}{\partial \theta} \right) = 0 \qquad (38)$$

so that z is no longer an ignorable coordinate. (Unless $\partial\psi/\partial\theta = 0$ when both ψ and z are ignorable.) Instead, the ignorable direction is a helix defined by $(\theta - \omega z) =$ constant. Thus, any non-axisymmetric solution of the equilibrium equation

$$\nabla_\perp^2 \psi = J_0 \exp\left[-\beta(\psi + \omega r^2/2)\right] \qquad (39)$$

will represent a *helical* preferred Tokamak profile!

Non-axisymmetric solutions of (39) were described by Smith and O'Neil whose work should be consulted for more information. Here we consider only the transition from a circular to a helical configuration. This transition is a bifurcation point for Eq. (39) just beyond which there will be two neighboring solutions with the same values of E and N — a symmetric solution $\psi_0(r)$ and a non-symmetric one $\psi_0(r) + \delta\psi(r, \theta)$.

As discussed by Smith and O'Neil, the first bifurcation is to the $m = 1$ azimuthal Fourier harmonic, $\delta\psi \sim \tilde{\psi}(r)\exp(i\theta)$. Then in a linear approximation it follows from (39) that

$$\mathcal{L} \cdot \tilde{\psi} \equiv \left[\frac{1}{r}\frac{d}{dr}r\frac{d}{dr} - \frac{1}{r^2} + \beta\, J(\psi_0)\right]\tilde{\psi}(r) = 0 \tag{40}$$

and $\tilde{\psi}(r)$ must vanish on the boundary $r = a$. (In the linear approximation the constraints of fixed E and N are automatically satisfied for any $m \neq 0$ harmonic.) Noting that

$$\mathcal{L} \cdot \frac{d\psi_0}{dr} = -\beta\,\omega r\, J(\psi_0) \tag{41}$$

it is clear, as first pointed out by Levy,[20] that

$$\tilde{\psi}(r) = \frac{d\psi_0}{dr} + \omega r \tag{42}$$

is a solution of (40). This solution vanishes on the boundary if

$$\omega = -\left[\frac{1}{r}\left(\frac{d\psi}{dr}\right)\right]_{r=a} = \frac{I}{2\pi a^2} \tag{43}$$

Hence, bifurcation to the $m = 1$ harmonic, and spontaneous symmetry breaking, occur when $\omega = I/2\pi a^2$.

Since Eq. (40) also describes neighboring equilibria in ideal MHD, this bifurcation point is also the point at which the axisymmetric natural profile (36) becomes unstable on the conventional MHD model. (However, it does so in a somewhat unusual manner since the resonant surface, $(k_z\, B_z + m\, B_\theta/r) = 0$, at which the pitch of the perturbation matches that of the unperturbed magnetic field, occurs at the boundary of the system.)

VIII Conclusions

There is an exact mechanical analog of the low-β Tokamak equilibrium problem if one assumes that, because the field lines are slightly chaotic, the current has a microscopic filamentary structure. The *evolution* of this mechanical analog represents the configuration of the current filaments at a *fixed* time. Therefore, just as there is a natural thermal equilibrium for the mechanical analog, so there will be a natural current profile for the Tokamak. This may account for the phenomenon of profile consistency[2, 3, 4, 5, 6] in Tokamaks.

The existence of a realisable dynamical analog for Tokamak equilibrium also allows us to use conventional statistical mechanics to find the natural Tokamak profiles. For a model in which the current filaments have equal fixed strength, the natural current profiles are given by

$$J = J_0 \exp(-\lambda\psi) \tag{44}$$

and are independent of the number and strength of the filaments. For a circular cross-section Tokamak this corresponds to a radial profile

$$J = \frac{J_0}{(1 + \alpha r^2/a^2)^2} \ . \tag{45}$$

In a more elaborate model of the filaments, in which they may merge and break-up, the natural profiles are slightly more general:

$$J = J_0 \, e^{-\lambda\psi}(1 - k e^{-\lambda\psi})^{-2} \ . \tag{46}$$

If the boundary of the Tokamak is strictly a circular cylinder, there is a constraint on the filaments equivalent to angular momentum conservation. In this case the natural current profiles become (for fixed strength filaments)

$$J = J_0 \exp(-\lambda\psi - \omega r^2/2) \tag{47}$$

and are no longer a function of ψ alone. This implies that large aspect-ratio, circular cross-section Tokamaks (where the angular momentum constraint is most likely to be valid) may have a greater variety of preferred current profiles than small aspect-ratio or non-circular machines.

However the most interesting consequence of (47) is that, even in a circular, cylindrical boundary, it may lead to natural current profiles *which are not axisymmetric.* Instead, they are helical. This spontaneous symmetry breaking is associated with instability of the axisymmetric profile and is reminiscent of similar behavior in the relaxed states of the toroidal pinch.[1]

In addition to providing a justification for the natural profiles, which are similar to ones proposed previously on the basis of ad hoc principles, the present work also provides an interpretation of magnetic field perturbations in terms of their effective temperature (the 'magnetic temperature'). This is related to the interaction energy of the filaments and determines the current-peaking factor α (Eq. (45)). When the magnetic temperature $\rightarrow \infty$, $\alpha \rightarrow 0$ and the current profile is flat, corresponding to the fully relaxed state of a Tokamak after disruption. When the magnetic temperature falls to a critical value T_c, $\alpha \rightarrow 0$ and the current profile is highly peaked on axis. Positive temperatures below T_c are not accessible. Negative magnetic temperatures, which are accessible, (through $T \rightarrow \infty$) correspond to a 'hollow' current profile.

Finally, the filamentary model predicts that there will be an inward flux of heat (a "heat-pinch" with velocity ν) and of particles (a "particle pinch" with velocity u). These inward-pinch velocities are related to the local current density and the thermal diffusivity χ or diffusion coefficient D by,

$$\frac{\nu}{\chi} = \frac{u}{D} = \frac{-1}{J}\frac{dJ}{dr} .$$

In this connection it may be noted that an inward flux of heat appears necessary to account for the behavior of certain Tokamaks[21, 22] and an inward particle flux is a well-known feature seen on many Tokamaks.[23, 24]

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