Amplification Mechanism of Ion-Ripple Lasers and its Possible Application

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Abstract

The IRL is an advanced scheme for generating coherent high-power radiation, in which a relativistic electron beam propagates obliquely through an ion ripple in a plasma. Its amplification mechanism is described by a low gain theory while the linear growth rate is given by the dispersion relation. The efficiency of the lasing is determined by the nonlinear saturation mechanism discussed. By proper choice of device parameters, sources of microwaves, optical, and perhaps even X-rays can be made. The availability of tunable sources for wide wavelength regimes, coherence and high-power, as well as lower cost and simplicity of equipment are emphasized.

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I. Introduction

The basic idea of an Ion-Ripple Laser (IRL) [1]–[3] is to use a relativistic electron beam propagating obliquely across an ion ripple, as shown in Fig. IV. The first step is to create a plasma density ripple in which the ion ripple is shielded by plasma electrons. In order to produce radiation from the ion ripple, a relativistic electron beam is injected into the plasma at an angle $\theta$ with respect to the ripple. As long as the beam density, $n_b$, is equal to, or higher than, the plasma density, $n_0$, the plasma electrons are expelled from the path of electron beam [4, 5] by its space charge. The ion ripple will then be seen as a stationary undulating force by the beam electrons. The electron oscillation frequency caused by the undulation force is $k_{ir}v_0\cos\theta$, where $k_{ir}$ is the undulation wave number, and $v_0$ is the beam velocity which closes to the speed of light and is much greater than the ion acoustic velocity. For a relativistic electron beam, the oscillation frequency is almost independent on the beam energy. The angle between the beam and the ion ripple is essential for the generation of electromagnetic radiation. Shorter undulation wavelength and stronger undulation force may make the IRLs be superior to the Free Electron Lasers (FELs) [6]–[9].

The electric field induced by this ion ripple is that produced by unshielded ions because the plasma electrons are expelled and the beam is so stiff it does not shield the ions. Subjected to the transverse electric field of the ion ripple, the beam electrons execute transverse oscillations. These transverse oscillations are the source of the energy needed to produce electromagnetic radiation [6]. We note that the driving force from the transverse electric field causes the electrons’ transverse energy to vary and the radiation comes from the transverse energy; for the FEL, the magnetic field converts energy from axial to transverse motion such that the radiation energy comes from the axial energy. This makes a very fundamental difference in the amplification mechanisms of the IRL and the FEL.
To illuminate the amplification mechanism, we study the Ion-Ripple Laser with a low gain theory [6] in the next section. The IRL share some similarities with the FEL, while some differences are also discussed here. However, the amplification mechanism of the IRL is quite different with the Ion-Channel Laser (ICL) [10]. In Sec. III, we write down the dispersion relation and growth rate for the high-gain regimes. The efficiency is estimated from the nonlinear saturation mechanism. Possible applications are discussed and comparisons made with the Free Electron Laser. The Summary is given in Sec. IV.

II. Amplification Mechanism

In the low-gain regime, the energy gain of wave fields is small compared with the initial energy of the waves. The amplitude of wave fields can be assumed to be constant during the interaction. This is usually true for early times in the interaction. The interaction can be studied as an initial value problem. The studying of the low-gain regime is for understanding the amplification mechanism.

Ion-Ripple lasers can only be linearly polarized. The component of the electric field of an ion ripple transverse to the beam direction can be expressed by

$$\mathbf{E}_t = \frac{4\pi n_0 e}{k_{ir}} \epsilon_{ir} \sin \theta \cos(k_{ir} z \cos \theta) \hat{x},$$

where $n_0$ is the average ion density, and $\epsilon_{ir}$ is the ripple of the ion density. If $\epsilon_{ir} \ll 1$ and the initial beam Lorentz factor $\gamma_b \gg 1$,

$$\mathbf{v}_\perp = -v_\perp \sin(k_{ir} z \cos \theta) \hat{x}$$

$$= -v_0 \frac{\omega_{pe}^2 / \gamma_b}{(\omega_{pe}^2 / \gamma_b) \beta_0^2 + (k_{ir} \beta_0 \cos \theta)^2} \epsilon_{ir} \cos \theta \sin \theta \sin(k_{ir} z \cos \theta) \hat{x},$$

where $\beta_0 = v_0 / c$.

We consider a linearly polarized plane electromagnetic wave; its Doppler shifted frequency
is comparable to the electron oscillation frequency:

\[ \mathbf{E}_1 = E_1 \hat{x} \cos(\omega t - kz), \]

\[ \mathbf{B}_1 = B_1 \hat{y} \cos(\omega t - kz), \]

where \( E_1(B_1) \) is the amplitude of wave electric (magnetic) field, \( \omega \) is the wave frequency, \( k = k \hat{z} \) is the wave number, \( z \) is the axial spatial position, and \( v_{ph} = \omega / k \) is the wave phase velocity.

The governing equations for an electron interacting with the wave are

\[ \frac{d}{dt}(\gamma m_e c^2) = -\frac{e E_1 v_\perp}{2} \sin(\omega t - kz - k_i r \cos \theta) \]

\[ \frac{d}{dt} P_z = -\frac{e}{c} (v_\perp \times \mathbf{B}_1) \]

\[ \frac{d}{dt} z = v_z \]

where \(-e\) is the electron charge, \( P_z = \gamma m_e v_z \) is the electron axial momentum, and we only keep the slow variation term. Since \( B_1 = E_1(c/v_{ph}) \), the axial momentum equation can also be related to the energy equation as

\[ \frac{d}{dt} P_z = \frac{1}{v_{ph}} \frac{d}{dt} \gamma m_e c^2. \]

We note that \( v_{ph} \) in the above equation becomes \( \omega / (k + k_u) \) for the FELs because of the additional ponderomotive force of a magnetic undulator, where \( k_u \) is the FELs' undulator wavenumber.

Since \( E_1 \) is a small parameter, \( eE_1 v_\perp / (\gamma m_e c^2) \ll 1 \), the governing equations can be solved by iteration. To zeroth order, Eqs. (5)-(7) reduce to

\[ \gamma = \gamma_0, \]

\[ v_z = v_{0z}, \]

\[ z = v_{0z} t + z_0, \]
where \( \gamma_0 = (1 - v_{0z}/c^2 - v_{0x}/c^2)^{-1/2} \approx \gamma \) is the unperturbed electron \( \gamma \), \( v_{0z} = v_0 \gamma_0 / \gamma_0 \approx v_0 \), and \( z_0 \) is the initial electron axial spatial position.

To obtain the first order, in \( E_1 \), solution for \( \gamma \) and \( v_z \), we substitute Eqs. (9) and (11) into Eqs. (5) and (6) and then, integrate them to obtain

\[
\gamma = \gamma_0 + \frac{e E_1 v_\perp}{2mc^2} \frac{\cos \psi - \cos \psi_0}{\omega - kv_{0z} - k_{ir}v_{0z} \cos \theta},
\]

\[
v_z = v_{0z} + \frac{e E_1 v_\perp}{2\gamma_0 mc^2} \left[ \frac{c^2}{v_{ph} - v_{0z}} \right] \frac{\cos \psi - \cos \psi_0}{\omega - kv_{0z} - k_{ir}v_{0z} \cos \theta},
\]

where \( \psi = (\omega - kv_{0z} - k_{ir}v_{0z} \cos \theta)t - (kz_0 + k_{ir}z_0 \cos \theta) \), and \( \psi_0 = kz_0 + k_{ir}z_0 \cos \theta \).

Substituting Eq. (13) into Eq. (7), and integrating it yield the expression for the perturbed amplitude of the axial phase position:

\[
z = z_0 + v_{0z}t + \left[ \frac{c^2}{v_{ph} - v_{0z}} \right] \frac{e E_1 v_\perp}{2\gamma_0 mc^2} \left\{ \frac{\sin \psi + \sin \psi_0}{(\omega - kv_{0z} - k_{ir}v_{0z} \cos \theta)^2} - \frac{t \cos \psi_0}{\omega - kv_{0z} - k_{ir}v_{0z} \cos \theta} \right\}.
\]

Although the first-order solution gives no net energy exchange between the wave and electrons, as can be seen by averaging Eq. (12) over the \( z_0 \) of all the electrons (assuming they are uniformly distributed), it illustrates that the electrons are bunched in the axial position.

Substituting Eq. (14) into Eq. (5) and averaging over initial axial positions, we obtain the rate of net electron energy change at second order:

\[
\left\langle \frac{d}{dt} \gamma_m c^2 \right\rangle \approx -\frac{e^2 E_1^2 c^4}{8\gamma_0 m_e c^2} \left[ -(k + k_{ir} \cos \theta) \left( \frac{c^2}{v_{ph} - v_{0z}} \right) \right]
\]

\[
\left[ \frac{\sin(\omega - kv_{0z} - k_{ir}v_{0z} \cos \theta)t}{(\omega - kv_{0z} - k_{ir}v_{0z} \cos \theta)^2} - \frac{t \cos(\omega - kv_{0z} - k_{ir}v_{0z} \cos \theta)t}{\omega - kv_{0z} - k_{ir}v_{0z} \cos \theta} \right].
\]

The wave growth saturates when \( \left\langle \frac{d}{dt}(\gamma_m c^2) \right\rangle = 0 \); that is, a saturation time \( T_{sat} \approx 4.4/(\omega - kv_{0z} - k_{ir}v_{0z} \cos \theta) \).
Integrating Eq. (15), we have the time-dependent change of the mean electron energy. Then, from energy conservation, we change the sign, multiply it by the total number of beam electrons, \( n_b V \), and divide it by the total wave energy \( 2E_1^2 V/8\pi \), where \( V \) is the volume of the interaction regime, to yield the gain expression:

\[
\text{Gain} \simeq -\frac{\omega_{pb}^2 v_0^2}{4\gamma_0 c^2} \left[ -(k + k_{ir}\cos\theta) \left( \frac{c^2}{v_{ph}^2} - v_{0z} \right) \right] \frac{\cos\delta t - 1 + \frac{\delta t}{2} \sin\delta t}{\delta^3} \tag{15}
\]

\[
\delta = \omega - kv_0 - k_{ir}v_{0z}\cos\theta , \tag{16}
\]

where \( \delta \) is the IRL frequency mismatch factor, and \( \omega_{pb} = (4\pi n_b e^2/m_e)^{1/2} \) is the beam electron plasma frequency.

For \( \delta t \ll 1 \), Eqs. (15) and (15) can be written as

\[
\left\langle \frac{d}{dt} \gamma m_e c^2 \right\rangle \simeq -\frac{\omega_{pb}^2 v_0^2 \omega}{24\gamma_0 m_e c^2} t^3 B\delta , \tag{17}
\]

\[
\text{Gain} \simeq \frac{\omega_{pb}^2 v_0^2 \omega}{96\gamma_0 c^2} t^3 B\delta , \tag{18}
\]

\[
B = -\left( \frac{c^2}{v_{ph}^2} - \frac{v_{0z}}{v_{ph}} \right) , \tag{19}
\]

where \( B \) is the IRL bunching parameter, and we assume \( k \gg k_{ir}\cos\theta \). As is the case for the FEL, there is no transverse bunching. This is different from the Ion-Channel Laser [10], in which the transverse bunching competes with the axial bunching. The condition for radiation is \( B\delta > 0 \). The bunching parameter and the mismatch factor are required to have same sign.

Since there is no ponderomotive force of the magnetic undulator as in an FEL, the interaction of the wave and the electrons in the axial direction, Eq. (8), is

\[
\frac{d}{dt} v_z = c \left( \frac{c}{v_{ph}} - \frac{v_0}{c} \right) \frac{d}{dt} \gamma . \tag{20}
\]

For FELs, the radiation energy comes from the axial beam energy by slowing the beam down. But, for the IRL, if \( c^2 < v_{ph}v_0 \) as might be the case if the system is in a wave guide, the
conventional $\mathbf{v} \times \mathbf{B}$ force is smaller than the mass variation effect in the axial bunching process. The radiation energy is from the transverse beam energy provided by the ion ripple and the beam electrons are speeded up in the axial direction. For this reason, the IRL frequency mismatch factor can be positive to radiate if $c^2 < v_{ph}v_0$ (i.e., the bunching parameter is positive). However, for the FELs, $v_{ph}$ in the bunching parameter is $\omega/(k + k_u)$ instead of the wave phase velocity $\omega/k$. The FELs' bunching parameter is always negative such that their mismatch factor is required to be also negative for radiation.

III. Possible Applications

The dispersion relation of the Ion-Ripple Laser can be derived [3] from Maxwell equations, the continuity equation, and the equation for electron motion; that is,

$$\varepsilon_{em}\varepsilon_{es} = C_f$$  \hspace{1cm} (21)

$$\varepsilon_{em} = \omega^2 - k^2c^2 - \frac{\omega_{pb}^2}{\gamma_0}$$  \hspace{1cm} (22)

$$\varepsilon_{es} = (\omega - k_p v_0)^2 - \frac{\omega_{pb}^2 + 3k_p^2v_f^2}{\gamma_0^2}$$  \hspace{1cm} (23)

$$C_f = \frac{(k - \frac{\omega_{pb}^2}{\gamma_0k_tr c^2 \cos \theta})k_p \beta_a^2 \omega_{pb}^2}{4\gamma_0^2} \left[ 1 - \frac{\omega - k_p v_0}{k_p c} \left( \frac{\omega_{pb}^2}{\gamma_0^2 \beta_0} + \frac{\omega_{pb}^2}{\gamma_0^2 k_tr v_0 c \cos^2 \theta} \right) \right]$$  \hspace{1cm} (24)

$\beta_a = -v_\perp/c$, $\varepsilon_{em} = 0$ is the dispersion relation for electromagnetic modes in a uniform plasma; $\varepsilon_{es} = 0$ is the dispersion relation for electrostatic modes for wavenumbers $k_p = k + k_{ir} \cos \theta$ (conservation of momentum) in a uniform plasma. The wave frequency can be determined by the intersection of the electromagnetic and the electrostatic dispersion curves; that is, $\omega_{em} = \omega_{es}$ (conservation of energy), where $\omega_{em} = (k^2c^2 + \omega_{pb}^2/\gamma_0)^{1/2}$ is the frequency of the EM mode, and $\omega_{es}(\pm) = k_p v_0 + S$, $S = \pm(\omega_{pb}^2 + 3k_p^2v_f^2)^{1/2}/\gamma_0^{3/2}$, is the frequency of fast (+) and slow (−) electrostatic beam modes. $C_f$ in Eq. (21) is understood
as the coupling factor of EM modes and ES modes through the ion ripple pump mode. In the Raman regime, the space charge effect is important; this separates the slow and the fast space charge modes. We know that the radiant wave frequency can be estimated from the intersection of electromagnetic and electrostatic dispersion curves. For $\gamma \gg 1$, the wavenumber of the EM mode is $k \sim 2\gamma_0^2(k_{ir}v_0 \cos \theta + S)/c$. This reveals the scaling of the radiation frequency, including Doppler-shifts and space charge effects. The radiation frequency of the backward Raman scattering, $\omega = \omega_r + i\omega_i$, is

$$\omega_r = \omega_{em} - \frac{\Delta}{2}$$  \hspace{1cm} (25)

$$\omega_i = \frac{1}{2} \left[ -\Delta^2 - \frac{C_f}{S\omega_{em}} \right]^{1/2},$$  \hspace{1cm} (26)

where $\Delta = \omega_{em} - \omega_{es}$. We define $\delta_r = \omega_r - \omega_{es} = \Delta/2$ as the mismatch factor including space charge effects. The maximum growth rate is at $\Delta = 0$; that is,

$$\omega_{i,\text{max}} = \frac{1}{2} \left( \frac{C_f}{S\omega_{em}} \right)^{1/2}.$$  \hspace{1cm} (27)

For an energetic electron beam, $\gamma \gg 1$, the maximum growth rate is

$$\frac{\omega_{i,\text{max}}}{\omega_{pb}} = \frac{1}{2} \left( \frac{\beta_0^2 \sqrt{\gamma_0 k_{ir} c \cos \theta}}{\omega_{pb}} \right)^{1/2}$$  \hspace{1cm} (28)

where the growth rate depends on the transverse oscillation velocity. If $k_{ir} c \cos \theta \gg \omega_{pb}/\gamma_0^{1/2}$, the maximum growth rate becomes

$$\frac{\omega_{i,\text{max}}}{\omega_{pb}} \sim \left( \frac{\omega_{pb}}{\sqrt{\gamma_0 k_{ir} c \cos \theta}} \right)^{3/2} \frac{k_{ir} \cos \theta \sin \theta}{2\sqrt{2}}$$  \hspace{1cm} (29)

the growth rate is inversely proportional to $\gamma_0^{3/4}$.

In the coherent Compton regime, the slow and fast electrostatic beam modes both are involved in the instability; they are strongly coupled and grow together. The condition is $|\omega - k_p v_0| \gg |S|$. At the maximum growth rate, the radiation frequency is

$$\omega - k_p v_0 = -\frac{1}{2} C_{fc}^{1/3} + i\frac{\sqrt{3}}{2} C_{fc}^{1/3}$$  \hspace{1cm} (30)
where
\[
C_{fc}^{1/3} = \left( \frac{\beta_u^2 \omega_{pb}^2 k_{ir} c \cos \theta}{4 \gamma_0} \right)^{1/3}.
\]

Then, we can rewrite the condition to be \((\beta_u^2 \omega_{pb}^2 k_{ir} c \cos \theta / 4 \gamma_0)^{1/3} \gg \omega_{pb} / \gamma_0^{3/2}\) or \(\beta_u \gg \left(4 \omega_{pb} / k_{ir} c \cos \theta \gamma_0^{7/2}\right)^{1/2}\). If this condition is satisfied, the scaling law of the high-gain coherent Compton regime is applicable and its maximum growth rate is
\[
\frac{\omega_{i,\text{max}}}{k_{ir} c \cos \theta} = \frac{\sqrt{3}}{2} \left( \frac{\beta_u^2}{4} \frac{\omega_{pb}^2}{\gamma_0 k_{ir}^2 c^2 \cos^2 \theta} \right)^{1/3}.
\]

If \(k_{ir} c \cos \theta \gg \omega_{pb} / \gamma_0^{1/2}\), the maximum growth rate becomes
\[
\frac{\omega_{i,\text{max}}}{k_{ir} c \cos \theta} \sim \frac{\sqrt{3}}{2} \left( \frac{e_{ir} \cos \theta \sin \theta}{2} \right)^{2/3} \frac{\omega_{pb}^2}{\gamma_0 k_{ir}^2 c^2 \cos^2 \theta}.
\]

The nonlinear saturation mechanism of the Ion-Ripple Laser is expected to be due to trapping of beam electrons in the electrostatic potential wells of the beam plasma wave. The mean velocity of the beam electrons after trapping is the phase velocity of the slow space charge wave. The average energy of the beam after trapping will be \(\gamma_{ph} = (1 - v_{ph}^2 / c^2)^{-1/2}\), where \(v_{ph} = v_0 - |S| / k_p\) is the phase velocity of the slow electrostatic beam mode. The efficiency for a cold beam may be estimated to be \(\eta = (\gamma_0 - \gamma_{ph}) / (\gamma_0 - 1)\).

For \(\gamma_0 \gg 1\), the efficiency of the Raman scattering is
\[
\eta \sim \frac{\omega_{pb}}{2 \gamma_0^{3/2} k_{ir} c \cos \theta}.
\]

It is insensitive to the undulator velocity and is inversely proportional to \(\gamma_0^{3/2}\).

For the high-gain coherent Compton scattering, from Eq. (30), we know the phase velocity of the beam mode for the maximum growth rate mode is \(v_{ph} = v_0 - C_{fc}^{1/3} / k_p\), where the amplitude of the mismatch factor is used. If \(C_{fc}^{1/3} / k_{ir} c \cos \theta \ll 1\), the efficiency is
\[
\eta \sim \frac{1}{2} \left( \frac{\beta_u^2}{4} \frac{\omega_{pb}^2}{\gamma_0 k_{ir}^2 c^2 \cos^2 \theta} \right)^{1/3}.
\]
The scaling law of the growth rates and the efficiencies given here has been checked by our simulation [3].

Conventional FELs have technical limitations of undulator wave lengths (e.g., $\geq 1$ cm) and magnetic field strength (e.g., $< 1$ T) such that it requires a very high $\gamma$ (e.g., $\sim 10^3$) electron beam to produce short wavelength (e.g., $\lambda \sim 500 \AA$) and operate there with a small growth rate and a low efficiency. In addition, this increases many beam requirements (e.g., higher energy, higher current and higher quality) and the magnet requirements (e.g., stronger and more precise magnetic field, very accurate undulator wave lengths and alignment). Although some non-conventional FELs [7, 8] can provide short undulator wave lengths, these have their limitations.

By contrast, the IRLs can have shorter undulator wave length (e.g., $\lambda_{ir} < 0.01$ cm$\sim 2\pi c/\omega_{pb}$ for $n_0 = 1 \times 10^{17}$ cm$^{-3}$) and it can easily be adjusted; the ion-ripple field is essentially steady and very high (e.g., $|E_{ir}| \sim 100$ KG for $n_0 = 1 \times 10^{17}$ cm$^{-3}$, $k_{ir}c = 2\omega_{pb}$, and $\epsilon_{ir} = 0.2$). The ion ripple is produced in a neutral plasma, it should be easy to generate and requires little energy; there is no need for an external magnet system. Thus the IRLs using relatively low energy beams can provide the same frequencies with higher growth rate and higher efficiencies than standard FELs. Alternatively, an IRL using the same energy beam as an FEL can produce shorter wavelength coherent radiation.

Beam quality is a major concern for coherent radiation sources, especially at short wavelengths. The ion-channel in the Ion-Ripple Laser provides some advantages for the beam emittance requirement. The ion-focusing force can prevent the increase of the electron beam diameter. The radiation can be confined to some degree by dielectric guiding by the plasma since the dielectric constant in the beam is larger than that of the outside plasma due to the relativistic mass increase of its electron. These effects may help with emittance requirements. Also, beam emittance gives an effective beam energy spread. The relation is $\Delta \gamma_z / \gamma_0 \simeq 0.5 (\epsilon_n/a)^2$, where $\epsilon_n = \beta_0 \gamma_0 \epsilon$ is the normalized beam emittance, and $a$ is the radial
size of the beam.

The fractional spread of axial beam energy is required to be small compared to the efficiency to ensure a coherent lasing. The efficiency in the coherent Compton regime is proportional to the undulation velocity. In the IRLs, the driving force is stronger and, thus, provides a larger oscillation velocity than in an FEL with the same undulator wavelength. Thus, IRLs have higher efficiency and hence allow a larger energy spread. Also, IRLs requires lower beam energy than FELs for the same wavelength radiation. The efficiency for both the coherent Compton and the Raman regime is inversely proportional to $\gamma$ such that a lower beam energy means a larger beam energy spread is allowed.

We give some numerical examples for the Ion-Ripple Lasers' scaling from the scaling laws for backward Raman scattering and for high-gain coherent Compton scattering in Tables I and II, respectively. The parameters of the electron beam required are accessible with current technology. $E$ is the beam energy, $\Delta E$ is the energy spread allowed, $I$ is the beam current, the beam density, $n_b$, is assumed to be the same as the plasma (ion) density, $\lambda_{ir}$ is the ripple length, $\lambda$ is the radiation wavelength, and $P$ is the peak output power.

The electron beam parameters are the same for both the Raman scattering and the Compton scattering. The fractional ripple in the ion density and the angle between the electron beam and the ion ripple remain the same for both ($\varepsilon_{ir} = 0.2$ and $\theta = 45^\circ$). We changed the wavelength of the ion ripple to satisfy the condition for the high gain coherent Compton regime. The IRLs numerical examples of both coherent Compton and Raman regimes show that high power, high efficiency, and short wavelength lasers may be achieved with relative low energy and quality beams. For instance, to have UV lasing in the coherent Compton regime, the axial beam energy spread is required to be smaller than .33%; that is, for a beam $\gamma = 60$, the energy spread is required to be smaller than 100 keV. If the beam employed for the UV lasing in Tables I and II has a normalized emittance $\epsilon_n = 3 \times 10^{-4} \pi$-cm-rad, the axial energy spread caused by the emittance is about 9 keV (or $\Delta \gamma_\varepsilon/\gamma_0 \sim 3.1 \times 10^{-4}$);
it is smaller than that required for the application in both the Raman and the coherent Compton regimes. For the X-ray case in the coherent Compton regime, the beam normalized emittance is required to be smaller than $1 \times 10^{-4} \pi$-cm-rad.

IV. Summary

The amplification mechanism of a new laser concept, which we call the Ion-Ripple Laser (IRL), has been studied. Its scaling laws are given. Its possible applications are also discussed. The laser uses a plasma ion ripple for the undulator; the electron beam propagates at an angle to the $k$ vector of the ion ripple. Since ion ripples can be created with very short wave lengths and the effective undulator field is quite high, we expected the IRL to be a realistic means for generating short wave length tunable coherent lasers; it may even prove practical to produce X-rays by this means. Furthermore, the channel effect can reduce emittance requirement and the high efficiency can allow larger axial energy spread; that is, the requirements on beam quality may be lower than for FELs. Thus, we believe that the IRL can provide a simple system with high frequency output, high efficiency and output power at relatively modest energies and beam quality; in many situations it may be superior to FELs. Proof of principal experiments are called for; these may begin with low frequency devices.

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References


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<th>X-ray</th>
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<td>$\Delta E$ (keV)</td>
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<td>&lt; 36</td>
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<td>1</td>
<td>1</td>
<td>4</td>
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<tr>
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Table I. Numerical Examples of Ion-Ripple Laser Scalings
in the High Gain Raman Regime

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<td>$E$ (MeV)</td>
<td>5</td>
<td>30</td>
<td>$2 \times 10^2$</td>
</tr>
<tr>
<td>$\Delta E$ (keV)</td>
<td>&lt; 220</td>
<td>&lt; 100</td>
<td>&lt; 60</td>
</tr>
<tr>
<td>$I$ (kA)</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>$n_b$ (cm$^{-3}$)</td>
<td>$4 \times 10^{13}$</td>
<td>$4 \times 10^{14}$</td>
<td>$4 \times 10^{15}$</td>
</tr>
<tr>
<td>$\lambda_{ir}$ (cm)</td>
<td>1</td>
<td>0.2</td>
<td>0.05</td>
</tr>
<tr>
<td>$k_{ir}c/\omega_{pb}$</td>
<td>0.53</td>
<td>0.84</td>
<td>1.1</td>
</tr>
<tr>
<td>$\omega_{i,\text{max}}/k_{ir}c$</td>
<td>$5.4 \times 10^{-2}$</td>
<td>$4.0 \times 10^{-3}$</td>
<td>$3.8 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$4.4 \times 10^{-2}$</td>
<td>$3.3 \times 10^{-3}$</td>
<td>$3.1 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\lambda$ (Å)</td>
<td>$6 \times 10^6$</td>
<td>$4 \times 10^3$</td>
<td>23</td>
</tr>
<tr>
<td>$P$ (MW)</td>
<td>222</td>
<td>99</td>
<td>248</td>
</tr>
</tbody>
</table>

Table II. Numerical Examples of Ion-Ripple Laser Scalings
in the High Gain Coherent Compton Regime
Figure Captions

1. A relativistic electron beam propagates through an ion ripple with an angle $\theta$. The radiation direction is the beam direction, $\vec{z}$. 
Fig. 1.