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Institute for Fusion Studies  
The University of Texas at Austin  
Austin, Texas 78712

CONFINEMENT SYSTEMS

THE THEORY OF NON-AXISYMMETRIC

ON

SUMMARY OF U.S. - JAPAN WORKSHOP

IFSR #5

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Institute for Fusion Studies  
University of Texas  
Austin, Texas

December 8-12, 1980

CONFINEMENT SYSTEMS  
THE THEORY OF NON-AXISYMMETRIC

ON

U.S. - JAPAN WORKSHOP

SUMMARY OF

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Contents of U.S. - Japan Workshop Summary

Program of U.S.-Japan Workshop on

"The Theory of Non-Axisymmetric Confinement Systems"  
December 8-12, 1980, Austin, Texas

Monday, 2:00 PM OPENING ADDRESS: Prof. J.A. Wheeler

2:30-5:30 PM EQUILIBRIUM, STABILITY AND TRANSPORT

Chairman: W. Horton

1. M. Wakatani "Interchange Ballooning Mode in Stellarators"

2. H. Strauss "Limiting  $\beta$  of Stellarators with No Net Current"

3. H. Grad "3-D Transport and Code Development"

4. T. Watanabe "Numerical Scheme for Stability Analysis Using Integral Equation in Wave Number Space"

Tuesday, 9:00AM-12:00PM

TANDEM MIRRORS

Chairman: D. Ross

1. T. Kamimura "Monte Carlo Simulation of Radial Transport in Nonaxisymmetric Tandem Mirrors"

2. T. Kaiser "Analytic Equilibria for Tandem Mirrors"

3. R. Cohen "Tandem Mirror Resonant Transport"

4. T. Katayama "Simulation on Thermal Barrier in Tandem Mirror"

2:00-5:00 PM

WORKING SESSIONS

Stellarators and Heliotrons Study Group  
Co-Chairmen: H. Grad, M. Wakatani and S. Yoshikawa

Tandem Mirror Study Group  
Chairman: D. Pearlstein

Bumpy Torus Study Group  
Co-Chairmen: D. Spong and H. Berk

Wednesday, 9:00AM-12:00PM

BUMPY TORUS

Chairman: J. Van Dam

1. D. Spong "Low and High Frequency Stability in EBT"

Wednesday (Cont.)  
9:00 AM-12:00 PM

- 2. J. Todoroki "Transport Theory of Non-Symmetric Torus"
- 3. H. Sanuki "Numerical Analysis for Stability in Bumpy Tori"
- 4. H. Berk "EBT Stability Theory"

2:00-5:00 PM

WORKING SESSIONS

Stellarators and Heliotrons Study Group  
Co-Chairmen: H. Grad, M. Wakatani and S. Yoshikawa

Bumpy Torus Study Group  
Co-Chairmen: L. Hedrick and H. Berk

"Monte Carlo Simulation of Nagoya Bumpy Torus"  
Comment: T. Kamimura

Tandem Mirror Study Group  
Co-Chairmen: T. Kaiser and T. Katsuma

Thursday

9:00 AM-12:00 PM

- 1. S. Yoshikawa "Stellarator Theory and Experiments"
- 2. A. Boozer "Transport in Asymmetric Devices"
- 3. R. Hazeltine "On Omnigenity"
- 4. J. Derr "Ripple Trapping in Modular Stellarators"

2:00-5:00 PM

HELIOTRON, STELLARATORS AND HELICAL PINCH  
Chairman: T. Watanabe

- 1. M. Wakatani "Monte Carlo Simulation of NBI Heating in Heliotron-E"
- 2. C. Chu "Helical Pinch Equilibria with Pitch Reversal (OHTE)"
- 3. M. Schmidt "FLR Stabilization of Diffuse Profile in High Beta Stellarators"
- 4. D. Barnes "Nonlinear Stability Calculation for a Compact Torus"

CLOSING REMARKS - T. Kamimura and M.N. Rosenbluth

Stellarator and Heliotron Working Session Report  
H. Grad, M. Wakatani and S. Yoshikawa

Tandem Mirror Working Session Report  
T. Kaiser, T. Katayama, W. Horton

Bumpy Torus Working Session Report  
T. Hedrick, J. Todoroki, H. Berk

Friday  
9:00 AM-12:00 PM

DISCUSSION, SUMMARY, AND FUTURE PLANNING  
Co-Chairmen: T. Kamimura and M.N. Rosenbluth

Additional Workshop Activities

Wednesday  
3:30-5:00 PM

Introduction of the Japanese Delegation to the  
Chairman of the Physics Department, Professor  
T.A. Griffy and to the Physics Department at the  
colloquium.

Thursday  
7:00-10:00 PM

Workshop Banquet hosted by the Institute for  
Fusion Studies and sponsored by TRACOR, Inc.

FORMAT OF U.S.-JAPAN WORKSHOP ON NON-AXISYMMETRIC SYSTEMS, DEC. 8-12, AUSTIN, TEXAS

<p>Monday Dec. 8</p>	<p>Tuesday Dec. 9</p>	<p>Wednesday Dec. 10</p>	<p>Thursday Dec. 11</p>	<p>Friday Dec. 12</p>
<p>1:30 Opening Address-Prof. J.A. Wheeler</p>	<p>1:30 Opening</p>	<p>1:30 Opening</p>	<p>1:30 Opening</p>	<p>1:30 Opening</p>
<p>EQUILIBRIUM, STABILITY AND TRANSPORT</p>	<p>WORKING GROUPS</p>	<p>WORKING GROUPS</p>	<p>HELLOTRON, STELLARATORS AND HELICAL PINCH</p>	<p>WORKING GROUPS</p>
<p>5:00 Closing</p>		<p>4:00 Physics Colloq. Don Dubois, IASL Plasma Turbulence Theory</p>	<p>7:00 Workshop Dinner</p>	
<p>9:00 Opening</p>	<p>9:00 Opening</p>	<p>9:00 Opening</p>	<p>9:00 Opening</p>	<p>TANDEM MIRRORS</p>
<p>12:00 Closing</p>	<p>12:00 Closing</p>	<p>12:00 Closing</p>	<p>BUMPY TORUS</p>	<p>TRANSPORT AND GENERAL THEORY</p>
<p>12:00 Closing</p>	<p>12:00 Closing</p>	<p>DISCUSSION, SUMMARY AND FUTURE PLANNING</p>	<p>12:00 Closing</p>	<p>12:00 Closing</p>

U.S.-Japan Workshop on  
 "Theory of Non-Axisymmetric Confinement Systems"

The University of Texas at Austin

December 8-12, 1980

Organizing Committee

W. Horton, Scientific Secretary, IFS, University of Texas  
 D. W. Ross, Assistant Director, IFS, University of Texas

Participants

JAPAN

A. Fukuyama

T. Kamimura

I. Katayama

H. Sanuki

J. Todoroki

M. Wakatani

T. Watanabe

UNITED STATES

D. Barnes

A. Bayliss

H. Berk

A. Boozer

C. Chu

R. Cohen

J. Derr

H. Grad

A. Hasegawa

R. Hazeltine

L. Hedrick

D. Kaiser

G. Miller

D. Pearlstein

T. Cayton

M. N. Rosenbluth

M. Schmidt

K-C Shain

D. Spong

H. Strauss

J. Van Dam

S. Yoshikawa

OTHER

E. Maschke

Fontenay-aux-Roses, FRANCE

IFS, University of Texas  
 Courant Institute, New York University  
 IFS, University of Texas  
 Princeton Plasma Physics Laboratory  
 General Atomic Co.  
 Lawrence Livermore National Laboratory  
 University of Wisconsin  
 Courant Institute, New York University  
 Bell Telephone Laboratories  
 IFS, University of Texas  
 Oak Ridge National Laboratory  
 Lawrence Livermore National Laboratory  
 Los Alamos Scientific Laboratory  
 Lawrence Livermore National Laboratory  
 Los Alamos Scientific Laboratory  
 Director, IFS, University of Texas  
 Science Applications, Inc.  
 University of Wisconsin  
 Oak Ridge National Laboratory  
 Courant Institute, New York University  
 IFS, University of Texas  
 Princeton Plasma Physics Laboratory  
 Los Alamos Scientific Laboratory  
 Lawrence Livermore National Laboratory

Okayama University and IFS, University of  
 Texas  
 Institute of Plasma Physics, Nagoya University  
 Plasma Research Center, Tsukuba University  
 Institute of Plasma Physics, Nagoya University  
 Institute of Plasma Physics, Nagoya University  
 Plasma Physics Lab, Kyoto University  
 Institute for Fusion Theory, Hiroshima Univ.



Recent important developments in MHD codes point to where accurate values of the critical plasma pressure  $\beta_c$  will soon be available. Present preliminary estimates were reported to range from 0.5% to 5% for a range of unoptimized profiles and system parameters. Two codes, by Wakatani and by Strauss, are based on the same theoretical approximations of low  $\beta$  and large aspect ratio (stellarator expansion) with a linear initial value solution of the reduced equations; but the codes are sufficiently different in the numerical algorithms to provide an accurate check on each code, if they are both run for the same equilibrium. This check will be made soon by Strauss. The third code by Betancourt-Garabedian is sufficiently different, including finite geometry without a parameter expansion and based on the method of steepest descent instead of initial value solution, to provide a more sophisticated check on the general reproducibility of the MHD stability of the different models. Without having the codes run for identical profiles, the present "discrepancy" of a factor of two between codes is held to be irrelevant, as noted by Grad.

Since toroidal stellarator MHD stability requires three-dimensional code calculations, the physical meaning of the differences in the codes cannot be easily interpreted. Thus

an understanding of the critical  $\beta$  for straight stellarators may be necessary for the prediction of stellarator equilibrium and stability, as noted by S. Yoshikawa. The straight stellarator suffers from bad curvature ( $V'' > 0$ ), but has shear. Also, the ballooning mode is not dangerous as the distance between good and bad curvature regions is of the order of the pitch length, which Yoshikawa notes is not long enough to have significant ballooning.

Neutral beam heating experiments with 2.5 MW that begin next year will give some information about  $\beta$  limits in the Heliotron-E device.

Recent Monte Carlo simulations of neoclassical transport by M. Wakatani suggest that helical ripple effects are not as large as expected from analytic theory. More intensive studies of transport will be required before the details of the radial loss rates are understood.

Numerical computation of stellarator equilibria with finite parameters is in a well-developed state using the Garabedian-Betancourt code. A modification of this code to include classical Braginsky transport (and incidentally speed up the code) will soon be available (Bayliss, Betancourt, Grad).

The current status of MHD equilibrium and stability

studies at Livermore was summarized by T.D. Pearlstein and

T. Kaiser. It is now possible to calculate three-dimensional

anisotropic equilibria for straight-axis large-aspect-ratio

configurations to the first order in  $\beta$ . These equilibria are

essentially analytic in that the only numerical calculations

required are quadratures of known vacuum field quantities. The

ballooning stability of these equilibria is determined by

Pearlstein and Kaiser using a computer code which solves on

each field line the eigenmode equation derived from the guiding

center energy principle. Localization to a field line is the

result of using an eikonal representation of the waves perpen-

dicular to the magnetic field. In more recent calculations,

finite-Larmor-radius effects have been included according to

the kinetic theory of Tang and Catto and found to be strongly

stabilizing in the zero-perpendicular-wavelength limit.

The most serious deficiencies of the MHD calculations to

date are acknowledged to be the limitation of the equilibrium

calculation to first order in  $\beta$ , and of the stability calculation

to infinite  $k_{\perp}$ . Several approaches to overcoming these diff-

iculties have been suggested and are under active consideration

by the Livermore group.

Regarding the equilibrium calculations, the general equa-

tions (that is, to all orders in  $\beta$ ) have been derived by Pearlstein, Kaiser and Newcomb; but the solution of the reduced equation remains elusive. One promising approach is an iterative one in which an equilibrium calculated by present techniques for a particular value of  $\beta$  would be linearly perturbed to obtain another equilibrium of incrementally higher  $\beta$ . This procedure could be repeated, thereby generating a succession of equilibria of increasing  $\beta$ . Another interesting approach is to minimize the magnetic field and plasma energy in the large-aspect-ratio limit with appropriate constraints, as in the Betancourt and Garabedian calculation for toroidal equilibria. The real importance of FLR stabilization cannot be assessed until the stability of modes with  $k_{\perp} r_p \sim 1$  can be investigated. The most promising approach for going beyond the infinite- $k_{\perp}$  limit appears to be a two-dimensional WKB treatment in which Einstein-Billouin-Keller quantization techniques would be employed; this line is currently being pursued at Livermore. Other possibilities include construction of a PEET-like code for tandem mirrors, and solution of the nonlinear MHD equations in the large-aspect-ratio limit as an initial value problem.

The first work on radial transport in the tandem mirror reported at this workshop was by Kamimura using Monte Carlo simulations of the nonaxisymmetric system. The objective of these simulations was to investigate resonant particle diffusion, which will be a large loss mechanism as long as

nonaxisymmetric plug coils are used. In the simulation model, azimuthal drift  $\Delta\psi$  is independent of the radial distance from the axis. The results of the Monte Carlo simulations show that the observed diffusion is in agreement with the theoretical resonant diffusion formula. However, the confinement time in the TMX experiment is longer than that of the simulation when the ambipolar potential  $\Phi_0$  of the experiment is used. The simulation results described by Kamimura show that the radial diffusion coefficient becomes large with large  $\Phi_0$ . Hence, the next subject to investigate is the potential profile dependence of the radial diffusion, which will be carried out soon at Nagoya.

Work on the simulation on the physics of the thermal barrier in tandem mirrors was reported by Katsuma. The object of this work is to investigate the barrier physics using self-consistent one-dimensional particle simulation codes. To include mirror effects partly in the one dimensional simulation, a force term is added to the electron and ion equations of motion. The system thus includes the acceleration and deceleration regions due to the mirror forces. The results of the simulation as reported by Katsuma, show that in a steady state, a large potential dip is observed in the mirror region along with an expected dip of the plasma density, although a sheath-like profile of the potential at the edge of the mirror was also observed.

The next step will be to undertake a more quantitative investigation of this result by using 2D or 3D particle simulation codes that will include more realistic effects. Analytic theory of radial transport in the tandem mirror system was reported by Cohen. For given axial magnetic and potential profiles, the guiding center equations of motion can be integrated approximately over one axial bounce period to determine the successive radial and azimuthal positions. In this manner Cohen derived mapping equations with an extrinsic stochasticity parameter  $\sigma$  for collisions and a nonlinear resonance overlap parameter  $K$ . In the asymptotic limits of small and large  $\sigma$  and  $K$ , analytic formulas were obtained for the guiding center diffusion. In the transition regions where  $\sigma$  and  $K$ , which are functions of energy and magnetic moment, are of order unity, the analytic formulas are joined by interpolation. To facilitate comparison of the simulation transport and the analytic transport, it was suggested that the accuracy of the mapping representation of the continuous dynamics should be tested for samples of particles which contribute strongly to the loss rates. Complete radial transport equations for the ion and electron densities and temperatures are obtained from the test particle diffusion by integrating over the Maxwellian distributions. Solutions of the transport equations were discussed by Cohen, who pointed out the occurrence of a bifurcation in the rotational states obtained by radial charge balance. The multiple rotational states lead to difficulties similar to

those already encountered in bumpy forus transport studies. The tandem mirror group and the bumpy forus group compared their experiences in this area.

## SUMMARY OF BUMPY TORUS SESSIONS

L. Hedrick, D. Spong, J. Todoroki, and H.L. Berk

The bumpy torus working sessions began with a summary

of the various areas in which theoretical work is underway and of the people involved in each area in the U.S. and Japan.

During the workshop, talks concerning calculations in the areas of equilibrium, stability, and transport were presented.

Hedrick described a number of important results that have been obtained at Oak Ridge. Data from recent toroidal experiments on EBT and NBT, as well as from the linear STM experiment and several simple mirror experiments at Oak Ridge and Nagoya, in both the fundamental and standard resonance modes, were noticed to all fit the empirical relationship  $p/L \approx \text{constant}$  (to within 20%), with  $p$  the ring electron gyroradius and  $L$  a typical equilibrium scale length. It was suggested that this may be related to a recent finding from numerical computations, which is that at high energies approximately equal to the experimentally observed hot electron ring temperature ( $\approx 500$  keV), the magnetic moment  $\mu$  is no longer an invariant but begins to fluctuate in a way that tends to prevent a hot electron from reaching the resonant zone.

Particle motion studies in time varying electromagnetic

fields have also found that the lower energy electrons exhibit large periodic charges in energy. A hybrid treatment using

Monte Carlo techniques for low energy electrons and quasilinear



theory for high energy electrons thus appears to be a more adequate approach than existing quasilinear heating theories for treating the formation of the hot electron rings from the much cooler toroidal core plasma.

The main confinement features of the toroidal core plasma are reasonably well described by neoclassical transport calculations, partly because the dominant electron transport is better understood than ion transport. Recent 1D transport studies that include both normal and reversed gradient regions were similar in global features to earlier 1D results.

The ions observed experimentally in EBT have an enhanced population at large energies. Using ad hoc ion tails in the 1D transport codes has yielded increased ambipolar potentials, closer in magnitude to those seen in the experiment. Furthermore, neoclassical Fokker-Planck calculations have indicated that very little power (e.g.,  $10^{-3}$  of that supplied to the torus by ECH) is needed to create a substantial tail by either classical or ad hoc non-classical mechanisms. The latter could represent ion heating by fluctuations (e.g., internal generation of ICRH or lower hybrid heating). Theoretical and experimental investigation of the significance of such non-classical mechanisms is underway at Oak Ridge.

Very recent Monte Carlo transport calculations were reported by Kamimura. These preliminary calculations in vacuum magnetic fields are for ions alone and neglect the ambipolar electric field. Subsequent work on including the effects of the finite

beta hot electron rings and the ambipolar electric field is planned in Japan.

Todozoki reported progress in the general theory of transport in three dimensional systems with anisotropic pressures. The theory presented uses natural magnetic coordinates for the high beta equilibrium and yields general but complicated formulas for the transport equations.

Boozer described recent work on the high beta, three dimensional transport problem using fluid theory and non-single-valued magnetic flux coordinates.

A numerical analysis of bumpy torus equilibrium using time dependent fluid equations with tensor pressure was presented by Sanuki. The initial position of the hot electron ring was found to shift inward to approach equilibrium when the ring pressure was increased for fixed bulk plasma pressure. However, for fixed ring pressure, the shift was inward for low bulk pressure, but outward for higher pressure, with no shift when the ratio of ring to bulk pressure was about 7 - 9.

Stability calculations being carried out at Oak Ridge and at the Institute for Fusion Studies were reported by Spong and Berk, respectively. Both groups agreed on the existence of a new instability due to coupling of the core plasma Alfvén wave with the free energy of the hot electron ring. The Alfvén mode leads to an upper limit on the core plasma density, whereas interchange stability of the hot electrons leads to a lower limit on the core density. Theory indicates a stability window consistent

with present EBT operation. Another interchange mode limits the core plasma pressure, especially during strong heating. This background interchange mode and the Alfvén mode are generally exclusive: one will be unstable where the other is stable. For present and near-term devices, theory predicts the Alfvén limit to be most relevant. In reactor devices, the Alfvén mode poses no difficulty, but background interchange stability may be restrictive. Also, although the Alfvén mode has features similar to the T-M transition, there are discrepancies with the power and ring beta scalings.

The current calculations are based on either slab geometry approximations or expansions in the ratio of the magnetic gradient scale length to the radius of curvature. More sophisticated theories have been formulated, involving finite geometry, finite hot electron and ion gyroradius corrections, radial mode structure, relativistic effects, and bounce motion and longitudinal dependence. A result has been obtained indicating that Larmor radius effects, above a certain level, could improve the interchange stability limit. Also, a consideration of the Alfvén mode radial dependence has shown that either bulk modes, where the perturbed field varies negligibly over the annulus, or body modes, whose wavelength is less than the annulus width, are possible, with the body modes being more difficult to stabilize. The ORNL and IFS groups agreed to continue work together on the radial structure problem.

U.S. - JAPAN JOINT WORKSHOP

INSTITUTE FOR FUSION STUDIES, AUSTIN, TEXAS

DECEMBER 8-12, 1980

GUIDE TO GROUP PHOTOGRAPH

R. Cohen, H. Berk, T. Cayton, G. Miller, A. Bayliss, D. Pearlstein, S. Yoshikawa, H. Strauss, T. Kaiser  
J. Van Dam, R. Hazeltine, K-C Shainy, M. Schmidt, A. Boozer, D. Spong, E. Maschke, C. Chu  
T. Watanabe, A. Fukuyama, J. Todoroki, M. Wakatani, D. Ross, T. Kamimura, H. Sanuki, W. Horton

