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Fast Ion-Driven Bernstein Instabilities

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Abstract

We investigate a new mechanism, the two-energy-stream cyclotron instability, for fast ions (e.g., fusion products) to drive electrostatic waves and to slow down. The instability comes from a relativistic effect, which dominates conventional phase overtaking as the axial phase velocity exceeds the speed of light. Both a single particle model and a dispersion relation are developed in order to illuminate the physics insights and scaling laws. We present numerical results and discuss nonlinear processes. The mechanism is essential for the dynamics of the fast ions in both D-D and D-T devices.

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Isotropic fast ions produced by thermonuclear fusion reaction are much more energetic than the background plasma. The total energy of the fast ions is required to be, at least, on the same order as the total plasma's energy for ignition [1, 2]. Thus, the dynamics of the fast ions and the instabilities driven by the fast ions can be critically important for the performance of fusion devices. In this letter, we study for the first time a new mechanism for the slowing down of the fast ions and show that this induces instabilities. This can not be put into the known collective wave/particle interaction: Landau type or beam (in real space) type [3]–[6]. The mechanism arises from the relativistic mass variation of the fast ions immersed in a magnetized plasma: namely, a two-energy-stream cyclotron instability. The background plasma plays a crucial role on the instability. A single particle model is developed to acquire more novel physics insights. We give a dispersion relation derived from kinetic theory to illuminate the scaling laws and properties of the instability. We also present numerical results (both D-D and D-T cases) from the dispersion relation and discuss nonlinear processes for estimating the saturation level of a single wave. The D-T case has a characteristic difference with the D-D case.

In the single particle model, we consider an ion gyrating along a uniform external magnetic field $B_o\,\widehat{z}$. The ion motion is $\mathbf{v} = v_\perp(\widehat{x}\cos\phi - \widehat{y}\sin\phi) + v_z\widehat{z}$ and $x = \rho\sin\phi + x_o$, where $v(v_\perp, v_z)$ is the ion (azimuthal, axial) velocity, $\rho(\phi)$ is the ion gyroradius (gyrophase), and x_o is the ion guiding center position. Then, there is an electrostatic wave $\mathbf{E} = (E_x\widehat{x} + E_z\widehat{z})\cos(\omega t - k_x x - k_z z)$ and $E_z = E_x(k_z/k_x)$, where E is the electric field of the wave, ω is the wave frequency, and k_x and k_z are the components of the wave number. The wave/ion interaction can be described by $(d/dt)\gamma m_i c^2 = q(\mathbf{v}\cdot\mathbf{E})$, $d\mathbf{P}_z/dt = q\mathbf{E}_z$, $d\phi/dt = \omega_c = \Omega_c/\gamma$, and $dz/dt = v_z$, where γ is the ion Lorentz factor, m_i is the ion rest mass, c is the speed of light, q is the ion charge, P_z is the ion axial momentum, and $\omega_c = \Omega_c/\gamma$ with $\Omega_c = qB_o/m_i c$ the nonrelativistic ion cyclotron frequency.

Only inertial phase bunching [4] and resonant interaction are of concern. Substituting the ion motion into the governing equations, we obtain

$$\frac{d}{dt}\gamma \simeq \frac{qv_\perp E_x}{m_i c^2}\frac{nJ_n(k_x\rho)}{k_x\rho}\left(1+\frac{k_z v_z}{n\omega_c}\right)\cos\psi \ ,$$

$$\frac{d}{dt}v_z \simeq \frac{qv_\perp E_x}{\gamma m_i c^2}\frac{nJ_n(k_x\rho)}{k_x\rho}\left[\frac{k_z c^2}{n\omega_c} - v_z\left(1+\frac{k_z v_z}{n\omega_c}\right)\right]\cos\psi \ ,$$

for the n-th harmonic interaction, where $\psi = \omega t - k_z z - n\phi - k_x x_o$, $\rho = v_\perp/\omega_c$, and J_n is the Bessel function of the first kind of order n.

The above equations give a resonance condition $\delta_n = \omega - \omega_D \sim 0$, where $\omega_D = k_z v_z + n\Omega_c/\gamma$ is the Doppler-shifted ion frequency, and δ_n is the frequency mismatch. The interaction is characterized by the time dependence of ω_D:

$$\frac{d}{dt}\omega_D = \frac{qE_x}{\gamma m_i c^2}\frac{J_n}{k_x}\left[k_z^2c^2 - (n\omega_c + k_z v_z)^2\right]\cos\psi \ ,$$

where the second term in the square bracket ($\simeq \omega^2$) is considered due to the effect of the relativistic ion mass variation. The relativistic effect dominates as $\omega/k_z > c$, which is independent of the ion energy. The relativistic effect has to be included in the calculation of the wave/ion interaction even for $\gamma \sim 1$. This is true at both linear and nonlinear stages. There are no net energy and momentum changes to first order for uniformly distributed ions.

By employing perturbation theory with small E_x assumption, we calculate to second order and then average over a uniform phase (i.e., $k_x x_o$, $k_z z_o$, or $n\phi_o$) to obtain the rate of net ion energy change

$$\frac{d}{dt}\gamma m_i c^2 = \frac{q^2 v_{o\perp}^2 E_x^2}{2\gamma_o m_i c^2}\left(\frac{nJ_n}{k_x\rho}\right)^2 \frac{n\omega_c + k_z v_z}{n^2\omega_c^2}\left[k_z^2c^2 - (n\omega_c + k_z v_z)^2\right]\left[\frac{\sin\delta_n t}{\delta_n^2} - \frac{t\cos\delta_n t}{\delta_n}\right] , \quad (1)$$

where $[\sin\delta_n t/\delta_n^2 - t\cos\delta_n t/\delta_n] \simeq \delta_n t^3/3$ for $\delta_n t \ll 1$. Here, we see the same factor for the competition between the relativistic mass variation and the conventional phase overtaking. As $\omega/k_z > c$, the relativistic effect dominates and the instability condition is $\omega > k_z v_z + \Omega_c/\gamma$, which is in contrast to that when conventional phase overtaking dominates (e.g., inverse Landau damping).

We will study the collective ion behavior self-consistently. For a uniform plasma and $k^2c^2 \gg \omega_p^2$, the dispersion relation for an electrostatic wave near harmonic cyclotron frequency can be derived from the relativistic Vlasov and Poisson equations by kinetic theory [5, 7] and is given by

$$0 = k^2 - \sum_{e,i} \omega_p^2 \sum_n \text{pr. v.} \int\int 2\pi p_\perp dp_\perp dp_z \frac{f_o}{\gamma} \left\{ \frac{\gamma k_x^2}{2\Omega_c \left(\omega - \frac{k_z p_z}{\gamma m} - \frac{n\Omega_c}{\gamma}\right)} \left[J_{n-1}^2(k_x\rho) - J_{n+1}^2\right] \right.$$
$$\left. + \frac{J_n^2}{\left(\omega - \frac{k_z p_z}{\gamma m} - \frac{n\Omega_c}{\gamma}\right)^2} \left[k_z^2 - \frac{1}{c^2}\left(\frac{n\Omega_c}{\gamma} + \frac{k_z p_z}{\gamma m}\right)^2\right] \right\} + \text{residues} , \qquad (2)$$

where $\omega_p = 4\pi N q^2/m$ is the nonrelativistic plasma frequency, N is the ion (i) or electron (e) density, pr. v. represents the Cauchy principal value of the integral, and the *residues* are contributed by the poles along the Landau contour [$\propto (\partial f_o/\partial p)_{\delta=0}$]. Equation (2) shows the same factor for the competition. For an electron beam, the dispersion relation recovers an earlier result [5] for electrostatic cyclotron instabilities. As $k_x = 0$ and $\rho = 0$, we recover the two-stream instabilities for a beam-plasma system in which Langmuir waves are excited. As $k_z = 0$ for the case of fast ions with $\gamma =$ constant and a background plasma (Maxwellian and with $\gamma \simeq 1$), we obtain the dispersion relation for the new mechanism (a two-energy-stream cyclotron instability in energy space) in which ion Bernstein waves are excited:

$$0 = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} - \sum_s \sum_{n_s=-\infty}^{\infty} \left\{ \frac{\omega_{ps}^2 n_s \omega_{cs}}{\omega - n_s\omega_{cs}} \frac{m_s}{k_x^2 \kappa T_\perp} I_{n_s}(k_x^2\rho_s^2) e^{-k_x^2\rho_s^2} - \frac{\omega_{ps}^2 n_s^2 \omega_{cs}^2}{k_x^2 c^2 (\omega - n_s\omega_{cs})^2} I_{n_s} e^{-k_x^2\rho_s^2} \right\}$$
$$- \sum_f \sum_{n_f=-\infty}^{\infty} \left\{ \frac{\omega_{pf}^2 \left\langle J_{n_f-1}^2 - J_{n_f+1}^2 \right\rangle}{2\omega_{cf}(\omega - n_f\omega_{cf})} - \frac{\omega_{pf}^2 n_f^2 \omega_{cf}^2 \left\langle J_{n_f}^2 \right\rangle}{k_x^2 c^2 (\omega - n_f\omega_{cf})^2} \right\} , \qquad (3)$$

where the subscripts e, s, and f are for the electrons, slow ions, and fast ions, respectively; $\kappa T_\perp$ is the slow ion temperature; I_{n_s} is the modified Bessel function of the first kind; $\left\langle J_{n_f}^2 \right\rangle = \int 2\pi p_\perp dp_\perp \int dp_z f_o J_{n_f}^2(k_x\rho_f) = \frac{1}{2}\int_0^\pi \sin\theta d\theta J_{n_f}^2(k_x\rho_{f_o}\sin\theta)$, where $\rho_{fo} = p_o/m_i\Omega_{cf} = v_o/\omega_{cf}$

for an isotropic distribution $f_o = \delta(\mathbf{p} - p_o)/4\pi p_o^2$, and $\left\langle J_{n_f}^2 \right\rangle = J_{n_f}^2(k_x \rho_f)$ for $f_o = \delta(p_\perp - p_{o\perp})\delta(p_z - p_{oz})/2\pi p_{o\perp}$ (e.g., neutral beam injection). The *residues* have been neglected since the fast ions have $\gamma_f \simeq$ constant and since the limit $N_f/N_s \ll 1$ is of interest here. Also, $k_x\rho_f \sim O(1)$ and $k_x \rho_s \ll 1$.

For one fast ion species with a harmonic n_f in resonance, the dispersion relation becomes

$$\mathcal{A} + \frac{\mathcal{D}}{\mathcal{W} + \Delta} + \frac{\mathcal{B}}{\mathcal{W}} + \frac{\mathcal{C}}{\mathcal{W}^2} \simeq 0 \, ,$$

where $\mathcal{W} = \omega^2/(n_f^2 \omega_{cf}^2) - 1$ with $\Delta = 1 - \gamma_f^2$, $\mathcal{A} \sim 1 + (\omega_{pe}^2/\omega_{ce}^2) - \sum_s \omega_{ps}^2 \mathcal{S}_1/\,(n_f^2 \Omega_{cf}^2/\gamma_f^2 - \Omega_{cs}^2)\,|_{\Omega_{cs} \neq n_f \Omega_{cf}}$, $\mathcal{D} = -\sum_s \omega_{ps}^2\, \mathcal{S}_{n_s}/n_f^2 \omega_{cf}^2\,|_{n_s \Omega_{cs} = n_f \Omega_{cf}}$, $\mathcal{B} = -\sum_f \omega_{pf}^2\,[n_f \left\langle J_{n_f-1}^2 - J_{n_f+1}^2 \right\rangle]/n_f^2 \omega_{cf}^2$, $\mathcal{C} = \sum_f 4\omega_{pf}^2 \left\langle J_{n_f}^2 \right\rangle / k_x^2 c^2$, and $\mathcal{S}_{n_s} = 2n_s^2\,(\omega_{cs}^2 m_s/\kappa T_\perp k_x^2)\, I_{n_s} \exp(-k_x^2 \rho_s^2)$.

In the case of $|\mathcal{D}| \ll |\mathcal{B} + \mathcal{A}\Delta|$, the instability is quadratic. There are two conditions for instability. The condition $n_f^2 \omega_{cf}^2 > \omega_{ps}^2 \mathcal{S}_1/(1 + \omega_{pe}^2/\omega_{ce}^2) + \omega_{cs}^2$ shows that there is a n_f threshold. The condition $(16 n_f^2 \omega_{cf}^2/k_x^2 c^2)\mathcal{A}\left\langle J_{n_f}^2 \right\rangle / \left\langle J_{n_f-1}^2 - J_{n_f+1}^2 \right\rangle^2 > \omega_{pf}^2/\omega_{cf}^2$ indicates that the waves at certain wavelengths become stable as the fast ion density goes higher. The peak growth rate, for $\mathcal{B}^2 \ll 4\mathcal{A}\mathcal{C}$, is $\omega_{i,\max}/\omega_{cf} \sim (\omega_{pf}/\omega_{ps})(v_o/c)(n_f \sqrt{\left\langle J_{n_f}^2 \right\rangle}/k_x \rho_{fo})(\mathcal{A}\omega_{cf}^2/\omega_{ps}^2)^{-1/2}$ and the real part of the wave frequency is $\omega_r/\omega_{cf} \sim n_f + n_f(-\mathcal{B}/2\mathcal{A})$. We note that $\omega_{i,\max} \propto \sqrt{N_f/N_s}$.

When $|\mathcal{D}| \simeq |\mathcal{B} + \mathcal{A}\Delta|$, the instability is cubic, with $\omega_{i,\max}/\omega_{cf} \sim 3^{1/2}/2^{4/3}\,(\omega_{pf}/\omega_{ps})^{2/3}\,(v_o/c)^{4/3}\, n_f^{1/3}\,(-\mathcal{A}\omega_{cf}^2/\omega_{ps}^2)^{-1/3}\,(n_f \sqrt{\left\langle J_{n_f}^2 \right\rangle}/k_x \rho_{fo})^{2/3}$ and $\omega_r/\omega_{cf} \sim n_f + (2/\sqrt{3})(\omega_{i,\max}/\omega_{cf})$. The peak growth rate is scaled as $(N_f/N_s)^{1/3}$ and $(v_o/c)^{4/3}$. Also we find $\delta_n \propto \omega_{i,\max}$.

If $|\mathcal{D}| \gg |\mathcal{B} + \mathcal{A}\Delta|$, which may be true when $1 = n_s \simeq n_f \Omega_{cf}/\omega_{cs}$, we have a coupled quadratic instability. For $4\mathcal{D}\mathcal{C}\Delta > (\mathcal{C} + \mathcal{B}\Delta)^2$, we find $\omega_{i,\max}/\omega_{cf} \sim n_f(\omega_{pf}/\omega_{ps})(v_o/c)^2\,(n_f \sqrt{\left\langle J_{n_f}^2 \right\rangle}/k_x \rho_{fo})\; \mathcal{S}_{n_s}^{-1/2}$ and $\omega_r/\omega_{cf} \sim n_f + 2n_f(\omega_{pf}^2/\omega_{ps}^2)\,(v_o/c)^2\,(n_f^2 \left\langle J_{n_f}^2 \right\rangle / k_x^2 \rho_{fo}^2)\; \mathcal{S}_{n_s}^{-1}$. The peak growth rate is $\omega_{i,\max} \propto \sqrt{N_f/N_s}$ and $(v_o/c)^2$, which is small because $v_o/c \ll 1$. Also we find $\delta_n \propto N_f/N_s$.

The efficiency η is defined as the fraction of the fast ion energy transferred to a single wave: that is, $\eta = (\gamma_o - \langle\gamma_f\rangle_s)/(\gamma_o - 1)$, where $\langle\gamma_f\rangle_s$ is γ_f averaged at the saturation. If the time of the interaction obeys $t < \pi/(n_f\delta_n)$, the saturation may be caused by the change of the fast ion cyclotron frequencies during their slowing down. From the dispersion relation we know the initial frequency mismatch is $\delta_o \simeq \omega_r - n_f\Omega_{cf}/\gamma_o > 0$, where γ_o is the initial γ_f. The interaction stops when the frequency mismatch becomes $\delta_s = \omega_r - n_f\Omega_{cf}/\langle\gamma_f\rangle_s \sim 0$. Thus, we find $\eta \simeq \delta_o/n_f\omega_{cf}(\gamma_o - 1)$.

Two cases are numerically investigated: (1) D+D$\longrightarrow$ p(3.02 MeV)+T(1.01 MeV) in which the fast ion (proton) has $\gamma_f \simeq 1.00322$; and (2) D+T$\longrightarrow$ α(3.5 MeV) +n(14.1 MeV) in which the α particle has $\gamma_f \simeq 1.00093$. In both cases, we use $B_o = 5$T, $N_s = 1 \times 10^{13}\text{cm}^{-3}$, $\kappa T_\perp = 5\,\text{keV}$, and $N_f/N_s = 0.001$, and the fast ion distribution is taken to be isotropic. For the D-D case, the maximum gyroradius of the protons is $\rho_{fo} \simeq 5\,\text{cm}$ and $\Omega_{cp} = 2\Omega_{cD}$. Figure 1 shows the growth rates and the quantity δ_1/ω_{cf} of the cubic instability. The higher the value of n_f, the shorter the wavelength, λ, of the unstable modes. For $n_f = 1$, the instability peaks at $\lambda \sim 14\,\text{cm}$ and $\eta \sim 20\%$; that is, the wave can significantly gain energy from the protons. At higher harmonics, the instability becomes quadratic. The growth rates are shown on Fig. 2. The spectrum slowly moves to shorter wavelength and smaller growth rate as n_f increases. The maximum growth rate $\omega_{i,\max}$ is larger than that of the cubic instability. However, its efficiency is smaller (e.g., $\eta \sim 10\%$ for $n_f = 7$). We check the dependence of ω_i upon N_f/N_s for $n_f = 1$ (not shown), which verifies the cubic scaling law (i.e., $\omega_{i,\max} \propto N_f^{1/3}$). Figure 3 verifies the quadratic dependence of $\omega_{i,\max}$ on N_f for $n_f = 7$. We also observe that the wave is stable in the wavelength interval $4\,\text{cm} < \lambda < 5\,\text{cm}$ as the fast ion density increases (i.e., $N_f/N_s = 0.005$). The growth rate of the cubic instability depends on the slow ion temperature; in contrast, the quadratic instability is insensitive to $\kappa T_\perp$ (not shown). As the temperature is increased, the spectrum becomes broader, the wavelength at the peak growth rate is longer, and the growth rate is higher. When we decrease the magnetic field or increase the slow ion density (at N_f/N_s = constant), the n_f threshold for the quadratic instability becomes higher; however, the cubic instability is insensitive to these parameters (not shown). The growth rates become higher, while the width of the spectrum is determined by n_f (e.g., the spectrum of $n_f = 10$ for $B_o = 3$T has the same λ/ρ_{fo} width as the one on Fig. 2., $B_o = 5$T.) and part of it may become stable due to being unable to satisfy the instability condition.

For the D-T case, the α particle has parameters $\rho_{fo} = 5.4\,\mathrm{cm}$, $N_D = 9N_T$, and $N_s = N_D + N_T$. We note that $\Omega_{c\alpha} = \Omega_{cD}$, and hence the first harmonic interaction may become the coupled quadratic instability, which is much weaker. The results are shown in Fig. 4. The growth rate is proportional to $\sqrt{N_f/N_s}$ whereas the real part has a linear dependence. Other harmonics behave similar to the D-D case.

The instability analysis here is for a uniform plasma and electrostatic waves. The contribution from the electromagnetic component remains to be explored. The mechanism may be modified by including the effects of a nonuniform magnetic field such as adding *residues* or Dopple-shift effects. The slow ions can gain energy from this process as indicated by the single particle model. A sheared magnetic field can introduce a finite k_z to the waves as they propagate. This gives plasma electrons an opportunity to gain energy by Landau damping. The electric field associated with the ion Bernstein waves is perpendicular to the external magnetic field. It may become a sheared radial electric field in a realistic geometry such as torus. The beating of the waves can generate low frequencies. Also, a ponderomotive force can be produced in a nonuniform plasma. These and other collective effects (e.g., turbulence and anomalous transport) affected may be important for plasma confinement [2, 8, 9], especially for a burning plasma. It seems to us that, as a new concept, this mechanism may be relevant to the ion cyclotron emission observed from JET experiments (both D-D and D-T) [10, 11]. The mechanism may be used for a plasma diagnostic. These points need more study.

In summary, we have found a new mechanism for wave-particle interaction which is a two-energy-stream cyclotron instability from the mass variation effect of the fast ions immersed in a magnetized plasma. A single particle model has been developed to show that the relativistic effect dominates as $\omega/k_z > c$ and to gain more physics insights. The dispersion relation verifies the dominance and gives the scaling laws for the growth rate and wave frequency in three different regimes. Numerical results have also shown the parameter dependence. The first harmonic for the protons in the D-D case is the cubic instability and is stronger than other harmonics. In contrast, the first harmonic of α particle in the D-T case is the coupled quadratic instability and is much weaker. This makes a very fundamental difference between the D-D and D-T cases. We have also shown that the energy of the fast ions can be transferred to the waves effectively. This interaction may happen before other proposed possible interactions [1, 12] since the time scale is much shorter. Simulations are being studied. We suggest that large scale simulations for fusion study should include this mechanism. We expect that the mechanism can also be used for the generation of coherent radiation and applied to space physics. The importance of this study for basic plasma physics on the understanding of collective wave/particle interactions and its potential applications for fusion are emphasized.

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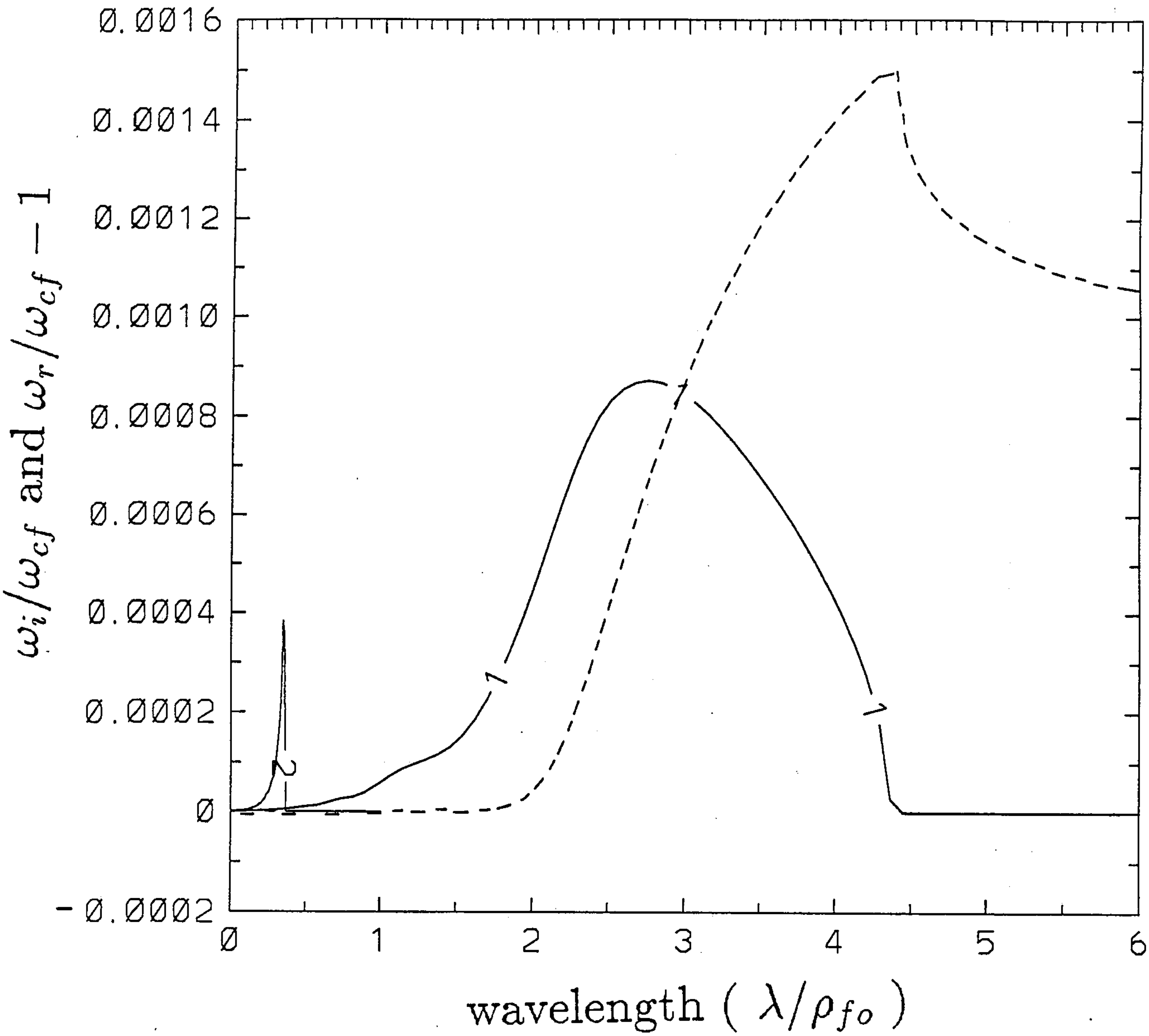
Figure Captions

1. Growth rate (ω_i/ω_{cf}) of $n_f = 1$ and 2, and $\omega_r/\omega_{cf} - 1$ of $n_f = 1$ (dashed line) vs. wavelength (λ/ρ_{fo}) for the D-D case. The instabilities are cubic.

2. Growth rate (ω_i/ω_{cf}) of $n_f = 7, 8, 9$, and 10, and $\omega_r/\omega_{cf} - 7$ of $n_f = 7$ (dashed line) vs. wavelength (λ/ρ_{fo}) for the D-D case. They are the quadratic instabilities.

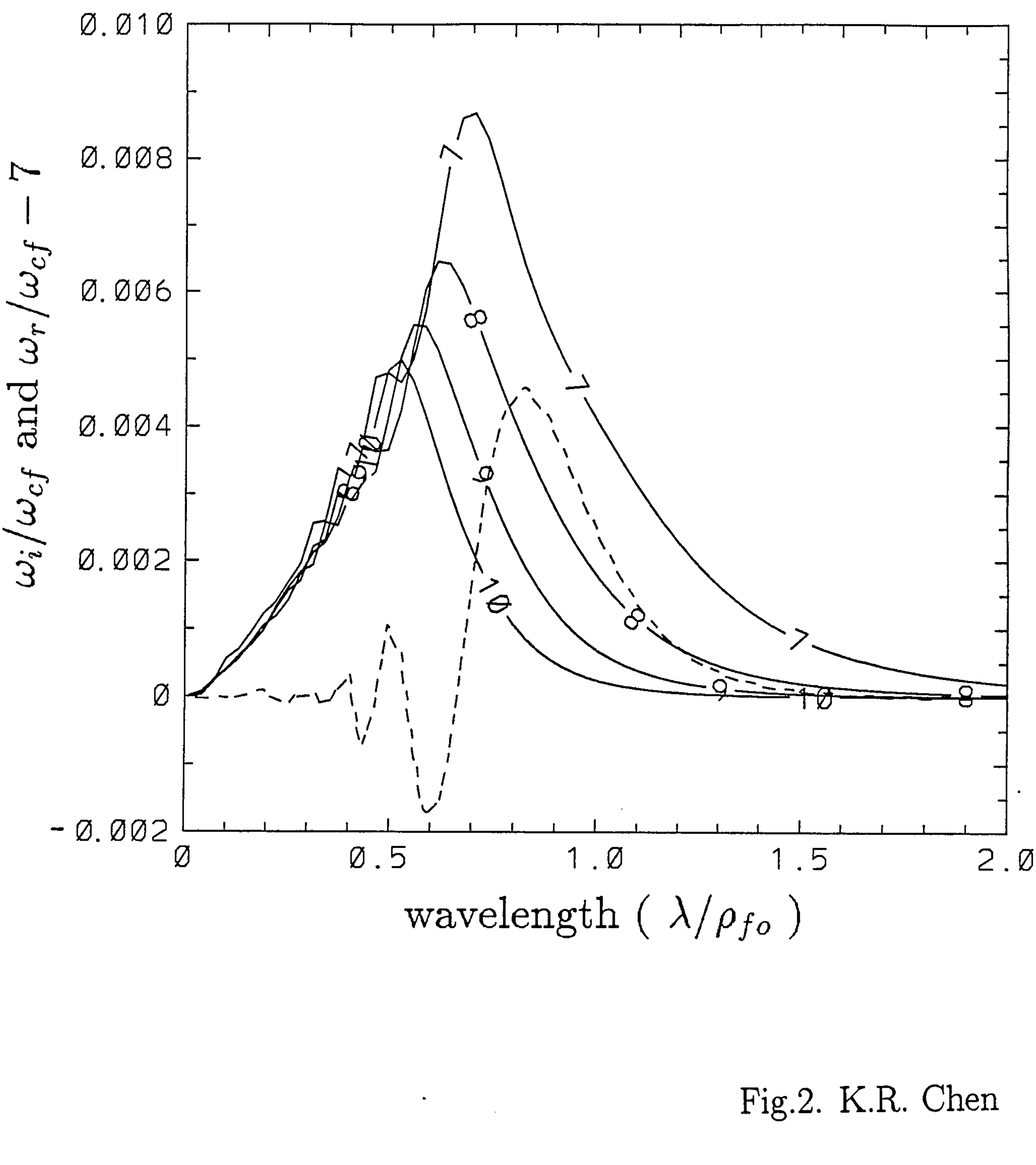
3. Growth rate (ω_i/ω_{cf}) of $n_f = 7$ vs. wavelength (λ/ρ_{fo}) for the D-D case of $N_f/N_s = 0.005, 0.001, 0.0001$, and 0.00001, respectively.

4. Growth rate (ω_i/ω_{cf}) and $\omega_r/\omega_{cf} - 1$ (with dashes) of $n_f = 1$ vs. wavelength (λ/ρ_{fo}) for the D-T case of $N_f/N_s = 0.01$ and 0.001, respectively. This is the coupled quadratic instability.



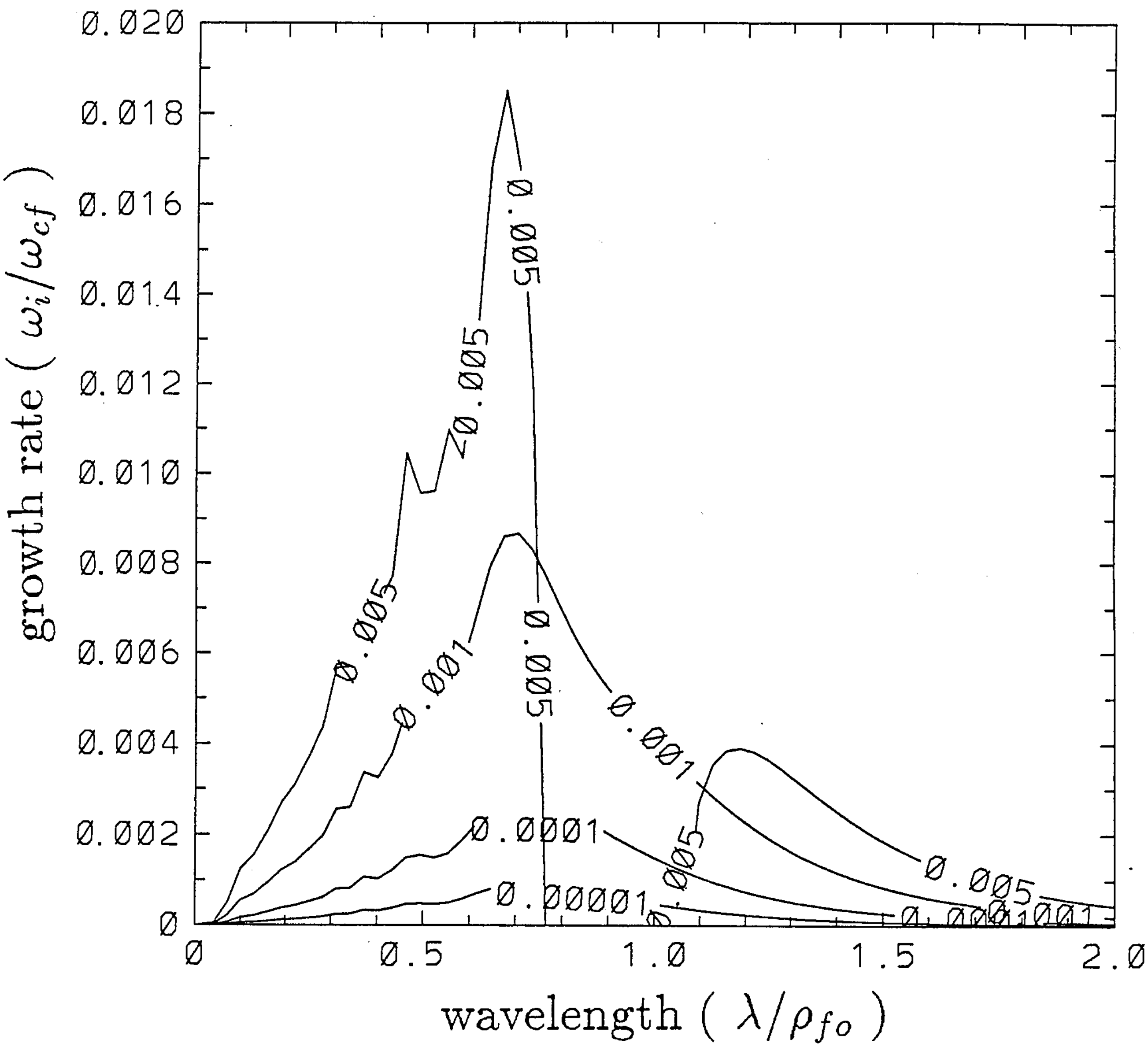
0.0016
0.0014
0.0012
0.0010
0.0008
0.0006
0.0004
0.0002
0
-0.0002
0
1
2
3
4
5
6
ω_i/ω_{cf} and $\omega_r/\omega_{cf} - 1$
wavelength (λ/ρ_{fo})
1
1
1
2

Fig.1. K.R. Chen



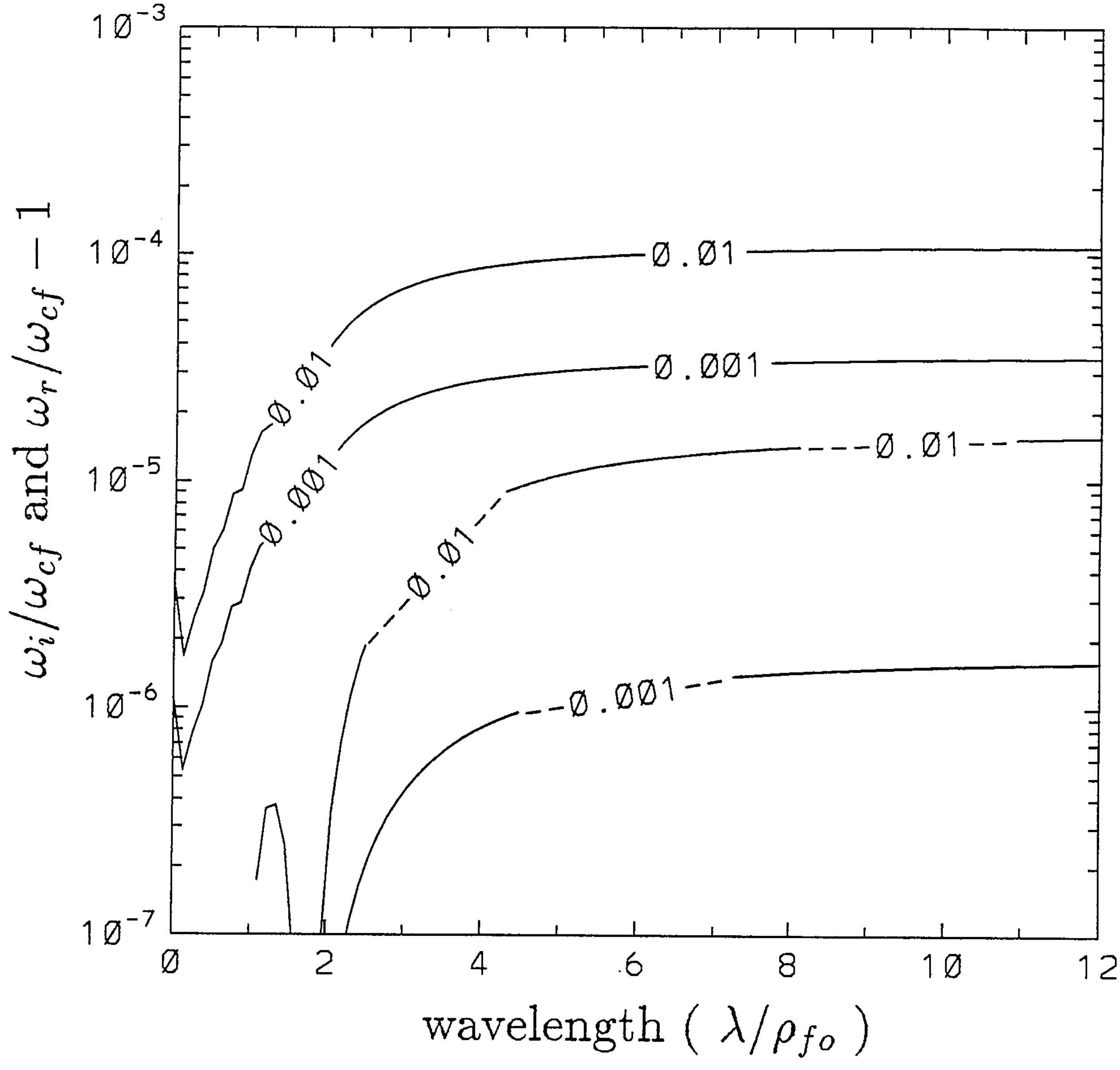
0.010
0.008
0.006
0.004
0.002
0
-0.002
0
0.5
1.0
1.5
2.0
ω_i/ω_{cf} and $\omega_r/\omega_{cf} - 7$
wavelength (λ/ρ_{fo})
7
8
9
10

Fig.2. K.R. Chen



growth rate (ω_i/ω_{cf})
wavelength (λ/ρ_{fo})
0.020
0.018
0.016
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0
0.5
1.0
1.5
2.0
0.005
0.001
0.0001
0.00001

Fig.3. K.R. Chen



ω_i/ω_{cf} and $\omega_r/\omega_{cf} - 1$
wavelength (λ/ρ_{fo})
10^-3
10^-4
10^-5
10^-6
10^-7
0
2
4
6
8
10
12
0.01
0.001
0.01
0.001
0.01
0.001
0.01
0.001

Fig.4. K.R. Chen