INSTITUTE FOR FUSION STUDIES

DOE/ET-53088-548 IFSR #548

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July 1992

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Abstract

Using the electron drift-kinetic equation and a hydrodynamic description for the ions new nonlinear vortex equations are derived taking into account the parallel trapping of the electrons in the positive potential regions. It is shown that the usual integration procedure for finding the coherent vortex structures for the $\mathbf{E} \times \mathbf{B}$ flows in the fluid description can be generalized to include the parallel acceleration $eE_{\|}\, \partial f / \partial v_{\|}$ producing the electron holes in the phase space. An example is considered in some detail.

I. Introduction

In the usual weak turbulence picture, the fluctuating plasma state is described as a superposition of finite amplitude waves with random phases in the lowest order, with the weak interactions between the waves and with the plasma particles calculated as perturbations. Relevant plasma transport properties, such as the particle and energy fluxes, the efficiency of RF plasma heating, and other transport quantities can then be, in principle, calculated from the spectra of such weakly turbulent fluctuations. In practice, however, the solution of the wave kinetic equation for the turbulent spectrum is a difficult problem[1] even for the simple two-dimensional description of drift waves.

However, if the level of the fluctuations is high (above a relatively low level), such a simple picture breaks down, because of the strong correlations required to account for non-wave-like fluctuations. Examples of such non-wave fluctuations are known in the hydrodynamics of neutral fluid and in plasma from particle simulations. In fluid experiments[2,3] it is well known that the convective nonlinearities give rise to the presence of convective cells, or coherent vortices, and some plasma simulations show a similar behavior. For larger moderate values of the Reynolds number, fluid behavior is governed by the presence of eddies, which are localized fluctuations of vorticity. Eddies can be regarded as dynamic, or transitional, elements of a turbulent cascade, having a finite lifetime, but being constantly produced by some nonlinear process.

Plasma counterparts of such fluid phenomena are known as plasma clumps, holes, and vortices. Clumps[4] are the lowest order non-Gaussian correction to the energy distribution of plasma fluctuations. They are localized in the phase space and their lifetime can exceed the correlation time of fluctuations if they are spatially localized with the size smaller than the correlation length of the fluctuations.

Holes,[5] or phase space vortices, consist of particles trapped in a self-consistent potential well. They can be found as a spatially localized BGK solution of the Vlasov-Poisson system. Since a local potential minimum can trap only particles whose kinetic energy is not exceeding the depth of the potential well, holes are localized also in the velocity space. The estimated lifetime of an isolated hole is longer than the ion bounce time (for holes having negative potential), see Ref. 5 and references therein, but in a system of many interacting holes the lifetime may be shorter.

Magnetized plasmas allow for the existence of yet another type of localized structures. As a result of the dominance of the convective nonlinearity, with the leading order fluid velocity being given by $\mathbf{v}_E = \mathbf{E} \times \mathbf{B}/B^2$ the plasma is expected to self-organize into pairs of vortex tubes, which are strongly elongated along the magnetic field and propagating in the perpendicular direction with the velocity proportional to their amplitude. Such vortex solutions are found for a large class of model equations describing plasma dynamics in various regimes and plasma geometries.[6–8] Although they are not solitons in the strict sense,[9] possessing only a finite number of integrals of motion, plasma vortices are remarkably robust objects, and it is though that the turbulent plasma state can be represented as a superposition of weakly correlated turbulent fluctuations and plasma vortices[10] arising due to some self-organization process. Such self-organization of flute modes in the presence of plasma vortices created by external electrodes have been observed experimentally.[11] Similar coherent vortex structures are also found in the simulations of the drift-wave[12] and Kelvin-Helmholtz[13] instabilities. Analytically, similar phase space vortex structures are predicted to develop in electron-beam drives[15] instabilities and parametric[16] instabilities.

In the presence of spatially localized electrostatic potentials in magnetized plasma, particle trapping in the direction parallel to the magnetic field is accompanied by the particle $\mathbf{E} \times \mathbf{B}$ drift around the local maxima and minima of the potential. As a consequence, strong coupling between electron holes and plasma vortices may be expected. The first attempt to study such hole-vortex structures is reported in Ref. 14. However, the Terry *et al.*[14] work does not take into account the usual fluid convective derivative, and their results are applicable only to holes which are not moving relative to the surrounding plasma. Furthermore, the drift-hole presented in Ref. 14 contains a physically unjustifiable surface current at the hole edge, and thus this structure is expected to be highly unstable.

In the present work we develop the theory of plasma vortices in low beta plasmas, accounting for kinetic effects associated with the electron motion parallel to the magnetic field line. In an earlier paper[15] it was demonstrated that shear-Alfvén vortices may be efficiently Landau damped by the resonant electrons, and as a consequence, the vortex speed is decelerated, and the damping also leads to the spatial spreading of the vortex. In other words, the standard hydrodynamic vortices are unstable on the Landau damping time scale.

On the electron bounce time scale, however, we show that a new type of stationary solutions become possible which are hybrid vortex/electron hole structures. In a simple case, we find one such solution analytically in the form of a "rider" monopole electron hole, superimposed on a hydrodynamic double vortex. Vortex/hole structures are expected to be stable on the electron time scale, $\tau_{be} \sim \frac{1}{k_{\|}}\left(\frac{m_e}{e\phi}\right)^{1/2}$ and to play an important role in the anomalous transport in open ended magnetic confinement devices.

II. Basic Self-Consistent Field Equations

We study electromagnetic fluctuations in a weakly inhomogeneous plasma with unperturbed density $n_0(x)$, immersed in a homogeneous magnetic field $B_0\,\mathbf{e}_z$. We assume cold ions and warm electrons, but the electron pressure is taken to be smaller than the magnetic pressure:

$$\beta = \frac{2n_0\,T_e}{c^2\,\varepsilon_0\,B_0^2} \lesssim \frac{m_e}{m_i}\,. \tag{1}$$

Here m_e, m_i denote electron and the ion mass, and T_e is the electron temperature.

For the perturbations which are both slowly varying in time, compared to the ion gyrofrequency Ω_i, and weakly z-dependent:

$$\frac{\partial}{\partial t} \ll \Omega_i \, , \qquad \frac{\partial}{\partial z} \ll \nabla_\perp \tag{2}$$

electron dynamics is described by the guiding-center approximation, i.e. by the drift-kinetic equation.

In the low β regime, Eq. (1), we can neglect all the finite electron Larmor radius effects, as well as the compressional component of the magnetic field. Furthermore, due to the small electron mass we also neglect the electron polarization drift effects, which give a contribution of the order $\frac{1}{\Omega_e}\frac{d}{dt} \ll \frac{m_e}{m_i}$. Under these restrictions, the electron drift-kinetic equation is given by

$$\frac{\partial f}{\partial t} + \mathbf{V}\cdot\nabla f + \frac{e}{m_e}\frac{\mathbf{E}\cdot\mathbf{B}}{|\mathbf{B}|}\frac{\partial f}{\partial v_\parallel} = 0 \, . \tag{3}$$

Here $e (e < 0)$ is the electron charge, $f = f(t, \mathbf{r}, v_\parallel)$ denotes the electron distribution function integrated for the velocity components perpendicular to the local magnetic field line, $v_\parallel$ is the parallel electron velocity, and $\mathbf{V}$ is the guiding-center velocity:

$$\mathbf{V} = v_\parallel \frac{\mathbf{B}}{|\mathbf{B}|} + \frac{\mathbf{E}\times\mathbf{B}}{B^2} \, . \tag{4}$$

We also assume small perturbations of the electron density and the magnetic field:

$$\begin{aligned} \delta n_e &= n_e - n_0 \ll n_0 \\ |\delta\mathbf{B}| &= |\mathbf{B} - \mathbf{B}_0| \ll B_0 \end{aligned} \tag{5}$$

and rewrite the drift-kinetic equation (3) as

$$\begin{aligned} &\left\{ \frac{\partial}{\partial t} + v_\parallel \frac{\partial}{\partial z} + \frac{1}{B_0}\left[\mathbf{e}_z \times \nabla(\phi - v_\parallel A_z)\right]\cdot\nabla \right\} f \\ &- \frac{e}{m_e}\frac{\partial f}{\partial v_\parallel}\left\{ \left[\frac{\partial}{\partial t} + \frac{1}{B_0}(\mathbf{e}_z\times\nabla\phi)\cdot\nabla\right] A_z + \frac{\partial \phi}{\partial z} \right\} = 0 \, . \end{aligned} \tag{6}$$

Here the electric and magnetic fields are expressed in terms of the electrostatic potential ϕ and the z-component of the vector potential $\mathbf{A}$:

$$\begin{aligned} \mathbf{E} &= -\nabla\phi - \mathbf{e}_z \frac{\partial A_z}{\partial t} \\ \mathbf{B} &= B_0\,\mathbf{e}_z - \mathbf{e}_z \times \nabla A_z \, . \end{aligned} \tag{7}$$

Note that the compressional component of the magnetic field $\delta\mathbf{B}_z = \nabla_\perp \times \mathbf{A}_\perp$ is neglected due to the low β assumption, Eq. (1).

Ions are assumed to be cold, and we can describe them by the hydrodynamic equations. Furthermore, we study modes whose parallel phase velocity is exceeding the sound speed, and the ions can be considered as strictly two-dimensional, with their fluid velocity being given by the sum of the $\mathbf{E} \times \mathbf{B}$ and polarization drifts:

$$\mathbf{v}_{\perp i} = \frac{1}{B_0}\left\{\mathbf{e}_z \times \nabla\phi - \frac{1}{\Omega_i}\left[\frac{\partial}{\partial t} + \frac{1}{B_0}(\mathbf{e}_z \times \nabla\phi)\cdot\nabla\right]\nabla\phi\right\} . \tag{8}$$

Substituting Eq. (8) into the ion continuity equation, we readily obtain to the leading order

$$\left[\frac{\partial}{\partial t} + \frac{1}{B_0}(\mathbf{e}_z \times \nabla\phi)\nabla\right]\left[\log n_0(x) + \frac{\delta n_e}{n_0} + \frac{e}{m_i}\left(\frac{1}{\Omega_i^2} + \frac{1}{\omega_{pi}^2}\right)\nabla^2\phi\right] = 0 . \tag{9}$$

In the usual way, perturbation of the electron density δn_e in Eq. (9) is expressed in terms of the distribution function f as

$$\delta n_e = \int_{-\infty}^{\infty} dv_{\|}\, f - n_0 . \tag{10}$$

Finally, system of equations is closed by the parallel component of the Amperé's law:

$$j_{\|} = e\int_{-\infty}^{\infty} dv_{\|} v_{\|}\, f = -c^2\varepsilon_0\, \nabla_\perp^2\, A_z \tag{11}$$

where we have neglected the ion contribution to the parallel current, being of the order m_e/m_i.

III. Stationary Solutions

We seek a solution which is time stationary, and z independent in the reference frame moving with the velocity

$$\mathbf{v}_{ph} = u\left(\mathbf{e}_y + \alpha^{-1}\,\mathbf{e}_z\right) , \qquad \alpha \ll 1 \tag{12}$$

i.e. we assume all perturbed quantities to be dependent only on x, and $y' = y + \alpha z - ut$. In other words, we have a two-dimensional elongated structure moving with the velocity u along x axes, and making a pitch angle α to the z axes.

Using Eq. (12) we can rewrite the ion continuity equation as a complete vector product, and integrate it, yielding

$$-\frac{1}{\rho_s^2}\frac{\omega_{pi}^2}{\Omega_i^2}\,B_0\,v_d x+\frac{e}{\varepsilon_0}\,\delta n_e+\left(1+\frac{\omega_{pi}^2}{\Omega_i^2}\right)\nabla^2\phi=G(\phi-B_0\,ux)\;. \tag{13}$$

Here $\rho_s^2=\frac{T_e}{m_i\,\Omega_i^2}$ is the ion inertial scale length, $v_d=-\frac{T_e}{eB_0}\frac{d}{dx}\log n_0(x)$ is the zero-order diamagnetic drift velocity, and G is an arbitrary function of its argument.

Similarly, in the travelling case the electron drift-kinetic equation can be written as

$$(\mathbf{e}_z\times\nabla\psi)\cdot\nabla f-\frac{\partial f}{\partial v_{\|}}(\mathbf{e}_z\times\nabla\psi)\cdot\nabla\frac{e}{m_e}\frac{\varphi}{v_{\|}-u_z} \tag{14}$$

where

$$\begin{aligned} u_z&=\frac{u}{\alpha}\\ \varphi&=\phi-u_z\,A_z\\ \psi&=\phi-v_{\|}\,A_z+(v_{\|}-u_z)\alpha\,B_0x\;. \end{aligned} \tag{15}$$

Equation (14) can easily be integrated[15] if we neglect the contribution of resonant particles, whose velocity parallel to the magnetic field is close to that component of the phase velocity, $v_{\|}\simeq u_z$. For nonresonant particles, satisfying $|v_{\|}-u_z|\gg\Delta v$ where the width of the resonant region Δv is given by

$$\Delta v=\left|-\frac{2e}{m_e}\varphi\right|^{1/2}=\left|\frac{2|e|\varphi}{m_e}\right|^{1/2} \tag{16}$$

for $\varphi>0$ and $\Delta v=0$ for $\varphi<0$, we may apply the Landau-type linearization:

$$\frac{\partial f}{\partial v_{\|}}\simeq\frac{\partial}{\partial v_{\|}}\,f_0(x,v_{\|}) \tag{17}$$

where

$$f_0(x,v_{\|})=n_0(x)\left(2\pi\,v_{Te}^2\right)^{-1/2}\exp\left(-\frac{v_{\|}^2}{2v_{Te}^2}\right) \tag{18}$$

is the unperturbed electron distribution function, and readily integrate Eq. (14) using the properties of the Poisson bracket, and neglecting small terms of the order $\frac{dn_0(x)}{n_0\,dx}\,\frac{\varphi}{B_0u}$

$$f=f_0(x,v_{\|})+\frac{e}{m_e}\frac{\partial f_0}{\partial v_{\|}}\left[\frac{\varphi}{v_{\|}-u_z}+H\left(\frac{\varphi}{v_{\|}-u_z}+\alpha B_0\,x-A_z\;,\quad v_{\|}\right)\right]\;. \tag{19}$$

Here, similarly to Eq. (13), H is an arbitrary function of its arguments. However, if the potentials ϕ, A_z are adiabatically "switched on" at $t = -\infty$, the explicit $v_\parallel$ dependence of the function H must be removed.

A complete nonlinear solution of the drift kinetic equation, which takes into account also the contribution of resonant particles, $v_\parallel \simeq u_z$, can be found by the method of characteristics. There are two integrals of motion (or characteristics) of the drift-kinetic equation (14), which are found from

$$\frac{dx}{-\frac{\partial\psi}{\partial y}} = \frac{dy}{\frac{\partial\psi}{\partial x}} = \frac{-m_e(v_\parallel - u_z)dv_\parallel}{e(\mathbf{e}_z \times \nabla\psi)\cdot\nabla\varphi} \tag{20}$$

and they can be identified as the electron energy and parallel momentum in the moving reference frame:

$$\begin{aligned} W &= \frac{m_e}{2}(v_\parallel - u_z)^2 + e\varphi \\ P &= m_e(v_\parallel - u_z) - e(\alpha B_0 x - A_z)\ . \end{aligned} \tag{21}$$

Any stationary electron distribution function can now be written in terms of the characteristics (21) as $f = F(W, P)$. The function F is determined so that asymptotically, for free particles, the electron distribution is given by Eq. (19).

Noting that for nonresonant particles we have

$$\left(\frac{2}{m_e}W\right)^{1/2} \simeq |v_\parallel - u_z| + \frac{e}{m_e}\frac{\varphi}{|v_\parallel - u_z|} \tag{22}$$

we can readily write one particular distribution function, among many possible ones which have the correct asymptotics:

$$f = f_0(X, V_\parallel)\ , \qquad (v_\parallel - u_z)^2 > \frac{2|e|\varphi}{m_e} \tag{23}$$

where f_0 is the unperturbed distribution function, Eq. (18). The characteristics $X, V_\parallel$ are expressed in terms of particle energy W and momentum P as:

$$\begin{aligned} X &= \frac{1}{\alpha\Omega_e}\left[\mathrm{sign}\,(v_\parallel - u_z)\left(\frac{2}{m_e}W\right)^{1/2} - \frac{P}{m_e}\right] \\ V_\parallel &= u_z + \mathrm{sign}\,(v_\parallel - u_z)\left(\frac{2}{m_e}W\right)^{1/2} + \frac{e}{m_e}H(\alpha B_0 X)\ , \end{aligned} \tag{24}$$

and H is the arbitrary function introduced in Eq. (19). As indicated in Eq. (23), in the case when the effective electrostatic potential in the moving frame is positive, $\varphi = \phi - u_z A_z > 0$, expressions (22)–(24) are not defined for resonant particles, i.e. those whose velocity $v_{\|}$ is within the interval $v_{\|} \in (u_z - \Delta v, u_z + \Delta v)$, where the width Δv of the resonant region is given by Eq. (16), namely

$$\Delta v = \left[\frac{-2e}{m_e}\varphi\right]^{1/2} .$$

Particles whose velocities lie within the above resonant region are trapped, and their distribution may be taken independently of free particles, Eq. (23). We choose the trapped particle distribution in the form of a hole, whose relative depth is determined by the parameter a:

$$f = (1-a) f_0 \left(X_{\text{Res}}, V_{\|\text{Res}}\right) , \qquad (v_{\|} - u_z)^2 < \frac{2|e|}{m_e}\varphi , \tag{25}$$

where

$$X_{\text{Res}} = \frac{-1}{\alpha \Omega_e} \frac{P}{m_e}$$

$$V_{\|\text{Res}} = u_z + \frac{e}{m_e} H \left(\alpha B_0 X_{\text{Res}}\right) . \tag{26}$$

The value of the parameter a which defines the hole depth is determined by the mechanism of the hole production at $t \to -\infty$, and other physical processes not studied here, such as collisions, turbulent diffusion, etc. The hole would be completely empty, $a = 1$, in the idealized case of perfectly absorbing boundaries at $z = \pm\infty$, and in the absence of processes which contribute to its filling up (e.g. collisions, etc.).

As it is stated above, the resonant distribution function, Eq. (25), applies only if the potential in the moving frame is positive, $\varphi > 0$. Negative potentials, naturally, do not trap electrons, and for $\varphi = \phi - u_z A_z < 0$ electrons are distributed according to Eq. (23) for all velocities. The distribution function defined by Eqs. (23) and (25) is continuous at the boundaries of the hole, given by $v_{\|} = u_z \pm \Delta v$, which eliminates certain instabilities of the electron hole.

Distribution function given by Eqs. (23) and (25) allows us to calculate the electron distribution δn_e, and parallel current $j_{\|}$, and to find the potentials ϕ, A_z in a self-consistent way. The calculation for δn_e and $j_{\|}$ can be done analytically in the limits of cold plasma $v_{Te} \ll u_z$, and of thermalized electrons, $v_{Te} \gg u_z$.

IV. Torsional Alfvén Holes

In the cold plasma limit $v_{Te} \ll u_z$, within the accuracy to first order of the small parameter v_{Te}^2/u_z^2 we can calculate the electron density and current using the distribution Eqs. (23) and (25)

$$\begin{aligned}
\delta n_e &= \frac{n_0 e}{m_e u_z^2}\left[\phi \frac{v_d}{u}\frac{u_z^2}{v_{Te}^2} + (\phi - u_z A_z)\left(\frac{v_d}{u} + 1 + H'\right)\left(1 + 3\frac{v_{Te}^2}{u_z^2}\right)\right. \\
&\quad \left. -\frac{3}{2}\frac{v_{Te}^2}{u_z^3}(\phi - u_z A_z)^2\left(H'' - H'''\frac{\phi - u_z A_z}{3u_z}\right)\right] - n_h \\
j_{\|} &= \frac{n_0 e^2}{m_e u_z}\left[(\phi - u_z A_z)\left(\frac{v_d}{u} + 1 + H'\right)\left(1 + 3\frac{v_{Te}^2}{u_z^2}\right) - (\phi - u_z A_z)H'\right. \\
&\quad \left. - u_z H\left(\alpha B_0 x - \frac{\phi}{u_z}\right) - \frac{3}{2}\frac{v_{Te}^2}{u_z^2}(\phi - u_z A_z)^2 H''\right] - e u_z n_h
\end{aligned} \tag{27}$$

where $H^{(n)}$ is the n-th derivative of the function H, and n_h is the density of the particles which are "missing" from the electron hole:

$$n_h = 2a\, f_0(u_z)\Delta v \ . \tag{28}$$

In the standard vortex scenario, and in the simple geometry studied here (no magnetic shear), arbitrary functions G, H are adopted in the form of linear functions

$$\begin{aligned}
G(\xi) &= (\xi - \xi_0)G' \\
H(\xi) &= (\xi - \xi_0)H' \ , \qquad G', H' = \text{const}
\end{aligned} \tag{29}$$

allowing for different slopes $G^{\rm in}, H^{\rm in}$, and $G^{\rm out}, H^{\rm out}$ inside and outside of the vortex core, and ξ_0 is the value of the argument at the core edge.

We will further simplify our equations neglecting the density inhomogeneity (i.e. setting $v_d = 0$), since finite v_d does not contribute to any new physical effects. However, we must keep small thermal corrections of the order v_{Te}^2/u_z^2 for principal reasons, since they are of the same order as the terms related to particle trapping, n_h.

From Eqs. (13) and (27) we see that in order to have finite potentials ϕ, A_z for $x \to \infty$, we must set $H^{\rm out} = G^{\rm out} = 0$.

Substituting the electron density and current, Eq. (27) into the ion continuity and Ampere's law, Eqs. (11) and (13), we obtain the following two coupled equations:

$$(1+H')\left(1+3\frac{v_{Te}^2}{u_z^2}\right)(\phi - u_z A_z) - \frac{e u_z^2}{\varepsilon_0\,\omega_{pe}^2} n_h + \frac{u_z^2}{c_A^2}\frac{c^2}{\omega_{pe}^2}\nabla^2\phi = \frac{u_z^2}{\omega_{pe}^2}(\phi - B_0\, ux - \Phi_0)G' \quad (30a)$$

$$\frac{c^2}{\omega_{pe}^2}\nabla^2 u_z A_z = (1+H')\left(1+3\frac{v_{Te}^2}{u_z^2}\right)(\phi - u_z A_z) - \frac{e u_z^2}{\varepsilon_0\,\omega_{pe}^2} n_h - (B_0\, ux - u_z A_z - \Phi_0)H' \quad (30b)$$

where $\Phi_0 = (\phi - B_0\, ux)_{r=R(\theta)}$ is the value of the stream function $\phi - B_0\, ux$ at the edge of the vortex core. Equations (30) are readily decoupled yielding the following nonlinear partial differential equation of the fourth order

$$(\nabla^2 + \kappa_1^2)(\nabla^2 + \kappa_2^2)\varphi - \gamma(c_1 + c_2\nabla^2)\varphi^{1/2} = 0 \quad (31)$$

where $\varphi = \phi - u_z A_z$, and the nonlinear term $\varphi^{1/2}$ exists only if $\varphi > 0$, i.e. when the electron trapping takes place. Coefficients appearing in Eq. (31) are determined from the slopes of linear functions G and H:

$$\kappa_1^2 + \kappa_2^2 = \frac{1}{\widetilde{\lambda}^2}\frac{c_A^2}{u_z^2}\left(\widetilde{H} - \frac{u_z^2}{\omega_{pe}^2}\widetilde{G} + 1 - \frac{u_z^2}{c_A^2}\right)$$

$$\kappa_1^2 - \kappa_2^2 = \frac{1}{\widetilde{\lambda}^4}\frac{c_A^2}{u_z^2}\left(\widetilde{H}^2 + \widetilde{H} + \frac{u_z^2}{\omega_{pe}^2}\widetilde{G}\right)$$

$$c_1 = \frac{1}{\lambda^4}\frac{c_A^2}{u_z^2}\left(H' + \frac{u_z^2}{\omega_{pe}^2}G'\right)$$

$$c_2 = \frac{1}{\lambda^2}\frac{c_A^2}{u_z^2}\left(1 - \frac{u_z^2}{c_A^2}\right)$$

$$\gamma = 2a\, f_0(u_z)\frac{u_z^2}{n_0}\left(\frac{2m_e}{|e|}\right)^{1/2} \quad . \tag{32}$$

In the above, tildae denote that the appropriate thermal corrections are included in the corresponding quantities:

$$\widetilde{H} = \frac{H'}{1+\eta}\,, \qquad \widetilde{G} = \frac{G'}{1+\eta}\,, \qquad \widetilde{\lambda}^2 = \frac{c^2}{\omega_{pe}^2}\frac{1}{1+\eta}$$

where

$$\eta = 3\,\frac{v_{Te}^2}{u_z^2}\,(1 + H')$$

and only leading order corrections in the small parameter v_{Te}^2/u_z^2 are kept.

Equation (31) has been extensively studied in Refs. 15 and 17, in the limit $a = \gamma = 0$, i.e. in the absence of the particle trapping, and it was shown that it possess a spatially localized solution in the form of a double vortex. For the details of this solution and its derivation we refer the reader to Refs. 15 and 17, and here we only present the expression for its electrostatic potential $\phi^{(1)}$ (where the superscript "1" denotes the first cylindrical harmonic):

$$\phi^{(1)} = B_0\, uR\cos\theta \cdot \begin{cases} \dfrac{r}{R} + \alpha_1\, J_1(\kappa_1 r) + \alpha_2\, J_1(\kappa_2 r)\,, & r < R \\ \dfrac{R}{r}\,, & r > R\,. \end{cases} \tag{33}$$

In Eq. (33) we used $r = (x^2 + y^2)^{1/2}$, $\theta = \mathrm{arctg}\, y/x$, the vortex core $r \leq R(\theta)$ is adopted as a circle with the radius R, J_1 is the Bessel function of the first order, and the effective wavenumbers κ_1, κ_2 are related through the following nonlinear dispersion relation

$$\alpha_1\, J_1(x_1 R) = \alpha_2\, J_1(\kappa_2 R) = 0 \tag{34}$$

where α_m, if different than zero, is given by

$$\alpha_m = \frac{-2(1+\widetilde{\lambda}^2\kappa_m^2)^{1/2}}{J_0(\kappa_m R)} \cdot \Bigg\{ \left[(1+\widetilde{\lambda}^2\,\kappa_m^2)^{1/2} + (1+\widetilde{\lambda}^2\kappa_n^2)^{1/2}\right]^{-1}$$

$$+\left[\frac{c_A - u_z}{u_z\tilde{\lambda}^2(\kappa_n^2 - \kappa_m^2)} \cdot (1 + \tilde{\lambda}^2\,\kappa_n^2)^{1/2}\right]\Bigg\} \qquad m = 1, 2\ .$$

It is convenient to adopt κ_1, κ_2 as our free constants of integration, and then the slopes H', G' are determined from κ_1, κ_2 using Eq. (32). The effective collisionless skin depth $\tilde{\lambda}$ in Eq. (34) can then be expressed as

$$\tilde{\lambda} = \frac{c^2}{\omega_{pe}^2}\frac{1}{1+\eta}$$

$$\eta = 3\,\frac{v_{Te}^2}{u_z\,c_A}\left(1 + \frac{c^2}{\omega_{pe}^2}\,\kappa_1^2\right)^{1/2}\left(1 + \frac{c^2}{\omega_{pe}^2}\,\kappa_2^2\right)^{1/2}\ . \tag{35}$$

In the presence of the electron trapping, $a \neq 0$, nonlinear wave equation (31) is very difficult to solve due to the presence of the nonlinear term $\gamma\sqrt{\varphi}$. It can be seen, however,that a simple one-dimensional, cylindrically symmetric localized solution (monopole) is not existing. Namely, cylindrically symmetric solution is possible only if $G' = H' = 0$ on the whole x-y-plane, but the nonlinear term, $\varphi^{1/2}$, is not strong enough to produce the localization (nonlinearity of the power $\gtrsim \frac{3}{2}$ is required for that).

However, there is one particular case which can be treated analytically. For the following choice of parameters:

$$\begin{aligned}\kappa_1 &= \frac{j_{1,n}}{R}\\ \kappa_2^2 &= \frac{c_A^2}{u_z^2}\frac{j_{1,n}^2}{R^2} - \frac{\omega_{pe}^2}{c^2}\left(1 - \frac{c_A^2}{u_z^2}\right)(1+\eta)\end{aligned} \tag{36}$$

where $j_{1,n}$ is the n-th zero of the Bessel function, $J_1(j_{1,n}) = 0$, and the thermal correction η is given by

$$\eta = 3\,\frac{v_{Te}^2}{u_z^2}\left(1 + \frac{c^2}{\omega_{pe}^2}\,\kappa_1^2\right)$$

the dipole solution, Eq. (33), has the potential in the moving frame equal to zero,

$$\varphi^{(1)} = \phi^{(1)} - u_z\,A_z^{(1)} = 0 \tag{37}$$

and consequently *it is not interacting* with particles. This permits us to separate equations for the dipole, and monopole parts of the moving frame potential φ and to calculate the cylindrically symmetric (monopole) part $\varphi^{(0)}$ (assuming $\varphi^{(0)} > 0$) from

$$
\begin{aligned}
(\nabla_\perp^2 + \kappa_2^2)\varphi^{(0)}(r) - \gamma\rho^2\sqrt{\varphi^{(0)}(r)} = 0 \ , \ r < R \\
(\nabla_\perp^2 - \rho^2)\varphi^{(0)}(r) - \gamma\rho^2\sqrt{\varphi^{(0)}(r)} = 0 \ , \ r > R
\end{aligned}
\tag{38}
$$

where κ_2 is defined in Eq. (36), and

$$
\rho^2 = \frac{\omega_{pe}^2}{c^2}\left(1 - \frac{c_A^2}{u_z^2}\right)(1+\eta) \ . \tag{39}
$$

Electrostatic potential $\phi^{(0)}(r)$ can be expressed in terms of the "moving" potential $\varphi^{(0)}(r)$ using Eqs. (30), and for our choice of parameters κ_1, κ_2, we obtain

$$
\phi^{(0)}(r) = \frac{c_A^2}{c_A^2 - v_z^2}\,\varphi^{(0)}(r) + \phi^{(0)}(R)\cdot
\begin{cases}
0 \ , & r > R \\
1 - \dfrac{J_0\left(j_{1,n}\,\dfrac{r}{R}\right)}{J_0(j_{1,n})} \ , & r < R \ .
\end{cases}
\tag{40}
$$

Thus, if the function $\varphi^{(0)}(r)$ is continuous and smooth at $r = R$, the same continuity conditions are automatically fulfilled also for the potential $\phi^{(0)}(r)$, owing to the following property of the Bessel function J_0

$$
\left.\frac{\partial}{\partial r} J_0\left(j_{1,n}\,\frac{r}{R}\right)\right|_{r=R} = -\frac{j_{1,n}}{R}\,J_1(j_{1,n}) = 0 \ . \tag{41}
$$

A numerical, cylindrically symmetric solution of Eq. (38) is shown in Fig. 1. It exists as a "rider" superimposed on the dipole, Eq. (33), which determines its radial scale length, and the velocity of propagation.

It is instructive also to present an approximative monopole solution. Linearizing Eq. (38) around $\varphi^{(0)} = 0$ outside, and around $\varphi^{(0)} = \frac{\gamma^2}{w}\frac{\rho^4}{\kappa_2^4}$ inside the vortex core, we obtain

$$
\varphi^{(0)}(r) \simeq \frac{\gamma^2}{w}\frac{\rho^4}{\kappa_2^4}\cdot
\begin{cases}
w + \rho K_1(\rho R) J_0(\kappa^* r) \ , & r < R \\
\kappa^* J_1(\kappa^* R) K_0(\rho r) \ , & r > R
\end{cases}
\tag{42}
$$

where $\kappa^* = \kappa_2/\sqrt{2}$, and w is the Wronskian:

$$w = \kappa^* J_1(\kappa^* R) K_0(\rho R) - \rho J_0(\kappa^* R) K_1(\rho R) \ . \tag{43}$$

Monopole solution (42) exists only in the presence of particle trapping, $a \neq 0$. It has a good radial localization, $\rho^2 > 0$, $\kappa_2^2 > 0$, if the following condition is satisfied:

$$j_{1,n} \frac{\widetilde{\lambda}^2}{R^2} \geq \frac{u_z^2 - c_A^2}{c_A^2} \geq 0 \ . \tag{44}$$

The above indicates that the phase velocity u_z bigger than the Alfvén speed c_A is required, and that the radial scale R must be of the same order as the collisionless skin depth.

The ratio of amplitudes of monopole and dipole potentials can also be estimated from Eq. (42):

$$M = \frac{\gamma^2}{B_0 u R} \simeq 16\pi\, a^2 \sqrt{\frac{m_e}{m_i}} \frac{c}{\omega_{pe} R} \frac{1}{a} \frac{u_z}{c_A} \frac{u_z^2}{v_{Te}^2} \exp\left(\frac{-u_z^2}{2 v_{Te}^2}\right) \tag{45}$$

The monopole may dominate, for $\frac{c}{\omega_{pe} R} \gtrsim 1$, provided the plasma is not too cold, i.e. that the resonant velocity $v_\| = u_z$ is not too far in the tail of the Maxwellian distribution function. From Eq. (45) we may also estimate the electron bounce period as

$$\tau_{be} \simeq \left(\frac{u}{R}\, \Omega_e\, a^2 M\right)^{-1/2} . \tag{46}$$

V. Drift-Wave Hole

In the domain of drift-wave phase velocities, $u_z \ll v_{Te}$, electron density and current can be calculated from the distribution function f, Eqs. (23) and (25) as

$$\delta n_e = \frac{-n_0 e}{m_e v_{Te}^2} \cdot \left\{ (\phi - u_z A_z)(1 + H') \left(1 - \frac{u_z^2}{v_{Te}^2}\right) - \frac{v_d}{u} \left[u_z A_z + (\phi - u_z A_z) \frac{u_z^2}{v_{Te}^2} \right] \right\} - n_h$$

$$j_\| = \frac{-n_0 e^2}{m_e u_z} \left[(B_0 u x - u_z A_z) H' - (\phi - u_z A_z)(1 + H') \frac{u_z^2}{v_{Te}^2} \right.$$
$$\left. + \frac{v_d}{u} (\phi - u_z A_z) \frac{u_z^2}{v_{Te}^2} \right] - e u_z n_h \tag{47}$$

where the density dip n_h due to the electron hole is given by Eq. (28).

In the purely electrostatic limit, $A_z = H' = 0$, the first of Eqs. (47) gives the usual expression for the electron Boltzmann distribution, and after the substitution into the ion continuity we recover the Hasegawa-Mima equation, but with a new nonlinear term $\phi^{1/2}$, which is existing only if $\phi > 0$

$$(\rho_s^2\nabla^2 - 1)\phi - B_0 v_d x + \frac{v_{Te}^2}{u_z^2}\, a\phi^{1/2} = \frac{v_{Te}^2}{u_z^2}\,(\phi - ux - \Phi_0)G' \,. \tag{48}$$

Although similar to Eq. (38), the drift-wave hole equation (48) does not possess the same kind of solution. Namely, finiteness of the potential at $x \to \infty$ sets in the exterior region the function G' to $G^{\rm out} = \frac{u_z^2}{v_{Te}^2}\frac{v_d}{u}$, but if we keep this value on the whole x-y-plane, it is not possible to have a spatially localized potential, as discussed in Sec. IV. On the other hand, allowing for a different value of the slope within the core, $G^{\rm in} \neq G^{\rm out}$, inevitably introduces a dipole part in the potential, which prohibits the simple monopole solution, similar to Eq. (38), which is existing for shear Alfvén perturbations.

The authors of Ref. 14 presented a localized monopole type of solution adopting $G' = \frac{u_z^2}{v_{Te}^2}\frac{v_d}{u}$ on the whole x, y plane, which corresponds to the neglect of the convective nonlinear term $\delta\mathbf{v}_{\perp i}\cdot\nabla$ in the continuity equation (3). However, they adopted a discontinuous hole density ($n_h = 0$ outside, and $n_h =$ const $\neq 0$ inside) which, naturally, lead them to a localized structure. The same trivial solution is obtained also in the hydrodynamics, without any particle trapping effects involved, but allowing for a discontinuous function $G(\phi - ux)$. However, such a choice of the functions G, and n_h, is not allowed, since it corresponds to an unphysical surface flow at the edge of the vortex core.

VI. Conclusions

In this work we show how the well-known integration of the 2D convective or Poisson-bracket nonlinearity for the hydrodynamic plasma motions extends to the drift-kinetic Vlasov equation that includes the parallel $\dot{v}_{||} = (e/m_e)E_{||}$ acceleration of the electrons. For the finite amplitude vortex solutions the positive potential regions form electron trapping in the z-$v_{||}$-phase space. These phase space structures called electron holes then co-exist with the $\mathbf{E} \times \mathbf{B}$ cross-field spatial trapping of the plasma in the solitary wave structures.

New nonlinear vortex equations for these $\mathbf{E} \times \mathbf{B}$ drift-electron hole coherent structures are given in Eq. (30). While no general solution is found, we develop the circular boundary solutions of radius R. For finite plasma beta $\beta_e \lesssim m_e/m_i$ the solutions change character for $R \gtrless c/\omega_{pe}$ due to electromagnetic screening of the finite mass electrons. For weak electron trapping the usual dipole vortex construction is carried out.

In the presence of significant electron trapping the vortex becomes a mixture of the dipole and monopole components. The ratio of the two components is estimated in Eq. (45). The dipole component determines the scale size and the speed of propagation of the structure. The monopole component provides the self-organization from the parallel phase space trapping.

We suspect that these new 3D coherent, localized structures are robust and stable to vortex-vortex interactions just as in the case of the 2D hydrodynamic vortices. Clearly, new hybrid ion fluid and drift-kinetic particle simulations will be required to fully investigate the new vortex-hole structures presented here.

Acknowledgments

The work was supported by the U.S. Department of Energy contract #DE-FG05-80ET-53088.

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Figure Captions

1. Radial profile of the shear-Alfvén hole potential $\varphi^{(0)}(r)$, Eqs. (38), in the case of

   a) small vortices $R^2 = 0.1\,\frac{c^2}{\omega_{pe}^2}$

   b) large vortices $R^2 = 10\,\frac{c^2}{\omega_{pe}^2}$.

   In both cases the phase velocity is adopted to be $u_z = 1.25\,c_A$, and the dashed line indicates the edge of the core.

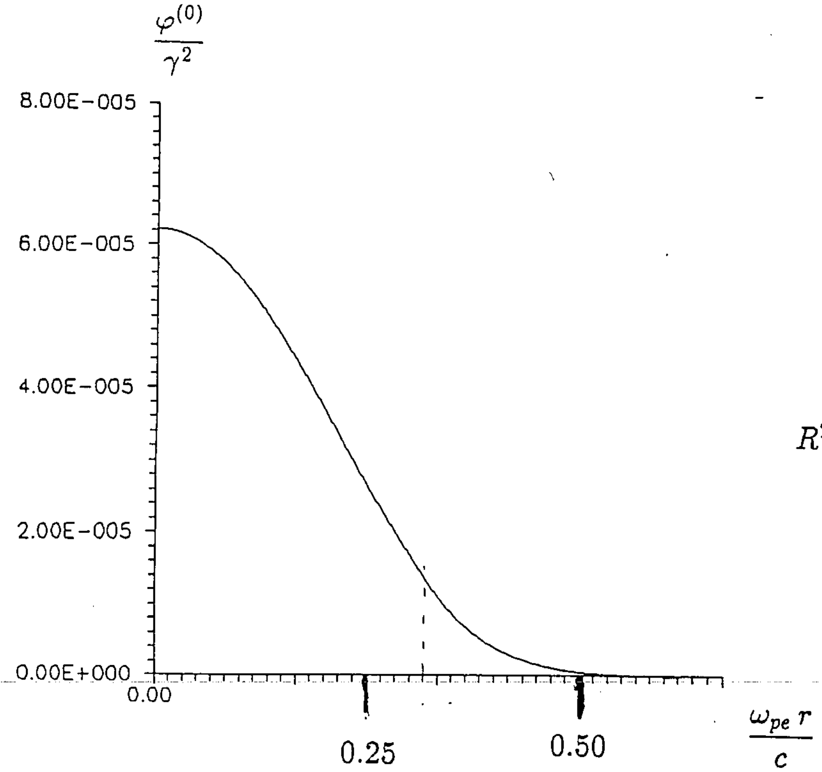

Fig. 1a

$$R^2 = 0.1\,\frac{c^2}{\omega_{pe}^2}$$

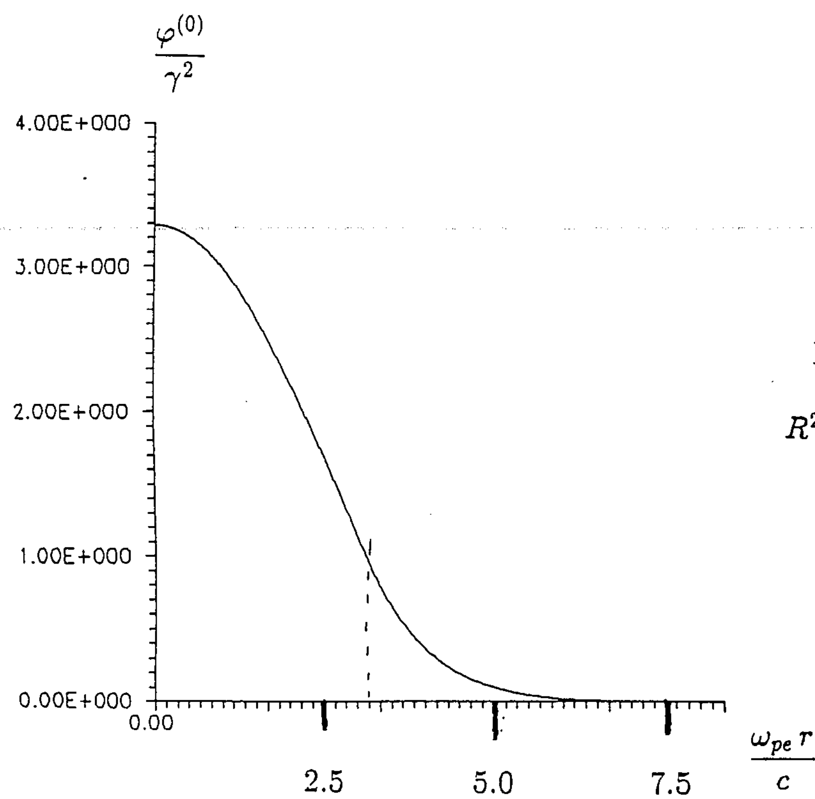

Fig. 1b

$$R^2 = 10\,\frac{c^2}{\omega_{pe}^2}$$