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P.M. VALANJU, M.D. CALVIN,
R.D. HAZELTINE, and E.R. SOLANO
Institute for Fusion Studies and
Fusion Research Center
The University of Texas at Austin
Austin, Texas 78712

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Institute for Fusion Studies and Fusion Research Center
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Abstract

We give a simple drift-kinetic derivation of the expressions for the poloidal ion and impurity flows in the presence of charge-exchange drag and ion-impurity collisions.

Recent observations[1] in DII-D have identified a sudden increase in plasma rotation as a signature of L to H transition. The discrepancies between experimental observations[2] and neoclassical predictions[3] of plasma flows are generally attributed to neutrals causing drag on ion rotation through charge-exchange (CX) collisions. Kim *et al.*[4] have derived neoclassical expressions for ion flows using the moment approach, but without CX effects. We show that a simpler derivation (including CX effects) is possible directly from the drift-kinetic equation without having to use the moment approach.

For simplicity, consider a plasma of ionized hydrogen (i) with a small concentration of neutrals (n), and one fully ionized impurity species (z). The results can be easily extended to multiple species. Also make the physically plausible assumptions that the ion density $n_i \gg n_z, n_n$, and that the CX mean-free-path λ_x and gyroradius $\rho \ll$ density scale length L.

The steady-state force balance equations for the three species are

$$\nabla \cdot \mathbf{P}_i = en_i \left(\mathbf{E} + \frac{\mathbf{V}_i \times \mathbf{B}}{c} \right) = \mathbf{F}_{iz} + \mathbf{F}_{in} \ , \tag{1}$$

$$\nabla \cdot \mathbf{P}_z - zen_z \left(\mathbf{E} + \frac{\mathbf{V}_z \times \mathbf{B}}{c} \right) = -\mathbf{F}_{iz} \ , \tag{2}$$

$$\nabla \cdot \mathbf{P}_n = -\mathbf{F}_{in} \ , \tag{3}$$

where $\mathbf{F}, \mathbf{P}$, and $\mathbf{V}$ are the friction force, pressure, and velocity. If the mass flow is much smaller than the thermal speed for each species s, $\mathbf{P}_s = \mathbf{I} p_s$. Rearranging the force-balance equations we write the system to be solved as

$$\nabla(p_i + p_n) - en_i \left(\mathbf{E} + \frac{\mathbf{V}_i \times \mathbf{B}}{c} \right) = \mathbf{F}_{iz} \ , \tag{4}$$

$$\nabla p_z - zen_z \left(\mathbf{E} + \frac{\mathbf{V}_z \times \mathbf{B}}{c} \right) = -\mathbf{F}_{iz} \ . \tag{5}$$

For large ion-impurity collisionality, we approximate the ion-impurity friction force as

$$\mathbf{F}_{iz} = -\mathbf{F}_{zi} = m_z n_z \nu_z \Delta \mathbf{V} \ , \tag{6}$$

where $\Delta \mathbf{V} \equiv \mathbf{V}_i - \mathbf{V}_z$ and ν_z is the ion-impurity collision frequency. Appropriate manipulation of (4)–(6) gives

$$(1 - q\mathbf{b}\times)\Delta \mathbf{V} = \mathbf{Q} , \tag{7}$$

where $\mathbf{b} \equiv \mathbf{B}/B$,

$$q \equiv \left(\frac{\Omega_z}{\nu_z}\right) \left[\left(1 + \frac{zn_z}{n_i}\right)\right]^{-1} \approx \frac{\Omega_z}{\nu_z} , \tag{8}$$

and

$$\mathbf{Q} \equiv \left(\frac{c}{eB}\right) q \left[\left(\frac{1}{n_i}\right) \nabla(p_i + p_n) - \left(\frac{1}{zn_z}\right) \nabla p_z\right] . \tag{9}$$

The solution to (7) can be written as

$$\Delta \mathbf{V} = (1 + q^2)^{-1}(1 + q\mathbf{b}\times)\mathbf{Q} . \tag{10}$$

We find that, due to small parallel pressure gradients,

$$Q_{\|} = \left(\frac{c}{eB}\right) q \left[\left(\frac{1}{n_i}\right) \nabla_{\|}(p_i + p_n) - \left(\frac{1}{zn_z}\right) \nabla_{\|} p_z\right] \tag{11}$$

is very small for all q, and therefore

$$V_{\|i} \approx V_{\|z} . \tag{12}$$

However, the perpendicular flows do not equilibrate. For the physical case of large $q(\Omega_z \gg \nu_z)$, we have

$$\begin{aligned} \Delta \mathbf{V}_\perp &= \left(\frac{1}{q}\right) \mathbf{b} \times \mathbf{Q}_\perp + \mathcal{O}(q^{-2}) \\ &\approx \left(\frac{c}{eB}\right) \mathbf{b} \times \left[\left(\frac{1}{n_i}\right) \nabla(p_i + p_n) - \left(\frac{1}{zn_z}\right) \nabla p_z\right] , \end{aligned} \tag{13}$$

whose poloidal component is given by

$$\Delta V_p = (\Delta \mathbf{V}_\perp)_p = \left(\frac{c}{eBn_i}\right) \left[p_i' + p_n' - \left(\frac{n_i}{zn_z}\right) p_z'\right] , \tag{14}$$

where primes indicate radial gradients. This result corresponds simply to independent diamagnetic (and $\mathbf{E} \times \mathbf{B}$) motion of each species. The physical point is very simple: large

$\nu_z(q \to 0, Q \to 0)$ wants the two species to move together, while large $\Omega_z(q \to \infty)$ wants the perpendicular velocities to be independent and diamagnetic.

Our goal is to calculate the net poloidal impurity flow

$$V_{pz} = V_{pi} - \Delta V_p \ . \tag{15}$$

Since $\mathbf{F}_{iz}$ does not affect perpendicular ion motion, (4) gives

$$\mathbf{V}_{\perp i} = \left(\frac{c}{eBn_i}\right) \mathbf{b} \times \left[\nabla(p_i + p_n) + en_i \nabla\Phi\right] \ . \tag{16}$$

Note that the neutral pressure gradient term contributes additively to the ion diamagnetic flow.

We also need $\mathbf{V}_{\|i}$ to compute V_{pi}. Using velocity variables $(v, \xi \equiv v_{\|}/v)$, we write the linearized ion drift-kinetic equation as

$$\omega \frac{\partial g}{\partial \theta} + Mg - Cg - Xg = -\mathbf{V}_d \cdot \nabla f_M - M f_d + X f_d \ . \tag{17}$$

Here θ is the poloidal angle, $\mathbf{V}_d$ is the guiding-center drift,

$$f \equiv f_M + f_d + g \tag{18}$$

is the ion distribution function,

$$f_d \equiv \left(\frac{2V_{i\|} v \xi}{v_{ti}^2}\right) f_M \tag{19}$$

is the first order perturbation of a displaced Maxwellian,

$$\omega \equiv \left(\frac{v\xi}{qR}\right) \tag{20}$$

is the transit frequency,

$$Mf = -\left(\frac{r}{2R}\right)\left(\frac{\omega}{\xi}\right) \sin\theta (1 - \xi^2) \frac{\partial f}{\partial \xi} \tag{21}$$

is the mirror force,

$$Xf = -\nu_x \left[f - \left(\frac{f_n}{n_n}\right) \int d^3v \, f \right] \tag{22}$$

is the CX operator, ν_x is the ion CX collision frequency, and $C = C_{ii}$ is the ion-ion collision operator.

For plateau ions, we order our operators according to

$$\omega > C + X > \dot{M} \ . \tag{23}$$

To lowest order, for small ν/ω, the solution to (17) is

$$g = (\nu^2 + \omega^2)^{-1}(\nu \sin\theta - \omega\cos\theta) Q_s f_M \ , \tag{24}$$

where $\nu = \nu_c + \nu_x$, and

$$Q_s = \left[\frac{v^2}{(2\Omega R)}\right](1 - \xi^2)\left[\frac{p_i'}{p_i} + \frac{e\Phi'}{T_i} + (s^2 - 5/2)\frac{T_i'}{T_i} + \frac{2\Omega r}{qR}\frac{V_{i\|}}{v_{ti}^2}\right] . \tag{25}$$

Using (24), we return to the "exact" drift-kinetic equation (17), multiply by $m_i v\xi$, integrate over velocity and perform a flux-surface average (here equivalent to a θ-average). We obtain for the parallel ion flow

$$V_{\|i} = D \cdot U_{\text{neo}} \ , \tag{26}$$

with the CX damping coefficient, D, given by

$$D = \left[1 + \left(\frac{2}{\sqrt{\pi}}\right)\left(\frac{\nu_x}{\omega}\right)\left(\frac{R}{r}\right)^2\right]^{-1} . \tag{27}$$

Here U_{neo} denotes the conventional neoclassical parallel flow

$$U_{\text{neo}} = -\left(\frac{T_i}{m_i \Omega_p}\right)\left(\frac{p_i'}{p_i} + \frac{e\Phi'}{T_i} - k\frac{T_i'}{T_i}\right) \ , \tag{28}$$

$k = -1/2$ for plateau ions, and Ω_p is the poloidal gyrofrequency.

From (16) and (26)–(28), we obtain the net poloidal ion flow

$$V_{pi} = \left(\frac{c}{eBn_i}\right)\left[p_n' + (1 - D)\left(\frac{p_i' + e\Phi'}{T_i}\right) + \frac{DkT_i'}{T_i}\right] \ , \tag{29}$$

and from (14) , (15) and (29), the net poloidal impurity flow

$$V_{pz} = -\left(\frac{c}{eB}\right)\left[DT_i\left(\frac{n_i'}{n_i}\right)\right.$$

$$+ (1-k)DT_i' + (D-1)e\Phi' - T_z\left(\frac{n_z'}{zn_z}\right) - \left(\frac{1}{z}\right)T_z'\bigg] \,. \tag{30}$$

We find that CX with neutrals causes the poloidal ion and impurity flows to depend on the radial electric field. Note that D approaches 1 in the limit of no CX. In this limit, our results agree with those of Kim, *et al.*[4] Charge-exchange introduces a neutral pressure gradient term into the ion diamagnetic flow. Since the neutral pressure gradient opposes the ion pressure gradient in much of the edge region, this should result in diminished ion diamagnetic rotation.

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