DOE/ET/53088-53 IFSR #53

PROCEEDINGS OF

US-JAPAN WORKSHOP

DRIFT WAVE TURBULENCE

JANUARY 11-15, 1982

AUSTIN, TEXAS

PREFACE

The Drift Wave Turbulence Workshop held at the University of Texas, January 11-15 was organized by the Institute for Fusion Studies under the agreement between the United States and Japan for the exchange of information in theoretical plasma physics. There were approximately thirty scientific participants: six from Japan and twenty-four from the U.S. There were six sessions with formal presentations and two sessions for informal discussions of key issues in the problem of drift wave turbulence and anomalous transport.

There were two major goals of the Workshop. The first goal being to advance the effort of identifying the more important anomalous transport mechanisms taking place in high temperature magnetic confinement systems. The second goal was the exchange of recent results between theorists, computer simulationists and experimenters. Leading scientists representing these three approaches (theory, experiment and simulation) made presentations reviewing the state-of-the-art in their respective areas. From these presentations a better sense of understanding emerged as a primary product of the Workshop. Some of the problems reported and issues discussed were the following.

The experimenters reported improved resolution in the $\underset{\sim}{k},\omega$ space of the measured fluctuations, extension of the fluctuation measurements to new machines, and measurements of magnetic fluctuations in low temperature experiments. Theorists presented alternative explanations for the character of these fluctuation spectra based on renormalized drift wave turbulence theories and the solution of nonlinear dissipative systems exhibiting intrinsic chaotic behavior. In contrast to the weakly correlated turbulence theories, the Japanese emphasized the importance of large scale correlated structures such as convective cells and solitons. An intermediate point of view of the turbulence was introduced by a theory containing a large number of randomly distributed solitons forming an ideal gas of strongly correlated objects. Simulations were presented for the collisions between drift wave solitons. A theoretical picture based on strong phase space correlations called clumps gave an alternative formulation of the fluctuation spectrum.

Simulations of drift wave turbulence above (strong gradients) and below (weak gradients) the ion cyclotron frequency and the measured transport of particles and thermal energy were reported by the Japanese and U.S. A new simulation technique offering the possibility of greatly extended parameter variations and long time runs was presented.

Discussions on the formulas for anomalous tranpsort centered on the fact that notwithstanding the importance of empirical scaling laws that synthesize by a particular parameterization large amounts of experimental data, the approach of characterizing the confinement by a formula for the global energy replacement time τ_E is an oversimplification of the issue since transport, atomic physics and heating mechanisms are interelated in power balance.

In the Wednesday discussion session the question was debated as to whether it is now timely to assemble the present differentiated areas of knowledge from these studies into an integrated data base giving the present understanding in the field of anomalous transport. Although no consenses was apparent on this issue, there was a sentiment in that direction implying that the present understanding may well be stronger than is generally realized. It is also clear, however, that many new and difficult problems continually emerge, and their solutions will change our understanding of the problem of anomalous transport.

Wendell Horton
Austin, Texas
January 1982

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US-JAPAN WORKSHOP PROGRAM

DRIFT WAVE TURBULENCE

JANUARY 11-15, 1982 AUSTIN, TEXAS

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2:30 PM	THEORETICAL AND EXPERIMENTAL REVIEWS CHAIRMAN: W. HORTON
W.M. TANG	REVIEW OF LINEAR MICROINSTABILITIES
T. SATO	NONLINEAR DRIFT WAVES IN A SHEARED MAGNETIC FIELD
P. LIEWER	A SURVEY OF ELECTROSTATIC FLUCTUATION MEASUREMENTS IN TOKAMAKS AND OTHER TOROIDAL EXPERIMENTS
S. ITOH	MICROINSTABILITY, ENTROPY-PRODUCTION AND PLASMA CONFINEMENT

TUESDAY, JANUARY 12, 1982

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T. TANIUTI	COLLISIONAL DRIFT WAVES
S. ZWEBEN	MEASUREMENTS OF TOKAMAK EDGE PLASMA FLUCTUATIONS AND TRANSPORT
L. CHEN	NONLINEAR GYROKINETIC EQUATIONS FOR LOW-FREQUENCY ELECTRO-MAGNETIC WAVES IN GENERAL PLASMA EQUILIBRIA APPLIED TO DRIFT WAVES
K. SWARTZ	NONLINEAR ELECTRON RESPONSE TO DRIFT WAVE FLUCTUATIONS IN TOROIDAL GEOMETRY
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TUESDAY AFTERNOON

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C. BEASLEY, K. MOLVIG, W. VAN RIJ	UNIVERSAL MODE WITH DIFFUSIVE ELECTRONS: LINEAR STABILITY AND NONLINEAR SATURATION
R. HATAKEYAMA	EXPERIMENTS ON LOW-FREQUENCY INSTABILITIES AND DIFFUSION IN FINITE-BETA PLASMA
R. WALTZ	NUMERICAL STUDY OF DRIFT WAVE TURBULENCE: SIMPLE MODELS FOR WAVE-WAVE NONLINEAR COUPLING
P.W. TERRY	TURBULENT SPECTRA FROM THE INTERACTION OF THREE DRIFT WAVES

TUESDAY EVENING

7:00 PM - 10:00 PM	WORKSHOP BANQUET, SPONSORED BY TRACOR, INC.

WEDNESDAY, JANUARY 13, 1982

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T. SATO	LOWER HYBRID DRIFT INSTABILITY: ANOMALOUS RESISTIVITY, TRANSPORT AND HEATING
T. GLADD	LOWER HYBRID DRIFT TURBULENCE AND ANOMALOUS TRANSPORT
W. NEVINS	SIMULATION OF DRIFT-CONE TURBULENCE IN A NEUTRAL-BEAM DRIVEN MIRROR MACHINE
T. TANIUTI	DYNAMICS OF TWO-DIMENSIONAL SOLITARY VORTICES IN A LOW-BETA PLASMA WITH CONVECTIVE MOTION (MOVIE)
J.D. MEISS	DRIFT WAVE TURBULENCE FROM A GAS OF SOLITONS
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WEDNESDAY AFTERNOON

2:00 PM	DISCUSSION OF ISSUES; SHORT CONTRIBUTIONS CHAIRMAN: R. WALTZ

THURSDAY, JANUARY 14, 1982

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P. SIMILON	NONLINEAR BEHAVIOR OF UNSTABLE TOROIDALLY INDUCED DRIFT MODES IN TOKAMAK GEOMETRY
E. MAZZUCATO	RECENT OBSERVATIONS ON MICROTURBULENCE IN PLT
A. HASEGAWA	TURBULENCE EXCITED BY BAROCLINIC VECTOR
T. HATORI	BIFURCATION AND CHAOS IN THE COLLISIONAL DRIFT INSTABILITY
P. DIAMOND	THEORY OF TWO-POINT CORRELATION FOR TRAPPED ELECTRONS AND THE FREQUENCY SPECTRUM DRIFT WAVE TURBULENCE IN TOKAMAKS

THURSDAY AFTERNOON

1:30 PM	CHAIRMAN: A. HASEGAWA
C. SURKO	SOME ASPECTS OF DENSITY FLUCTUATIONS IN ALCATOR C AND PDX
S. ITOH	KINETIC THEORY OF BALLOONING MODE
. REWOLDT	ELECTROMAGNETIC KINETIC TOROIDAL EIGENMODES FOR GENERAL MHD EQUILIBRIA
H. ABE	PARTICLE SIMULATION OF THE DRIFT WAVE INCLUDING THE ELECTROMAGNETIC EFFECT
W. HORTON	ANOMALOUS ION THERMAL CONDUCTIVITY

FRIDAY, JANUARY 15, 1982

9:00 AM	SUMMARIES AND DISCUSSION CO-CHAIRMEN: T. SATO AND M.N. ROSENBLUTH

REVIEW OF LINEAR MICROINSTABILITIES

W.M. TANG

PRINCETON UNIVERSITY

Tang

LINEAR KINETIC STABILITY STUDIES

SCOPE OF TALK:

- **NOT** detailed survey of linear μ-instab. theory but a brief overview of our current understanding of some important physics questions in this area
- consider Toroidal Systems (not Mirrors)

I INTRODUCTORY COMMENTS:
- Motivation + Objectives
- General Approach + Models Adopted

II ELECTROSTATIC MODES:
- Electron + Ion Drift Waves

III ELECTROMAGNETIC MODES:
- Shear-Alfvén Waves (Ballooning Modes)

IV PRESENT CONCLUSIONS:
- Practical Consequences

MOTIVATION:

(i) Anomalous Energy Transport [$\chi_e^{Exp} \gtrsim 10^2 \times \chi_e^{Neo}$]
- low freq. μ-instab. remain plausible major contributing factor

(ii) Possible Beta Limit [ISX-B, PDX]
- kinetic effects can significantly modify ideal MHD estimates of β_c
- could be related to (i) in enhancing already present χ_e^{Anom}

OBJECTIVES: (Linear Theory)

(i) Determine realistic threshold conditions for instab.
- how do threshold + localization charac. compare with experimental observations?
- how does worst γ scale?

(ii) Obtain properties of linear spectrum
- establish viable "sources and sinks" in spectrum for use in NL-mode-coupling theories

GENERAL APPROACH :

- SOLVE GYROKINETIC EQ. FOR f AND USE IN MAXWELL'S EQ'S (QN, $\parallel$ and $\perp$ AMPERE'S LAW)
- ORDERING ASSUMES

  $\rho/L \,,\; \omega/\Omega \,,\; k_\parallel/k_\perp \ll 1$ and $k_\perp \rho_i \sim 1$

- AXISYMMETRY $\Rightarrow$ 2D Problem
- "BALLOONING REPRESENTATION" $\Rightarrow$ 1D Problem (along equilib. $\underline{B}$-field line)
  - radial corrections higher order in $1/n$
  - applicability verified by fully-2D calc.

EQUILIBRIUM MODELS :

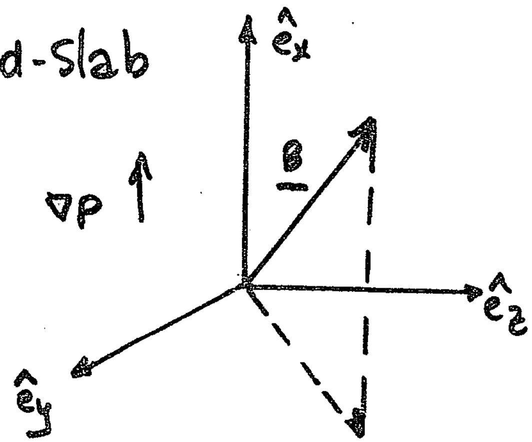
(i) LOCAL

(ii) SHEARED SLAB

(iii) CIRCULAR, CONCENTRIC SURFACE TOROIDAL

(iv) EXACT NUMERICAL EQUILIB. FROM MHD CODES

(ii) Sheared-Slab



$$\underline{B} = B_0\left(\hat{e}_z + \frac{x}{L_s}\hat{e}_y\right)$$

$\Rightarrow$ { 1D radial eigenmode problem in x-direction

e.g. Electrostatic Modes : $\left[\frac{d^2}{dx^2} + Q(x,\omega)\right]\phi(x) = 0$

For $Q = A + Bx^2$ "Weber Eq"

$\Rightarrow \phi \sim H_n(\sigma^{1/2}x)\exp(-\sigma x^2/2)$

with $n = 0, 1, 2, \ldots$

(iii) Circular Concentric Toroidal Model : (OK for $\beta \ll 1$)

$$\begin{cases} \underline{B} = B_T\hat{e}_\zeta + B_p\hat{e}_\theta \\ |B| \simeq B_0\left(1 - \frac{r}{R}\cos\theta\right) \end{cases}$$

(iv) Exact Equilib. $\Rightarrow$
- Shafranov shift + noncircular shape for $\beta \neq 0$
- significant effect on curvature

[ELECTROSTATIC INSTABILITIES $(\beta \to 0)$]

- Gyrokinetic eq $\to f$ + QN $\Rightarrow$ eigenmode eq.

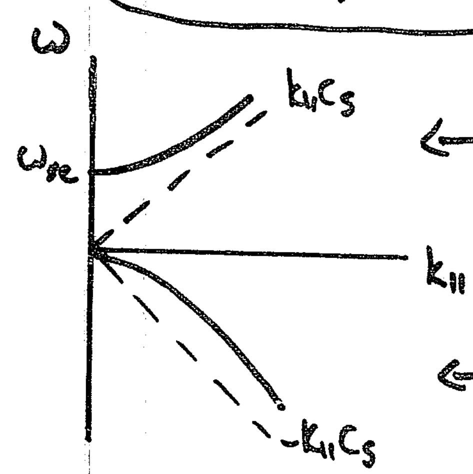
DRIFT WAVES { Low-freq. instab. driven by expansion FE in confined plasma

(includes TP modes)

- FE Source $\to$ ∇P
- Accessibility Mechanisms $\to$
  - collisional dissipation
  - wave-particle resonances
  - fluid-like interchange motion

Simplest Example : $v_{Ti} < \frac{\omega}{k_\parallel} < v_{Te}$; $k_\perp \rho_i \ll 1$

QN $\Rightarrow$ $\omega^2 - \omega\omega_{*e} - k_\parallel^2 c_s^2 = 0$



$\longleftarrow$ Electron Branch { "Universal", Dissipative, TEM, TIM

$\longleftarrow$ Ion Branch { η_i-modes (Ion-Temp-Gradient), Impurity, TIM

SHEARED-SLAB

- Universal Mode, Collisional D.M. → STABLE*
- Ion-Temp-Grad (η_i) Mode, Impurity D.M. → UNSTABLE

* EXCEPTIONS:
(i) Peaked Equilib. Profiles
(ii) Moderate collisionality + $\eta_e \neq 0$ for CDW

GALEEV MODEL (TEM) – Trapped-electrons with $\bar{\phi} = \phi$ (Weber Eq. for radial eigenmodes)

- Horton's mode-coupling calc.
- Mahajan's adaptation of H-M nonlinear calc.
- Coupling to shear-stabilized radial eigenmodes (Nishikawa)

TOROIDAL MODEL

- Univ. Mode, Collisional D.M. → UNSTABLE by ion-∇B-drift effects

Taylor, et. al. ; Chen + Cheng

[TRAPPED-PARTICLE MODES]

K-P Picture $\Rightarrow \gamma \sim \frac{\omega_*^2}{\nu_{eff}} \sim \frac{T^{7/2}}{n_0}$

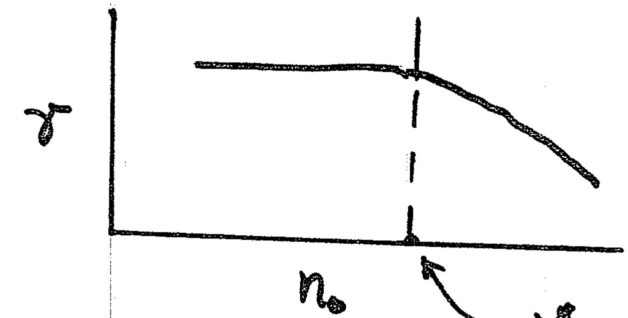
(ignores ∇B-drift effects)

- TEM ($\omega_{bi} < \omega < \omega_{be}$) with $\nu^* \equiv \nu_{eff}/\omega_b < 1$

$$\left\langle \frac{\omega - \omega_*^T}{\omega - \omega_D + i\nu_{eff}} \right\rangle \Rightarrow$$

collisionless drive (under $\omega - \omega_D$)

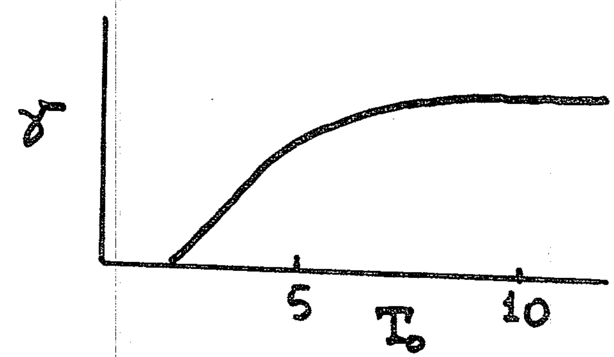
(Landau Res. and/or Interchange)



[Transition betw. regimes of different scaling with density (n_0)]

- TIM ($\omega < \omega_{bi}$)

$$\left\langle \frac{\omega - \omega_*^T}{\omega - \omega_D + i\nu_{eff} - k_\parallel v_\parallel} \right\rangle \Rightarrow$$

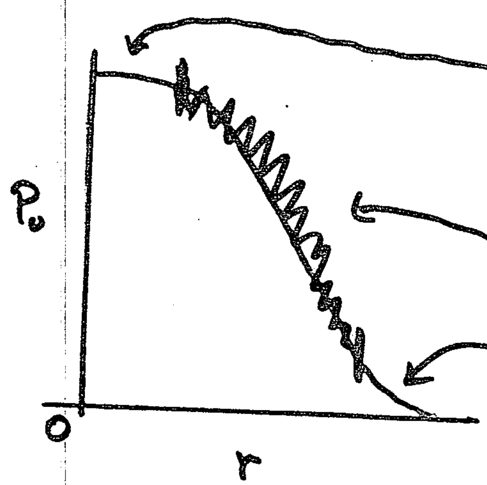


[weak T-dependence @ high temps.]

EXPERIMENTAL OBSERVATIONS + LINEAR THEORY

- THRESHOLD CONDITIONS GENERALLY CONSISTENT
  - broad spectrum of unstable DW
  - ∇T_i-driven TIM in PLT w/ threshold @ $T_i \sim 2-4$ KeV

- LOCALIZATION ("BALLOONING") IN BAD CURVATURE REGION CONSISTENT WITH FLUCTUATION MEASUREMENTS ON PLT (MAZZUCATO)

- 

  INTERIOR REGION where $q \lesssim 1$ (sawtooth oscill., weak ∇P_0, few TP)

  μ INSTAB REGION

  EDGE REGION (impurities, etc.)

$\Rightarrow$ NEED FOR EXPERIMENTAL INFORMATION ON LOCAL TRANSPORT PROPERTIES (CORRELATION BETW. OBSERVED FLUCTUATIONS AND ENHANCED TRANSP.?)

e.g. PLT (Innsbruck) { no change in $T_i(0)$ but fluctuations measured @ $r = 22$ cm

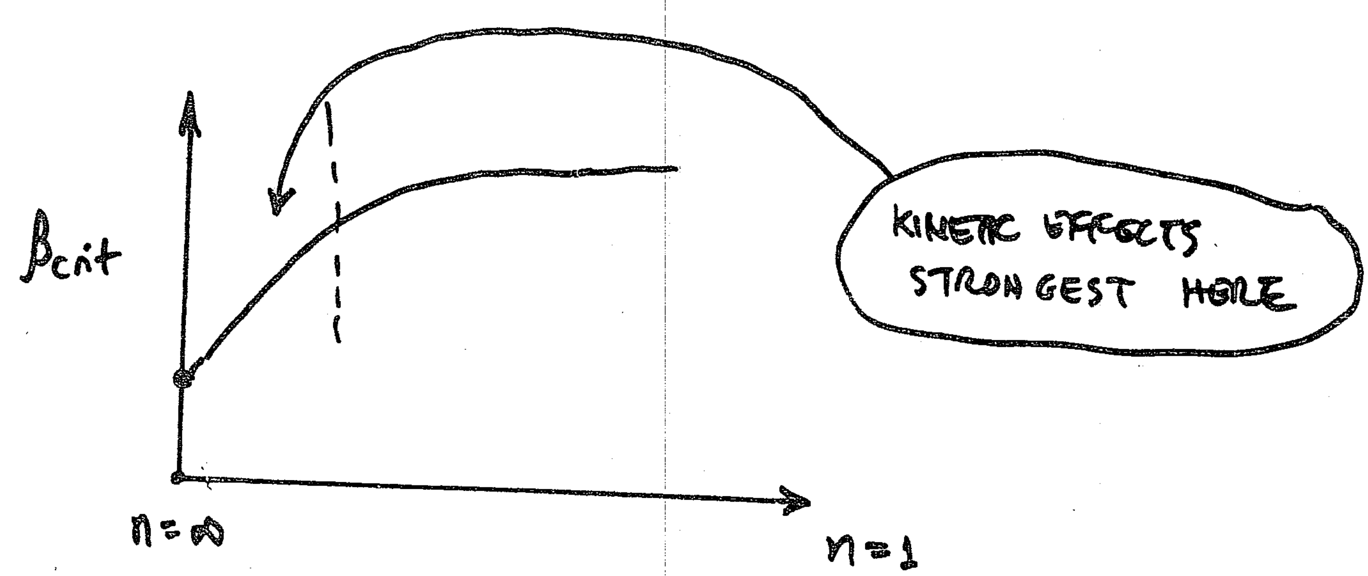
ONGOING TOROIDAL μ-INSTABILITY STUDIES (ES + EM)
[ref. - talk by G. Rewoldt]

- Interface w/ Exact Equilibrium
  - broad spectrum of unstable modes (PDX, ISX-B, PLT)
  - growth rates can differ by more than factor of 2 when compared w/ model equilib. (circular, concentric surf.)
- Beam Effects (separate beam component w/ anisotropic F_0^{beam})
  - stabilizing for LF modes [implications for τ-scaling ?]
  - possible to excite HF modes [Chen-Tsai formalism]
- Alpha Particle Effects (treat like beam effects)
  - destab. of LF Alfvén modes in <u>slab-like</u> or <u>cylindrical</u> geom. [Ros.-Ruth., Sigmar, et. al.] ⇒ toroidal <u>geometry</u> ?
  - destab. of HF modes ? [local studies of Mikhailovskii, et. al.]

KINETIC MHD STUDIES:

PRESENT ESTIMATES OF LIMITING β COME FROM IDEAL MHD BALLOONING MODE STAB. CRITERIA $\Rightarrow$ MOST PESSIMISTIC β FOR LARGE n (largest toroidal-mode-number perturbations)

e.g.



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[TOKAMAK ANALYSIS]

GOVERNING EQS – 3 COUPLED INTEGRO-DIFFERENTIAL EQS.

- ALL 3 EQ'S NEEDED TO RECOVER MHD LIMIT (cannot ignore $A_{\parallel}$)
- SINGLE 2ND ORDER DIFFERENTIAL EQ. OBTAINED BY IGNORING T.P. AND TREATING (ω/ω_{tj}) AS EITHER SMALL OR LARGE

CALCULATIONS :

(1) IDEAL MHD + ION DIAMAGNETIC DRIFTS (ω_{*i})

(2) FULL FLR + ION ∇B-DRIFT RESONANCES ADDED TO (1)

(3) RADIAL CORRECTIONS TO (1)

EQUILIBRIA CONSIDERED :

(1) MODEL ANALYTIC EQUILIB.* (CIRCULAR SURFACES)

(2) ACTUAL EQUILIBRIA (FROM EXPERIMENT AND NUMERICAL MHD CALC.)

* J.B. TAYLOR, LORTZ-NUHRENBERG, etc.

IDEAL MHD + ω_{*i}

(MODEL EQUILIBRIUM CALCULATION)
[radially local]

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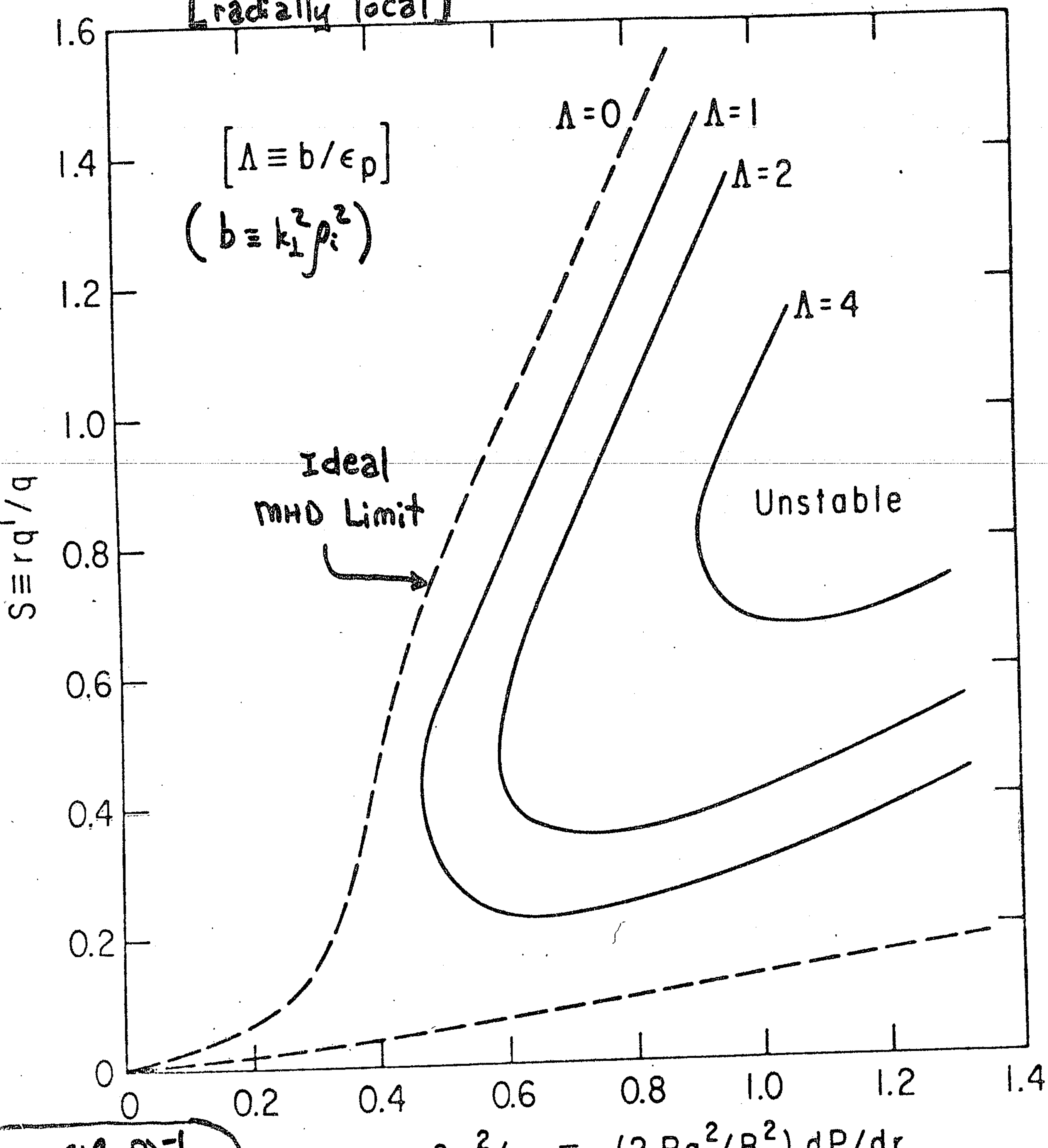

$\epsilon_p \sim \frac{1}{R}\left(\frac{d\ln P}{dr}\right)^{-1}$

[Ref. - W.M. Tang, J.W. Connor, R.B. White, Nucl. Fus. 21, 891 (1981)]

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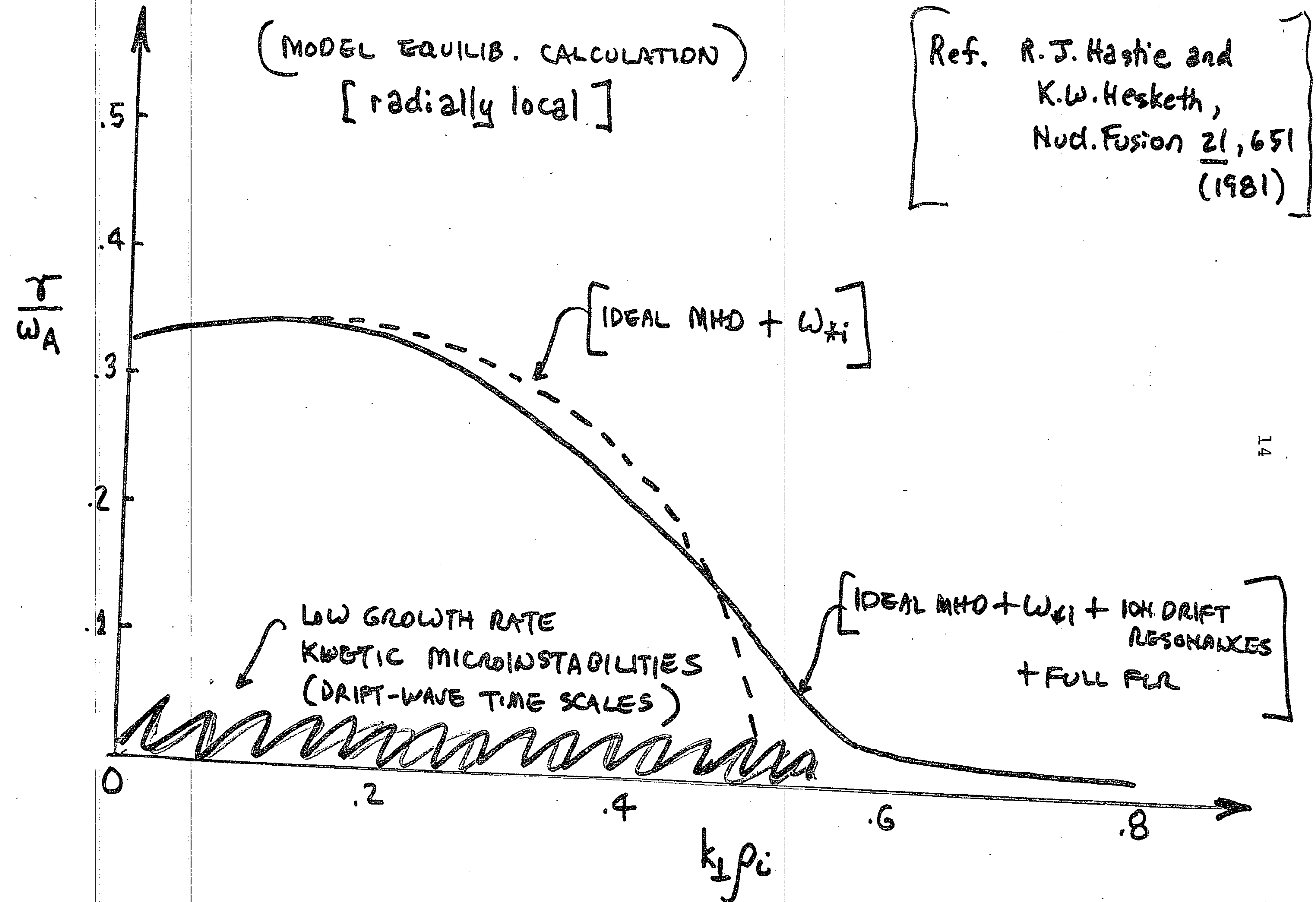
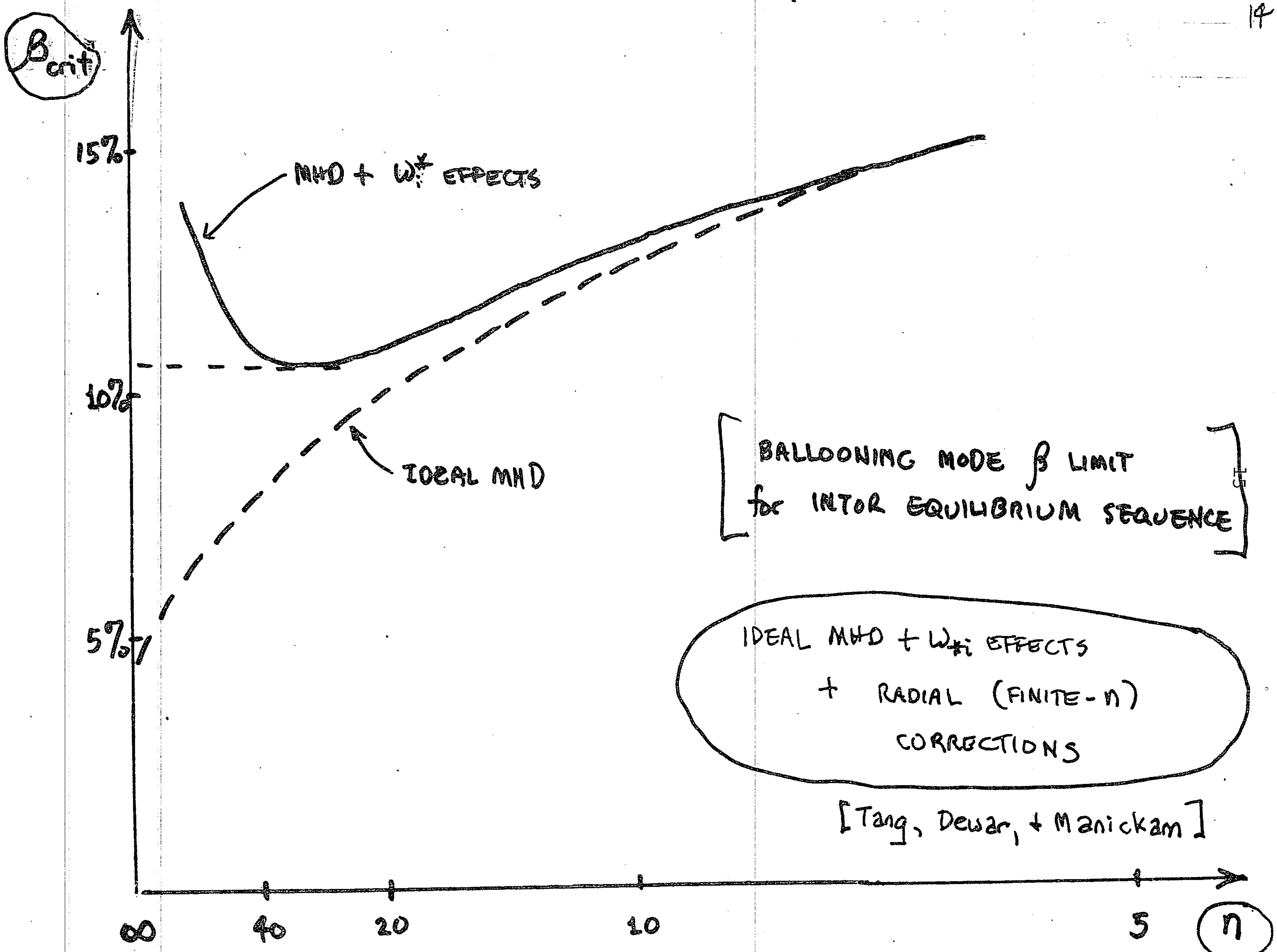

(MODEL EQUILIB. CALCULATION)
[radially local]
Ref. R. J. Hastie and K.W. Hesketh, Nucl. Fusion 21, 651 (1981)
γ / ω_A
.5
.4
.3
.2
.1
0
.2
.4
.6
.8
$k_\perp \rho_i$
[IDEAL MHD + ω_{*i}]
[IDEAL MHD + ω_{*i} + ION DRIFT RESONANCES + FULL FLR]
LOW GROWTH RATE KINETIC MICROINSTABILITIES (DRIFT-WAVE TIME SCALES)



[BALLOONING MODE β LIMIT
for INTOR EQUILIBRIUM SEQUENCE]

MAIN CONCLUSIONS FOR TOKAMAK STUDIES :

- INCLUSION OF KINETIC EFFECTS CAN LEAD TO SIGNIFICANTLY MORE OPTIMISTIC β_{CRIT} <u>FOR</u> <u>PARTICULAR</u> <u>EQUILIBRIA</u>

  e.g. { for INTOR EQUILIB. SEQUENCE , IMPROVEMENT OVER IDEAL MHD IS BETTER THAN A FACTOR OF 2

- KINETIC EFFECTS ON β_{CRIT} ARE SIGNIFICANT ONLY FOR EQUILIBRIA WHERE β_{CRIT} AT $n \gg 1$ IS <u>APPRECIABLY</u> <u>LOWER</u> THAN β_{CRIT} AT $n \lesssim 10$

- RESIDUAL MICROINSTABILITIES OCCURING ON THE DRIFT-WAVE TIME SCALE ARE LIKELY TO PERSIST

  [FULL KINETIC CALCULATION]

PRACTICAL CONSEQUENCES : μ INSTABILITIES VERY LIKELY TO BE UNAVOIDABLE

⇒ NEED TO IDENTIFY WORST FORMS AND ASSESS IMPACT ON CONFINEMENT

IMPORTANT LINEAR THEORY ISSUES

(1) Establish (with aid of exp. observations) whether meaningful β_{crit} exists for Bal. Modes ; i.e., threshold where these modes lead to anomalous transp. levels greater than that due to already present ES modes ($\beta \to 0$)

(2) Beam and Alpha Particle Effects

(3) Need for simplified linear models to interface with nonlinear and transport code studies

- should reflect proper scaling of γ with T, n_0, B, etc.

NONLINEAR DRIFT WAVE IN A SHEARED MAGNETIC FIELD

T. TANGE, M. SHIGETA, AND K. NISHIKAWA

HIROSHIMA UNIVERSITY

Nonlinear Drift Wave in a Sheared Magnetic Field

Toshio TANGE
Mitsuhiro SHIGETA
Kyoji NISHIKAWA

Institute for Fusion Theory
Hiroshima University
Hiroshima Japan

(Read by T. Sato)

Purpose :

Nonlinear effects of electrons on stability of monochromatic drift wave in a sheared magnetic field

Method :

Perturbation theory

- Expand in powers of wave potential
- Sum secular terms
- Derive nonlinear eigenvalue eq.
- Find growth rate & wave structure numerically

Slab Model

$$\mathbf{B}_0 = B_0 \left(0, \frac{x}{L_s}, 1 \right)$$

$$\kappa \hat{x} = -\frac{1}{n}\frac{\partial n}{\partial x}\hat{x}$$

$$\frac{\partial}{\partial z} = 0$$

$$f = f^{(0)} + f^{(1)} + f^{(2)} + f^{(3)} + \cdots$$

$$f^{(n)} \propto \phi^n$$

$$\langle f \rangle = f^{(0)} = \underline{\text{fixed}}$$

$$= \left(\frac{m}{2\pi T}\right)^{\frac{3}{2}} n(X_G)\, e^{-\frac{m}{2T}\left[v_\perp^2 + (v_\parallel - u)^2\right]}$$

$$f^{(n)} = \int_{-\infty}^{t} dt' \frac{q}{m} \nabla' \varphi \cdot \frac{\partial}{\partial v'} f^{(n-1)}$$

$$- \langle \ \ldots \ \rangle \qquad n > 1$$

(3)

Ions

$$\varphi_{k\omega}(x') = \varphi_{k\omega}(x) + (x'-x)\frac{d}{dx}\varphi_{k\omega}(x) + \frac{1}{2}(x'-x)^2\frac{d^2}{dx^2}\varphi_{k\omega}(x)$$

- $l_x \ll \rho_i$
- $\omega/k_\parallel \gg v_{ith} \rightarrow k_\parallel \simeq kx/L_s$

Obtain

$$n_i^{(1)} = -n_i\frac{e_i}{T_i}\sum_k\int\frac{d\omega}{2\pi}e^{iky-i\omega t}\left\{\varphi_{k\omega}(x)\right.$$

$$\left.+\frac{\omega-k_yu_i-\omega_{*i}}{\sqrt{2}|k_\parallel|v_{ith}}Z\left(\frac{\omega}{\sqrt{2}|k_\parallel|v_{ith}}\right)\left[\Lambda\varphi_{k\omega}-\Lambda'\rho_i^2\frac{d^2}{dx^2}\varphi_{k\omega}\right]\right.$$

Electrons

- $\varphi(x') \simeq \varphi(x)$
- $(\kappa L_s)^{\frac{1}{2}} \ll (m_i/m_e)^{\frac{1}{2}}$
- $u_e \ll v_{eth}$
- $\kappa \rho_i u_e v_{ith} / v_{eth}^2 \ll 1/\kappa L_s$

$$f_e^{(n)}(k, \omega, x, v) = \sum_{k^{(1)}+\cdots+k^{(n)}=k} \cdots \sum f_0^{(0)} \frac{i}{\omega - k_\parallel v_\parallel - \sum_1^n h(k^{(j)}) + i0}$$

$$\times \left[\frac{1}{\omega - k_\parallel v_\parallel + i0} \frac{e}{m} k^{(1)} \varphi_{k^{(1)}} \frac{\partial}{\partial v_y} \right.$$

$$\times \frac{1}{\omega - k_\parallel v_\parallel - h(k^{(1)}) + i0} \frac{e}{m} k^{(2)} \varphi_{k^{(2)}} \frac{\partial}{\partial v_y}$$

$$\times \cdots\cdots$$

$$\vdots$$

$$\left. \times \frac{1}{\omega - k_\parallel v_\parallel - \sum_{j=1}^{n-1} h(k^{(j)}) + i0} \frac{e}{T_e} \varphi_{k^{(n)}} \right]$$

$$\times (u_e k^{(n)} - u k^{(n)} - \omega_{*(n)})$$

(5)

<u>most secular terms</u>

$$f_e^{(2\nu+1)} = f_e^{(0)} (\omega_* k - \omega k_\parallel - \omega) \Big[- \frac{1}{\omega - k_\parallel v_\parallel}$$

$$\times \frac{\partial}{\partial v_y} \sum_q \frac{1}{\omega - k_\parallel v_\parallel + \omega_q - q_\parallel v_\parallel} \left(\frac{e}{m}\right)^2 q^2 |\varphi_q|^2 \frac{\partial}{\partial v_y} \Big]^\nu$$

$$\times \frac{1}{\omega - k_\parallel v_\parallel} \frac{e}{T} \varphi_{k\omega}$$

Sum up all odd terms

$$\tilde{f}_e = f_0^{(0)} \frac{e}{T} \varphi_{k\omega} \Big[1 + G(\cdots) \frac{\omega_* + k_\parallel u - \omega}{\omega_*} \Big]$$

$$G(k, \omega, x, v)$$

$$= \omega_* \sum_{\nu=0}^{\infty} \Big[- \frac{1}{\omega - k_\parallel v_\parallel} \frac{\partial}{\partial v_y} \sum_q \frac{1}{\omega - k_\parallel v_\parallel + \omega_q - q_\parallel v_\parallel}$$

$$\times \left(\frac{e}{m}\right)^2 q^2 |\varphi_q|^2 \frac{\partial}{\partial v_y} \Big]^\nu \frac{1}{\omega - k_\parallel v_\parallel}$$

It turns out that G satisfies

$$G(k,\omega,x,v) = \frac{1}{\omega - k_{\parallel} v_{\parallel}}$$

$$\times \left[\omega_{*} - \frac{\partial}{\partial v_y} \sum_{q} \frac{1}{\omega - k_{\parallel} v_{\parallel} + \omega_q - q_{\parallel} v_{\parallel}} \left(\frac{e}{m}\right)^2 q^2 |\varphi_q|^2 \frac{\partial}{\partial v_y}\right.$$

$$\left. \times\, G(k,\omega,x,v) \right]$$

<u>For a single mode</u>

$$\frac{1}{4\xi}\frac{\partial}{\partial \xi}\frac{1}{\xi}\frac{\partial}{\partial \xi} g(\xi) + g(\xi) = \frac{1}{\xi}$$

$$\xi = (\omega - k_{\parallel} v_{\parallel}) / \omega_{*} a$$

$$g(\xi) = a G$$

$$a = 2^{\frac{1}{6}} \left[\frac{T_i}{T_e} \frac{v_n}{v_i} \frac{1}{\kappa L_s \kappa \rho_i} \frac{e|\varphi_k|}{T_e} \right]^{\frac{1}{2}}$$

G : renormalized propagator

General solution

$$g(\zeta) = i e^{-i\zeta^2} \int_0^{\zeta} dt\, e^{it^2} - i e^{i\zeta^2} \int_0^{\zeta} dt\, e^{-it^2}$$

$$+ C_1 e^{-i\zeta^2} + C_2 e^{i\zeta^2}$$

Boundary condition

$$g(\zeta) \to \frac{1}{\zeta} \qquad |\zeta| \to \infty$$

(upper half plane)

$$C_1 = i C_2 = e^{-\frac{\pi}{4} i} \frac{\sqrt{\pi}}{2}$$

For large ζ

$$g(\zeta) \sim \sum_{m=0}^{\infty} \frac{(-1)^m (4m-1)!!}{2^{2m} \zeta^{4m+1}}$$

(8)

Nonlinear Eigenvalue Equation

$$n_e \approx n_i$$

$$\varphi_{k\omega}(x) + \frac{\omega + \omega_* \frac{T_i}{T_e}}{\omega} \zeta_i Z(\zeta_i) \left[\Lambda \varphi_{k\omega}(x) - \Lambda' \rho_i^2 \frac{d^2}{dx^2} \varphi_{k\omega}(x) \right]$$

$$+ \frac{T_i}{T_e} \varphi_{k\omega}(x) \left[1 + \int du \frac{\omega_* + k_\parallel u - \omega}{\omega_*} \frac{1}{n_e} f_e G \right] = 0$$

(9)

Lowest nonlinearity

$$G \sim \frac{\omega_*}{\omega - k_\parallel v_\parallel} - \frac{3}{4}\frac{\omega_*^5 a^4}{(\omega - k_\parallel v_\parallel)^5} + \frac{105}{16}\frac{\omega_*^9 a^8}{(\omega - k_\parallel v_\parallel)^9} - \cdots$$

$\propto |\phi|^2$

linear lowest

Lowest term can be easily obtained by the usual perturbation theory

$$\left(f = f^{(0)} + f^{(1)} + f^{(2)} + f^{(3)} \right)$$

$$\left[\rho_i^2 \frac{d^2}{dx^2} + P_L(k,\omega,x) + P_N(k,\omega,x)|\phi|^2 \right]\phi(k,\omega) = 0$$

$$P_L = \frac{\Lambda}{\Lambda'} + \frac{(1+\tau)|k_\parallel| v_i}{(\omega\tau) + \omega_e^*)Z(\xi_i)\Lambda'} + \frac{\omega - \omega_e^* - \omega_i^*}{\omega\tau + \omega_e^*}\frac{v_i Z(\xi_e)}{v_e Z(\xi_i)\Lambda'}$$

$$P_N = \frac{T_e}{n_0}\frac{|k_\parallel| v_i}{\omega\tau + \omega_e^*}\frac{\omega_e^* - \omega}{Z(\xi_i)\Lambda'}\left(\frac{c}{B}\right)^2 \frac{3}{2}\frac{k^4}{L_s^4}$$

$$\cdot \int dv \frac{v_\parallel^2 f_e^{(0)}}{(\omega - k_\parallel v_\parallel)^5}$$

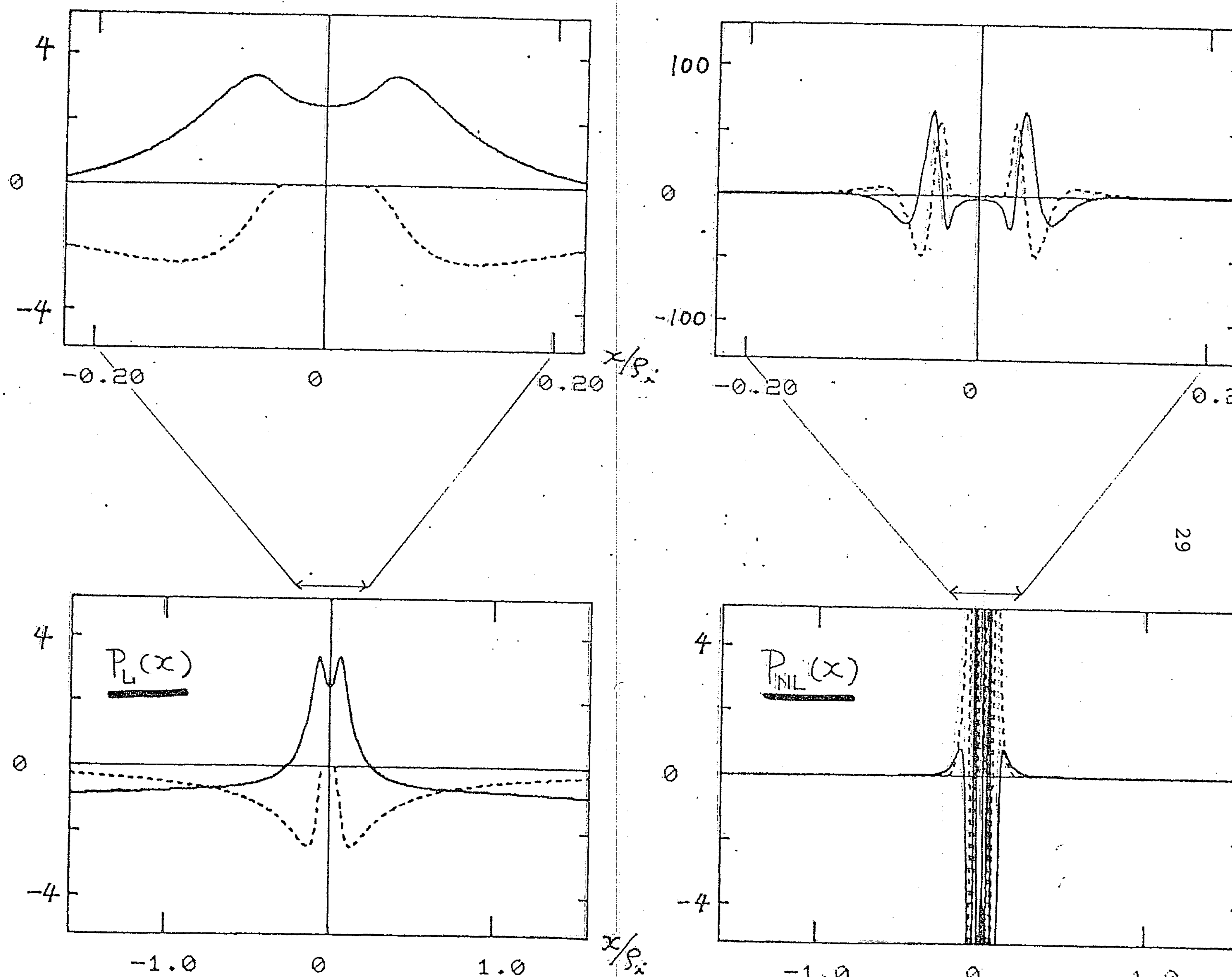
$\varkappa L_s = 32$

$T_e/T_i = 1$

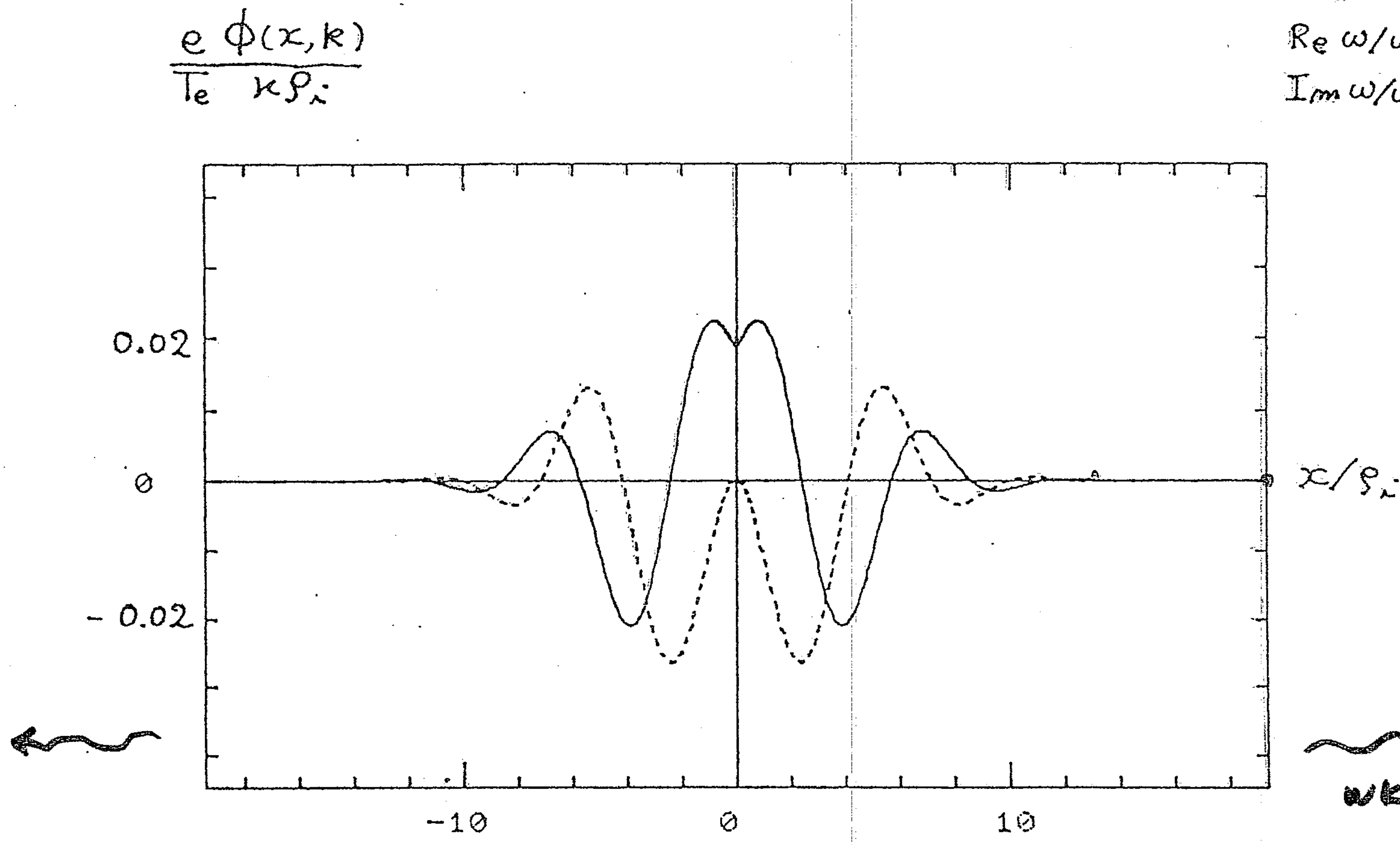
$u = 0$

$k\rho_i = 1$

$|\varphi(0)| = 0.019$

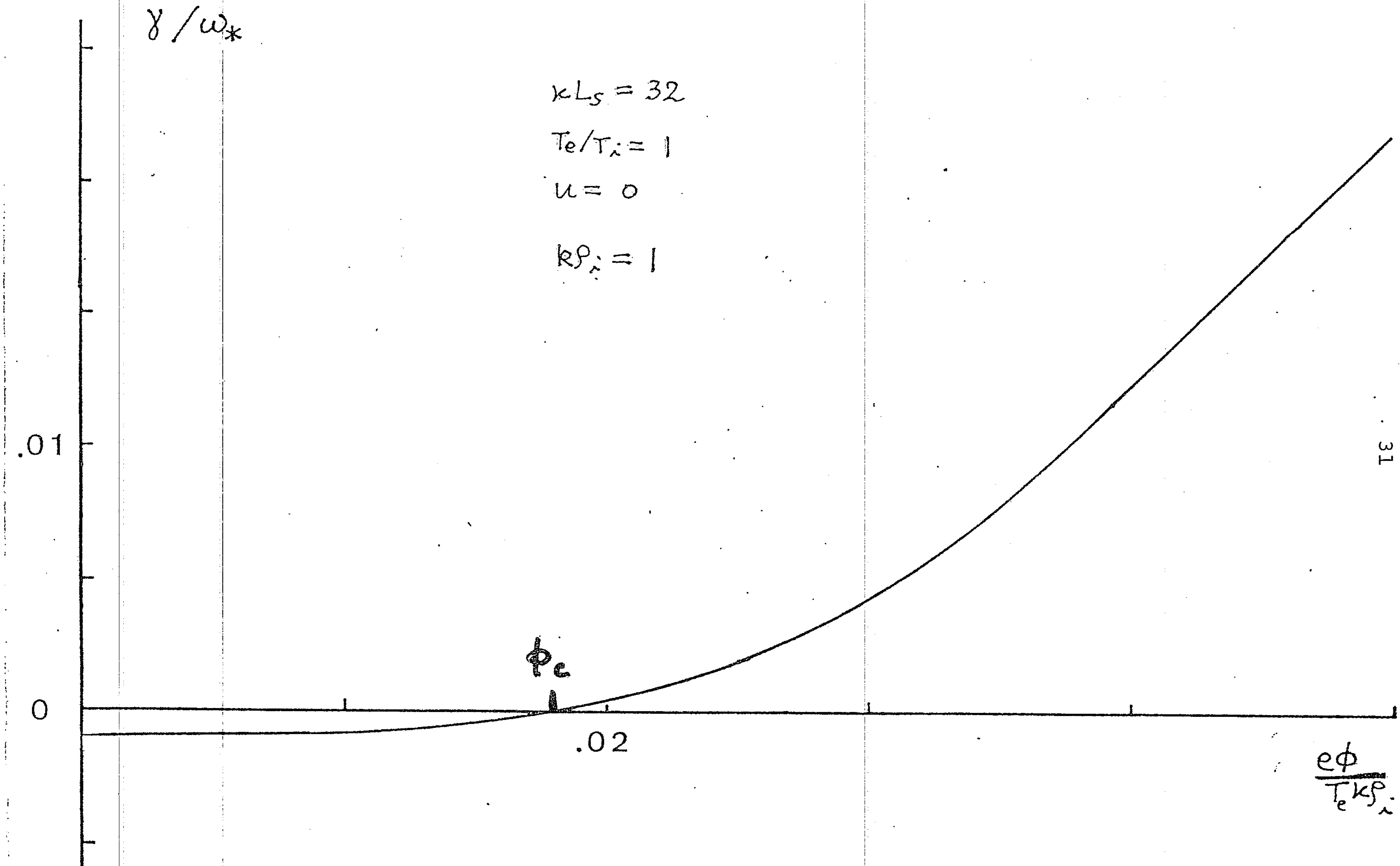


$\kappa L_s = 32$
$T_e/T_i = 1$
$u = 0$
$k\rho_i = 1$
$Re\, \omega/\omega_* = 0.175$
$Im\, \omega/\omega_* = 0.518 \times 1$

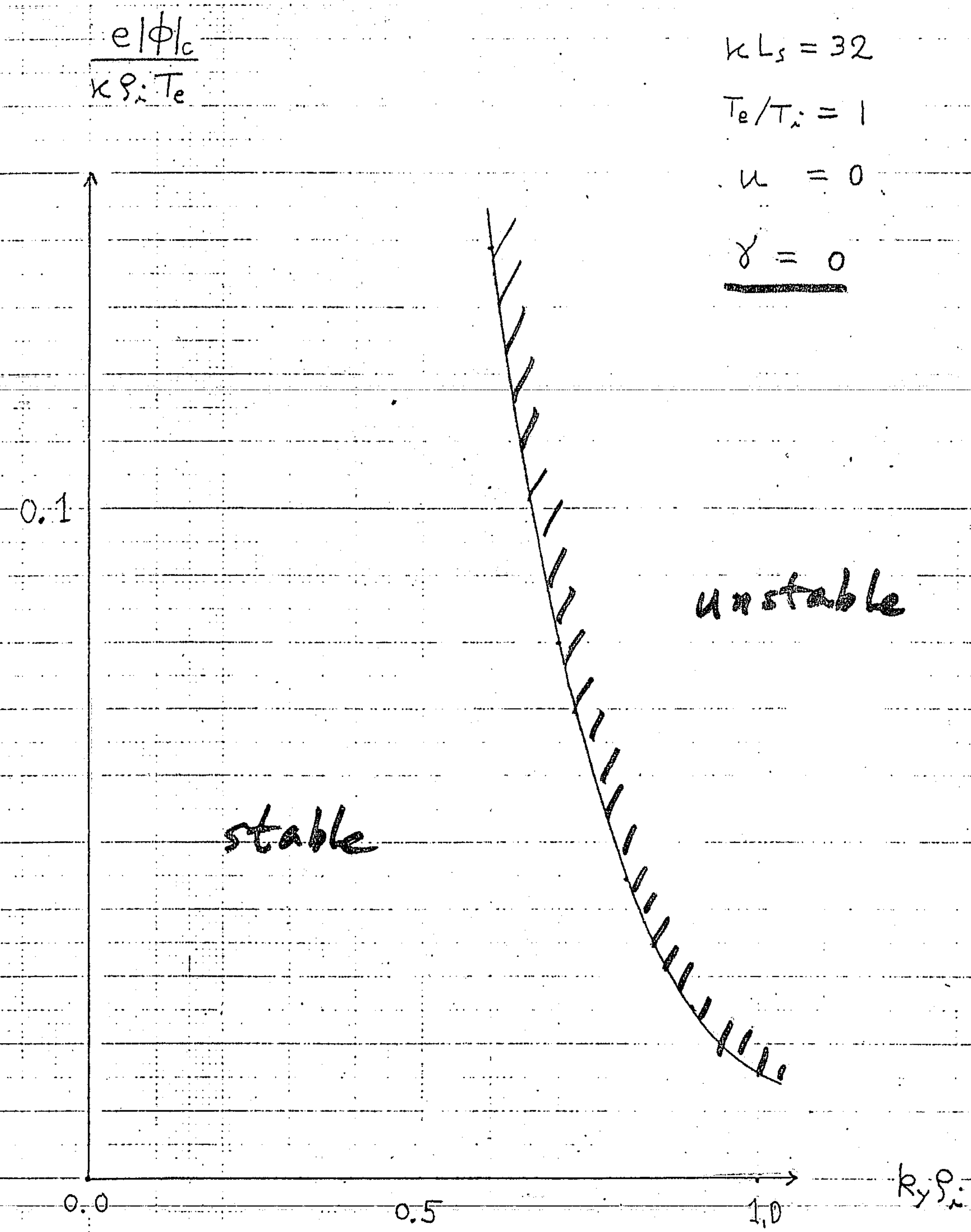


outgoing WKB solution
$\phi(x_0)e^{-\int_{x_0}^{x}\sqrt{P_L}}$

(obtained by a shooting integration)


γ/ω_*
$\kappa L_s = 32$
$T_e/T_i = 1$
$u = 0$
$k\rho_i = 1$
.01
0
ϕ_c
.02
$\frac{e\phi}{T_e k\rho_i}$

(13)



$\frac{e|\phi|_c}{\kappa \rho_i T_e}$
$\kappa L_s = 32$
$T_e/T_i = 1$
$u = 0$
$\gamma = 0$
0.1
unstable
stable
0.0
0.5
1.0
$k_y \rho_i$

SURVEY OF LOW FREQUENCY ELECTROSTATIC FLUCTUATION MEASUREMENTS IN TOKAMAKS AND OTHER TOROIDAL EXPERIMENTS

P.C. LIEWER AND S.J. ZWEBEN

CALIFORNIA INSTITUTE OF TECHNOLOGY

Survey of Low Frequency ($\omega \ll \omega_{ci}$) Electrostatic Fluctuation Measurements in Tokamaks + Other Toroidal Experiments

P.C. Liewer + S.J. Zweben, Caltech

Motivation: Energy Confinement Time often dominated by anomalous electron thermal conduction - generally attributed to low frequency turbulence

In tokamaks, no really conclusive link between fluctuations ($\tilde{n}$ or $\tilde{B}$) and transport

Here: Survey measurements in tokamaks + some other experiments

Look for

1) Character of spectra - can any modes be identified? is it turbulent? what kind of turbulence)

2) Any links between fluctuations + transport

Outline + Summary

I. Survey Tokamaks
PLT, ATC, Alcator A, Microtor, WT-1, TFR, Macrotor, Caltech
(In progress - PLT (Mazzucato)
Alcator-C + PDX (Surko)
Caltech (Zweben)

A. General features - broad spectra with no clearly defined "modes"
e.g. for fixed k $\Delta\omega \gtrsim \omega$ very turbulent

isotropic in $\underset{\sim}{k}_\perp$ $\underset{\sim}{k}_\perp \gg k_\parallel$

B. Summarize results + compare parameters
(ν_e^*, v_e/v_{ti}, $\tilde{n}/n$, ρ_s/L_n, L_s/L_n, T_e/T_i)

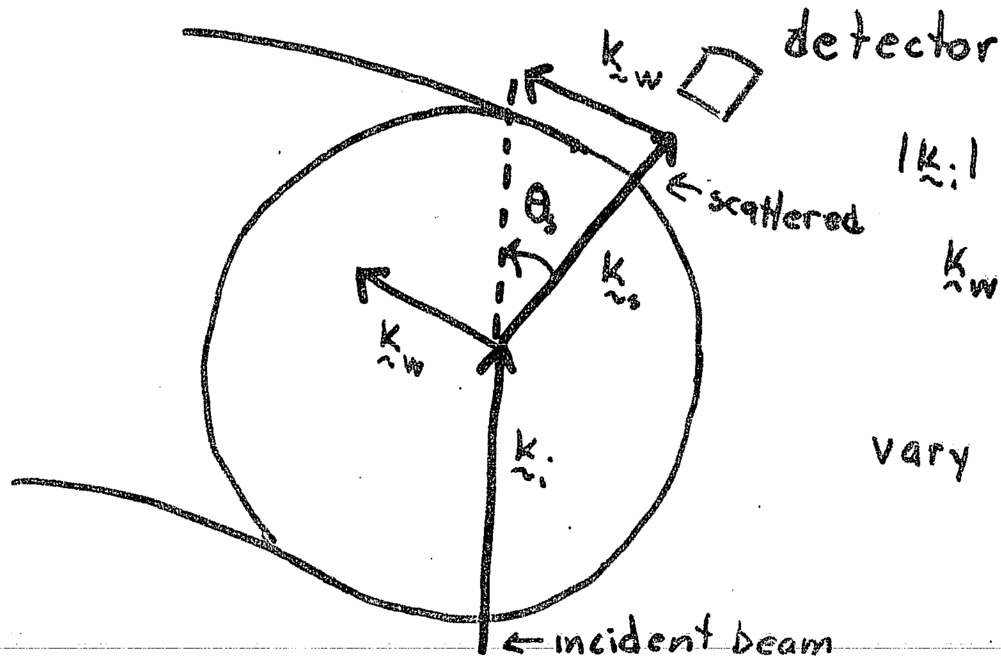
II. Multipoles
A. 2 octapoles (GA + Wisc) where saw large amplitude drift waves but no anomalous transport

B. FM-1 spherator - saw both single-mode + turbulent spectra by going from strong to weak shear + correlation with transport

III Wendelstein VII A - broad spectra + some correlations with transport

IV. References for RFP's, EBT, linear and others
Conclusions

Brief introduction to scattering experiments

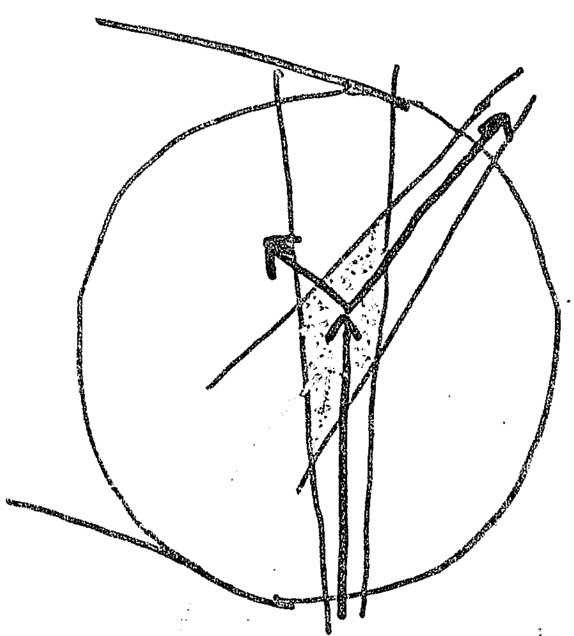


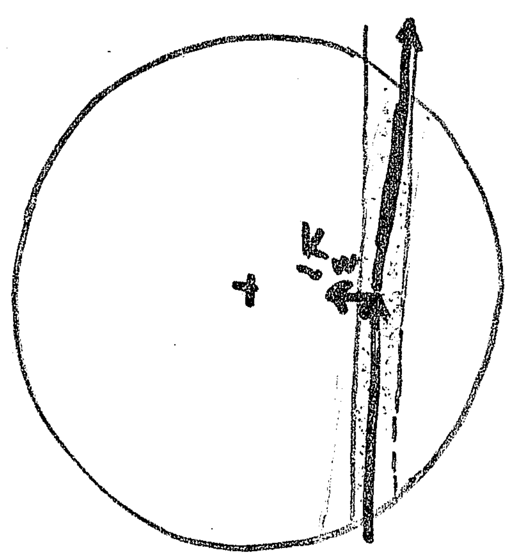
$|k_i| = |k_s|$

k_w - k of density fluctuation

vary k_w by varying θ_s

Resolution depends on angular size of beam + detector + k_i + k_w





k_w is k_θ or k_r or mixture depending on location of scattering

Tokamak Scattering Experiments

ATC E. Mazzucato PRL 36 (1976) 793 μwave
Surko + Slusher PRL 37 (1976) 1747 CO_2
PLT Mazzucato PF 21 (1978) 1063 + PPPL-1653 μwave
Mazzucato + Semet BAPS 26 (1981) 981
Alcator A - Surko + Slusher PRL 40 (1978) 400 CO_2
Phys Fluid 23 (1980) 2425
Microtor - Semet, Mase, Peebles, Luhmann + Zweben FIR
PRL 45 (1980) 445
TFR - 8th Int Conf. Brussels 1980 IAEA-CN-38/N-5
WT-1 (Kyoto) Saito, Hamada, Yamashita, Ikeda, Nakamura, Tanaka
Nuc Fus 21 (1981) 1005 FIR

Alcator-C Watterson, Gentile, Slusher, Surko
BAPS 26 (1981) 885
PDX - Slusher, Surko, Motley BAPS 26 (1981) 999

General Features
(not all exps show all features)

* Broad frequency spectra at each $\underset{\sim}{k}$
$\Delta\omega \gtrsim \omega$

width increases with increasing $\underset{\sim}{k}$

** isotropic in $\underset{\sim}{k}_\perp$ $\bar{k}_r \sim \bar{k}_\theta \gg k_\parallel$
$S(k_r)$ similar to $S(k_\theta)$

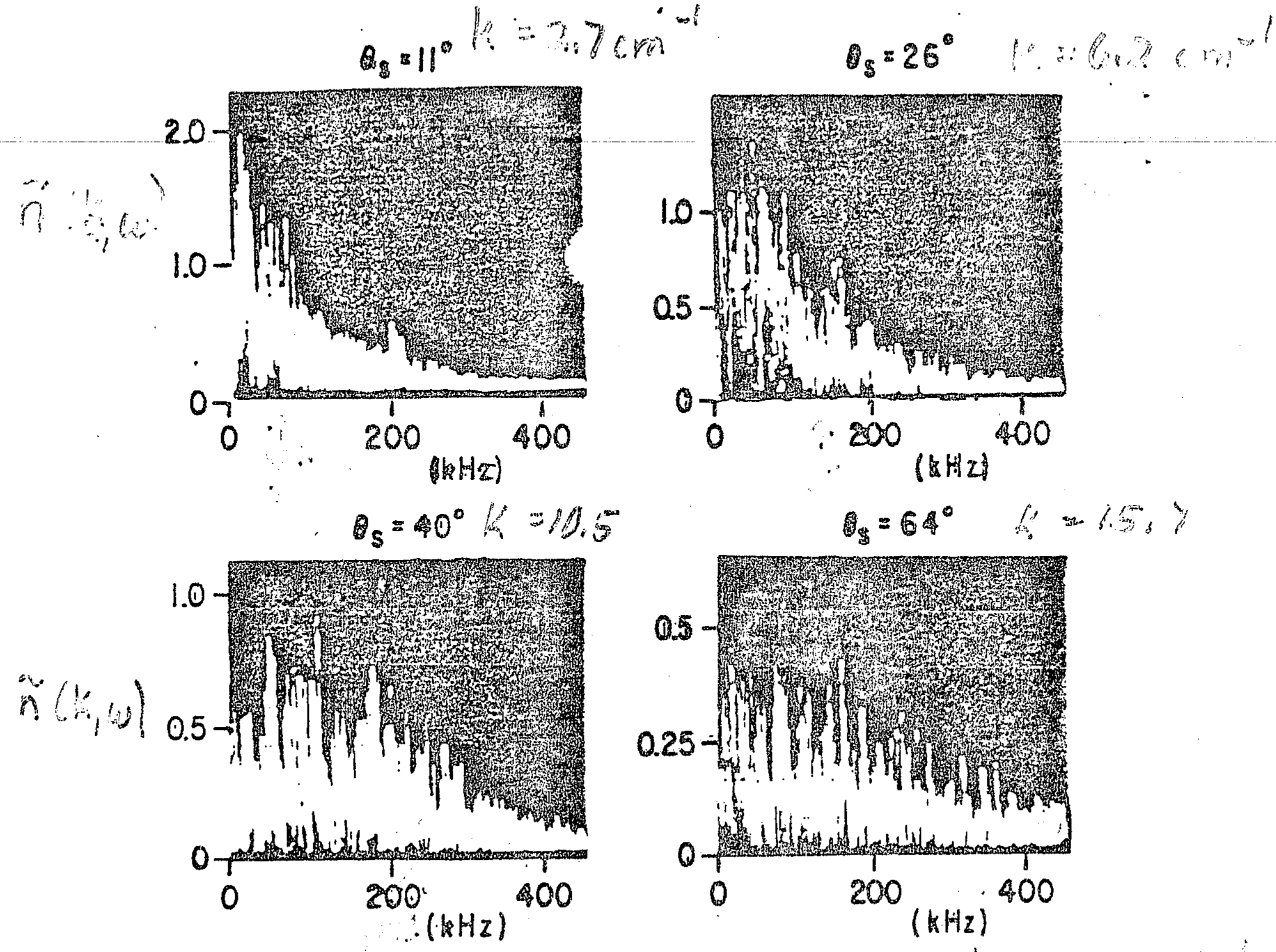
* often fluctuations $\tilde{n}/n$ largest at edge

* fluctuation levels are large enough to cause observed transport (using rough estimates!)

E. Mazzucato – first observed low frequency density fluctuations in tokamak ATC using microwave

* Saw broad frequency spectrum $\tilde{n}(\underset{\sim}{k},\omega)$ for each $\underset{\sim}{k}$ (vol. resolution $\Delta r \simeq a$)

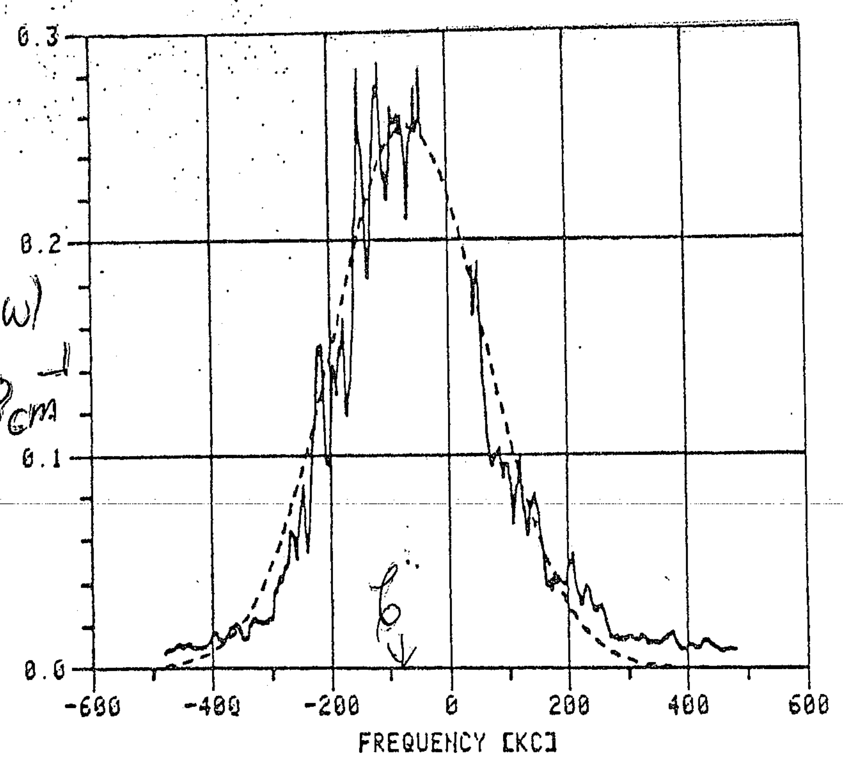
* width of spectrum Δf increased with increasing $\underset{\sim}{k}$



$f \rightarrow$ $\qquad f = \frac{\omega}{2\pi}$

$\tilde{n}/n \simeq 10^{-2} - 5\times10^{-3}$

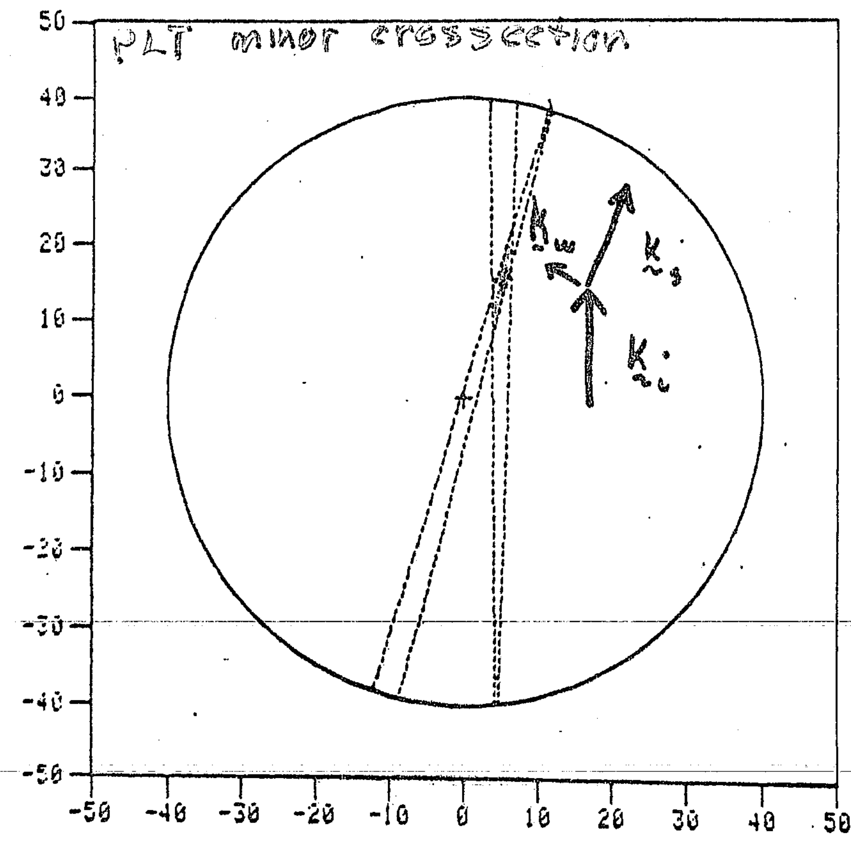
Excellent spatial resolution & still very broad frequency spectrum for fixed K



spectrum peaked at

$\omega \simeq \omega^A$

but $\Delta\omega \gg \omega$



shows small scattering volumn

$\Delta r \sim \frac{a}{5}$

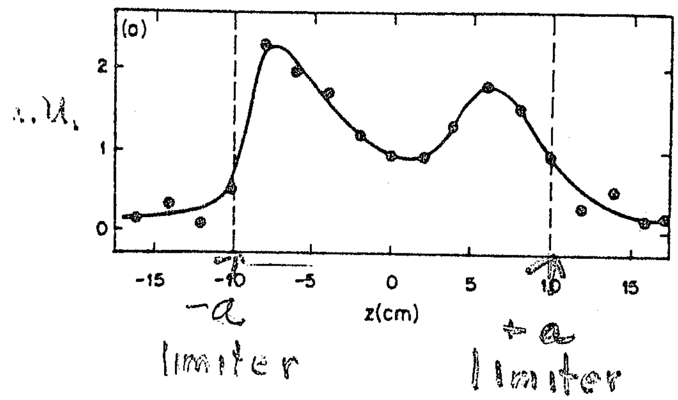
urko + Slusher - study radial distribution of $\tilde{n}/n$ by crossed-beam correlation (Alcator A, C, PDX)

In Alcator A - they found 2 distinct density regimes

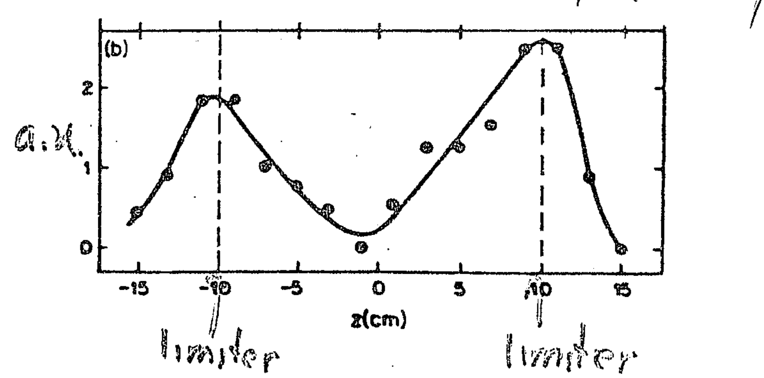
low density
$\bar{n} < 5 \times 10^{13}$
$\frac{\tilde{n}}{n(r)}$ peaks in interior $r \simeq 7$cm
($a = 10$ cm)
$\frac{\tilde{n}}{n}\Big|_{max} \simeq .07 \pm .03$
$\bar{k} \simeq 6\ cm^{-1}$ ($\bar{k}\rho_s = 0.3$)

high density
$\bar{n} > 7 \times 10^{13}\ cm^{-3}$
$\frac{\tilde{n}}{n(r)}$ peaks at limiter
$\frac{\tilde{n}}{n(a)} \sim 0.3 - 1$
$\bar{k} = 14$ ($\bar{k}\rho_s = .13$)

$\tilde{n}$ vs. radius (bottom to top).

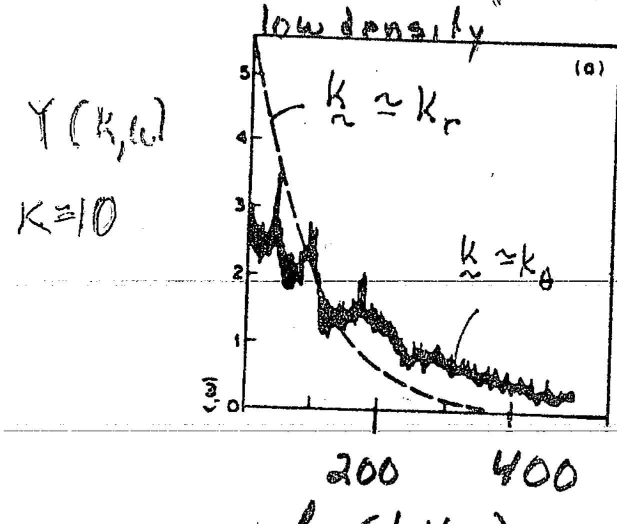


$\tilde{n}$ vs. radius for high density

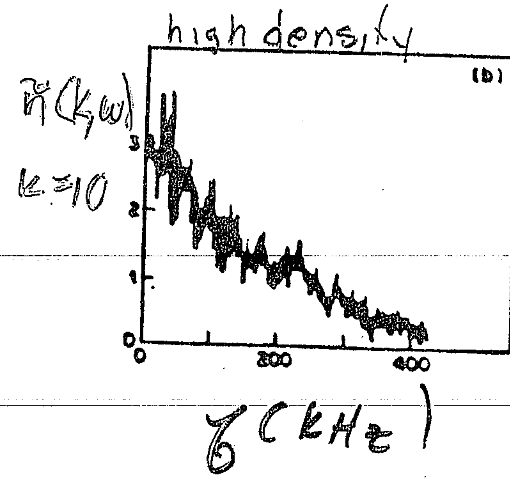


Spectra for $\underset{\sim}{k} \simeq k_\theta = 10\ cm^{-1}$

low density



high density



+ spectrum does not change with $\bar{n}$

WT-1 Tokamak - Kyoto Univ.

Saito, Hamada, Yamashita, Ikeda, Nakamura, Tanaka

$a = 5\,cm$, $R = 28\,cm$, $B \sim 7\,kG$, $\bar{n} \sim 0.5 - 1.0 \times 10^{13}\,cm^{-3}$ $T_e \lesssim 200$ eV

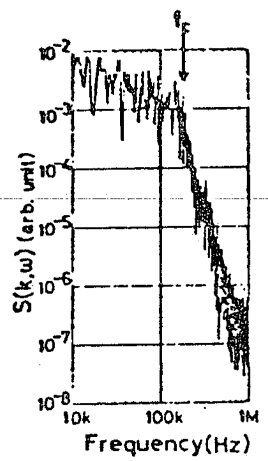
Study chord-integrated spectra with FIR laser

Nuc. Fusion 21 (1981) 1055

frequency spectrum

$\log \tilde{n}^2(k,\omega)$

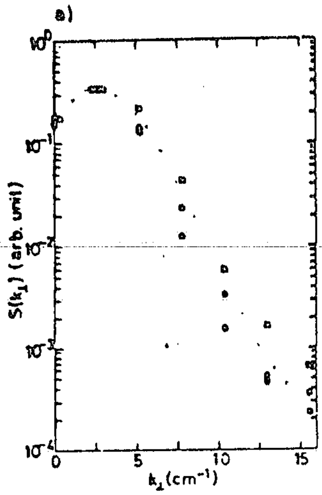
$k = 2.6\,cm^{-1}$



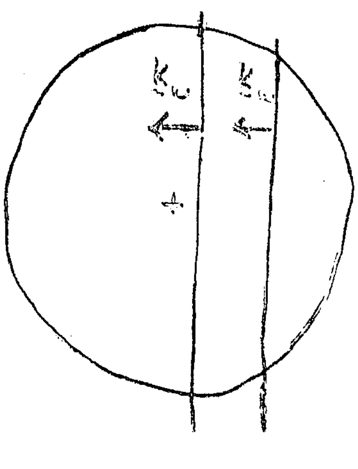
log f

S(k) (integrated over θ)

S(k)



k



* S(k) peaks at $k\rho_s \sim 0.3$

* S(k) has almost same shape for all radii (all chords)

... $k_\omega \sim k_\theta$ + $k_\omega \sim k_r$

so turbulence isotropic in $\underset{\sim}{k}_\perp$

* Also $k_\perp \gg k_\parallel$

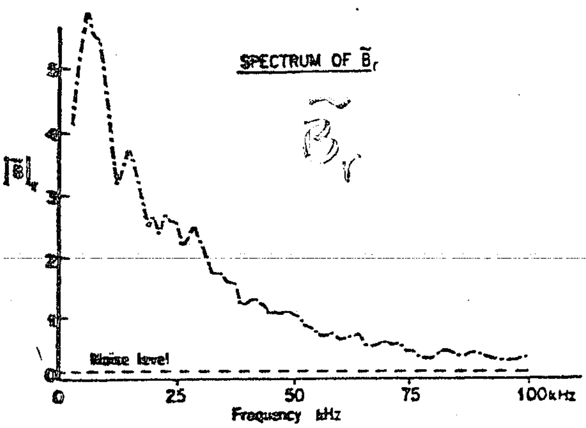
(Same features as seen by Surko & Slusher + as seen on Microtor

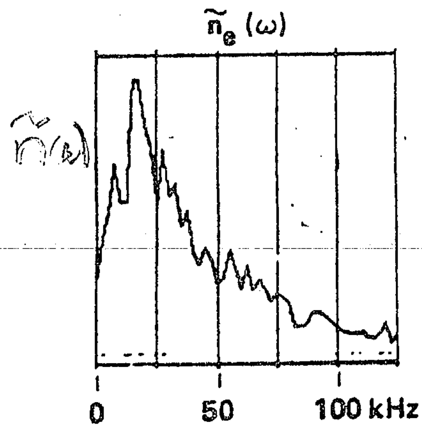
\+ If use $D \sim \frac{\gamma}{k_\perp^2}$ $\gamma \sim \omega_c$, get proper D for observed τ_E

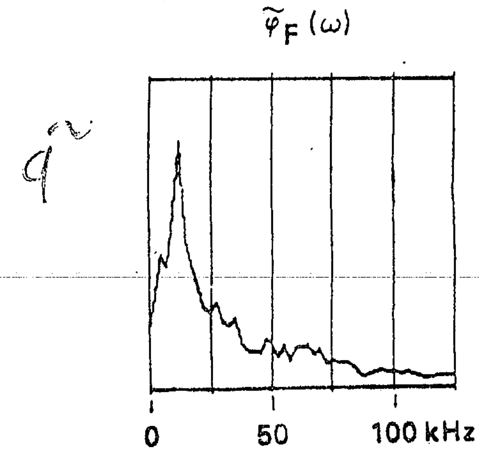
Fluctuation spectra also measured with Probes

Can measure, $\tilde{n}, \tilde{\phi}, \tilde{E}, \tilde{B}$; Macrotor (Zweben + Taylor)
(int. over all k; good vol. resolution) Caltech Tokamak Hedemann, Gould (thesis) + Zweben

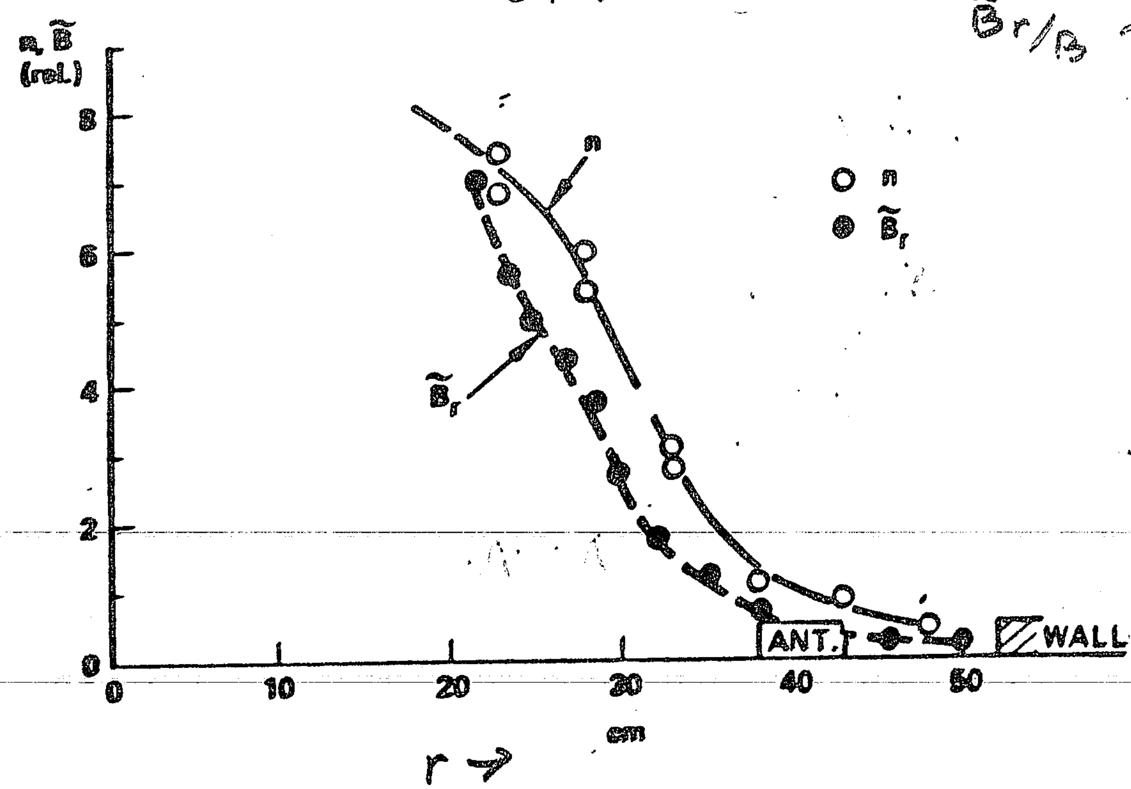
Here, results from Macrotor (Nuc. Fus. 21, 1981 p. 193)
Spectra of $\tilde{B}, \tilde{n}, \tilde{\phi}$ all similar + all insensitive to radial position $r \approx 0.6a - a$







Radial distribution: $\tilde{n}/n$ peaks at wall $\tilde{n}/n \sim 0.5$
$\tilde{B}/B$ increases as move into plasma
$\tilde{B}_r/B \sim 10^{-4}$ at $r \simeq 20$



Parameter Estimates - All Ohmic
(all are approximate)

Exp.	$\frac{V_c}{V_{te}}$	$\frac{V_c}{C_s}$	ν_{*e}	$\frac{L_s}{L_n}$	$\frac{\rho_s}{L_n}$
PLT	.09	5	.13	15	.005
ATC	.13	8	.3	17	.015
Alc-A					
high dens	.02	2	7	19	.05
low dens	.15	12	.8	29	.07
Microtor	.2	13	.8	15	.02
TFR					
moderate $\bar{n}$	.04	2	.6	20	.01
high $\bar{n}$	.02	1	.2	11	.01
Macrotor	.17	10	1.3	14	.015
Caltech	.4	20	3	11	.025
Alcator-C(?)	.013	0.8	5	26	.003
WT-1	.3	18	4	28	.03

Some Results of Fluctuation Measurements in Ohmic Tokamaks

	$\tilde{n}/n$	ρ_s/L_n	where is $\tilde{n}/n$ max?	size of scattering volumn	isotropic in $k_\perp$? $k_\parallel \ll k_\perp$?	was a peak in $S(k)$ seen?
PLT						
Mazzucato	~.005-.01	~.005				
ATC						
μ (Mazz.)	~.005-.01	~.015	-	~a/2		yes $k\rho_i \simeq 0.5$
O_2 (S&S)	~.03	~.015	edge (?)	chord	yes $S(k_r, \omega_0) \simeq S(k_\theta, \omega)$ $k_\parallel \ll k_\perp$	
Alcator A - Surko & Slusher						
density	~.07	~.07	at r≈.7a	chord	yes yes	no
CO_2						
high density	~0.3-1	~.05	at r=a	chord	yes yes	no
icrotor	~.04	~.02	-	chord	yes yes	no
Semet el.al. (FIR)						
JT-1 ... ito el-al (FIR)	~.05	~.03	edge(?)	chord	yes yes	yes $k\rho_i \simeq 0.3$
TFR (superf μwave)	.002-.005†	~.01		~a/4	- $k_\parallel \ll k_\perp$	
icrotor ...man & Taylor probes	.1 r≈.6a .5 edge	~.015	edge	probe size (mm's)	radial + pol correlation lengths different	—

† TFR saw a correlation of $\tilde{n}/n$ dropping & anomalous losses dropping in their high density regime IAEA-CN-38/N-5 (Brussels)

Typical Parameters for Tokamak Scattering Experiment (my estimates - for comparison only)

1. Drift parameters: (I assume $q_0 = 1$) $\frac{V_c}{V_{te}} = \frac{j(0)}{ne\sqrt{\frac{2T_e}{m_e}}}$ $\quad \frac{V_c}{C_s} = \frac{V_c}{V_{te}}\sqrt{\frac{2m_i}{m_e}}$

2. Electron Collisionality: if $Z_{eff} = 1$ $\quad \nu_{*e} = \nu_{eff}\,\tau_b = \sqrt{2}\left(\frac{R}{r}\right)^{3/2} \frac{q R_0}{V_{te}} \frac{1}{\tau_e}$

3. Shear $\quad \frac{L_s}{L_n} = \frac{q(a)}{a} R$

4. gradient $\quad \frac{\rho_s}{L_n} = c\,\frac{\sqrt{T_e m_i}}{eB}\,\frac{2}{a}$

* $\frac{\tilde{n}}{n}$ compared to $\frac{\rho_s}{L_n}$ measures "nonlinearity"

If $\frac{\tilde{n}}{n} \geq \frac{\rho_s}{L_n}$, then $\frac{c\,\delta E}{B} \geq V_{de}$ for $k\rho_s = 1$

Also for $\frac{\tilde{n}}{n} \geq \frac{\rho_s}{4L_n}$, nonlinear mode-coupling term in Hasegawa-Mima eqn. greater than linear term ($i\omega_*$) for $k\rho_s = 1$

Other toroidal experiments

Drift waves & quasilinear transport have been seen in Linear devices [illegible], discussed by Horton [illegible]

First, Two octapoles where very large amplitude drift waves identified, but no correlation with transport

I Wisconsin Octopole. Navratil & Post Phys Lett 64A (1977) [illegible]

$n \simeq 5 \times 10^{11}\ cm^{-3}$ $T_i \approx T_e \sim 0.2\ eV$ (no temp gradients)

$B_t \ll B_p$ $B_p \sim 750\ G$

using probes

* Saw single mode

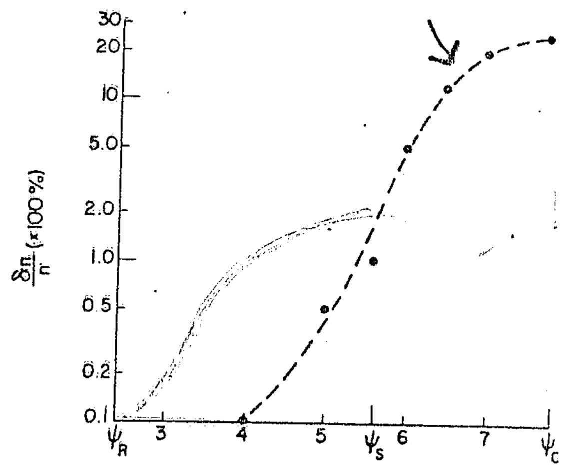
$k_\perp \rho_i \approx 0.3$ & $\omega \simeq \omega_*$

V_p - in elec. diamag. direction

measure $k_\parallel$: $V_{ti} < \frac{\omega}{k_\parallel} < V_{te}$

identify resistive drift mode

density fluctuation amplitude (%)



But observed diffusion agrees well with classical diffusion

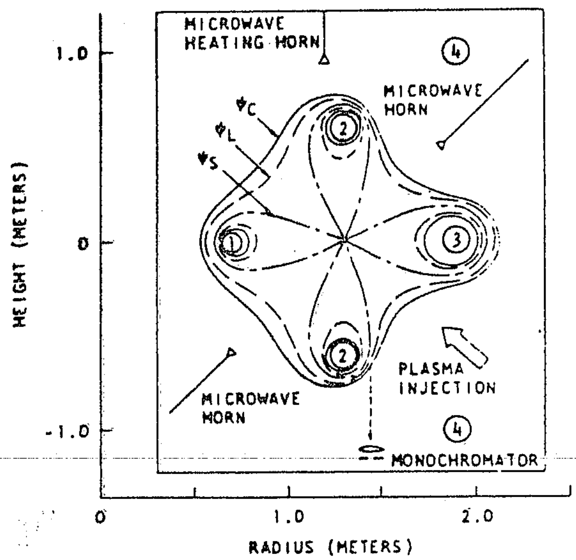
* also measure phase angle between $\tilde{n}$ & $\tilde{\phi}$: $\sin \psi_{n\phi}$

& obtain fluctuation-induced flux $\Gamma_\psi = \sum_k k_\phi \frac{\tilde{n} \tilde{\phi}}{B} \sin \psi$

& this predicts diffusion much faster than observed classical diffusion

(3) GA Octapole Prater, Ejima, Ohkawa, Wong

Saw highly coherent, monochromatic drift wave on outside of plasma profile where $\frac{dq}{dr}$ is small



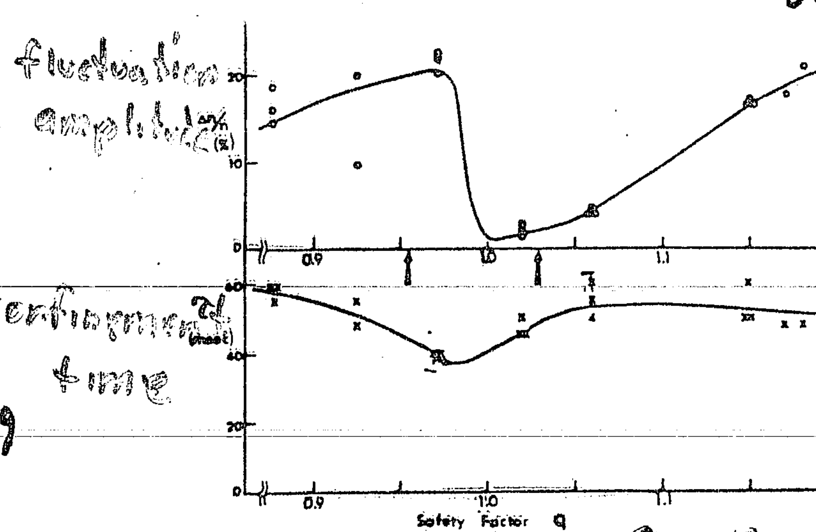
He $n \simeq 10^{11}\ cm^{-3}$

$T_i \simeq .05$ T_e ...

$B_p \simeq 120$ G

- fluctuation level high $\tilde{n}/n \lesssim 0.5$
  measured $k_\perp$ gives frequency & phase velocity reasonably close (factor of 2) to drift wave prediction
- $\tilde{n}/n$ drops for rational q values
- No correlation of transport & observed variation in $\tilde{n}/n$ (Diffusion was $D_\perp \sim \frac{1}{500} D_{Bohm}$ & not understood

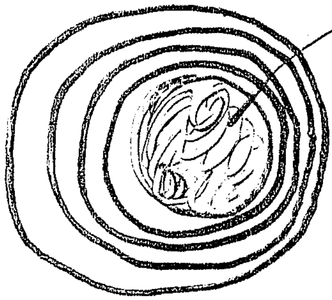
Here, compare $\tilde{n}/n$ & τ



(Explanation for quenching of drift wave given by Okuda

FM-1 Spherator - Okabayashi + Arunasalam
Nuc. Fus. 17 (1977) 497

μwave scattering measurements of density fluctuations

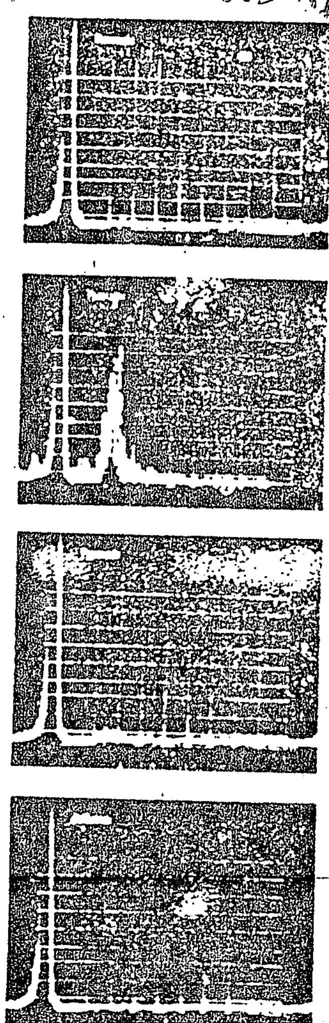


Superconducting ring R = 90cm
ring diam. = 20cm

Non-ohmic parameters
$n \simeq 5 \times 10^{11}$ $T_e \sim 5ev$ $T_i \sim 2ev$
$B_t \sim \frac{1}{5} - \frac{1}{7} B_p$

Spectra changed from single-mode (well defined drift wave) to turbulent as shear decreased

$S(k,\omega)$ for various $k_\perp$



$S(k,\omega)$ — $k_\perp\rho_i = .25$
$k_\phi \rho_i = 0.25$
$(k_\phi = 2 k_0 \cos 75°)$

$S(k,\omega)$ — $k_\perp\rho_i = .5$
$k_\phi \rho_i = 0.5$
$(k_\phi = 2 k_0 \cos 60°)$

$S(k,\omega)$ — $k_\perp\rho_i \simeq .7$
$k_\phi \rho_i = 0.71$
$(k_\phi = 2 k_0 \cos 45°)$

$S(k,\omega)$ — $k_\perp\rho_i \sim 1$
$k_\psi \rho_i = 0.97$
$(k_\psi = 2 k_0 \cos 30°)$

$\omega \rightarrow$

- Here, well defined single drift mode $k_\perp\rho_i = 0.5$, $\Delta\omega \ll \omega$
- high shear, $\frac{L_s}{L_n} \sim 7$ in Helium
  mode is P.B. stable
  (authors term this situation drift wave stable regime)
- mode localized radially in region of steepest density gradient

(Okabayashi & Arunasalam - cont. FM-1 spherator)

* For lower shear - L_s a factor of 10 longer ($L_s/L_n \sim 70$) (He)

See 1) much broader drift wave peak $\Delta\omega \sim \omega$

2) appearance of peak in spectra at $\omega \simeq 0$

$k\rho_i = 0.25$

$k_\phi\rho_i = 0.5$

$k_\phi\rho_i = 0.7$

$k_{\parallel}\rho_i \sim 1$ $k_r\rho_i \sim 1$

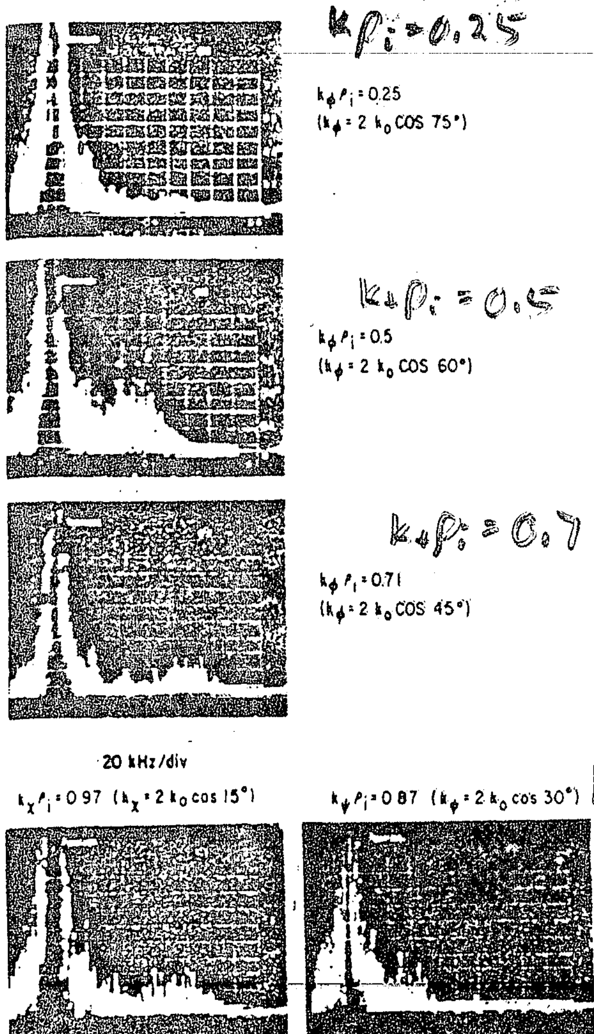


FIG.3. The frequency power spectrum observed by probe and by microwave scattering in the medium shear configuration in a helium plasma. Again $\delta f \cong 2$ kHz.

3) fluctuations seen in all 3 components of $\underset{\sim}{k}$ - moving towards isotropic

* This spectrum change is correlated with transport:

$D, K \propto \frac{L_s}{a}$

(found previously)

confinement worse for less shear

* If further reduce shear - spectrum become of strong turbulence type
  1) no peak at drift frequency
  2) peak at ω=0 (power law ω^{-6})
  3) spectrum same for all $k_\perp$

S(K,f) for various θs   log $\hat{n}^2$ vs. f

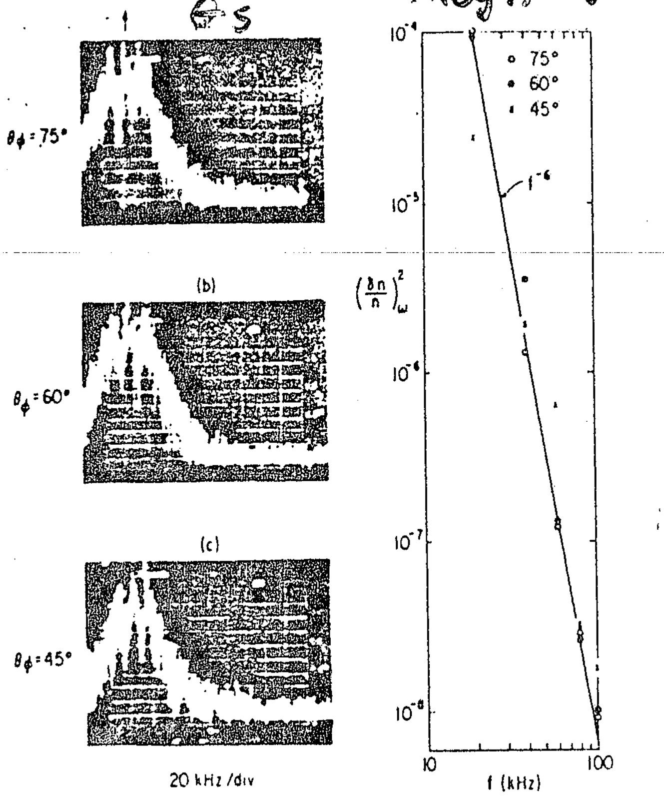


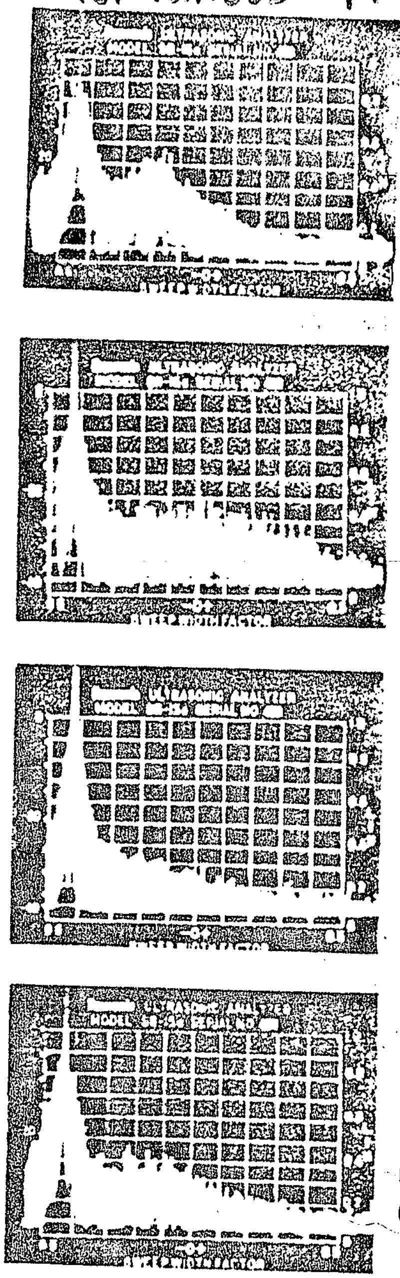
FIG.5. On the left is the frequency power spectrum observed by microwave scattering in the low shear configuration in an argon plasma. On the right we show the power law dependence of the observed fluctuation spectrum at high frequencies. $\delta f \approx 2$ kHz.

4) indications of spectrum isotropic in $k_\perp$ & $k_\parallel$

* Okabayashi & Arunasalam found could obtain strong turbulent type spectra under any shear conditions if electrons heated by ohmic ($V_d/C_s \gtrsim .1$) or neutral beams

FM-1 continued

S(k, ω) for various $k\rho_i$



$k_\perp\rho_i = .13$

$k_\phi \rho_i = 0.13$
($k_\phi = 2\,k_0 \cos 75°$)

$k\rho_i = .25$

$k_\phi \rho_i = 0.25$
($k_\phi = 2\,k_0 \cos 60°$)

$k\rho_i = .36$

$k_\phi \rho_i = 0.36$
($k_\phi = 2\,k_0 \cos 45°$)

$k_r\rho_i = .97$

$k_\chi \rho_i = 0.97$
($k_\chi = 2\,k_0 \cos 15°$)

10 kHz/div

FIG. 4. The frequency power spectrum observed by microwave scattering in the high shear configuration in a hydrogen plasma. $\delta f \approx 2$ kHz.

* they see stabilization (transition from $\Delta f \sim f$ to $\Delta f \ll f$)

scaling as $\frac{a}{L_s} \propto \left(\frac{m_e}{m_i}\right)^{1/3}$

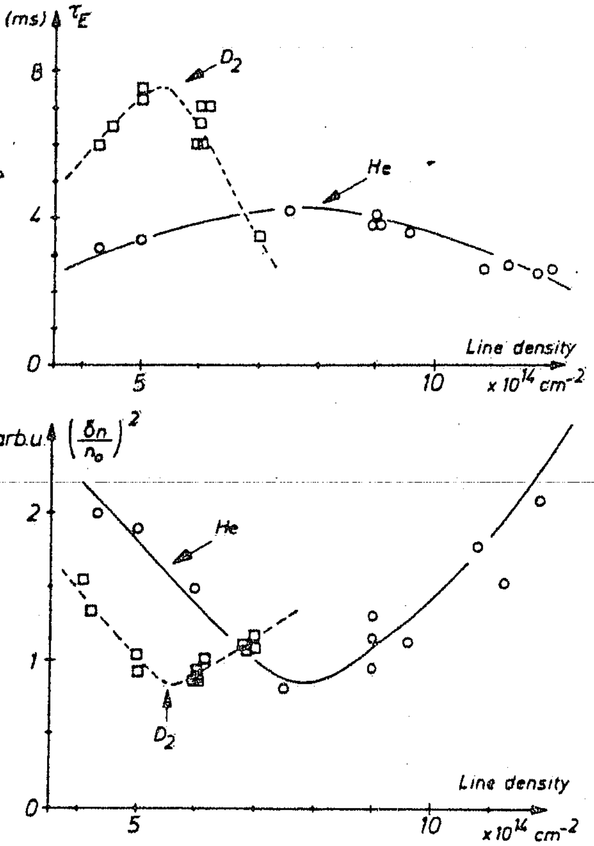
as in Perlstein-Berk

Wendelstein VII-A "Garching Stellarator"

Meyer + Mahn PRL 46 (1981) 1206 - CO_2 scattering

Saw close relation between fluctuation level + energy confinement time

τ_E



$\frac{n_k}{n}$

← τ_E increases, then decreases with $\bar{n}$

← correlates with density fluctuation level

$\frac{\delta n_k}{n}$ for $k = 125 cm^{-1}$

(So this is fluctuation level for $k = 125 cm^{-1}$ ($k\rho_s \sim 7$) integrated over all ω - not total $\tilde{n}/n$)

$\int n dl$ →

× Spectrum seen is broad : slight peak at $f \sim 50-100$ kHz, $\Delta f \sim 100$ kHz

W VII A Parameters: $R = 200$ $a = 10$ $B_0 \leq 35$ kG

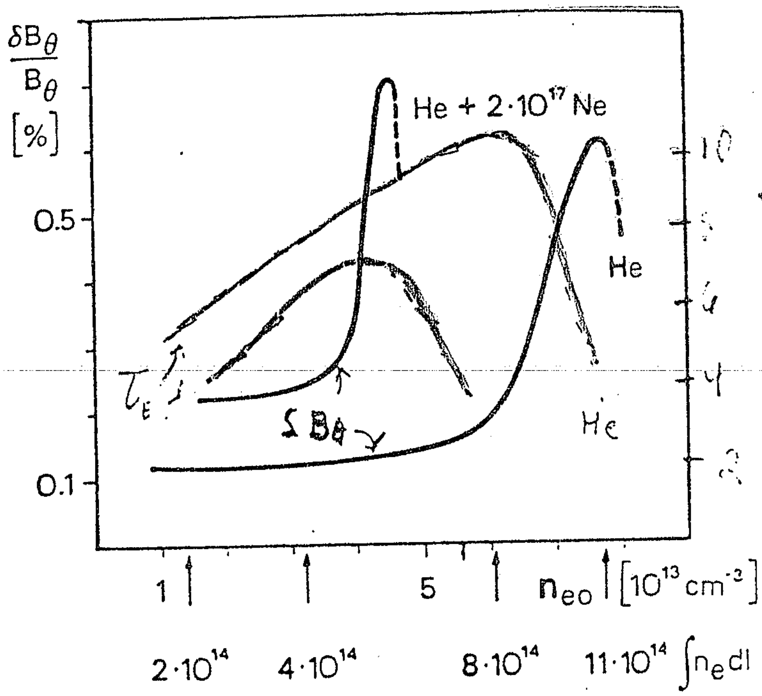
$T_0 \sim 300-500$ eV $I \sim 20$ kA

helical windings $l = 2, m = 2$ shearless ext. transform $\iota = .23$]

Wendelstein VII A – Poster APS-NYC – WVIIA-Team

Also see deterioration of confinement at high $\bar{n}$ correlated with MHD activity – (2,1) + (3,2)

(tearing modes, deformation + ergodization)



τ_E msec

in low density regime, find

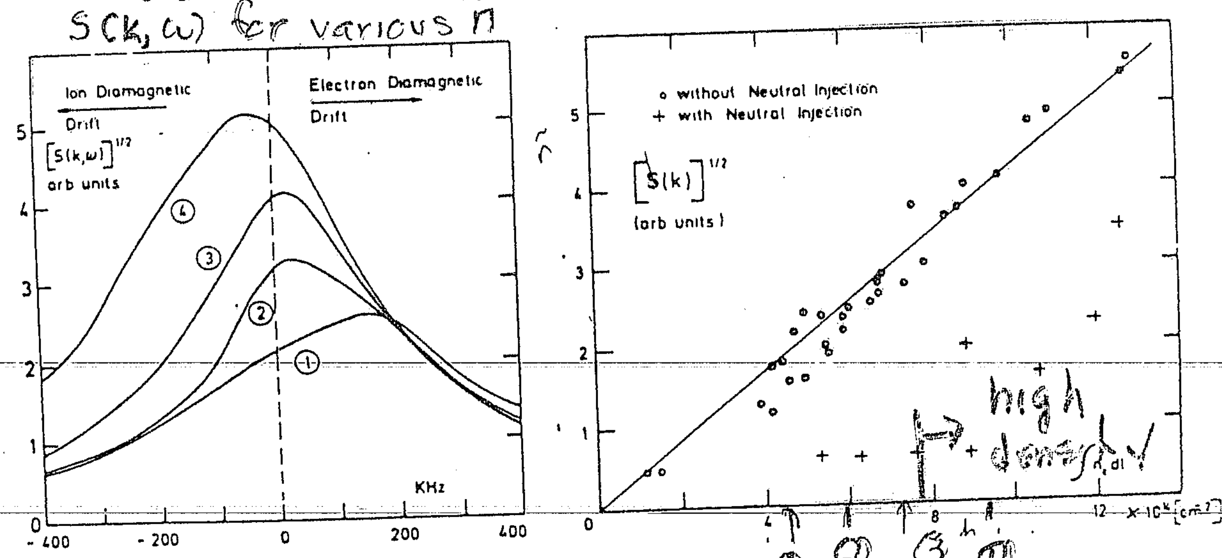
$n\tau_E \simeq \frac{3\times10^{18}}{T_e^{2/3}}$ cm^{-3}sec (T_e in eV)

(lower than INTOR

Microwave scattering (WVIIA-Team IAEA-CN-38/H-1) $k_\perp \simeq 6$-25 cm^{-1} $k\rho_s \sim 1$) showed no correlation of $\delta\tilde{n}_k$ and τ_E

S(k,ω) for various $\bar{n}$

S(k,ω)



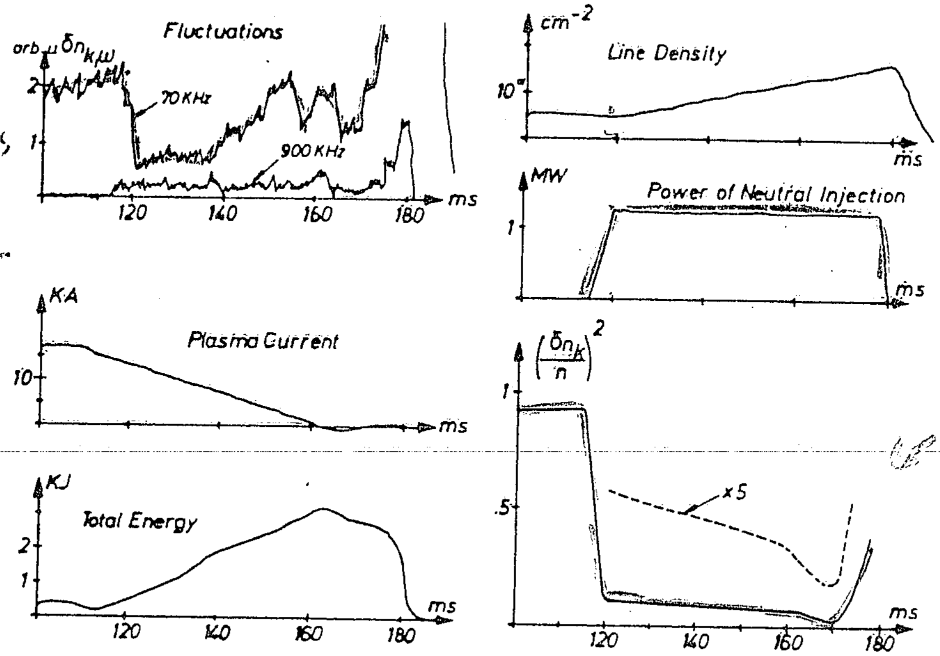
f →

density $\int n dl$

Wendelstein VII A - Neutral Beam injection

* Sharp drop in $\tilde{n}_k/n$ + change in spectra within 1 msec of start of injection - faster than profiles change
* seen in both CO_2 + μwave spectra
* Confinement is better during beam injection

low freq. fluctuations drop - 70 kHz



Here, fig. from Meyer + Hahn CO_2

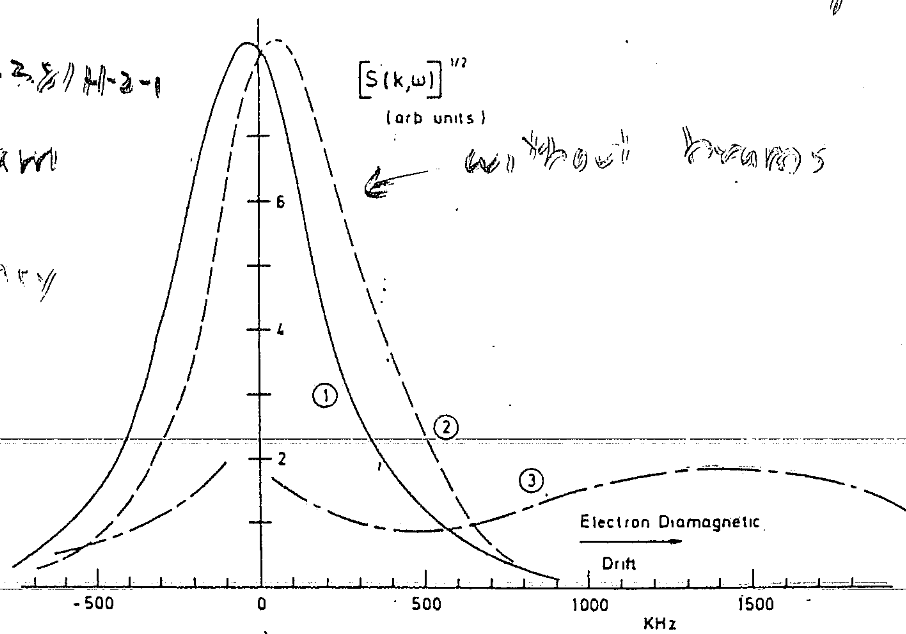
level drops by 10

Meyer + Hahn suggest that since fluctuations change before any global parameters or profiles change, subsequent improved confinement is due to lower $\tilde{n}$ level

μwave scattering spectra change during beam injection

IAEA-CN-38/H-3-1
WVIIA Team

low frequency fluctuation level drops



a)

Fluctuations & Transport have also been studied in

RFPls

Zeta - Robinson & Rusbridge Plasma Phys 11 (1969) 73 & refs therein

Rusbridge Plasma Phys 22 (1980) 331

ZT-40 Jacobson Appl. Phys. Lett 39 (1981) 795

Plasma Phys 23 (1981) 927

EBT's Roth, Krawczonek, Powers, Hong & Kim PRL 40 (1978) 1195

Linear

Boisser Nuc. Fus 18 (1978) 867

Carter, Edwards, Mossack, Rusbridge, Hastie Plasma Phys 23 (1981) 819

Hendel, Chu, Politzer P.F. 11 (1968) 2426

Other Multipoles - Yoshikawa Review Nuc Fus 13 (1973) 432

Culham Levitron IAEA Brussels

Conclusions

1. Ohmic Tokomaks have broadband, turbulent spectra, $\Delta\omega > \omega$ for frequencies $\omega \lesssim \omega_*$

2. Similar spectra seen in other experiments under a wide variety of conditions

3. Not clear what this low frequency turbulence is - "drift-wave"? "convective cell"? "MHD"? etc.

4. Probably turbulence is related to transport

Is mode coupling important for fluctuation levels $\tilde{n}/n$ measured in tokamaks?

DEFINE $\left.\frac{\tilde{n}}{n}\right|_{crit.}$ - turbulence level such that

nonlinear modecoupling term (n.linear $E \times B$) equal to linear (ω^*) term for $k\rho_s = 1$

then, from Hasegawa Mima eqn.

$$\left.\frac{\tilde{n}}{n}\right|_{crit} \approx \frac{\rho_s}{4L_n} \sim \frac{\rho_s(T_e(0))}{2a}$$

Tokamak	$\left.\frac{\tilde{n}}{n}\right\|_{crit}$	$\left.\frac{\tilde{n}}{n}\right\|_{measured}$	Ratio $\frac{\tilde{n}_m}{\tilde{n}_c}$ (if $\gtrsim 1$ mode coupling impt.)
PLT	10^{-3}	.005-.01	4-8
Alcator-A (low density)	4×10^{-3}	$.07 \pm .03$	17
ATC $\begin{cases} \mu \\ co^2 \end{cases}$	$4\text{-}5 \times 10^{-3}$	$\begin{cases} .005-.01 \\ .03 \end{cases}$	1 6
Macrotor	8×10^{-3}	.1	12
Microtor	5×10^{-3}	~.04	8
TFR $\tilde{n} = 5 \times 10^{13}$, $\bar{n} = 10^{14}\,cm^{-3}$	2×10^{-3}	$\begin{cases} 5 \times 10^{-3} \\ 2 \times 10^{-3} \end{cases}$	2.5 1

In all cases, mode coupling is important

MICROINSTABILITY, ENTROPY-PRODUCTION AND PLASMA CONFINEMENT

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HIROSHIMA UNIVERSITY

Jan. 1982

Microinstability, Entropy-production and Plasma Confinement

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Institute for Fusion Theory, Hiroshima Univ.

A Non-equilibrium thermodynamic approach

Motivations and Status

Anomalous plasma transport

...... explain Scaling Laws ?

Previous. traditional analyses

- Force - Balance equilibrium

↓

- Microinstability analysis

↓

- given profiles, calculate fluxes (induced)

↓

< Saturation level, spectrum analyses >

quasilinear, non-linear (turbulence weak, strong etc.)

↓

$\Gamma, Q, J \rightarrow \tau_p, \tau_E$??

These are depending on geometric factors!

↓

- How can we determine geometric factors such as , χ, χ_T . ...

Confined Plasma

- Non-thermal equilibrium (inhomogeneities)

  κ, κ_p ....

- Steady-state with External Supplies

(Anomalous) Loss →

← Input Source { particle, ohmic, neutral beam, RF heating, Current Drive etc.

← balance. →

Steady-State

Losses induced by fluctuations

balance with External Source

- Internal fluctuation level, spectrum

  should be determined by External Source Condition!

☆ Similar.

- Pseudo-classical, NeoBohm Yoshikawa
- Profile Consistency , B. Coppi

a method to determine plasma losses

in Non-equilibrium steady-state w external supply

"Ansatz"

"relevant state has the minimum entropy production rate (irreversible), under the conditions imposed to the system"

$\vec{\Gamma}, \vec{Q}, \vec{J}$ ...(flow) ← induced by fluctuations, I_k

↓

$\dot{S}$ < Gibbs' relation
local. quasi-static change >

↓

$\left.\frac{dS}{dt}\right)_{irr}$ $(\kappa, \kappa_T, \vec{\Gamma}, \vec{Q}\ I_k)$

∘ Variational Principle < Wave-Kinetic Eq. >

∘ Conditional Minimum (density, energy conservations)

↓

Determine, $\vec{\Gamma}, \vec{Q}, \kappa, \kappa_T, \int I_k$ ...
simultaneously.

Basic Equations and Induced Fluxes

$$m_j n_j \frac{d\vec{V}_j}{dt} - \frac{q_j}{c}\vec{\Gamma}_j \times \vec{B} = q_j \langle \tilde{n}_j \tilde{\vec{E}}_j + \frac{1}{c}\tilde{\vec{\Gamma}}_j \times \tilde{\vec{B}} \rangle + n_j q_j \vec{E} - \nabla p_j \, , \tag{1}$$

$$\frac{\partial}{\partial t}(3 n_j T_j) + \nabla \cdot \vec{Q}_j + 2 p_j \nabla \cdot \vec{V}_j + \nabla p_j \cdot \vec{V}_j = q_j \langle \tilde{\vec{\Gamma}}_j \cdot \tilde{\vec{E}} \rangle + \dot{P}_j , \tag{2}$$

where $\vec{\Gamma} = n\vec{V}$, $Q_{ijk} = \frac{1}{2} \int m_j (v_i - V_i)(v_j - V_j)(v_k - V_k) f d\vec{v}$ is the heat conduction, $Q_k = 3T\Gamma_k + \sum_i Q_{kii}$ is the total heat flux

$$\Gamma_r = \frac{c}{B} \langle (\tilde{n}\tilde{\vec{E}} + \frac{1}{c}\tilde{\vec{\Gamma}} \times \tilde{\vec{B}})_\theta \rangle \tag{3}$$

$$\sum_i Q_{rii} = \sum_i \frac{c}{B} [\langle (\tilde{\vec{E}} + \frac{\vec{V} \times \tilde{\vec{B}}}{c})_\theta \tilde{P}_{ii} \rangle + 2 \langle (\tilde{\vec{E}} + \frac{\vec{V} \times \tilde{\vec{B}}}{c})_i \tilde{P}_{i\theta} \rangle - \frac{B}{nc} p_{ii} \Gamma_r$$
$$- \frac{1}{n} p_{i\theta} \langle (\tilde{n}\tilde{\vec{E}} + \frac{\tilde{\vec{\Gamma}} \times \tilde{\vec{B}}}{c})_i \rangle + \frac{1}{c} \langle \tilde{B}_r \tilde{Q}_{zii} - \tilde{B}_z \tilde{Q}_{rii} \rangle] . \tag{4}$$

for cylindrical plasma, κ, κ_T, u, EM fluctuations (low freq.)

☆ $$\Gamma_r = \sum_k [\frac{\omega - k_{\parallel} u}{\omega} - \frac{\omega_{*j}}{\omega} \{ 1 - \frac{\kappa_T}{\kappa}(1 - 2\xi^2) \}] R_k \, , \tag{3'}$$

☆ $$Q_r = \sum_k [(\frac{\omega - k_{\parallel} u}{\omega} - \frac{\omega_{*j}}{\omega} \{ 1 - \frac{\kappa_T}{2\kappa}(1 - 2\xi^2) \}) \{ m(\frac{\omega}{k_{\parallel}} - u)^2 + 2T \} - 2T \frac{\omega_*}{\omega} \frac{\kappa_T}{\kappa}] R_k \tag{4'}$$

and

$$R_k = \frac{n q_j c}{B k_\theta T} \, \mathrm{Im} (\frac{\omega}{\sqrt{2} |k_{\parallel}| v_T} Z(\xi)) \, | \tilde{E}_\theta + \frac{\omega}{k_{\parallel} c} \tilde{B}_r |_k^2 , \qquad k \equiv (\vec{k}, \omega)$$

where $\kappa_T = -\nabla T / T$, $\omega_* = -\kappa k_\theta c T / q_j B$, $\xi = (\omega - k_{\parallel} u) / \sqrt{2} |k_{\parallel}| v_T$, $v_T^2 = T/m$

Induced Flows v.s. Force acting on Plasma

$$\begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{pmatrix} = \begin{pmatrix} -K_1, & \frac{1}{2}K_1, & K_2, & K_4 \\ \frac{1}{2}K_1, & -\frac{5}{4}K_1, & -\frac{1}{2}K_2, & \frac{1}{2}K_4 \\ K_2, & -\frac{1}{2}K_2, & -K_3, & K_5 \\ 0, & 0, & 0, & T/\eta_c \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} \tag{14}$$

where the "flow" $(J_1 \cdots J_4)$ is $(\Gamma_r, J_{sr}, \dot{W}, TJ_{0/\!/})$ and the "force" $(X_1 \cdots X_4)$ is $(\kappa, \kappa_T, 1/T, \ddot{E}_{/\!/})$, $\dot{W} \equiv q\langle \vec{\vec{\Gamma}} \cdot \vec{\vec{E}} \rangle$ is the energy exchange with the wave, $\bar{\xi} \equiv \omega/\sqrt{2}|k_{/\!/}|v_T$,

$$K_1 = n \int dk \ \mathrm{Im}\bar{\xi}Z \ \frac{I_k c^2}{B^2 \omega} \tag{15-1}$$

$$K_2 = n \int dk \ \mathrm{Im}\bar{\xi}Z \ \frac{eI_k c^2}{Bk_\theta}, \tag{15-2}$$

$$K_3 = n \int dk \ \mathrm{Im}\bar{\xi}Z \ \frac{e^2 \omega}{k_\theta^2} I_k c^2, \tag{15-3}$$

$$K_4 = n \int dk \ \mathrm{Im}\bar{\xi}Z \ \frac{k_{/\!/} c^2 I_k}{\eta_c \omega k_\theta BT}, \tag{15-4}$$

$$K_5 = n \int dk \ \mathrm{Im}\bar{\xi}Z \ \frac{ek_{/\!/} c^2 I_k}{\eta_c k_\theta^2 T}, \tag{15-5}$$

$I_k = |\tilde{E}_\theta + \omega \tilde{B}_r / k_{/\!/} c|^2_k$ and η_c is the classical resistivity*).

Definition of Entropy production rate

Gibbs' relation . quasi-static change

$$\frac{d}{dt}U = T\frac{d}{dt}s - p\frac{d}{dt}\left(\frac{1}{\rho}\right) + \sum_k \mu_k \frac{d}{dt}\left(\frac{\rho_k}{\rho}\right) \qquad (5)$$

Entropy balance eq.

$$\rho\frac{ds}{dt} + \nabla\cdot\vec{J}_s = \frac{dS}{dt}\Big)_{irr} \qquad (6)$$

Irreversible Entropy production rate (i & e)

$$\frac{dS_j}{dt}\Big)_{irr} = \frac{1}{T_j}\{-T_j\vec{\Gamma}_j\cdot\frac{\nabla p_j}{p_j} - \vec{J}_s\cdot\nabla T_j + q_j\langle\vec{\tilde{\Gamma}}_j\cdot\vec{\tilde{E}}\rangle + \dot{P}_j - \frac{5}{2}T_jS_{pj}\} \qquad (7)$$

Wave Entropy

$$S_k = (N_k+1)\,ln(N_k+1) - N_k\,ln(N_k) \qquad S_w = \sum_k S_k \; . \qquad (9)$$

Total Entropy (local)

$$\frac{dS}{dt}\Big)_{irr} = \frac{dS_e}{dt}\Big)_{irr} + \frac{dS_i}{dt}\Big)_{irr} + \frac{dS_W}{dt}\Big)_{irr} \qquad (10)$$

Principle of Minimum Entropy Production Rate

$$\sigma \equiv \int d\vec{r} \; \left.\frac{dS}{dt}\right)_{irr} \tag{11}$$

Constraints

$$\nabla \cdot \vec{\Gamma}_j = S_{pj} \; , \tag{12}$$

$$\nabla \cdot \vec{Q}_j + 2p_j \nabla \cdot \vec{V}_j + \nabla p_j \cdot \vec{V}_j = q_j \langle \vec{\tilde{\Gamma}}_j \cdot \vec{\tilde{E}} \rangle + \dot{P}_j \; . \tag{13}$$

Steady state, wave energy conservation

$$\sum_j q_j \langle \tilde{\Gamma}_j \cdot \tilde{E} \rangle + \nabla \cdot (\tilde{E} \times \tilde{B}) = 0 \tag{8}$$

A model example for electrons (point)

$$L \equiv \frac{dS_e}{dt}\Big]_{irr} = -(X_1^2 + \frac{3}{2}X_2^2)K_1 + 2K_2X_1X_3 - K_3X_3^2 + (\tau \;\; - \frac{5}{2})S_p. \qquad (17)$$

Conditions of the particle and energy balances in a steady state are rewritten as

$$\frac{1}{R}\{-K_1X_1 + \frac{1}{2}K_1X_2 + K_2X_3\} = S_p \qquad (18)$$

and

$$2K_1(X_2 - \frac{1}{R})X_1 + K_2(\frac{2}{R} - \frac{3}{2}X_2 - X_1)X_3 + K_3X_3^2 = \tau S_p, \qquad (19)$$

$$\dot{P} = \tau S_p, \quad \nabla X_3 = -X_2 X_3, \quad \frac{1}{r}\frac{\partial}{\partial r}r \sim \frac{1}{R}$$

introduce Lagrange's indeterminate coefficients λ_{1_0} and λ_{2_0}, and obtain the functional $\hat{L}$ as

$$\hat{L} = -K_1(X_1^2 + \frac{3}{2}X_2^2) - K_3X_3^2 + 2K_2X_1X_3 - \lambda_1(-K_1X_1 + \frac{1}{2}K_1X_2 + K_2X_3 - RS_p)$$

$$- \lambda_2\{2K_1(X_2 - \frac{1}{R})X_1 + K_2(\frac{2}{R} - \frac{3}{2}X_2 - X_1)X_3 + K_3X_3^2 - \tau S_p\}. \qquad (20)$$

Find solutions

☆ $\delta\hat{L}/\delta X_i = 0$ $(i=1,2,3)$ (23)

and

☆ $\delta\hat{L}/\delta K_i = 0$ $(i=1,2)$. (24)

Remark Ⓐ $\circ\ \delta\hat{L}/\delta K_i$ $\circ\ \delta K_i/\delta I_k$ → $\delta\hat{L}/\delta I_k \doteq 0$

Ⓑ $\{ X_1, X_2, K_1, K_2 X_3, K_3 X_3^2 \}$ indep.

X_1, X_2 →(Table) determined ! x. Kr.

K_1 → dissipation rate

$$n \int dk\ \mathrm{Im}\bar{\xi} Z \frac{c^2 I_k}{B^2 \omega R^2} = C\ S_p \tag{25}$$

⟶ inelastic collision freq. (e ↔ wave)

$n I_k \xi Z \propto n_r (I_k/\omega_k \text{ or } N_k)$, $\propto \langle \tilde{n} \hat{E}^3 \rangle$

Wave fluctuation of Finite Amplitude is dictated by Source condition !

Solutions for various values of τ

τ	$(R\kappa, R\kappa_T)_1$	$(R\kappa, R\kappa_T)_2$	$(R\kappa, R\kappa_T)_3$	$(R\kappa, R\kappa_T)_4$
0.1		(3.0, -0.87)	(-1.7, 3.1)	
0.5	(0.43, 1.34)	(3.0, -0.87)	(-0.87, 1.8)	(0., 0.83)
1.0	(0.63, 1.67)	(2.9, -0.88)		
2.0	(0.85, 2.06)	(2.8, -0.89)	(1.3, -1.6)	(-2.3, -0.47)
3.0	(1.0, 2.35)	(2.7, -0.9)	(3.1, -4.4)	(-2.6, -0.51)

Wave Spectrum

$$K_1 = n\int dk\ \mathrm{Im}\bar{\xi}Z\ \frac{I_k c^2}{B^2\omega}$$

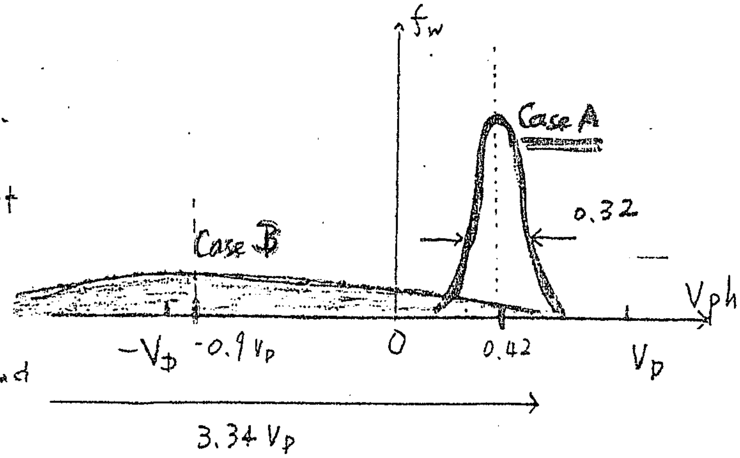
$$K_2 = n\int dk\ \mathrm{Im}\bar{\xi}Z\ \frac{eI_k c^2}{Bk_\theta},$$

$$K_3 = n\int dk\ \mathrm{Im}\bar{\xi}Z\ \frac{e^2\omega}{k_\theta^2} I_k c^2,$$

	$R\kappa$	$R\kappa_T$	K_1/R^2S_p	K_2X_3/RS_p	$K_3X_3^2/S_p$	$\Gamma.Q$	> out
(A)	.85	2.06	1.67	.71	.34	$\Gamma>0$ $Q<0$	
(B)	1.29	−1.63	−.332	.30	−.93	$\Gamma, Q, >0$	

$$K_2X_3/K_1 = \left\langle \frac{\omega}{k_\theta}\Big/\frac{cT}{eBR} \right\rangle$$ 1st

$$K_3X_3^2/K_1 = \left\langle \left(\frac{\omega}{k_\theta}\Big/\frac{cT}{eBR}\right)^2 \right\rangle$$ 2nd



rough estimation of saturation (ES)

$$\int dk_\perp d\omega (k_\theta/\omega) I_k \sim (\int dk_\perp I_k)/(\overline{\omega/k_\theta}), \quad \mathrm{Im}\,\xi Z \sim O(1)$$

$$\left| \tilde{E}_\theta + \frac{\omega}{k_\parallel c}\tilde{B}_r \right|^2 \sim C' \frac{S_p}{n} \frac{\kappa^2 T^2 N}{\omega_* e^2} \qquad (26)$$

$$\left(\frac{e\phi}{T}\right)^2 \sim \frac{\kappa^2}{k_\perp^2}\left(\frac{C' S_p N}{n\omega_*}\right) \qquad (27)$$

Summary and Discussions.

- A method to analyze the plasma confinement is proposed.

  * Minimum entropy production rate

  < in non-equilibrium steady-state, Loss induced by fluctuations balances with external source >

  Variational ; Conditional minimum

  ↓

  particle, energy conservations.

$\left.\frac{d\dot{S}}{dt}\right)_{irr}$ min $\longrightarrow$ $\varkappa$, $\varkappa_T$, Γ_r, Q_r, $\int I_k$ $\int V_p I_k$ $\int V_p^2 I_k$ determined simultaneously.

; plasma loss ← Source condition

; plasma profile

; saturation ← Source

; Spectrum Information $\langle V_{ph}\rangle$, $\langle V_{ph}^2\rangle$

- Without Source

  $S_p = \tau S_p = E_{\parallel} = 0$

  $\left.\frac{dS}{dt}\right)_{irr} \geq 0$ for any low-freq. EM mode.

- A model example
  - electron, energy exchange with wave
  - $x_1, x_2, K_1, K_2 x_3, K_3 x_3^2$ obtained.
  - dissipation rate K_1 is balancing with S_p.
  - expected spectrum profile obtained.

- Different Input Source scheme.
  < RF, NB, Ohmic, Thermonuclear etc >
- Wave-Kinetic (Non linear, propagator ....)
  within weak turbulence (e.K.)
- randomization, thermalization of B (mag.)

- $\delta\sigma = 0$; with constraints!

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MEASUREMENTS OF TOKAMAK EDGE FLUCTUATIONS AND TRANSPORT

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UCLA

topics

- Significance & Properties of EDGE
- Particle diffusion measurements
- SPECTRUM OF EDGE FLUCTUATIONS
- $\tilde{E}_p \times B_T$ INDUCED Transport (JUST $\tilde{E}_p$)

[MAGNETIC FLUCTUATIONS : UCLA Articles + M. HEDEMENN Thesis, CIT]

Significance of EDGE

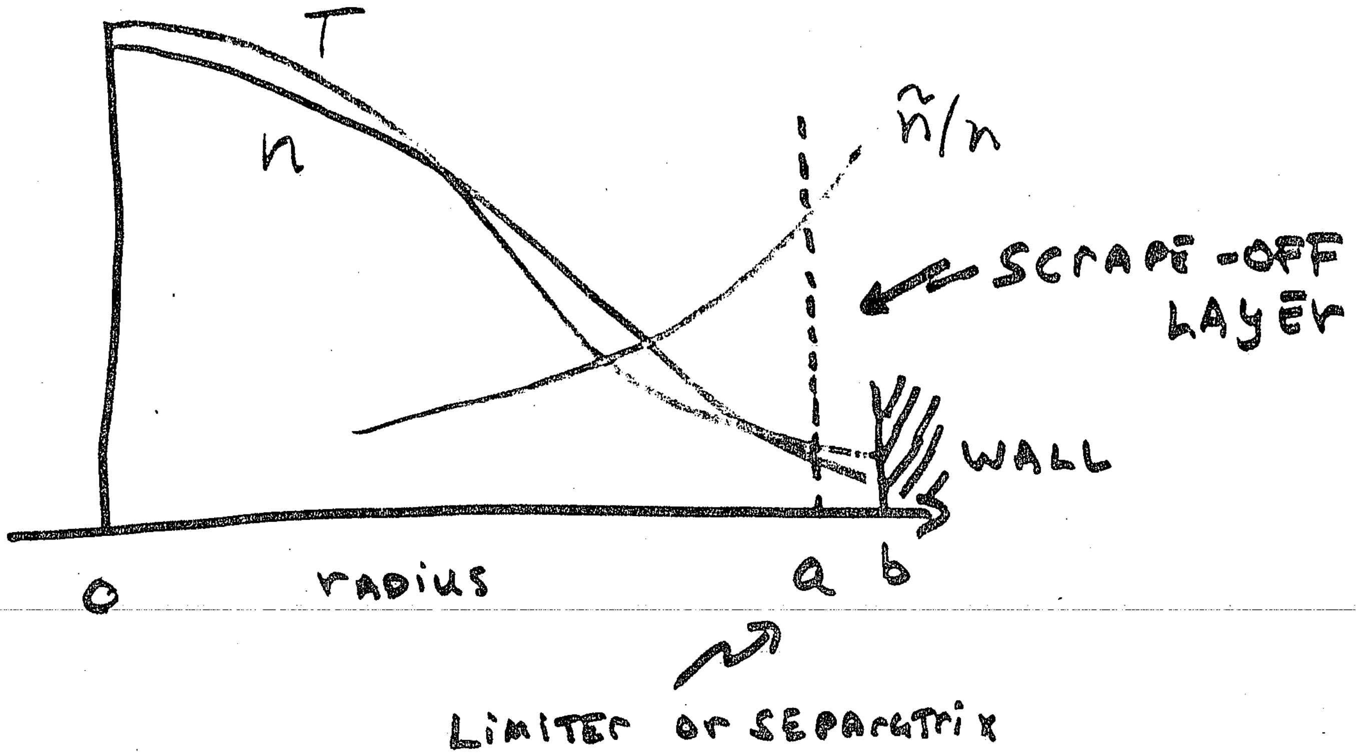
- EXAMPLE of strongly turbulent SYSTEM

- many tokamak EDGE MEASUREMENTS give ~ similar results [UCLA, CIT, ISX-B, PDX, Alcator, ATC, DITE, JFT-II, DIVA ...]

- important for reactors:
  - heat + particle removal (large D, χ good)
  - impurity release + shielding
  - divertor design
  - WAVE propagation

ALSO – relatively EASY to STUDY with probes

What is "EDGE"



EDGE ~ region in contact w/ WALL

$$\begin{cases} r/a \gtrsim .8 \ (?) \\ n \simeq .1\, n_0 \sim 10^{11}\,cm^{-3} - 10^{12}\,cm^{-3} \\ T_e \simeq 5-30\ eV \end{cases}$$

How far does influence of WALL (SEPARATRIX) extend inward past scrape-off layer?

[Is noise at EDGE heard in interior?]

4a

EDGE PROPERTIES

What makes EDGE TURBulent?

	EDGE	CIT CENTER	HOT CENTER
n	$\sim 10^{12}$	$\sim 10^{13}$	$\sim 10^{14}$
T	20	100	1000

- drift parameter $\xi \propto T/n$ not UNUSUAL
- Collisionality $\propto n/T^2$ relatively high
- density relatively low (β low also)
- possible atomic physics ($n_0 \gtrsim 10^{11}$ (?))
- relatively large ∇n (not ∇T)
- OPEN FIELD LINES (?)
- LARGE Diffusion: $D_\perp \sim D_{Bohm}$ (Alcator $\propto \frac{1}{n}$?)

(5)

PARTICLE DIFFUSION

RELATIVELY EASY TO MEASURE IN LIMITER SHADOW (SCRAPE-OFF LAYER):

- DISTANCE TO LIMITER ALONG B IS "L" ($L \sim 4\pi R$)
- FLOW SPEED $\sim C_s \sim \bar{v}_i$
- measure $n(r)$ in shadow region

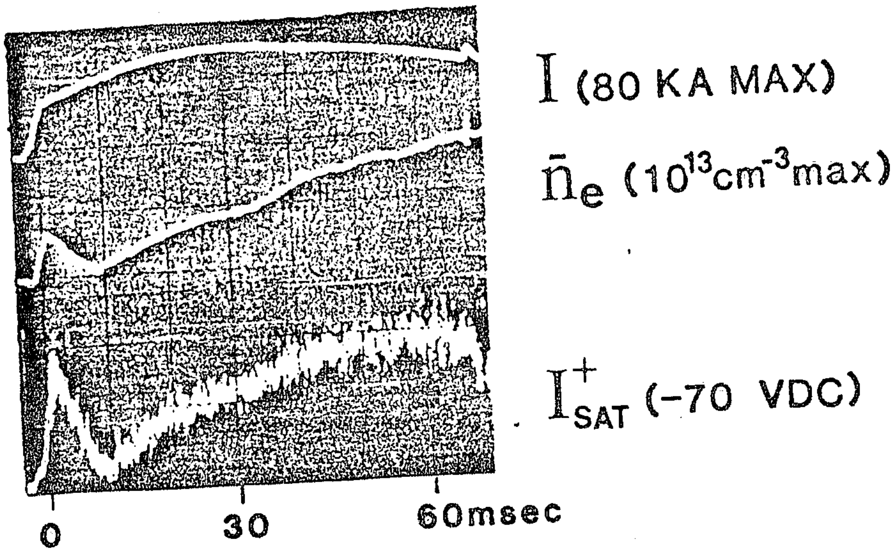
USUALLY $n(r) \propto e^{-r/\lambda_\perp}$

$$\therefore \; D_\perp \approx \lambda_\perp^2 / (L/C_s)$$

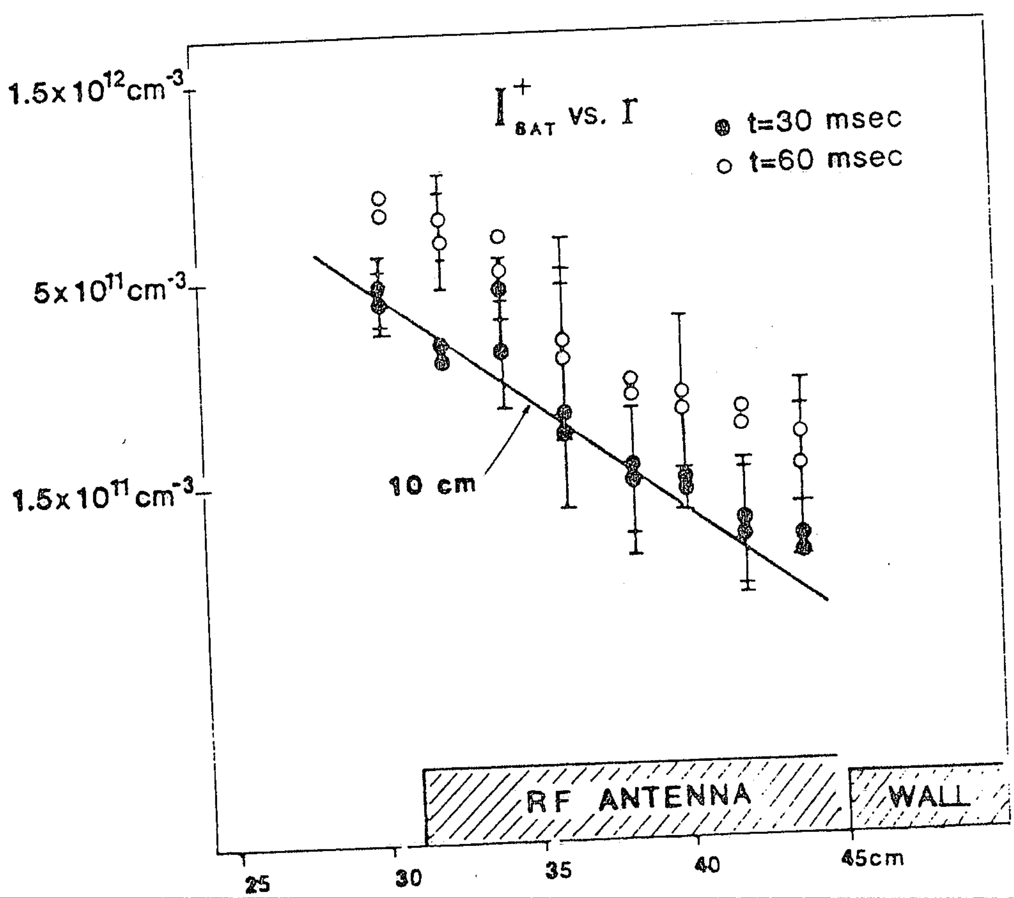
[ASSUME NO IONIZATION, recomb., reflection]

(6)

MACROTOR (2.5kg)



120° away from LIMITER



$\lambda_\perp \sim 10$cm

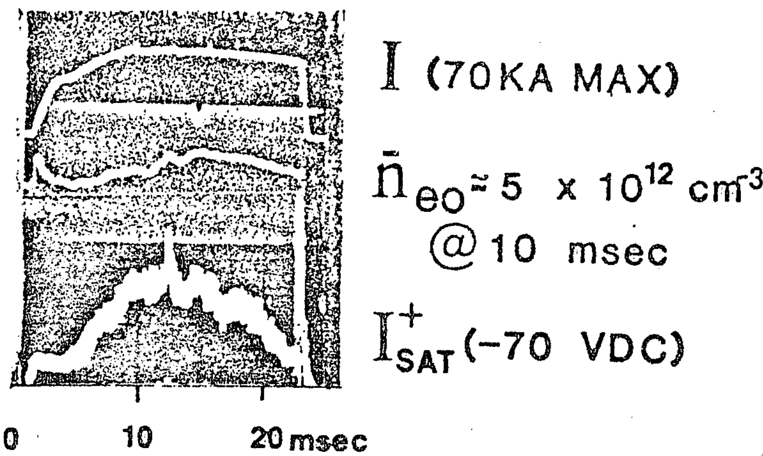
$D_\perp \sim 10^5$ cm^2/sec

$D_\perp \sim 2 D_{bohm}$

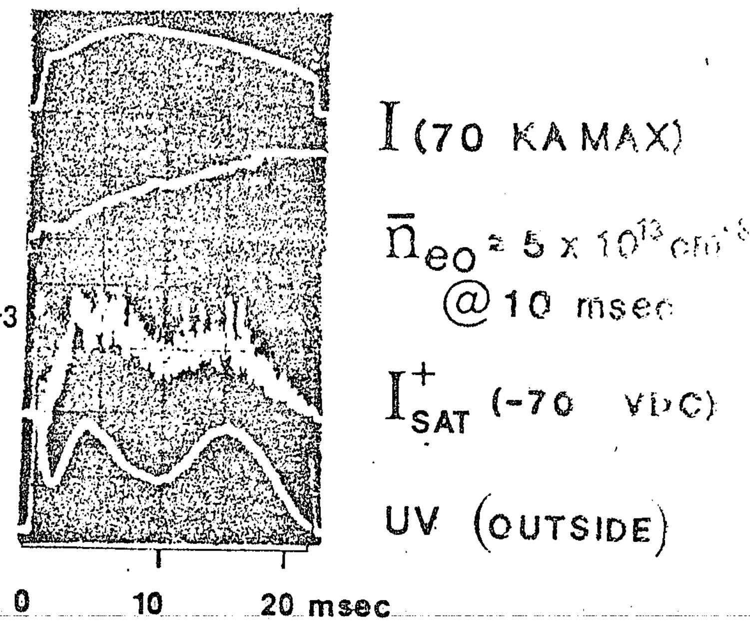
Fig. 5b. Radial profile of I_{SAT}^+ 120° toroidally from a grounded RF antenna which acts as a rail limiter.

MICROTOR (20kg)

LOW DENSITY

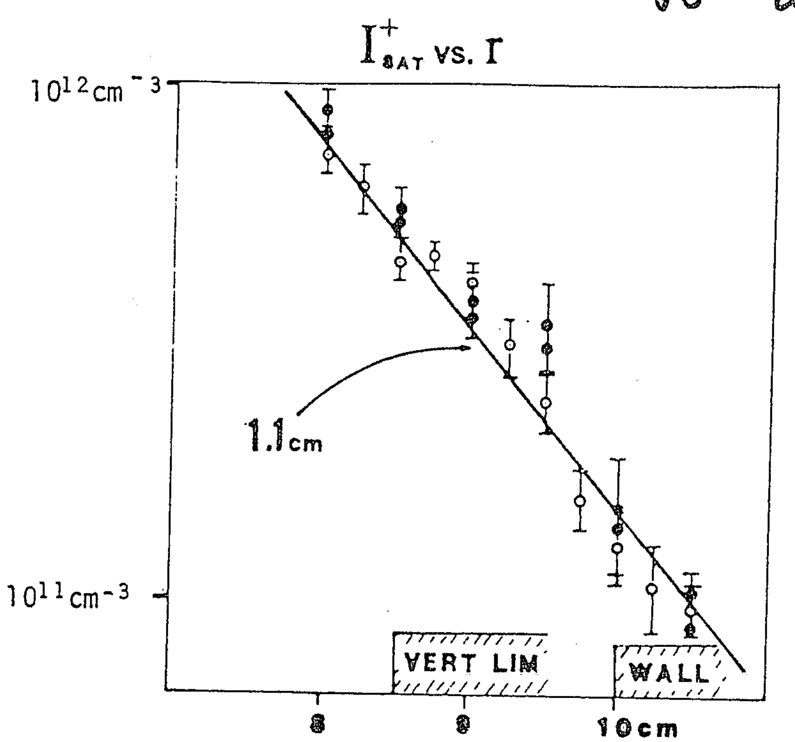


HIGH DENSITY



$\lambda_\perp \sim 1$ cm $\Rightarrow 5 \times 10^3$ cm²/sec $\sim .5\, D_{Bohm}$

~60° away from Limiter



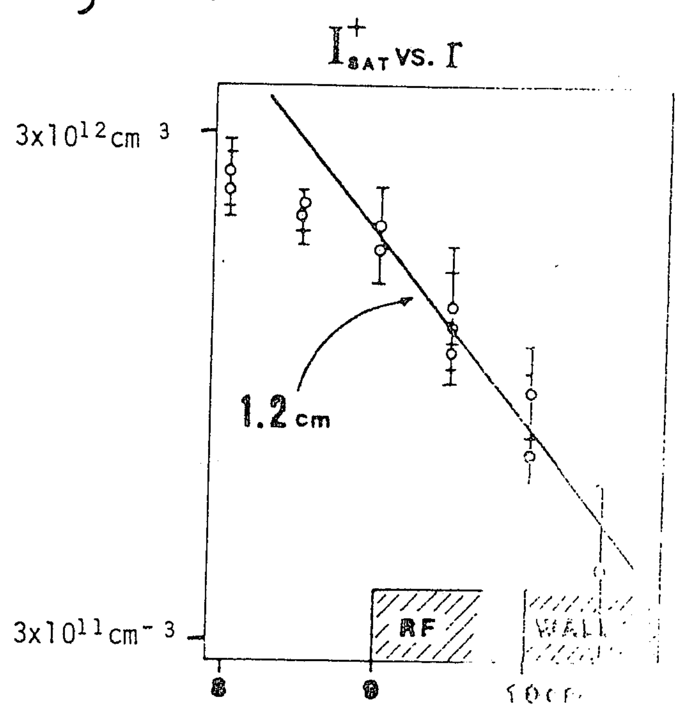
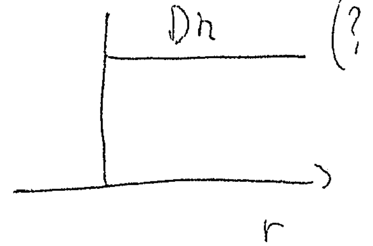


Fig. 6. Radial profiles of I_{SAT}^+ for low (left) and high (right) density Microtor discharges. The two types of points at left show the reproducibility from one day to the next. A "T" shaped langmuir probe was used for this data (the calculated n axis is only approximate).

Cause of $D_\perp$?



MAGNETIC:

$$\lambda_B < \left(\frac{\tilde{B}_r}{B_T}\right) L \ll \lambda_\perp$$

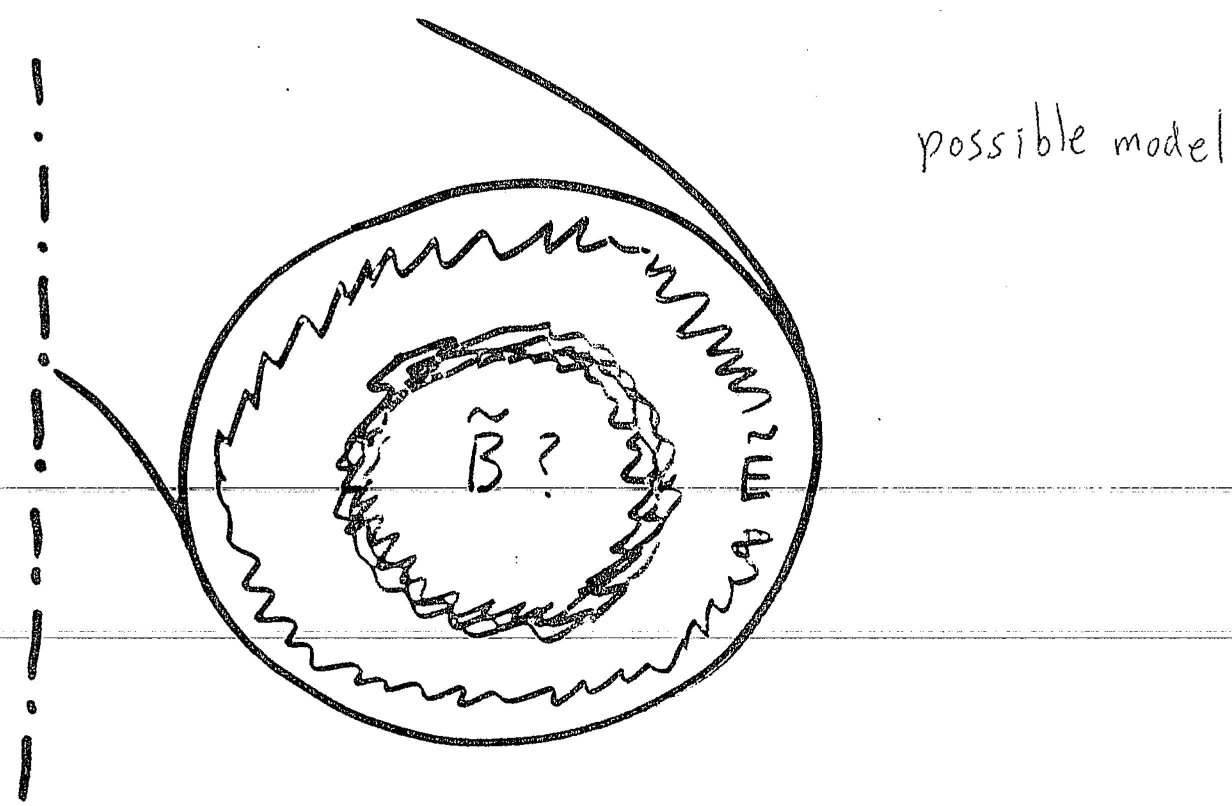
10^{-5} macrotor

probably negligible at Edge

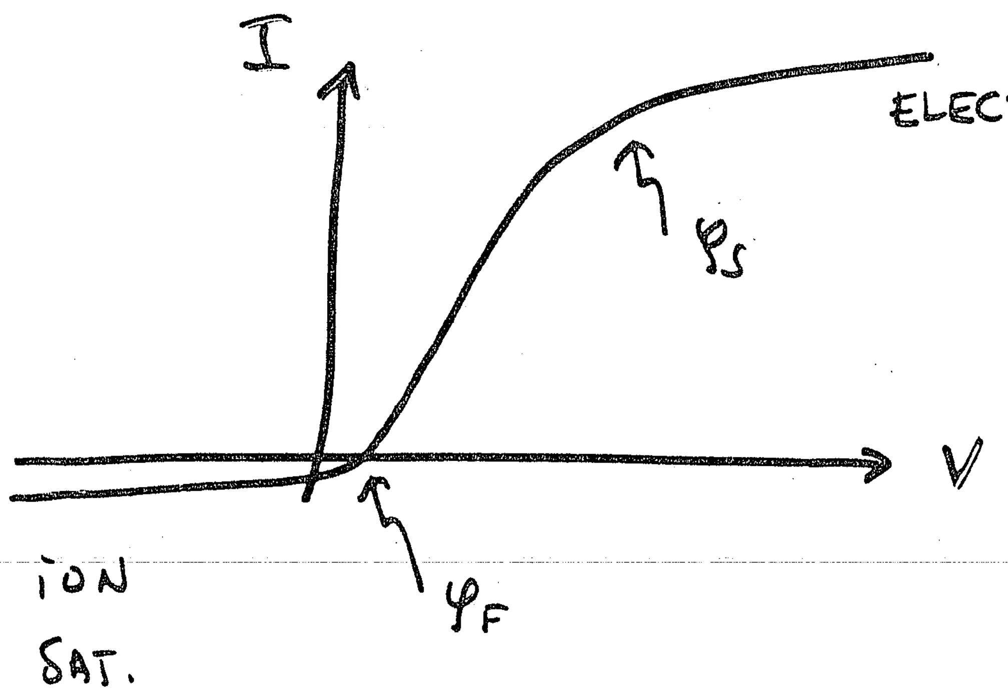
Electric:

$$\tilde{n}/n \sim .1 \rightarrow .3 \Rightarrow D_{Bohm}\ (?)$$

probably does cause EDGE Diffusion



FLUCTUATION MEASUREMENTS



LANGMUIR PROBE CHARACTERISTIC

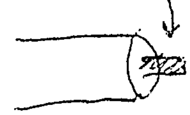
roughly:

$$\begin{cases} \varphi_f = \varphi_s - 3.6\,kT_e \\ I_i = ne\sqrt{T_e/m_i}\,A_i \\ I_e = ne\sqrt{T_e/m_e}\,A_e \end{cases}$$

($\tilde{T} = 0$)

n.b. { possible probe PERTURBATIONS (seems ok)
difficult to INTERPRET EXACTLY
(e.g. possible $\tilde{T}$?)

probe tip .1cm x .3cm



(10)

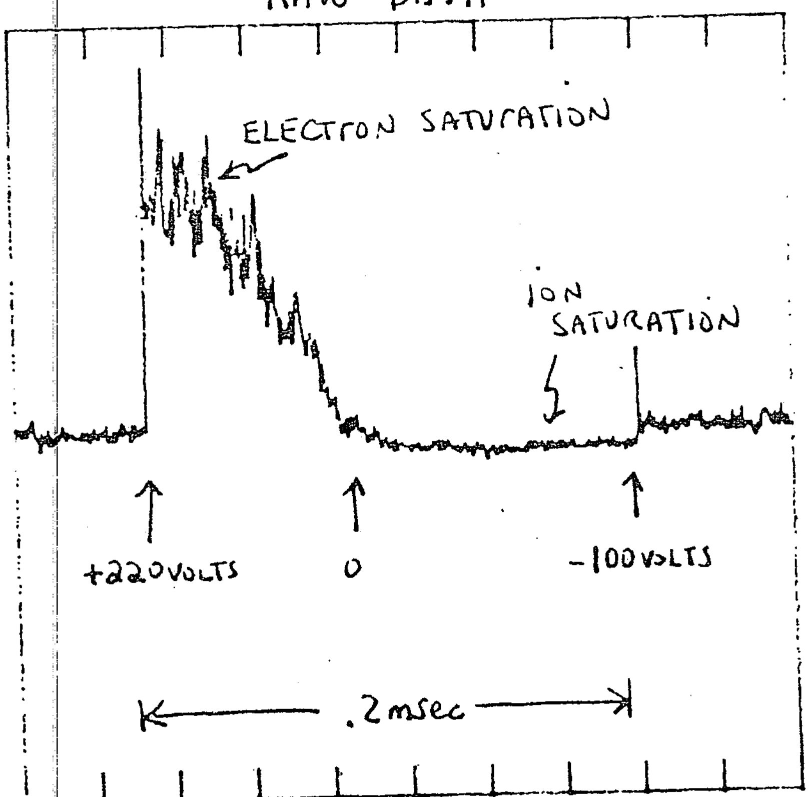
CALTECH

L14

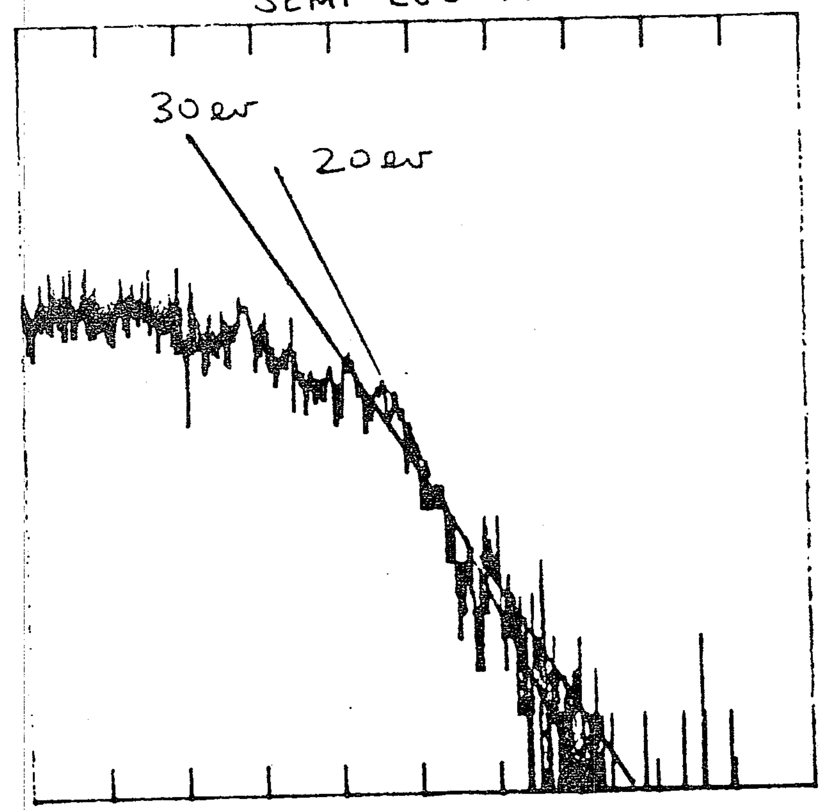
LANGMUIR PROBE SWEEP

SPACE POTENTIAL ~ +100V; $T_e \sim 25$ eV ; $e\tilde{\varphi}/kT \sim .4$

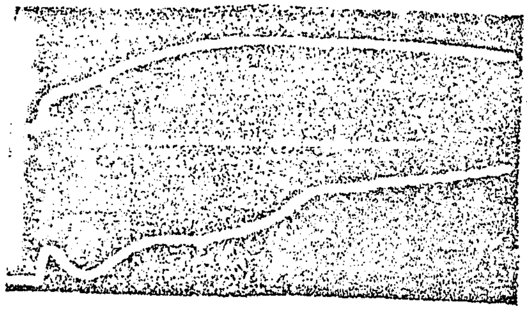
RAW DATA



SEMI-LOG PLOT

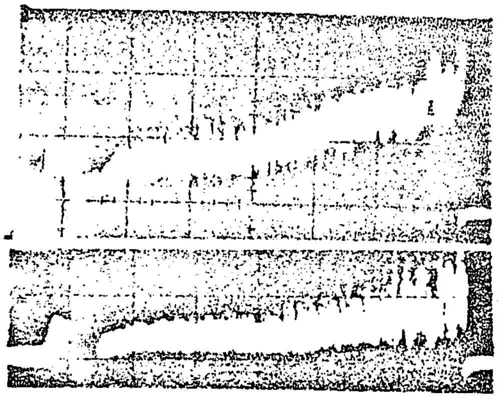


MACROTOR (NO LIMITER) 16



I
(30 KA MAX)

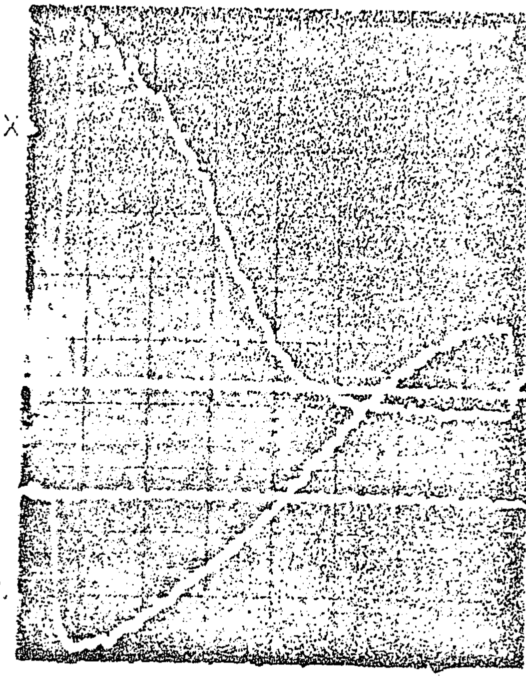
$\bar{N}_e$
(2 x 10^{13} CM^{-3} MAX)



middle
I^+_{SAT} (R=31 CM)
(.4 AMP/CM)

I^+_{SAT} (R=40CM)
edge

+120 V.



-125V
0 CURR
0 VOLT
(50V/DIV

↑ 12 MSEC/DIV'N

FLUCTUATIONS WITH NO LIMITER ≃ SAME AS THOSE IN LIMITER SHADOW.

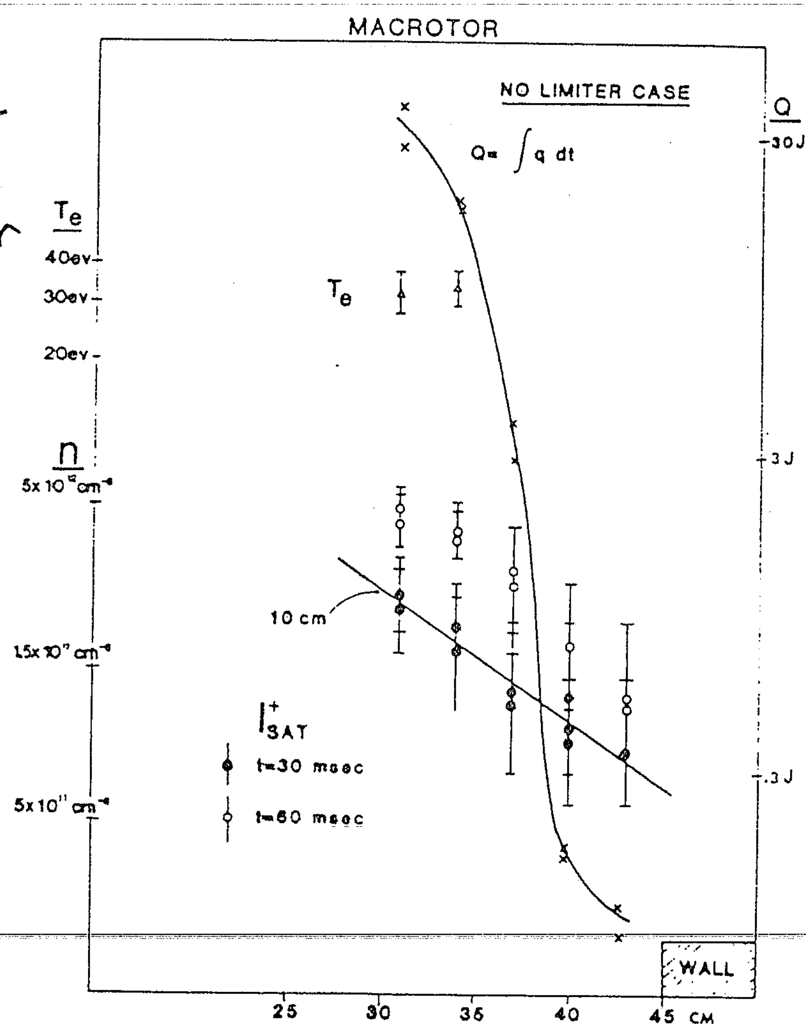


Fig. 9. $I_{SAT}{}^+$, T_e, and Q for a limiterless Macrotor discharge. The profile of $I_{SAT}{}^+$ is similar to that in the limiter cases of Figs. 4 and 5. The Langmuir probe trace at top was made at r=31 cm.

(12)

FLUCTUATION SPECTRA

$\tilde{\varphi}_F$ IN CALTECH TOKAMAK EDGE:

1) vs. TIME during shot
2) vs. Adjacent TIMES
3) vs. radius in Edge region
4) with/without gas puff (vs. n)
5) dirty/clean (vs. Z)
6) with strong MHD Mode

$\tilde{n}_e$ vs. DENSITY IN MACROTOR

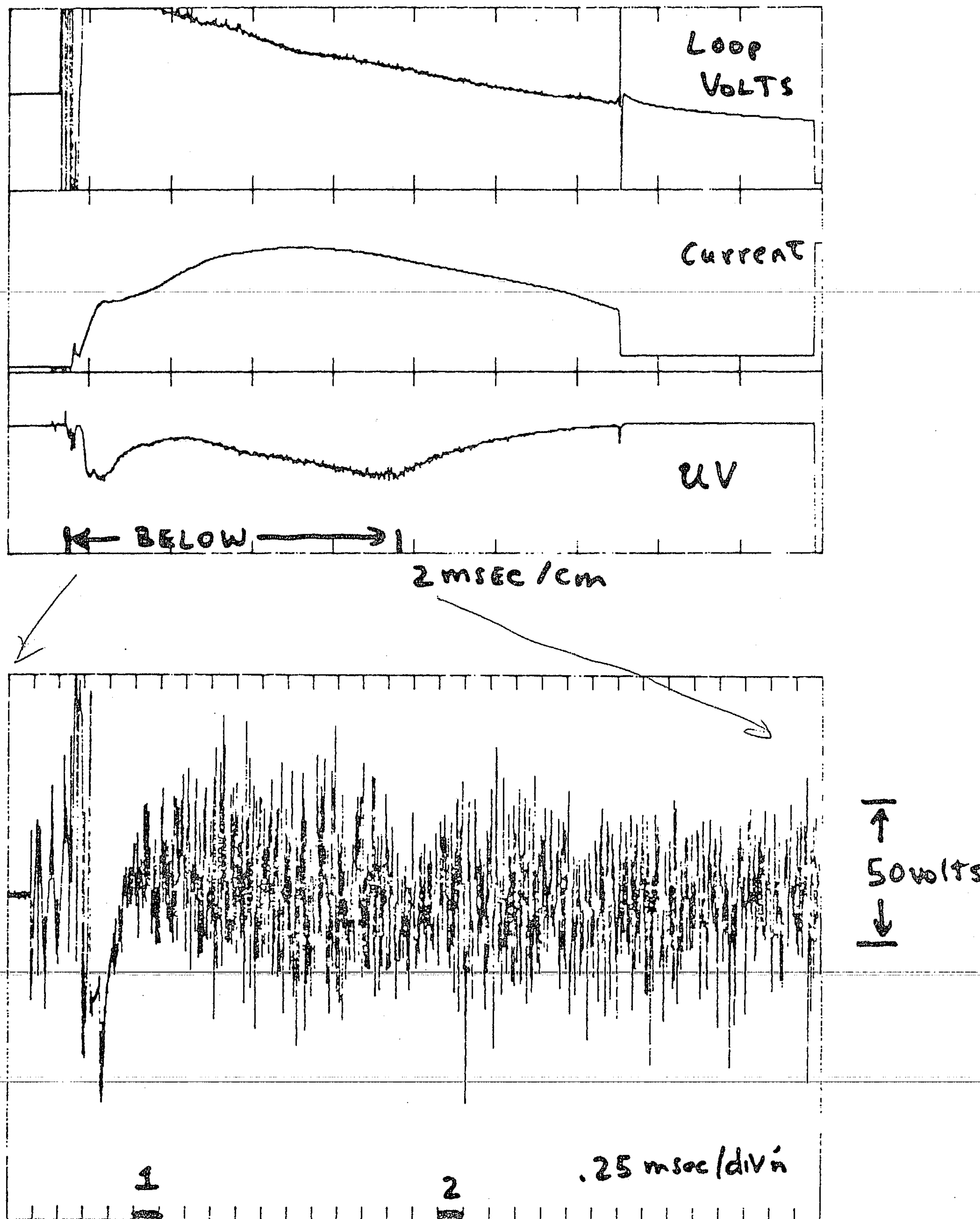
[if $\tilde{T} \cong 0$ ∴ $\tilde{\varphi}_f = \tilde{\varphi}_s$]

$\tilde{\varphi}_F$ vs. Time

[r/a = .9
CIT TOK]

(13)

EARLY TIMES

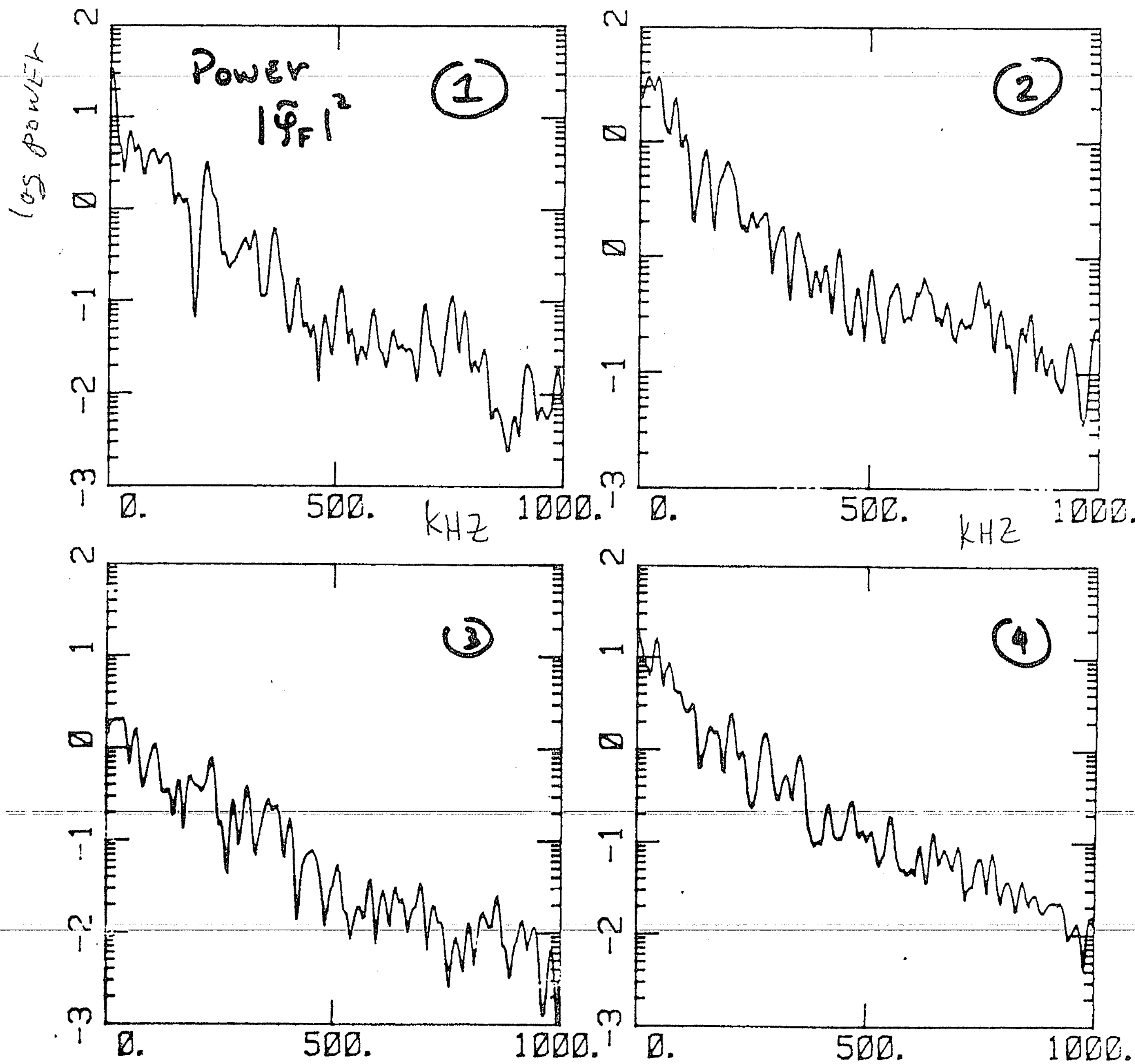


(14)

$\tilde{\varphi}_F$ SPECTRA vs. TIME

$r/a = .9$ (1.5 cm in)

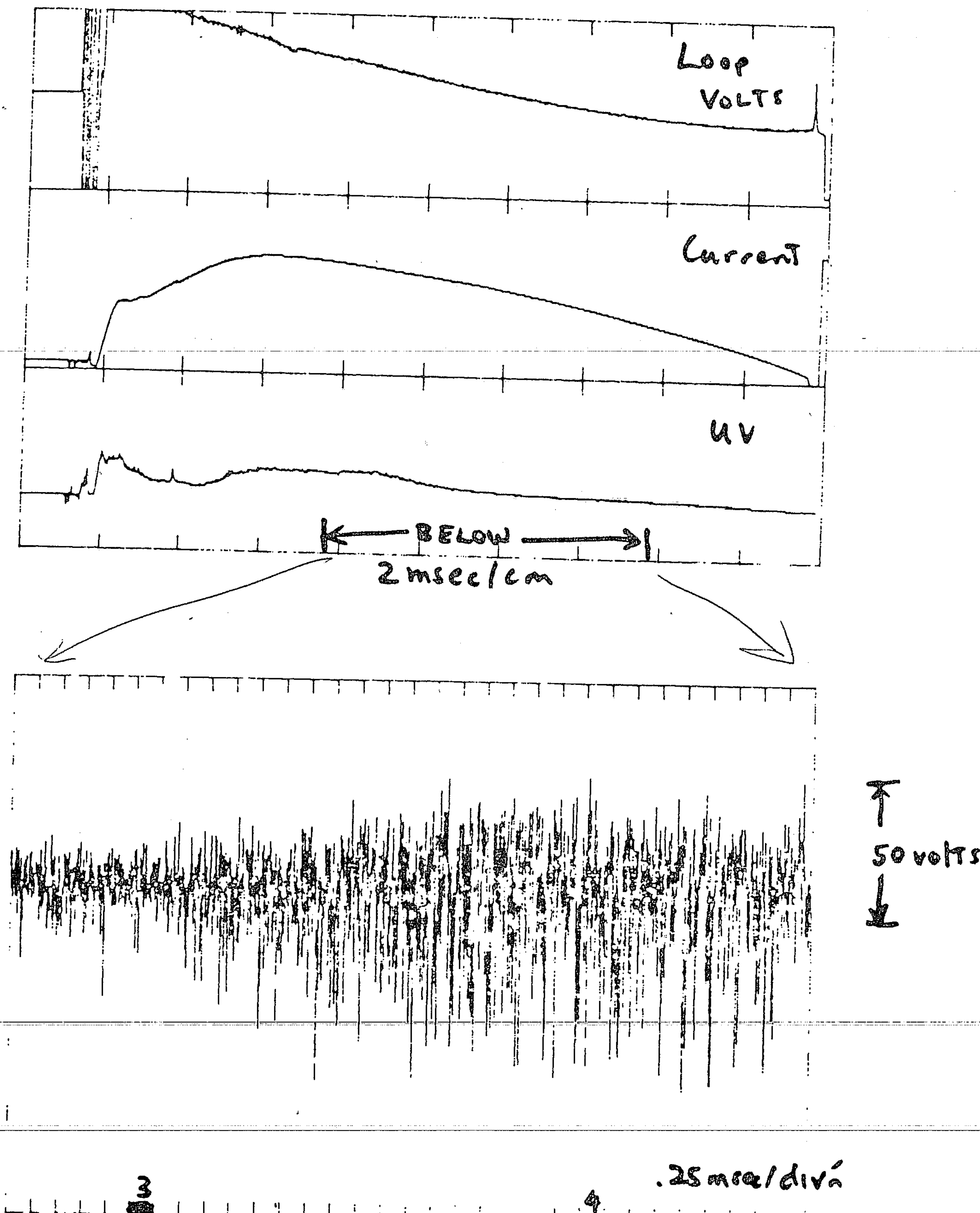
1) current rise
2) high I, low n (before puff)
3) high I, high n (puff)
4) low I, low n



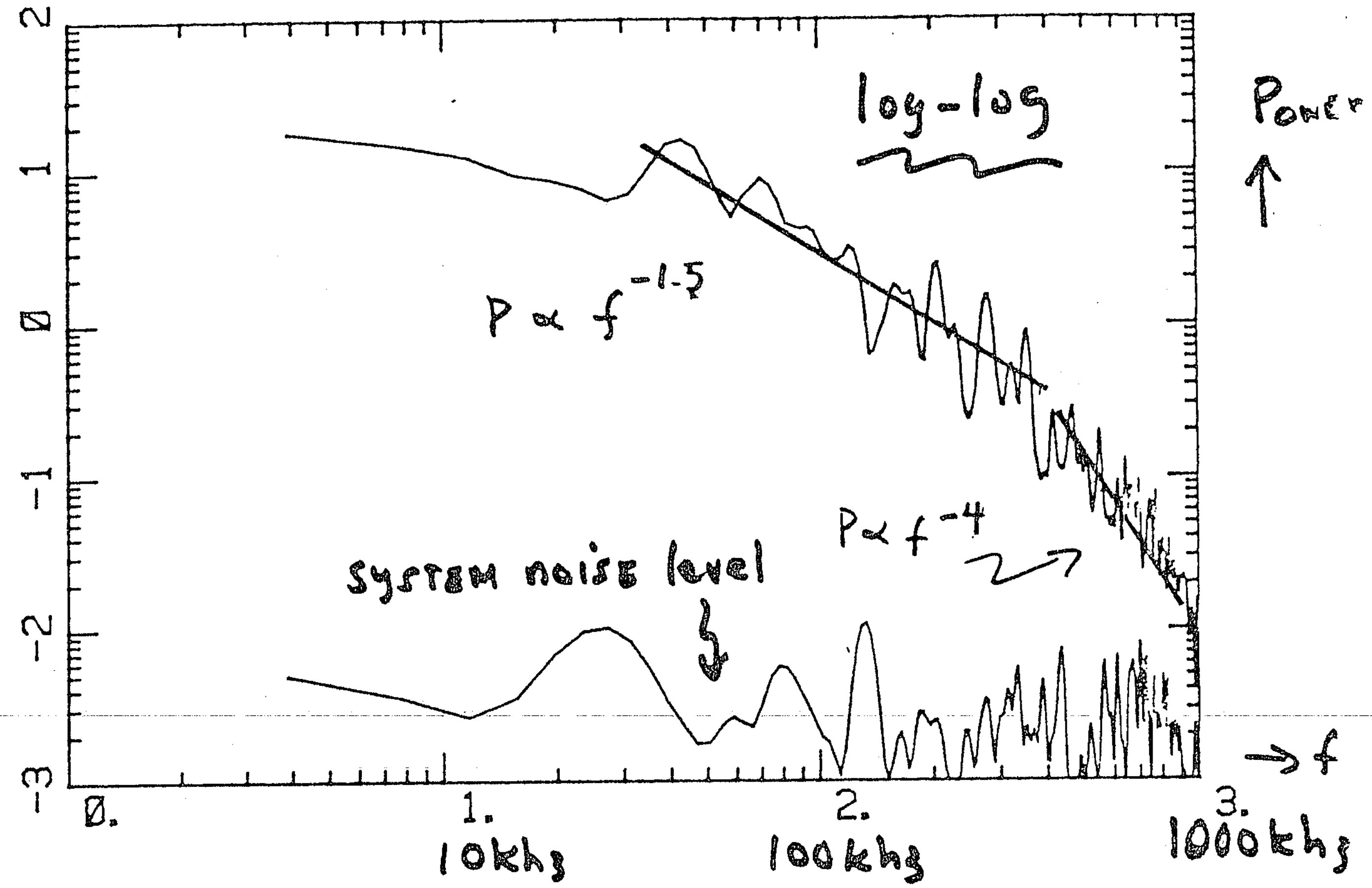
ρ_F vs. TIME

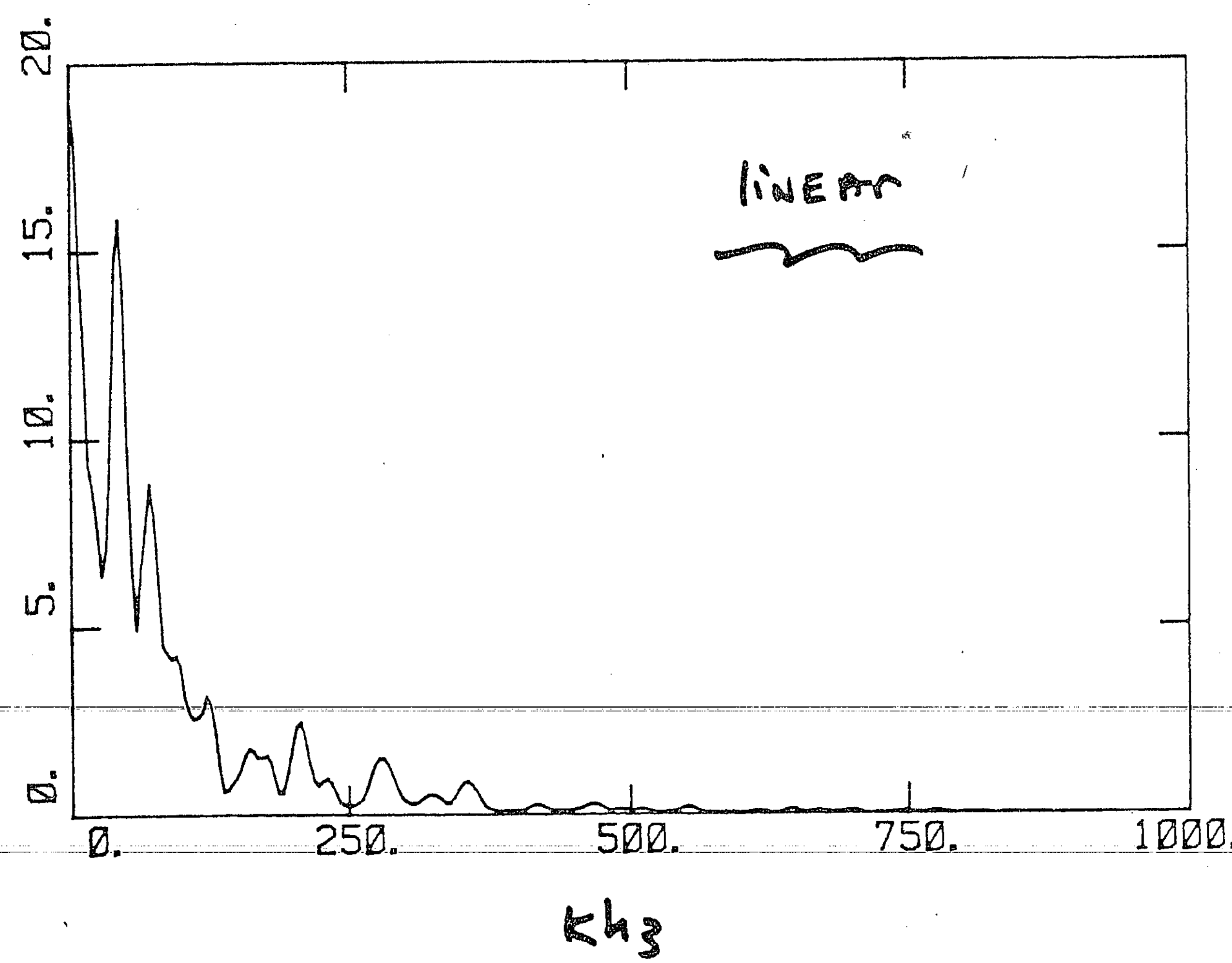
$\left[r/a = .9 \right.$ CIT TOK$\left. \right]$ (15)

LATE TIMES

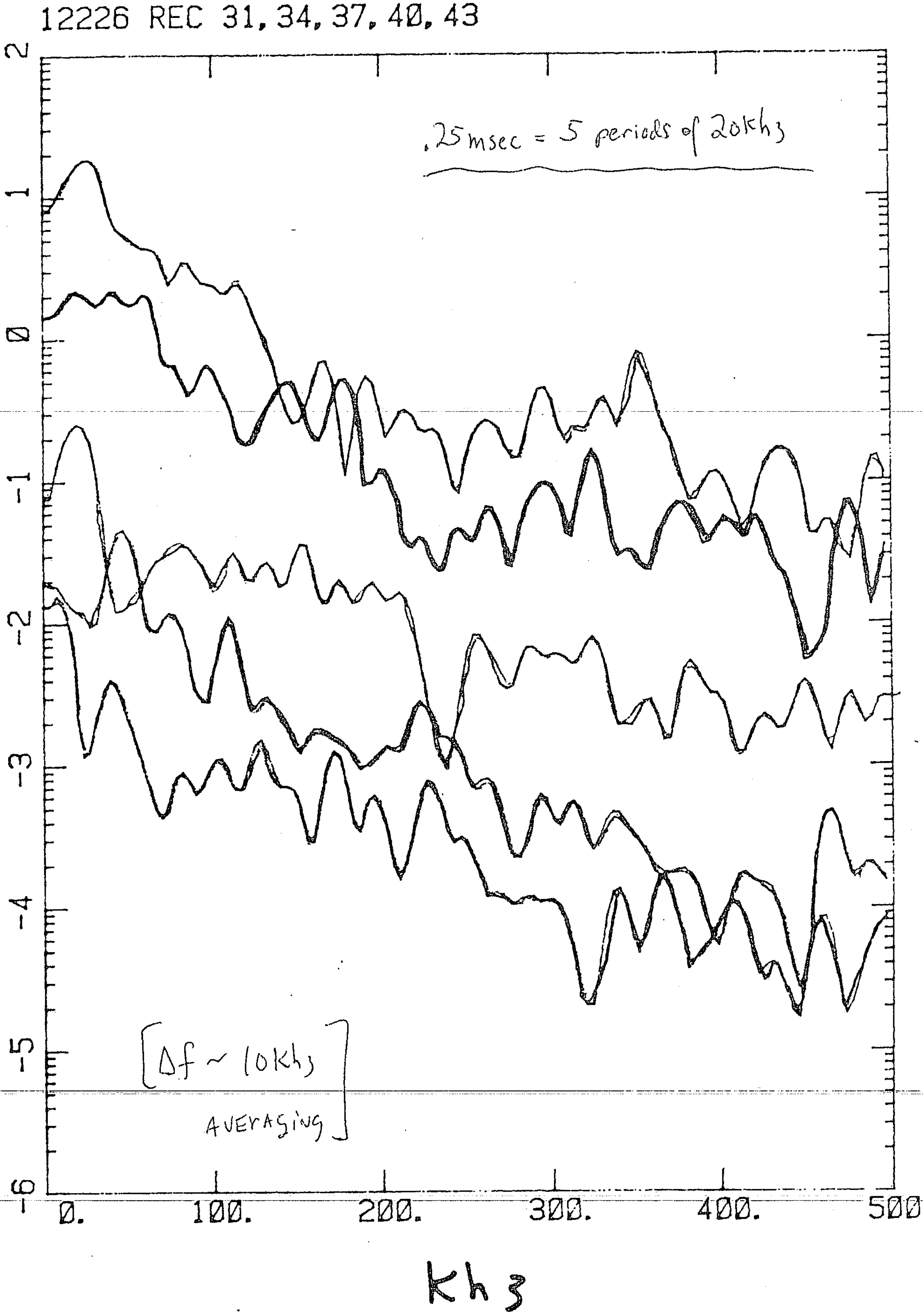


DIFFERENT PRESENTATION of (4) (16)





$|\tilde{\varphi}_F|^2$ for ADJACENT .25 msec INTERVALS (17)



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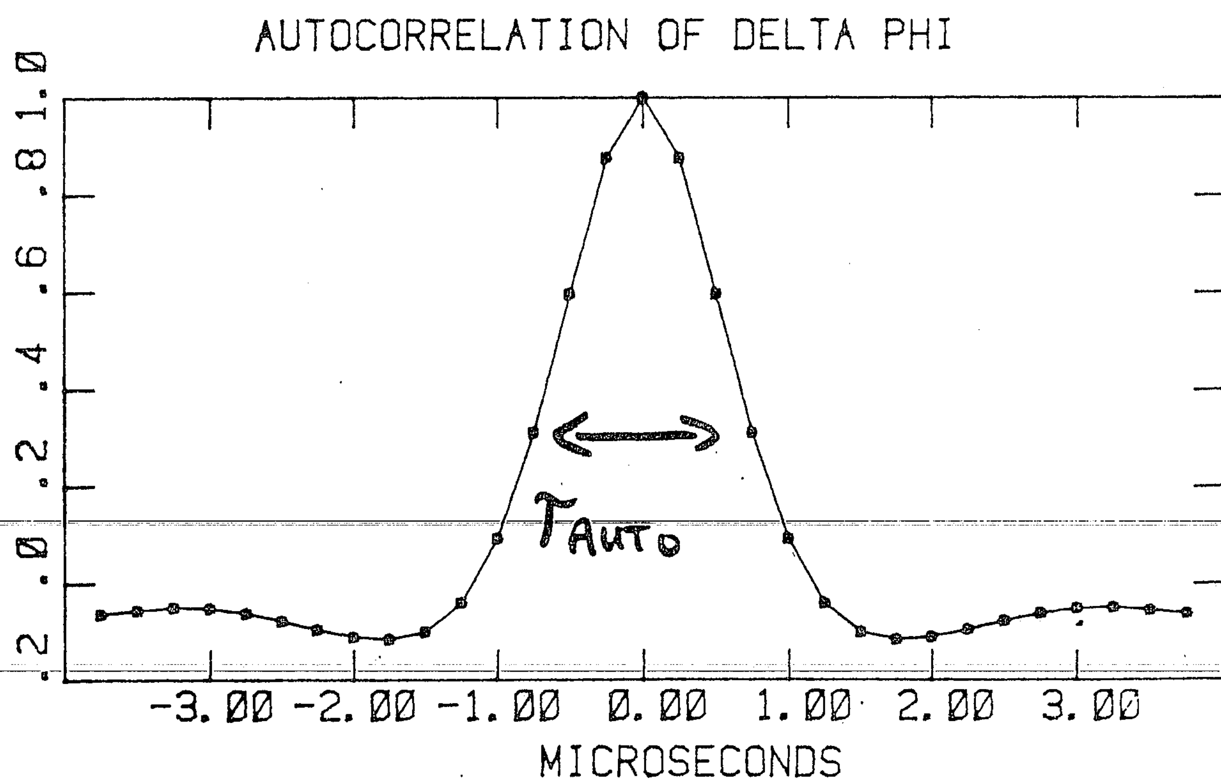
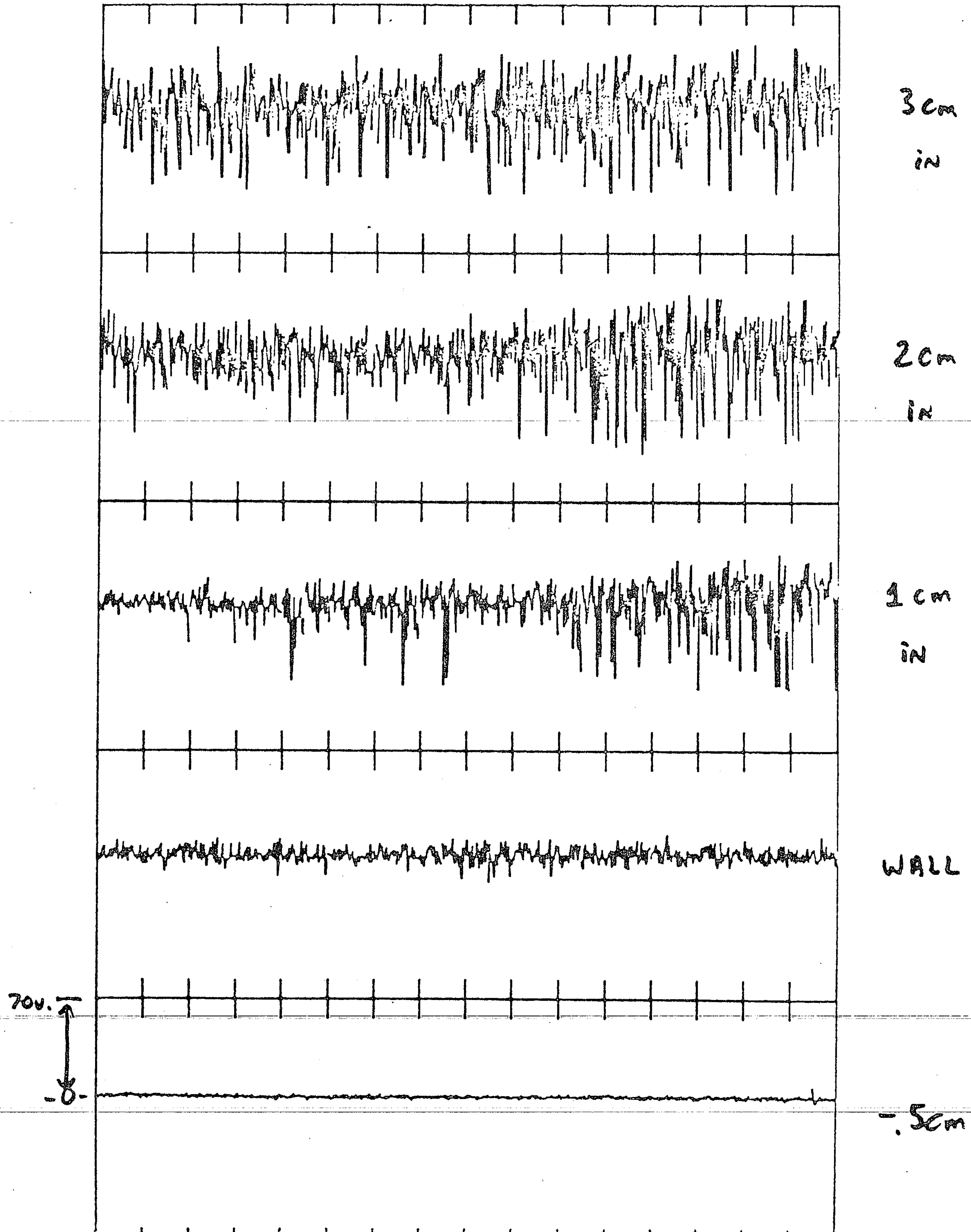
AUTOCORRELATION of $\Delta\varphi$

$$C(\tau) = \frac{\frac{1}{T}\int_0^T \Delta\tilde{\varphi}(t)\,\Delta\tilde{\varphi}(t-\tau)\,dt}{\frac{1}{T}\int_0^T (\Delta\varphi(t))^2\,dt}$$

Here $\tau_{AUTO} = C(0) \approx 2\,\mu\text{SEC}$

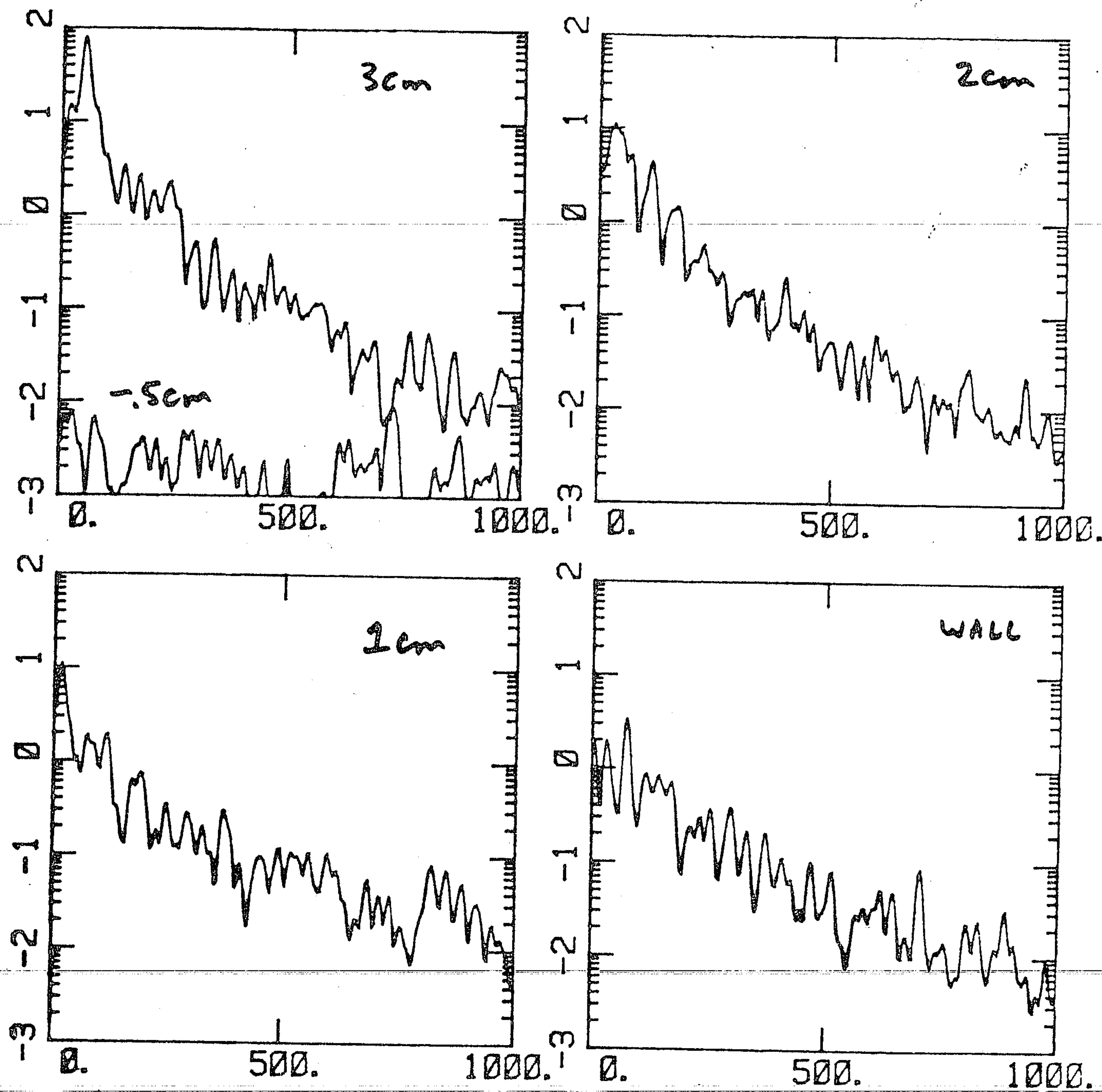


$\tilde{\varphi}_{IF}$ vs. RADIUS near Edge



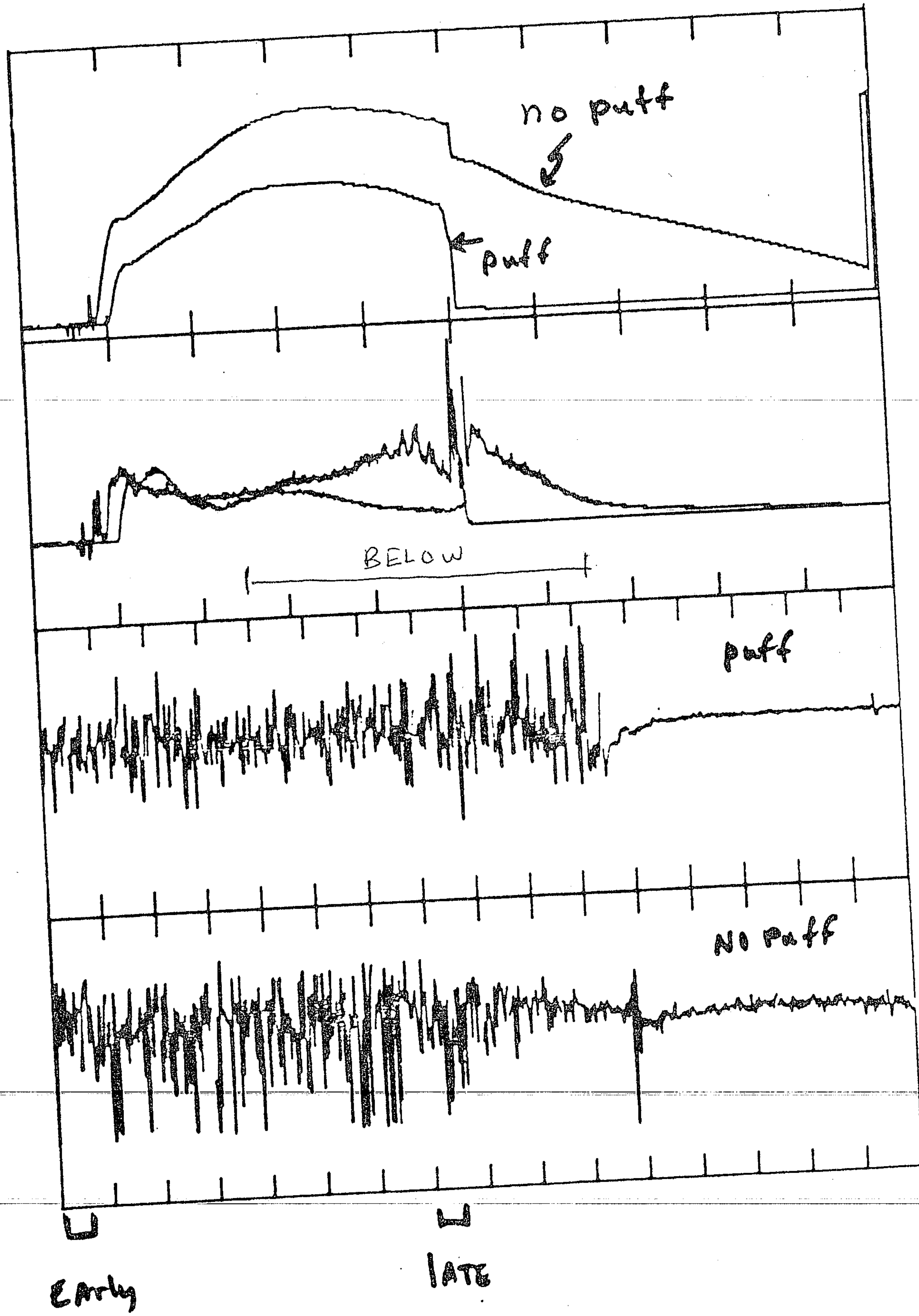
$\tilde{\varphi}_F$ SPECTRA vs. r

(20)



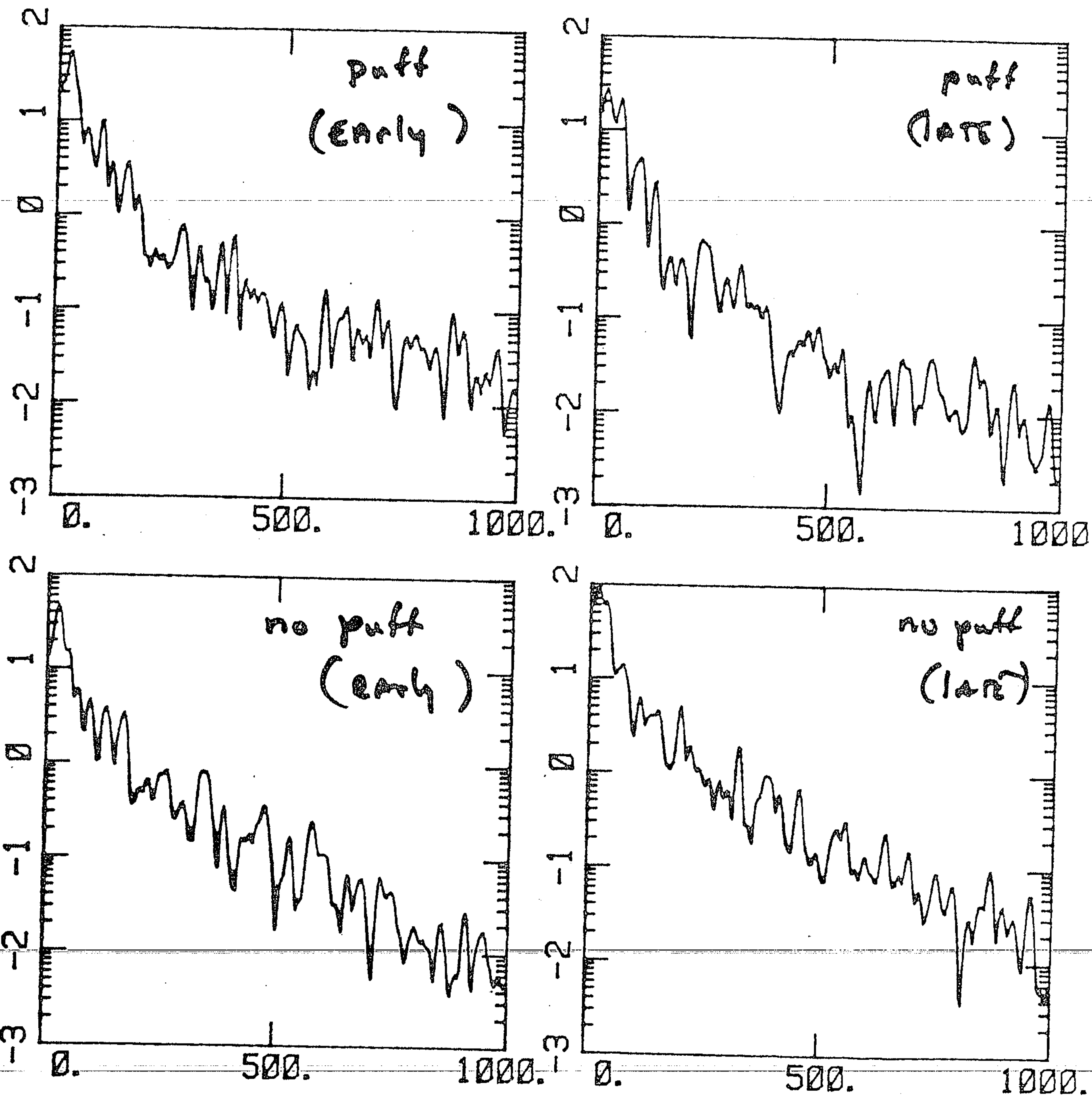
21

$\tilde{\varphi}_f$ w/wo GAS Puff



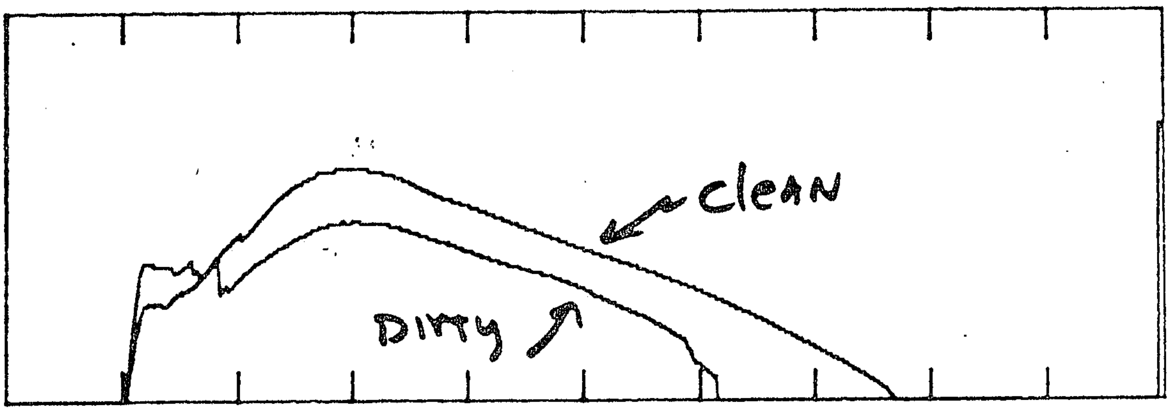
$\tilde{\varphi}_F$ Spectra w/w o Puff

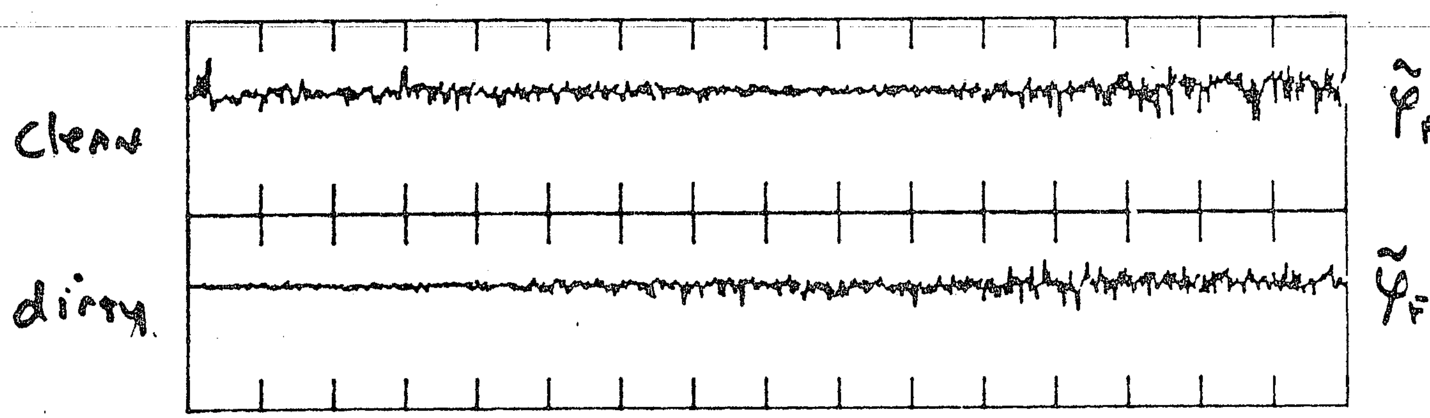
Difference in $\bar{n}_{\sim}$ ×5

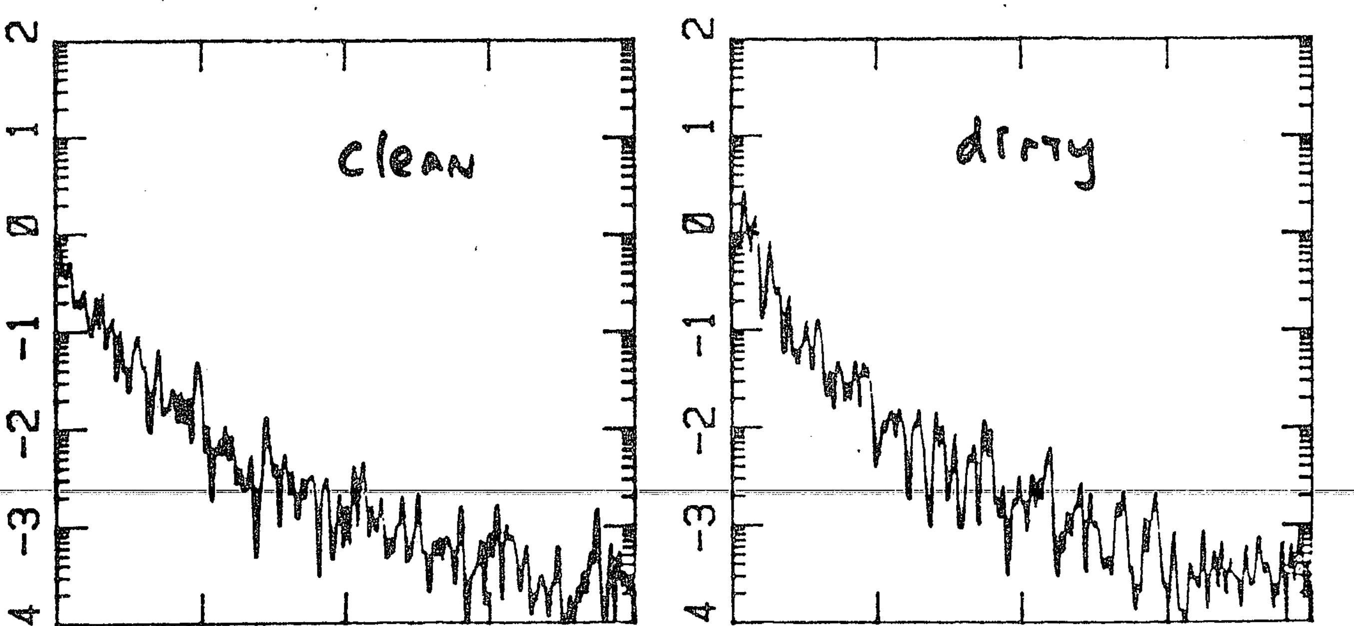


$\tilde{\varphi}_F$ dirty/clean

(23)



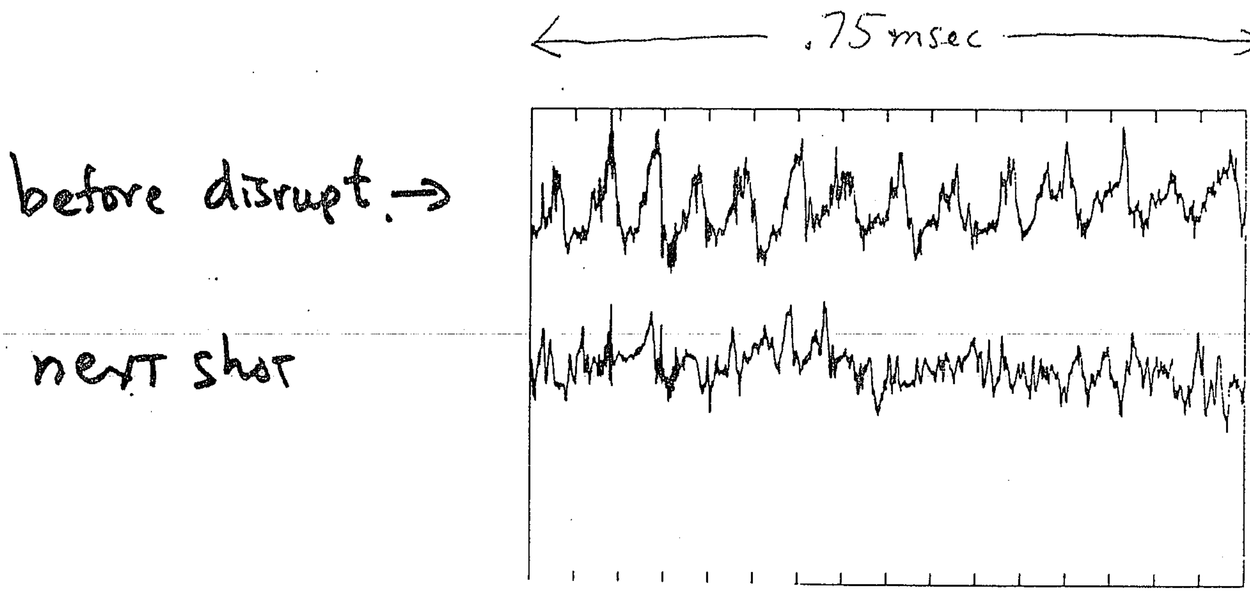


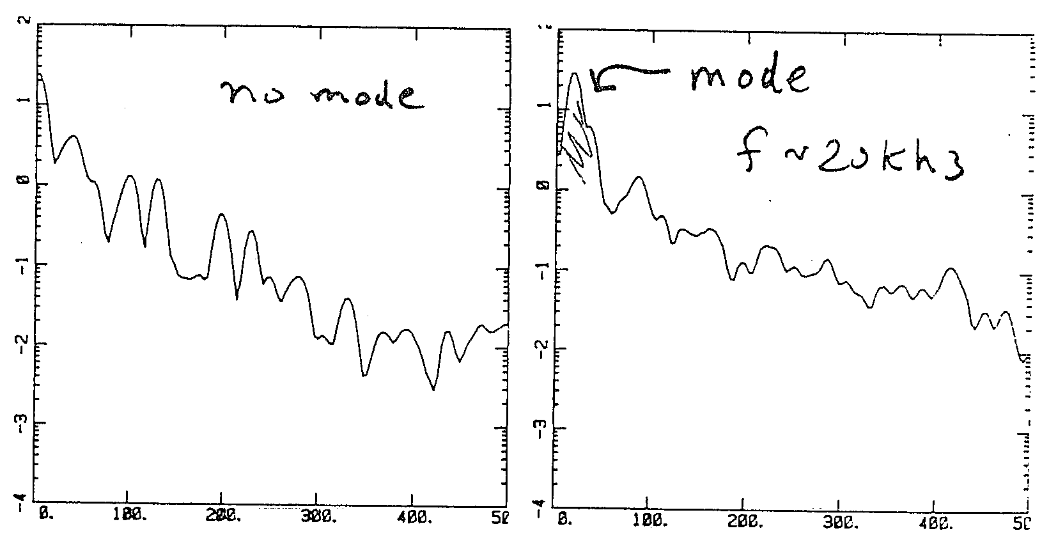


(24)

COHERENT $\tilde{\varphi}_f$ w/ MHD MODE

ONLY CASE of single mode





(25)

MACROTOR

UCLA report
PPG-~~548~~ 463

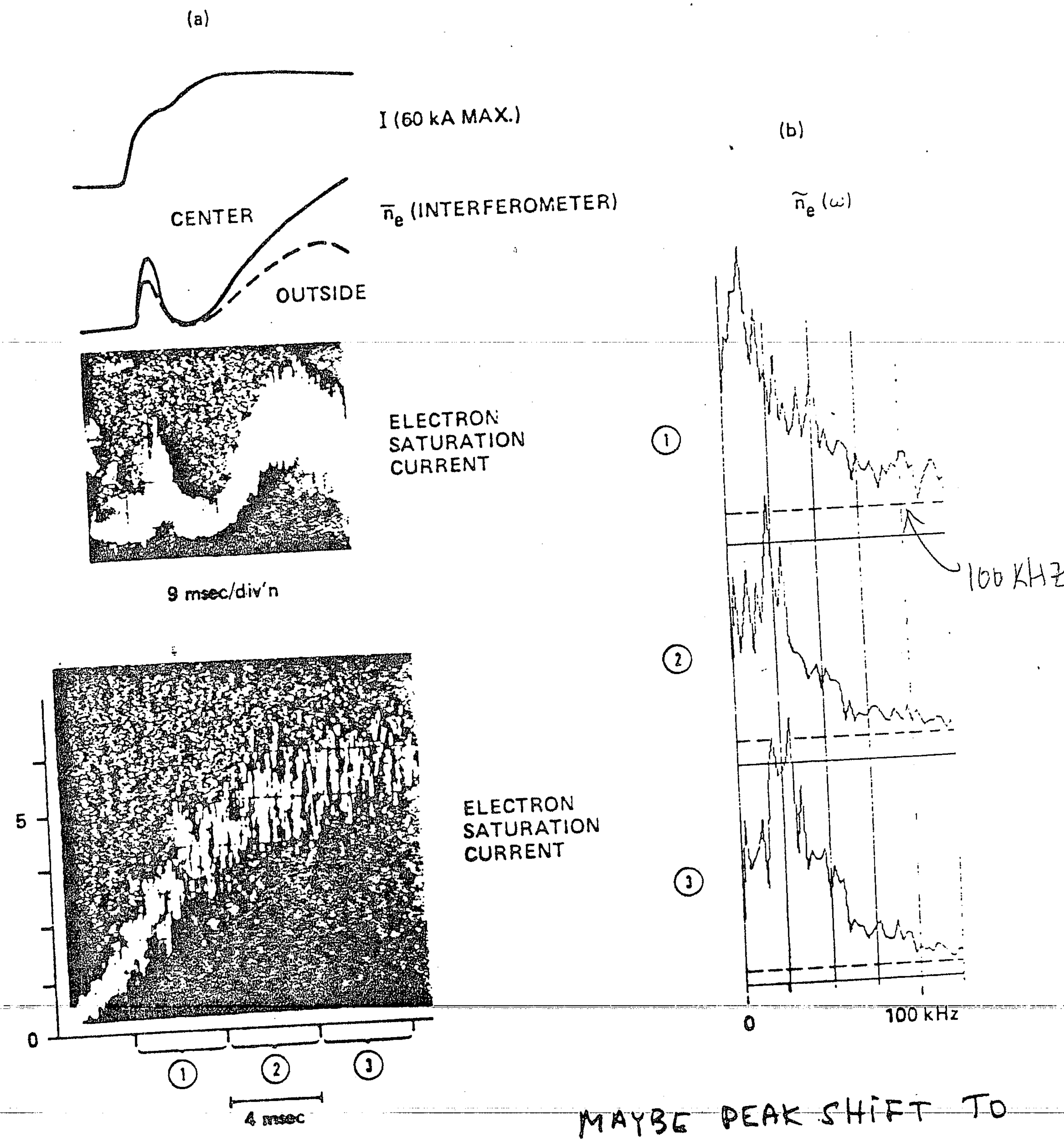


FIGURE 11.

MAYBE PEAK SHIFT TO Higher freq. w/ n

Double-Probe Measurements

- $(\tilde{\varphi}, \tilde{\varphi})$, $(\tilde{n}, \tilde{n})$ correlation lengths (UCLA)
- $\tilde{E}_p = \Delta\tilde{\varphi}_f / \Delta X_{pol}$ (assuming $\tilde{T}/T \ll \tilde{\varphi}_s/\varphi_s$)
- better $\tilde{n}$ from floating double-probe
- $(\tilde{n}, \tilde{\varphi})$ Transport (?) $\tilde{E}_r \sim \tilde{E}_p$

Triple-Probe

$(\tilde{E}_p, \tilde{n}) \rightarrow$ radial flux $\Gamma_r = \langle \tilde{n}, \tilde{v}_r \rangle$

assuming $\vec{E} \times B$ drifts

(2P/6)

Electric Field CIT

$\tilde{E}_p = \Delta\tilde{\varphi}_f / \Delta X_{pol} \lesssim$ 10 volts/cm

{ CONSISTENT with $\tilde{\varphi}_f = 10V$ and poloidal correlation length ~ 1cm

$$\tilde{v}_r(\vec{E}\times B) = 10^8 \frac{E}{B} \cong 3\times10^5 \text{ cm/sec}$$

$$v_i \simeq C_s \cong 5\times10^{5} \text{ cm/sec}$$

(5 = 6 MISTAKE IN TALK)

$$v_* \simeq 5\times10^5 \text{ cm/sec} \quad \propto \frac{T}{B}\frac{1}{\lambda_n}$$

probable EXISTENCE of $\tilde{E}_r \simeq \tilde{E}_p$ makes interpretation of $(\tilde{n}, \varphi)$ difficult (better $\tilde{n}, \tilde{E}_p$)

CONCLUSIONS

1) EDGE is "TURBULENT"

- Broad SPECTRA $\sim f^{-1.5} \rightarrow f^{-4}$
- large level $e\tilde{\varphi}/kT$ & $\tilde{n}/n \sim .1 \rightarrow .3$
- large diffusion $\sim D_{Bohm}$

2) It is not clear why This is So

what dimensionless number is large [$\sim R$?]

3) It is not clear what effect Edge conditions have on overall confinement

ANALOGOUS to fluid boundary layer turbulence?

maybe not simply $\tau \propto a^2$

Some References

FLUCTUATIONS:

old {
- Robinson & Rusbridge (ZETA) PF 14, '71, 2499
- K.M. YOUNG (Stellerator) PF 10, '67, 213
- K. BOL (Stellerator) PF 7, 64, 1855

SEMET, et.al. → P.R.L. 45 ('80) 445

ZWEBEN + TAYLOR, N.F. 21 ('81) P.193

EDGE PLASMA:

ZWEBEN + TAYLOR (UCLA) REPORT PPG-543 ('81)

UEHARA, et.al. (JFT-2) Plasma Phys. 21, '79, 89

GOMAY, etal, (JFT-2) NF 18, '78, 849

DIVA Group, NF 18, 1619

CALLEN, et.al, IAEA BRUSSELS ('80) CN-38/Y-3

OGDEN, et.al., (1-D code) IEEE Plasma Science Dec 81, 274

TFR group, EUR-CEA-FC-1059 ('80)

DOE/TIC-11600 FED REPORT IV sec 4.

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old { Robinson & Rusbridge (ZETA) PF 14, '71, 2499
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NONLINEAR GYROKINETIC EQUATIONS FOR LOW-FREQUENCY ELECTRO-MAGNETIC WAVES IN GENERAL PLASMA EQUILIBRIA

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NONLINEAR GYROKINETIC EQUATIONS FOR LOW-FREQUENCY ELECTRO-MAGNETIC WAVES IN GENERAL PLASMA EQUILIBRIA

by

E. A. Frieman and Liu Chen

* PPPL-1834 (1981). To appear in Phys. Fluids, March, 1982.

(I) MOTIVATION

(i) Linear microinstability theory depends crucially on

◎ Finite Larmor radius effects : $k_\perp \rho_i \sim 1$.

◎ Geometrical effects : magnetic trapping, magnetic drifts, shear, plasma inhomogeneities, toroidal effects, etc.

(ii) Experimental observations :
$\delta n/n \sim e\delta\phi/T \sim \rho_i/L_n \Rightarrow \omega_{nl} \sim \omega_l \sim \omega_* \Rightarrow$ strong turbulence.

(iii) A credible nonlinear formalism must retain these important features.

p.2

(II) Theoretical Approach

◎ Extension of the linear gyrokinetic formalism (Rutherford, Frieman ; Taylor, Hastie).

◎ Guiding-center transformation :

$$\underset{\sim}{X} = \underset{\sim}{x} + \underset{\sim}{v} \times \underset{\sim}{e}_{\parallel} / \Omega ,$$

$$\underset{\sim}{V} = \underset{\sim}{V} (\varepsilon, \mu, \alpha)$$

$\varepsilon = v^2/2 + q\Phi_0/m$, $\mu = V_\perp^2/2B$, $\underset{\sim}{e}_{\parallel} = \underset{\sim}{B}/B$,

α : gyrophase angle

$(\underset{\sim}{x}, \underset{\sim}{v}) \Longrightarrow (\underset{\sim}{X}, \underset{\sim}{V})$ $\qquad \underset{\sim}{p}(\underset{\sim}{x}, \underset{\sim}{v}) \Longrightarrow \hat{\underset{\sim}{p}}(\underset{\sim}{X}, \underset{\sim}{V})$

◎ Two Spatial scales :

- microscopic scale $\underline{X_{10} \sim \rho}$
- macroscopic scale L

$\lambda \equiv \rho/L \ll 1$ smallness parameter

P3

$$\hat{\underset{\sim}{p}} = \hat{\underset{\sim}{P}} + \delta\hat{\underset{\sim}{P}} \quad , \quad \hat{\underset{\sim}{P}} \equiv \langle \hat{\underset{\sim}{p}} \rangle_{\underset{\sim}{X}_{\perp 0}}$$

◎ <u>Two time scales</u>

<u>micro</u> time scale $\sim O(\omega_*^{-1})$

<u>macro</u> time scale $\sim O(\text{transport time scale})$

$$\boxed{(\text{transport time scale})^{-1} \ll \omega_* \ll \Omega}$$

(III) <u>Theoretical Analyses</u>

§1. <u>Vlasov Eqn. in $(\underset{\sim}{X}, \underset{\sim}{V})$ phase space</u>

◎ $\mathcal{L}_g \hat{f} = -(q/m)(\delta \hat{R}\, \hat{f})$

$$\begin{aligned} \circ\ \mathcal{L}_g = {} & \partial_t + \partial_t \underset{\sim}{V} \cdot \partial_{\underset{\sim}{V}} + \partial_t \underset{\sim}{X} \cdot \partial_{\underset{\sim}{X}} + V_\parallel \underset{\sim}{e}_\parallel \cdot \partial_{\underset{\sim}{X}} \\ & + \underset{\sim}{v} \cdot (\underset{\sim}{\lambda}_{B1} + \underset{\sim}{\lambda}_{B2}) + (q/m)(\underset{\sim}{E} - \underset{\sim}{E}_0) \cdot \underset{\sim}{v}\, \partial_\varepsilon \\ & + (q/m) \underset{\sim}{E} \cdot [(\underset{\sim}{v}_\perp / B)\partial_\mu + (\underset{\sim}{e}_\alpha / v_\perp)\partial_\alpha] + \underset{\sim}{V}_E \cdot \partial_{\underset{\sim}{X}} \\ & - \Omega\, \partial_\alpha \end{aligned}$$

"unperturbed" propagator

$$\circ\ \underset{\sim}{\lambda}_{B1} = \underset{\sim}{V} \times \nabla_{\underset{\sim}{X}} (\underset{\sim}{e}_\parallel / \Omega) \cdot \partial_{\underset{\sim}{X}} \ ; \ \underset{\sim}{\lambda}_{B2} = (\partial_{\underset{\sim}{X}} \mu)\partial_\mu + (\partial_{\underset{\sim}{X}} \alpha)\partial_\alpha$$

- $\delta \hat{\underset{\sim}{R}} = \delta \hat{\underset{\sim}{a}} \cdot \underset{\sim}{\partial}_V + \delta \hat{\underset{\sim}{a}} \times \underset{\sim}{e}_\parallel / \Omega \cdot \underset{\sim}{\partial}_X$, $\delta \underset{\sim}{R} = \delta \underset{\sim}{a} \cdot \underset{\sim}{\partial}_v$

$\delta \underset{\sim}{a} = \delta \underset{\sim}{E} + \underset{\sim}{v} \times \delta \underset{\sim}{B} / c$.

$\underset{\sim}{E}_0 = - \underset{\sim}{\partial}_x \Phi_0$, $\underset{\sim}{V}_E = c \underset{\sim}{E} \times \underset{\sim}{e}_\parallel / B$.

◎ <u>spatial averaging $\langle \ \rangle_{\underset{\sim}{X}_{\perp 0}}$</u>

$$\boxed{\mathcal{L}_g \hat{F} = -(q/m) \langle \delta \hat{R} \, \delta \hat{F} \rangle_{\underset{\sim}{X}_{\perp 0}}}$$ <u>Macro</u>

$$\boxed{\mathcal{L}_g \delta \hat{F} = -(q/m) [\delta \hat{R} \hat{F} + \delta \hat{R} \delta \hat{F} - \langle \delta \hat{R} \delta \hat{F} \rangle_{\underset{\sim}{X}_{\perp 0}}] .}$$ <u>Micro</u>

◎ Ordering

- <u>Macro</u>

$\underline{|\partial_t| / |\Omega| \sim O(\lambda^3)}$, $|\rho \underset{\sim}{\nabla}_X| \sim O(\lambda)$

- Micro

$|\partial_t| / |\Omega| \sim |\rho \, \partial_{X_\parallel}| \sim \underline{|\delta \hat{F} / \hat{F}| \sim |q \delta \hat{\Phi} / T| \sim |\delta \hat{B} / \hat{B}|}$

$\underline{\sim |V_E| / |V_t| \sim O(\lambda)}$; $|\rho \underset{\sim}{\partial}_{X_{\perp 0}}| \sim O(1)$

p.5

- $\underline{|\delta R\, \delta F| \sim O(\lambda^2) \sim |\partial_t \delta F|}$

  nonlinear time scale $\sim$ linear time scale

§2. Solution of $\hat{F}$

◎ $\hat{F} = \hat{F}_0 + \hat{F}_1 + \cdots$

- $\hat{F}_0 = \hat{F}_0(\varepsilon, \mu, \underset{\sim}{X}_\perp) \qquad \partial_{X_\parallel} \hat{F}_0 = 0$

- $\tilde{F}_1 = (\tilde{\beta}/B)(\partial \hat{F}_0 / \partial \mu)$

$$\tilde{\beta} = -\left\{ \underset{\sim}{v}_\perp \cdot \underset{\sim}{v}_D + \int^{\alpha} (d\alpha'/\Omega) \left[v_\parallel (\underset{\sim}{v}_\perp \cdot \underset{\sim}{\partial}_X \underset{\sim}{e}_\parallel \cdot \underset{\sim}{v}_\perp) - v_\perp^2 \underset{\sim}{\partial}_X \cdot \underset{\sim}{e}_\parallel / 2 \right] \right\}$$

- $\underset{\sim}{v}_D = \underset{\sim}{v}_d + \underset{\sim}{v}_{E_0}$

$$\underset{\sim}{v}_d = \underset{\sim}{e}_\parallel \times \left[(v_\perp^2/2)\, \underset{\sim}{\partial}_x \ln B + v_\parallel^2\, \underset{\sim}{e}_\parallel \cdot \underset{\sim}{\nabla}\, \underset{\sim}{e}_\parallel \right] / \Omega$$

For our purpose, $\hat{F}_0$ and $\tilde{F}_1$ are sufficient.

§3. Nonlinear gyrokinetic equation

◎ To $O(\lambda^2)$

$$\mathcal{L}_g \delta\hat{F} \simeq \mathcal{L}_{gf}\delta\hat{F} = -(q/m)[\delta\hat{R}\hat{F} + \delta\hat{R}\delta\hat{F} - \langle\delta\hat{R}\delta\hat{F}\rangle_{X_{10}}]$$

$$O(\lambda) \qquad\qquad O(\lambda) \quad O(\lambda^2) \quad O(\lambda^2)$$

$$\mathcal{L}_{gf} = \partial_t + v_\parallel \partial_{X_\parallel} + \underset{\sim}{v}\cdot(\underset{\sim}{\lambda}_{B1} + \underset{\sim}{\lambda}_{B2})$$

$$+ (q/m)\underset{\sim}{E}_0\cdot[(\underset{\sim}{v}_\perp/B)\partial_\mu + (\underset{\sim}{e}_\alpha/v_\perp)\partial_\alpha]$$

$$+ \underset{\sim}{V}_{E_0}\cdot\underset{\sim}{\partial}_X - \Omega\,\partial_\alpha$$

$O(1)$ the rest is $O(\lambda)$

◎

$$\boxed{\delta\hat{F} = (q/m)[\delta\hat{\Phi}\,\partial_\varepsilon + (\delta\hat{\Phi} - v_\parallel\delta\hat{A}_\parallel/c)\frac{1}{B}\partial_\mu]\hat{F}_0 + \delta\hat{G}}$$

To remove $O(\lambda)$ term in the R.H.S.

◎ $\delta\hat{G} = \delta\hat{G}_0 + \delta\hat{G}_1 + \cdots$

∘ $\underline{\partial_\alpha \delta\hat{G}_0 = 0}$

- $\langle \mathcal{L}_{gf} \rangle_\alpha \, \delta \hat{G}_0 = -(q/m) \langle R_l + R_{nl} \rangle_\alpha$

$\langle \mathcal{L}_{gf} \rangle_\alpha = \partial_t + v_\parallel \partial_{\bar{X}_\parallel} + \underset{\sim}{V}_D \cdot \underset{\sim}{\partial}_{\bar{X}_{\perp 0}}$

$R_{nl} = \delta \hat{R} \delta \hat{F} - \langle \delta \hat{R} \delta \hat{F} \rangle_{\underset{\sim}{X}_{\perp 0}}$

R_l : usual linear term

◎

- $\delta \hat{G}_0 = -(q/m) \langle \delta \hat{L} \rangle_\alpha \frac{1}{B} \partial_\mu \hat{F}_0 + \delta \hat{H}_0$

$\delta \hat{L} = \delta \hat{\Phi} - \underset{\sim}{v} \cdot \delta \hat{\underset{\sim}{A}} / c$

- WKB ansatz

$$\Rightarrow [\partial_t + v_\parallel \underset{\sim}{e}_\parallel \cdot \underset{\sim}{\partial}_{\bar{X}} + i \underset{\sim}{k}_\perp \cdot \underset{\sim}{V}_D] \, \delta \bar{H}_0(k_\perp)$$
$$+ \frac{c}{B} \sum_{k_\perp' + k_\perp'' = k_\perp} [\underset{\sim}{e}_\parallel \cdot (\underset{\sim}{k}_\perp' \times \underset{\sim}{k}_\perp'')] \langle \delta \bar{L} \rangle_\alpha (k_\perp') \, \delta \bar{H}_0 (k_\perp'')$$
$$= -(q/m) S_l (k_\perp)$$

- $\langle \delta \bar{L} \rangle_\alpha = (\delta \bar{\Phi} - v_\parallel \delta \bar{A}_\parallel / c) J_0(\gamma) + v_\perp J_1 \delta \bar{b}_\parallel / k_\perp c$

$\gamma = k_\perp v_\perp / \Omega$

- $S_l = (\partial_t \partial_\varepsilon \hat{F}_0 + i \underset{\sim}{e}_\parallel \times \underset{\sim}{k}_\perp / \Omega \cdot \underset{\sim}{\partial}_{\bar{X}} \hat{F}_0) \langle \delta \bar{L} \rangle_\alpha$

∘ Effective $\delta \underset{\sim}{E} \times \underset{\sim}{B}$ coupling : <u>nonlinear polarization drift</u> exhibits via <u>FLR</u> effects.

(IV) <u>Axisymmetric Tokamaks</u>

(i) Ballooning-mode representation

◎ $\underset{\sim}{B} = \underset{\sim}{\nabla}\zeta \times \underset{\sim}{\nabla}\psi + I(\psi,\chi)\underset{\sim}{\nabla}\zeta$

ψ (poloidal flux), ζ (toroidal angle), χ (poloidal)

◎ $\hat{F}_0 = \hat{F}_0(\psi, \mu, \varepsilon)$

◎ $\delta H_0 = \sum_n \sum_m \delta h_{n,m} \exp[i(n\zeta - m\chi + n\int^{\psi} k\, d\psi]$

∘ $\delta h_{n,m}(\psi, \underset{\sim}{v}) = \int d\hat{\theta}_n\, \delta \bar{h}_n(\psi, \hat{\theta}_n, \underset{\sim}{v})\, g_{n,m}(\hat{\theta}_n)$

∘ $g_{n,m} = \exp[i(m\hat{\theta}_n - n\int^{\hat{\theta}_n} \nu\, d\hat{\theta}]$

∘ $\nu = IJ/R^2$, $J^{-1} = \underset{\sim}{\nabla}\psi \times \underset{\sim}{\nabla}\zeta \cdot \underset{\sim}{\nabla}\chi$

(ii) nonlinear gyrokinetic equation

◎ $\left[\partial_t + \frac{v_\parallel}{JB}\partial_{\hat{\theta}_n} + i\,\underset{\sim}{k}_\perp \cdot \underset{\sim}{v}_D\right]\delta \bar{h}_n = -(q/m)\left[\bar{S}_{gn} + (\bar{R}_{ng})_n\right]$

◎ $\underset{\sim}{k}_\perp = \frac{nB}{RB_\chi}\,\underset{\sim}{e}_\parallel \times \underset{\sim}{e}_\psi - nRB_\chi\left[\int_0^{\hat{\theta}_n} \nu'_\psi\, d\hat{\theta} - k\right]\underset{\sim}{e}_\psi$

◎ $$(\bar{R}_{ng})_n = -\pi \sum_{\substack{n'+n'' \\ = n}} W_{n',n''} \sum_{p=-\infty}^{\infty} \exp(-i n'' 2\pi p Q)$$

$$C_{n',n''} \int d\hat{\theta}_{n'}\, \delta(\hat{\theta}_n - \hat{\theta}_{n'}) \int d\hat{\theta}_{n''}\, \delta(\hat{\theta}_{n''} - \hat{\theta}_n - 2\pi p)$$

$$\cdot\,(\delta \bar{L}_{n'}\, \delta \bar{h}_{n''} - \delta \bar{L}_{n''}\, \delta \bar{h}_{n'})$$

- $W_{n',n''} = \exp\left[i\int^{\psi} (n'k' + n''k'' - nk)\, d\psi\right]$
- $C_{n',n''} = (Bn'n''/\Omega)\left[2\pi p Q'_\psi + k' - k''\right]$
- $Q = \oint \nu\, d\hat{\theta}$

◎ Maximum coupling for moderately ballooning modes.

(iii) Electrostatic Drift Waves

◎ Model :

- adiabatic electrons , cold fluid ions
- concentric, circular magnetic surfaces
- ignore radial global wavenumber

$$k = k' = k'' = 0$$

◎ Tokamak version of Hasegawa-Mima Eqn.

$$\left[(1+k_\perp^2 \rho_s^2)(\partial_t) + i\omega_{*e} - i\bar{\omega}_{de} T_c(\hat{\theta})\right]\delta\psi(n,\hat{\theta})$$

$$= \pi \Omega_i \hat{s} \sum_{n'+n''=n} \sum_p e^{-in''2\pi p Q} \rho_s^4 (k_{n'} k_{n''} 2\pi p)$$

$$\cdot\left[k_\perp''^2 - k_\perp'^2\right] \delta\psi(n',\hat{\theta})\,\delta\psi(n'',\hat{\theta}+2\pi p)$$

- $T_c = \cos\hat{\theta} + \hat{s}\,\hat{\theta}\sin\hat{\theta}$, $\hat{s} = rQ'/Q$
- $k_n = nQ/r$, $k_\perp'^2 = k_{n'}^2 (1+\hat{s}^2\hat{\theta}^2)$

$$k_\perp''^2 = k_{n''}^2 \left[1 + \hat{s}^2(\hat{\theta}+2\pi p)^2\right]$$

- conserve energy + enstrophy.

(V) Other Applications

(i) Multiple-scale gyrokinetic particle simulation in general geometries.

(ii) Nonlinear theory for kinetic drift-tearing or MHD ballooning modes.

NONLINEAR ELECTRON RESPONSE TO DRIFT WAVE FLUCTUATIONS IN TOROIDAL GEOMETRY

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1. STUDY NONLINEAR MODIFICATIONS OF TRAPPED ELECTRON ELECTROSTATIC DRIFT WAVE (CATTO-TSANG 1978) SIMPLIFY TO $\eta_e = 0$.

2. RESONANCE: BANANA CENTER DRIFT

$$\frac{\omega}{k_T} = v_D$$

$$\omega = \omega(k_\theta) \; ; \; k_T = n/R \; ; \; v_D = \varepsilon (\rho/r)(v_\parallel/v_e)^2 v_e$$

3. HELICITY OF Φ_{mn} WAVEFRONT $\neq$ FIELDLINE (PARTICLE) HELICITY $\Rightarrow$ RESPONSE RADIAL LOCALIZATION $\Delta = (nq_b')^{-1}$

4. TIME SCALES: $\left.\begin{array}{l} \omega \ll \omega_b \ll \Omega \\ (\omega_b \tau_c^r) > 1 \end{array}\right\} \Rightarrow$

GYRO-, BOUNCE- AVERAGED, DIA RESPONSE

5. <u>CONCLUDE</u>: RADIAL AND POLOIDAL DIFFUSION AND OTHER NONLINEARITIES CAN BE TREATED AS PERTURBATIONS OF LINEAR TRAPPED ELECTRON EFFECTS

ACTION-ANGLE VARIABLES

$$(\underline{J}, \underline{\Theta}) = (M, J, p\,; \Theta_g, \Theta, \varphi) \quad \text{(KAUFMAN 1972)}$$

$M = \frac{m^2 c}{e}\mu \propto$ MAGNETIC MOMENT

$J = \frac{e}{c}\oint \frac{d\beta}{2\pi}\, \alpha(\beta; H_0, p, M) \propto$ TOROIDAL FLUX

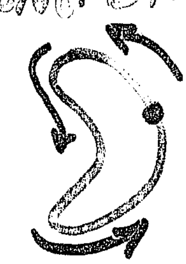
$p = m\dot{\varphi}R^2 - \frac{e}{c}\psi \propto$ TOROIDAL ANGULAR MOMENTUM

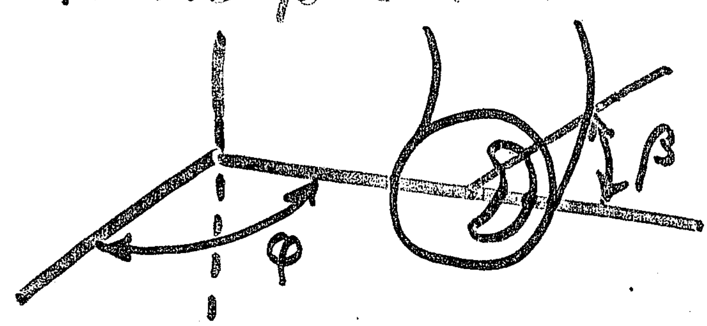
BANANA TIP: $\dot{\varphi} = 0$; RADIUS $\alpha_0 \doteq \alpha_0(p)$

$\Theta_g \doteq$ BOUNCE AVERAGED GYROANGLE

Θ = "POLOIDAL" VARIABLE: TIME ALONG BANANA

φ = TOROIDAL ANGLE WHERE BANANA INTERSECTS $\beta = 0$ PLANE





FROM HAMILTONIAN: VLASOV EQUATION

$$\boxed{\frac{\partial f}{\partial t} + \frac{\partial}{\partial \underline{\Theta}}\cdot(\dot{\underline{\Theta}} f) + \frac{\partial}{\partial \underline{J}}\cdot(\dot{\underline{J}} f) = 0}$$

TIMESCALES

$$\underline{\omega}_0 = \frac{\partial H_0}{\partial \underline{J}} = (\Omega, \omega_b, \omega_D)$$

Ω = BOUNCE AVERAGED GYROFREQUENCY

ω_b = BOUNCE FREQUENCY

ω_D = TOROIDAL DRIFT FREQUENCY

1. **ORDERING**: $\left.\begin{array}{r} \omega_D \ll \omega_b \ll \Omega \\ \omega \ll \omega_b \end{array}\right\} \Longrightarrow$

COMPUTE GYRO- AND BOUNCE-AVERAGED RESPONSE

2. BOUNCE AVERAGED EFFECTS NOT DESTROYED BY RADIAL DIFFUSION - SHEAR DECORRELATION

$$\tau_c^r = \left[2D^r (k_{\parallel}' v_e)^2\right]^{-1/3} \quad \text{(HIRSHMAN-MOLVIG 1979)}$$

$$(\omega_b \tau_c^r) = \left(\frac{M}{m}\right)^{1/2} \left(\frac{\pi}{q\sqrt{2}\,K(k)}\right) \epsilon^{3/2} (\omega \tau_c^r) > 1$$

TRUE FOR MOST VALUES OF TRAPPING PARAMETER: $0 < K < 1$

AVERAGED, NONLINEAR VLASOV EQUATION

FOURIER TRANSFORM VLASOV EQUATION:

$\underline{\ell} = (\ell, m, n)$ = HARMONICS IN $\underline{\theta}$

$$-i(\omega - \underline{\ell}\cdot\underline{\omega}_0) f_{\underline{\ell}\omega} + N_{\underline{\ell}} = i\underline{\ell} H_{\underline{\ell}} \cdot \frac{\partial f_0}{\partial \underline{J}}$$

↑ NONLINEAR TERM ($N_{\underline{\ell}}$)

HAMILTONIAN

$$H(\underline{J}, \underline{\theta}) = H_0(\underline{J}) + \sum_{\underline{\ell} \neq \underline{0}}{}' e^{i\underline{\ell}\cdot\underline{\theta}} H_{\underline{\ell}}(\underline{J})$$

$H_0(\underline{J})$: UNPERTURBED MOTION

$H_{\underline{\ell}} = -e\Phi_{\underline{\ell}}$

GYRO & BOUNCE AVERAGES: $\underline{\ell} \rightarrow (0, 0, n)$

$\ell = 0 \equiv$ AVERAGE OVER θ_g

$m = 0 \equiv$ AVERAGE OVER θ

n = HARMONIC OF TOROIDAL DRIFT MOTION

NONLINEAR TERM: EXTRACT COHERENT PIECE $\propto f_{\underline{\ell}}, H_{\underline{\ell}}$

$$N_{\underline{\ell}} = \underbrace{C_{\underline{\ell}} f_{\underline{\ell}} + D_{\underline{\ell}} H_{\underline{\ell}}} + \underbrace{Z_{\underline{\ell}}}_{\text{INCOHERENT}}$$

COHERENT PIECE $\Rightarrow$ NL DIELECTRIC $\epsilon(\Phi_{\underline{\ell}})$

DIA PROCEDURE

$$N_{\underline{\ell}} = i \sum_{\substack{\underline{\ell}',\omega' \\ \underline{\ell}' \neq 0, \underline{\ell}}} \left[\underline{\ell} \cdot \underline{\omega}_{\underline{\ell}'} f_{\underline{\ell}-\underline{\ell}'} - \underline{\ell}' \cdot \frac{\partial}{\partial \underline{J}} H_{\underline{\ell}'} f_{\underline{\ell}-\underline{\ell}'} \right]$$

$$\left(\omega_{\underline{\ell}} \equiv \frac{\partial H_{\underline{\ell}}}{\partial \underline{J}} \right)$$

NOTE

$$f_{\underline{\ell}-\underline{\ell}'} = G_{\underline{\ell}-\underline{\ell}'} Z_{\underline{\ell}-\underline{\ell}'} + \cdots$$

GREEN FUNCTION: $G_{\underline{\ell}} \equiv \left\{ -i(\omega - \underline{\ell} \cdot \underline{\omega}_0) + C_{\underline{\ell}} \right\}^{-1}$

CHOOSE TERMS IN $Z_{\underline{\ell}-\underline{\ell}'} \Rightarrow f_{\underline{\ell}-\underline{\ell}'}$ SUCH THAT $N_{\underline{\ell}}$ ABOVE IS COHERENT.

$$N_{\underline{\ell}-\underline{\ell}'} = i \sum_{\substack{\underline{\ell}'' \\ \underline{\ell}'' \neq 0, \underline{\ell}-\underline{\ell}'}} \left\{ (\underline{\ell}-\underline{\ell}') \cdot \underline{\omega}_{\underline{\ell}''} f_{\underline{\ell}-\underline{\ell}'-\underline{\ell}''} - \underline{\ell}'' \cdot \partial_{\underline{J}} H_{\underline{\ell}''} f_{\underline{\ell}-\underline{\ell}'-\underline{\ell}''} \right\}$$

TERMS $\underline{\ell}'' = \underline{\ell}; \underline{\ell}'' = -\underline{\ell}'$ "DIRECTLY INTERACT" WITH $H_{\underline{\ell}'}$ TERM IN $N_{\underline{\ell}}$ TO GIVE COHERENT RESULT. THEN EASILY FIND

$$N_{\underline{\ell}} = C_{\underline{\ell}} g_{\underline{\ell}} + D_{\underline{\ell}}(H_{\underline{\ell}}, g_{\underline{\ell}}) H_{\underline{\ell}} + O\left(\frac{\text{MODE SCALE}}{\text{GLOBAL SCALE}} \right)$$

$$g_{\underline{\ell}} = f_{\underline{\ell}} + \frac{f_0}{T} H_{\underline{\ell}} \quad ; \quad f_0 = e^{-H_0/T} F(p)$$

$g_{\underline{\ell}}$: NON ADIABATIC; $F(p)$: L_n

RESULTING EQUATION: $\left(\omega_{*n} \equiv \left(\frac{nq}{r}\right)\left(\frac{\rho_e v_e}{L_n}\right)\right)$

$$\left[-i(\omega - n\omega_0) + C_n\right] g_n = -\left[i\frac{f_0}{T}(\omega - \omega_{*n}) + D_n\right] H_n$$

DRIFT RESONANCE (under $-i(\omega - n\omega_0)$); DIFFUSION (under C_n); LINEAR RESPONSE (under $\omega - \omega_{*n}$); BACKGROUND RENORM. (under D_n)

POISSON: RELATE (Θ, J) TO

φ = TOROIDAL ANGLE, β = POLOIDAL ANGLE

$$K^2 \equiv \left[\frac{H_0 + e\Phi_0 - m B_M(1-\epsilon)}{2\epsilon m\mu B}\right]$$

TRAPPING PARAM. $0 < K < 1$

SOURCE TERM:

$$H_n = -e \sum_m{}' \Phi_{nm}(r) \left\langle e^{i\beta(nq-m)} \right\rangle_b$$

ORBITAL HARMONIC (for n); SPATIAL HARMONICS: φ, β (for nm); BOUNCE AVERAGE (for b)

STRONG WAVE-PARTICLE COUPLING REQUIRES

1. RESONANCE: $\frac{\omega}{k_T} = \frac{\omega R}{n} = v_D$
2. HELICITY MATCHING: $\Phi_{mn} \propto e^{-in\varphi} e^{im\beta}$

   PARTICLE: $(\Delta\varphi / \Delta\beta) = q \Rightarrow e^{-in\varphi} e^{inq\beta}$ $\Bigg\} \Rightarrow$

   $e^{i\beta(nq-m)}$ MEASURES MOTION $\perp$ WAVEFRONT

3. BOUNCE LENGTH: $(k_\parallel)$(BOUNCE LENGTH) < 1

$$\left\langle e^{i\beta(nq-m)} \right\rangle_b \underset{K \ll 1}{\approx} J_0\left(\frac{2K x}{\Delta}\right) \Rightarrow \text{RESONANCE WIDTH } \Delta/K$$

MODEL FOR DRIFT MODES

TRANSLATIONAL SYMMETRY: $\Delta / L_n \ll 1 \Rightarrow$

$$\Phi_{m+p,n}(x+p\Delta) \simeq \Phi_{mn}(x) \qquad x \equiv r - r_{mn}$$

GLOBAL DISTURBANCE, DIFFERENT m's IN PHASE

EIGENMODE SPECTRUM (1-POINT THEORY)

$$|\Phi_{mn}(x)|^2 = \left(\frac{L_s}{rRx_i}\right)\Phi_0^2 S(k_\theta) H\left(\frac{x}{x_i}\right)$$

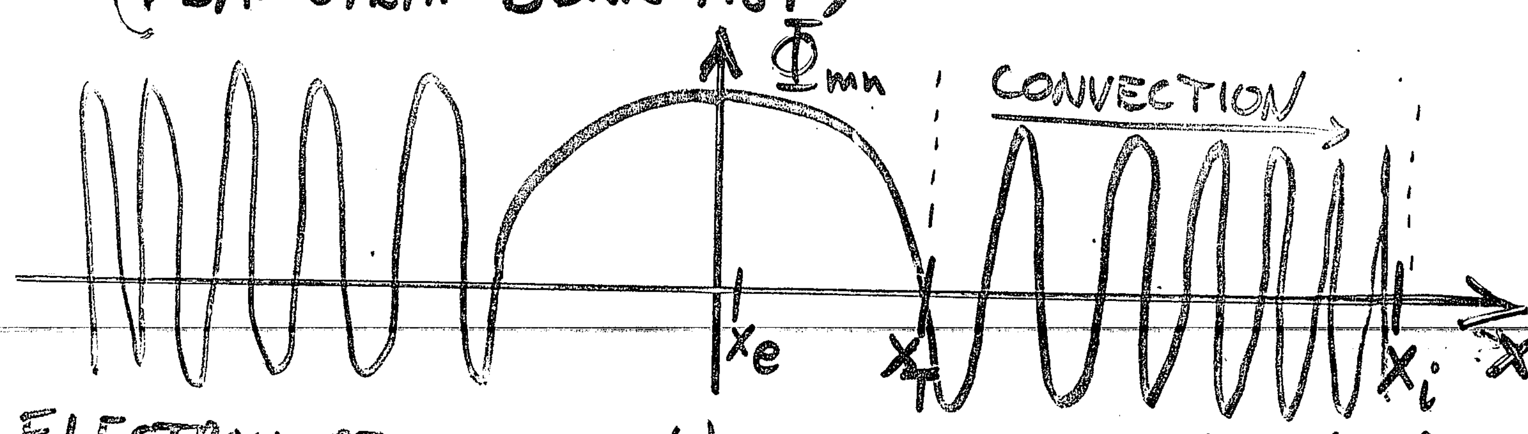
NORMALIZATION: $\sum'_{mn} |\Phi_{mn}|^2 = \Phi_0^2$

RADIAL WIDTH: $|x| < x_i \quad \left(H(x) = \begin{cases} 1 & |x| \le 1 \\ 0 & |x| > 1 \end{cases}\right)$

k_θ SPECTRUM: $S(k_\theta)$: $\Delta k_\theta \sim k_\theta$

MODE STRUCTURE: $\Phi_{mn}(x) = \Phi_0 e^{-i\mu x^2/2}$

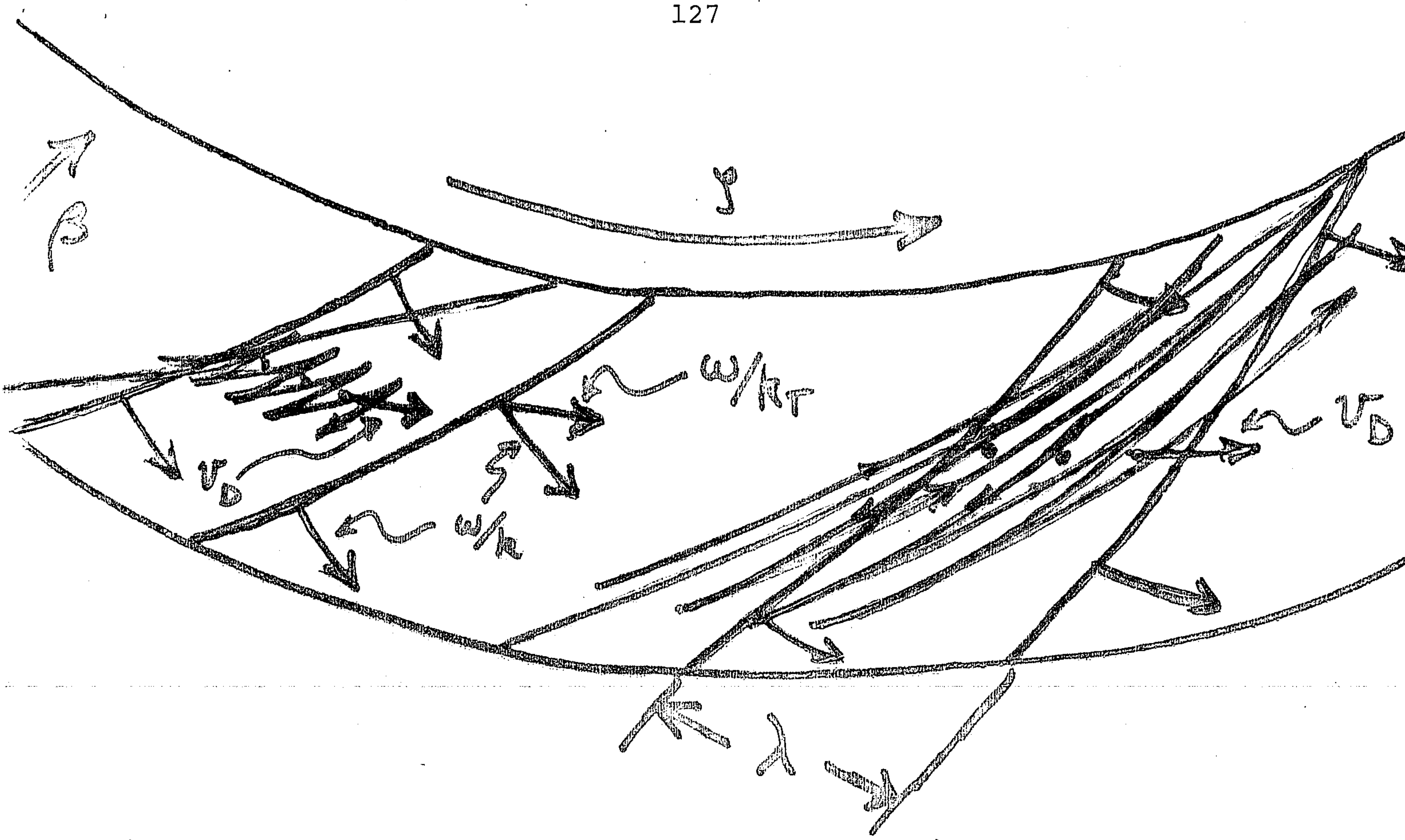
(PEARLSTEIN-BERK 1969)



ELECTRON RESONANCE: $\frac{\omega}{k_\parallel x_e} = v_e$ $\qquad x_i = \frac{\omega}{k_\parallel v_i} \approx \left(\frac{T_e}{T_i}\right)\left(\frac{L_s}{L_n}\right)\rho_i$

TURNING POINT: $x_T \simeq \left(T_e/T_i\right)^{1/2}\left(\frac{L_s}{L_n}\right)^{1/2}\rho_i$ $\qquad$ ION LANDAU DAMPING

DRIFT RESONANCE (CONT.)



BANANA ORBIT :

BANANA CENTER :

FIELD LINE :

MOVING WAVEFRONT :

TRAPPED ELECTRON ON RIGHT IS FOR CASE $\left(\frac{Kx}{\Delta}\right) > 1$. ELECTRON ON LEFT IS FOR $\left(\frac{Kx}{\Delta}\right) < 1$.

COMPUTATION OF C_n, D_n

$$C_n^{(r,r)} = -\left(\frac{c}{B}\right)^2 \sum_{n'} \left(\frac{m'}{r}\right)^2 G_{n-n'} \sum_{p=-\infty}^{+\infty}{}' I_p \frac{\partial^2}{\partial x^2}$$

$$I_p = \sum_{m'}{}' \Phi_{n'm'}(x)\, J_0\!\left(\frac{2Kx}{\Delta}\right) \Phi^*_{n',m'+p}(x)\, J_0\!\left(\frac{2K}{\Delta}(x-p\Delta)\right)$$

BOUNCE AVERAGE $\Rightarrow$ POLOIDAL HARMONIC COUPLING

EVALUATE WITH CONTINUOUS VARIABLES:

$$m' \to \underline{k'_\theta} r \;;\; n \to m'/q(r_{mn}+\underline{x}) \;;\; \sum_{m'n'}{}' \to \iint dx'\, dk'_\theta \frac{Rr}{L_s} |k'_\theta|$$

1. OSCILLATIONS IN $\Phi_{m'n'}(x-p\Delta)$ FOR $|x-p\Delta| > X_T \Rightarrow$

$$|p| < (X_T/\Delta) \equiv p_0 \sim 5$$

2. DOMINANT TERM IN $\sum_p{}'$ IS $p=0$; "DIAGONAL"

BECAUSE OF OSCILLATIONS IN p.

RESULT IS FAMILIAR FORM:

$$C_n \simeq \left(\frac{c\Phi_0}{B}\right)^2 \int dk'_\theta\, G_{n-n'} |k'_\theta| S(k'_\theta) \underline{\left(\frac{\Delta}{KX_i}\right)} \times$$

NOTE 1. FACTOR $\left(\frac{\Delta}{KX_i}\right)$: $\left[k_\theta^2 (uX_i)^2 \frac{1}{2\pi} - 2(k'_\theta)^2 \frac{\partial^2}{\partial x^2} \right]$

FRACTION OF SPATIAL EXTENT OF MODE SEEN BY e

2. $1. \Rightarrow \left(D_n^c / D_n^T\right) \sim \left(X_e/\Delta\right) < 1$

3. $\left[C_n^{(\theta,\theta)} / C_n^{(r,r)}\right] \sim (X_i/X_T)^2 \gg 1$: OSC. IN Φ.

NONLINEAR EFFECTS SMALL

1. RADIAL DIFFUSION

(arrow to id_n: POLOIDAL DIFFUSION)

$$g_n(x) \propto \int_0^\infty d\tau\, e^{i(\omega - n\omega_0 + id_n)\tau}$$
$$\times \left\langle \sum_p \Phi_{nm}(x - p\Delta + \delta x)\, J_0\!\left(\tfrac{2k}{\Delta}(x - p\Delta + \delta x)\right) \right\rangle_{\delta x}$$

$\langle\ \rangle_{\delta x}$ = STATISTICAL AVERAGE OVER δx SUCH THAT $\langle(\delta x)^2\rangle_{\delta x} = 2D_r\tau$; $\delta x(\tau)$ = ORBIT DIFFUSION

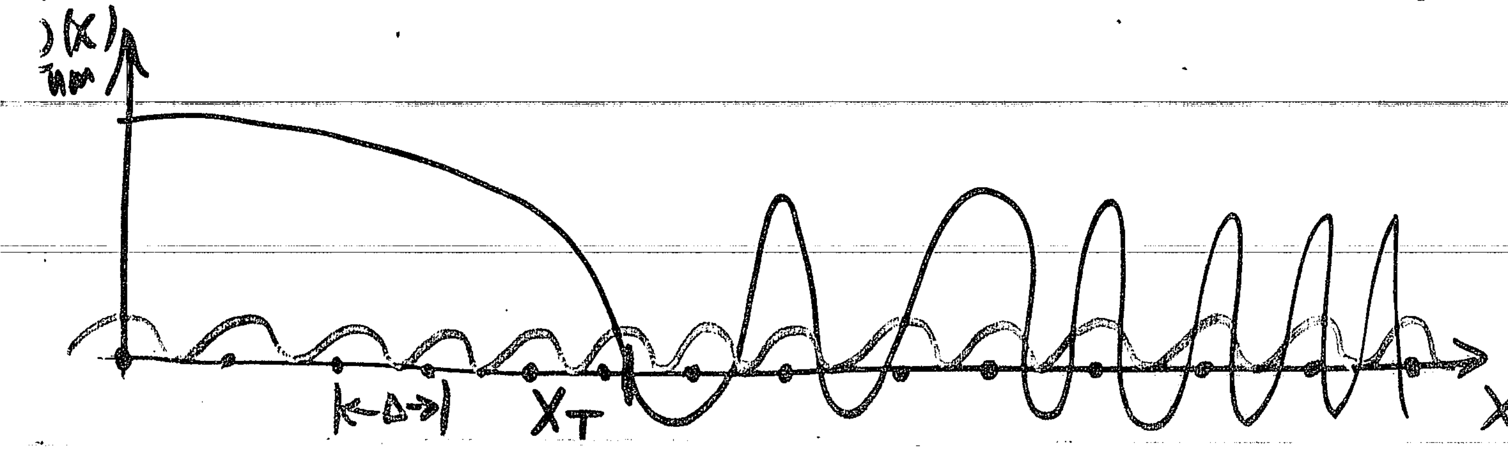
NOTE WANT $g_n(x)$ FOR $x \lesssim \Delta \ll x_T$ TO FIND $\rho_{nm}(\Phi_{nm}(x), x)$. AGAIN, $\rho < \rho_0 \Rightarrow \Phi_{nm}(x - p\Delta + \delta x) \approx \Phi_{nm}(x + \delta x)$. LEAVES $\sum_p J_0 = 1/k$.

FINALLY $\boxed{g_n \propto \frac{1}{k}\langle \Phi_{mn}(x + \delta x)\rangle_{\delta x} \approx \frac{\Phi_{nm}(x)}{k}}$

SINCE $\delta x \leqslant x_c^\theta = (D_r \tau_c^\theta)^{1/2} < x_T$

$$\tau_c^\theta \sim \left(\frac{1}{k_\theta^2 d_n}\right)$$

CONCLUDE g_n APPROX. INDEPENDENT OF δx



2. POLOIDAL DIFFUSION

LOOK AT RESONANCE : IN TAIL OF f_0 : RESONANT

$$\omega = \frac{k_\theta \rho_s v_s}{L_n(1+(k_\theta\rho_s)^2)} = n\omega_D = \left(\frac{k_\theta \rho_e v_e}{R}\right)\left(\frac{E_R}{E_{th}}\right)$$

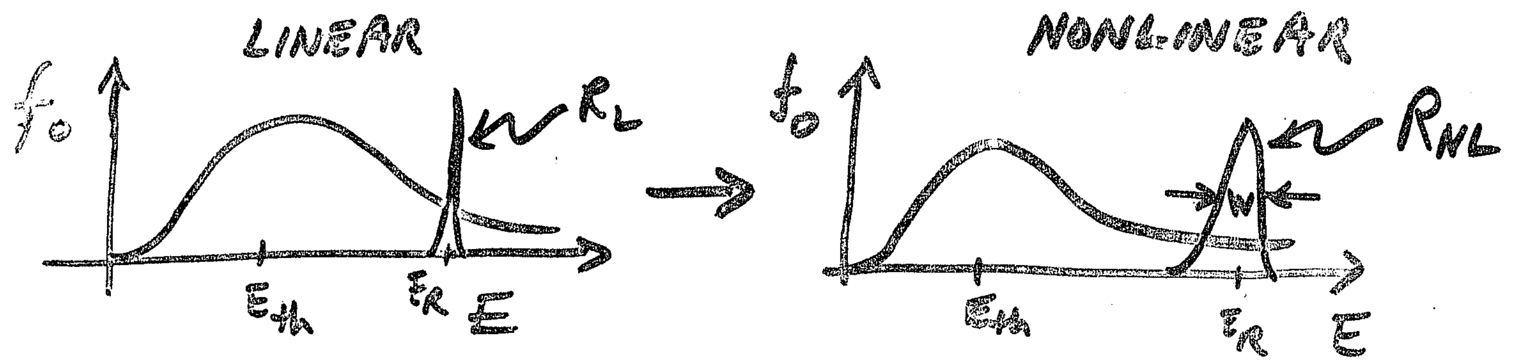
(E_{th}: THERMAL)

$k_\theta \rho_s \sim 1$

$$\Rightarrow \left(E_R/E_{th}\right) \approx \frac{1}{\epsilon}\left[\frac{1}{1+(k_\theta\rho_s)^2}\right] \sim 5/2$$

THUS DIVIDE RESPONSE INTO

A. RESONANT : $\omega = n\omega_D$: E IN TAIL



B. NON RESONANT : $E \sim E_{th} \Rightarrow n\omega_D \ll \omega$

REQUIRE : $W < (E_R - E_{th})$. THIS IS TRUE IF

$$\left(\frac{e\Phi_{mn}}{T_e}\right) < \left(\frac{e\Phi_{mn}}{T_e}\right)_{CRIT} \approx \left(\frac{L_s}{L_n}\right)^{1/2}\frac{\rho_s}{L_n} \sim 10^{-1}$$

THUS $(\omega \tau_c^\theta) < 1$, W HAS SMALL EFFECT $\Rightarrow$

TREAT C_n, D_n AS SMALL PERTURBATIONS IN (C_n/ω) AND $(\bar{D}_n/\omega)$

SUMMARY OF ELECTRON RESPONSE

PATCHED RESPONSE

$$g_n = -\frac{e f_0}{T_e}\left\{ \frac{\omega - \omega_{*n}}{\omega + n\omega_D + i\boxed{\frac{E}{E_R}} d_n^R} - i\frac{d_n^{NR}}{\omega}\left(1 - \frac{\omega_{*n}}{\omega}\right) - i\frac{b_n^{NR}}{\omega} \right\} \times \left\langle e^{in\beta q}\, \Phi_n(\alpha,\beta) \right\rangle_b$$

(① marks the first term; ② marks the remaining terms.)

① PRIMARY RESONANCE, BROADENED, PATCHED

② ELECTRON COMPTON SCATTERING: NONRESONANT

$$\omega - n\omega_D = \omega' - n'\omega_D \rightarrow \omega \approx \omega' \gg n'\omega_D$$

$$k_\theta \rho_s \sim k_\theta' \rho_s \quad \text{BEAT WAVE RESONANCE}$$

RESULTING DENSITY

$$\rho_{nm}(\Phi_{nm}, x) = \left(\frac{n_0 e^2}{T}\right)\left(\frac{\epsilon}{2}\right)^{1/2} \int_0^1 dK^2\, C_{mn}\, L \sum_p \Phi_{mn}(x - p\Delta)\, C^*_{mn p,l}$$

$$C_{mn} \equiv \left\langle e^{i\beta(m - nq)} \right\rangle_b$$

$$L = A_1(\omega - \omega_{*n}) + \left(\frac{c\Phi_0}{B}\right)^2 \int dk_\theta' \, |k_\theta'|\, S(k_\theta') \frac{1}{\omega^2}\left(\frac{\Delta}{K x_i}\right)$$

$$\times \left[(k_\theta')^2 2 - k_\theta^2 (\mu x_i)^2 \frac{1}{2\pi}\right]\left[(\omega' - \omega_{*n'})A_3 + [(\omega' - \omega) - (\omega_{*n'} - \omega_{*n})]A_2\right]$$

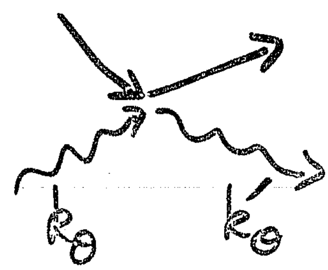
$$A_j \equiv -\left(\frac{2}{\omega_j}\right)(y_j)^2\left[1 + y_j Z(y_j)\right] : \; y_j \sim L, NL \text{ RESONANCES}$$

CIRCULATING ELECTRONS (SIMPLIFIED HIRSHMAN-MOLVIG): $\rho^c_{mn} \approx \frac{n_0 e^2}{T_e}\left(\frac{\omega_{*e}}{\omega} - 1\right) i \frac{\omega}{\omega_c} \Phi_{mn}$

ION RESPONSE

(DIAMOND-ROSENBLUTH 1981)

1. NONLINEAR ION GYROKINETIC EQ. (FRIEMAN-CHEN) AND WEAK TURBULENCE CALCULATION ⟹ ION COMPTON SCATTERING DOMINANT

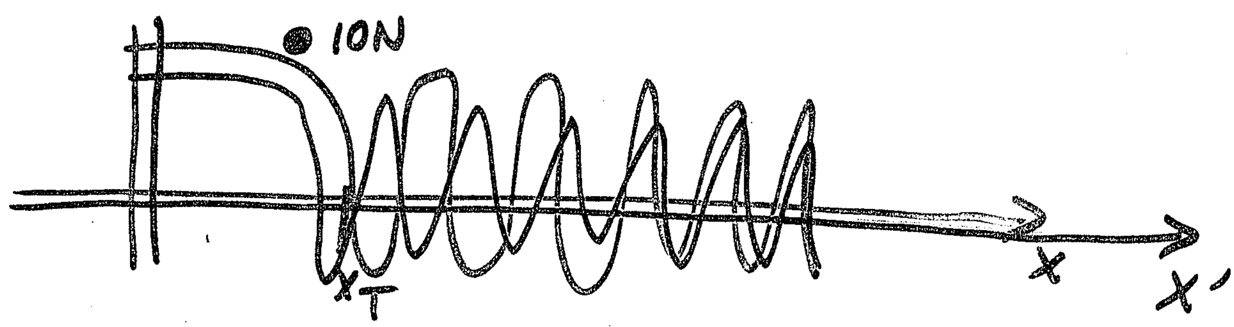


- IONS DAMP WAVES
- $k_\theta' < k_\theta$: SCATTER TO LONGER, STABLE, WAVELENGTH

2. NL RESONANCE : $\omega(k_\theta) - k_\parallel v_i = \omega(k_\theta') - k_\parallel' v_i$

$$k_\parallel^{(\prime)} = \frac{k_\theta^{(\prime)} x^{(\prime)}}{L_s} \Rightarrow x \sim x'$$

3. WAVES



ION DAMPING IMPORTANT FOR $0 < x < x_T$

$x > x_T$: CONVECTION ONLY

4. IONS : DISSIPATION OVER FULL SPATIAL WIDTH OF MODE : NO (Δ/x_i) FACTOR AS FOR TRAPPED ELECTRONS

VARIATIONAL FORMULATION

EIGENMODE EQUATION : QUASINEUTRALITY

$$\rho^{T}_{mn}(\Phi_{mn}(x),x)+\rho^{C}_{mn}(\Phi_{mn}(x),x)-\frac{n_0 e^2}{T_e}\Phi_{mn}(x)=\rho^{I}_{mn}(\Phi_{mn}(x),x)$$

TRAPPED ELECTRONS — CIRCULATING ELECTRONS — ADIABATIC — IONS

$\equiv \mathcal{L}(\Phi_{mn}(x),x)=0$ AND $\mathcal{L}$ IS SELF-ADJOINT

$$\Rightarrow S(\alpha)=\left[\frac{1}{\int dx\,(\varphi(x))^2}\right]\int dx\,\varphi(\alpha,x)\,\mathcal{L}(\varphi(\alpha,x),x)$$

EIGENMODE : $\frac{\partial S}{\partial \alpha}=0 \Rightarrow \alpha,\ \Phi(x)$

TRIAL $\varphi(x,\alpha)=e^{-\alpha x^2/2}$

DISPERSION RELATION : $S(\Phi(x))=0 \Rightarrow \gamma$

$$S(\alpha)\approx\underbrace{\left(\frac{\omega_{*e}}{\omega}-1\right)-(k_\theta\rho_s)^2}_{1}-\underbrace{\frac{\alpha}{2}\rho_s^2-H^{I}_{\theta}}_{2}+\underbrace{\frac{\mu^2\rho_s^2}{2\alpha}}_{3}$$
$$\underbrace{i\left(\frac{\omega}{\omega_c}\right)\left(\frac{\omega_{*e}}{\omega}-1\right)}_{4}+\underbrace{(2\pi\epsilon)^{1/2}L(k_\theta)\Delta\alpha^{1/2}\ln\left(\frac{1}{\Delta\alpha^{1/2}}\right)}_{5}$$

1. BASIC WAVE RESPONSE
2. NONLINEAR IONS
3. SHEAR
4. CIRCULATING ELECTRONS
5. TRAPPED ELECTRONS

DISPERSION RELATION

NEGLECTING $\epsilon^{1/2}$ TRAPPED ELECTRON DISTORTION GET P-B SOLUTION $\alpha = i\mu \Rightarrow$ SIMPLEST DISPERSION RELATION

$$S(\alpha = i\mu) \equiv \left(\frac{\omega_{*e}}{\omega} - 1\right) - (k_\theta \rho_s)^2 - \frac{i\mu}{2}\rho_s^2 - H_\theta^I - i\frac{\mu \rho_s^2}{2} + i\left(\frac{\omega}{\omega_c}\right)\left(\frac{\omega_{*e}}{\omega} - 1\right) + (2\pi\epsilon)^{1/2} L(k_\theta)\Delta(i\mu)^{1/2} \times \ln\left(\frac{1}{(i\mu)^{1/2}\Delta}\right) = 0$$

SATURATION AT MARGINAL STABILITY FROM

GROWTH $\longleftrightarrow$ DAMPING

1. CIRCULATING ELECTRONS
   TRAPPED ELECTRONS:
2. LINEAR
3. NONLINEAR

4. ION COMPTON SCATTERING
5. SHEAR DAMPING

1 AND 2 ARE COMPARABLE, 3 SMALL.

4 IS LARGER THAN 2 BY (X_i/Δ) FACTOR.

RESULT FOR DIFFUSION COEFFICIENT

$\gamma_{k_\theta} = 0 \Longrightarrow$ DIFFERENTIAL EQUATION FOR $S(k_\theta)$ DOMINATED BY ION NONLINEARITY

$S(k_\theta)$ IN TRAPPED ELECTRON RADIAL DIFFUSION $\Longrightarrow$

$$D_r(E/E_{th}) = \left\{ \left(\frac{\rho_s^2 c_s}{Rq}\right)\left(\frac{E_{th}}{\epsilon E}\right)^2 \left(\frac{E_{th}}{\epsilon E} - 1\right)^{1/2} \left(\frac{T_i^2}{T_e^2 \alpha^5 K_0}\right) I \right\}$$

$$I = \int_{\bar{b}}^{b_M} \frac{db}{b} \frac{|1-b^2|^2}{|1+b^2|^4} \left\{ \frac{\Delta}{X_T} \ln\left(\frac{X_T}{\Delta}\right) \left[4\epsilon^{1/2} y_0^3 e^{-y_0^2} + \left(\frac{\epsilon}{2\pi}\right)^{1/2} \right] - \left(\frac{L_n}{L_s}\right) \frac{(1+b^2)^2}{b^2} \right\}$$

$\bar{b} = \bar{k}_\theta \rho_s$, $\bar{k}_\theta$ = RESONANT $k_\theta = \bar{k}_\theta(E)$

$b_M = k_{\theta M} \rho_s$, $k_{\theta M}$ = MAXIMUM k_θ CUTOFF

$(y_0)^2 \equiv (\omega/\omega_D)$

TYPICALLY $(\omega_c/\omega) \sim 2$

GYROKINETIC APPROACH IN PARTICLE SIMULATION

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Gyrokinetic Approach in Particle Simulation

W. W. Lee

Princeton Plasma Physics Laboratory

I. Introduction

- Difficulties
- Remedies

II. Gyrokinetic Approach

- Aim
- Procedures

III. Gyrokinetic Particle Simulation Scheme

- Equations of Motion
- Field Equations

IV. Simulation Results

V. Conclusions

I. Introduction

- Difficulties in using conventional particle codes to study microinstabilities :
  1. Time step : limited by ω_{ce} or ω_{pe}
  2. Grid size : limited by λ_D
  3. Profile Modification : quasilinear diffusion

- Remedies
  1. Time step :
     a. Numerical approach : Implicit Methods
     b. Physical approach :
        Drift - kinetic electrons — ω_{LH}
        Adiabatic electrons — ω_{ci}
        * Gyrokinetic particles — ω^*
  2. Grid size : higher order intrapolation
  3. Profile Modification :
     * Gyrokinetic + Multiple Spatial Scale schemes

II. Gyrokinetic Approach

- Aim — to obtain a reduced Vlasov equation, which ~~retains all the relevant kinetic effects but~~ not the gyromotion, for particle pushing purpose.

- Procedures
  1. Gyrokinetic change of variables in the Vlasov equation from particle variables to guiding center variables [P. Catto, Plasma Phys. '78]
  2. Gyrokinetic ordering [Rutherford and Frieman, Phys. of Fluids '68 ; Taylor and Hastie, Plasma Phys. '68]
  3. Gyrophase averaging

- Development of Gyrokinetic Equation in Electrostatic Slab
  1. Vlasov Equation

$$\frac{\partial F}{\partial t} + \underset{\sim}{v} \cdot \frac{\partial F}{\partial \underset{\sim}{x}} + \frac{q}{m}\left(\underset{\sim}{E} + \frac{\underset{\sim}{v} \times \underset{\sim}{B}}{c}\right) \cdot \frac{\partial F}{\partial \underset{\sim}{v}} = 0$$

  2. Gyrokinetic Change of Variables

$$\underset{\sim}{x}, \underset{\sim}{v}, t \implies \underset{\sim}{R}, \mu, v_{\parallel}, \phi, t$$

$$\frac{\partial F}{\partial t} + \left(\underset{\sim}{v}_{\parallel} + \frac{q}{m}\frac{\underset{\sim}{E} \times \hat{\underset{\sim}{b}}}{\Omega}\right) \cdot \frac{\partial F}{\partial \underset{\sim}{R}} - \Omega \frac{\partial F}{\partial \phi}$$

$$+ \frac{q}{m}\underset{\sim}{E} \cdot \left(\frac{\underset{\sim}{v}_{\perp}}{B}\frac{\partial F}{\partial \mu} + \hat{\underset{\sim}{b}}\frac{\partial F}{\partial v_{\parallel}} + \frac{\hat{\underset{\sim}{b}} \times \underset{\sim}{v}_{\perp}}{v_{\perp}^2}\frac{\partial F}{\partial \phi}\right) = 0$$

where $\underset{\sim}{R} = \underset{\sim}{x} + \underset{\sim}{\rho}$, $\underset{\sim}{\rho} = \frac{\underset{\sim}{v}_{\perp} \times \hat{\underset{\sim}{b}}}{\Omega}$, $\hat{\underset{\sim}{b}} = \frac{\underset{\sim}{B}}{B}$, $\mu = \frac{v_{\perp}^2}{2B}$

$\underset{\sim}{v}_{\perp} = v_{\perp}(\cos\phi\, \hat{\underset{\sim}{e}}_1 + \sin\phi\, \hat{\underset{\sim}{e}}_2)$, $\underset{\sim}{v}_{\parallel} = v_{\parallel}\hat{\underset{\sim}{b}}$, $\Omega = \frac{qB}{mc}$

$\underset{\sim}{E} = -\partial\Phi(\underset{\sim}{x})/\partial \underset{\sim}{x}$

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3. Gyrokinetic Ordering

$$\frac{\omega}{\Omega} \sim \epsilon, \quad \frac{\rho}{L_{eq.}} \sim \epsilon, \quad L_{\parallel} \sim L_{eq.}, \quad \frac{e\Phi}{T} \sim \epsilon$$

a. Zeroth Order

$$\Omega \frac{\partial F}{\partial \phi} = 0 \quad \Longrightarrow \quad F = f + \epsilon g(\phi), \text{ where } f \neq f(\phi)$$

b. Order ϵ

$$\frac{\partial f}{\partial t} + [\underset{\sim}{v}_{\parallel} + \frac{q}{m} \frac{\underset{\sim}{E} \times \hat{b}}{\Omega}] \cdot \frac{\partial f}{\partial \underset{\sim}{R}} - \Omega \frac{\partial}{\partial \phi} [g - \frac{q}{m} \frac{\Phi}{B} \frac{\partial f}{\partial \mu}]$$

$$+ \frac{q}{m} \underset{\sim}{E} \cdot \hat{\underset{\sim}{b}} \frac{\partial f}{\partial v_{\parallel}} = 0$$

$$\Longrightarrow \quad g = \frac{q}{m} \frac{\Phi}{B} \frac{\partial f}{\partial \mu}$$

Here we used $\dfrac{\partial \Phi}{\partial \underset{\sim}{x}} = \dfrac{\partial \Phi}{\partial \underset{\sim}{R}}$, $\Omega \dfrac{\partial \Phi}{\partial \phi} = -\underset{\sim}{v}_{\perp} \cdot \dfrac{\partial \Phi}{\partial \underset{\sim}{R}}$

c. From a and b, we obtain

$$\boxed{F = f + \frac{q}{m} \frac{\Phi}{B} \frac{\partial f}{\partial \mu}}$$

4. Substituting the above expression into the ~~gyrokinetic equation and using~~

$$\frac{\partial \Phi}{\partial v_{\parallel}} = 0 \quad \text{and} \quad \frac{\partial \Phi}{\partial \mu} = -\frac{B}{v_{\perp}^2} \underset{\sim}{\rho} \cdot \frac{\partial \Phi}{\partial \underset{\sim}{R}}$$

We obtain

$$\frac{\partial F}{\partial t} + [\underset{\sim}{v}_{\parallel} + \frac{q}{m}\frac{\underset{\sim}{E}\times\hat{b}}{\Omega}]\cdot\frac{\partial F}{\partial \underset{\sim}{R}} + \frac{q}{m}\underset{\sim}{E}\cdot\hat{\underset{\sim}{b}}\frac{\partial F}{\partial v_{\parallel}}$$

$$+\frac{1}{2}\frac{\Omega}{B^2}\left(\frac{q}{m}\right)^2\frac{\partial^2 f}{\partial \mu^2}\frac{\partial \Phi^2}{\partial \phi} = 0$$

5. Gyrophase averaging

$$\frac{\partial\langle F\rangle}{\partial t} + \underset{\sim}{v}_{\parallel}\cdot\frac{\partial\langle F\rangle}{\partial \underset{\sim}{R}} - \frac{q}{m}\left\langle \frac{1}{\Omega}\frac{\partial \Phi}{\partial \underset{\sim}{R}}\times\hat{\underset{\sim}{b}}\cdot\frac{\partial F}{\partial \underset{\sim}{R}}\right\rangle$$

$$-\frac{q}{m}\left\langle \frac{\partial \Phi}{\partial \underset{\sim}{R}}\cdot\hat{\underset{\sim}{b}}\frac{\partial F}{\partial v_{\parallel}}\right\rangle = 0$$

where $\langle\ \ \rangle \equiv \frac{1}{2\pi}\int d\phi$

Using $\langle e^{i\underset{\sim}{k}\cdot\underset{\sim}{\rho}}\rangle = J_0\left(\frac{k_\perp v_\perp}{\Omega}\right)$

$$\langle\Phi\rangle = \sum_{\underset{\sim}{k}}\Phi(\underset{\sim}{k})\,J_0\left(\frac{k_\perp v_\perp}{\Omega}\right)e^{i\underset{\sim}{k}\cdot\underset{\sim}{R}}$$

$$\langle\Phi^2\rangle = \sum_{\underset{\sim}{k}}J_0\left(\frac{k_\perp v_\perp}{\Omega}\right)e^{i\underset{\sim}{k}\cdot\underset{\sim}{R}}\left[\sum_{\underset{\sim}{k}'}\Phi(\underset{\sim}{k}')\Phi(\underset{\sim}{k}-\underset{\sim}{k}')\right]$$

we have

$$\frac{\partial\langle F\rangle}{\partial t} + \underset{\sim}{v}_{\parallel}\cdot\frac{\partial\langle F\rangle}{\partial \underset{\sim}{R}} - \frac{q}{m}\frac{1}{\Omega}\frac{\partial\langle\Phi\rangle}{\partial \underset{\sim}{R}}\times\hat{\underset{\sim}{b}}\cdot\frac{\partial f}{\partial \underset{\sim}{R}} - \frac{q}{m}\frac{\partial\langle\Phi\rangle}{\partial \underset{\sim}{R}}\cdot\hat{\underset{\sim}{b}}\frac{\partial f}{\partial v_{\parallel}}$$

$$-\frac{q}{m}\frac{1}{\Omega}\frac{1}{2}\frac{\partial\langle\Phi^2\rangle}{\partial \underset{\sim}{R}}\times\hat{\underset{\sim}{b}}\cdot\frac{\partial}{\partial \underset{\sim}{R}}\left(\frac{q}{m}\frac{1}{B}\frac{\partial f}{\partial \mu}\right) - \frac{q}{m}\frac{1}{2}\frac{\partial\langle\Phi^2\rangle}{\partial \underset{\sim}{R}}\cdot\hat{\underset{\sim}{b}}\frac{\partial}{\partial v_{\parallel}}\left(\frac{q}{m}\frac{1}{B}\frac{\partial f}{\partial \mu}\right) = 0$$

where

$$\langle F\rangle = f + \frac{q}{m}\frac{\langle\Phi\rangle}{B}\frac{\partial f}{\partial \mu}$$

Let f be Maxwellian in $v_\perp$, we have

$$f = \frac{\langle F \rangle}{1 - q\langle \Phi \rangle / T}$$

$$\frac{q}{m}\frac{1}{B}\frac{\partial f}{\partial \mu} = -\frac{q}{T}\frac{\langle F \rangle}{1 - q\langle \Phi \rangle / T} = -\frac{q}{T} f \; ; \quad \text{note that } \frac{\partial \langle \Phi \rangle}{\partial \mu} = 0$$

Gyrokinetic Equation becomes

$$\frac{\partial \langle F \rangle}{\partial t} + \underset{\sim}{v}_{\parallel} \cdot \frac{\partial \langle F \rangle}{\partial \underset{\sim}{R}} - \frac{q}{m}\frac{1}{\Omega}\frac{\partial \langle \Phi \rangle}{\partial \underset{\sim}{R}} \times \hat{\underset{\sim}{b}} \cdot \frac{\partial f}{\partial \underset{\sim}{R}} - \frac{q}{m}\frac{\partial \langle \Phi \rangle}{\partial \underset{\sim}{R}} \cdot \hat{\underset{\sim}{b}} \frac{\partial f}{\partial v_\parallel}$$

$$+ \frac{q}{m}\frac{1}{\Omega}\frac{1}{2}\frac{\partial \langle \Phi^2 \rangle}{\partial \underset{\sim}{R}} \times \hat{\underset{\sim}{b}} \cdot \frac{\partial}{\partial \underset{\sim}{R}}\left(\frac{q}{T} f\right) + \frac{q}{m}\frac{1}{2}\frac{\partial \langle \Phi^2 \rangle}{\partial \underset{\sim}{R}} \cdot \hat{\underset{\sim}{b}} \frac{\partial}{\partial v_\parallel}\left(\frac{q}{T} f\right) = 0 \qquad (1)$$

where
$$f = \frac{\langle F \rangle}{1 - q\langle \Phi \rangle / T} = \frac{F}{1 - q\Phi / T}$$

6. Gyrokinetic Equation in terms of Guiding Center Variables

- Substituting f by $\langle F \rangle$ in the above equation [Eq. (1)] and Dropping Φ^3 terms, we have

$$\boxed{\frac{\partial \langle F \rangle}{\partial t} + \underset{\sim}{v}_{\parallel} \cdot \frac{\partial \langle F \rangle}{\partial \underset{\sim}{R}} - \frac{q}{m}\frac{1}{\Omega}\frac{\partial \Psi}{\partial \underset{\sim}{R}} \times \hat{\underset{\sim}{b}} \cdot \frac{\partial \langle F \rangle}{\partial \underset{\sim}{R}} - \frac{q}{m}\frac{\partial \Psi}{\partial \underset{\sim}{R}} \cdot \hat{\underset{\sim}{b}} \frac{\partial \langle F \rangle}{\partial v_\parallel} = 0} \qquad (2)$$

where $\langle F \rangle$ is related to original F by

$$\boxed{F = \frac{1 - q\Phi/T}{1 - q\langle \Phi \rangle / T}\langle F \rangle} \qquad (3)$$

- This equation is valid for arbitrary values of $k_\perp \rho_t$
- Ingenious schemes have yet to be devised to push particles using these equations.

7. Gyrokinetic Equation in terms of particle variables

- Substituting f and $\langle F \rangle$ by F in Eq. (1)
- Realizing that $\partial/\partial \underset{\sim}{R} = \partial/\partial \underset{\sim}{x}$ in slab geometry.
- Assuming that F is Maxwellian in $v_\perp$ [its consequence will be discussed later]
- Taking $\int v_\perp dv_\perp d\phi$ of Eq. (1) by holding $\underset{\sim}{x}$ constant
- Again dropping Φ^3 terms

Gyrokinetic Equation becomes

$$\frac{\partial \tilde{f}}{\partial t} + \underset{\sim}{v}_\parallel \cdot \frac{\partial \tilde{f}}{\partial \underset{\sim}{x}} - \frac{q}{m}\frac{1}{\Omega}\frac{\partial \tilde{\Psi}}{\partial \underset{\sim}{x}} \times \hat{\underset{\sim}{b}} \cdot \frac{\partial \tilde{f}}{\partial \underset{\sim}{x}} - \frac{q}{m}\frac{\partial \tilde{\Psi}}{\partial \underset{\sim}{x}} \cdot \hat{\underset{\sim}{b}}\, \frac{\partial \tilde{f}}{\partial v_\parallel} = 0 \qquad (4)$$

where

$$\tilde{\Psi} = \tilde{\Phi} + \frac{1}{2}\frac{q}{T}(\tilde{\Phi})^2 - \frac{1}{2}\frac{q}{T}\widetilde{(\Phi^2)}$$

$$\tilde{\Phi} = \sum_{\underset{\sim}{k}} \Phi(\underset{\sim}{k})\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}}\, \Gamma_0(k_\perp^2 \rho_t^2)$$

$$\widetilde{(\Phi^2)} = \sum_{\underset{\sim}{k}} \Gamma_0(k_\perp^2 \rho_t^2)\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}} \sum_{\underset{\sim}{k}'} \Phi(\underset{\sim}{k}')\, \Phi(\underset{\sim}{k}-\underset{\sim}{k}')$$

$$\Gamma_0(k_\perp^2\rho_t^2) = \frac{1}{v_t^2}\int_0^\infty e^{-\frac{v_\perp^2}{2v_t^2}} J_0^2\left(\frac{k_\perp v_\perp}{\Omega}\right) v_\perp dv_\perp$$

$$\rho_t = v_t/\Omega$$

Poisson's Equation becomes

$$\nabla^2\Phi - \underbrace{k_{Di}\frac{n_i}{n_0}\left[\Phi - \tilde{\Phi}\right]\left[1 + \frac{e\tilde{\Phi}}{T_i}\right]}_{\text{polarization term}} = -4\pi e(n_i - n_e) \qquad (5)$$

$$n = \int \tilde{f}\, dv_\parallel, \qquad k_{Di}^2 = 4\pi n_0 e^2/T_i$$

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- These equations can be obtained by
  a) substituting <F> by f in Eq. (1) and using the approximation

$$\frac{1}{v_t^2}\int_0^\infty e^{-\frac{v_\perp^2}{2v_t^2}} J_0\left(\frac{k_\perp v_\perp}{\Omega}\right) J_0\left(\frac{k_\perp' v_\perp}{\Omega}\right) v_\perp dv_\perp \simeq e^{-\frac{k_\perp''^2 \rho_t^2}{2}} J_0(k_\perp'^2 \rho_t^2)$$

  where $\underset{\sim}{k}_\perp = k_\perp' + k_\perp''$

  or

  b) Using Eqs (2) and (3) directly with similar approximations

- Therefore, Eqs (4) and (5) are valid for arbitrary of $k_\perp \rho_i$ linearly, and, nonlinearly for $k_\perp \rho_i < 1$.

- Particle simulation schemes can be easily derived from Eqs (4) and (5).

III. Gyrokinetic Particle Simulation Scheme

Let $$\tilde{f}(\underset{\sim}{X}, v_\parallel, t) = \sum_{j=1}^{N} \delta[\underset{\sim}{X} - \underset{\sim}{X}_j(t)]\,\delta[v_\parallel - v_{\parallel j}(t)]$$

- Equations of Motion

$$\frac{d\underset{\sim}{X}_i}{dt} = \underset{\sim}{v}_{\parallel j} - \frac{q}{m}\frac{1}{\Omega}\frac{\partial\tilde{\Psi}}{\partial\underset{\sim}{X}}\bigg|_{\underset{\sim}{X}_j} \times \hat{\underset{\sim}{b}}$$

$$\frac{dv_{\parallel j}}{dt} = -\frac{q}{m}\frac{\partial\tilde{\Psi}}{\partial\underset{\sim}{X}}\bigg|_{X_j} \cdot \hat{\underset{\sim}{b}}$$

$$\tilde{\Psi} = \tilde{\Phi} + \frac{1}{2}\frac{q}{T}(\tilde{\Phi})^2 - \frac{1}{2}\frac{q}{T}\widetilde{(\tilde{\Phi}^2)}$$

- Poisson's Equation in $\underset{\sim}{k}$-space

$$k^2 \Phi(\underset{\sim}{k}) + \frac{k_{0i}^2}{n_0} \frac{1}{V} \int n_i (\Phi - \tilde{\Phi}) e^{-i\underset{\sim}{k}\cdot\underset{\sim}{x}} d\underset{\sim}{x}$$

$$+ \frac{k_{0i}^2}{n_0} \frac{1}{V} \int n_i (\Phi - \tilde{\Phi}) \left(\frac{e\tilde{\Phi}}{T_i}\right) e^{-i\underset{\sim}{k}\cdot\underset{\sim}{x}} d\underset{\sim}{x}$$

$$= 4\pi e \left[n_i(\underset{\sim}{k}) - n_e(\underset{\sim}{k}) \right]$$

Fredholm Equation of the 2nd kind with a nonlinear term, which can be treated perturbatively, and

$$n = \sum_{j=1}^{N} \delta \left[\underset{\sim}{x} - \underset{\sim}{x}_j(t) \right]$$

- Simulation Model

2-½ D (x, y, v_x, v_y, v_z) Electrostatic slab

Bounded in x-direction, $\Phi = 0$ at boundaries

Periodic in y-direction

Magnetic field:

$$\underset{\sim}{B} = B_0 \hat{z} + B_y \hat{y}$$

$$\begin{cases} B_y = \text{constant} & \text{-- no shear} \\ B_y = \dfrac{x}{L_s} B_0 & \text{-- shear} \end{cases}$$

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IV. Simulation Results

- Three cases have been studied
  a. Density gradient with no shear
  b. Density gradient in sheared slab
  c. Density gradient and Ion temperature gradient in sheared slab (η_I - mode)

- In all three cases results agreed very well with those obtained earlier* in both linear and nonlinear stages of the instability. Time step used : $\omega_{pe} \Delta t \sim 100$.

  * a. W. W. Lee, Y. Y. Kuo and H. Okuda, Phys. Fluids 24, 617 (1978)
  b. W. W. Lee, W. M. Nevins, H. Okuda, R. White, Phys. Rev. Lett. 43, 347 (1979)
  c. W. W. Lee, M. S. Chance and H. Okuda, Phys. Rev. Lett. 46, 1675 (1981)

- Nonlinear $\underset{\sim}{E} \times \underset{\sim}{B}$ and nonlinear polarization (in Poisson's equation) have played an important part in the saturation of the instability.

V. Conclusions

1. Gyrokinetic code using particle variables works well at ~~considerable saving of computer resources for cases of $k_\perp \rho_i < 1$.~~

2. The code makes nonlinear physics more transparent and easier to interprete.

11

3. In conjunction with Multiple Spatial Scale scheme, it can be used to study steady-state turbulence phenomena.

4. Extension to 3D slab in straightforward and 3D toroidal code including finite-β effects is also possible

5. To study nonlinear effects for $k_\perp \rho_i \gtrsim 1$, gyrokinetic code using guiding center variables has to be developed.

UNIVERSAL MODE WITH DIFFUSIVE ELECTRONS: LINEAR STABILITY AND NONLINEAR SATURATION

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UNIVERSAL MODE WITH DIFFUSIVE ELECTRONS : LINEAR STABILITY AND NONLINEAR SATURATION

ORNL TM-8132

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Union Carbide Corporation.

SYNOPSIS

CONCEPT: Nonlinear effects through spatial diffusion resulting from stochasticity of electron orbits

EQUATIONS: Exact

Krook approximation

Velocity-dependent diffusion coefficient

Renormalization of finite-β equations

RESULTS: Comparison of KROOK vs diffusion

Velocity-dependent diffusion coefficient

Electron temperature gradient

Finite β

CONCLUSIONS:

ION GYROKINETIC EQUATION

[Differential ($k_x\rho_i \ll 1$)]

$$\left(\frac{\partial}{\partial t} + i k_\parallel' x v_\parallel \right) h_i$$

$$= -i \frac{e}{T_e} F_i \left\{ \omega_* \left[1 + \eta_i \left(\frac{v_\parallel^2}{v_i^2} - \frac{1}{2} \right) \right] + \tau k_\parallel' x v_\parallel \right\} \left(\phi_1 - \frac{v_\parallel}{c} A_1 \right)$$

$$+ i \eta_i \frac{e}{T_e} F_i \left(\phi_2 - \frac{v_\parallel}{c} A_2 \right)$$

$$h_i = g_i - \frac{e}{T_i} F_i \left(\phi_1 - \frac{v_\parallel}{c} A_1 \right)$$

$$g_i = f_i + \frac{e}{T_i} F_i \phi$$

$$F_i = \frac{n_0}{\sqrt{\pi} v_i} \exp\left(-\frac{v_\parallel^2}{v_i^2} \right), \quad \eta_i = \frac{d(\ln T_i)}{d(\ln n_0)}$$

$$v_i = \sqrt{\frac{2 T_i}{m_i}}, \quad \tau = \frac{T_e}{T_i}$$

$$(\phi_1, A_1) = \left[\Gamma_0 + (\Gamma_0 - \Gamma_1) \rho_i^2 \frac{\partial^2}{\partial x^2} \right] (\phi, A_\parallel)$$

$$(\phi_2, A_2) = \left\{ b_y (\Gamma_0 - \Gamma_1) - \left[\Gamma_0 - 2 b_y (\Gamma_0 - \Gamma_1) \right] \rho_i^2 \frac{\partial^2}{\partial x^2} \right\} (\phi, A_\parallel)$$

$$\rho_i = \frac{v_i}{\sqrt{2}\,\Omega_i}, \quad \Gamma_n = \exp(-b_y) I_n(b_y), \quad b_y = (k_y \rho_i)^2$$

DRIFTKINETIC EQUATION FOR ELECTRONS WITH DIFFUSION

$$\left(\frac{\partial}{\partial t} + i k_{\parallel}' x v_{\parallel} - D \frac{\partial^2}{\partial x^2}\right) h_e$$

$$= -i \frac{e}{T_e} F_e \left\{ \omega_* \left[1 + \eta_e \left(\frac{v_{\parallel}^2}{v_e^2} - \frac{1}{2}\right)\right] - k_{\parallel}' x v_{\parallel} - i D \frac{\partial^2}{\partial x^2} \right\} (\phi - \psi)$$

$$h_e = g_e + \frac{e}{T_e} F_e (\phi - \psi)$$

$$g_e = f_e - \frac{e}{T_e} F_e \left(\phi - \psi + \frac{1}{k_{\parallel}' c} \omega_* A_{\parallel}\right)$$

$$\psi = \frac{\omega}{k_{\parallel}' c} \frac{A_{\parallel}}{x} = \frac{i}{k_{\parallel}' c} \frac{1}{x} \frac{\partial A_{\parallel}}{\partial t}$$

$$k_{\parallel}' = \frac{k_y}{L_s}, \quad F_e = \frac{n_0}{\sqrt{\pi} v_e} \exp\left(-\frac{v_{\parallel}^2}{v_e^2}\right), \quad \omega_* = \frac{v_e^2 k_y}{2 \Omega_e L_n}$$

$$v_e = \sqrt{\frac{2 T_e}{m_e}}, \quad \eta_e = \frac{d(\ln T_e)}{d(\ln n_0)}$$

DIFFUSION COEFFICIENT

Correlation frequency : $\omega_c = \left[\frac{1}{3}(k_{\parallel}' v_e)^2 D\right]^{\frac{1}{3}}$

Krook approximation : $D \frac{\partial^2}{\partial x^2} \rightarrow -\omega_c$

$D(v_{\parallel})\ [\beta_i = 0]$: $D = D_0 \frac{v_e}{|v_{\parallel}|} \exp\left(-\frac{v_e^2}{v_{\parallel}^2}\right) + D_c$

QUASINEUTRALITY

$$\phi - \phi_1 = \frac{T_i}{e n_0} \int_{-\infty}^{\infty} dv_\parallel \, (h_i - h_e) - \frac{\omega_*}{\tau k_y c} \frac{1}{x} A_\parallel$$

AMPERE'S LAW

$$\left(b_y - \rho_i^2 \frac{\partial^2}{\partial x^2}\right) A_\parallel + \frac{1}{2} \beta_i A_1$$
$$= \beta_i \frac{m_i}{m_e} \frac{c T_e}{e n_0 v_e^2} \int_{-\infty}^{\infty} dv_\parallel \, v_\parallel (h_i - h_e)$$

ψ is calculated from Ampere's Law and the kinetic equation moments

$$\int_{-\infty}^{\infty} dv_\parallel v_\parallel \frac{\partial h_e}{\partial t} , \quad \int_{-\infty}^{\infty} dv_\parallel v_\parallel \frac{\partial h_i}{\partial t}$$

ASYMPTOTICS ($\beta = 0$)

$$\tilde{n}^{NA}(x) = i\,\frac{e}{T_e}\int_{-\infty}^{\infty} dv_{\parallel}\, F_e \left\{\omega - \omega_*\left[1+\eta_e\left(\frac{v_{\parallel}^2}{v_e^2}-\frac{1}{2}\right)\right]\right\}$$

$$\times \int_0^{\infty} d\tau\; \Theta(x;v_{\parallel},\tau)$$

$$\Theta = \int_{-\infty}^{\infty} dx'\, G(x,x';v_{\parallel},\tau)\,\phi(x')$$

$$G = \frac{1}{\sqrt{4\pi D\tau}}\, C_{k,\omega}(x;v_{\parallel},\tau)\exp\left[-\frac{(x-x'-iDk_{\parallel}'v_{\parallel}\tau^2)^2}{4D\tau}\right]$$

$$C_{k,\omega} = \exp\left[i(\omega - k_{\parallel}' x v_{\parallel})\tau - \frac{1}{3}(k_{\parallel}'v_{\parallel})^2 D\tau^3\right]$$

$$G(x,x';v_{\parallel},\tau) \approx G(x,x+\delta x;v_{\parallel},\tau)$$

$$\Theta \approx C_{k,\omega}\,\phi(x) + \frac{d}{dx}\left[D\tau\, C_{k,\omega}\,\frac{d\phi(x)}{dx}\right]$$

$$-\frac{1}{2}(Dk_{\parallel}'v_{\parallel}\tau^2)^2\, C_{k,\omega}\,\frac{d^2\phi(x)}{dx^2}$$

COMPARISON OF KROOK AND DIFFUSION RESULTS

GROWTH RATES: Larger with diffusion

SATURATION LEVEL: Larger with diffusion

EIGENMODES: Similar

PHYSICAL EXPLANATION:

Competition between ion shear damping and electron dissipation

$$\gamma = \gamma_e - \gamma_i$$

Shear damping independent of ω_e: γ_i

Electron dissipation proportional to τ_e: $\gamma_e = \frac{\alpha}{\omega_e}$

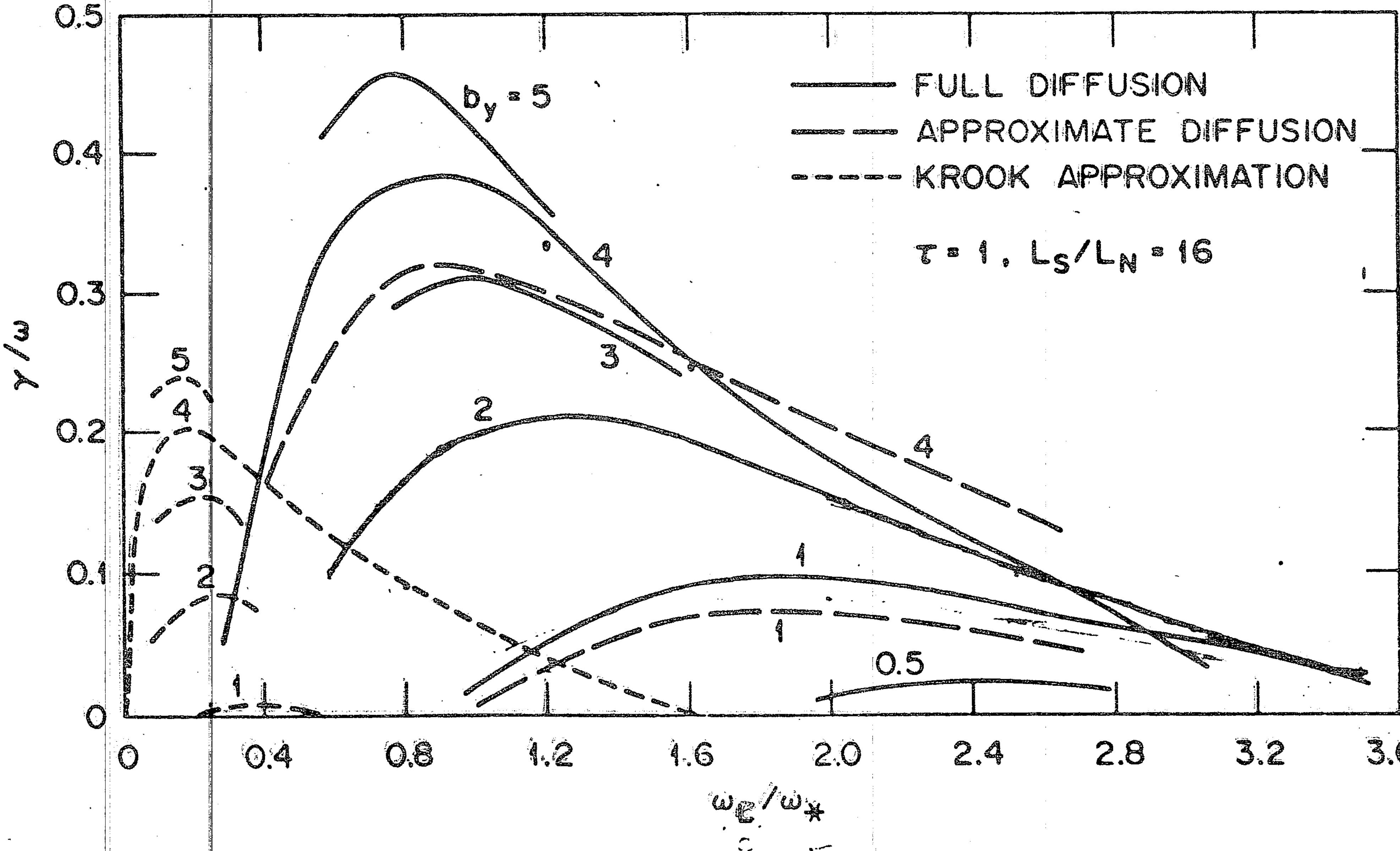
Asymptotic solution indicates factor of 2.8:

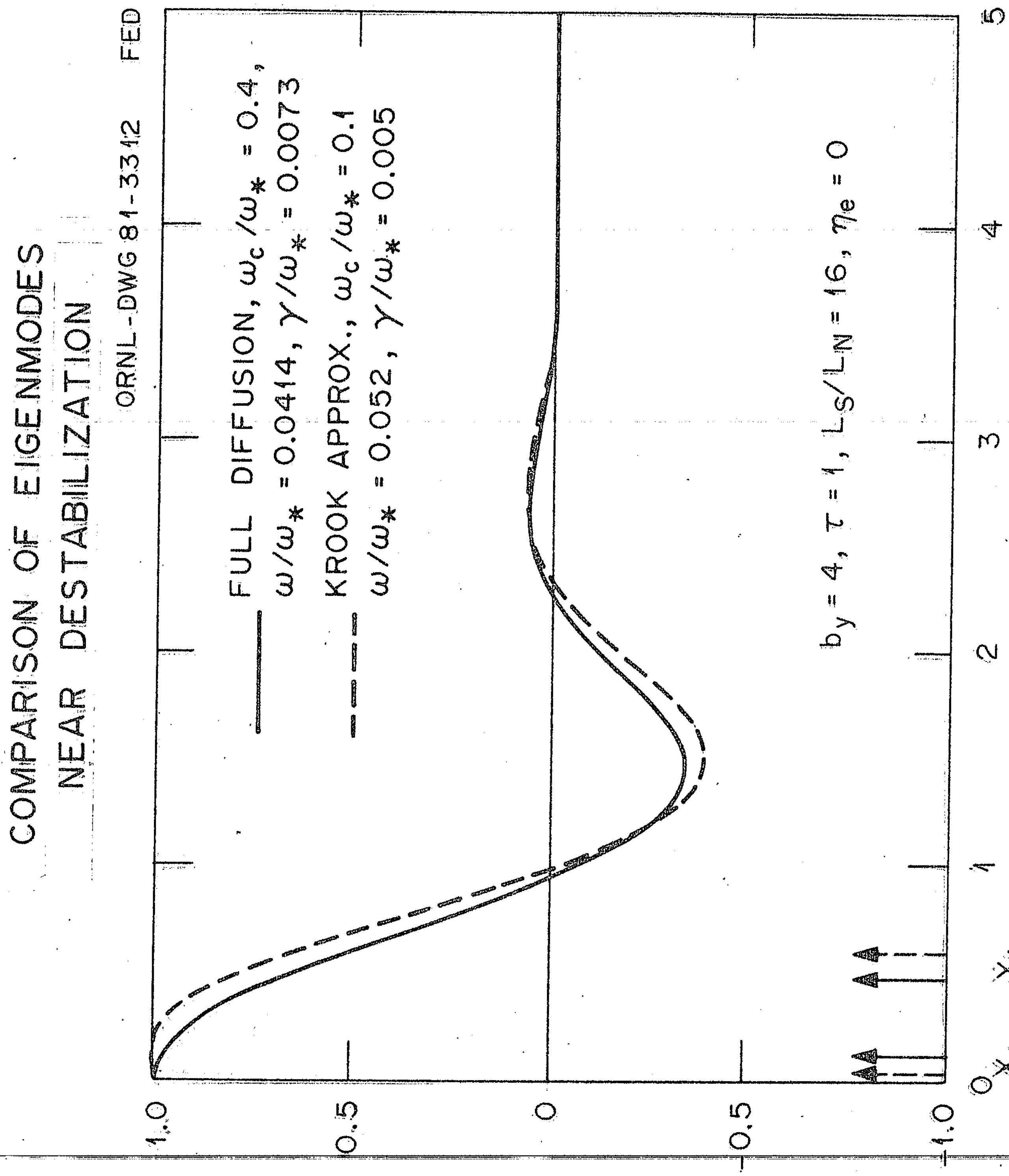
$$\gamma = \frac{2.8\alpha}{\omega_e} - \gamma_i$$

Other factors: frequency shift, modification of φ

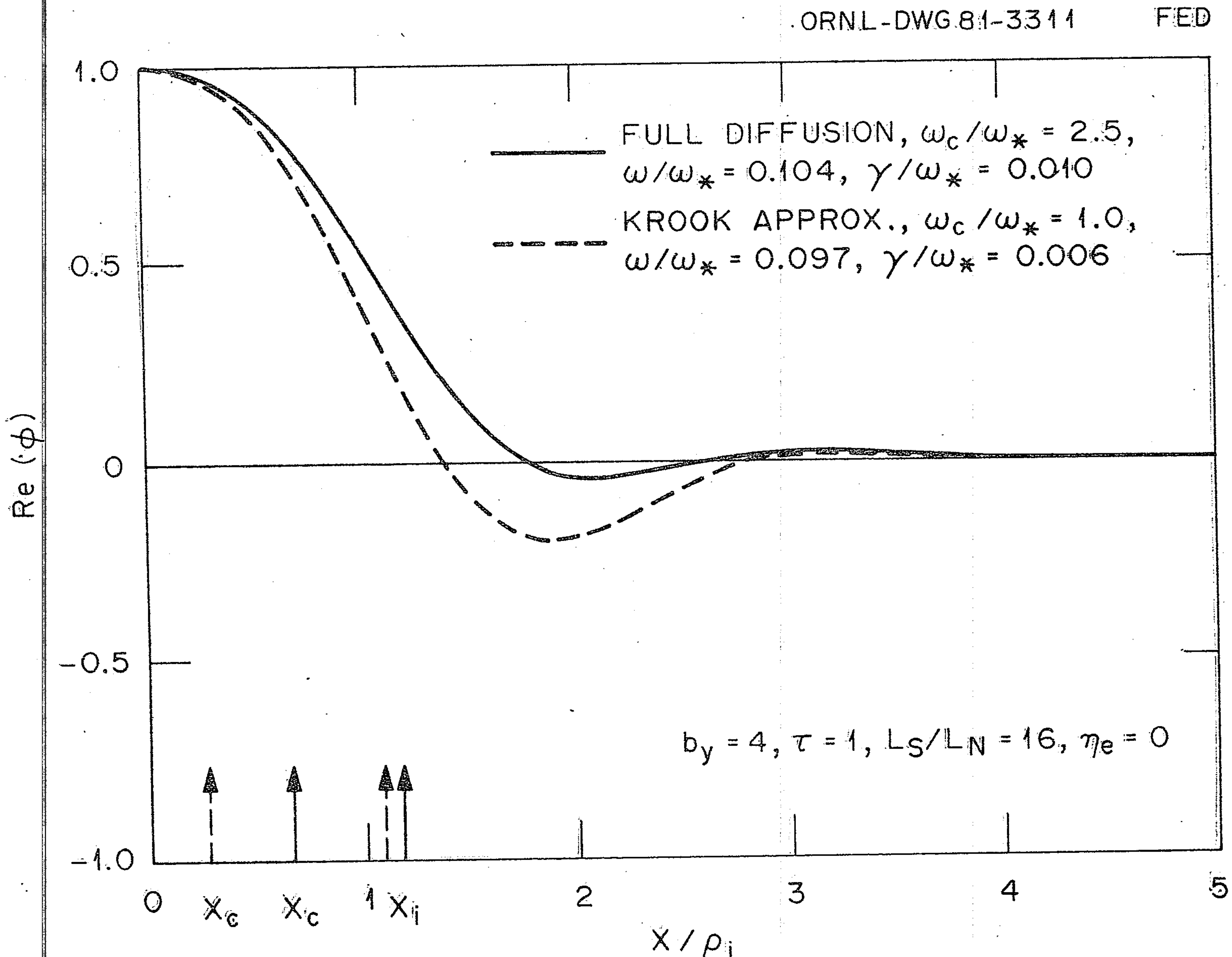
NONLINEAR GROWTH vs DIFFUSION FOR VARIOUS $b_y = k_y^2 \rho_i^2$

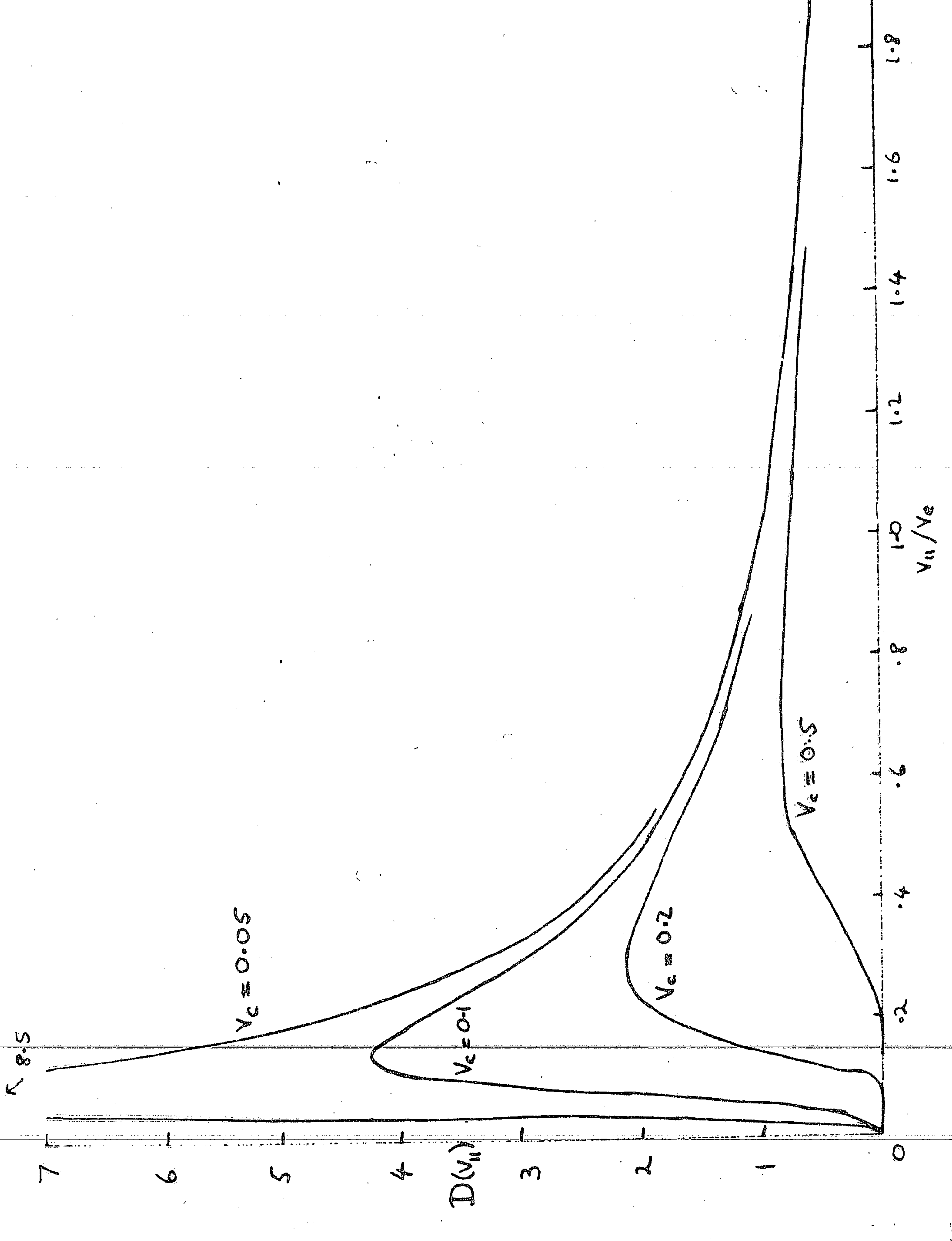
ORNL-DWG 81-2677 FED




COMPARISON OF EIGENMODES
NEAR DESTABILIZATION
ORNL-DWG 81-3312 FED
FULL DIFFUSION, $\omega_c/\omega_* = 0.4$,
$\omega/\omega_* = 0.0414$, $\gamma/\omega_* = 0.0073$
KROOK APPROX., $\omega_c/\omega_* = 0.1$
$\omega/\omega_* = 0.052$, $\gamma/\omega_* = 0.005$
$b_y = 4$, $\tau = 1$, $L_S/L_N = 16$, $\eta_e = 0$
Re (ϕ)
1.0
0.5
0
-0.5
-1.0
0
X_c
X_i
1
2
3
4
5
X/ρ_i

COMPARISON OF EIGENMODES NEAR SATURATION





← 8·5
$V_c = 0{\cdot}05$
$V_c = 0{\cdot}1$
$V_c = 0{\cdot}2$
$V_c = 0{\cdot}5$
$D(v_{\|})$
7
6
5
4
3
2
1
0
·2
·4
·6
·8
1·0
1·2
1·4
1·6
1·8
$v_{\|}/v_e$

FINITE β

EIGENMODE EQUATIONS

$$\frac{\partial}{\partial t} \rightarrow -i\omega \quad , \quad D\frac{\partial^2}{\partial x^2} \rightarrow -\omega_c$$

$$\left(\rho_i^2 \frac{d^2}{dx^2} - \xi\right)\phi = \left(\Lambda - \xi - \mu^2 x^2 + \frac{\sigma_e}{|x|}\right)(\phi - \psi)$$

$$\left(\rho_i^2 \frac{d^2}{dx^2} - b_y\right)x\psi = \frac{d x_A^2}{x}\left[\Lambda - \xi - \mu^2 x^2 + \frac{\sigma_e}{|x|}\left(1 + i\frac{\omega_c}{\omega}\right)\right](\phi - \psi)$$

$$\xi = \frac{1-\Gamma_0}{\Gamma_0 - \Gamma_1} \quad , \quad \Lambda = \frac{1}{d\omega}\left\{\left[1 + \tau(1-\Gamma_0)\right]\omega - \Gamma_0\omega_*\right\}$$

$$\mu^2 = \frac{\Gamma_0}{\Gamma_0 - \Gamma_1}\left(\frac{\omega_*}{\tau\omega}\frac{L_n}{L_s}\frac{1}{\rho_i}\right)^2 , \quad \sigma_e = \frac{(\omega - \omega_*)x_e}{d\omega} Z\left(\frac{x_e + i x_c}{|x|}\right)$$

$$d = (\Gamma_0 - \Gamma_1)\left(\tau + \frac{\omega_*}{\omega}\right), \quad x_e = \frac{\omega}{k_\parallel' v_e} , \quad x_c = \frac{\omega_c}{k_\parallel' v_e}$$

$$\psi \approx \frac{x_\beta^2}{x^2 + x_\beta^2}\phi \quad , \quad x_\beta^2 = d\, x_A^2 = d\left(\frac{\omega}{k_\parallel' v_A}\right)^2$$

$$\frac{d^2\phi}{dx^2} = \frac{\left(\Lambda - \mu^2 x^2 + \frac{\sigma_e}{|x|}\right)x^2 + \xi x_\beta^2}{x^2 + x_\beta^2}\phi$$

$$\Theta(x;v_{\parallel},\tau) \approx C_{k,\omega}(x;v_{\parallel},\tau)\,\phi(x)$$

$$\tilde{n}^{NA}(x) \approx \text{function}\left[\, I^{(n)}(x) \,\right]$$

$$I^{(n)}(x) = \frac{1}{n_0}\int_0^{\infty} d\tau \int_{-\infty}^{\infty} dv_{\parallel}\, F_e\, v_{\parallel}^{n}\, C_{k,\omega}(x;v_{\parallel},\tau)$$

$x > x_c$: $I^{(0)} \sim \tau_c \sqrt{\pi}\, \dfrac{x_c}{x}$

$x < x_c,\ \omega\tau_c < 1$: $I^{(0)} \sim \frac{1}{3}\tau_c\, B(\frac{1}{3},\frac{1}{6})$

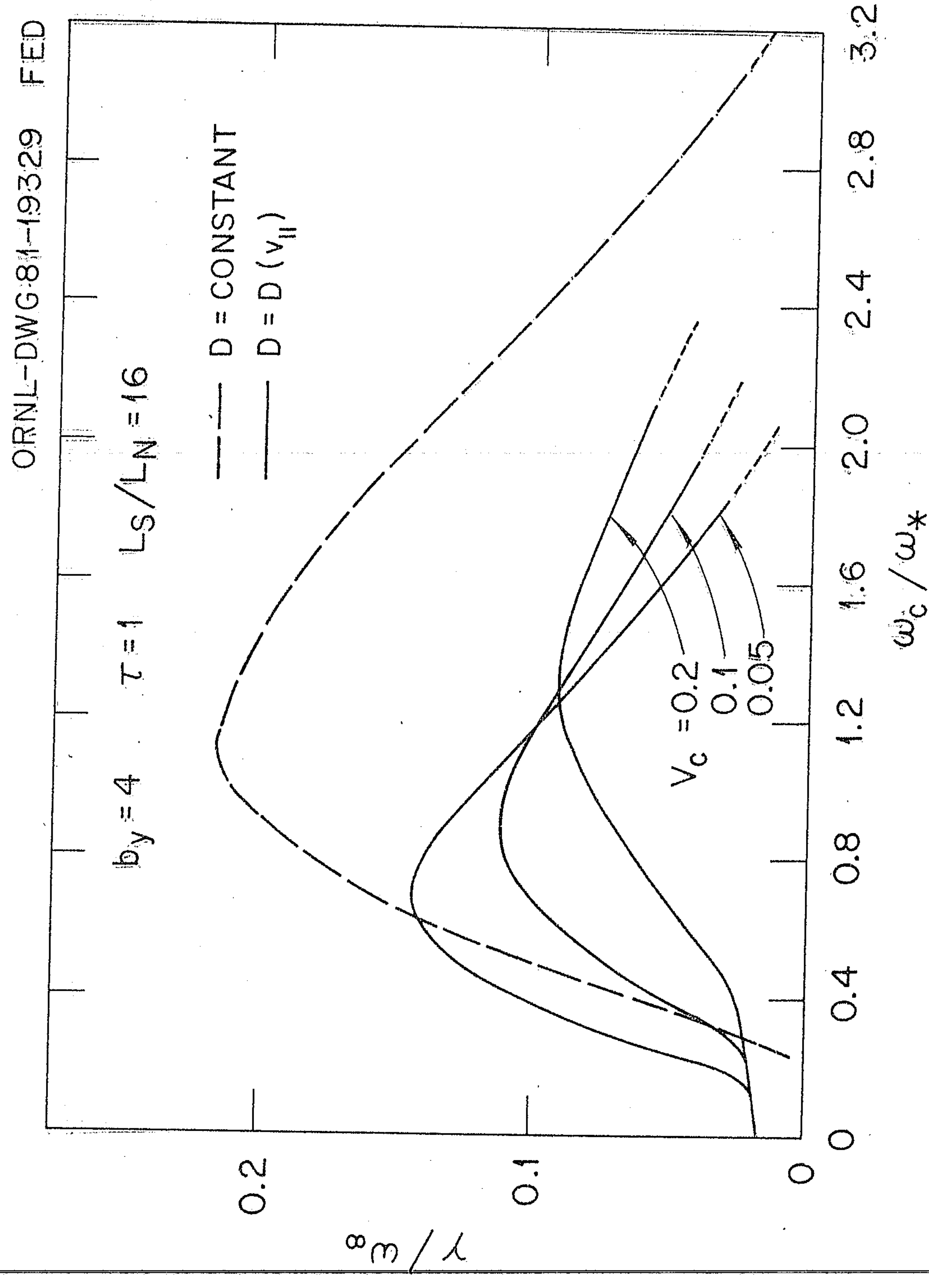
Interpolate : $I^{(0)} \sim \dfrac{2.80\,\tau_c}{1 + 1.58\,x/x_c}$

$$I^{(0)}(\text{Krook}) = \frac{-i}{k_{\parallel}'\, v_e\, |x|}\, Z\left(\frac{x_e + i x_c}{|x|}\right)$$

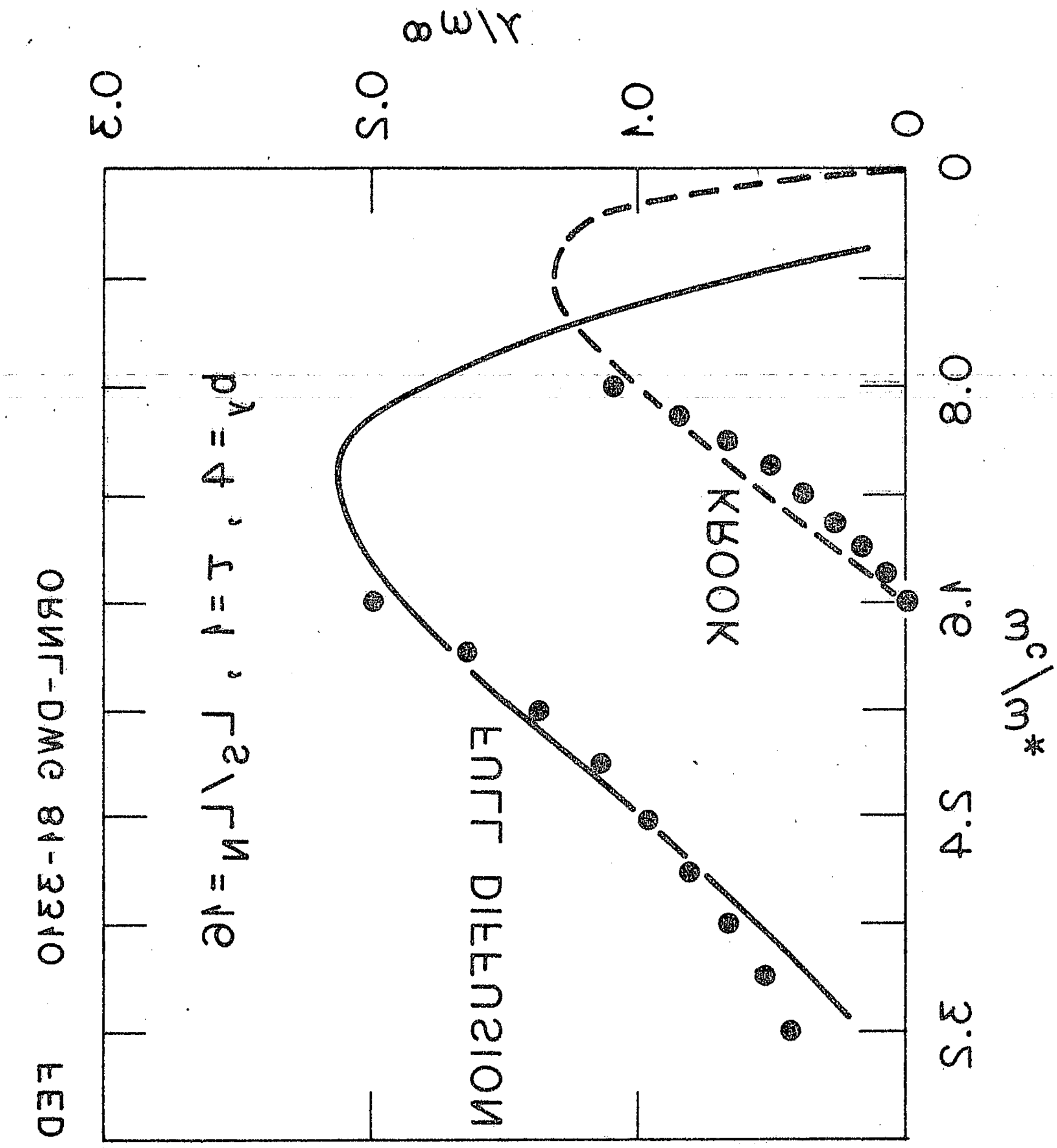
$x > x_c,\ \omega\tau_c < 1$: $I^{(0)}(\text{Krook}) \sim \tau_c \sqrt{\pi}\, \dfrac{x_c}{x}$

$x < x_c,\ \omega\tau_c < 1$: $I^{(0)}(\text{Krook}) \sim \tau_c$

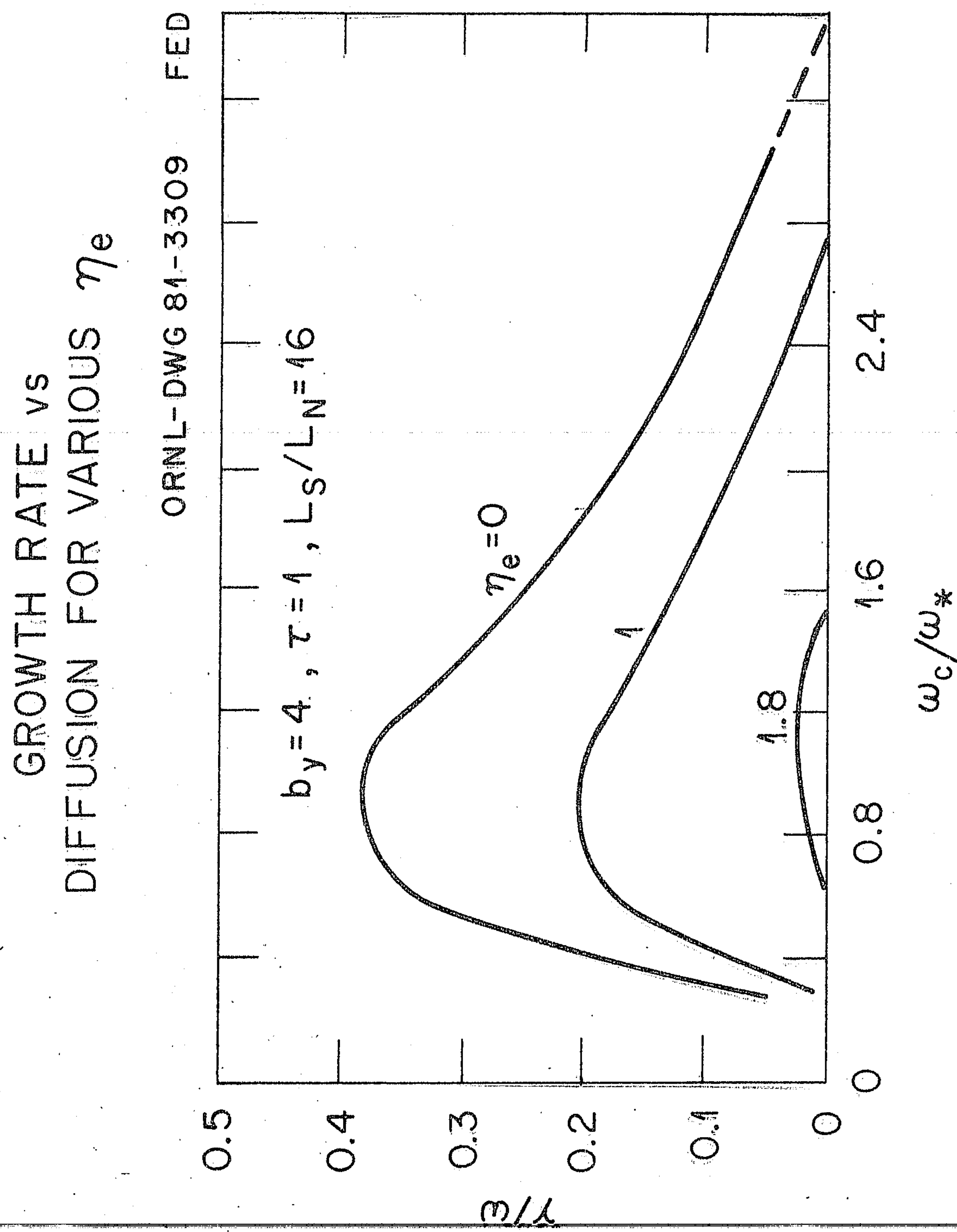
<u>Alternative</u>: $I^{(0)}(\text{interp.}) = \dfrac{-i}{k_{\parallel}'\, v_e\, |x|}\, Z\left(\dfrac{x_e + i x_c/2.80}{|x|}\right)$



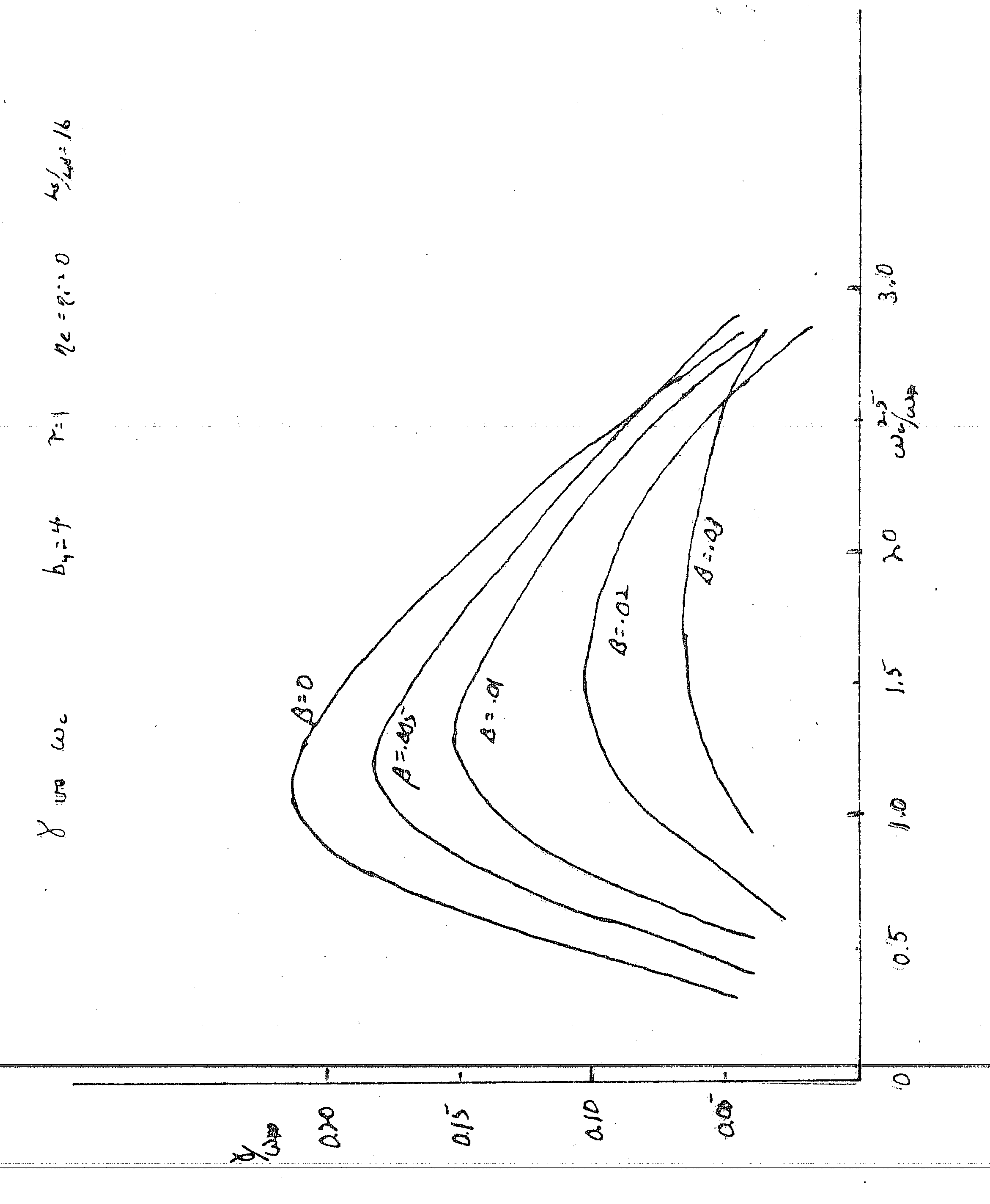
ORNL-DWG 81-19329 FED
$b_y = 4$
$\tau = 1$
$L_S/L_N = 16$
D = CONSTANT
D = D ($v_{\parallel}$)
$V_c = 0.2$
0.1
0.05
ω_c/ω_*
γ/ω_*
0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
0
0.1
0.2

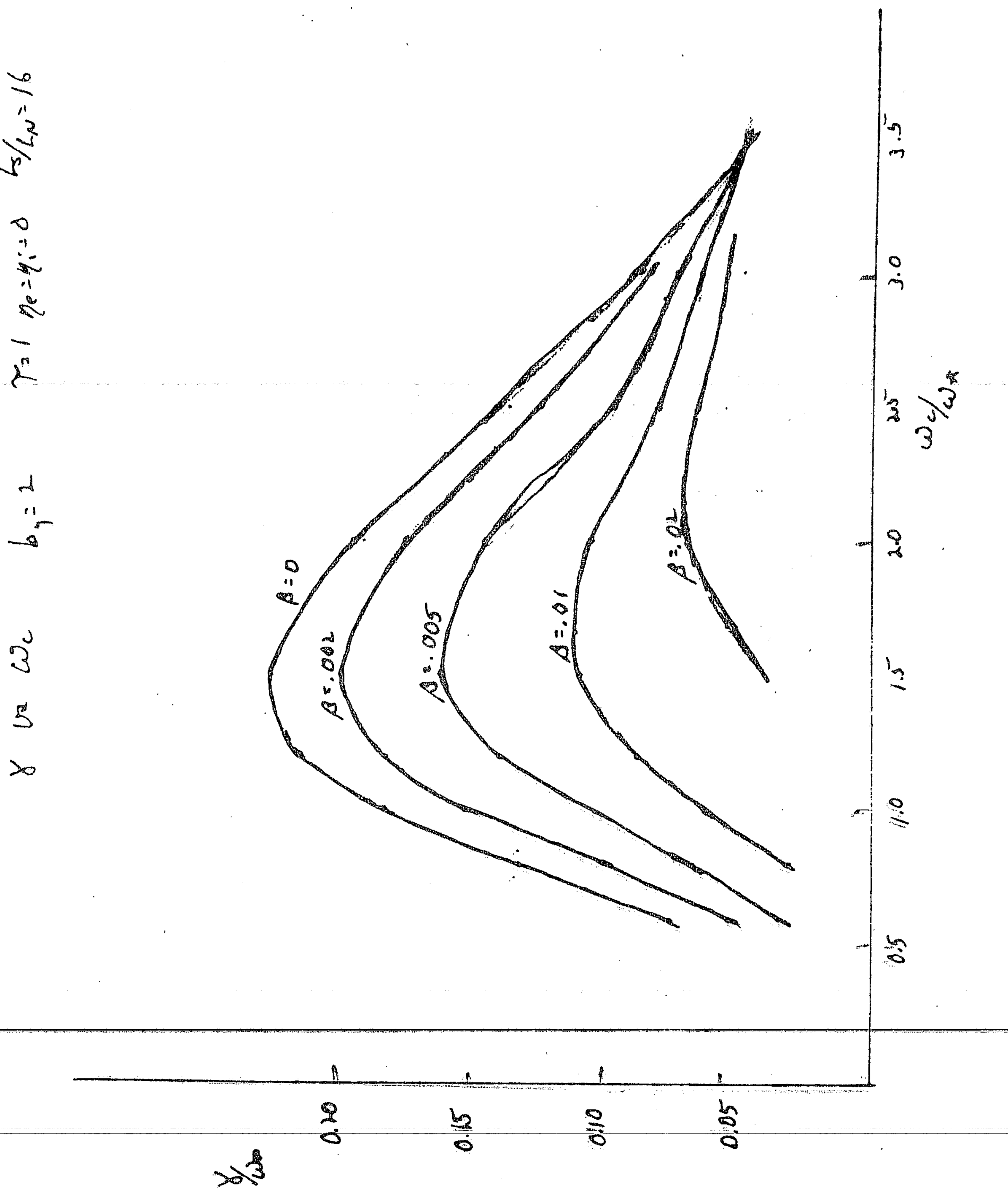


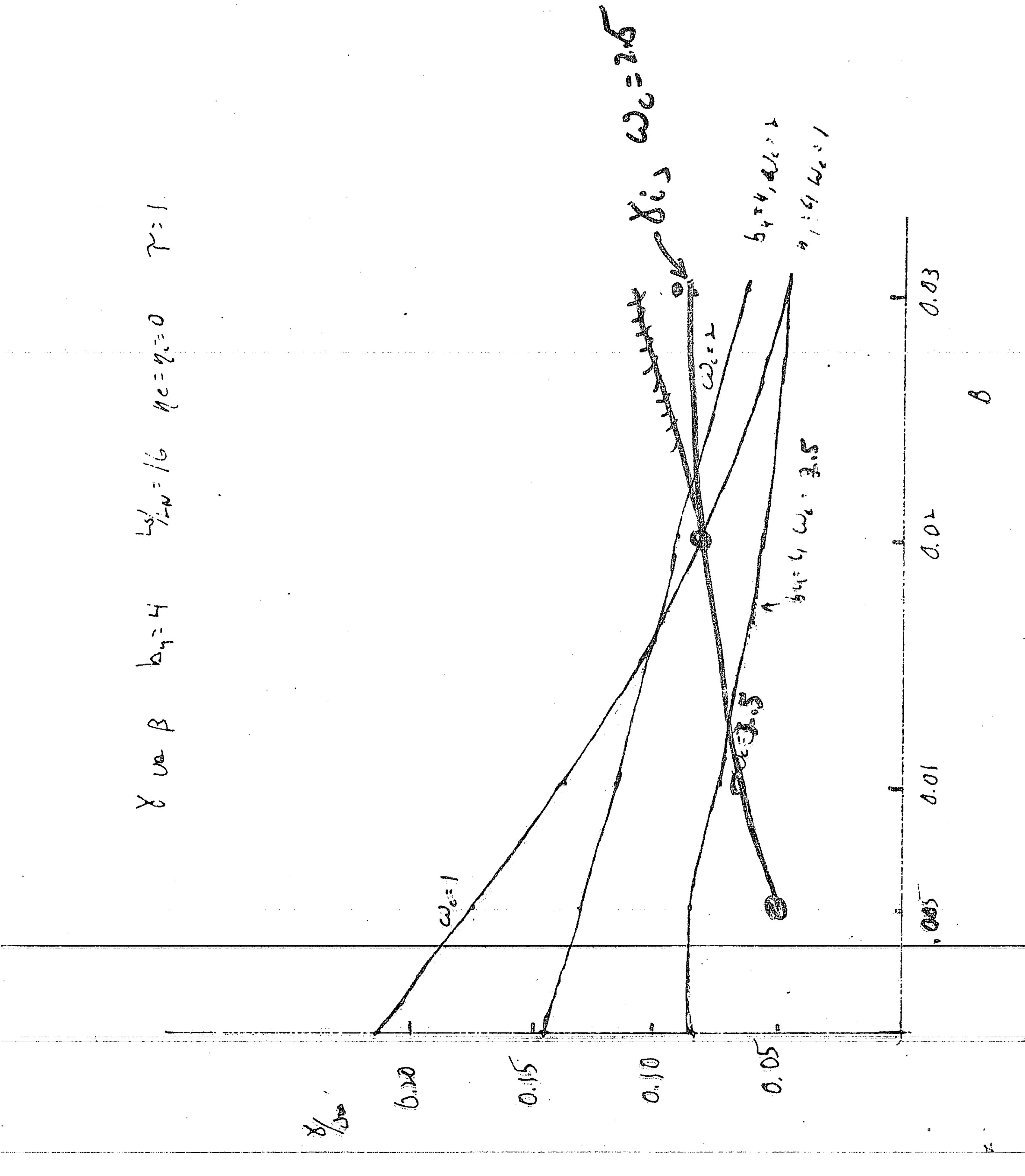
ASYMPTOTIC SCALING OF
GROWTH RATE WITH DIFFUSION

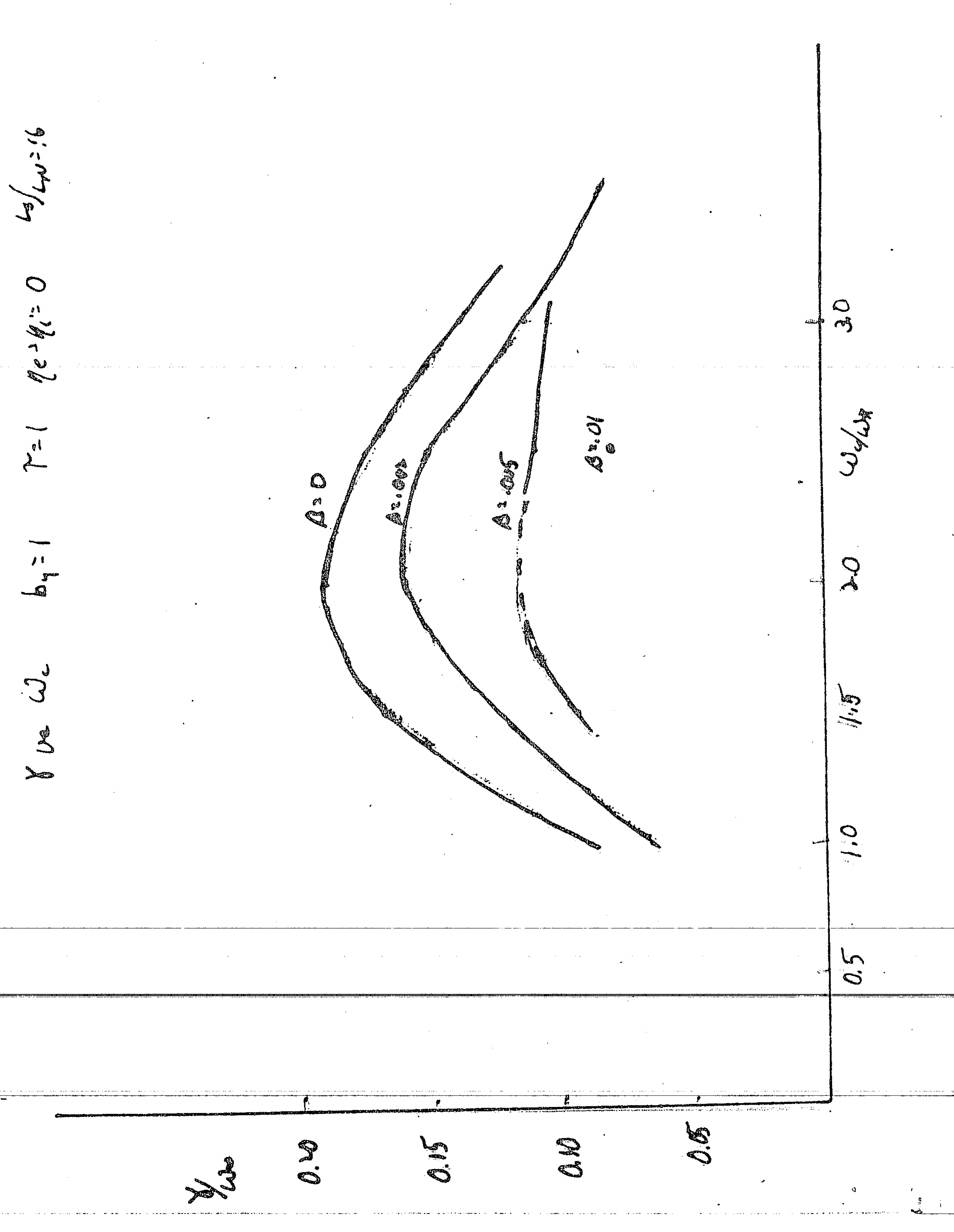


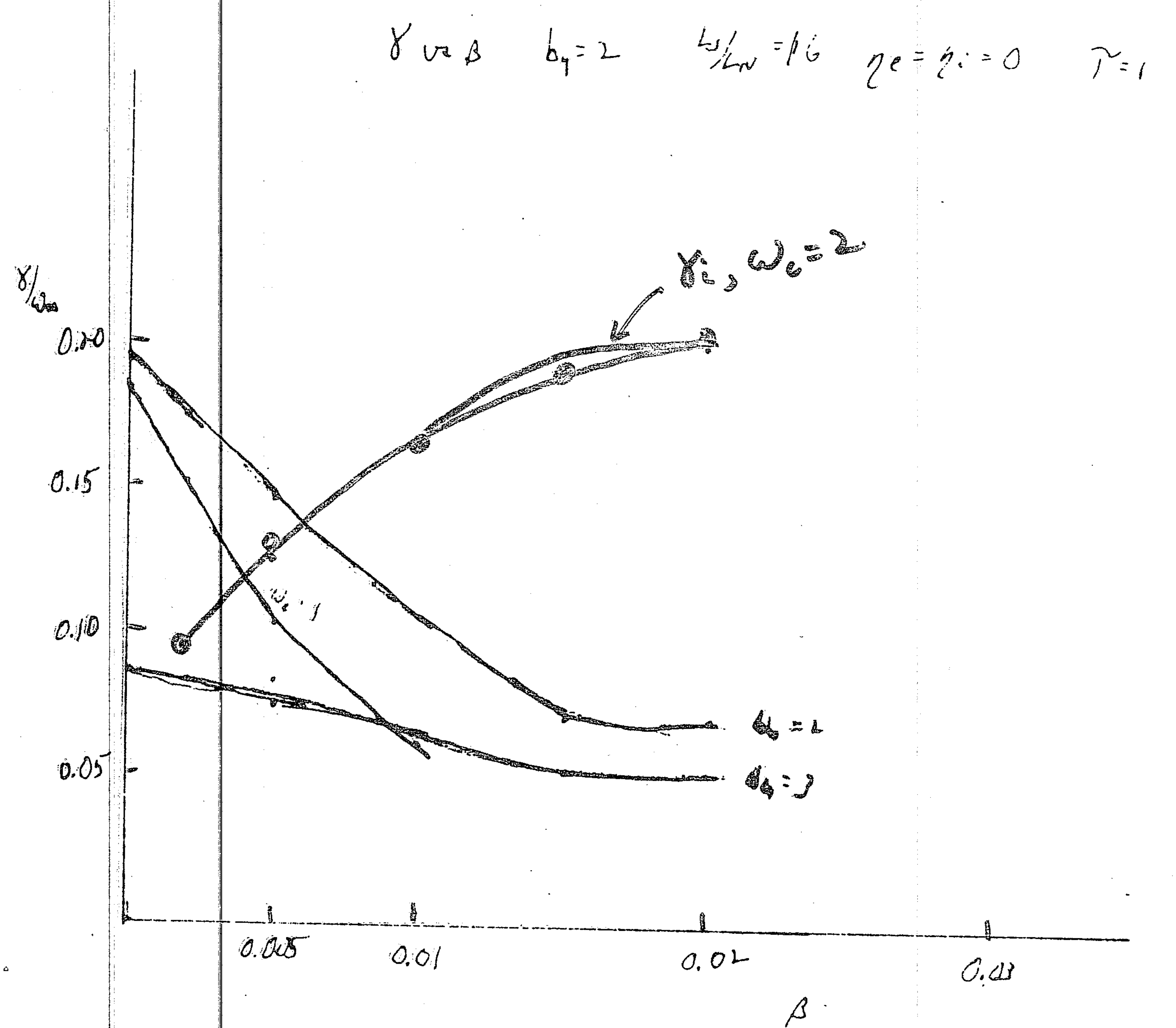
GROWTH RATE vs
DIFFUSION FOR VARIOUS η_e
ORNL-DWG 81-3309 FED
$b_y = 4$, $\tau = 1$, $L_S/L_N = 16$
$\eta_e = 0$
1
1.8
γ/ω
0.5
0.4
0.3
0.2
0.1
0
0
0.8
1.6
2.4
ω_c/ω_*


β=0
β=.005
β=.01
β=.02
β=.03
0
0.5
1.0
1.5
2.0
3.0
0.20
0.15
0.10
0.05
τ=1

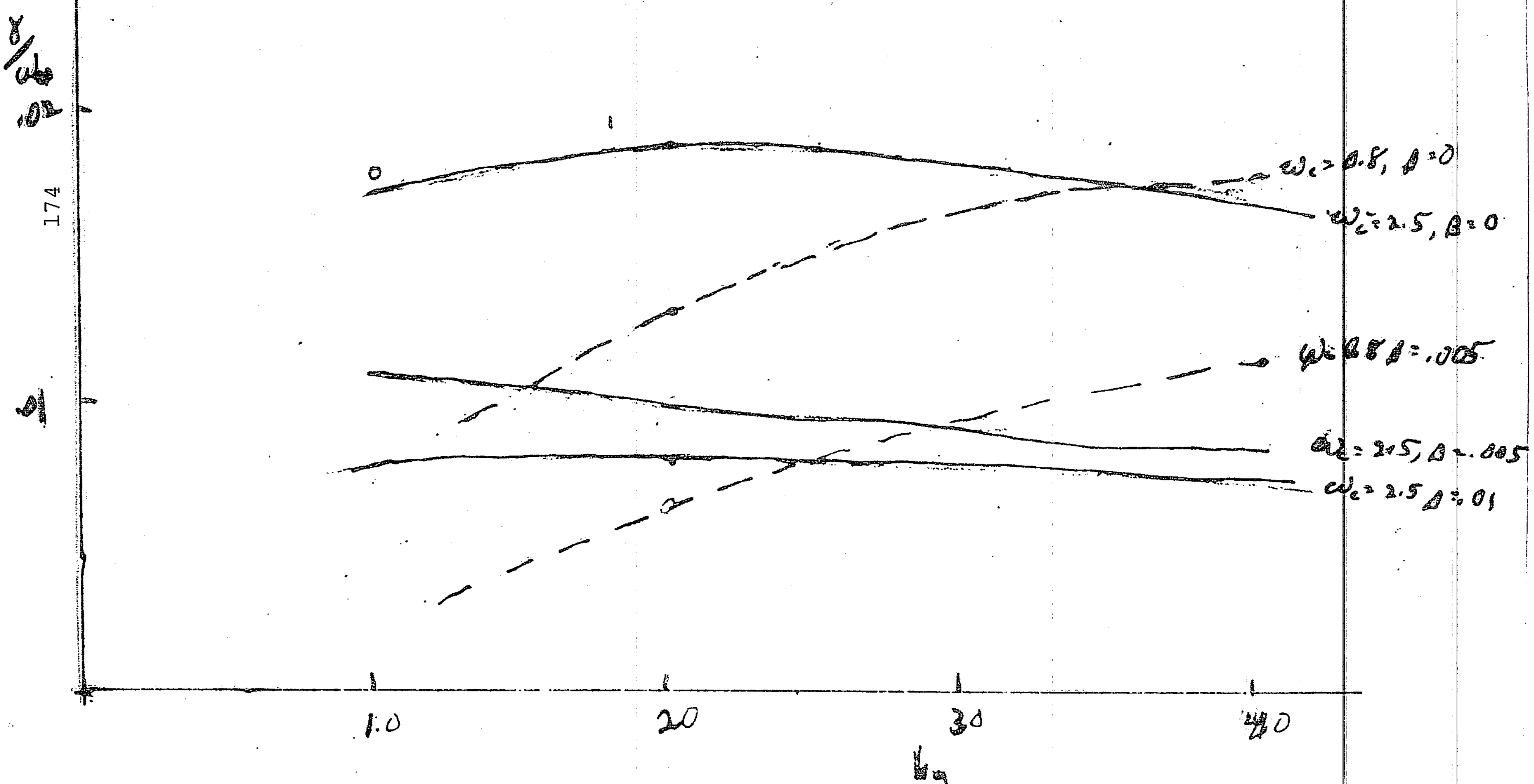

γ vs ωc
b_i = 2
τ=1 η_e=η_i=0
L_s/L_N=16
β=0
β=.002
β=.005
β=.01
β=.02
0.5
1.0
1.5
2.0
2.5
3.0
3.5
ω_c/ω_*
0.05
0.10
0.15
0.20
γ/ω_*


0.01
0.02
0.03
β
0.20
0.15
0.10
0.05


β=0
β=.005
β=.01
0.5
1.0
1.5
2.0
3.0
0.05
0.10
0.15
0.20


γ vs β   b_y = 2   ω/ω_n = 16   η_e = η_i = 0   τ = 1
γ/ω
0.20
0.15
0.10
0.05
γ_i, ω_e = 2
ω_e = 1
β
0.005
0.01
0.02
0.03

γ vs $b_y = k_y^2 \rho_i^2$ $\tau = 1$ $\eta_e = \eta_i = 0$ $L_s/L_n = 16$



Conclusions

1. $\beta = 0$

   a. Qualitatively HM ≡ Diffusion
      Quantitatively scaling OK

   b. $k_\theta \rho_i$ large range ⇒ $\delta\omega$ large

   c. $D(v_n)$ – parameters determine saturation
      D_e ⇒ self-starting

   d. $\chi_i \sim \chi_T$ $(b_y = 4)$

   e. η_e scaling of saturation: $(1 - \eta_e/g)$

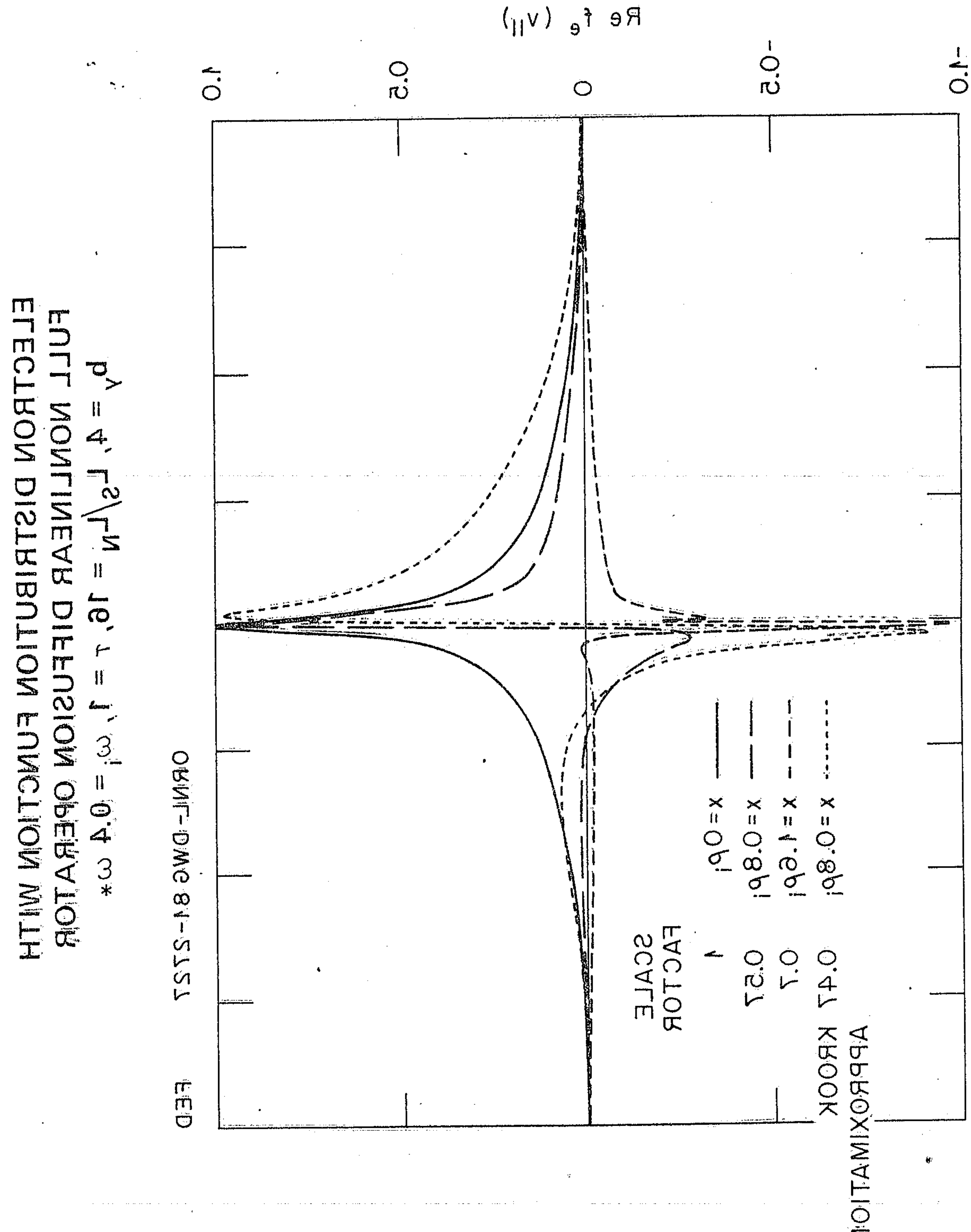
   f. v-space diffusion important to determine saturation

2. $\beta \neq 0$ (Thus far)

   a. Alcator scaling obtains

   b. $\psi \ll \phi$ at $x \sim 0$

   c. Under study: new renormalizations


ELECTRON DISTRIBUTION FUNCTION WITH
FULL NONLINEAR DIFFUSION OPERATOR
$b_y = 4, L_s/L_N = 16, \tau = 1, \omega_i = 0.4\,\omega^*$
ORNL-DWG 81-2727
FED
Re f_e ($v_{\parallel}$)
$v_{\parallel}/v_{th}$
SCALE FACTOR
x = 0ρ_i 1
x = 0.8ρ_i 0.57
x = 1.6ρ_i 0.7
x = 0.8ρ_i 0.47 KROOK APPROXIMATION

LOW-FREQUENCY MAGNETIC INSTABILITIES AND DIFFUSION

R. HATAKEYAMA

TOHOKU UNIVERSITY

Low-Frequency Magnetic Instabilities and Diffusion (Hatakeyama)

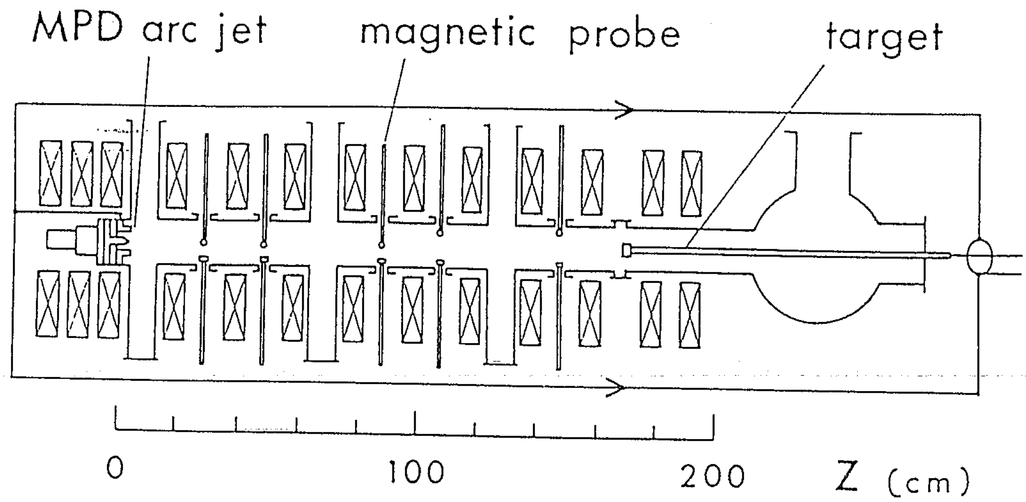
1) Broadband Shear Alfvén-Wave Instabilities driven by a Current and Resistivity Gradient

- $|\omega_e^*| < \omega < \omega_{ci}$, $\omega/k_z \simeq V_A$
- $\tilde{B}_\theta > \tilde{B}_z$
- $J_0 \times \vec{\nabla}\eta_0$ ($\vec{\nabla}T_e$) ⟶ Tokamak
- Diffusion

2) Magnetic Ballooning Instability

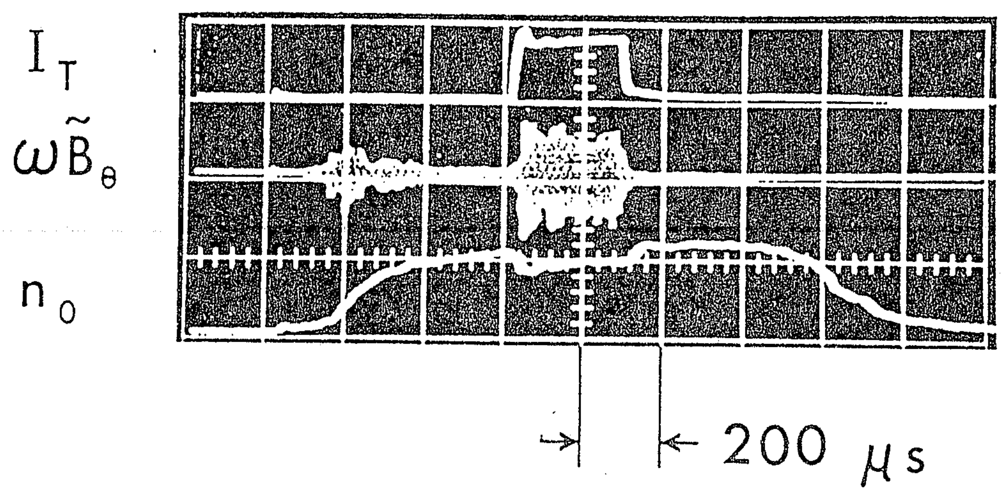
- $0.05 \lesssim \beta$ (< 0.35) , $\omega_r = \delta\,\omega_i^*$ $(0<\delta<1)$
- localized in radial and axial directions (excited in the form like standing wave)
- travels beyond the local mirror with $C_s < \omega/k_z < v_A$, $\lambda_z \simeq 2L_c$
- $\tilde{B}_z > \tilde{B}_\theta$ (compressional)
- diffuse pressure profile (similar to that caused by microturbulence)

Shear Alfvén-Wave Instability



$10^{14} \lesssim n_0 \lesssim 10^{15}\, cm^{-3}$, $T_e \simeq T_i \simeq 5\, eV$

$B_{0z} < 4\, KG$



I_T
$\omega\tilde{B}_\theta$
n_0
200 μs

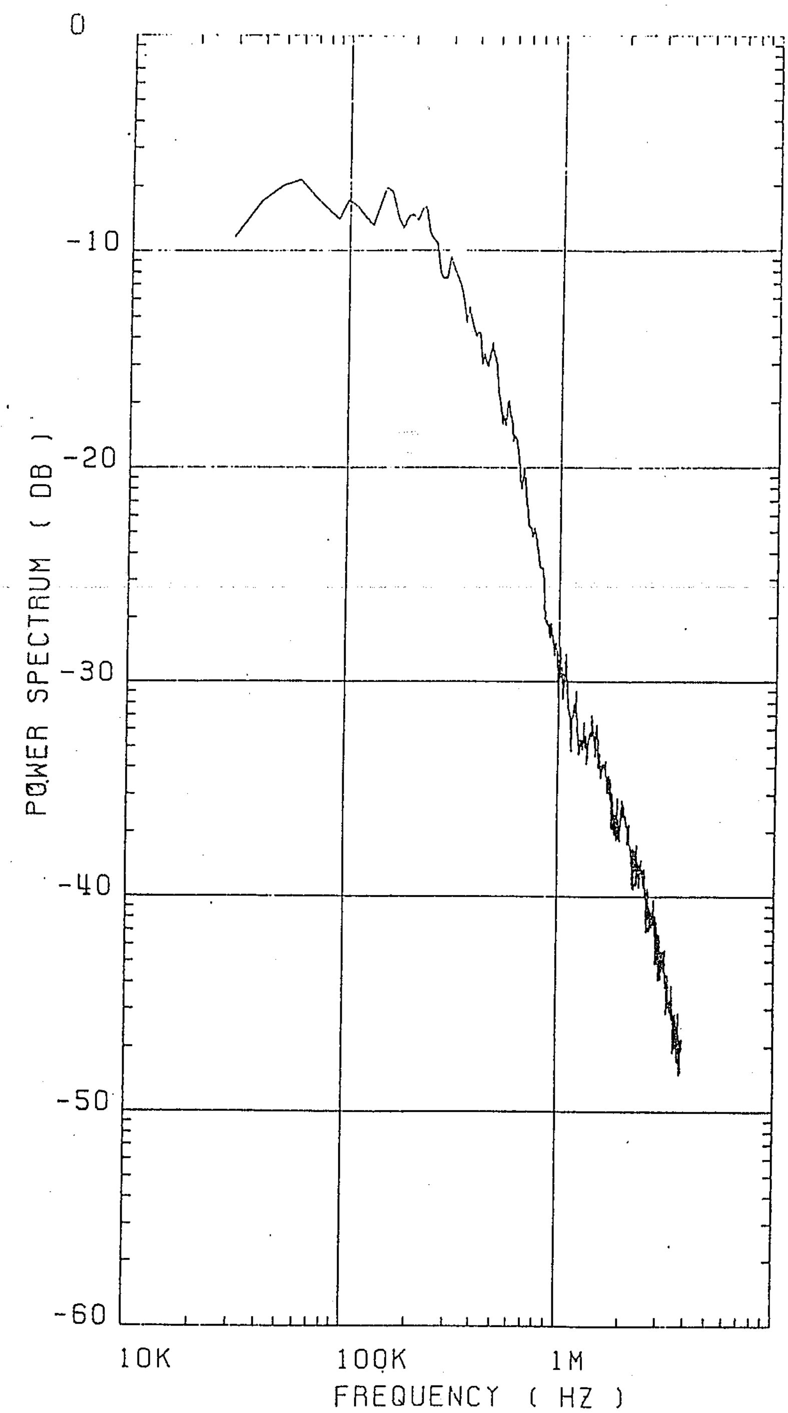
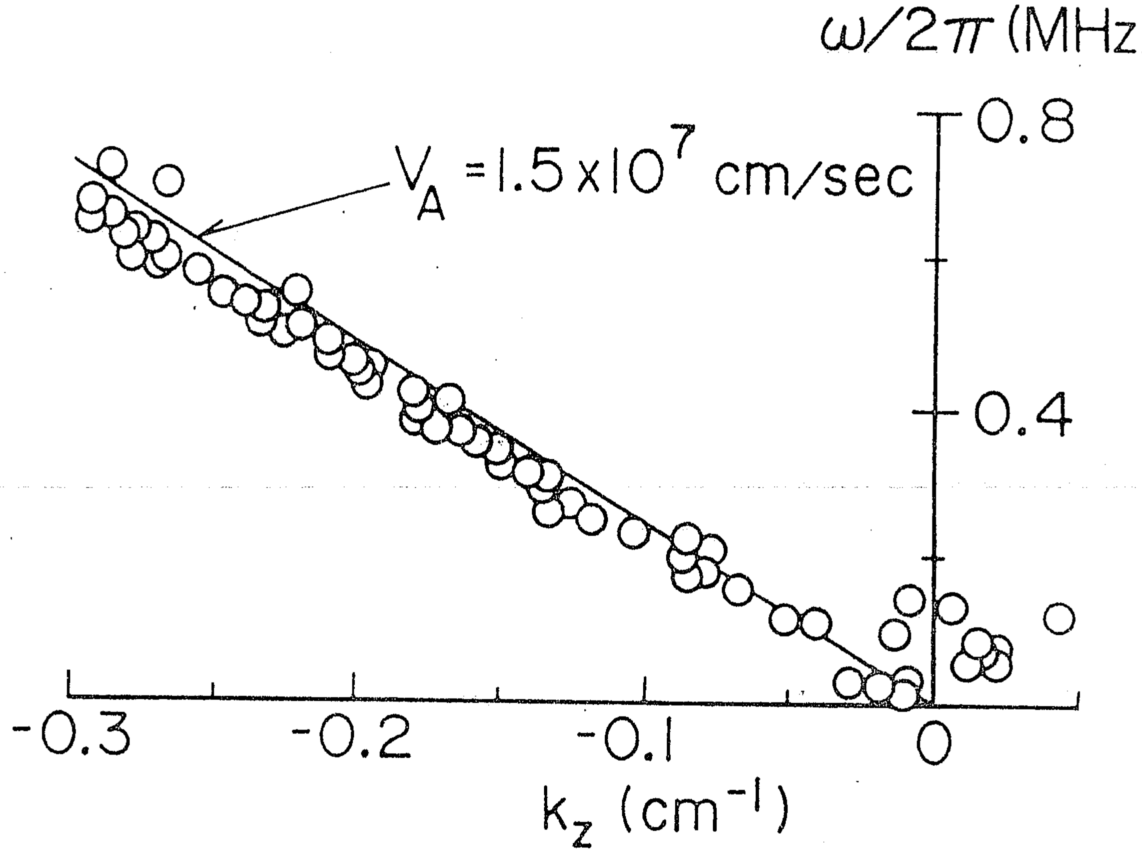
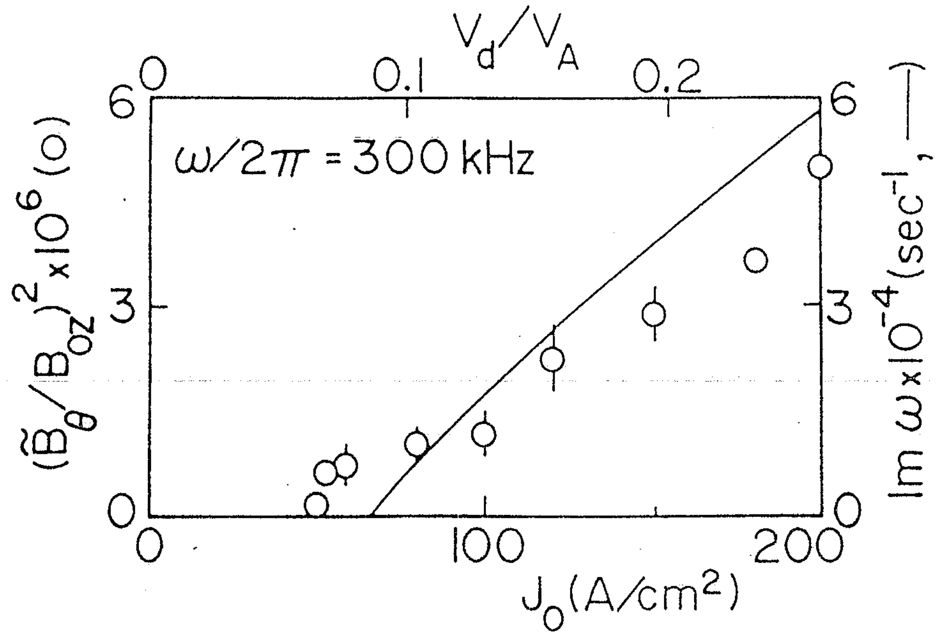


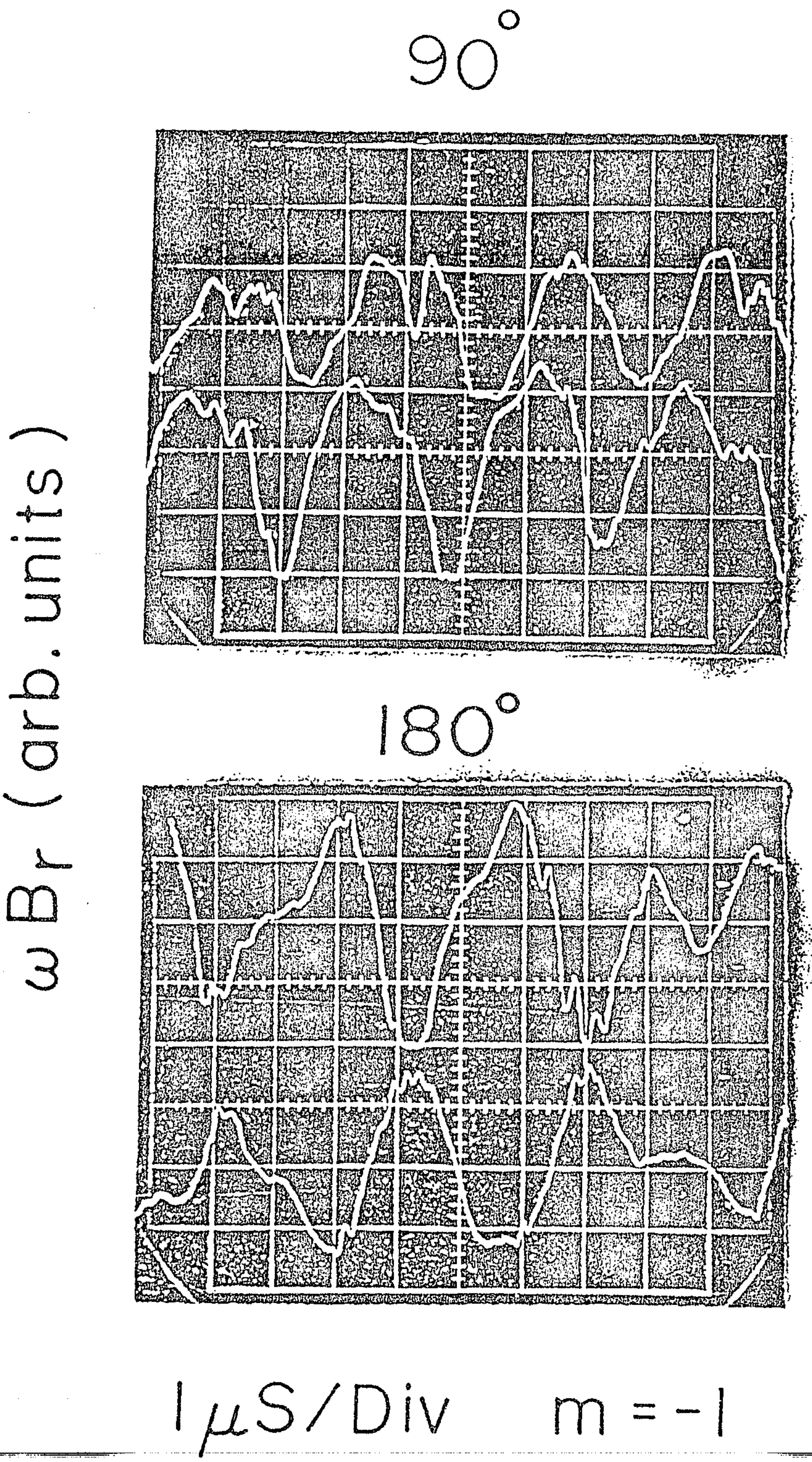
FIG. 1   531019 HE3-1-1-2/1   NO.6

$J_0 \approx 200\ A/cm^2$

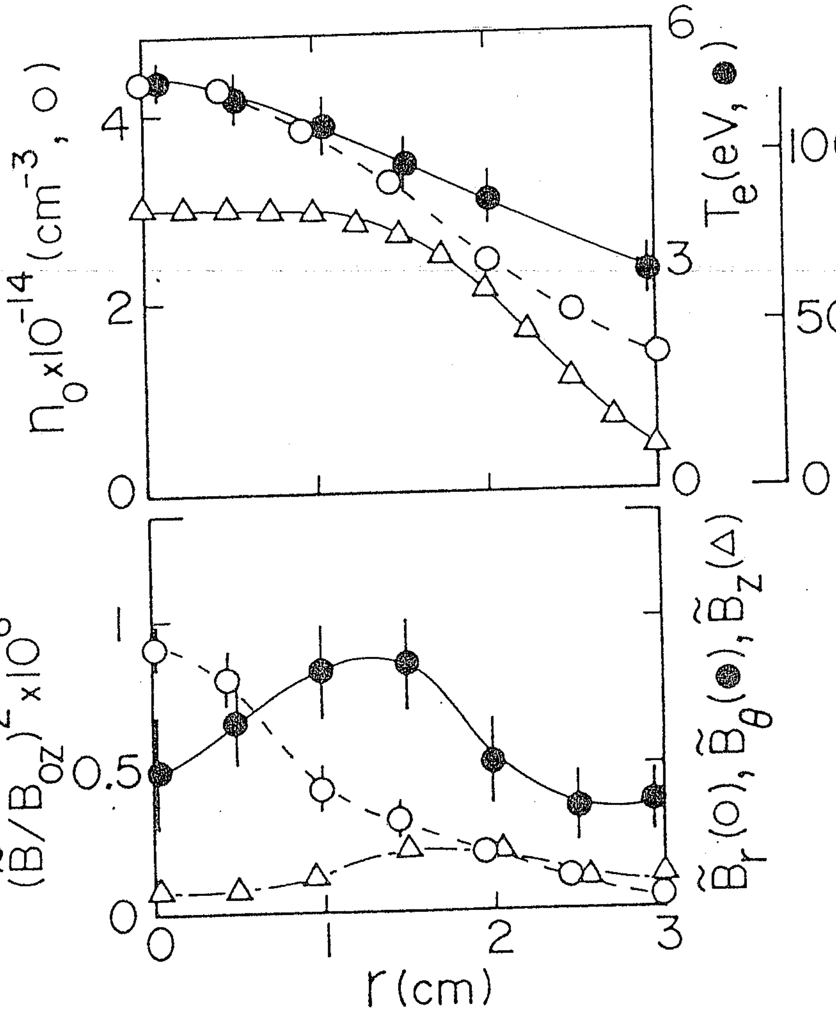


Probe distance ; 20 cm


V_d/V_A
0
0.1
0.2
6
3
0
$\omega/2\pi = 300$ kHz
$(\tilde{B}_\theta/B_{oz})^2 \times 10^6$ (o)
Im $\omega \times 10^{-4}$ (sec^{-1}, ——)
0
100
200
J_o(A/cm^2)

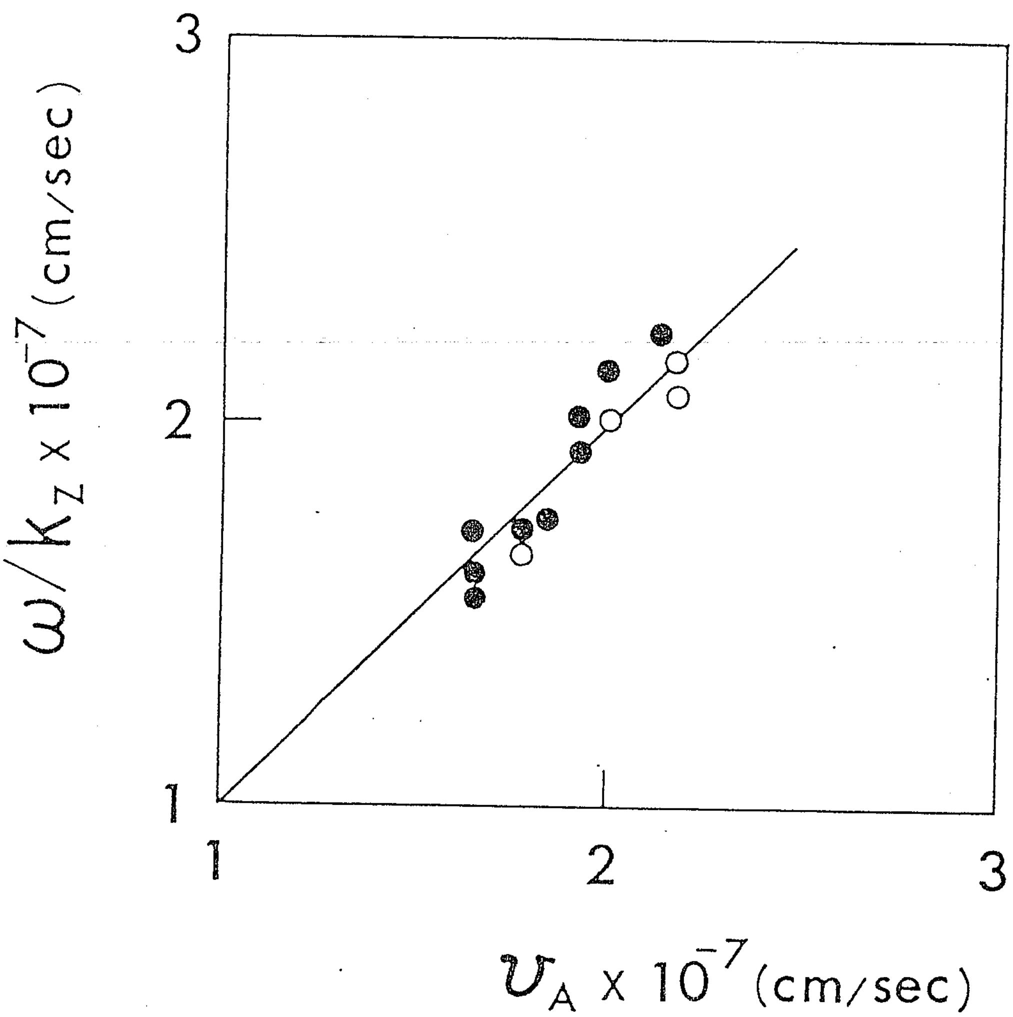


r = 1.5 cm

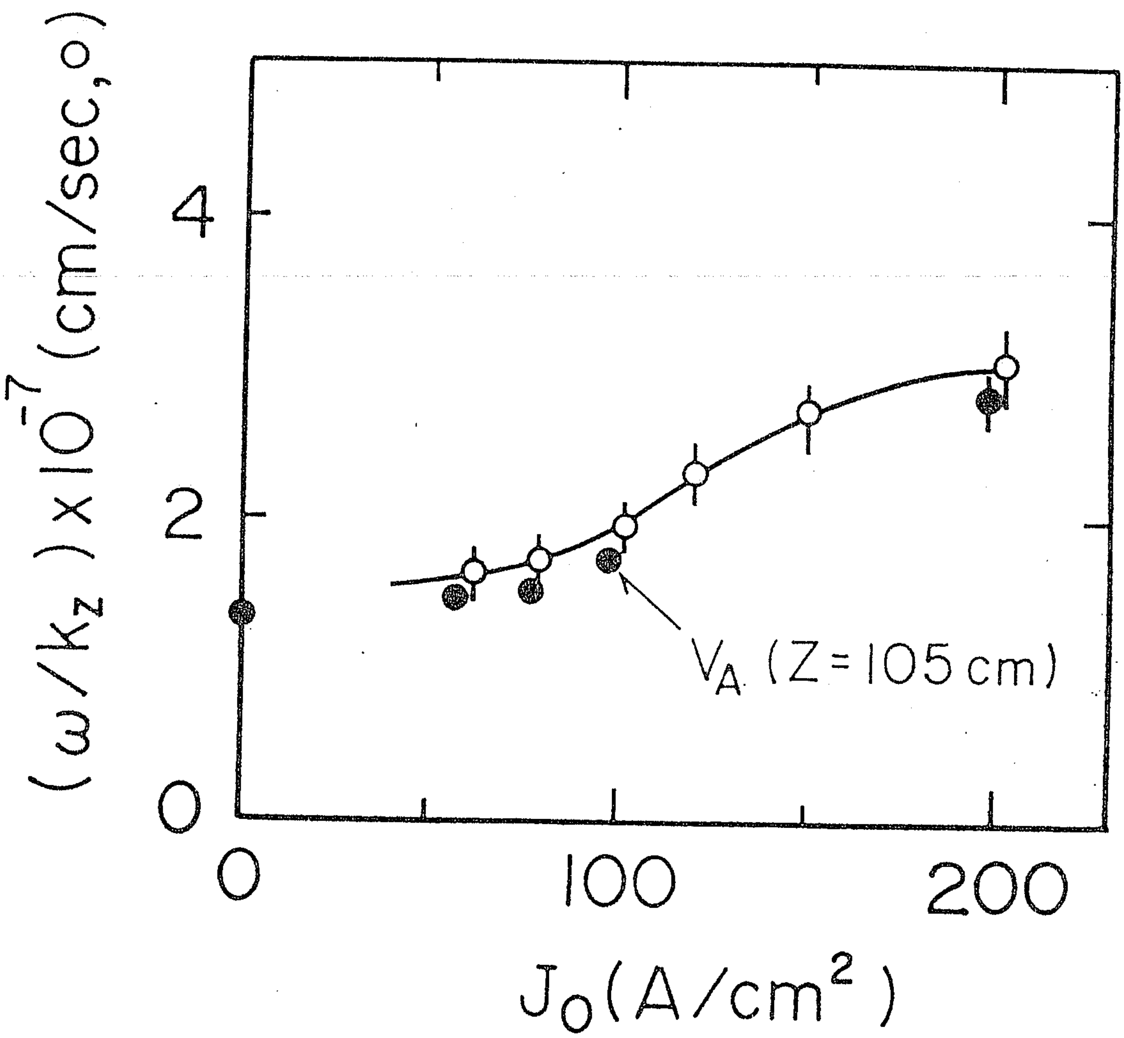


$n_0 \times 10^{-14}$ (cm^{-3}, ○)
T_e(eV, ●)
J_0(A/cm^2, △)
$(\tilde{B}/B_{0z})^2 \times 10^6$
$\tilde{B}_r$(○), $\tilde{B}_\theta$(●), $\tilde{B}_z$(△)
r (cm)

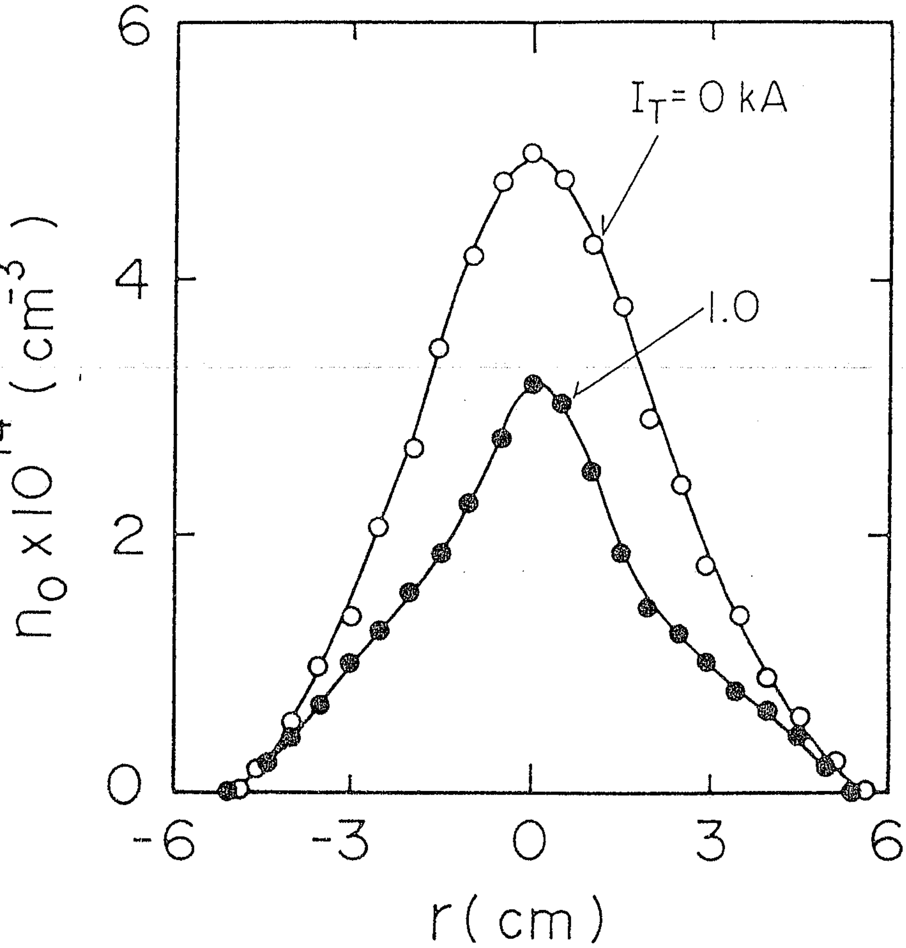
$I_T = 0.8$ KA



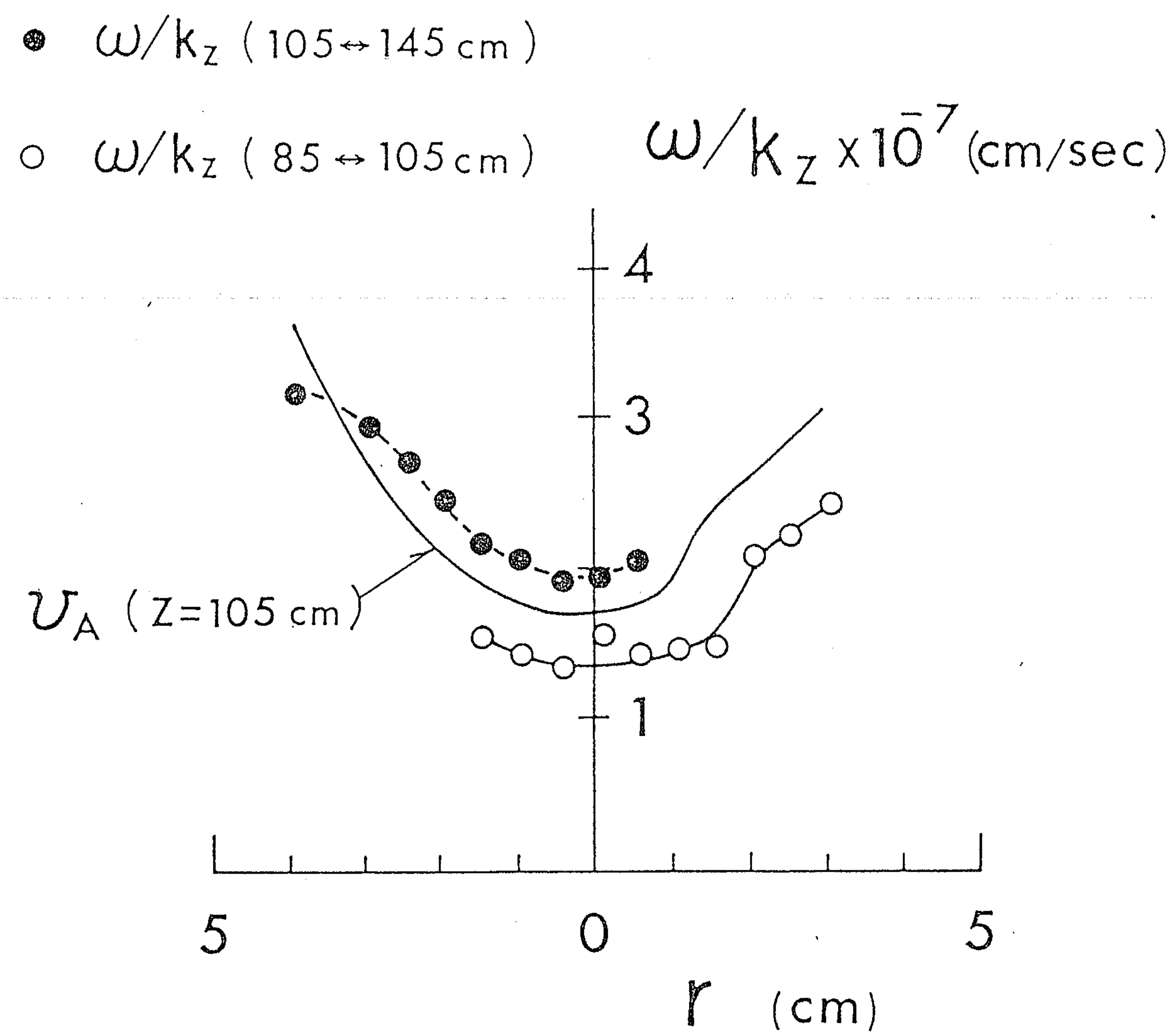
Closed Circle ; I_A
Open Circle ; I_B



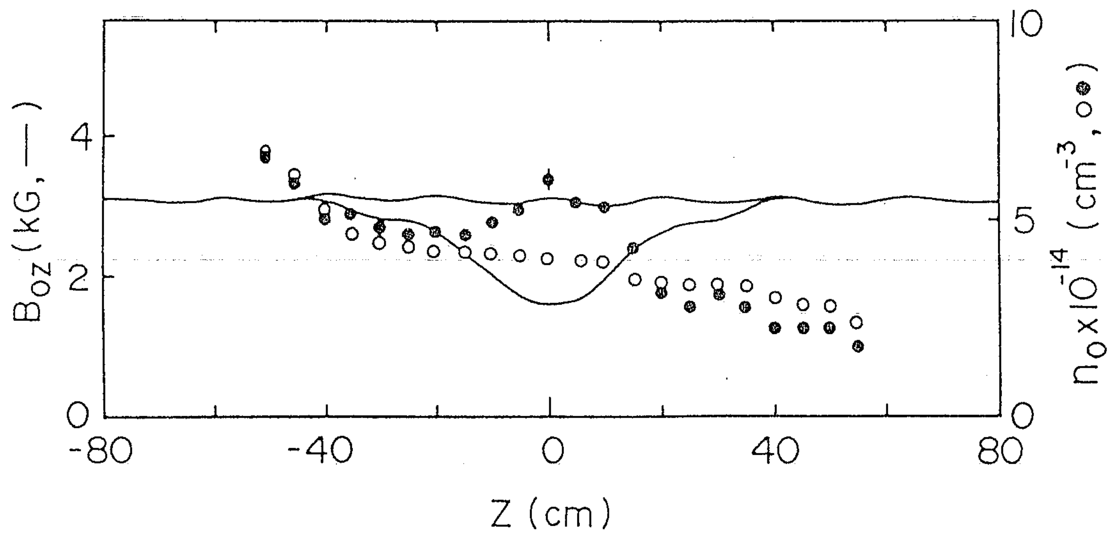
Open circle ; 105 ↔ 145 cm



z = 105 cm

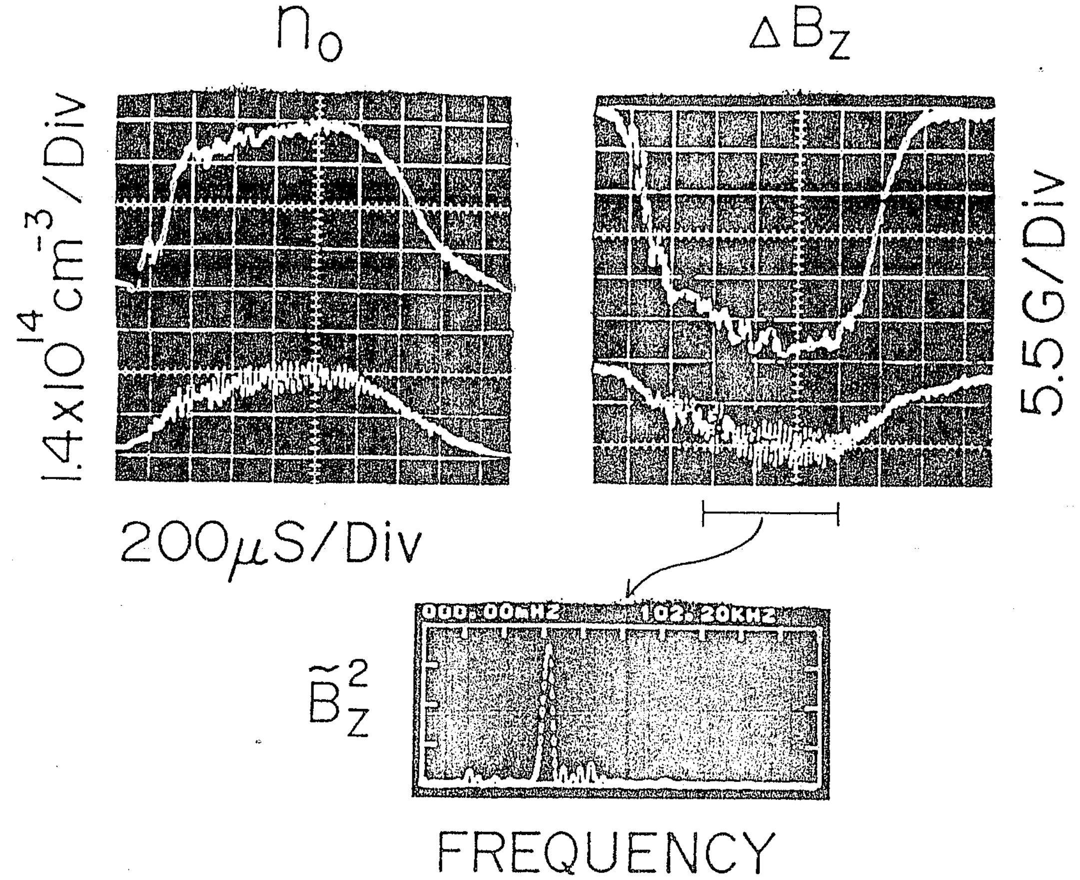

ω/k_z (105↔145 cm)
ω/k_z (85↔105 cm)
ω/k_z x10^-7 (cm/sec)
4
3
1
v_A (Z=105 cm)
5
0
5
r (cm)

Magnetic Ballooning Instability

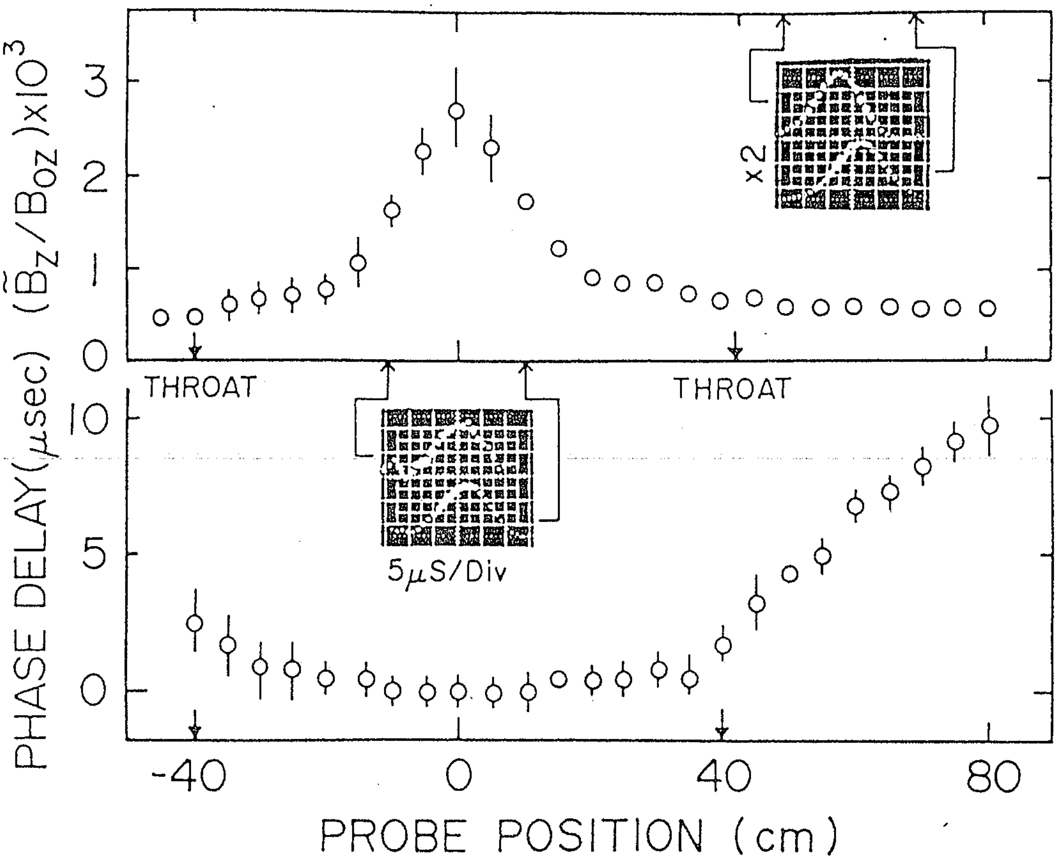


MPD Source ; Z = -115 cm
Conduc. Plate ; Z = +90 cm

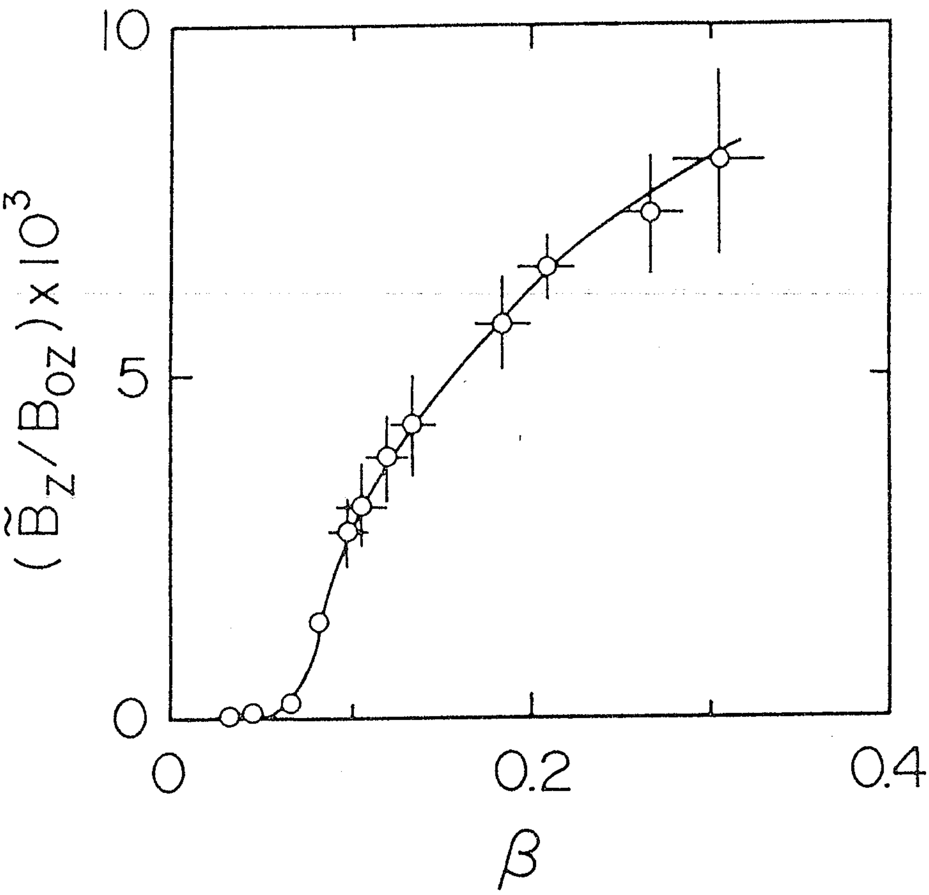
$I_A = 4$ kA , $r = 0$ cm

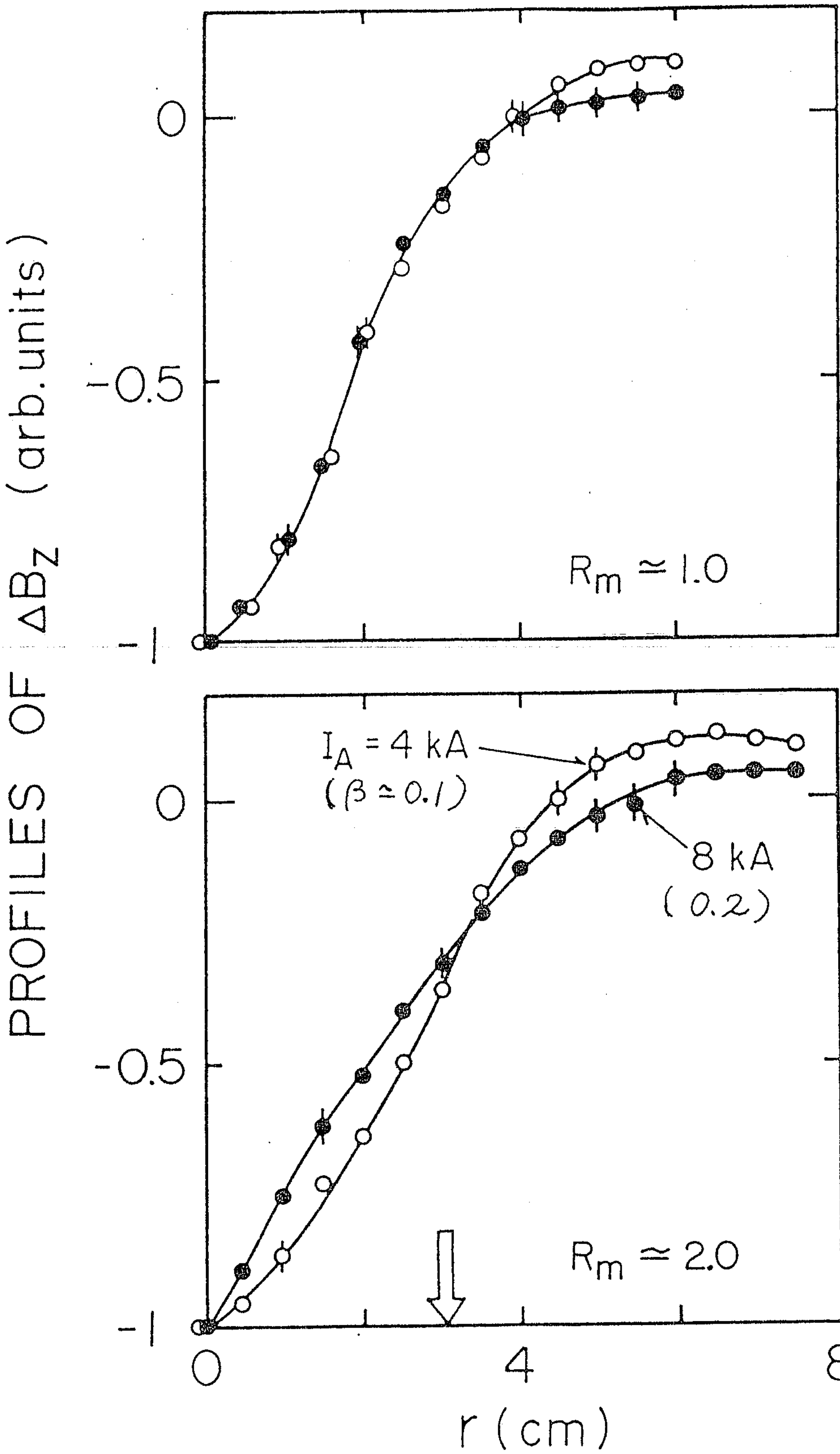


$R_m \simeq 2.0$, $Z = +10$ cm
upper ; $r = 0$ cm
lower ; $r = 2.5$ cm



$R_m \simeq 2.0$, r ; max (2.5 ~ 3.0 cm)


10
5
0
0
0.2
0.4
$(\tilde{B}_z/B_{oz}) \times 10^3$
β


PROFILES OF ΔB_z (arb. units)
0
-0.5
-1
$R_m \simeq 1.0$
$I_A = 4$ kA
($\beta \simeq 0.1$)
8 kA
(0.2)
$R_m \simeq 2.0$
0
4
8
r (cm)

z = 0 cm

NUMERICAL STUDY OF DRIFT WAVE TURBULENCE:

SIMPLE MODELS FOR WAVE-WAVE NONLINEAR COUPLING

R.E. WALTZ

GENERAL ATOMIC COMPANY

NUMERICAL STUDY OF DRIFT WAVE TURBULENCE: SIMPLE MODELS FOR WAVE-WAVE NONLINEAR COUPLING

R. E. WALTZ

GENERAL ATOMIC CO

US-JAPAN WORKSHOP ON
DRIFT WAVE TURBULENCE
JANUARY 11-15, 1982
AUSTIN, TEXAS

▶ MOTIVATION

- NEW SMALL VOLUME SCATTERING EXPERIMENTS ON TOKAMAKS INDICATE A BROAD FREQUENCY SPECTRUM FOR A GIVEN WAVE NUMBER ($k_\perp \rho_s \lesssim O(1)$)
$\omega_k \pm \Delta\omega_k \sim \omega_* \pm 2\omega_*$

- $\Delta\omega_k/\omega_k \sim O(1)$ INDICATES THAT TOKAMAKS ARE IN A <u>STRONG</u> <u>TURBULENCE</u> REGIME (REMINISCENT OF FLUID TURBULENCE) IN WHICH UNSTABLE MODES ARE STRONGLY COUPLED VIA NONLINEAR WAVE-WAVE INTERACTION TO STABLE MODES

- QUESTION: WHAT IS THE NONLINEAR COUPLING MECHANISM WHICH FOR A GIVEN (STRONG) LINEAR GROWTH-DAMPING RATE SPECTRUM γ_k, GIVES $\Delta\omega_k/\omega_k \sim O(1)$ CONSISTENT WITH $\tilde{n}/n \sim O(1\%)$ AND $D \sim O(10^4 cm^2/sec)$; AND HOW ARE SUCH QUANTITIES TO BE CALCULATED?

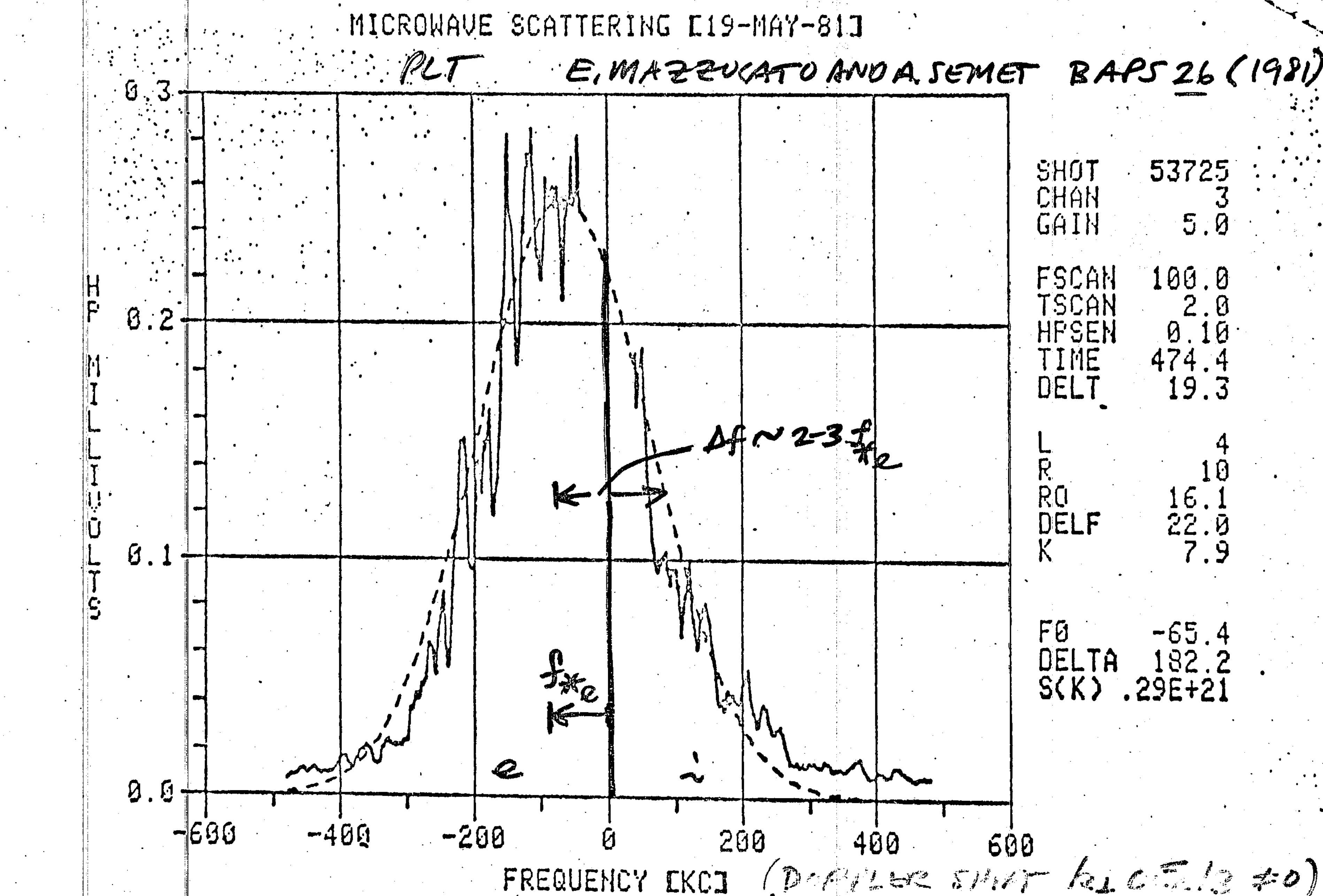


FIG. 11. FREQUENCY SPECTRUM OF FLUCTUATIONS WITH k = 7 cm^{-1} ± 1 cm^{-1} AT R = 18 cm ± 5 cm.

$k\rho_s \sim .7$

▶ OBJECTIVE

EXPLORE NUMERICAL AND MATHEMATICAL METHODS FOR TREATING STATIONARY HOMOGENOUS MICROTURBULENCE WITH VERY SIMPLE DRIFT WAVE PHYSICAL MODELS FOR NONLINEAR WAVE-WAVE COUPLING — THE ONLY SOURCE OF WAVE DECORRELATION $\Delta\omega$

▶ OUTLINE

- DRIFT WAVE PHYSICAL MODELS
- NUMERICAL SOLUTIONS
- ROUGH COMPARISON OF $\tilde{n}/n$, D, $\Delta\omega/\omega$ WITH EXPERIMENT

▶ SIMPLE PHYSICAL MODELS FOR $\tilde{E}_\perp$ DRIFT WAVES

IONS – NONLINEAR COLD FLUID IONS WITH NO PARALLEL MOTION

- $$\frac{\partial n_i}{\partial t} + \vec{\nabla}_\perp \cdot n_i \left(\vec{V}_{E_\perp} + \vec{V}_{p_\perp}\right) = 0$$

$$\vec{V}_{E_\perp} = -\frac{c}{B^2} \vec{\nabla}_\perp \Phi \times \vec{B}$$

$$\vec{V}_{p_\perp} = \frac{c}{\Omega_i B}\left[-\frac{\partial}{\partial t}\vec{\nabla}_\perp \Phi - \left(\vec{V}_E \cdot \vec{\nabla}_\perp\right)\vec{\nabla}_\perp \Phi\right]$$

(↑ NONLINEAR POLARIZATION DRIFT)

ELECTRONS – LINEAR HOT KINETIC ELECTRONS WITH FAST PARALLEL MOTION

- $$\frac{\tilde{n}_e}{n_0} = \frac{e\Phi}{T_e}(1 - i\delta) \qquad \delta \lesssim O(1)$$

($\frac{e\Phi}{T_e}$: "ADIABATIC" OR BOLTZMANN RESPONSE; $i\delta$: "NON-ADIABATIC" OR WAVE-PARTICLE RESPONSE)

QUASINEUTRALITY

- $\tilde{n}_e = \tilde{n}_i$

* UNITS AND EXPERIMENTAL MAGNITUDES

LENGTH $\rho_s = c_s/\Omega_i$; $c_s = \sqrt{T_e/m_i}$

TIME $(c_s/L_n)^{-1}$; $L_n^{-1} \equiv -(dn_0/dr)/n_0$

- $\phi = \frac{e\Phi}{T_e} / [\rho_s/L_n]$
- $\hat{D} = D / [\rho_s^2 c_s/L_n]$

PLT: $T_e = 1$ KeV, $n = 1.5\times10^{13}$, $B = 32$ kG, $L_n = 30$ cm
$\beta \sim .04\%$, $\nu_{*min} \sim .1$

- $\rho_s/L_n \sim .1/30 \sim .3\times10^{-2}$; $\tilde{n}/n \sim 1\% \Rightarrow$

$$\boxed{\phi_{exp} \sim O(3)}$$

- $\rho_s^2(c_s/L_n) \sim 10^4\ cm^2/sec$; $D = 1/4 \frac{K^{ALC}}{n} = \frac{1}{4}\frac{5\times10^{17}}{1.5\times10^{13}} \sim 10^4 \Rightarrow$

$$\boxed{\hat{D}_{exp} \sim O(1)}$$

* SCALED 2D-EQUATION OF MOTION

$$\mathbf{k} = (k_x, k_y)$$

$$\frac{d\phi_k}{dt} = (-i\omega_{0k} + \gamma_k)\phi_k + \frac{1}{2}\sum_{k_1,k_2}{}' \delta(k-k_1-k_2)\, V_{k k_1 k_2}\, \phi_{k_1}\phi_{k_2}$$

$$(\omega_{*k} = k_y)$$

$$\omega_{0k} = \frac{\omega_{*k}(1+k^2)}{(1+k^2)^2 + \delta_k^2} \; ; \; \gamma_k = \frac{\delta_k\, \omega_{0k}}{(1+k^2)} - \gamma_k^{DAMP}$$

AD HOC SHEAR DAMPING FROM NEGLECTED ION PARALLEL MOTION

HASEGAWA-MIMA NLPD

HORTON NLEXB

$$V_{k k_1 k_2} = \hat{z}\cdot \mathbf{k}_1 \times \mathbf{k}_2 \frac{[(k_2^2 - i\delta_{k_2}) - (k_1^2 - i\delta_{k_1})]}{[1+k^2 - i\delta_k]}$$

* TURBULENCE LEVEL ϕ

$$\frac{\tilde{n}/n_0}{[\rho_s/L_n]} = \phi_{rms} = \left(\int_0^{L_x} \frac{dx}{L_x} \int_0^{L_y} \frac{dy}{L_y} \; \phi^2 \right)^{1/2} = \left(\sum_k{}' \phi_k \phi_k^* \right)^{1/2}$$

* TRANSPORT $\hat{D}$

$$\frac{\partial n_0}{\partial t} + \frac{d}{dx} \Gamma_{x\,rms} = 0$$

$$\Gamma_{x\,rms} = \int_0^{L_x} \frac{dx}{L_x} \int_0^{L_y} \frac{dy}{L_y} \; \tilde{n}\,\tilde{v}_x = \sum_k{}' \tilde{n}\,\tilde{v}_{xk}^*$$

$$= D\, n_0/L_n = \hat{D}\; [\rho_s^2 c_s/L_n]\; n_0/L_n$$

$$\hat{D} = \sum_k{}' \delta_k k_y \phi_k \phi_k^* + \sum_k{}' i k_x \left(\frac{\partial}{\partial t} \phi_k\right)^* \phi_k \left(\frac{\rho_s}{L_n}\right)$$

EXB — IGNORABLE POLARIZATION DRIFT — $\left(\frac{\rho_s}{L_n}\right) \sim O(.003)$

* WHAT IS LEFT OUT ?

$$\frac{d\phi_k}{dt} = -i\omega_{0k}\phi_k + \gamma_k\phi_k + \frac{1}{2}\sum_{k_1 k_2}{}' \delta(k-k_1-k_2) V_{k k_1 k_2} \phi_{k_1}\phi_{k_2}$$

1) SPATIAL DIFFUSION CHANGES TO "EQUILIBRIUM" PARAMETERS

i.e. IT IS ASSUMED

$$\frac{1}{n_0}\left(\frac{dn_0}{dt}\right) \text{ AND } \frac{1}{\omega_*}\left(\frac{d\omega_*}{dt}\right) \ll \omega_k^* \ (\text{OR } \omega_{0k}, \gamma_k)$$

2) PARTICLE ORBIT VELOCITY SPACE DIFFUSION OR NONLINEAR WAVE-PARTICLE EFFECTS

$$\text{"}\gamma_k\text{"} \rightarrow \gamma_k^{\text{LINEAR}} + \underbrace{\frac{dE_k}{E_k\,dt} + \gamma_k^{WP}(|\phi|^2)}_{\sim \text{"}-k_\perp^2\hat{D}\text{"}}$$

"WEAK TURBULENCE" WHICH INCLUDES THESE EFFECTS BUT IGNORES WAVE-WAVE COUPLING

$$\gamma_k - k_\perp^2\hat{D} = 0 \quad \text{OR} \quad \hat{D} = \gamma_k/k_\perp^2$$

$$\Delta\omega/\omega \rightarrow 0 \quad \underline{\text{AND}} \quad \Delta k/k \rightarrow 0$$

CONSIDERING WAVE-WAVE COUPLING <u>ALONE</u> HAS ADVANTAGE OF SIMILARITY TO FLUID TURBULENCE PROBLEMS WHICH HAVE A LONG HISTORY 1883 – 1982

* CONSERVED QUANTITIES AND INERTIAL RANGES

$$\mathcal{E} = \sum_k{}' (1+k^2)\,\phi_k^*\phi_k = \sum_k{}' E_k \qquad \text{ENERGY}$$

$$\Omega_1 = \sum_k{}' k^2(1+k^2)\,\phi_k^*\phi_k = \sum_k{}' \Omega_{1k} \qquad \text{ENSTROPHY}$$

LEAVING OUT DISSIPATIVE NON LINEAR E×B

$$V_{k k_1 k_2} = \tfrac{1}{2}\,\hat{z}\cdot k_1\times k_2\,\frac{[(k_2^2-i\cancel{\delta_{k_2}})-(k_1^2-i\cancel{\delta_{k_1}})]}{[1+k^2-i\cancel{\delta_k}]}$$

$$\frac{d(\mathcal{E},\Omega_1)}{dt} = \sum_k{}' 2\gamma_k\,(E_k,\Omega_{1k})$$

i.e. NON-LINEAR COUPLING CONSERVES ENERGY AND ENSTROPHY

- STATISTICAL EQUILIBRIUM

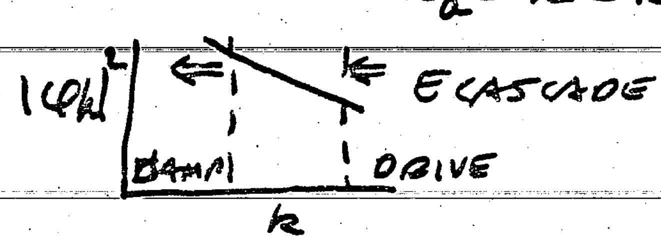
IF $\gamma_k = 0$ ALL k $\quad |\varphi_k|^2$ ASSUMES A GIBBS DIST.

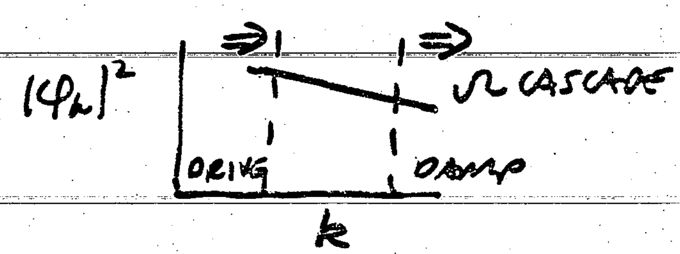
$$\mathcal{P}\{|\varphi_k|^2\} = \frac{\exp(-\beta E_k - \alpha\Omega_{1k})}{\exp(-\beta\mathcal{E}-\alpha\Omega_1)} \Rightarrow$$

$$\langle|\varphi_k|^2\rangle \propto \frac{1}{(1+k^2)(\beta+\alpha k^2)} \qquad (\downarrow \text{ AS } k\uparrow)$$

- INERTIAL RANGE

IF $\gamma_k = 0 \quad k_a < k < k_b$





▶ NUMERICAL SOLUTIONS

- ARE THERE STATIONARY STATES?

- HOW GOOD ARE APROXIMATE WEAK COUPLING THEORIES (DIA); ARE THEY COMPUTATIONALLY FASTER?

- HOW MANY MODES ARE NEEDED TO REPRESENT HOMOGENOUS HIGH-m TURBULENCE?

- HOW DO THINGS SCALE?

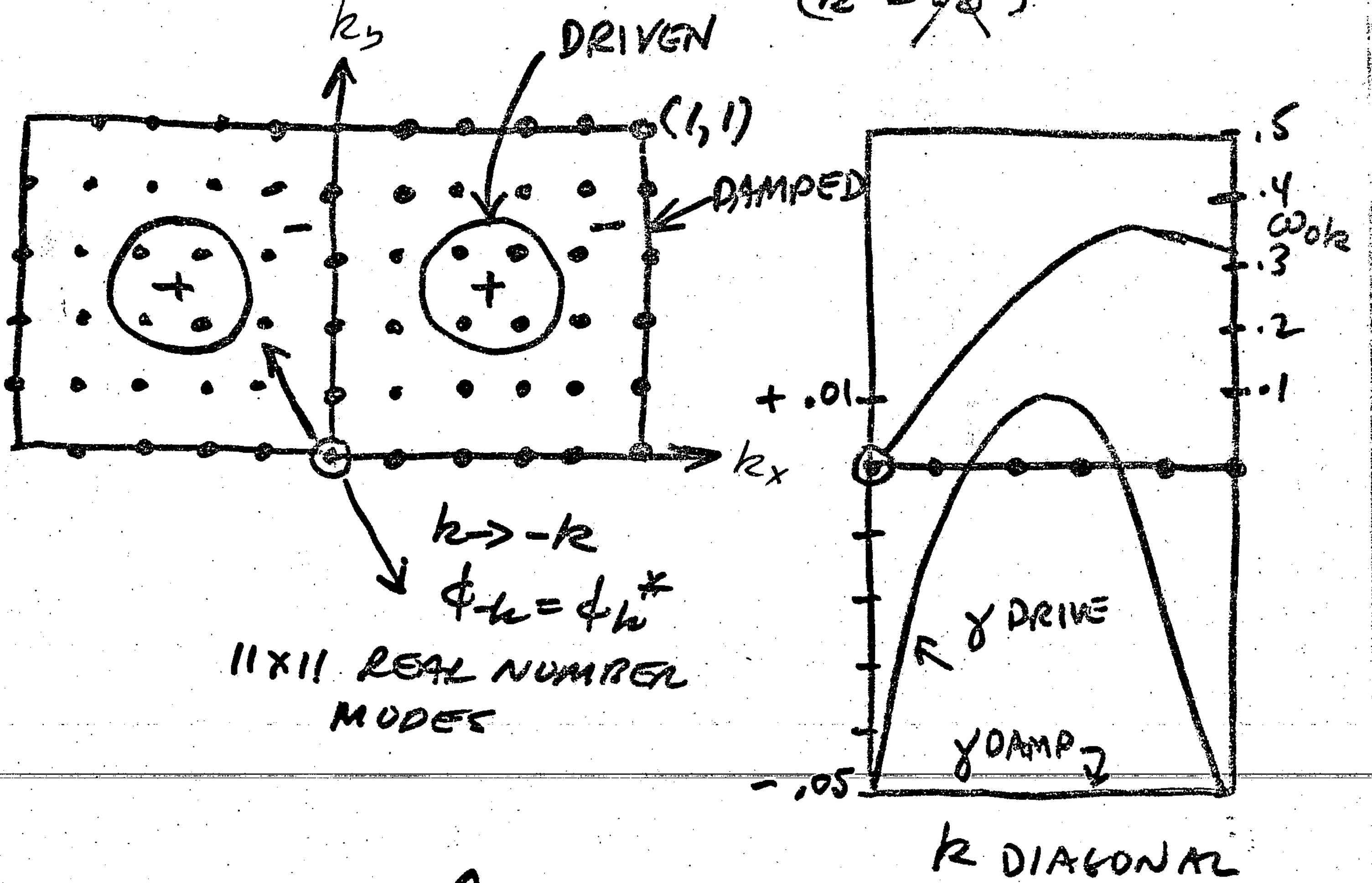
▶ "EXACT" NUMERICAL SOLUTION OF

$$\frac{d\phi_k}{dt} = -i\omega_{0k}\phi_k + \gamma_k\phi_k + \frac{1}{2}\sum_{k_1 k_2}{}' \delta(k-k_1-k_2) V_{k k_1 k_2} \phi_{k_1}\phi_{k_2}$$

OR WITH $\phi_k(t) = \hat{\phi}_k(t)\, e^{-i\omega_{0k} t}$

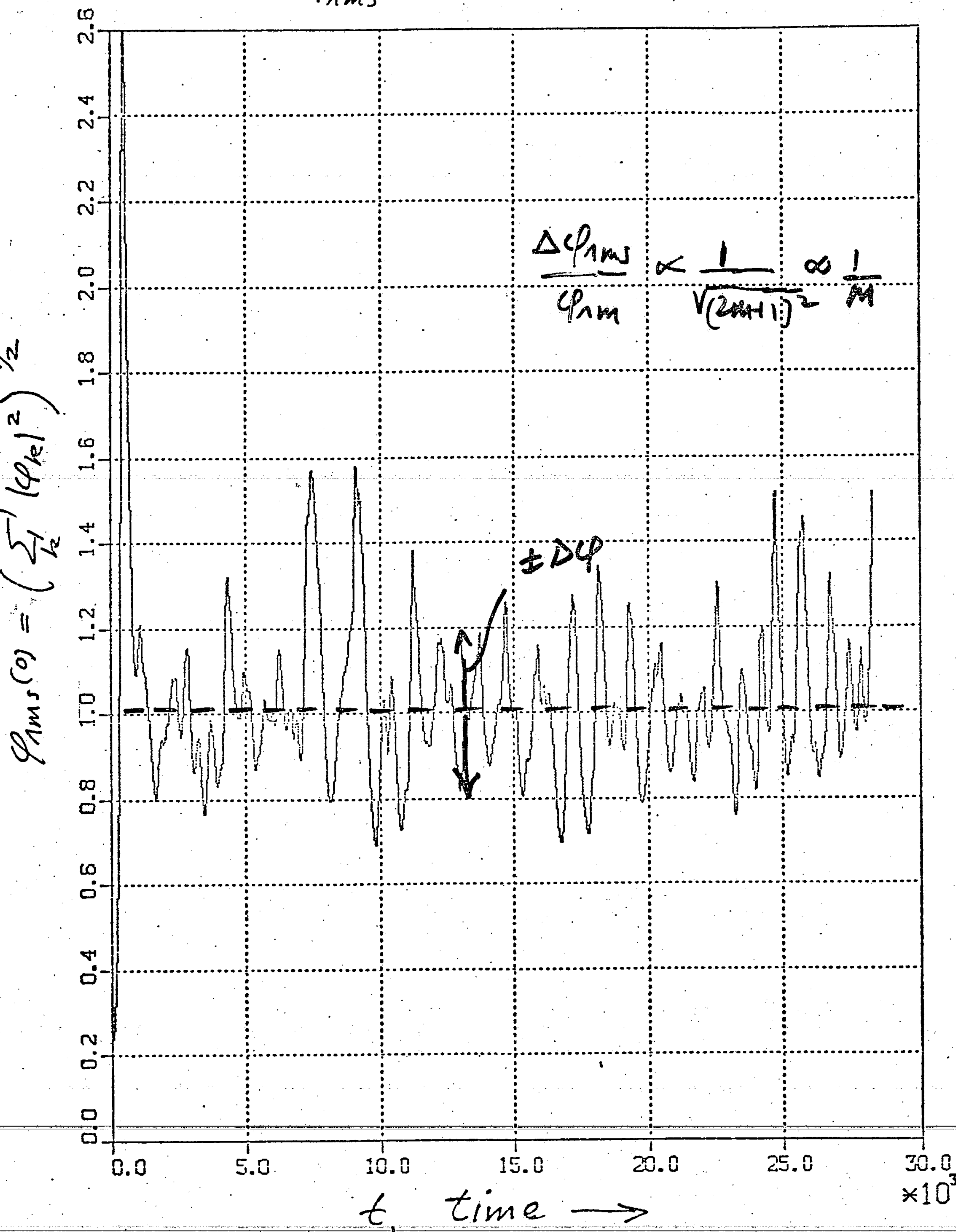
$$\frac{d\hat{\phi}_k}{dt} = \gamma_k\hat{\phi}_k + \sum_{k_1 k_2}{}' \delta(k-k_1-k_2)\, e^{+i(\omega_{0k}-\omega_{0k_1}-\omega_{0k_2})t}\, V_{k k_1 k_2}\hat{\phi}_{k_1}\hat{\phi}_{k_2}$$

* TEST PROBLEM (NONLINEAR EXB DROPPED)



SOLVE FOR $\hat{\phi}_k(t)$ IN <u>TIME</u>

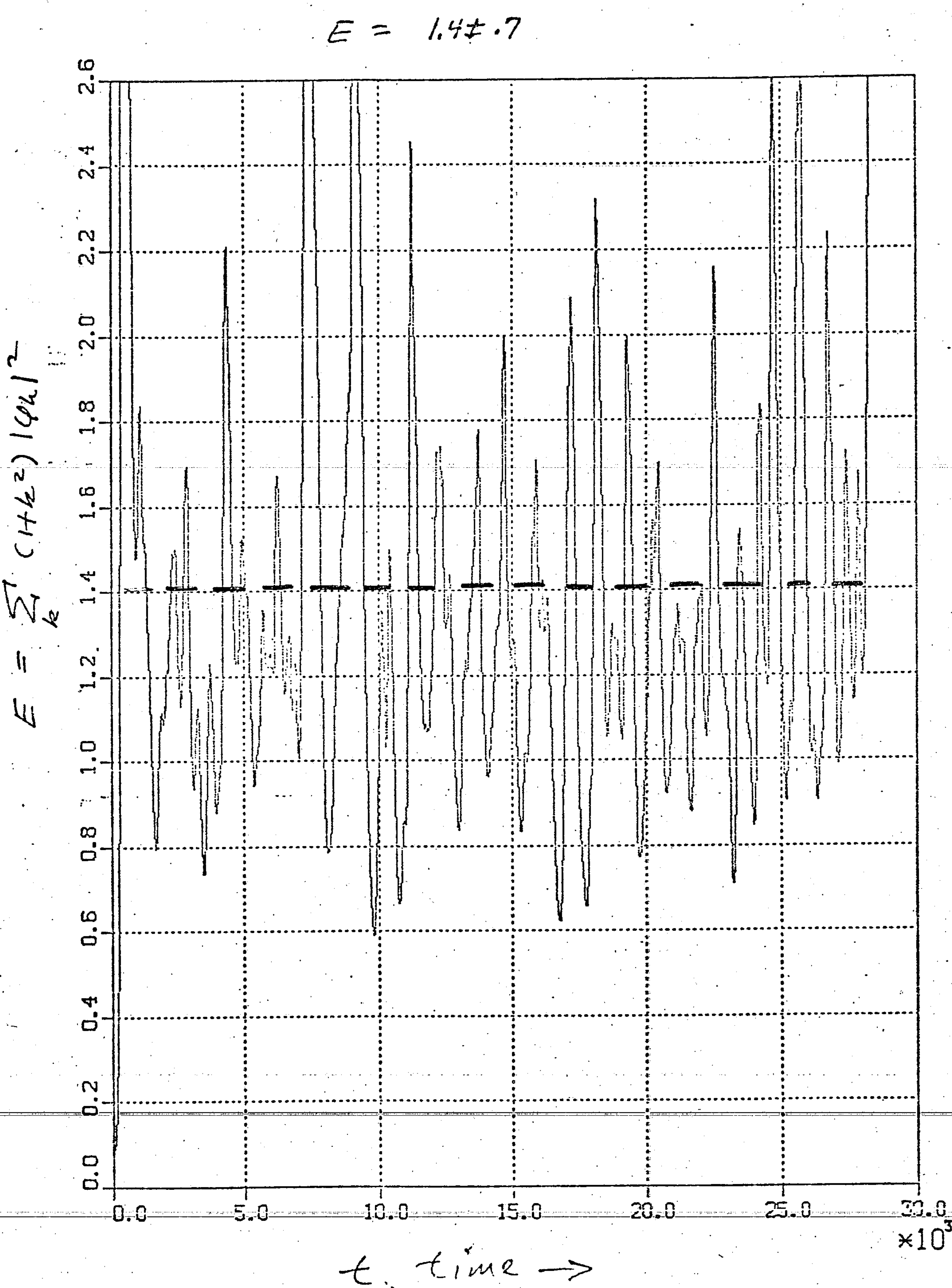
$\varphi_{rms}(0) = 1.0 \pm .2$

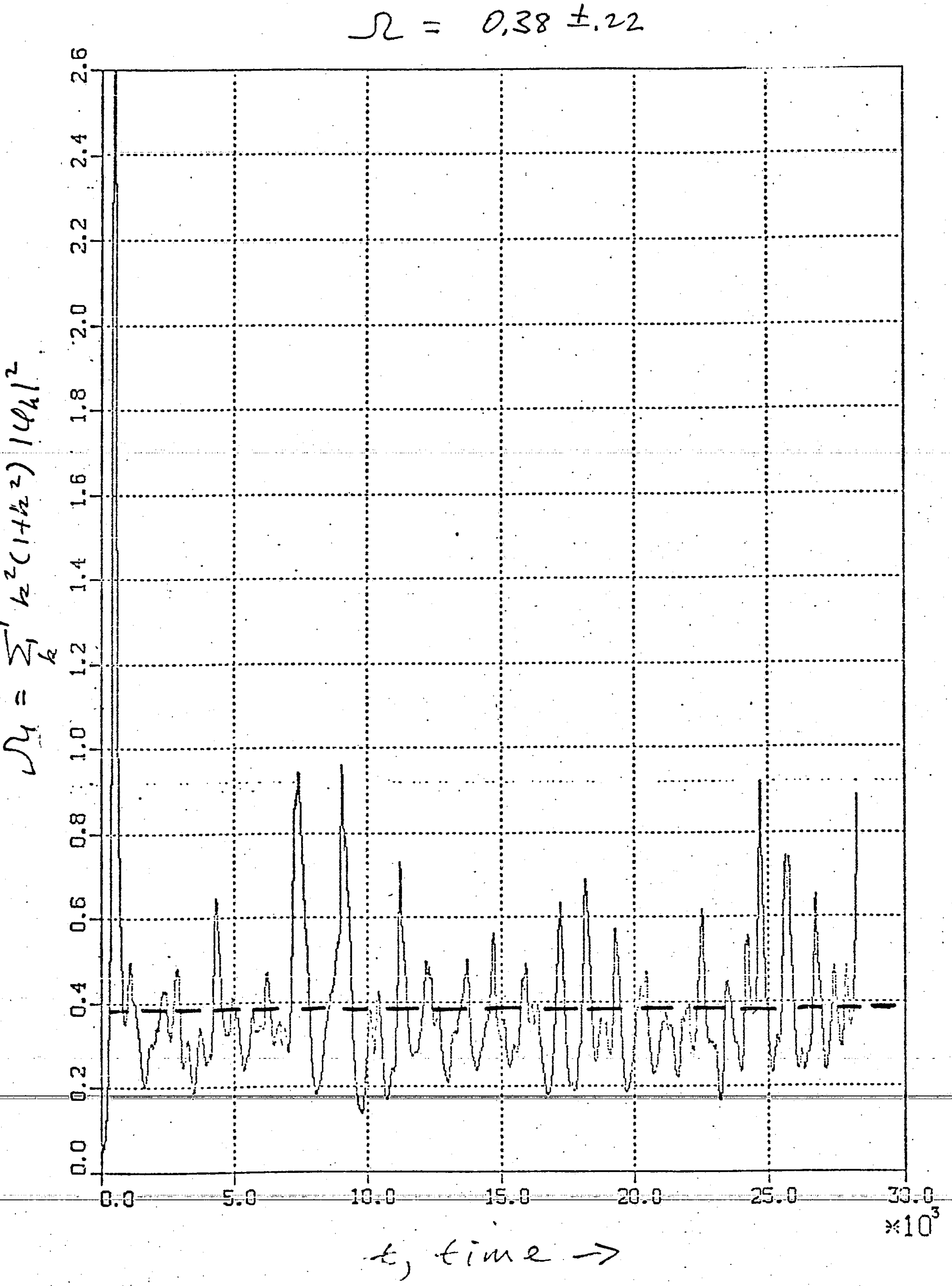


$T_{*min} = 2\pi/\omega_{*max} = 2\pi/1 = 6.28$

$\Delta t_{SAVE} = 2.36$ (12,000 STEPS); $\Delta t_{calc} = .785$ (36,000

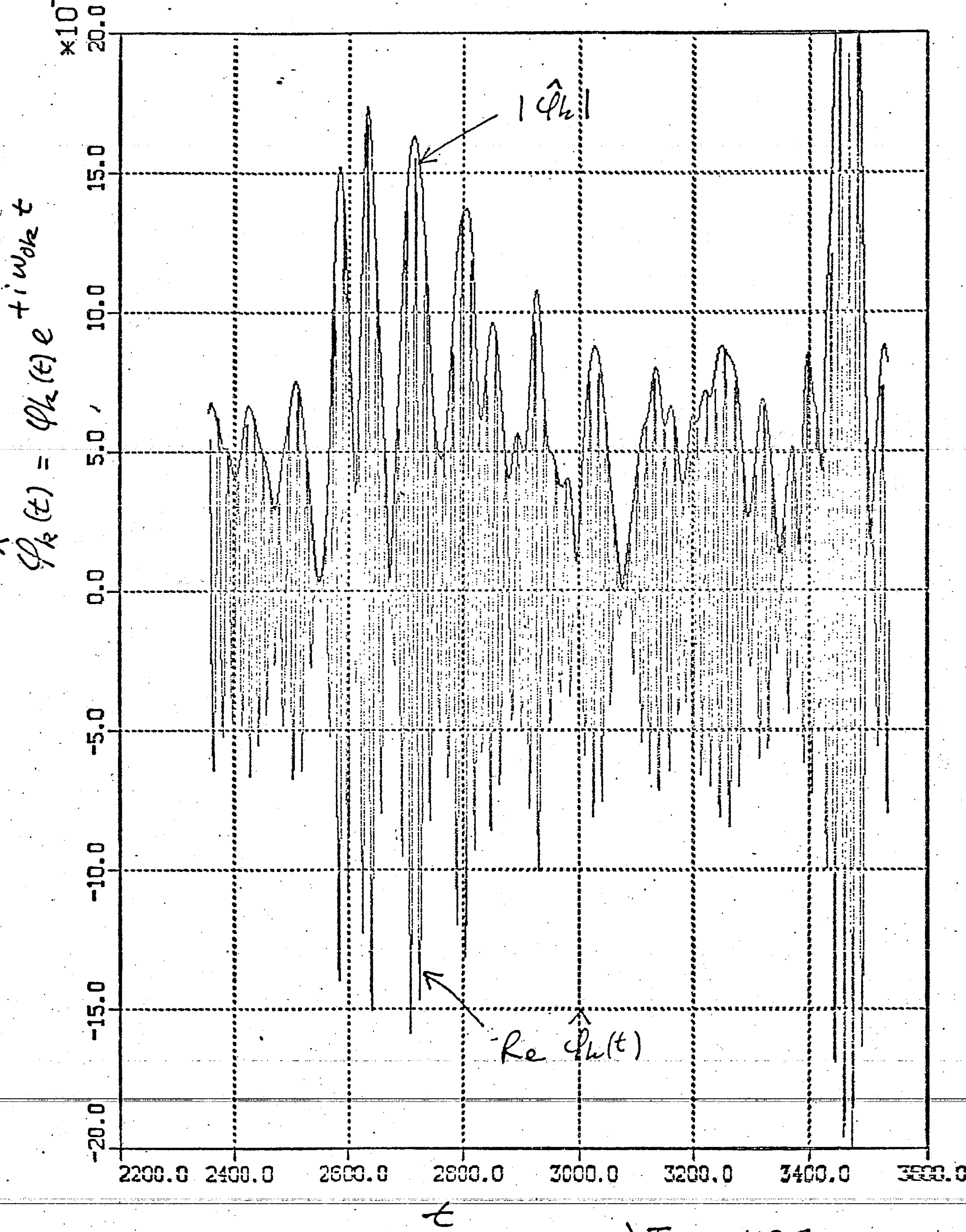
11X11


E = 1.4 ± .7
$E = \sum_k (1+k^2)|\varphi_k|^2$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
$\times 10^3$
t, time →


Ω = 0,38 ±.22
Ω₁ = Σ'ₖ k²(1+k²) |φₖ|²
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
5.0
10.0
15.0
20.0
25.0
30.0
×10³
t, time →

$k = (1.0, 1.0)$ 11

mode 11 11X11

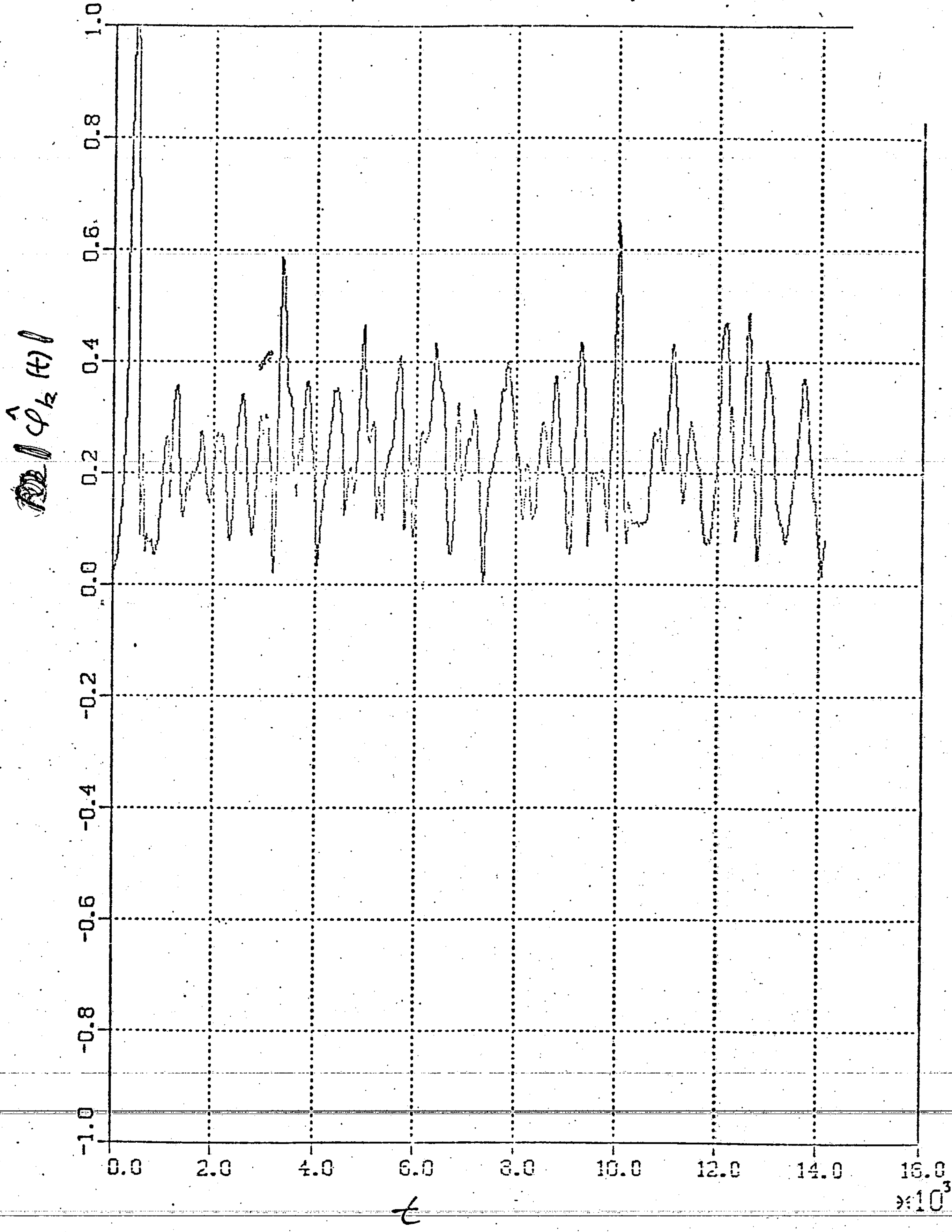


$\Delta T = 1178$

$\Delta T_{RUN} = 28272$

$|\varphi_k| = 0.23$ $\quad k^{212} = (0.4, 0.4)$

mode 44



$\Delta T = 14136$

$\Delta T_{cum} = 28272$

* STATISICAL DESCRIPTION

$$I_k \underset{T\to\infty}{=} \int_0^T \frac{dt}{T}\, \hat{\phi}_k(t)\, \hat{\phi}_k^*(t)$$

CORRELATION FUNCTION

$$f_k(\tau) \underset{T\to\infty}{=} \int_0^T \frac{dt}{T}\, \hat{\phi}_k(t+\tau)\, \hat{\phi}_k^*(t) / I_k$$

$$\simeq e^{-i\,\delta\tilde{\omega}_k \tau - \Delta\omega_k |\tau|}$$

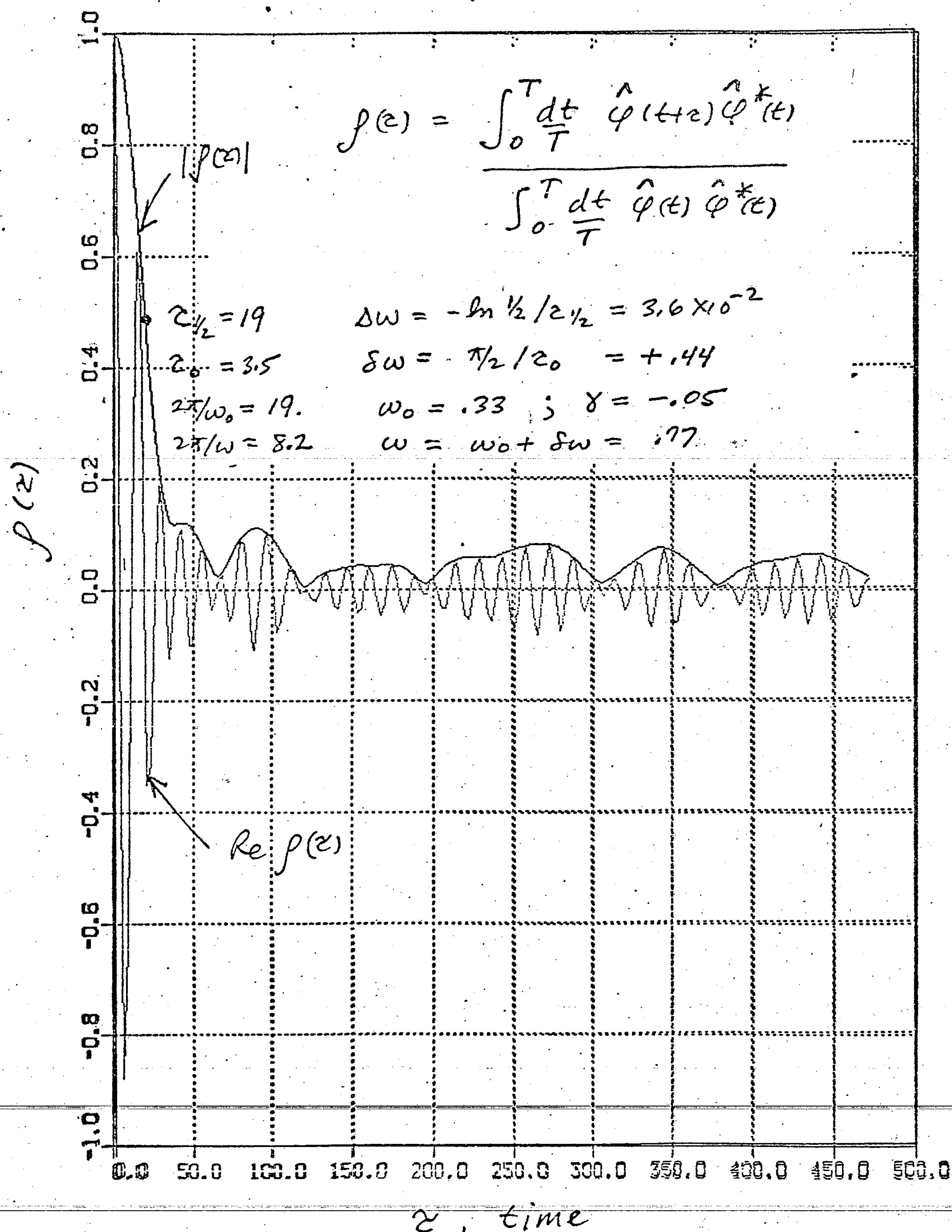
"RESONANCE APPROXIMATION"

$\Rightarrow$

$$I_{k,\omega} = \phi_k(\omega)\, \phi_k^*(\omega) \simeq I_k \frac{\Delta\omega_k}{(\omega - \omega_{0k} - \delta\tilde{\omega}_k)^2 + \Delta\omega_k^2}$$

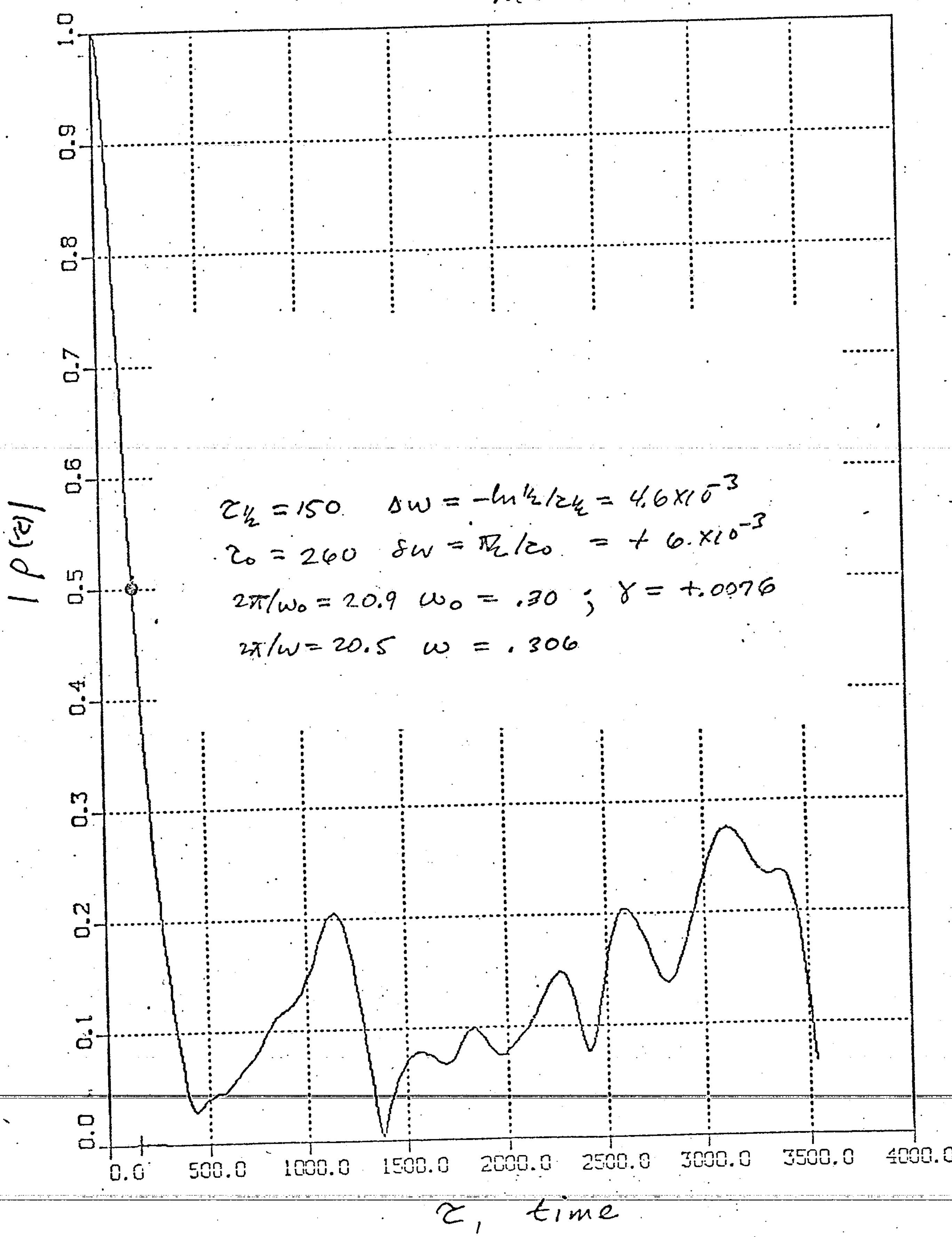
$|\varphi_k| = .0008$ $k = (1.0, 1.0)$ TITLE FOR PLOT

mode 11



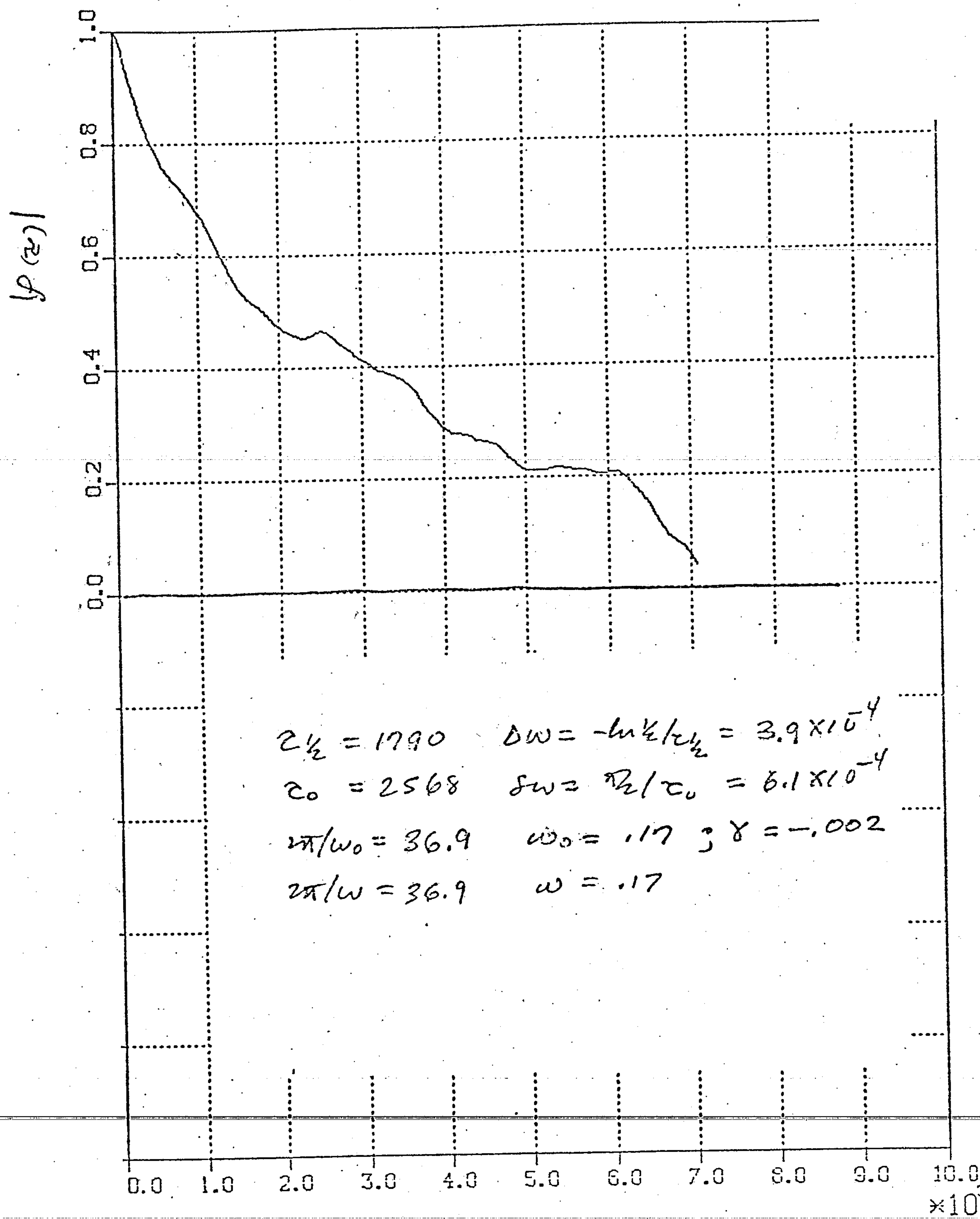
$|\phi_k| = 0.23$ $k = (0.4, 0.4)$

mode 44



$|\varphi_k| = 0.38$ 216 $k = (0.4, 0.2)$

mode 45



$\tau_{1/2} = 1790$

$\Delta T_{run} = 28272$

STATISTICAL THEORY

- DIRECT INTERACTION APPROXIMATION (DIA)

  KRAICHNAN, R. H. J. FLUID MECH 5 (1959) 497

- WEAK COUPLING APPROXIMATION

  KADOMTSEV, B.B. PLASMA TURBULENCE (1965) p49

IN THE POLE APPROXIMATION

$$I_{k,\omega} = |\varphi_{k,\omega}|^2 = \frac{I_k \, \Delta\omega_k}{(\omega-\tilde{\omega}_k)^2 + \Delta\omega_k^2} \qquad \tilde{\omega}_k = \omega_{0k} + \delta\omega_k$$

WAVE KINETIC (BALANCE) EQU.

$$(1)\quad 0 \simeq \dot{I}_k = \Big(2\gamma_k + 2\,\mathrm{Re}\sum_{k_1 k_2}{}' \delta(k-k_1-k_2)\, V_{k k_1 k_2} V^*_{k_1 k_2 k}\, I_{k_2}\, P_{k k_1 k_2}\Big) I_k + \sum_{k_1 k_2}{}' \delta(k-k_1-k_2)\, V_{k k_1 k_2} V^*_{k k_1 k_2}\, I_{k_1} I_{k_2}\, \mathrm{Re}\, P_{k k_1 k_2}$$

$$(2)\quad \tilde{\omega}_k = \omega_{0k} - \mathrm{Im}\sum_{k_1 k_2}{}' \delta(k-k_1-k_2)\, V_{k k_1 k_2} V^*_{k_1 k_2 k}\, I_{k_2}\, P_{k k_1 k_2}$$

$$(3)\quad \Delta\omega_k = -\gamma_k - \mathrm{Re}\sum_{k_1 k_2}{}' \delta(k-k_1-k_2)\, V_{k k_1 k_2} V^*_{k_1 k_2 k}\, I_{k_2}\, P_{k k_1 k_2}$$

$$\simeq \Delta\omega'_k \equiv \frac{1}{2}\sum_{k_1 k_2}{}' \delta(k-k_1-k_2)\, V_{k k_1 k_2} V^*_{k k_1 k_2}\, I_{k_1} I_{k_2}\, \mathrm{Re}\, P_{k k_1 k_2} / I_k$$

$$\geq 0$$

$$P_{k k_1 k_2} \equiv \frac{1}{-i(\tilde{\omega}_k - \tilde{\omega}_{k_1} - \tilde{\omega}_{k_2}) + (\Delta\omega_k + \Delta\omega_{k_1} + \Delta\omega_{k_2})}$$

NOTES

- $\mathrm{Re}\, P_{k k_1 k_2} \rightarrow 2\pi\, \delta(\omega_{0k} - \omega_{0k_1} - \omega_{0k_2})$

  "WEAK TURBULENCE" $\delta\omega_k, \Delta\omega_k \rightarrow 0$

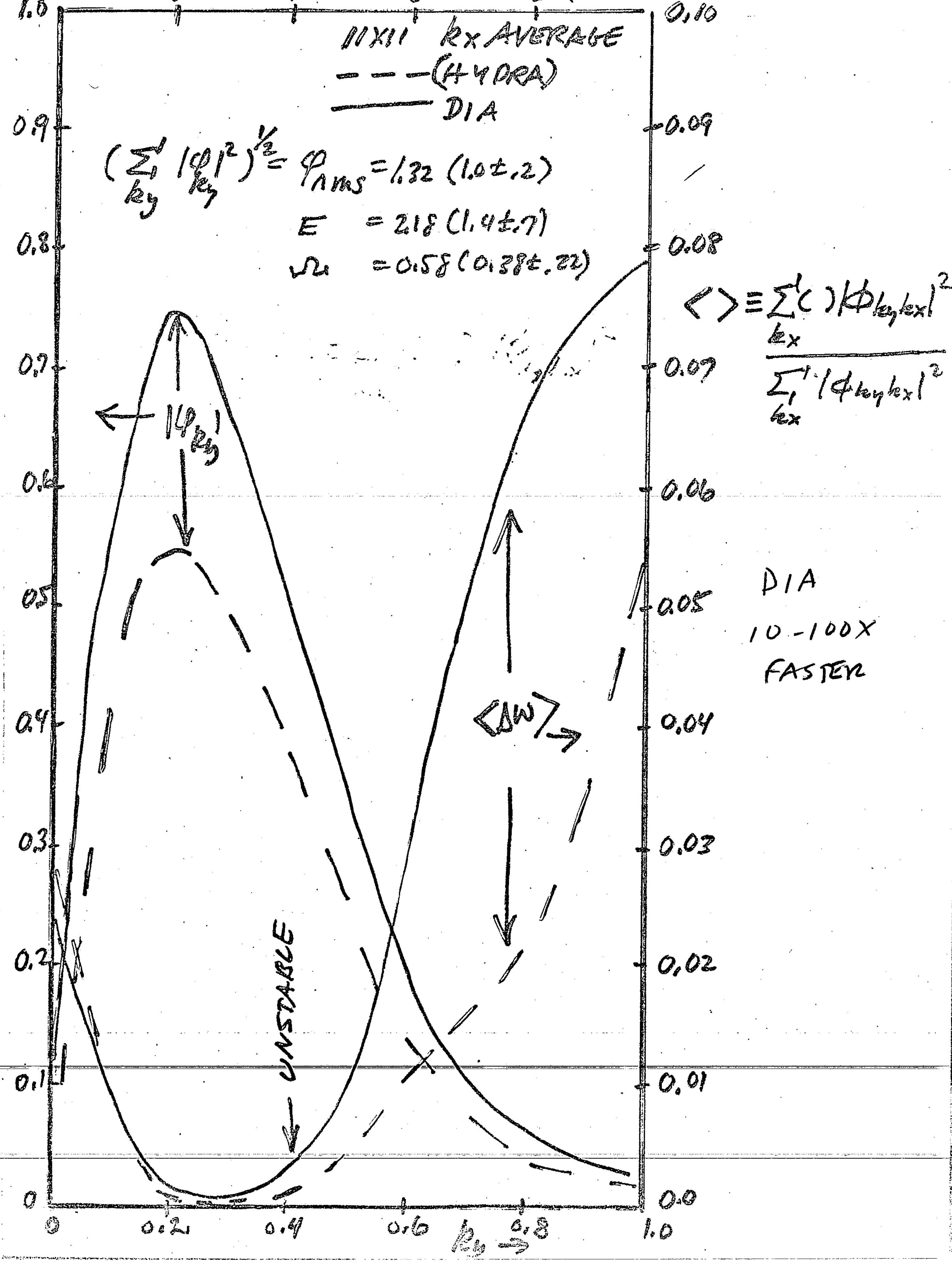
- (1)+(3) $\quad -(\gamma_k + \gamma_k^{nl})\, I_k \cong \Delta\omega_k I_k \simeq \Delta\omega'_k I_k \geq 0$

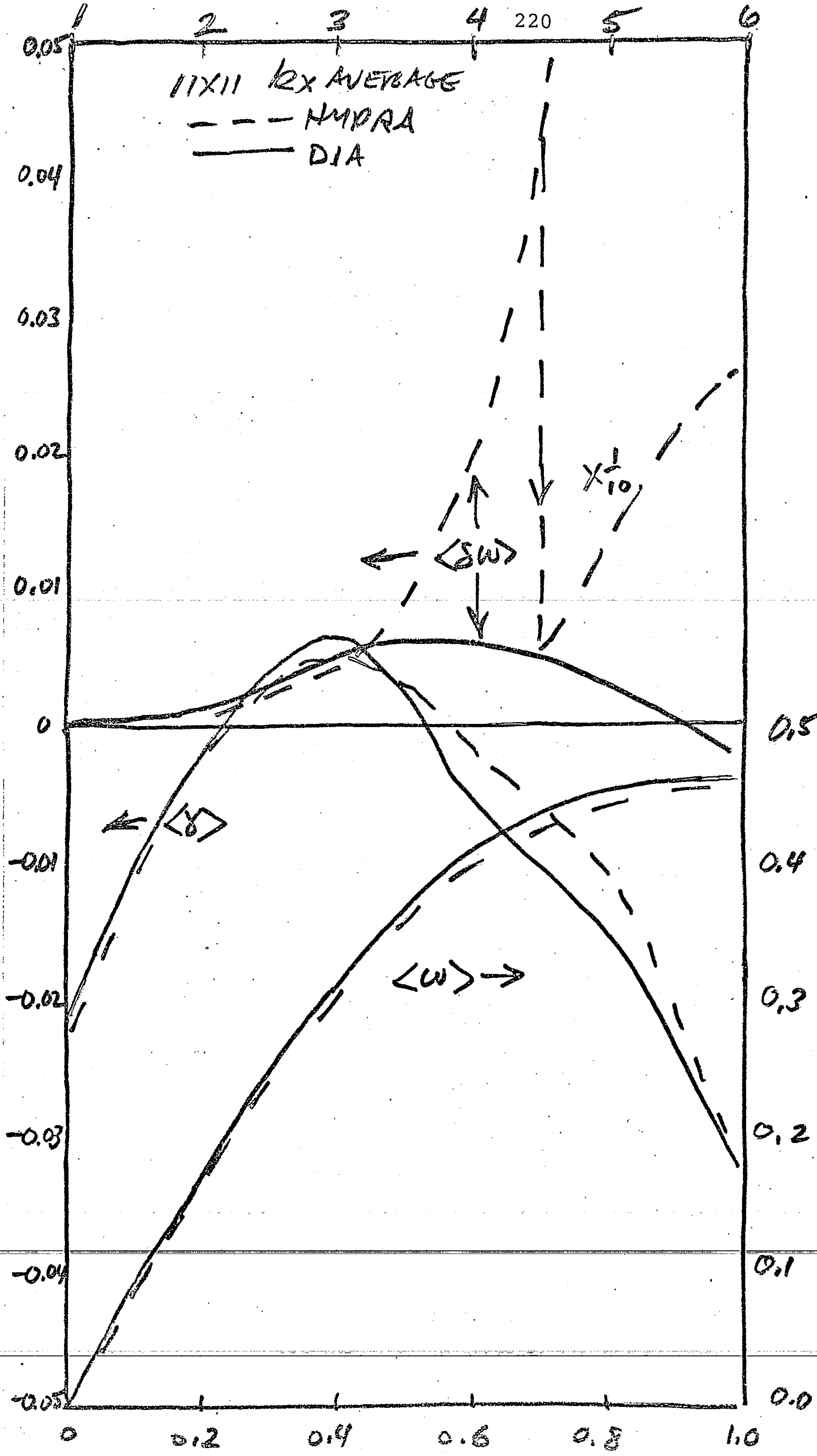
DIA (HYDR)
11 × 11 modes

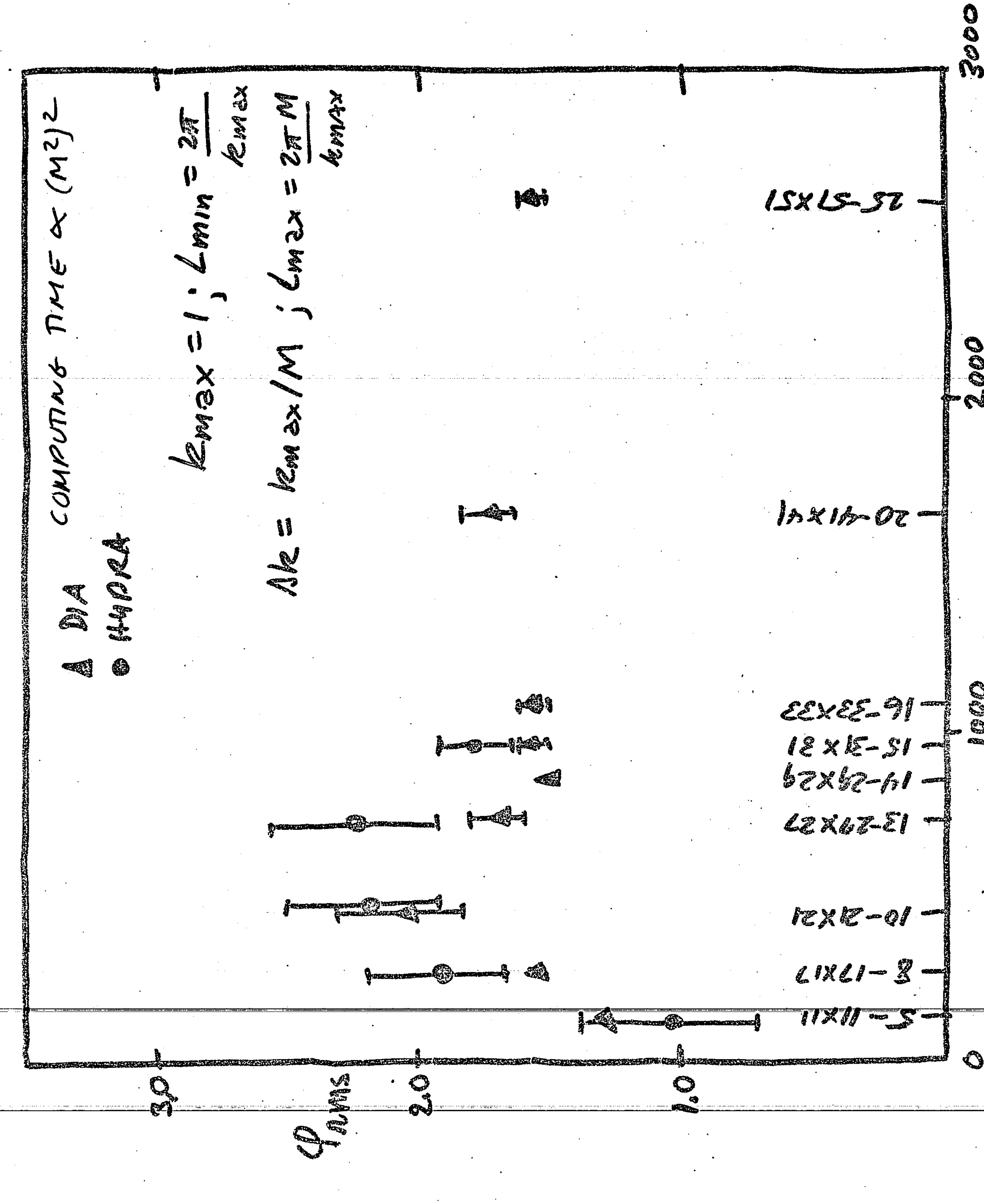
ϕ_{rms} = 1.32 (1.0 ± 0.2); E = 2.18 (1.4 ± 0.7); Ω = 0.58 (0.38 ± 0.22)

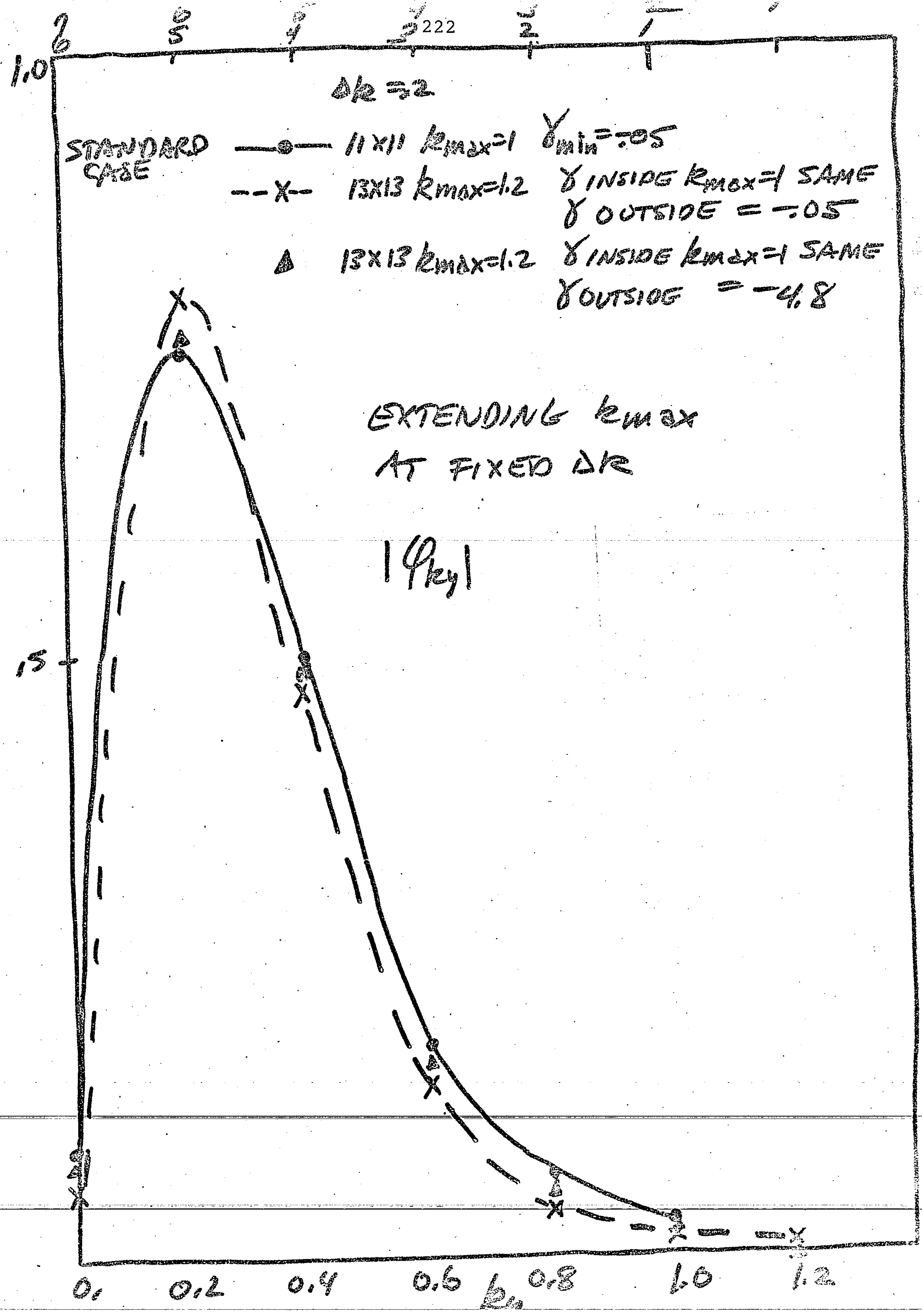
k_y ↑						
1.0	0.017 (0.008) 0.095 (0.014) −0.039 (+0.260) −0.05/0.5 61	0.0062 (0.0052) 0.082 (0.025) −0.043 (+0.260) −0.031/0.49 51	0.014 (0.005) 0.071 (0.033) −0.078 (+0.260) −.021/0.46 41	0.0041 (0.0032) 0.10 (0.35) −0.085 (+0.330) −0.021/0.42 31	0.0064 (0.0017) 0.038 (0.027) −0.012 (+0.330) −0.031/0.38 21	0.0017 (0.0008) 0.058 (0.036) −0.01 (+0.44) −0.05/0.33 11
0.8	0.040 (0.018) 0.059 (0.017) 0.0038 (0.1100) −0.031/0.49 62	0.012 (0.012) 0.055 (0.032) 0.008 (0.130) −0.012/0.48 52	0.030 (0.018) 0.078 (0.018) 0.001 (0.140) −0.002/0.44 42	0.0074 (0.0093) 0.110 (0.026) 0.00079 (0.20000) −0.002/0.40 32	0.013 (0.005) 0.051 (0.028) 0.047 (0.240) −0.12/0.35 22	0.0033 (0.0023) 0.094 (0.037) 0.049 (0.290) −0.031/0.30 12
0.6	0.11 (0.07) 0.028 (0.015) 0.00019 (0.03300) −0.021/0.44 63	0.026 (0.042) 0.019 (0.014) 0.011 (0.017) −0.002/0.43 53	0.084 (0.067) 0.037 (0.009) 0.021 (0) +0.0076/0.39 43	0.013 (0.018) 0.069 (0.023) 0.0072 (0.1100) +0.0076/0.35 33	0.024 (0.013) 0.051 (0.028) 0.030 (0.110) −0.002/0.30 23	0.0052 (0.0059) 0.090 (0.032) 0.035 (0.710) −0.021/0.25 13
0.4	0.044 (0.027) 0.022 (0.018) −0.0009 (−0.3100) −0.021/0.34 64	0.076 (0.120) 0.0013 (0.0020) 0.001 (0) −0.002/0.33 54	0.34 (0.23) 0.0026 (0.0046) 0.012 (0.006) +0.0076/0.30 44	0.020 (0.031) 0.032 (0.023) −0.0054 (+0.018) +0.0076/0.26 34	0.049 (0.040) 0.038 (0.010) 0.0020 (0.0510) −0.002/0.22 24	0.0070 (0.0099) 0.078 (0.028) 0.017 (0.052) −0.021/0.19 14
0.2	0.044 (0.021) 0.031 (0.067) −0.0004 (−0.0430) −0.031/0.19 65	0.020 (0.019) 0.0085 (0.0081) −0.0018 (−0.0023) −0.012/0.19 55	0.53 (0.38) 0.00028 (0.00039) 0.0015 (0.0006) −0.002/0.17 45	0.032 (0.040) 0.010 (0.010) −0.0034 (+0.0410) −0.002/0.14 35	0.084 (0.056) 0.0270 (0.0061) −0.0025 (+0.023) −0.012/0.12 25	0.009 (0.010) 0.710 (0.028) 0.008 (0.054) −0.031/0.098 15
0.0	$\|\phi\|$ $\Delta\omega$ $\delta\tilde{\omega}$ γ/ω	0.01 (0.01) 0.31 (0.020) 0 (−0.028) −0.031/0 56	0.054 (0.038) 0.022 (0.013) 0 (0.044) −0.021/0 46	0.032 (0.029) 0.022 (0.058) 0 (−0.11) −0.021/0 36	0.011 (0.017) 0.034 (0.019) 0 (0) −0.031/0 26	0.0085 (0.0010) 0.081 (0.033) 0 (0.042) −0.05/0 16
	0.0	0.2	0.4	0.6	0.8	1.0

k_x →

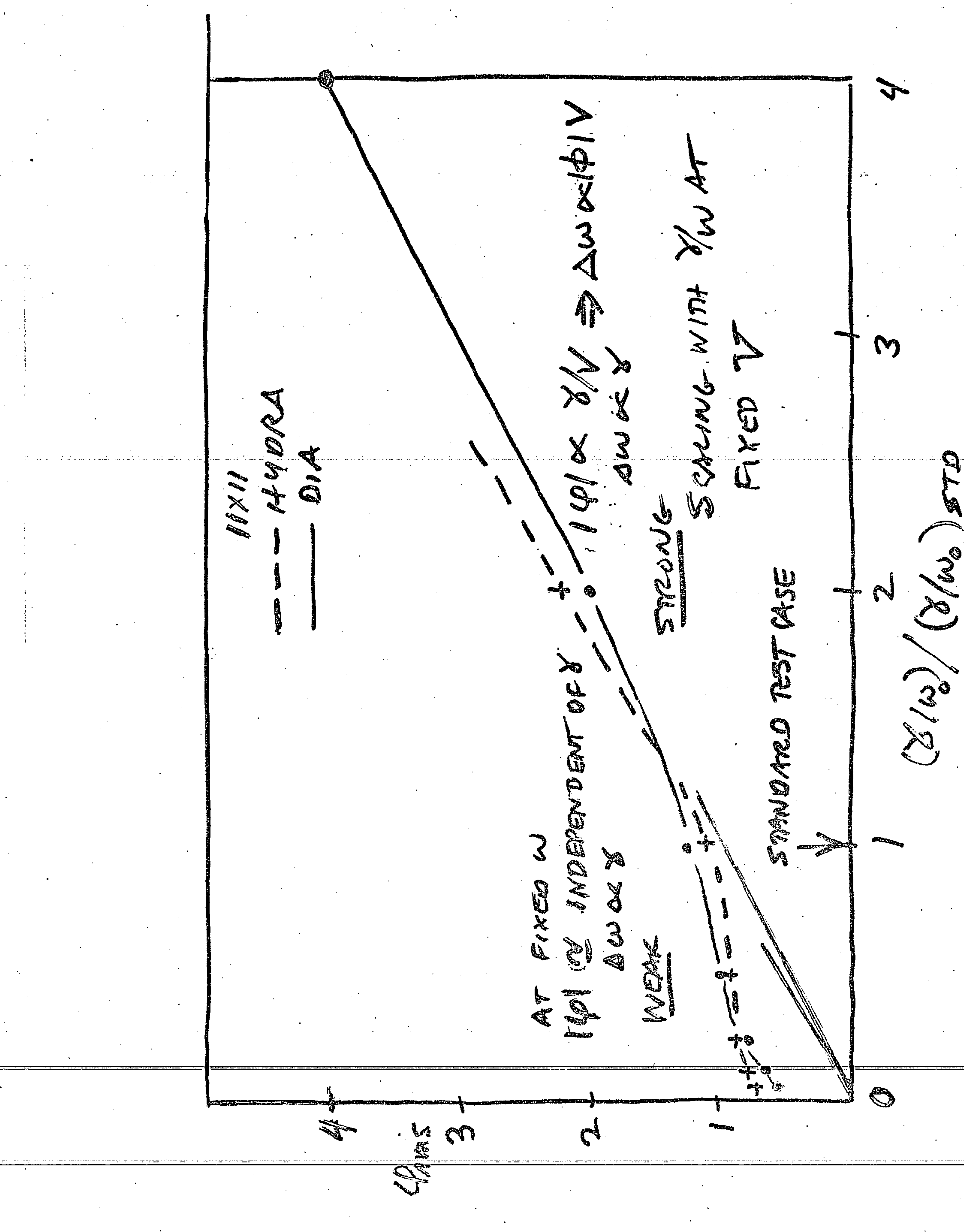

MODE #
6
5
4
3
2
1
11X11' kx AVERAGE
- - - (HYDRA)
—— DIA
$(\sum'_{k_y} |\varphi_{k_y}|^2)^{1/2} = \varphi_{rms} = 1.32\ (1.0 \pm .2)$
$E = 218\ (1.4 \pm .7)$
$\Omega_i = 0.58\ (0.38 \pm .22)$
$\langle\ \rangle \equiv \frac{\sum'_{k_x} (\ ) |\phi_{k_y,k_x}|^2}{\sum'_{k_x} |\phi_{k_y k_x}|^2}$
$|\varphi_{k_y}|$
$\langle \Delta\omega \rangle$
UNSTABLE
DIA
10-100X
FASTER
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.0
0
0.2
0.4
0.6
0.8
1.0
$k_y \rightarrow$


11X11 kx AVERAGE
HYDRA
DIA
⟨δω⟩
$\times\frac{1}{10}$
⟨γ⟩
⟨ω⟩
1 2 3 4 5 6
0.05 0.04 0.03 0.02 0.01 0 -0.01 -0.02 -0.03 -0.04 -0.05
0.5 0.4 0.3 0.2 0.1 0.0
0 0.2 0.4 0.6 0.8 1.0


▲ DIA
● HYDRA
COMPUTING TIME ∝ (M²)²
k_max = 1 ; L_min = 2π/k_max
Δk = k_max/M ; L_max = 2πM/k_max
φ_rms
1.0
2.0
3.0
0
1000
2000
3000
(2M+1)² # OF MODES
5-11x11
8-17x17
10-21x21
13-27x27
14-29x29
15-31x31
16-33x33
20-41x41
25-51x51



Δk = .2
STANDARD CASE —•— 11×11 kmax=1 γmin=−.05
--X-- 13×13 kmax=1.2 γ INSIDE kmax=1 SAME
γ OUTSIDE = −.05
▲ 13×13 kmax=1.2 γ INSIDE kmax=1 SAME
γ OUTSIDE = −4.8
EXTENDING kmax
AT FIXED Δk
|φky|
1.0
.5
0.
0.2
0.4
0.6
0.8
1.0
1.2
ky



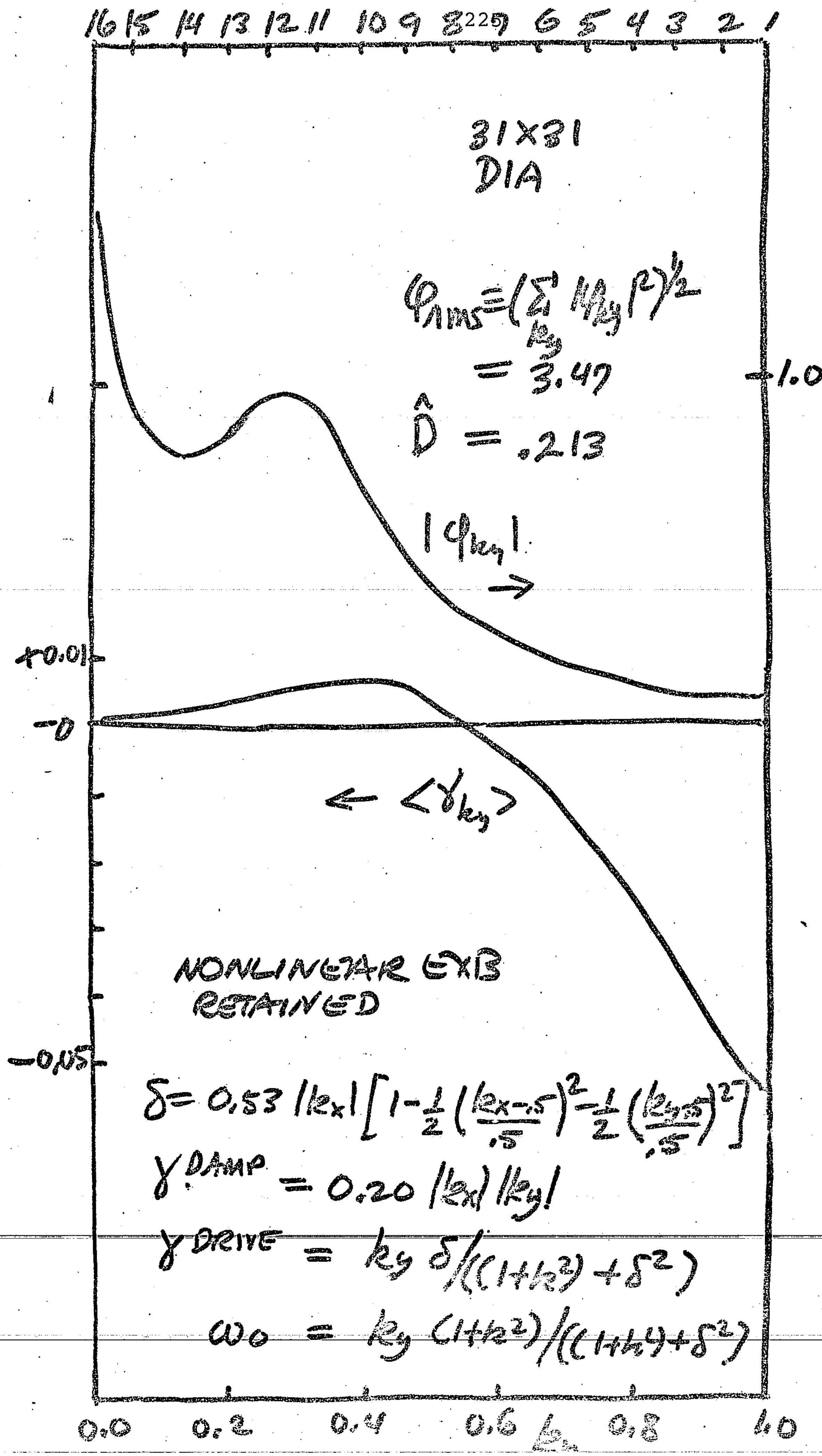
11X11
HYDRA
DIA
AT FIXED ω
|φ| ≈ INDEPENDENT OF γ
Δω ∝ γ
WEAK
|φ| ∝ γ/V ⇒ Δω ∝ |φ| V
Δω ∝ γ
STRONG
SCALING WITH γ/ω AT
FIXED V
STANDARD TEST CASE
0
1
2
3
4
(γ/ω₀)/(γ/ω₀)STD
φRMS

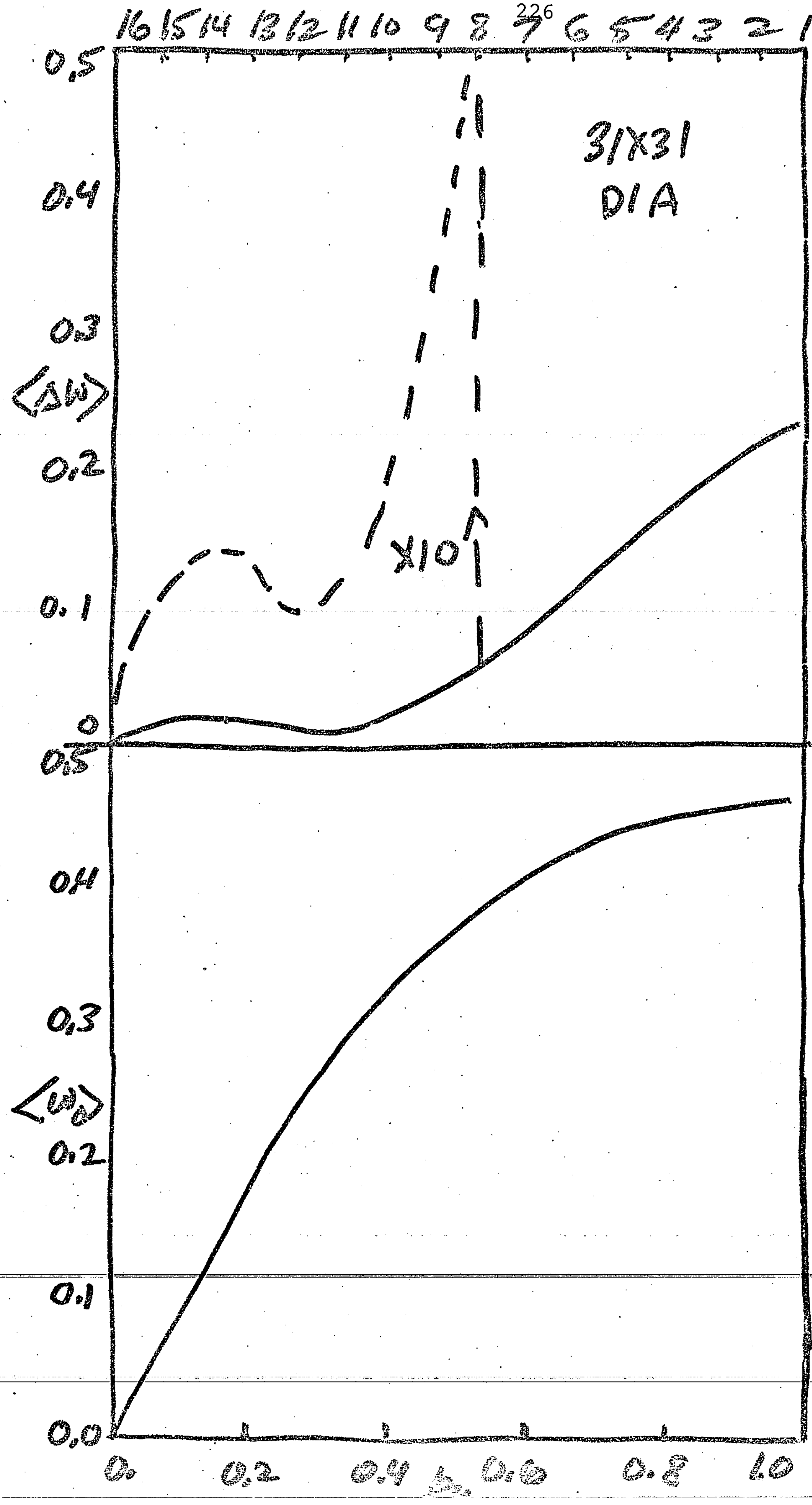
ROUGH COMPARISON TO EXPERIMENT

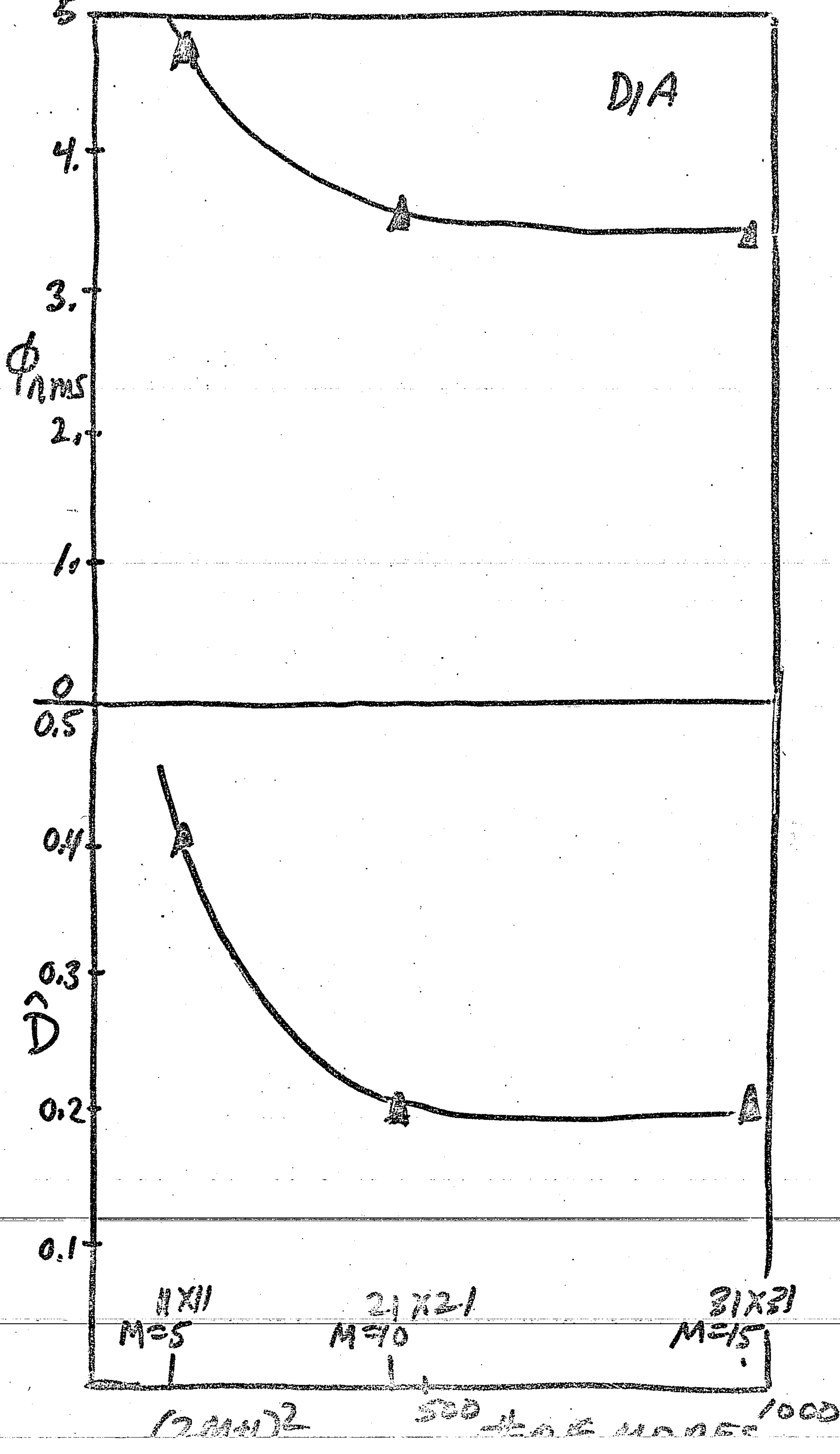
$\frac{\tilde{n}}{n_0} \sim 1\%$ $\quad \phi \sim O(3)$

$D \sim 10^4 \, cm^2/sec$ $\quad \hat{D} \sim O(1)$

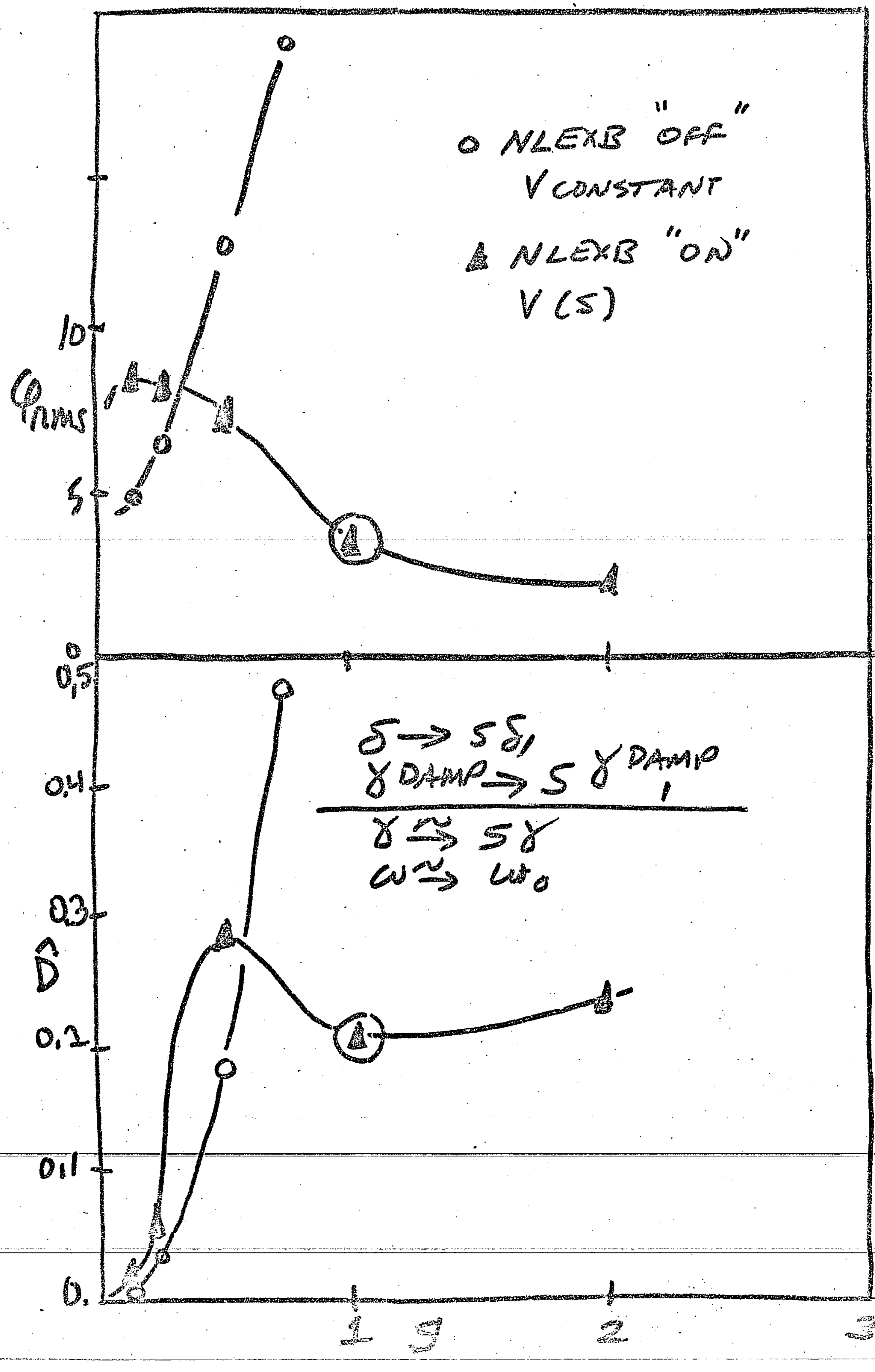
$\frac{\Delta\omega}{\omega} \sim O(1)$

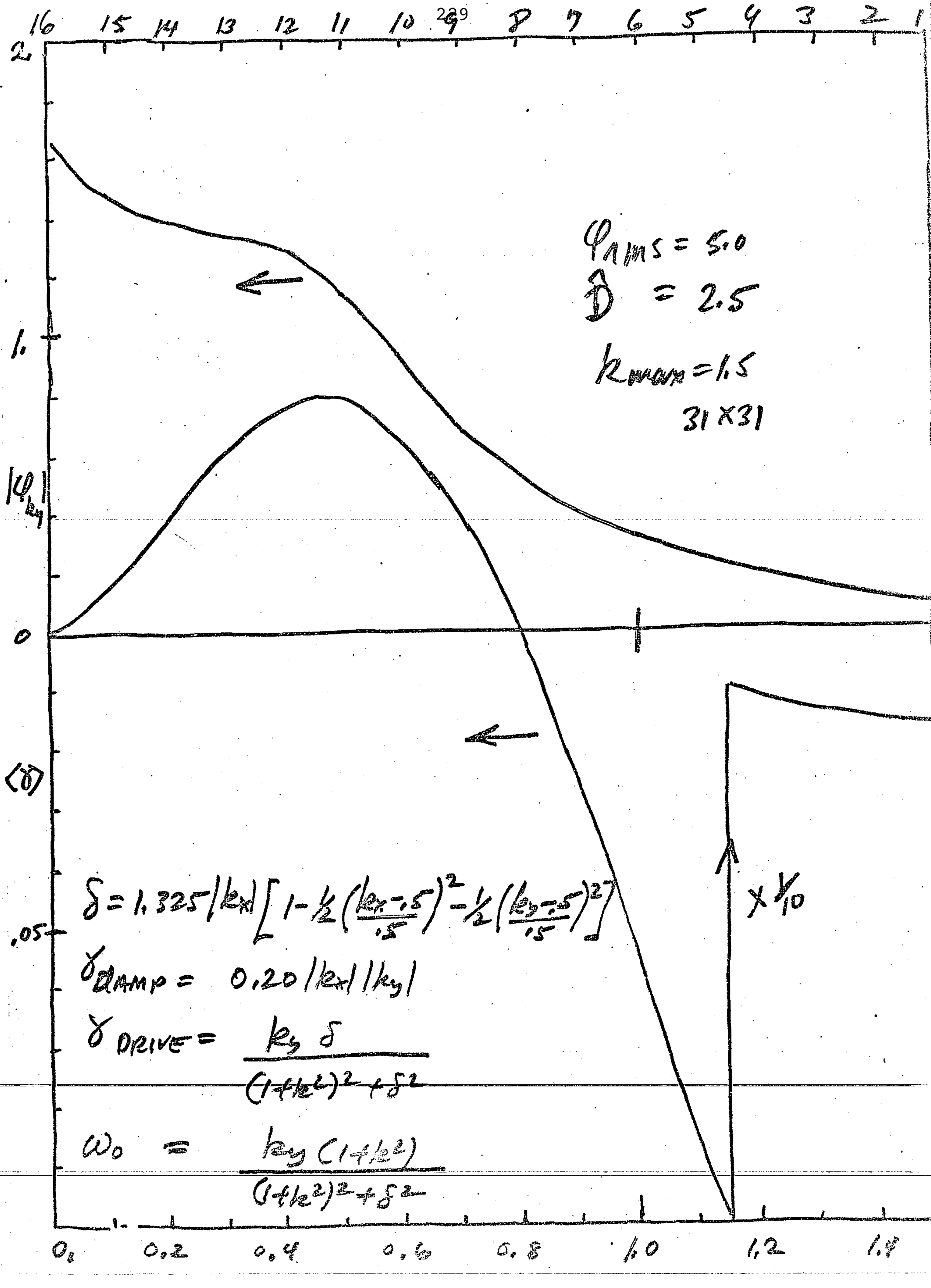

16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
31×31
DIA
$\varphi_{rms} = (\sum_{k_y}' |\varphi_{k_y}|^2)^{1/2}$
$= 3.47$
$\hat{D} = .213$
$|\varphi_{k_y}|$ →
1.0
+0.01
0
← $\langle \gamma_{k_y} \rangle$
NONLINEAR E×B
RETAINED
−0.05
$\delta = 0.53\,|k_x|\left[1 - \frac{1}{2}\left(\frac{k_x - .5}{.5}\right)^2 - \frac{1}{2}\left(\frac{k_y - .5}{.5}\right)^2\right]$
$\gamma^{DAMP} = 0.20\,|k_x|\,|k_y|$
$\gamma^{DRIVE} = k_y\,\delta / ((1+k^2)^2 + \delta^2)$
$\omega_0 = k_y\,(1+k^2) / ((1+k^2)^2 + \delta^2)$
0.0
0.2
0.4
0.6
k_x
0.8
1.0

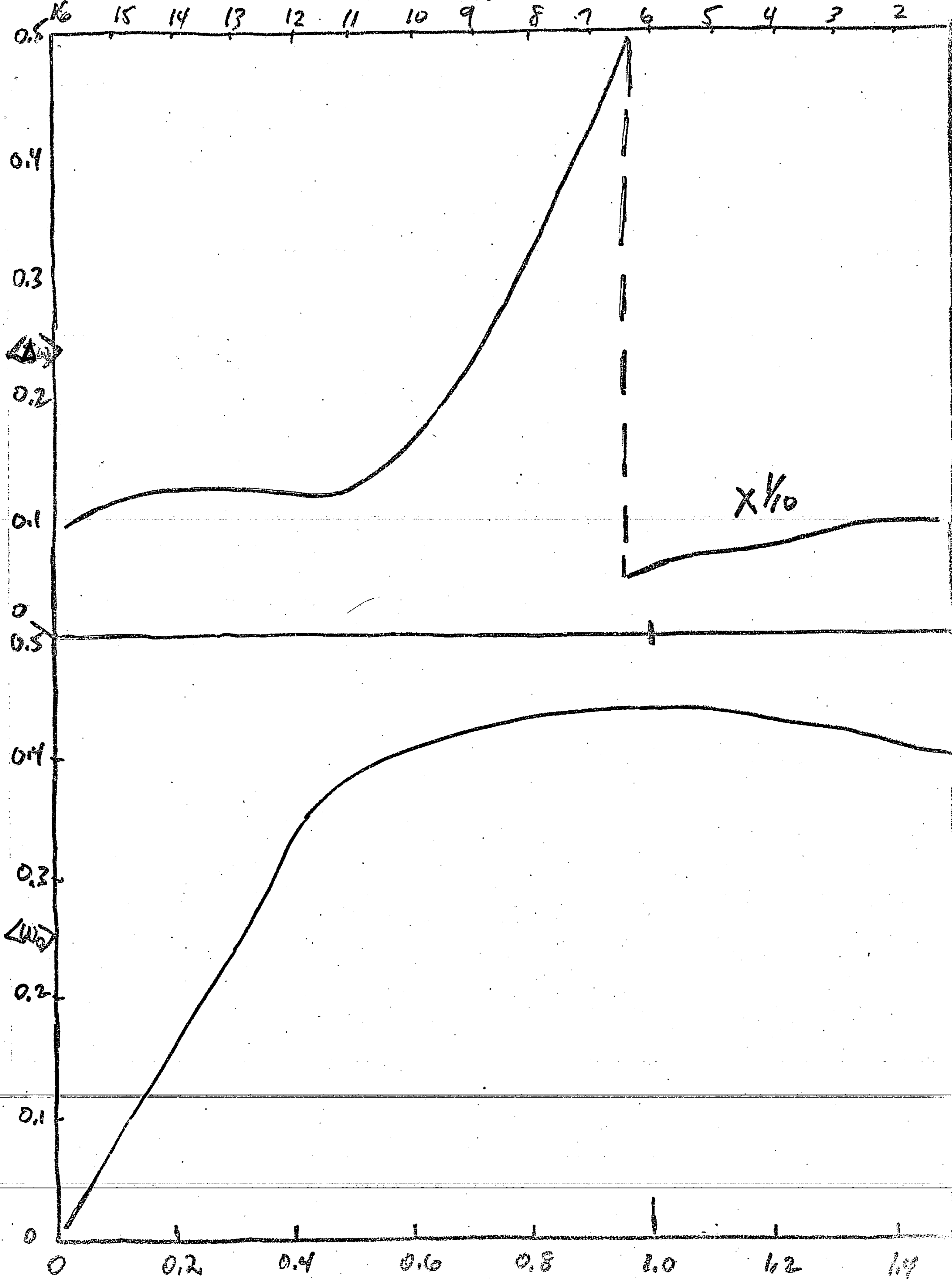

16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
31X31
DIA
x10
⟨ΔW⟩
⟨W₀⟩
0.5
0.4
0.3
0.2
0.1
0
0.0
0.2
0.4
0.6
0.8
1.0



D/A
5
4.
3.
ϕ_{rms}
2.
1.
0
0.5
0.4
0.3
$\hat{D}$
0.2
0.1
11X11
M=5
21X21
M=10
31X31
M=15
500
1000


o NLEXB "OFF"
V CONSTANT
▲ NLEXB "ON"
V(S)
φ_{rms}
10
5
0
0.5
0.4
0.3
0.2
0.1
0.
$\hat{D}$
$\delta \to 5\delta_1$
$\gamma^{DAMP} \to 5\gamma_1^{DAMP}$
$\gamma \to 5\gamma$
$\omega \to \omega_0$
1
2
3
S


16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
2
1.
$|\varphi_{k_y}|$
0
$\langle\delta\rangle$
.05
$\varphi_{rms} = 5.0$
$\hat{D} = 2.5$
$k_{max} = 1.5$
31 X 31
$\times \frac{1}{10}$
$\delta = 1.325|k_x|\left[1-\frac{1}{2}\left(\frac{k_x-.5}{.5}\right)^2-\frac{1}{2}\left(\frac{k_y-.5}{.5}\right)^2\right]$
$\gamma_{DAMP} = 0.20|k_x||k_y|$
$\gamma_{DRIVE} = \frac{k_y\,\delta}{(1+k^2)^2+\delta^2}$
$\omega_0 = \frac{k_y(1+k^2)}{(1+k^2)^2+\delta^2}$
0. 0.2 0.4 0.6 0.8 1.0 1.2 1.4


16 15 14 13 12 11 10 9 8 7 6 5 4 3 2
0.5
0.4
0.3
0.2
0.1
0
0.5
0.4
0.3
0.2
0.1
0
x 1/10
0 0.2 0.4 0.6 0.8 1.0 1.2 1.4

▶ CONCLUSIONS

- HOMOGENOUS STATIONARY TURBULENT STATES EXIST FOR LINEARLY DRIVEN-DAMPED SYSTEMS WITH (NLPD-ONLY) AND WITHOUT (NLEXB-INCLUDED) CONSERVATIVE NON LINEAR COUPLING

- APPROXIMATE WEAK COUPLING THEORIES (DIA) GIVE FAITHFUL DESCRIPTIONS (2X) OF TURBULENCE USING 10-100 X LESS COMPUTER TIME

- RELATIVELY FEW MODES ARE REQUIRED TO REPRESENT HIGH-m DRIFTWAVE HOMOGENEOUS TURBULENCE

- IGNORING WAVE-PARTICLE NONLINEARITY FLUID-LIKE SCALING PREVALES BEYOND SOME MODEST γ/ω

- CAN GET $\tilde{n}/n_0 = \Phi(\rho_s/L_n)$, $\hat{D}$, AND $\Delta\omega/\omega$ WITHIN 2X EXPERIMENTAL VALUES. HOWEVER $\Delta\omega \propto |\phi| \nu$ TENDS TO BE SOMEWHAT TOO SMALL SUGGESTING SIMPLE MODELS FOR ν ARE TOO WEAK

TURBULENT SPECTRA FROM THE INTERACTION OF THREE DRIFT WAVES

P.W. TERRY AND W. HORTON

UNIVERSITY OF TEXAS AT AUSTIN

TURBULENT SPECTRA FROM THE INTERACTION OF THREE DRIFT WAVES

P. W. TERRY

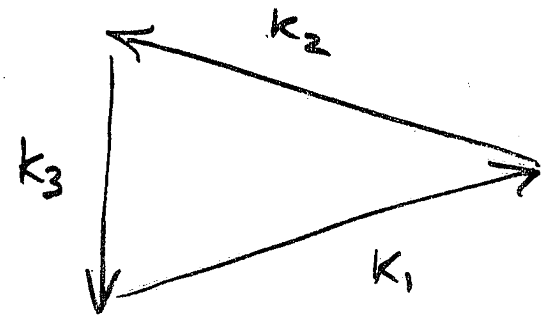
W. HORTON

CONDITIONS FOR CHAOTIC INTERACTION

SENSITIVITY TO MODELS

GENERAL CHARACTERISTICS OF "TURBULENCE"
ESP. SPECTRA

INTERACTION OF THREE WAVES



$\underset{\sim}{k}_1 + \underset{\sim}{k}_2 + \underset{\sim}{k}_3 = 0$

$$\dot{\varphi}_j^* = i(\omega_j - i\gamma_j)\varphi_j^* - A f_j \varphi_k \varphi_\ell$$

φ_j : COMPLEX ELECTROSTATIC POTENTIAL

ω_j : LINEAR FREQUENCY

γ_j : LINEAR GROWTH RATE

f_j : COUPLING COEFFICIENT

IN PREVIOUS WORK

$f_1 = -1$; $f_2 = 1$; $f_3 = 1$

$\gamma_1 > 0$; $\gamma_2 = \gamma_3 < 0$

$$\gamma_2 = \gamma_3 \quad \Rightarrow \quad |\varphi_2| \sim |\varphi_3| \qquad (t \to \infty)$$

BEHAVIOR OBSERVED:

FIXED POINT

LIMIT CYCLE

CHAOTIC OR STOCHASTIC BEHAVIOR

CONTROLLED BY

$$\Gamma = \frac{|\gamma_2|}{|\gamma_1|}$$

FOR STOCHASTIC BEHAVIOR Γ LARGE, ($\gtrsim$ 12)

CONSIDER 3 WAVE INTERACTION FOR DRIFT WAVES:

f_j, ω_j, γ_j ARE NOW CONSTRAINED BY PHYSICS OF DRIFT WAVE INTERACTION

IONS: E X B
POLARIZATION

ELECTRONS: ADIABATIC WITH NONADIABATIC CONTRIBUTION FROM DISSIPATIVE WAVE-PARTICLE INTERACTION

f_j IS COMPLEX

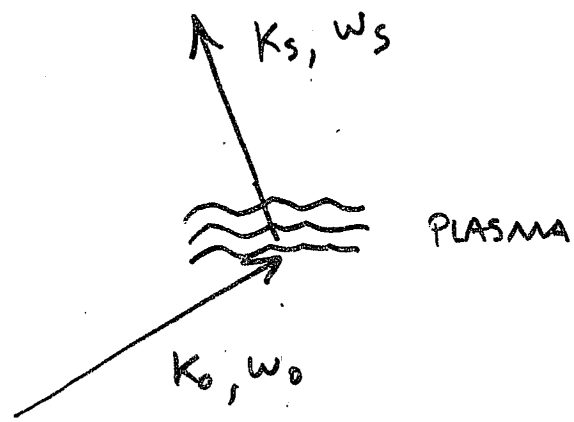
DYNAMICS ARE 4-D IN GENERAL

BEHAVIOR IS STOCHASTIC FOR TYPICAL DRIFT WAVE PARAMETERS

PREVIOUSLY TESTED VALIDITY OF R.P.A.

SPECTRUM OF FLUCTUATIONS

EXPERIMENTAL WORK



FIXED WAVENUMBER

EXPERIMENTAL FREQUENCY SPECTRA BROAD

DERIVATION OF THREE MODE EQUATION FOR DRIFT WAVES

IONS HYDRODYNAMIC :

$$\frac{\partial n}{\partial t} + \underset{\sim}{V}_E \cdot \nabla n + \nabla \cdot (n \underset{\sim}{V}_p) = 0$$

$$\underset{\sim}{V}_E = \frac{cT_e}{eB} \hat{b} \times \nabla \varphi$$

$$\underset{\sim}{V}_p = \underset{\sim}{V}_p^{(1)} + \underset{\sim}{V}_p^{(2)} = -\rho^2 \left(\frac{\partial}{\partial t} + \frac{cT_e}{eB} b \times \nabla \varphi \nabla_\perp \right) \nabla_\perp \varphi$$

ELECTRONS ADIABATIC

$$n(x,t) = n_0(r) \left[\varphi + \hat{\mathcal{L}}^a \varphi(x,t) \right]$$

$$\mathcal{L}^a(k) = i(\pi/2)^{1/2} \left[\omega_k - \omega_{*e} (1 - \tfrac{1}{2} \eta_e) \right] / k_\parallel V_e$$

$$\approx i\delta_0 k_y (k_\perp^2 - \tfrac{1}{2} \eta_e)$$

$$\delta_0 = (\pi/2)^{1/2} (m_e/m_i)^{1/2} / |k_\parallel r_n|$$

QUASINEUTRALITY

$$\left(1 + \mathcal{L}^a - \nabla_\perp^2\right)\frac{\partial \varphi}{\partial t} = -\frac{\partial \varphi}{\partial y} - \left[\varphi, \left(\mathcal{L}^a - \nabla_\perp^2 \varphi\right)\right]$$

~~REQUIRE~~ WHERE

$$[g, h] = \hat{z} \cdot \nabla_\perp g \times \nabla_\perp h = \frac{\partial g}{\partial x}\frac{\partial h}{\partial y} - \frac{\partial h}{\partial x}\frac{\partial g}{\partial y}$$

FOURIER TRANSFORM

$$(1 + \chi_k)\frac{d\varphi_k}{dt} = -ik_y\varphi_k + \frac{1}{2}\sum_{\substack{k_1+k_2\\=k}} \hat{z}\cdot(\underset{\sim}{k_1}\times\underset{\sim}{k_2})(\chi_{\underset{\sim}{k_1}} - \chi_{\underset{\sim}{k_2}})\varphi_{\underset{\sim}{k_1}}\varphi_{\underset{\sim}{k_2}}$$

$$\chi_k = k_\perp^2 - i\delta_0 k_y\left(k_\perp^2 - \tfrac{1}{2}\eta_e\right)$$

CONSIDER 3 MODES

$$k = -k_3$$

$$f_1 = F_1 - iG_1 = \chi_{k_2} - \chi_{k_3}$$

$$f_2 = F_2 - iG_2 = \chi_{k_3} - \chi_{k_1}$$

$$f_3 = F_3 - iG_3 = \chi_{k_1} - \chi_{k_2}$$

$$A = \frac{1}{2} K_1 \times K_2 \cdot Z \Big/ \sqrt{(1+k_1^2)(1+k_2^2)(1+k_3^2)}$$

$$\hat{\varphi}_k = \varphi_k \sqrt{1+k^2}$$

$$\Rightarrow \quad \frac{d\varphi_1^*}{dt} = (i\omega_1 + \gamma_1)\varphi_1^* - A f_1 \varphi_2 \varphi_3$$

$$\omega_1 + i\gamma_1 = K_{1y} / (1 + \chi_{\underset{\sim}{k}})$$

INTRODUCE AMPLITUDE AND PHASE

$$\hat{\varphi}_j(t) = a_j(t) \, \mathrm{EXP}\,(i\alpha_j(t))$$

$$\frac{da_j}{dt} = \gamma_j a_j - A a_k a_\ell \left[F_j \cos\alpha + G_j \sin\alpha\right]$$

$$\frac{d\alpha}{dt} = -\Delta\omega + A \sum \frac{a_k a_\ell}{a_j} \left[F_j \sin\alpha - G_j \cos\alpha\right]$$

WHERE

$$\Delta\omega = \omega_1 + \omega_2 + \omega_3$$

$$\alpha = \alpha_1 + \alpha_2 + \alpha_3$$

PROPERTIES OF 3 MODE EQUATIONS

VOLUME IN PHASE SPACE :

$$\frac{dV}{dt} = \sum_j \frac{1}{a_j}\frac{\partial}{\partial a_j}\left(a_j \frac{da_j}{dt}\right) + \frac{\partial}{\partial \alpha}\left(\frac{d\alpha}{dt}\right) = 2(\gamma_1+\gamma_2+\gamma_3)$$

ENERGY

$$W = \tfrac{1}{2}a_1^2 + \tfrac{1}{2}a_2^2 + \tfrac{1}{2}a_3^2$$

$$\frac{dW}{dt} = \gamma_1 a_1^2 + \gamma_2 a_2^2 + \gamma_3 a_3^2$$

ENSTROPHY

$$U_r = \tfrac{1}{2}\chi_1{}^r a_1^2 + \tfrac{1}{2}\chi_2{}^r a_2^2 + \tfrac{1}{2}\chi_3{}^r a_3^2$$

$$\frac{dU_r}{dt} = \gamma_1 \chi_1{}^r a_1^2 + \gamma_2 \chi_2{}^r a_2^2 + \gamma_3 \chi_3{}^r a_3^2$$

INTEGRABILITY :

IN ABSENSE OF DISSIPATION :

$$\underset{\sim}{\gamma} = 0 \quad ; \quad \underset{\sim}{G} = 0$$

TRANSFORM $I_j = \dfrac{a_j^2}{F_j}$

HAMILTONIAN:

$$H = -\omega_1 I_1 - \omega_2 I_2 - \omega_3 I_3$$

$$+ 2A\,(F_1 F_2 F_3)^{1/2}\,(I_1 I_2 I_3)^{1/2}\,\mathrm{SIN}\,(\alpha_1 + \alpha_2 + \alpha_3)$$

CONSTANTS OF MOTION: H, $I_1 - I_2 = m_{12}$, $I_1 - I_3 = m_{13}$

QUADRATURE:

$$\frac{dI_1}{dt} = -\frac{1}{3}\left\{ P_3(I_1) \right\}^{1/2}$$

$P_3(I_1)$ IS CUBIC POLYNOMIAL IN I_1

DISSIPATIVE CASE

FOR VOLUME CONTRACTING FLOW, $\gamma_1 > 0$, $\gamma_2, \gamma_3 < 0$

STUDY:

- FIXED POINT
- LIMIT CYCLES
- CHAOTIC BEHAVIOR

FIXED POINT :

$$\Delta \omega = \sum_j \left[\gamma_j \left(\frac{F_j \sin\alpha - G_j \cos\alpha}{F_j \cos\alpha + G_j \sin\alpha} \right) \right]$$

$$a_j^2 = \frac{\gamma_k \gamma_\ell}{A^2 \left[F_\ell F_k \cos^2\alpha + G_\ell G_k \sin^2\alpha + (F_\ell G_k + G_\ell F_k) \sin\alpha \cos\alpha \right]}$$

EXISTS ONLY FOR $a_j^2 > 0$

EXISTENCE CONDITION

$$Sg(\gamma_k) = -Sg(\gamma_\ell) = -Sg(\gamma_j)$$

$$Sg\left[\mathrm{Re}\, f_k e^{i\alpha}\right] = -Sg\left[\mathrm{Re}\, f_\ell e^{i\alpha}\right] = -Sg\left[\mathrm{Re}\, f_j e^{i\alpha}\right]$$

SAME AS PARAMETRIC DECAY CONDITION

FOR

$$f_j = F_j - iG_j$$ (COMPLEX SUSCEPTIBILITY : REACTIVE AND DISSIPATIVE CONTRIBUTION)

1 AND ONLY 1 FIXED POINT

$$f_j = F_j$$ ($G_j = 0$, REACTIVE CONTRIBUTION ONLY)

NO FIXED POINT

$$f_j = G_j$$ ($F_j = 0$, DISSIPATIVE CONTRIBUTION ONLY)

1 AND ONLY 1 FIXED POINT)

STABILITY OF FIXED POINT

1) $C_0 > 0$

2) $C_3 > 0$

3) $C_2 C_3 > C_1^2$

4) $C_1 (C_2 C_3 - C_1^2) > C_0 C_3^2$

$$C_3 = -2(\gamma_1+\gamma_2+\gamma_3)$$

$$C_2 = (\gamma_1+\gamma_2+\gamma_3)^2 - \Delta\omega^2 + 2\sum_j{}' \gamma_j^2 \left(\frac{F_j \sin\alpha - G_j \cos\alpha}{F_j \cos\alpha + G_j \sin\alpha}\right)^2$$

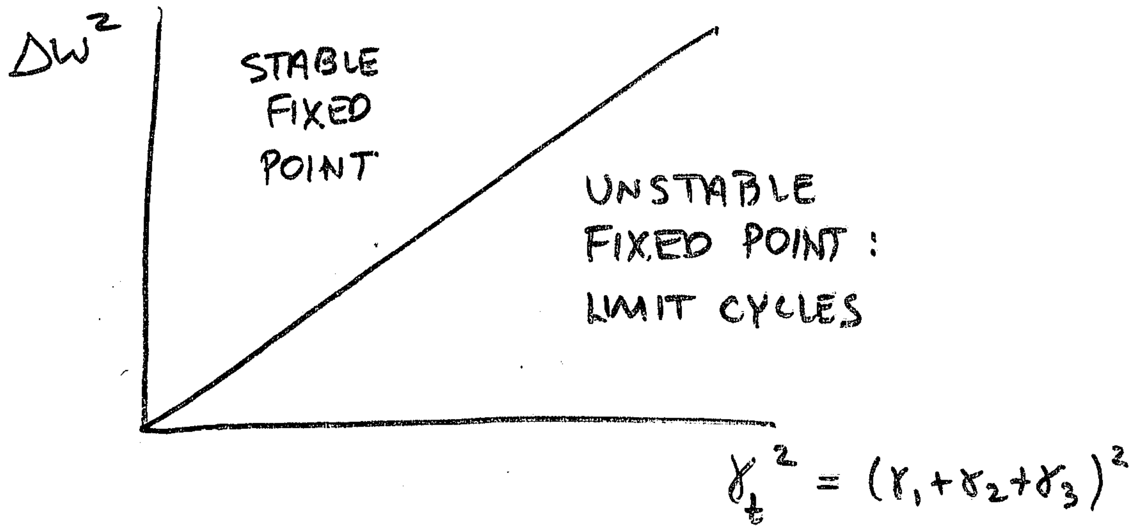
$$C_1 = 4\gamma_1\gamma_2\gamma_3 \left\{ 1 + \sum_{k,l}{}' \frac{(F_k \sin\alpha - G_k \cos\alpha)(F_l \sin\alpha - G_l \cos\alpha)}{(F_k \cos\alpha + G_k \sin\alpha)(F_l \cos\alpha + G_l \sin\alpha)} \right\}$$

$$C_0 = 4\gamma_1\gamma_2\gamma_3 \left\{ \gamma_1+\gamma_2+\gamma_3 - \sum_j{}' \gamma_j \left(\frac{F_j \sin\alpha - G_j \cos\alpha}{F_j \cos\alpha + G_j \sin\alpha}\right)^2 \right\}$$

FOR $|F| \ll |G|$

$C_0 > 0 \Rightarrow$

$$4\gamma_1\gamma_2\gamma_3(\gamma_1+\gamma_2+\gamma_3)\left[1 - \frac{\Delta\omega^2}{(\gamma_1+\gamma_2+\gamma_3)^2}\right]$$

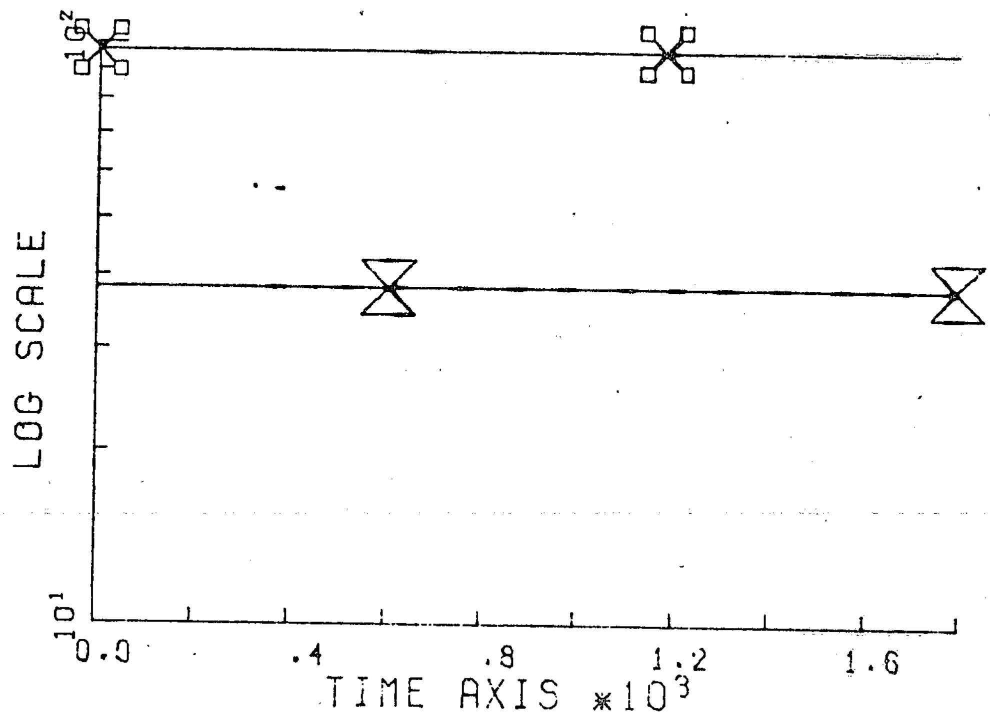


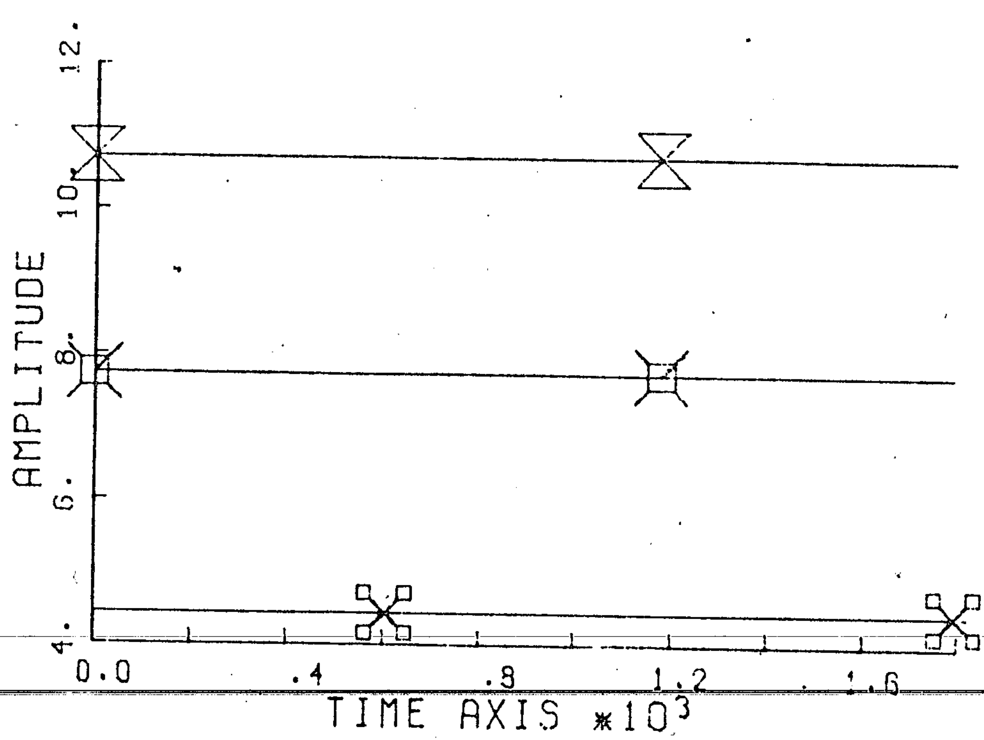
TRANSITION TO CHAOS:

1) $|F| \ll |G|$

INCREASE DAMPING RELATIVE TO GROWTH

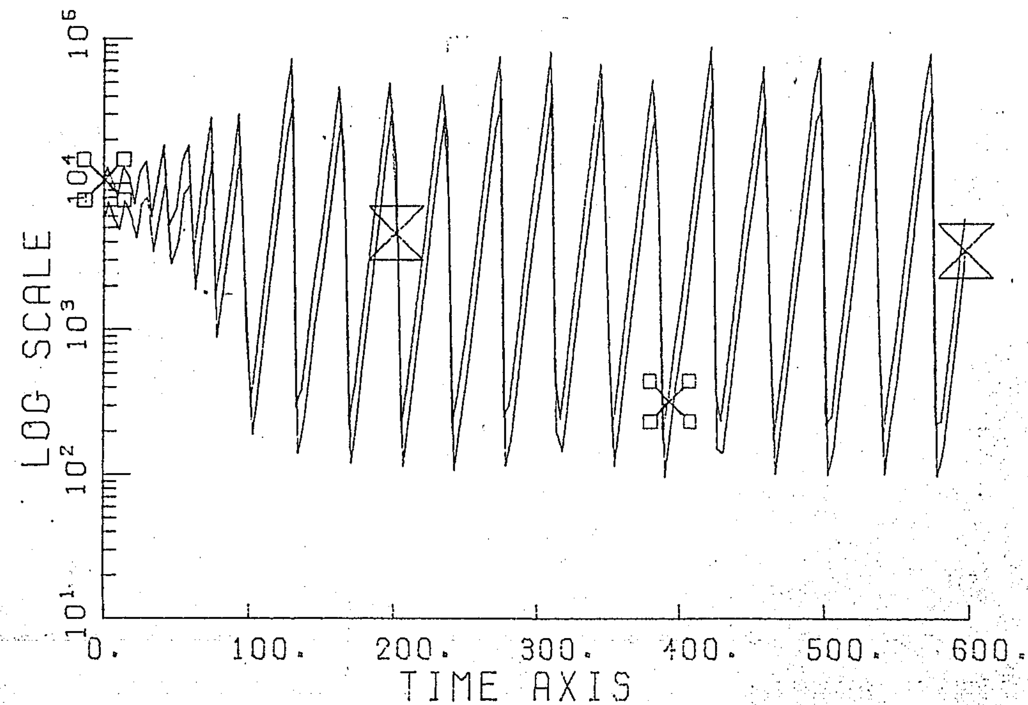
2.) INCREASE $|F|$ RELATIVE TO $|G|$

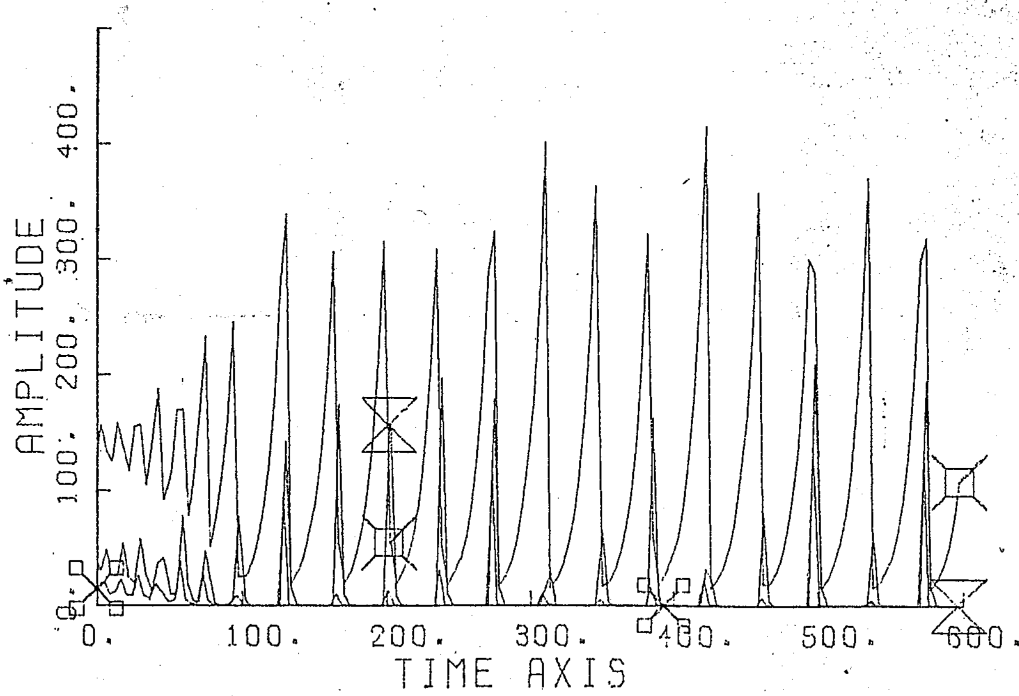




STABLE FIXED POINT (F = 0)

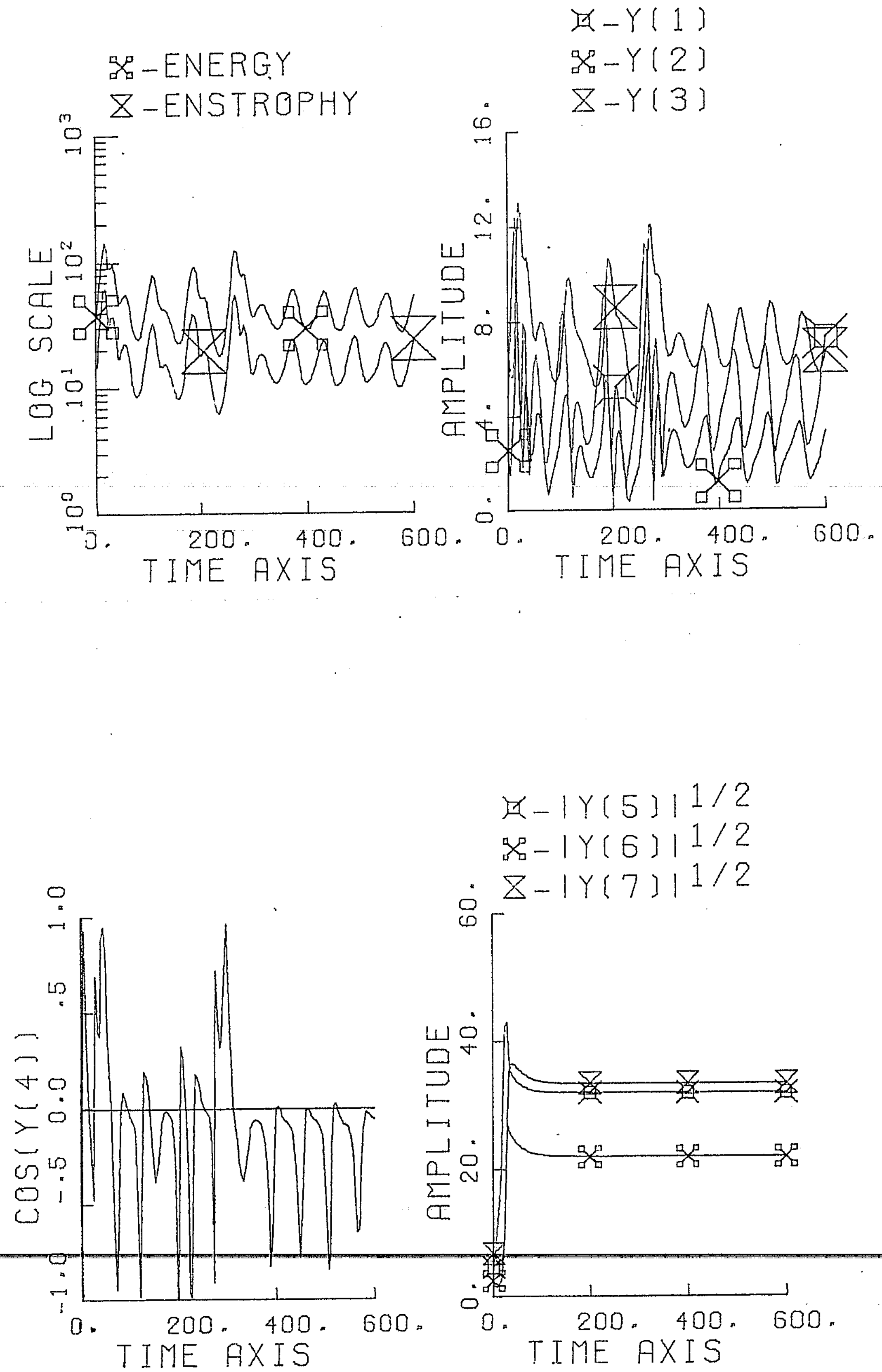
(Γ SMALL)


LOG SCALE
10^5
10^4
10^3
10^2
10^1
0.
100.
200.
300.
400.
500.
600.
TIME AXIS


AMPLITUDE
0.
100.
200.
300.
400.
0.
100.
200.
300.
400.
500.
600.
TIME AXIS

THREE MODE APPROXIMATION ELECTRON DRIFT WAVES

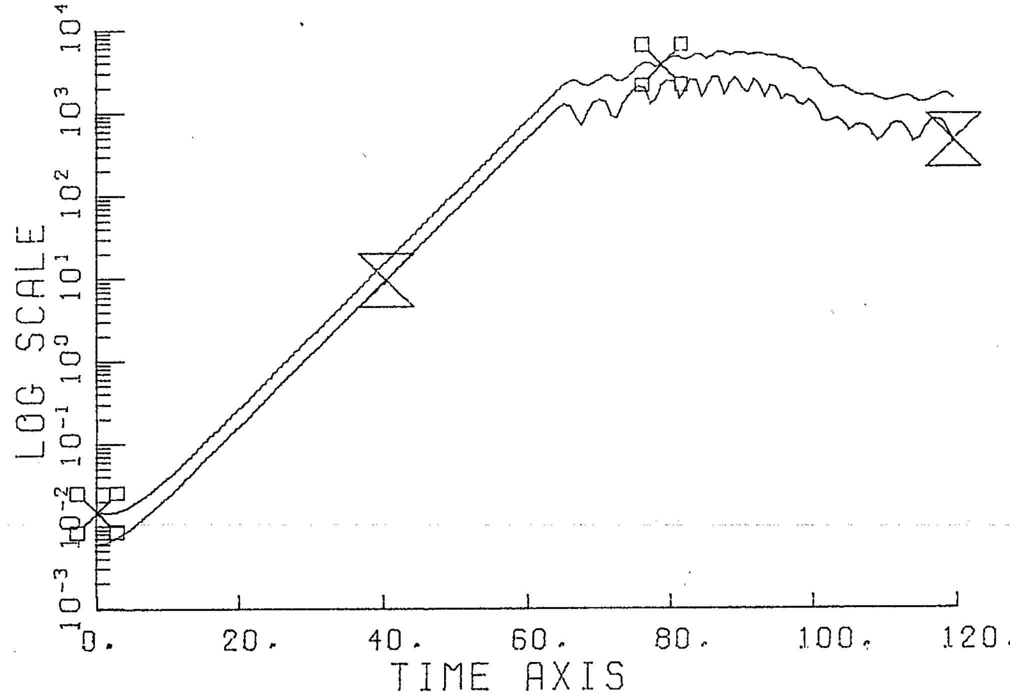
TAU=1.00 c = .3



1. STABLE LIMIT CYCLE, $(|F| \approx .3|G|; -\gamma_1 \approx \gamma_2 \approx \gamma_3)$

CHAOTIC REGIME $|F| \approx .8|G|$

$-\gamma_1 \approx \gamma_2 \approx \gamma_3$



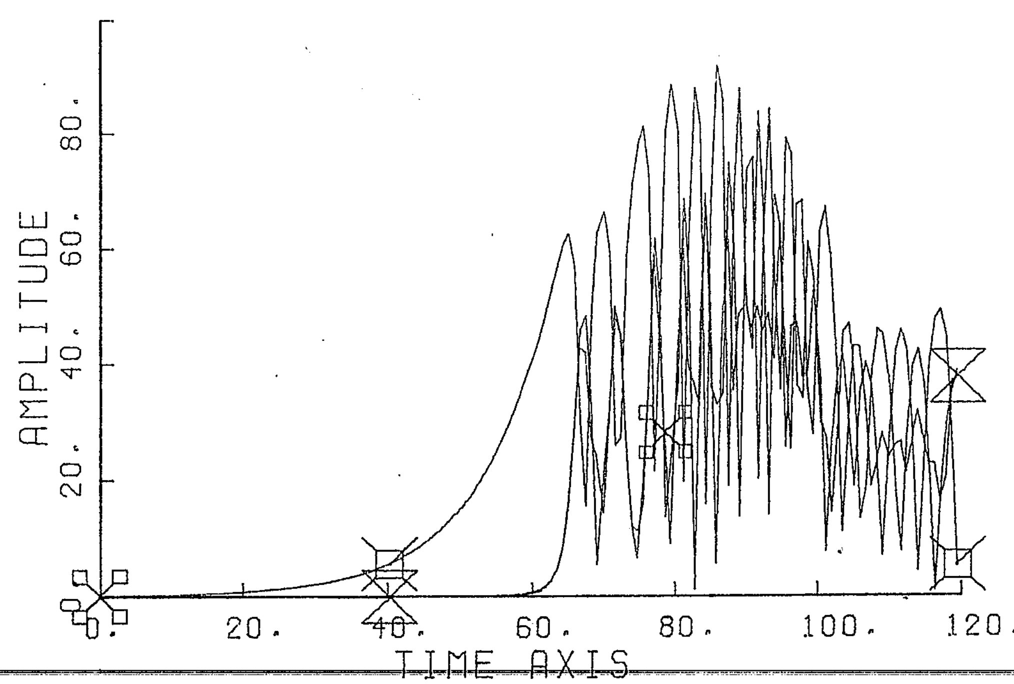


Fig 8

ω, γ, G same

$F = k F$ [illegible]

$k = .8$

AMPLITUDE AND PHASE EVOLUTION IN CHAOTIC REGIME

1) UNSTABLE (TRIVIAL LIMIT CYCLE)

2) "PARAMETRIC DECAY" OSCILLATION

CRITICAL AMPLITUDE:

$$a_j \approx \frac{\gamma_k \gamma_\ell}{A^2\left[F_k F_\ell \cos^2\alpha + G_k G_\ell \sin^2\alpha + (F_k G_\ell + F_\ell G_k)\sin\alpha\cos\alpha\right]}$$

APPROXIMATE FREQUENCY:

$$\Omega \approx 2A\sqrt{F_1 F_2 F_3 \; (a_1^2 - a_2^2)}$$

3) SATURATION

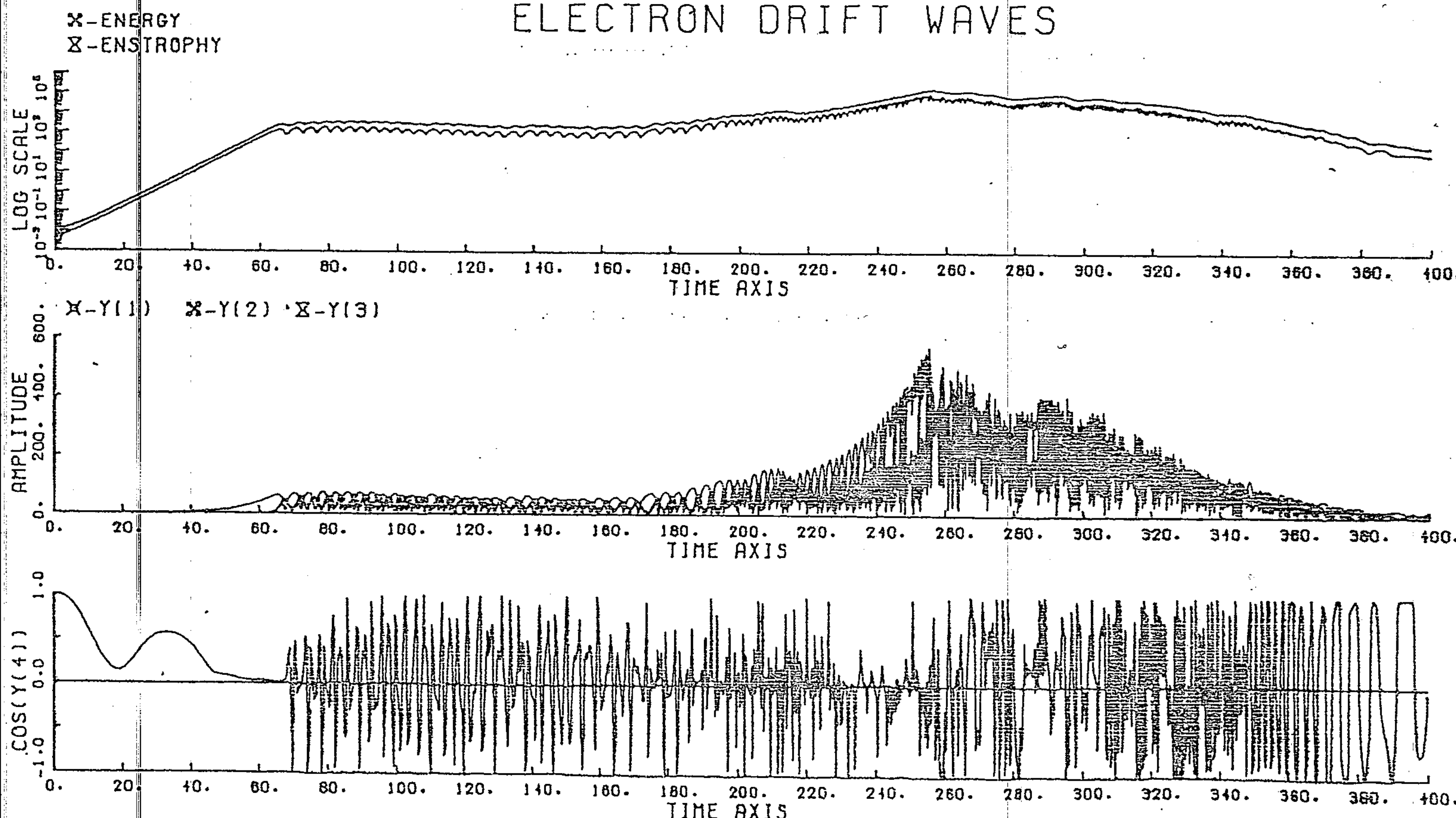
4) RANDOM PHASE

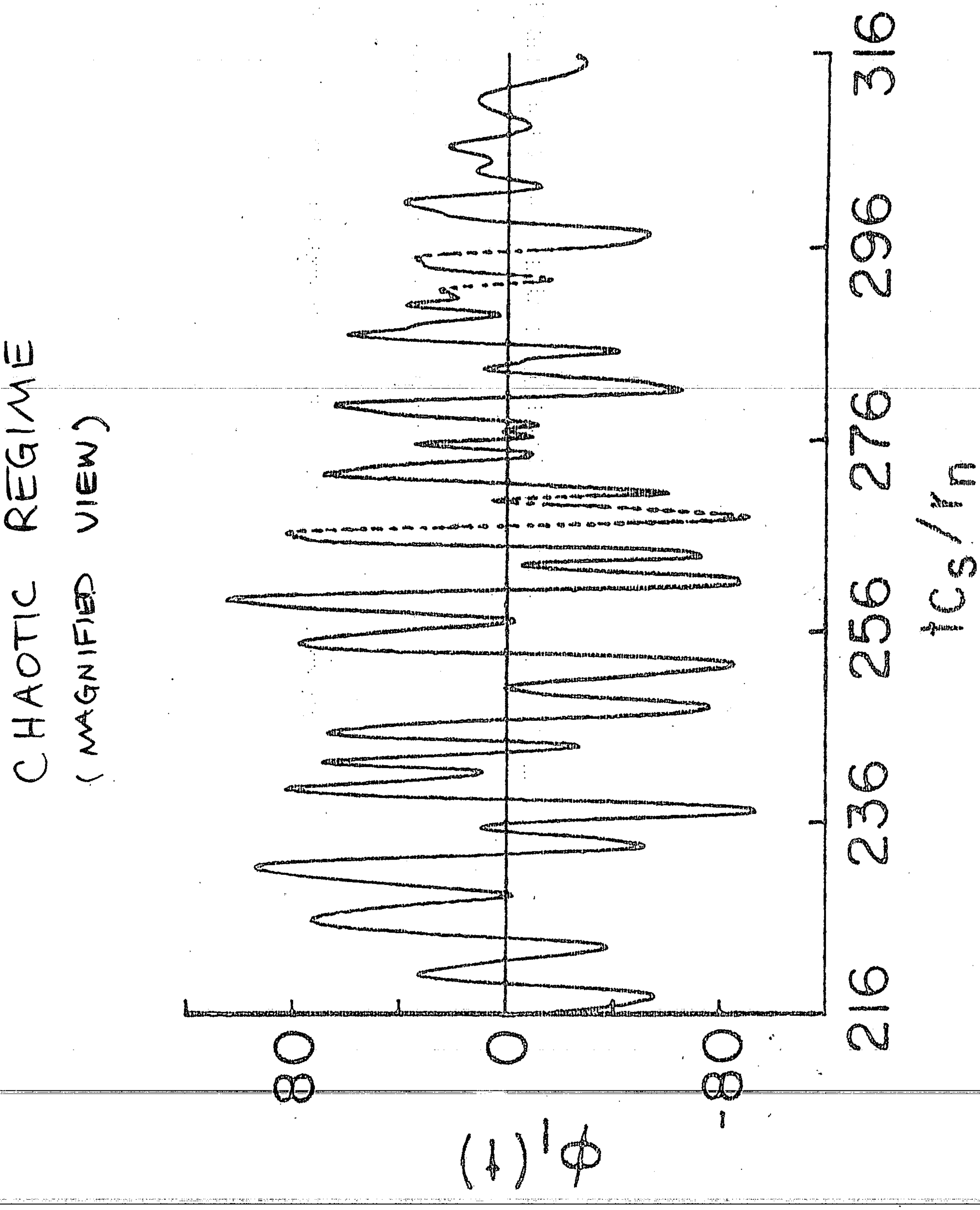
$$e^{i\alpha} = \int d\omega \; C^{1/2}(\omega)\, e^{i\omega t}\, e^{i\Theta(\omega)}$$

$$C(\omega) = \frac{\nu}{(\omega - \bar{\omega})^2 + \nu^2}$$

CHAOTIC REGIME

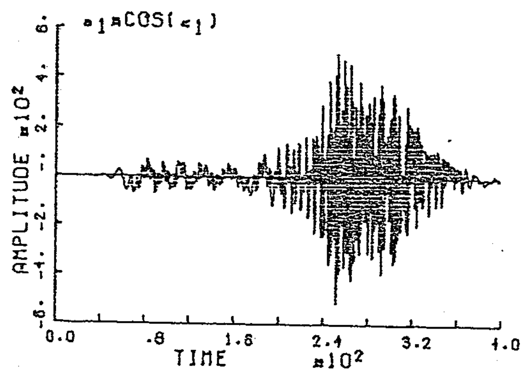
THREE MODE APPROXIMATION
ELECTRON DRIFT WAVES

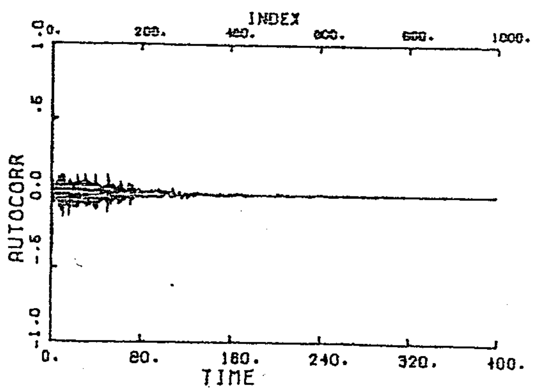




CHAOTIC REGIME
(MAGNIFIED VIEW)
80
0
-80
$\phi_1(t)$
216
236
256
276
296
316
tc_s/r_n

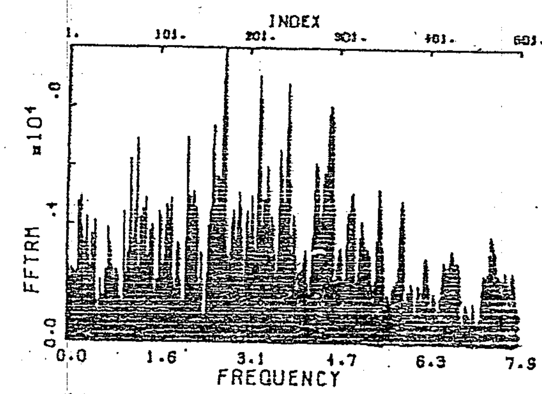
FOURIER TRANSFORM AND AUTOCORRELATION



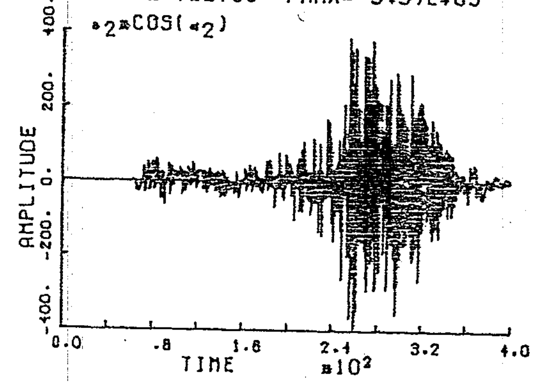


MEAN= 3.24E-01 VAR= 1.16E+04

FREQUENCY SPECTRUM

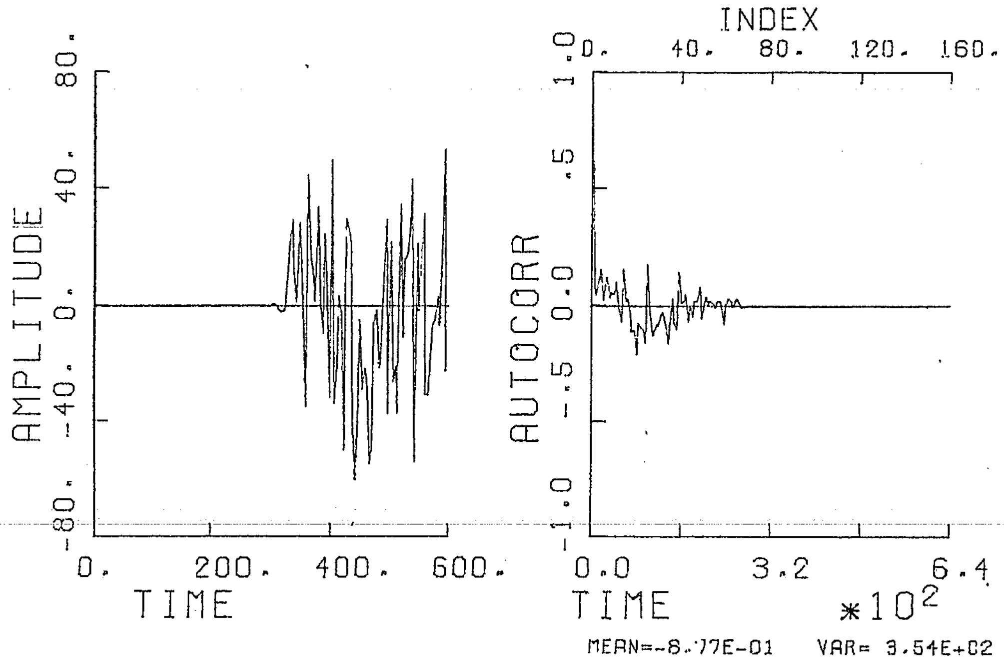


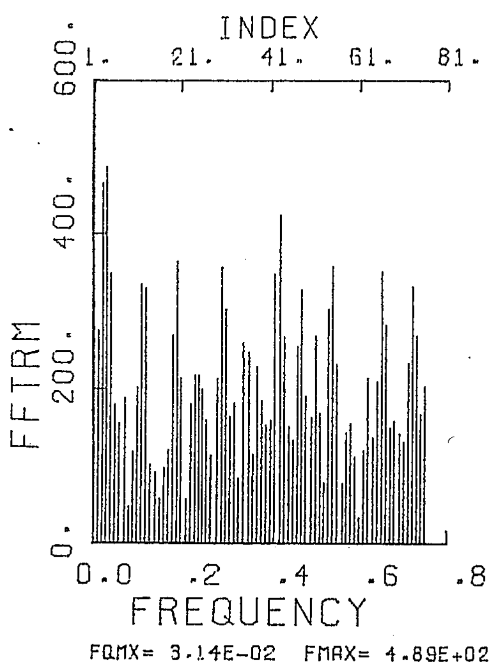
FQMX= 2.72E+00 FMAX= 9.97E+03



FREQUENCY SPECTRUM
(UNSTABLE MODE)

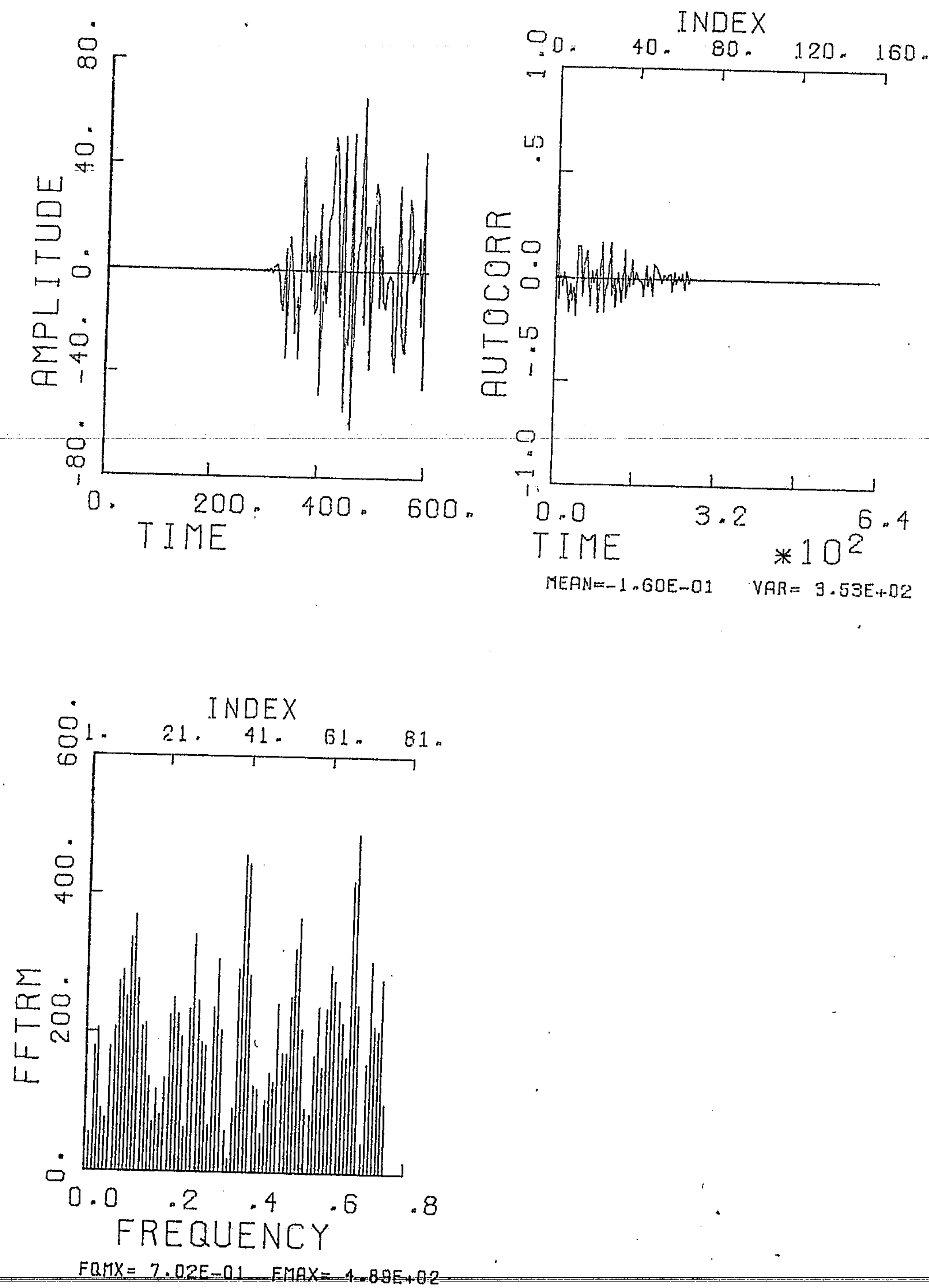
a3*COS(α3)





FREQUENCY SPECTRUM
(STABLE MODE)

$a_2 * COS(\alpha_2)$



LOWER HYBRID DRIFT INSTABILITY:

ANOMALOUS RESISTIVITY, TRANSPORT AND HEATING

M. TANAKA AND T. SATO

HIROSHIMA UNIVERSITY

Lower Hybrid Drift Instability: Anomalous Resistivity, Transport and Heating

by

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Hiroshima Univ., Hiroshima, Japan

①

Purpose :

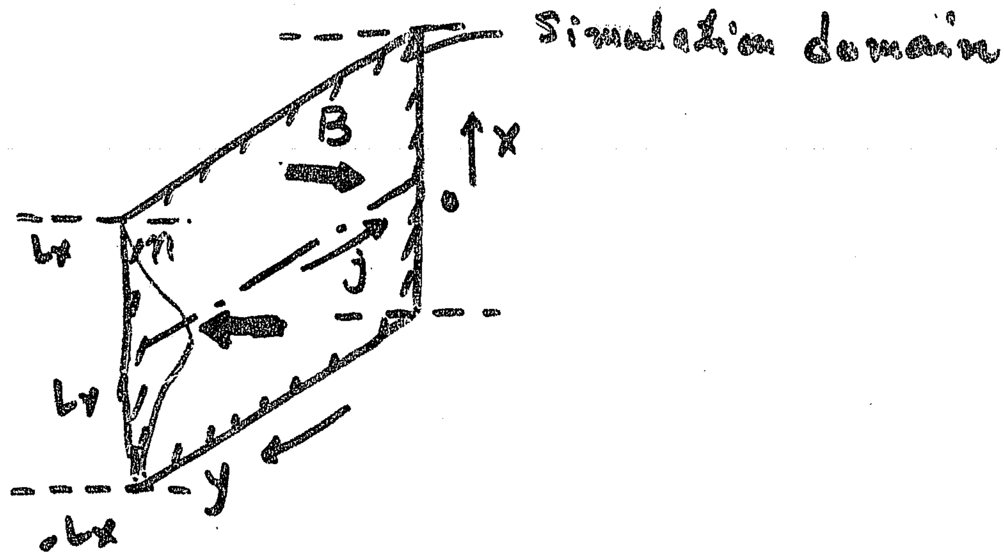
Nonlinear behaviors, such as resistivity, transport, heating, etc, in a field reversed plasma

Method :

2-D particle simulation by using magnetostatic code

(2)

Model



$$f_{0j}(x, v) = \frac{n(x)}{\pi v_{thj}^2} \exp\left\{-\frac{1}{v_{thj}^2}\left[v_x^2 + (v_y - V_{dj})^2\right]\right\}$$

$$n_e(x) = n_i(x) = n_0 \operatorname{sech}^2\left[(x - \tfrac{1}{2}L_x)/L_B\right]$$

$$B_z(x) = B_0 \tanh\left[(x - \tfrac{1}{2}L_x)/L_B\right]$$

$$(V_{de}/T_e = -V_{di}/T_i)$$

$x = \pm L_x$: conducting wall

$y = 0, L_y$: periodic

Parameters :

$L_x \cdots 64\Delta$, $L_y \cdots 128\Delta$ (32Δ)
($1.6\Delta \sim \lambda_D$)

$L_B / L_x \cdots \frac{1}{6}$ ($\frac{1}{5}$, $\frac{1}{4}$)

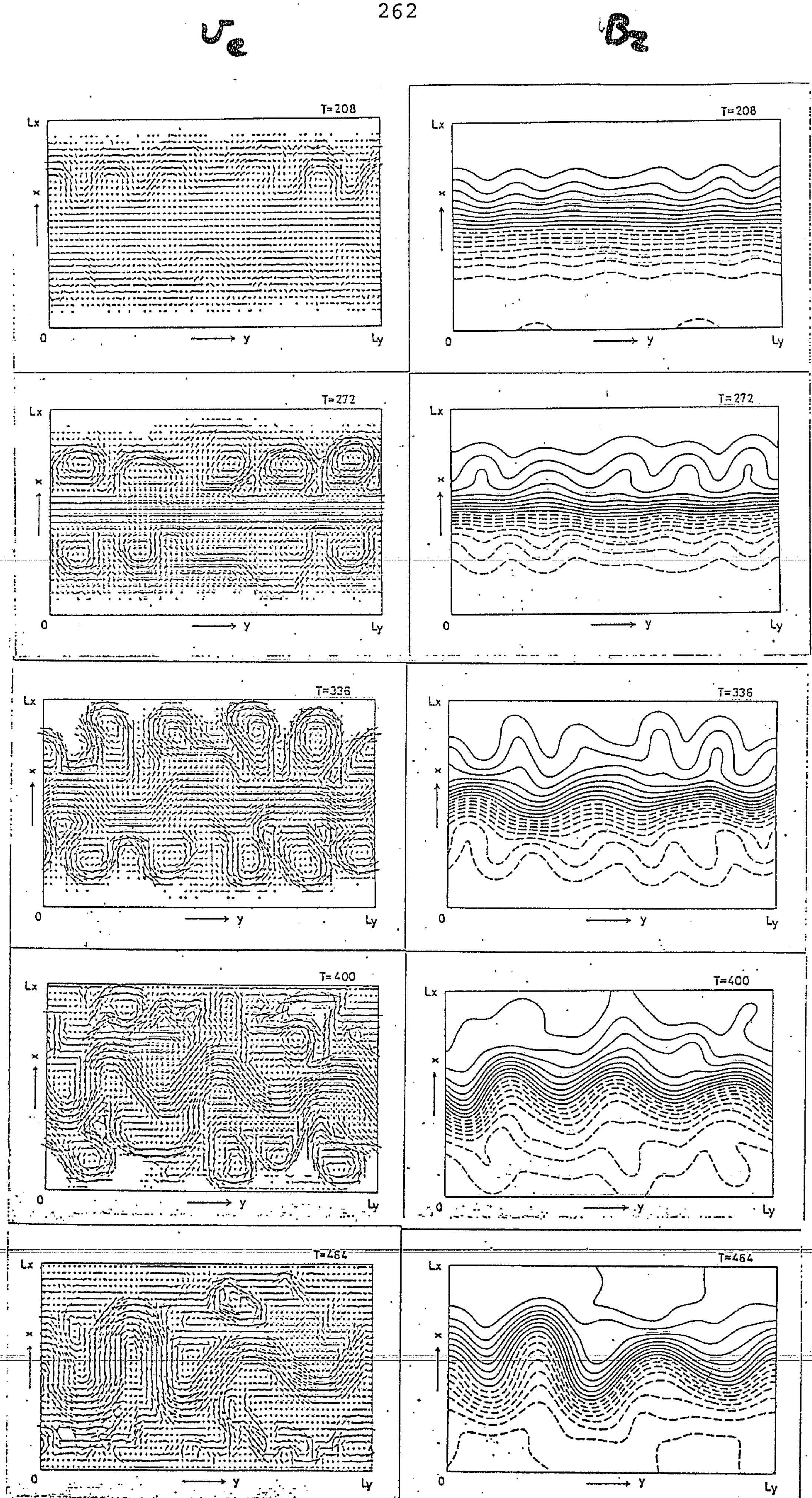
$m_i / m_e \cdots 185$ ($5 \sim 76$)

$(\omega_{pe}/\omega_{ce})^2 \cdots 0.55$ ($2 \sim 5$)

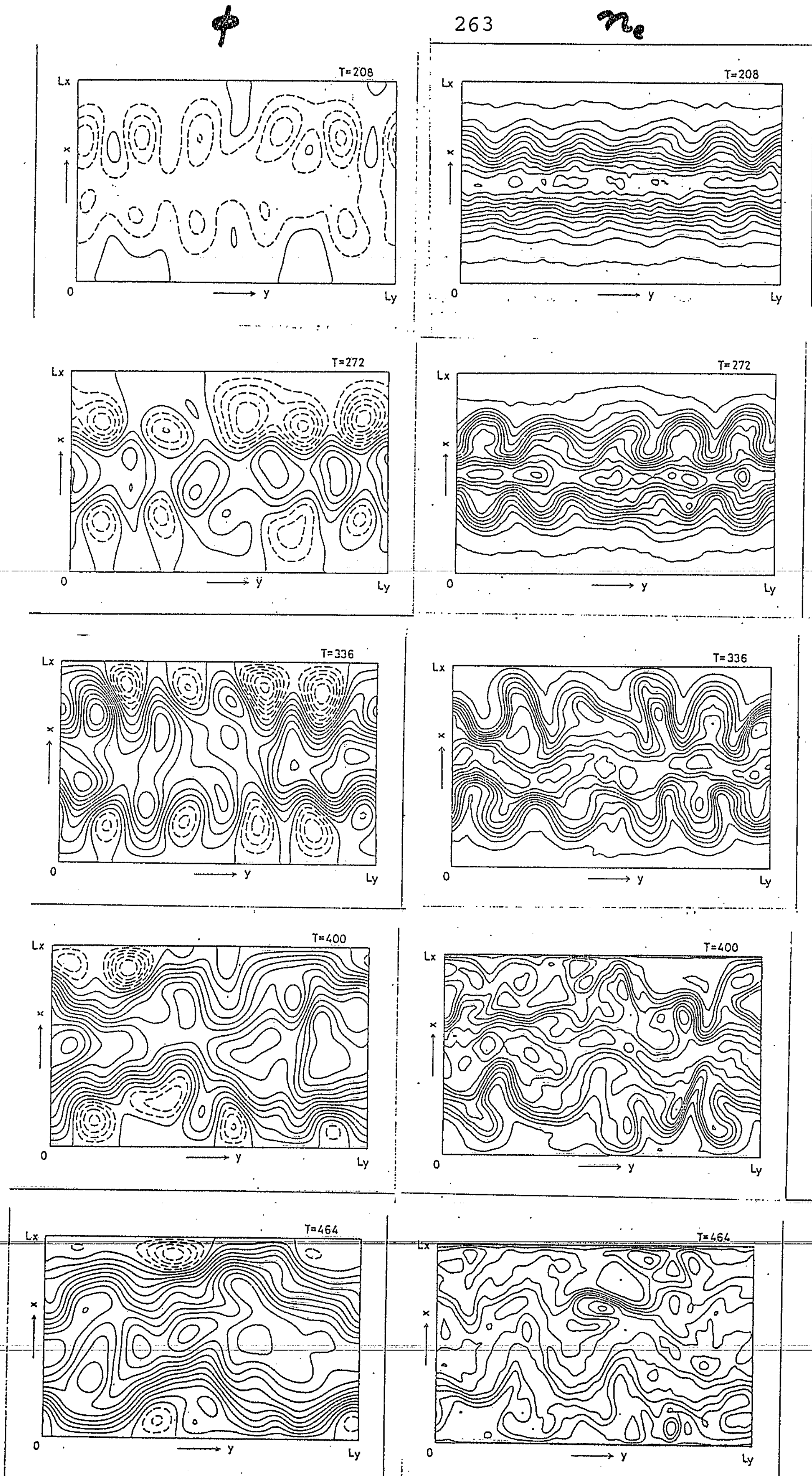
$c/\omega_{pe} L_B \cdots 0.48$ ($0.43 \sim 0.77$)

$T_i / T_e \cdots 2.0$ (2.0)

$V_{di} / v_{thi} \cdots 3.0$ ($0.7 \sim 2.9$)



Ve
Bz
T=208
T=272
T=336
T=400
T=464
Lx
x
0
y
Ly


φ
n_e
T=208
T=272
T=336
T=400
T=464
Lx
x
0
y
Ly

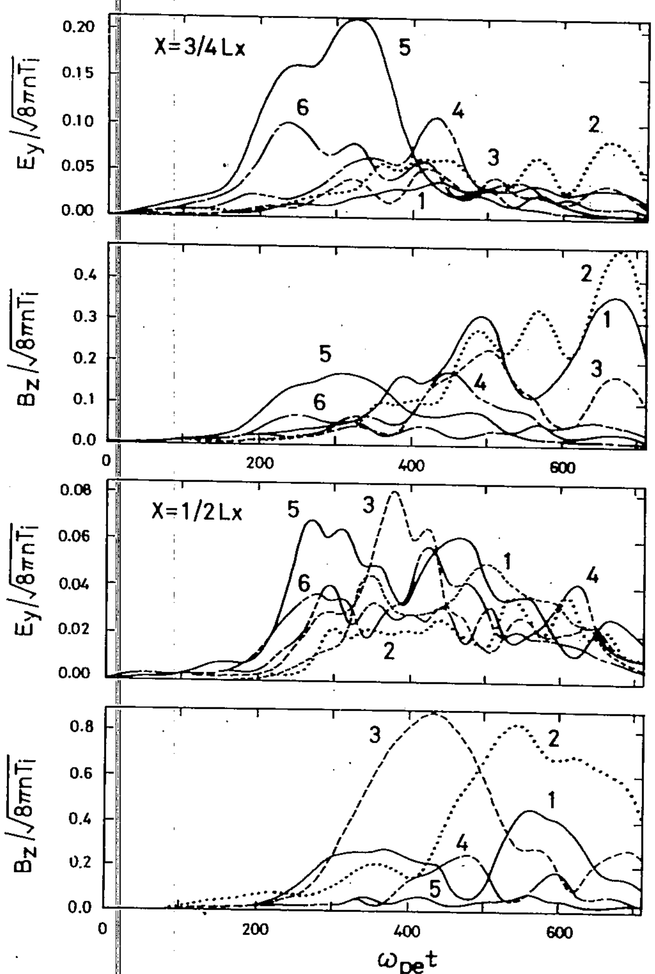

X=3/4Lx
X=1/2Lx
$E_y/\sqrt{8\pi nT_i}$
$B_z/\sqrt{8\pi nT_i}$
$\omega_{pe}t$

Fig. 5

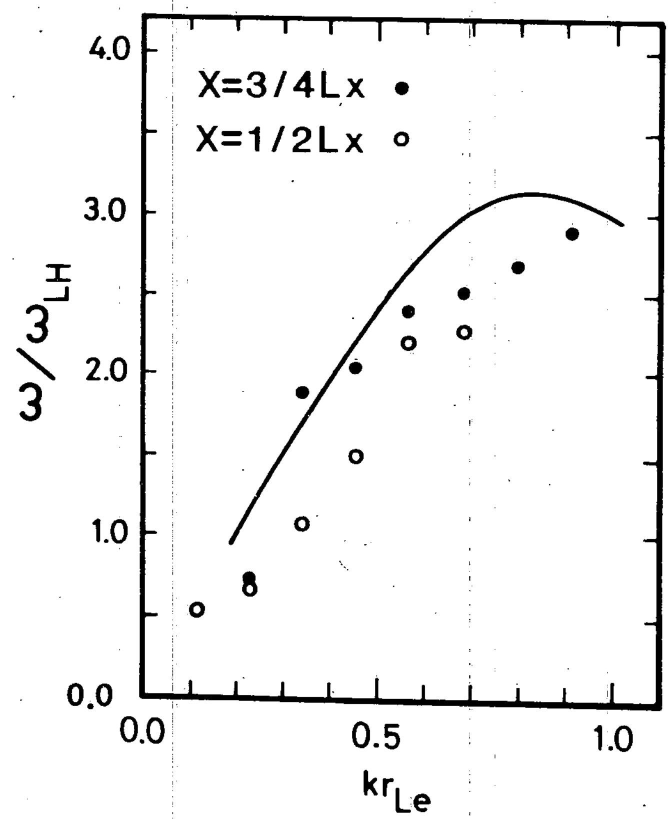

X=3/4Lx
X=1/2Lx
ω/ω_{LH}
kr_{Le}

Fig. 6

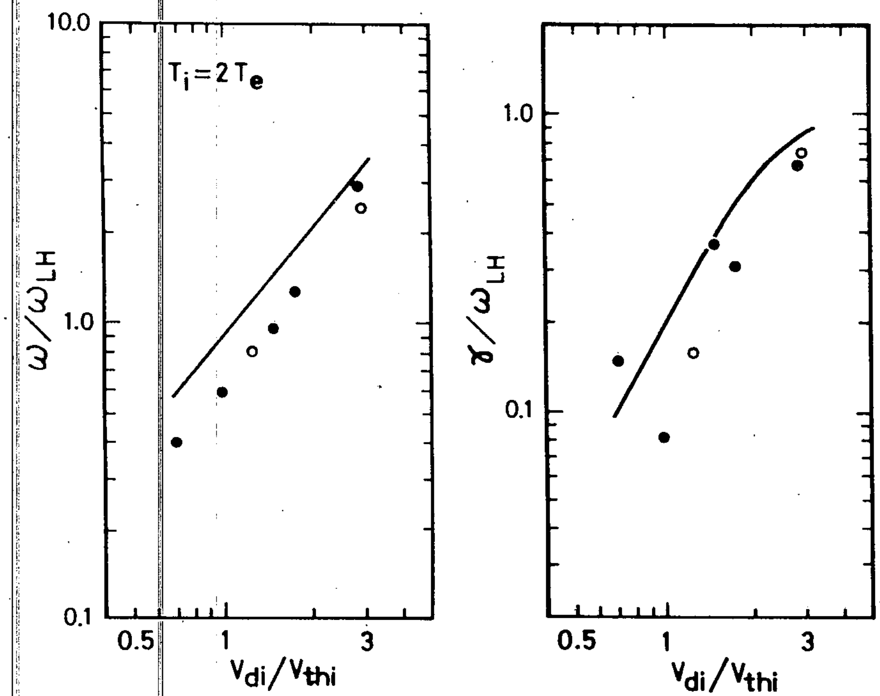

10.0
1.0
0.1
$T_i = 2T_e$
ω/ω_{LH}
0.5
1
3
V_{di}/V_{thi}
1.0
0.1
γ/ω_{LH}
0.5
1
3
V_{di}/V_{thi}

Fig. 7

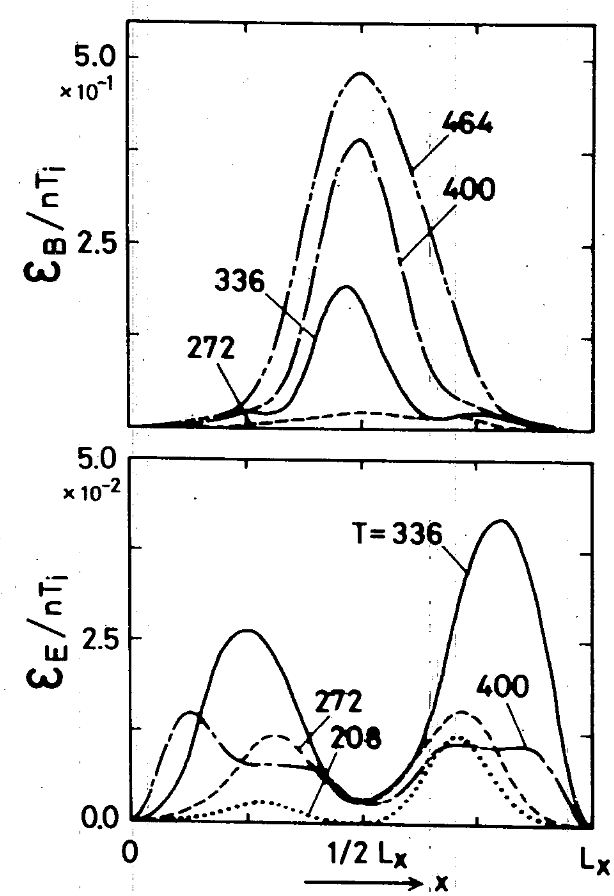

5.0
$\times 10^{-1}$
2.5
ε_B/nT_i
464
400
336
272
5.0
$\times 10^{-2}$
2.5
0.0
ε_E/nT_i
T=336
272
208
400
0
1/2 L_x
L_x
x

Fig. 8

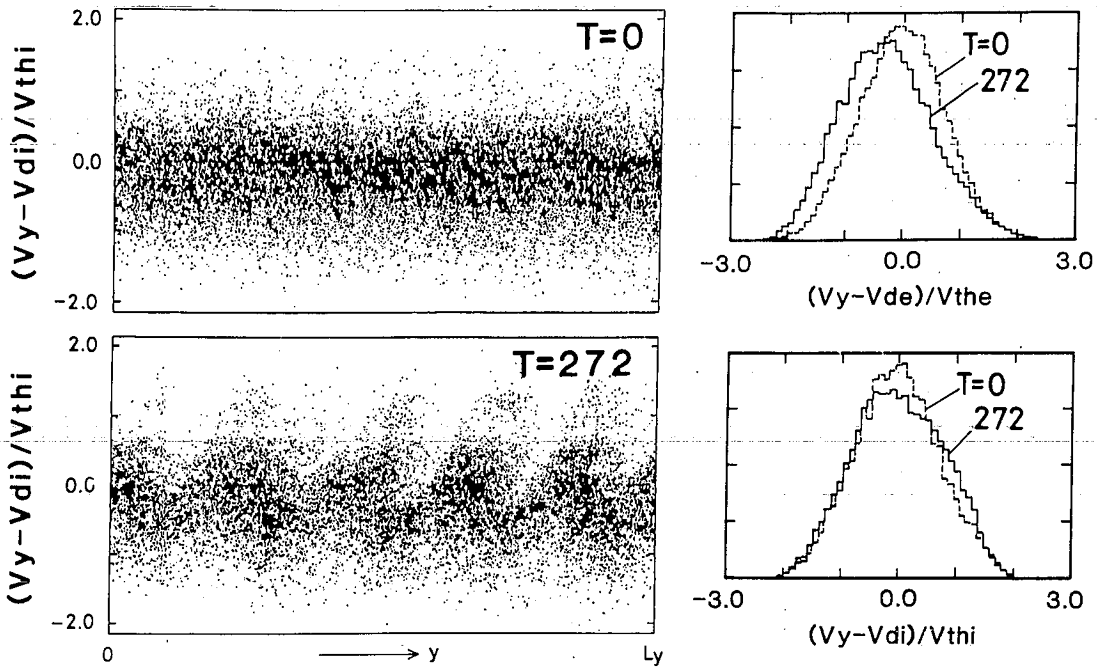

T=0
T=272
(Vy-Vdi)/Vthi
2.0
0.0
-2.0
0
y
Ly
T=0
272
-3.0
0.0
3.0
(Vy-Vde)/Vthe
(Vy-Vdi)/Vthi

Fig. 9

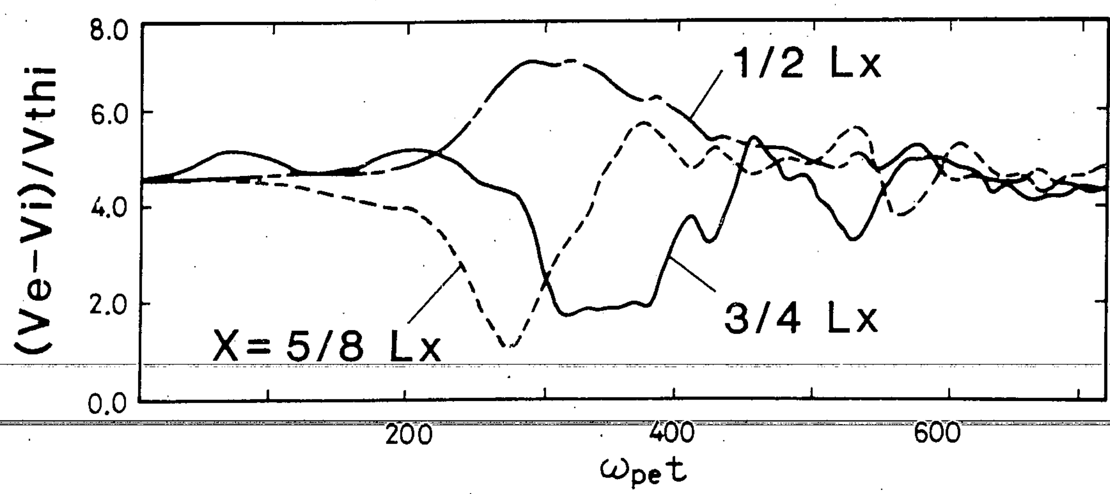

(Ve-Vi)/Vthi
8.0
6.0
4.0
2.0
0.0
1/2 Lx
3/4 Lx
X= 5/8 Lx
200
400
600
$\omega_{pe}t$

Fig. 10

(11)

Saturation

Saturation Energy $\mathcal{E}_E$

$$\mathcal{E}_E = 3.9 \times 10^{-2}\, nT_i \text{ (Observation)}$$

① Drift energy of electrons = wave energy

$$2\left(1+\frac{\omega_{pe}^2}{\omega_{ce}^2}\right)\mathcal{E}_E^{CR} = \frac{1}{2} m_e (V_{de}-V_{di})^2$$

$$\mathcal{E}_E^{CR} \approx \underline{3.5 \times 10^{-2}}\; nT_i$$

② Electron Resonance Broadening (Gary & Sanderson 1979)

$$(\mathcal{E}_E^{RB}/nT_i) \approx .1 \times \frac{\omega_{ce}^2}{\omega_{pe}^2}\frac{m_e}{m_i}\left(\frac{T_i}{T_e}\right)^{1/2}\left(\frac{2V_{di}}{v_{thi}}\right)^2 = \underline{4.2 \times 10^{-2}}$$

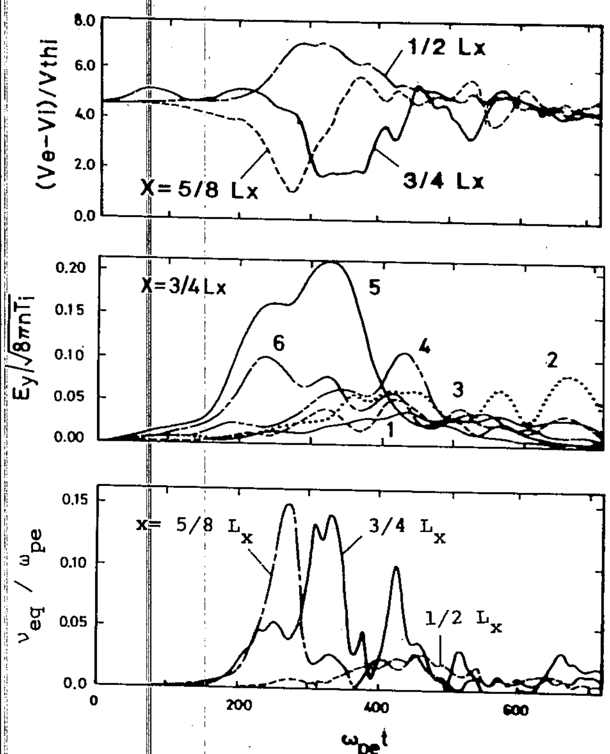

(Ve−Vi)/Vthi
8.0
6.0
4.0
2.0
0.0
1/2 Lx
3/4 Lx
X=5/8 Lx
Ey/√8πnTi
0.20
0.15
0.10
0.05
0.00
X=3/4Lx
5
6
4
3
2
1
νeq / ωpe
0.15
0.10
0.05
0.0
x= 5/8 Lx
3/4 Lx
1/2 Lx
0
200
400
600
ωpet

Fig. 12

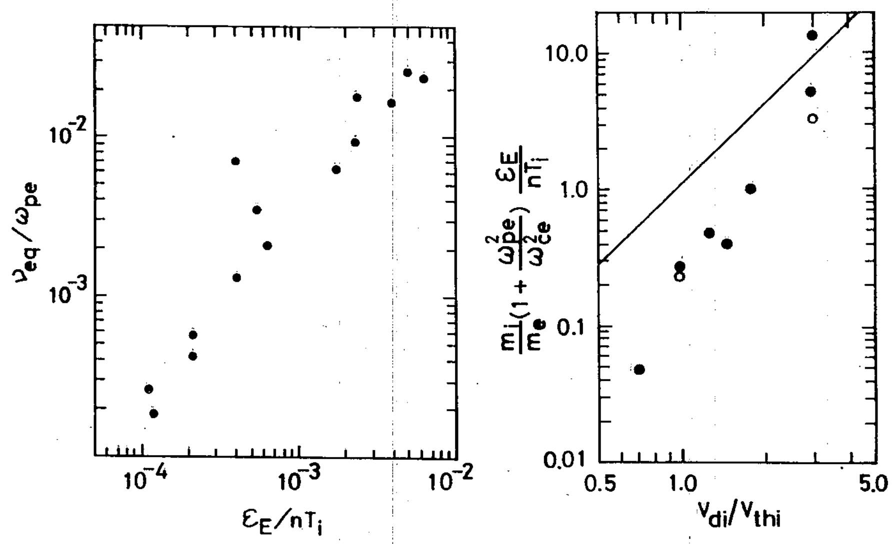

νeq/ωpe
10−2
10−3
10−4
10−3
10−2
ℰE/nTi
mi/me(1 + ω²pe/ω²ce) ℰE/nTi
10.0
1.0
0.1
0.01
0.5
1.0
5.0
vdi/vthi

Fig. 13

(13)

Anomalous Resistivity

Diagnosis of collision frequency ν_{eff}

$$\nu_{eff} = -\frac{e}{m_e}\langle \delta n_e \delta E_y - \frac{1}{c}\delta(n_e v_{ex})\delta B_z\rangle / \langle n_e\rangle\langle v_{iy} - v_{ey}\rangle$$

$$\eta = \frac{4\pi \nu_{eff}}{\omega_{pe}^2}$$

<Observation>

$$\eta_{x=\frac{3}{4}L_x} \approx .12\left(\frac{4\pi}{\omega_{pe}}\right)$$

$$\eta_{x=\frac{1}{2}L_x} \approx .03\left(\frac{4\pi}{\omega_{pe}}\right)$$

< Empirical formula > at $x = \frac{3}{4}L_x$

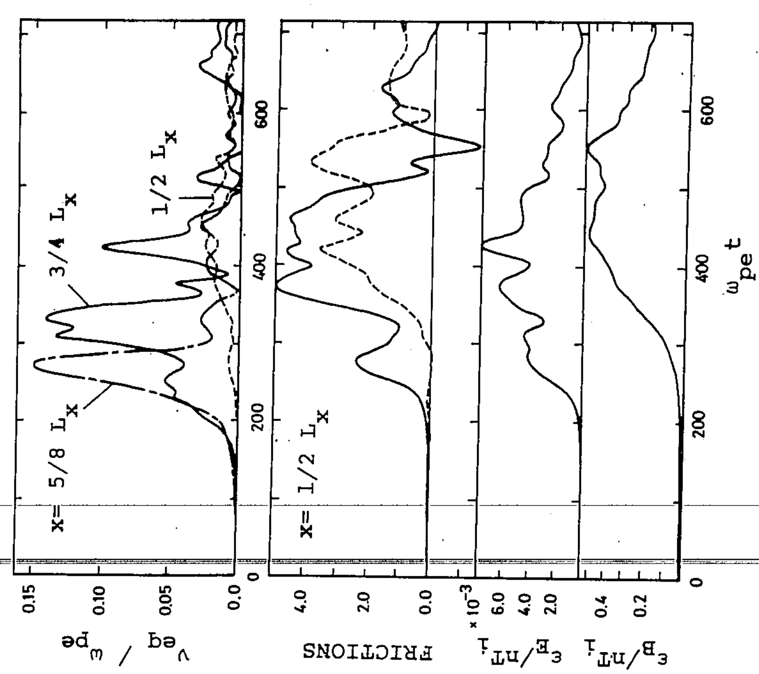
$$\frac{m_i}{m_e}\left(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\right)\frac{\mathcal{E}_E}{nT_i} \approx 0.1\left(\frac{V_{di}}{v_{thi}}\right)^{2.9}$$

(wave energy = lost drift energy)

$$\eta_{EP} = 1.2\frac{\omega_{LH}}{\omega_{pe}^2}\left(\frac{V_{di}}{v_{thi}}\right)^{1.9}$$

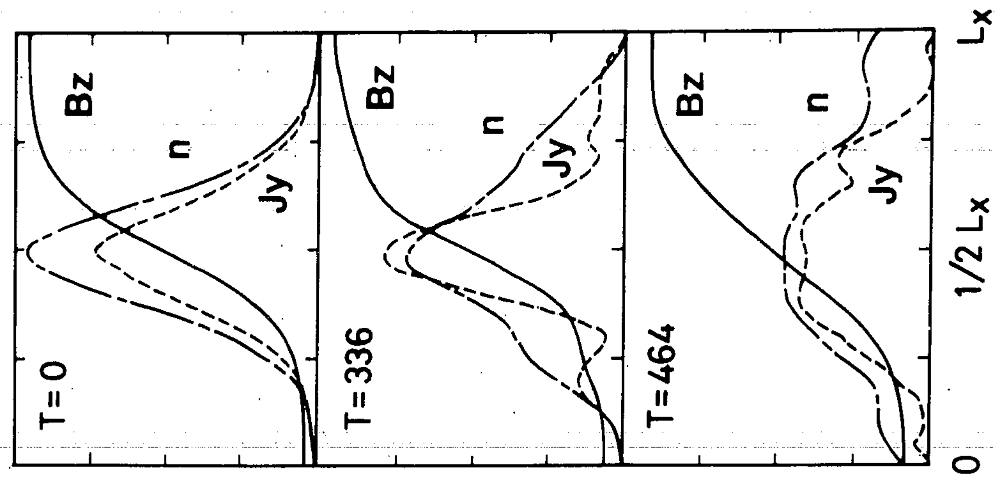
< Q.L. > (Davidson & Gladd, 1975)

$$\eta_{QL} \propto \left(\frac{V_{di}}{v_{thi}}\right)^2$$



x= 5/8 L_x
3/4 L_x
1/2 L_x
x= 1/2 L_x
ν_{eq} / ω_{pe}
FRICTIONS
ε_E/nT_i
ε_B/nT_i
$\times 10^{-3}$
0.15
0.10
0.05
0.0
4.0
2.0
0.0
6.0
4.0
2.0
0.4
0.2
0
200
400
600
$\omega_{pe}t$

Fig. 14



T=0
T=336
T=464
Bz
n
Jy
0
1/2 L_x
L_x

Fig. 17

15

At neutral sheet

$$\eta_{x=\frac{1}{2}L_x} = \lambda\,\mathcal{E}_E + \mu\,\mathcal{E}_B$$

< Observation >

$$\left(\frac{\lambda}{\mu}\right)_{obs} \approx 10^2$$

< Theory >

$$\nu_{eff} = -\frac{e^2}{m_e^2}\,\mathrm{Im}\left\langle \sum \frac{1}{k_y v_{the}^2}\left[\delta E_y^2 Z'(\xi_0) + \delta B_z^2 \frac{V_{de} v_{the}}{c^2} Z(\xi_0)\right]\right\rangle \Big/ \langle v_{iy} - u_{ey}\rangle$$

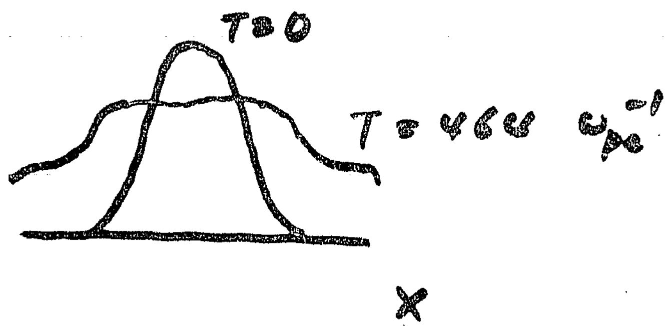
$$\approx -\frac{T_i}{T_e}\frac{\omega_{pe}^2}{k_y V_d}\left[\frac{\mathcal{E}_E}{n T_i}\,\mathrm{Im}\,Z'(\xi_0) + \frac{\mathcal{E}_B}{n T_i}\frac{V_{de} v_{the}}{c^2}\,\mathrm{Im}\,Z(\xi_0)\right]$$

($\mathrm{Im}\,Z'(\xi_0)$ term $\to \lambda$; $\frac{V_{de} v_{the}}{c^2}$ term $\to \mu$)

$$\left(\frac{\lambda}{\mu}\right)_{theory} \approx 0.6 \times 10^2$$

(15)

Particle Transport



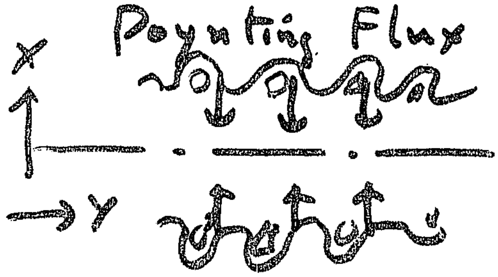
$\dfrac{\delta E \times \delta B}{B^2}$ vortices

$\langle v_{ex} \rangle_{obs} \approx 0.12\, \omega_{LH} L_B$

$\langle v_{ex} \rangle_{th} \approx 0.075\, \omega_{LH} L_B$

Heating at Neutral Sheet

Poynting Flux $0.1\, n T_{i0}$ per ω_{LH}^{-1}



neutral sheet

$$\frac{\partial \mathcal{E}_B}{\partial t} \approx 9.6 \times 10^{-2}\, n T_{i0}\, \omega_{LH}$$

Ohmic heating by anomalous resistivity

$$\frac{\partial \frac{1}{2} n (T_{ex} + T_{ey})}{\partial t} \approx 8.5 \times 10^{-2}\, n T_{e0}\, \omega_{LH}$$

$$\frac{\partial}{\partial t}\left(\frac{T_{ex}}{T_{e0}}\right) \approx 9.2 \times 10^{-2}\, \omega_{LH} \qquad (t \approx 200\text{–}500\, \omega_{pe}^{-1})$$

$\eta j^2 \approx 7 \times 10^{-2}\, n T_{e0}\, \omega_{LH}$

$$\frac{\partial}{\partial t}\left(\frac{T_{ey}}{T_{e0}}\right) \approx 7.8 \times 10^{-2}\, \omega_{LH}$$

$$\frac{\partial}{\partial t}\left[\frac{1}{2} n (T_{ix} + T_{iy})\right] / n T_{i0} \approx 3.2 \times 10^{-3}\, \omega_{LH}$$

Conclusions

1. LHD instability at the steepest density gradient. An electromagnetic instability is nonlinearly excited near neutral sheet.

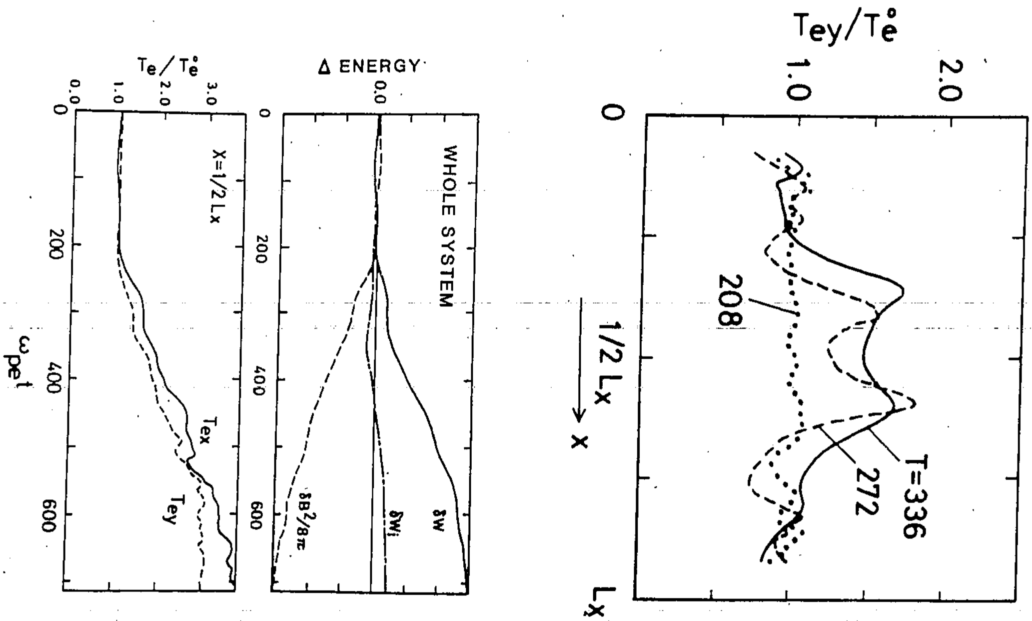
2. Electron diamagnetic drift is considerably reduced but electrons are accelerated at the neutral sheet ← dc electric field induced

3. $\eta \propto \left(\frac{V_{di}}{V_{thi}}\right)^{1.9}$

4. Outward plasma transport ← electron $E \times B$ vortices

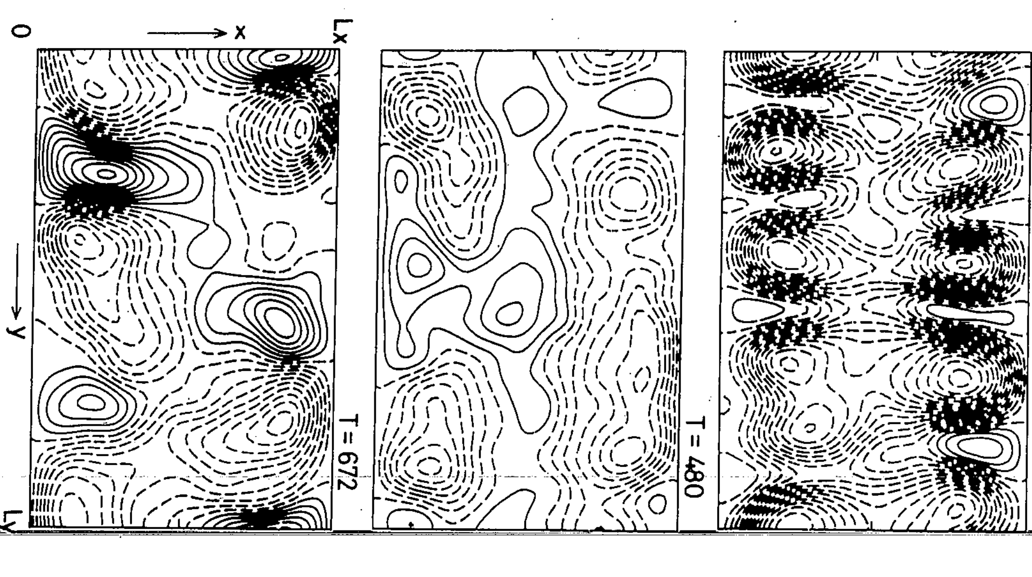
5. Saturation energy of LHD waves is bounded by electron directed energy

6. Electrons are heated considerably at the expense of the magnetic energy >> electron directed energy



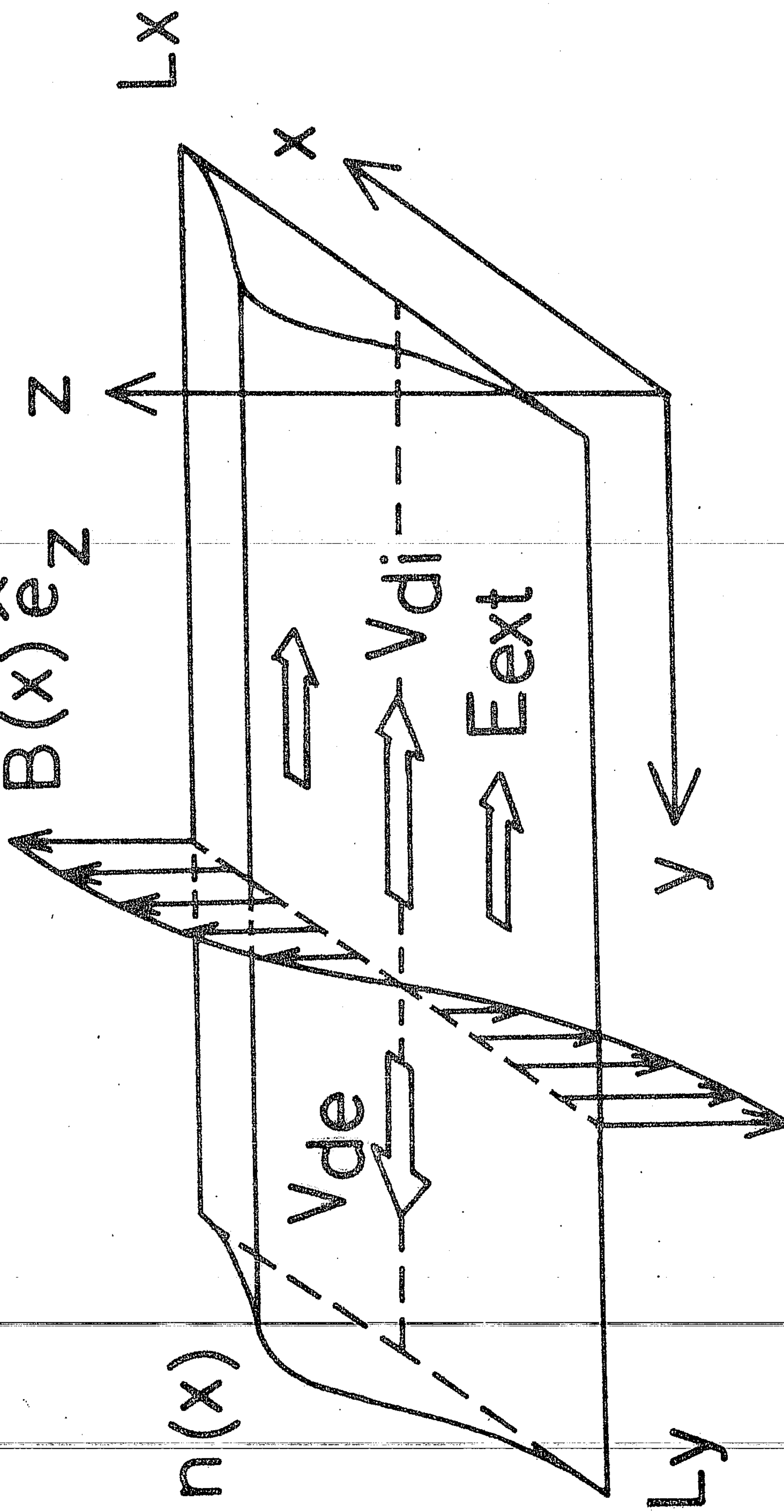
Tey/Te°
2.0
1.0
0
1/2 Lx
x
Lx
208
272
T=336
Δ ENERGY
WHOLE SYSTEM
δW
δWi
δB²/8π
Te/Te°
X=1/2Lx
Tex
Tey
200
400
600
ωpet

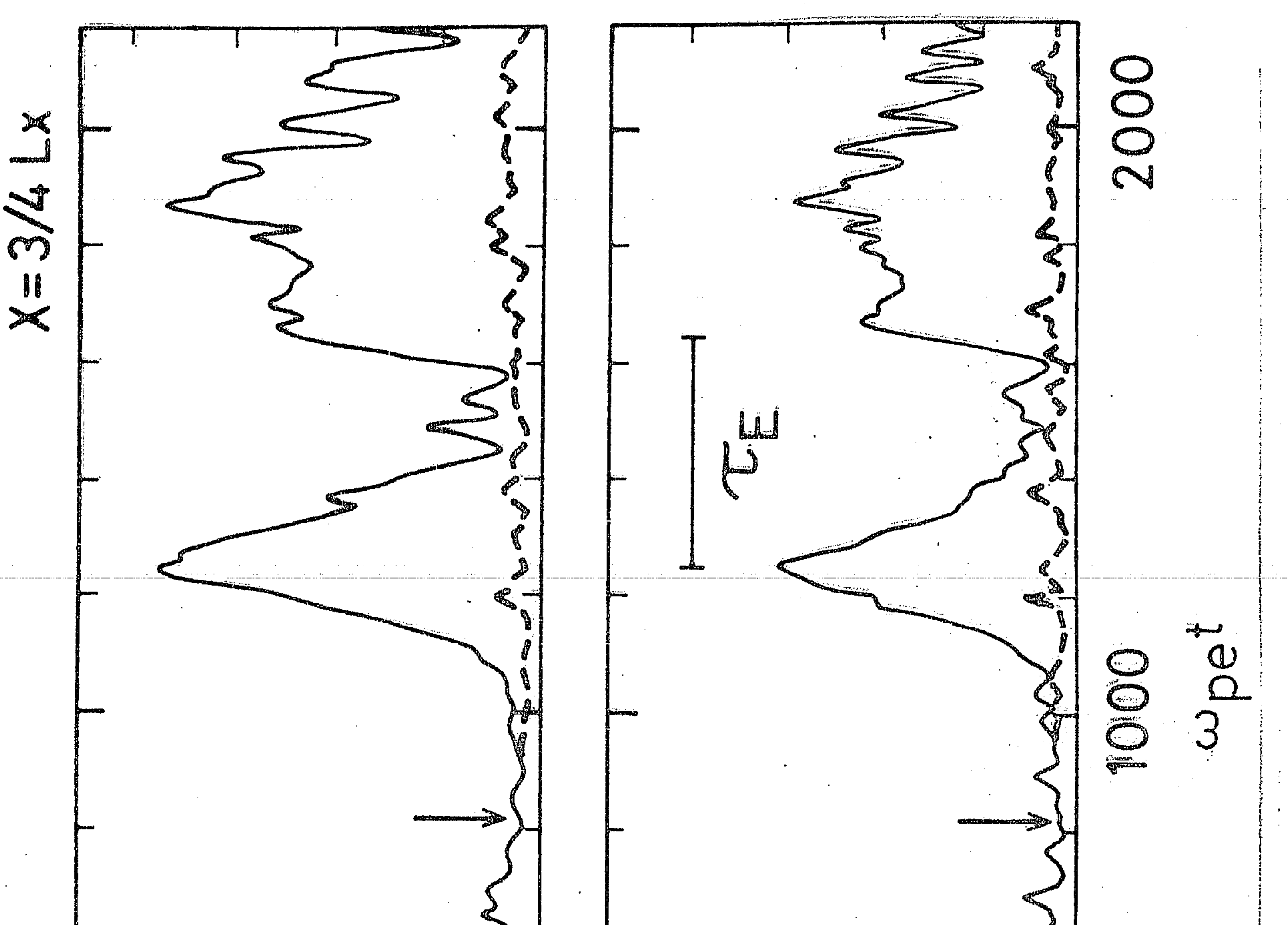
Fig. 18

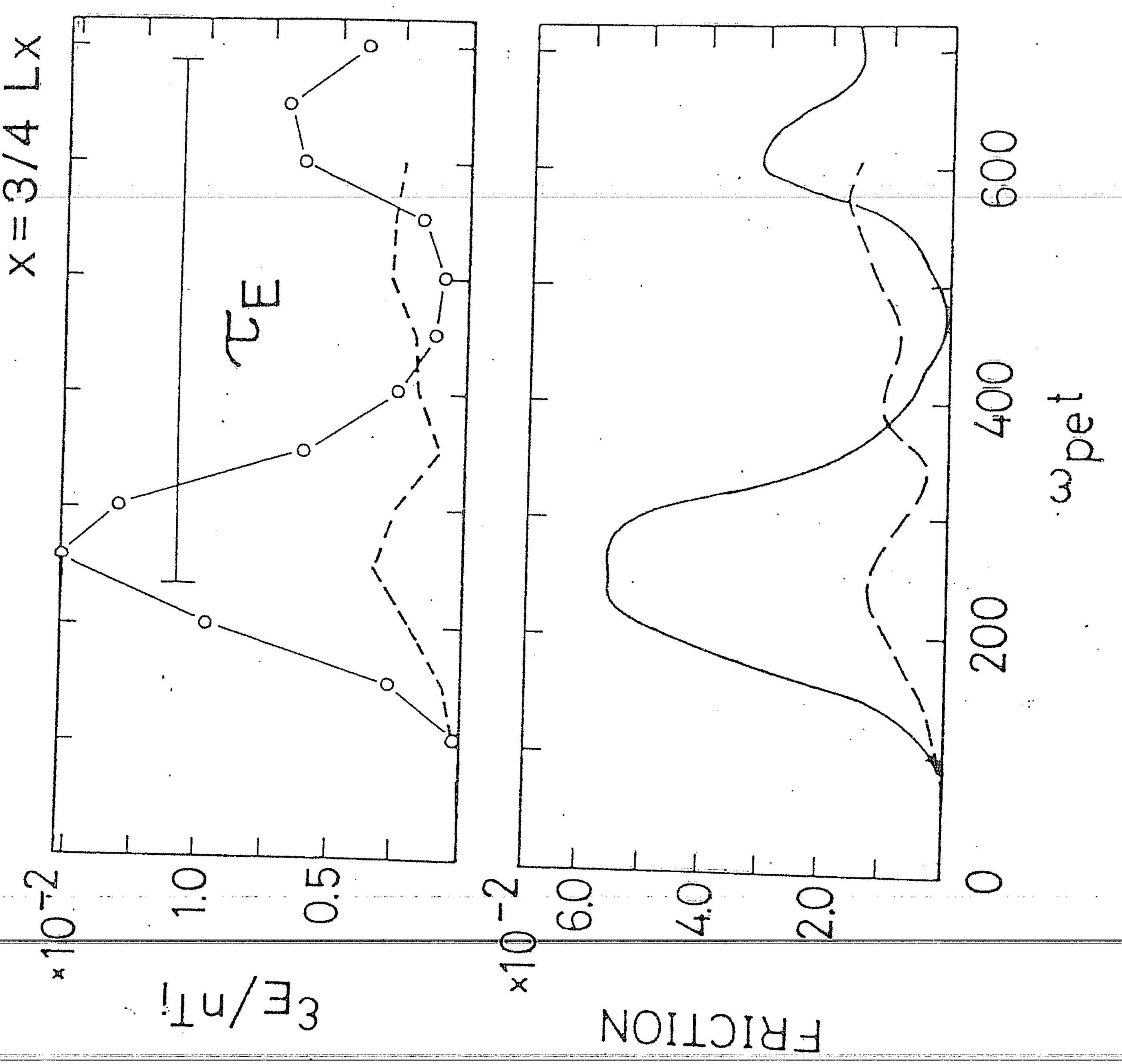


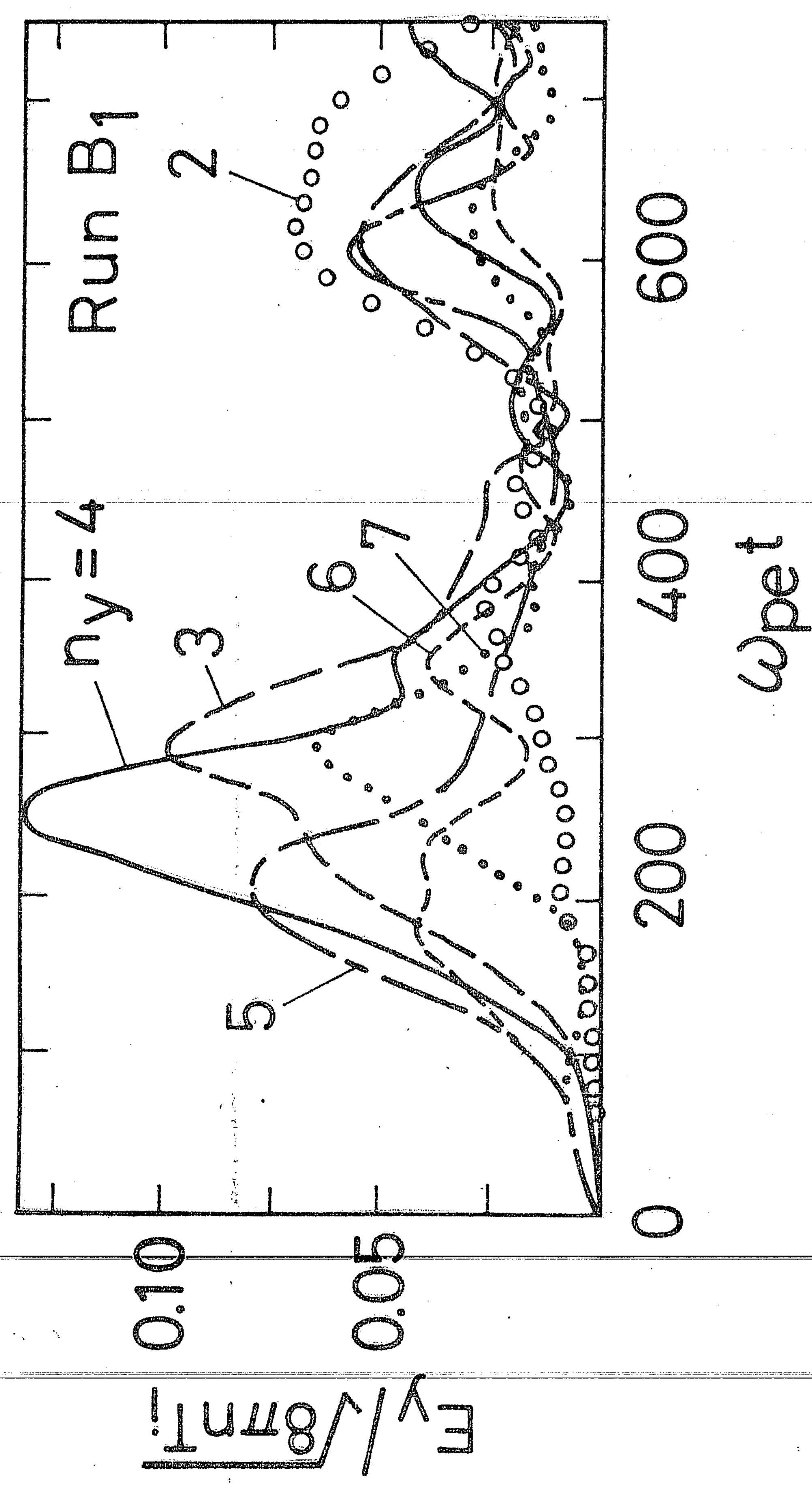
0
x
Lx
y
Ly
T=672
T=480
T=288

Fig. 22


Lx
x
z
B(x)ê_z
Vdi
Eext
y
Vde
n(x)
Ly


X = 3/4 Lx
$E_y/\sqrt{8\pi n T_i}$
0.20
0.10
$B_z/\sqrt{8\pi n T_i}$
0.10
0.05
0
1000
2000
$\omega_{pe}t$
τ_E


X = 3/4 Lx
τ_E
$\times 10^{-2}$
1.0
0.5
ε_E/nT_i
$\times 10^{-2}$
6.0
4.0
2.0
FRICTION
0
200
400
600
$\omega_{pe}t$


Run B1
ny = 4
3
5
6
7
2
0
200
400
600
ωpe t
0.10
0.05
Ey/√8πnTi

LOWER-HYBRID-DRIFT TURBULENCE AND ANOMALOUS TRANSPORT

N.T. GLADD

J A Y C O R

LOWER-HYBRID-DRIFT TURBULENCE AND ANOMALOUS TRANSPORT

N. T. Gladd
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The lower-hybrid-drift instability has been considered as a source of anomalous transport in plasmas with sharp gradients for a number of years. In this talk, a review is given of developments in the linear theory of the lower-hybrid-drift and various saturation mechanisms are discussed. A comparison is made of theoretically predicted rates of anomalous transport with transport rates deduced from simulation. Attention is focused on developments which have arisen as a consequence of recent detailed nonlocal analyses and simulations. The two-dimensional character of the wave spectrum and the importance of δEXB electron trapping are particularly considered. Finally, an outstanding problem in lower-hybrid-drift theory - the failure of lower-hybrid-drift turbulence to penetrate to the vicinity of a field null - is discussed.

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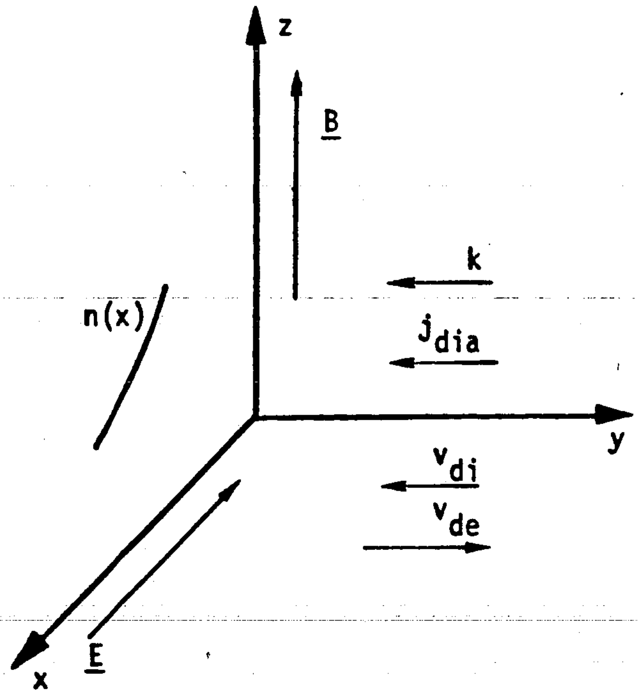
- Brief review of linear theory
- Saturation mechanisms
- Anomalous transport
- Recent developments
- Mention of new nonlinear theories
- An outstanding problem
- A new complication for nonlinear theories

Basic Lower-Hybrid-Drift Properties



$$\frac{\gamma_{MAX}}{\omega_{\ell h}} = \frac{1}{4}\sqrt{\frac{\pi}{2}}\left(\frac{V_{di}}{V_i}\right)^2 \sim \left(\frac{\rho_i}{L_N}\right)^2$$

$$\frac{\omega_{MAX}}{\omega_{\ell h}} = \frac{1}{2}\,\frac{k_{MAX} V_{di}}{\omega_{\ell h}} = \frac{1}{\sqrt{2}}\left(\frac{V_{di}}{V_i}\right) \sim \left(\frac{\rho_i}{L_N}\right)$$

$$\frac{k_{MAX} V_i}{\omega_{\ell h}} = \sqrt{2}$$

$$\underline{k}_{MAX} \cdot \underline{B} = 0$$

Discovered in America   Krall, Liewer   1971

Some Physical Properties

- Magnetized electron response $(\omega \ll \omega_{ce})$

$$\delta n_e \sim (k\lambda_{De})^{-2}\left\{1 + \left(\frac{\omega - \omega_{*e}}{k_z v_e}\right)\Gamma_0(b_e)\, Z\left(\frac{\omega}{k_z v_e}\right)\right\}$$

$T_e \to 0$

$$\delta n_e \sim \left(\frac{\omega_{pe}}{\omega_{ce}}\right)^2 + \left(\frac{1}{k\lambda_{De}}\right)^2\left(\frac{\omega_{*e}}{\omega}\right) - \left(\frac{k_z}{k}\right)^2\left(\frac{\omega_{pe}}{\omega}\right)^2$$

polarization, $\delta E \times B$ — first two terms: Neccesary for LHD

last term: retain for M2S

- Unmagnetized ions $\omega \gg \omega_{ci}$

$$\delta n_i \sim (k\lambda_{Di})^{-2}\left\{1 + \left(\frac{\omega - \omega_{*i}}{k v_i}\right) Z\left(\frac{\omega}{k v_i}\right)\right\}$$

$\frac{\rho_i}{L_N} \ll 1$

$$\delta n_i \sim (k\lambda_{Di})^{-2}\left\{1 + i\sqrt{\pi}\left(\frac{\omega - \omega_{*i}}{k v_i}\right)\right\}$$

- Instability also exists in fluid ion limit

$$(\omega^2 - \omega_{lh}^2)(\omega - \omega_{*i}) = \begin{cases} \nabla n & \Rightarrow \text{LHD} \\ k_z & \Rightarrow \text{M2S} \end{cases}$$

$(\omega^2 - \omega_{lh}^2)(\omega - \omega_{*i})$: origin of name

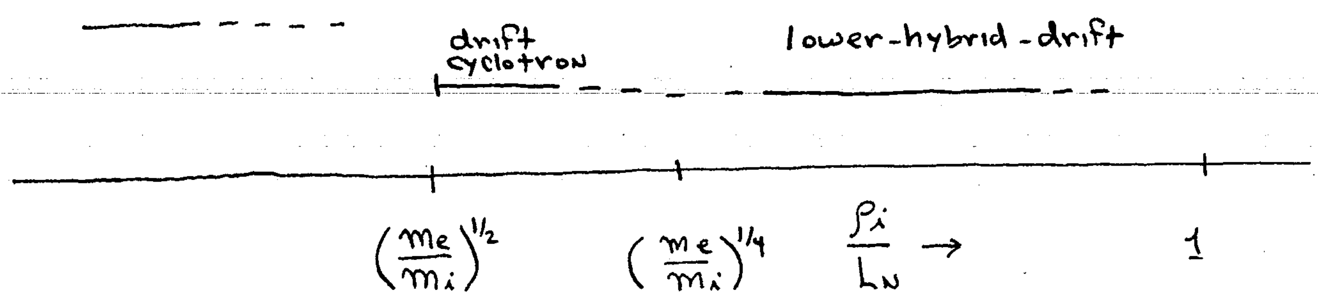
Relationship to Other Drift Modes

$k\rho_i \sim 1$
$\omega \ll \omega_{ci}$

$k\rho_e \sim 1$
$\omega \sim \omega_{ci}$

$k\rho_e \sim 1$
$\omega \gg \omega_{ci}$

universal



Tokamaks
modern RFP's
EBT core

Mirrors
theta-pinches
EBT boundary
Magnetotail
Magnetopause

Other Factors Which Significantly Affect LHD Growth

Magnetic field gradients

Huba, Wu 76
Davidson, Gladd, Wu, Huba 77
Huba, Gladd, Drake 80
Drake, Huba 81

$$\delta n_e = n_0 \frac{e\varphi}{T_e}\left\{1 - 2\int_0^\infty V_\perp dV_\perp\, e^{-V_\perp^2} J_0^2(aV_\perp)\left[\frac{\omega - \omega_{*e}}{\omega - k_y \bar{V}_{\nabla B} V_\perp^2}\right]\right\}$$

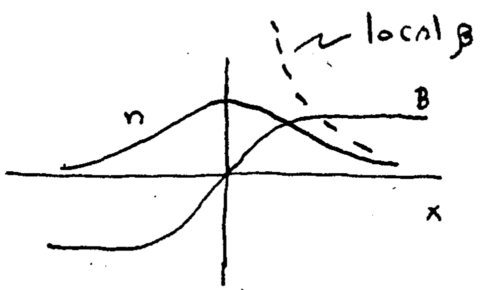
- For $T_e \to 0$ ∇B is destabilizing

$$\frac{\nabla n}{n} \to \frac{\nabla n}{n} - \frac{\nabla B}{B} \to \frac{\nabla n}{n}\left(1 + \frac{\beta_i}{2}\right)$$

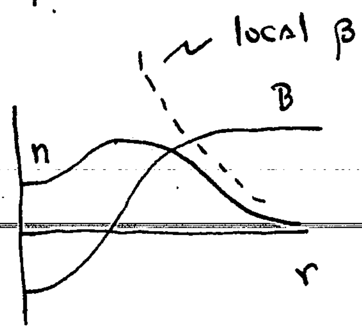
- For finite T_e ∇B is strongly stabilizing through the ∇B resonance
- Electromagnetic effects

Important effect in

Magnetotail



reversed field experiments



Magnetic Shear

Krall, 77
Gladd, Goren, Liu, Davidson 77
Davidson, Gladd, Goren 77
Tormac IAEA ····· Berk, ·····, Gladd 78

- shear stabilizing on LHD

$$\left.\frac{L_s}{L_N}\right|_{crit} < \left(\frac{\rho_i}{L_N} + \frac{L_N}{\rho_i} \right)$$

- details very different from universal mode
- outstanding problems remain

Other effects

Field curvature — Krall, McBride 78

Cold plasma — Gary, Ashour-Abdalla 80

Hot plasma — Gladd 81

∇T — McBride, Hamasaki 78; Gary, Sanderson 79; Gladd 81

Collisions — Huba, Ossakow 79; Sperling, Goldman 80

Saturation Mechanisms (< 80)

- Free energy bound
- Quasilinear plateau formation
- Resonance broadening
- Ion trapping

Free energy bound

Davidson, Gladd 75
Liewer, Davidson 77
Davidson, Krall 77

on fluctuations (Fowler bound)

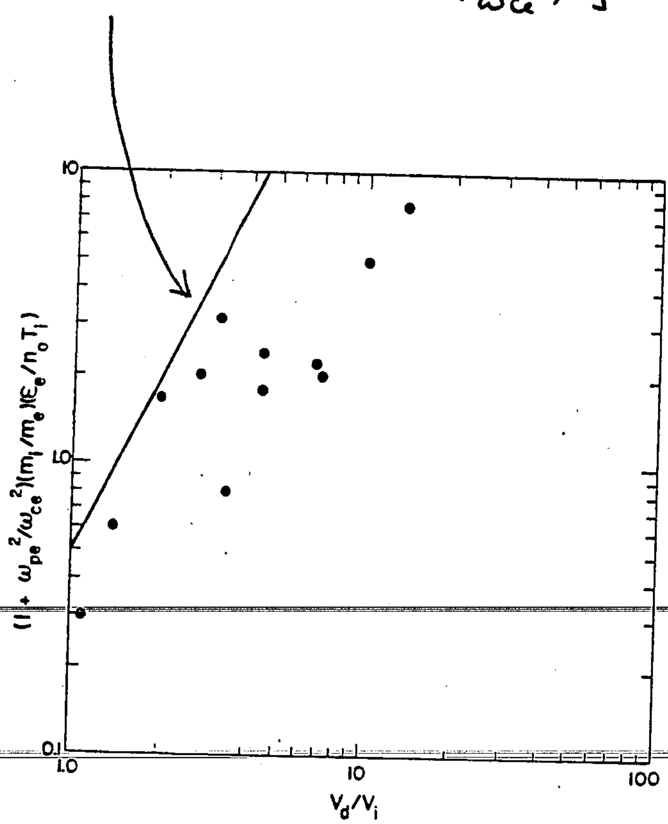
$$\mathcal{E}_{MAX}^{FLUCT} = n\left[\frac{1}{2} m_e V_d^2\right]$$

on waves

$$\mathcal{E}_{MAX}^{WAVE} = \left|\omega \frac{\partial}{\partial \omega} \mathrm{Re}\, D(\omega, k)\right|_{k=k_M} \mathcal{E}_{MAX}^{FLUCT}$$

for LHD

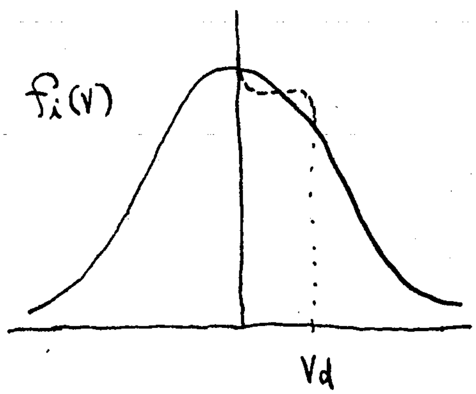
$$\mathcal{E}_{MAX}^{WAVE} = \frac{1}{2}\left[\frac{1}{1+\left(\frac{\omega_{pe}}{\omega_{ce}}\right)^2}\right] n\left[\frac{1}{2} m_e V_d^2\right]$$



Simulation by
Winske, Liewer 78

Quasilinear plateau formation

Davidson, 78



$$\mathcal{E}_{MAX}^{Q.L} = \frac{8}{45\sqrt{\pi}} \left(\frac{m_i}{m_e}\right) \left(\frac{V_d}{V_i}\right)^3 \mathcal{E}_{MAX}^{WAVE}$$

$$\simeq 180 \left(\frac{V_d}{V_i}\right)^3 \mathcal{E}_{MAX}^{WAVE}$$

valid for $\frac{V_d}{V_i} \ll 1$

Resonance Broadening

Huba, Papadoupoulous 78

Gary 80

Gary, Sanderson 80

$$\frac{\delta n_e}{n_0} \sim \left\{ 1 - 2 \int_0^\infty V_\perp dV_\perp \, e^{-V_\perp^2} J_0^2(aV_\perp) \left[\frac{\omega - \omega_{*e}}{\omega - k_y \bar{V}_{\nabla B} V_\perp^2 + i\,\Delta\omega_k(V_\perp)} \right] \right\}$$

$$\Delta\omega_k(V_\perp) = \frac{1}{2} \left(\frac{e}{m_e}\right)^2 \left(\frac{k}{\omega_{ce}}\right)^2 \sum_{k'} |\delta E_{k'}|^2 J_1^2(aV_\perp) \, R_e \left\{ \frac{i}{\omega - k'_y \bar{V}_{\nabla B} V_\perp^2 + i\,\Delta\omega_{k'}(V_\perp)} \right\}$$

$$\mathcal{E}_{MAX}^{RB} \simeq \frac{4}{5} \left(\frac{T_i}{T_e}\right)^{5/4} \mathcal{E}_{MAX}^{WAVE}$$ (estimate by Gary)

Ion trapping

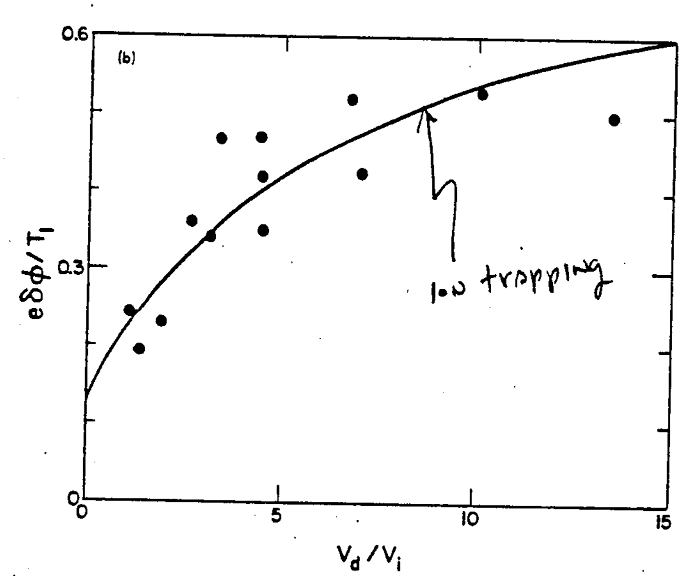
Winske, Liewer 78

Ion trapping occurs when

$$e\varphi \sim \frac{1}{2} m_i \left(\frac{\omega}{k}\right)^2$$

$$\Rightarrow \quad \frac{e\varphi_{MAX}^{I.T.}}{T_i} = C\left[1 + 2\left(\frac{\omega}{k v_i}\right)^2\right]$$

C = .133 (empirical)



Existence of lower-hybrid drift verified by direct CO_2 Laser scattering measurements in the INTEREX theta pinch

Fahrbach, et. Al. 1981

Similar experiment planned for RFX-C

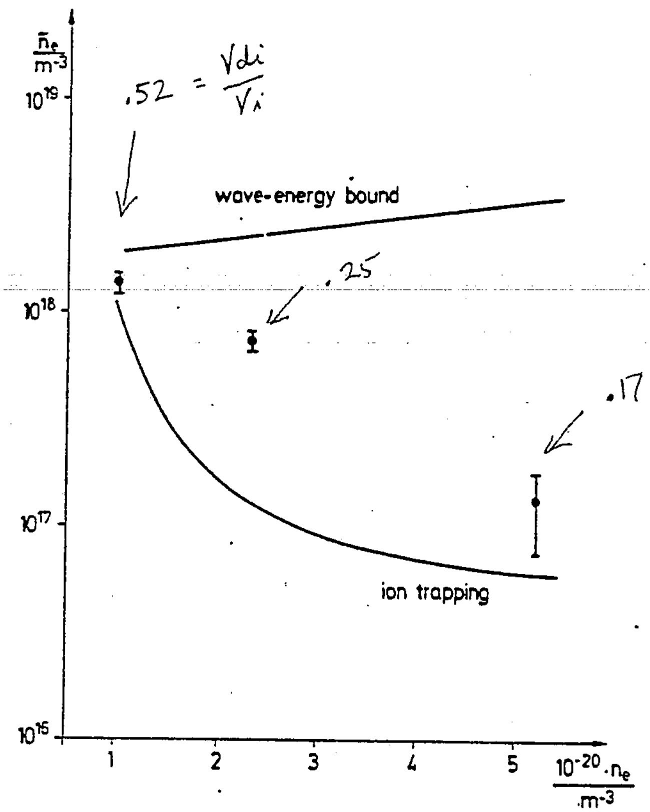


FIG.14. Density fluctuation amplitudes for 3, 5 and 8 mtorr discharges in comparison with saturated values from free-energy bound and ion trapping.

Other experimental data

Magnetotail

Huba, Gladd, Drake(78) analysis of sattelite data from Gurnett, et. Al. (76)

Magnetopause

Gurnett, et. Al. 79

Anomalous Transport

Quasilinear estimate

Davidson, Gladd 75
Davidson, Krall 77
Liewer, Davidson 77

$$\left[\frac{\partial}{\partial t} + V_x \frac{\partial}{\partial x} + \frac{q}{m}\left(\langle \delta \underline{E} \rangle + \frac{\underline{V} \times \langle \delta \underline{B} \rangle}{c}\right)\cdot \nabla_v\right] \delta f = -\frac{q}{m}\langle \delta \underline{E} \cdot \nabla_{v_y} f_0 \rangle$$

$$\Downarrow$$

Anomalous momentum transport, etc. energy

$$\left(\frac{\partial}{\partial t}\, n m V_y\right)_{AN} = 2 \int dk_y\, \mathcal{E}_{k_y}\, \mathrm{Im}\left[k_y \chi(\omega, k_y)\right]$$

$$\Downarrow$$

$$\eta^{AN} = \frac{m_e}{ne^2}\, \nu^{AN} = \frac{4\pi}{\omega_{pe}^2} \left\langle \mathrm{Im}\,(-k_y V_d)\, \chi_e) \right\rangle_k \frac{\mathcal{E}^{wave}_{max}}{n m_e V_d^2/2}$$

$$Q_i^{AN} = \left\langle 2\, \mathrm{Im}\,(\omega \chi_i) \right\rangle_k \mathcal{E}^{wave}_{max}$$

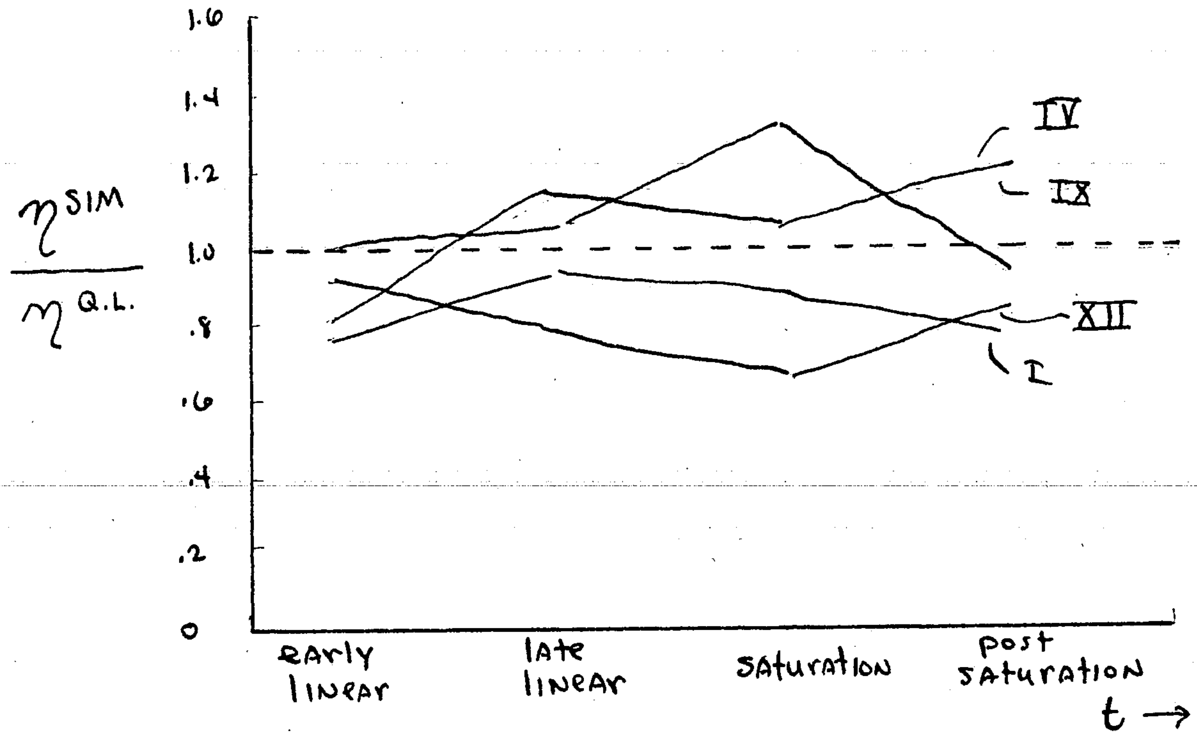
$$Q_e^{AN} = \left\langle 2\, \mathrm{Im}\,(\omega \chi_e) \right\rangle_k \mathcal{E}^{wave}_{max}$$

For LHD $\quad \nu^{AN} \approx \omega_{eh}$

$$\Downarrow$$

$$\eta^{AN} \gg \eta^{cl}$$

Quasilinear LHD anomalous resistivity and simulation



Taken from Table III of Winske and Liewer 78

Case	Vd/Vi
I	1.1
IV	2.7
IX	6.8
XII	13.5

Recent Developments

Nonlocal stability analyses

• Huba, Drake, Gladd	magnetotail
• Gladd	EBT boundary
• Aamodt, Catto, Myra	Mirror
• Chen, Birdsall, Nevins	mirror

New simulations

• Winske	reversed field geometry
• Tanaka and Sato	reversed field geometry
• Brackbill	reversed field geometry
• Chen, Birdsall, Nevins	

⇓

- nonlocal effects are important, especially the 2-D character of the LHD spectrum !

- electron trapping is important !

Example of lower-hybrid-drift eigenmodes

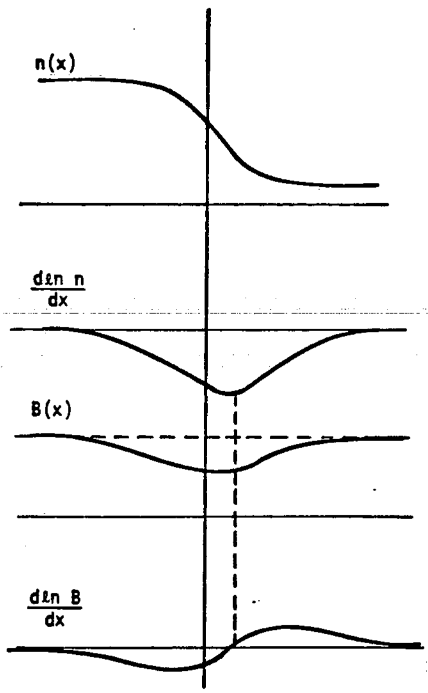


Figure 5. Model EBT density and field profiles.

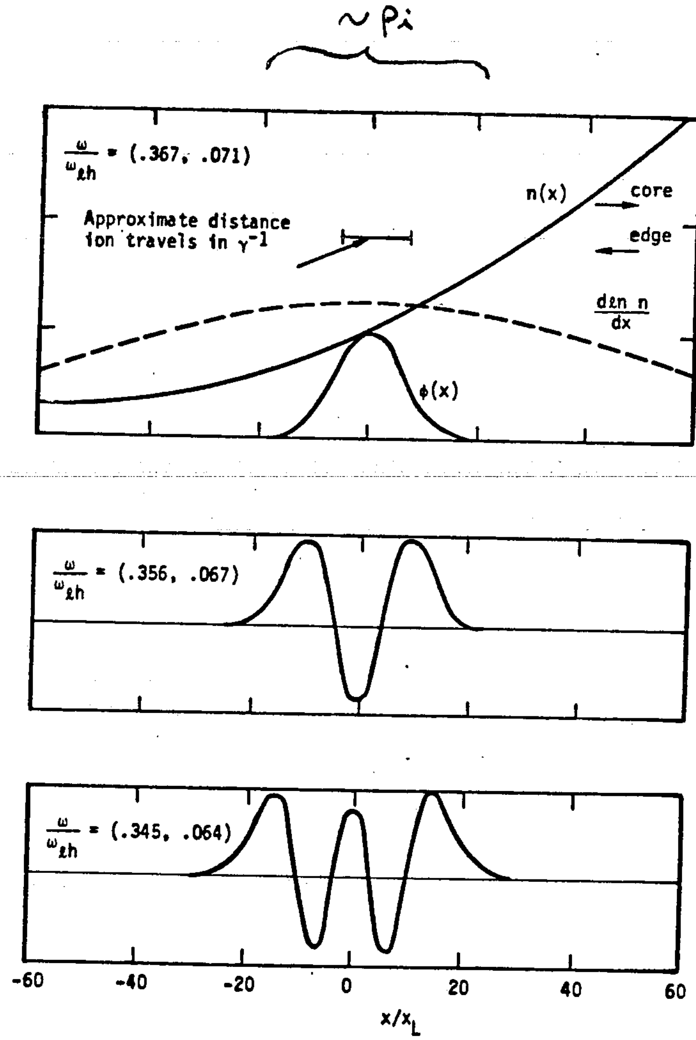


Figure 6. A sequence of lower-hybrid-drift eigenmodes (even) in the profile described by Eq. (44). Relevant parameters are ρ_i $d\ell n\, n/dx = 0.775$, $\omega_{pe}/\omega_{ce} = 10$, $T_e/T_i = 0.001$, $k_y x_L = 1.414$. Parameters are specified at $x = 0$ which is chosen to be the maximum of $d\ell n\, n/dx$.

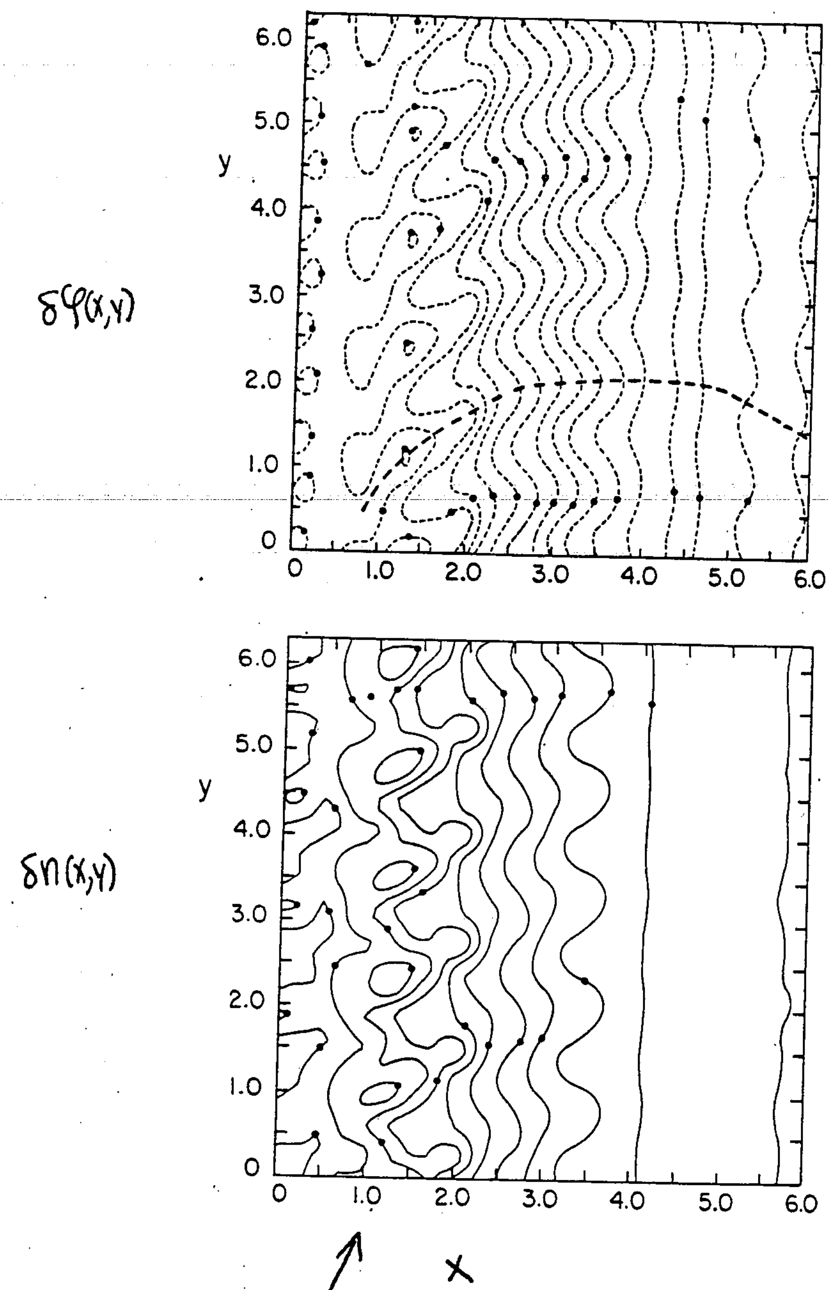
For this case

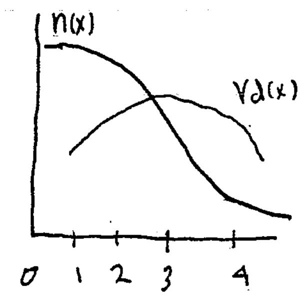
$k_y V_i/\omega_{\ell h} = 1.4$

$k_x V_i/\omega_{\ell h} \simeq .14$

Example of electron trapping

Chen, Birdsall, Nevins 81





saturation due to local current relaxation by $\delta \underline{E} \times \underline{B}$ trapping of electrons

Some new nonlinear theories

- Drake, Gladd, Huba 81 — Drift energy bound not valid in reversed field plasma (Field energy may be tapped) Flux transported toward null by LHD

- Drake and T.T. Lee 81 — Naive (i.e., 1-D) quasilinear theory does not yield anom. xport at low fluctuation amps.

- Drake and Huba 81 — 2-D wave spectrum and ∇B resonances very important because of $\delta E \times B$ trapping of resonant electrons

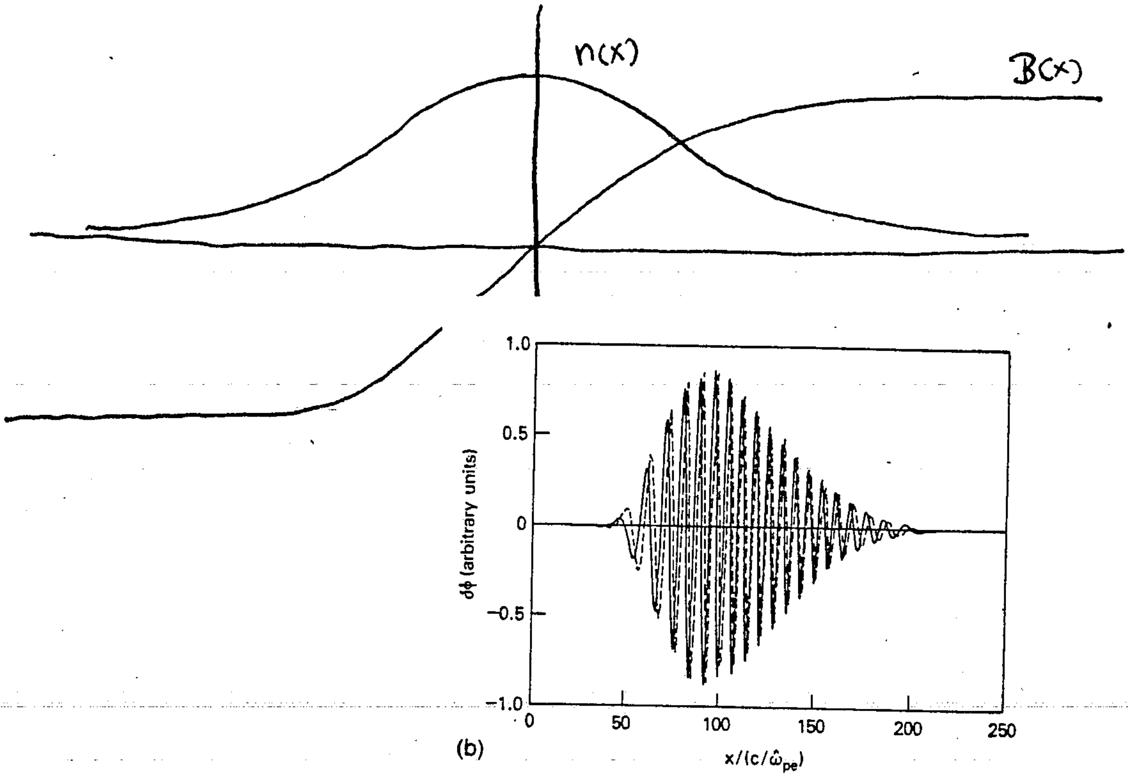
- Chen and Cohen 81 — Finite amplitude pertubation of ion orbits leads to weakly stabilizing nonlinear frequency shift

- Chen, Nevins and Birdsall 81 — Different sat. mechanisms in different regions! $\delta \underline{E} \times \underline{B}$ trapping in resonant electron region – ion q-lin diffusion near $[\nabla n/n]^{-1}_{max}$

- Myra and Aamodt 81 — Different rates of q-lin diffusion lead to electric field $\Rightarrow$ $E \times B$ drift which opposes driving drift $\Rightarrow$ saturation

- Diamond and Myra 81 — 2-D $\delta E \times B$ nonlinearity very important! - leads to Compton scattering induced cascade to long wavelengths. $\Rightarrow$ saturation

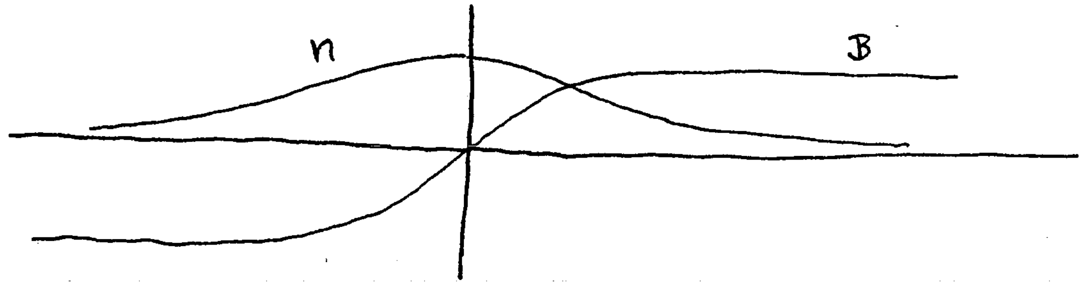
An Outstanding Problem

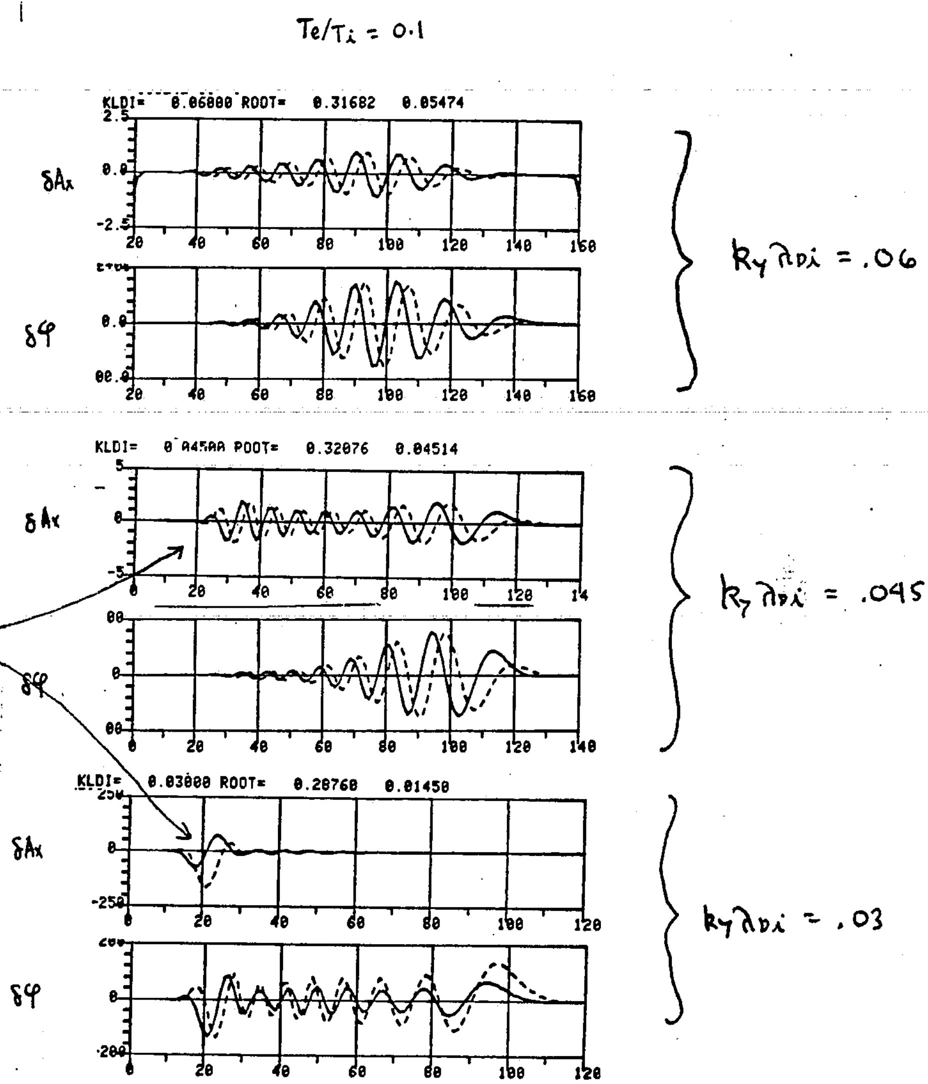


Nonlocal $T_e \to 0$, electromagnetic LHD calculation
Huba, Drake, Gladd 80

How does the LHD produce turbulence near a field null?

The trouble with finite Te





Evidence for electromagnetic turbulence near field Null?

Nonlocal, finite Te, electromagnetic calculation
Gladd, Huba (unfinished)

- Tendency of LHD to be predominantly electrostatic or electromagnetic in different spatial regions complicates the analysis of saturation mechanisms!

SIMULATION OF DRIFT-CONE TURBULENCE IN A NEUTRAL-BEAM DRIVEN MIRROR MACHINE

BRUCE I. COHEN, NEIL MARON, GARY R. SMITH, AND WILLIAM M. NEVINS

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LIVERMORE, CALIFORNIA 94550

PARTICLE SIMULATIONS OF MICROINSTABILITIES IN A MIRROR MACHINE

BRUCE I. COHEN, NEIL MARON, GARY R. SMITH, AND WILLIAM M. NEVINS
LAWRENCE LIVERMORE NATIONAL LABORATORY

- SIMULATIONS OF DRIFT-CYCLOTRON-LOSS-CONE (DCLC) AND ION BOUNCE MODES IN A NONUNIFORM FIELD $B_0(z)$
- INCLUSION OF MIRROR SOURCES AND SINKS -

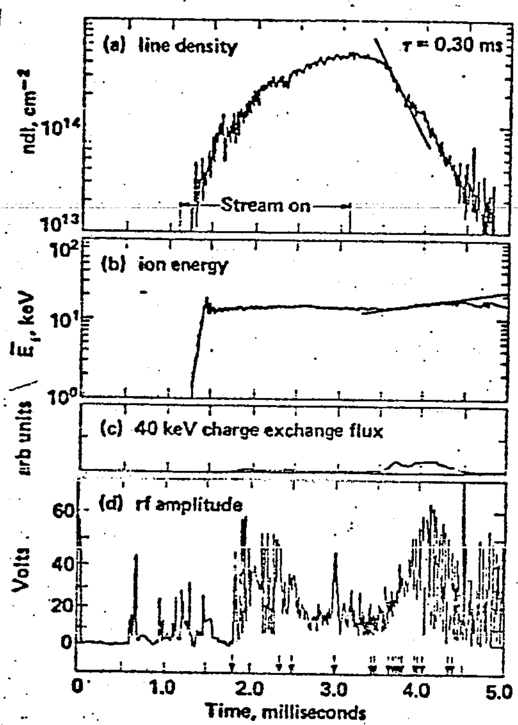
  AXIAL LOSS
  ION-ELECTRON DRAG
  AMBIPOLAR POTENTIAL
  CHARGE EXCHANGE BY NEUTRAL BEAM
  STREAM INJECTION AND NEUTRAL BEAM IONIZATION

- SELF-CONSISTENT SIMULATION OF ION TRAPPING, SUPERADIABATICITY, AND STOCHASTIC DIFFUSION DRIVEN BY DCLC
- NONLINEAR EVOLUTION TO STEADY STATE WITH A SINGLE COHERENT DCLC WAVE DOMINANT, DIFFUSING IONS INTO THE LOSS CONE. (LOW DENSITY)
- HIGH DENSITY SIMULATIONS -- LOWER AMPLITUDE AND MORE RESONANT ($\omega \sim \omega_{CI}$) TURBULENT FLUCTUATIONS
- ELECTRON AXIAL DISSIPATION REDUCES TURBULENCE AND SHIFTS SPECTRUM TO LONGER WAVELENGTHS

Experimental Observations of DCLC in 2XIIB Mirror Plasma

- Ion heating and diffusion correlate with DCLC activity

ref. W.C.Turner, J. de Physique 38, C6-121 (1977).



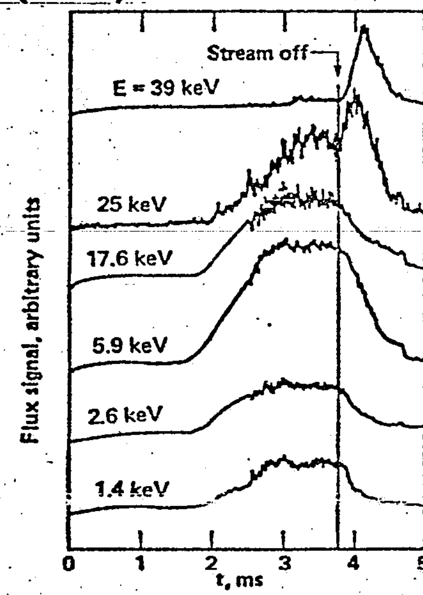
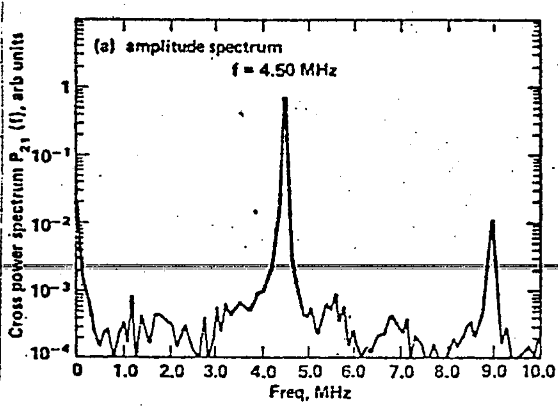
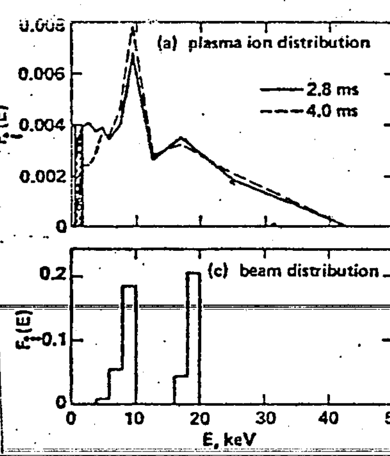


FIG. 10. — Charge exchange analyzer signals vs. time. Stream plasma is shut off at $t = 3.8$ ms.

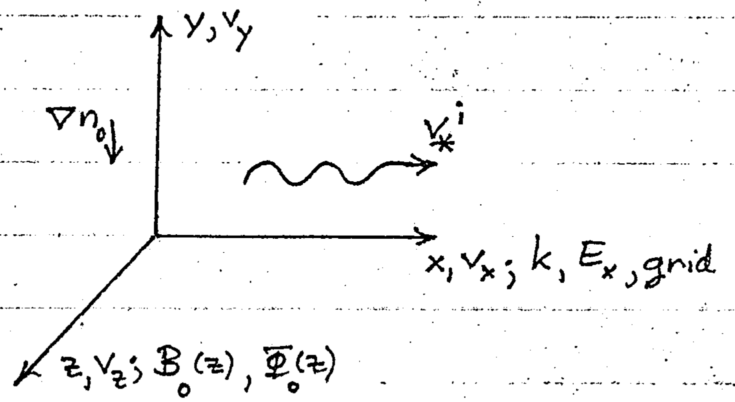
- Spread ion energy distribution

- Narrow frequency spectrum of rf





Hybrid Simulation Model of Ion-Cyclotron Flute Modes in Non-uniform B_0



Local slab model for drift-wave simulation

- particle ions

$$\frac{d}{dt}\vec{v}_\perp = \frac{e}{m_i}\left(\vec{E} + \frac{\vec{v}_\perp \times \vec{B}}{c}\right) - \nu_{drag}\vec{v}_\perp$$

$$\frac{d}{dt}\vec{x} = \vec{v} \qquad \frac{d}{dt}v_z = -\frac{v_\perp^2}{B_0}\nabla_z B_0(z) - \nu_{drag} v_z - \frac{e}{m_i}\nabla_z \Phi_0^{ambipolar}(z)$$

- linearized fluid electrons: $\vec{E}\times\vec{B}$ and $\frac{\partial \vec{E}}{\partial t}$ polarization drifts

- Poisson equation (flute averaged, $\vec{k}\cdot\vec{B}=0$)

$$\left(1+\frac{\omega_{pe}^2}{\omega_{ce}^2}\right) i\frac{\partial}{\partial t}\phi_{k_x} - \frac{\omega_{pe}^2}{\omega_{ce} k_x L_n}\phi_{k_x} = \frac{4\pi e}{k_x^2}\, i\frac{\partial}{\partial t} n_{i,k_x}$$

$n_i(x) \rightarrow n_{i,k_x}$ $\quad ik\phi_{k_x} \rightarrow E_x(x)$ using fast Fourier transforms

Simulation of DCLC and Ion Bounce Modes

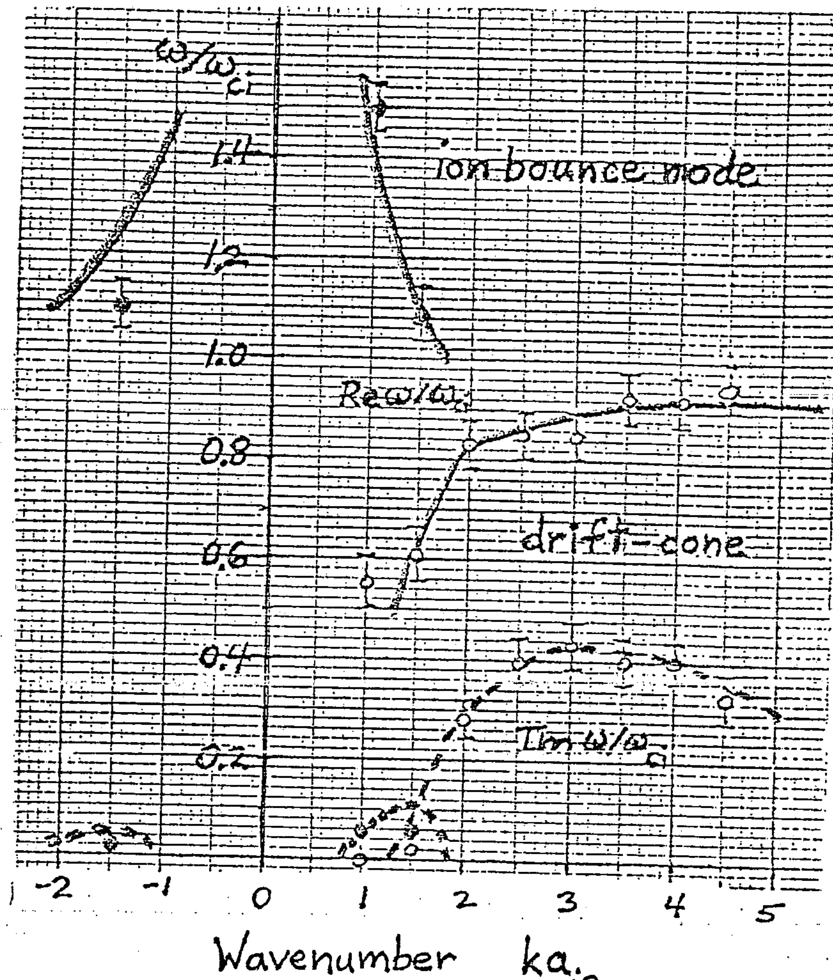
- Linear theory and simulations

$$f_{0i}(\vec{v}) = \frac{R}{(2\pi\Delta^2)^{1/2}} \frac{\exp(-v_{\parallel}^2/2\Delta^2 v_{\perp}^2)}{2\pi v_i^2 (R-\Gamma)} [\exp(-v_{\perp}^2/2v_i^2) - \Gamma \exp(-R v_{\perp}^2/2v_i^2)]$$

$$\frac{a_{io}}{r_p} = \frac{a_{io}}{L_B} = 0.2 \quad m_i/m_e = 3672$$

$$\Gamma = 1 \quad R = 2$$

$$\frac{\omega_{pi}}{\omega_{cio}} = \frac{\omega_{cio}}{\langle\omega_b\rangle} = 5 \quad \frac{T_{\parallel}}{T_{\perp}} = \Delta^2 = 0.1$$



- Ion bounce $|\mathrm{Re}\,\omega| > \omega_{cio}$, lower growth rates $\mathrm{Im}\,\omega < \bar{\omega}_b$, longer wavelengths, and propagate in $\pm v_*^i$ directions.

- Ion bounce modes saturate in simulations via $\Delta\omega_{ci}^{\text{nonlinear}} \propto |e\tilde{\phi}/T_i|^2 \ll 1$. DCLC modes grow to higher amplitudes causing ion trapping, stochastic diffusion and filling of loss cone in ion $f(v_{\perp})$.

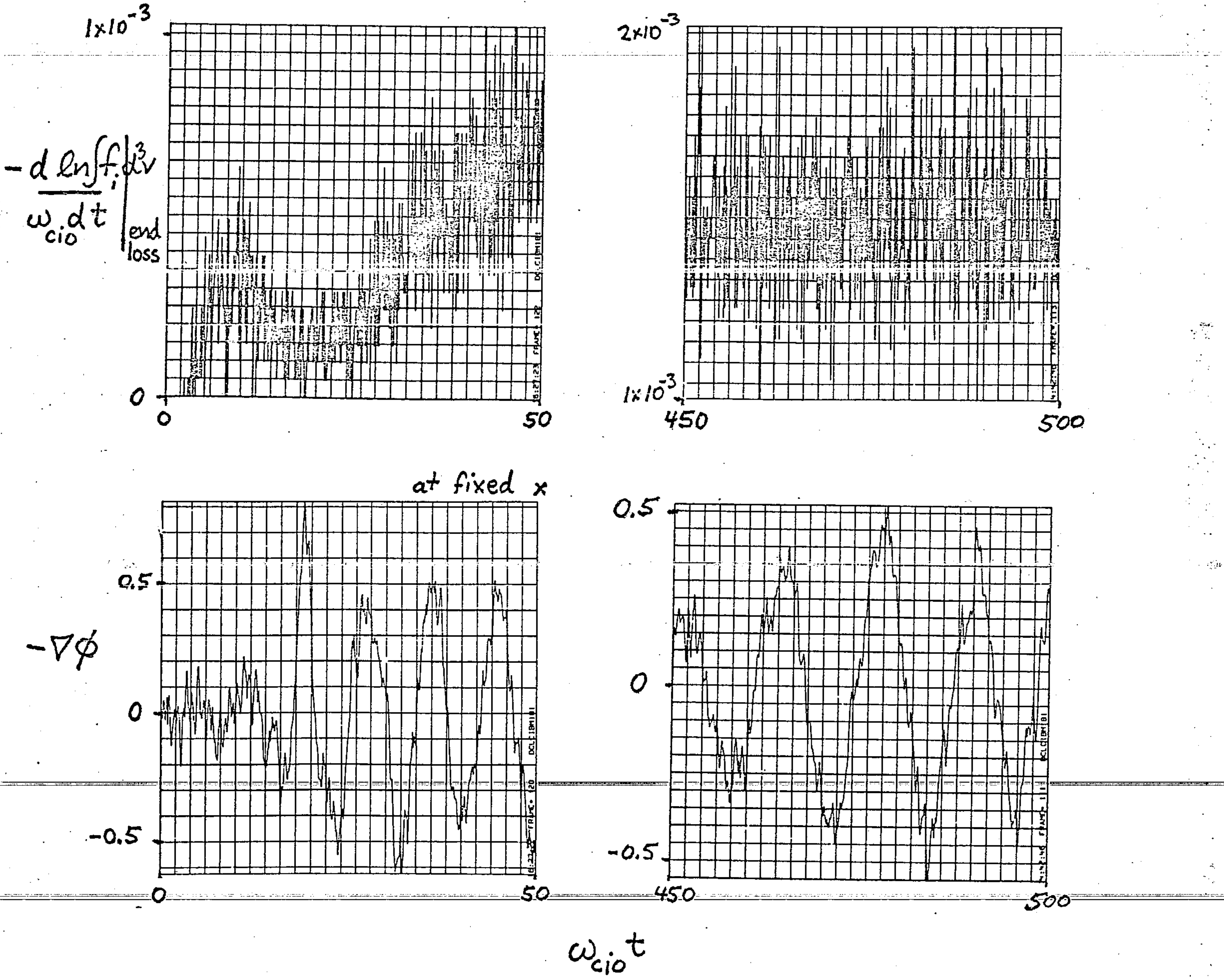
Simulation of DCLC with Ion Bouncing, Axial Loss & Injection

Parameters: $m_i/m_e = 3672$ $\omega_{pi}/\omega_{cio} = \omega_{cio}/\bar{\omega}_{bio} = 5 = R_p/a_{io} = L_B/a_{io}$

$e\Phi_o^{max} = 4T_e = 0.04\ T_{\perp io}$ $T_{\parallel io}/T_{\perp io} = 0.1$ $B_{max}/B_o = 2$

$\nu_{cx}/\omega_{cio} = 0.75\times10^{-2}$ $\nu_{drag}/\omega_{cio} = 0.75\times10^{-3}$ $i_{stream} \equiv i_{ioniz.} \equiv \frac{1}{2}$ end loss

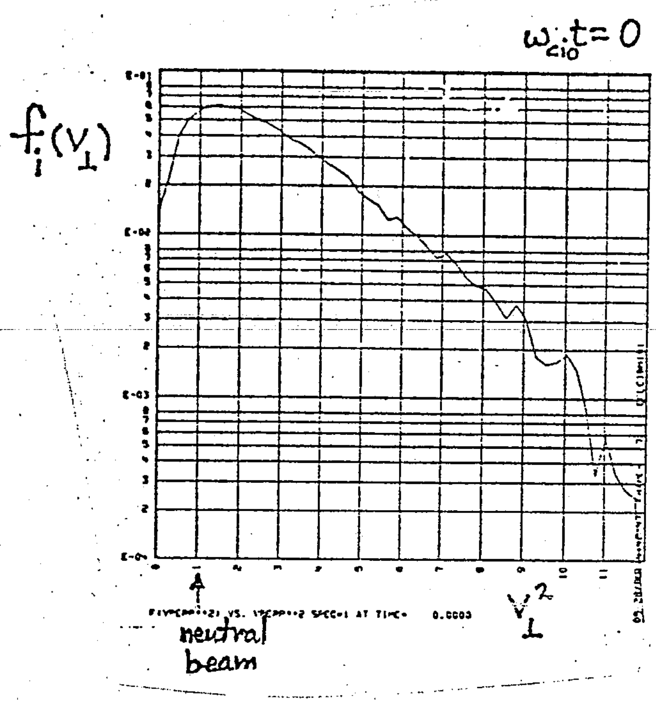
- Rapid early growth ($\mathrm{Im}\,\omega \sim .35\omega_{cio}$) of DCLC drives ions into loss region of velocity space; they transit the device and are lost.
- End loss approaches a steady rate accompanying wave saturation.

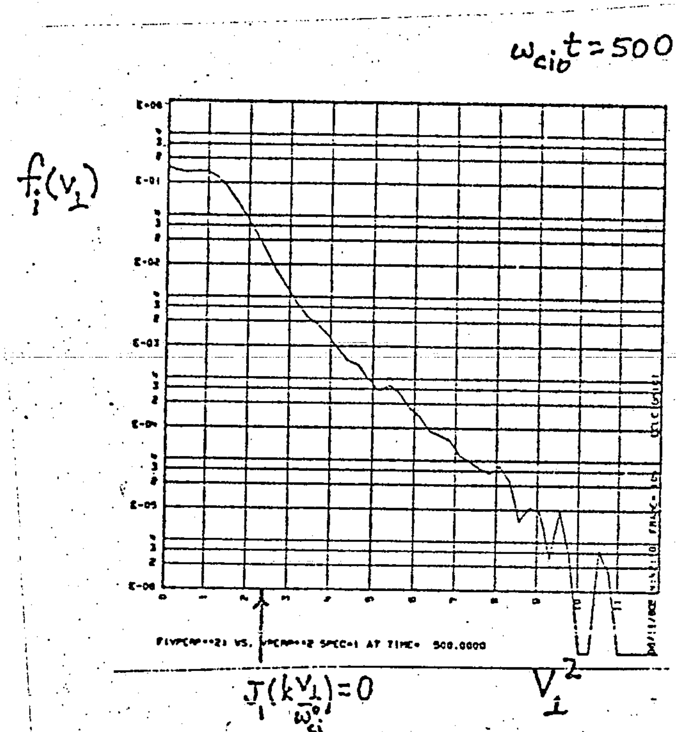


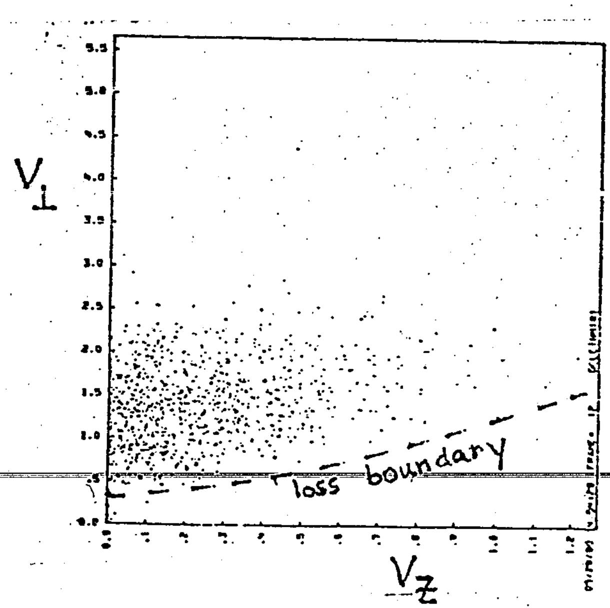
DCLC Drives Ion Velocity Distribution to Stability

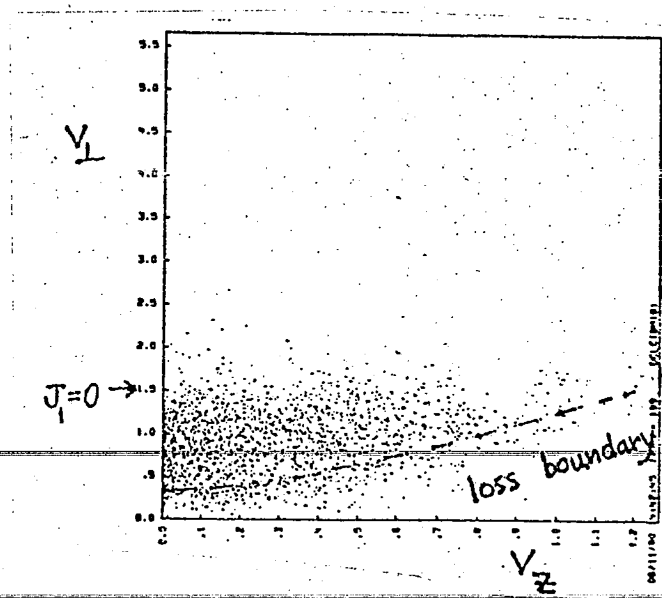
- Initially unstable loss-cone $f_i(v_\perp)$ $\rightarrow$ DCLC instability.

- DCLC turbulence, drag, neutral beam and stream evolve $f_i(v_\perp)$ to steady state.







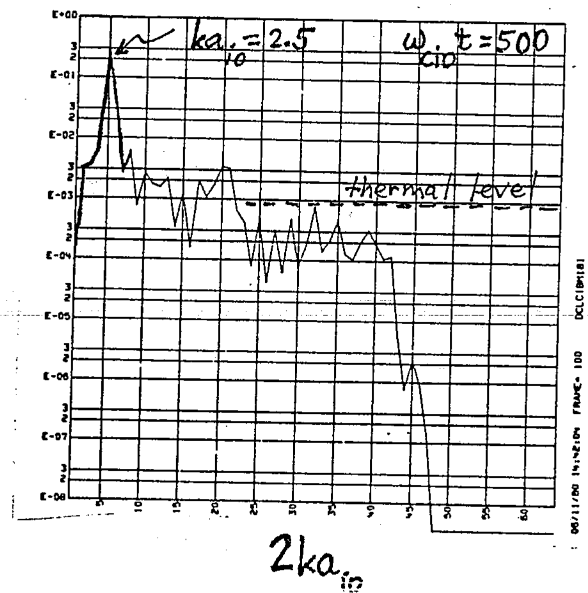


Most Unstable DCLC/Drift-Cyclotron Mode Dominates Spectrum

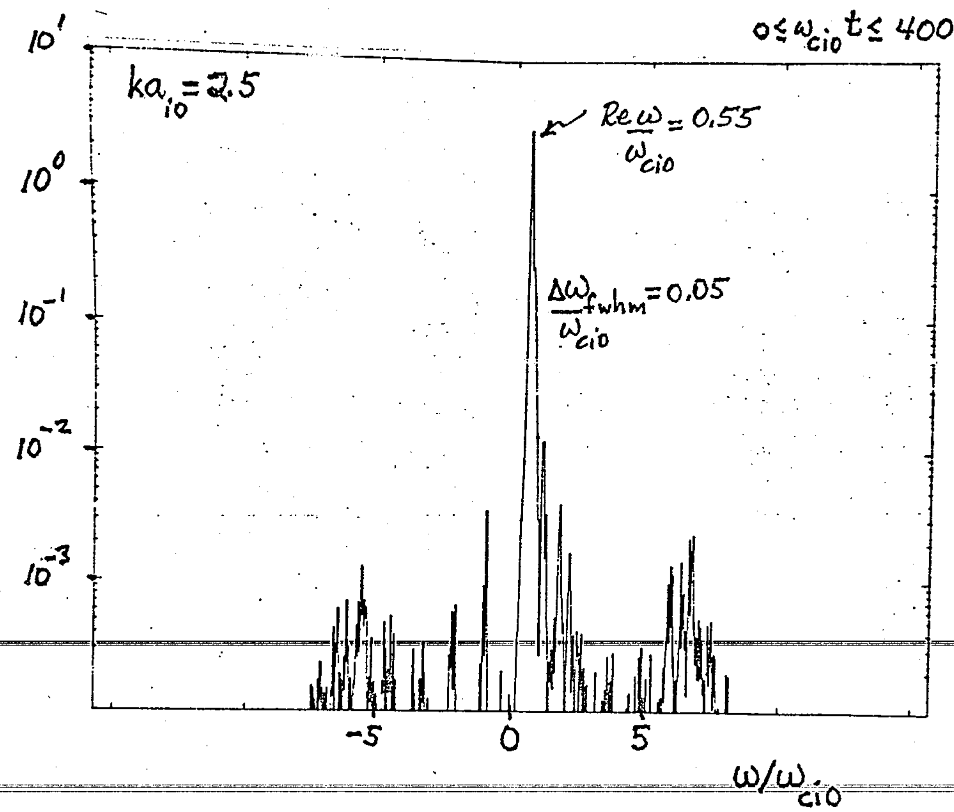
- DCLC turbulence evolves to coherent steady state for low density and discrete spectrum.
- Frequency and wavenumber of dominant mode are bracketed by DCLC, $f(v_\perp) \sim \delta(v_\perp - v_0)$, and DCI, $f(v_\perp) \sim \exp -v_\perp^2/2v_0^2$, marginally stable modes:

$$ka_{io}^{DCLC} = 2 < ka_{io}^{obs} = 2.5 < ka_{io}^{DCI} = 5 \qquad \omega^{DCLC} = 0.39\,\omega_{ci} < \omega^{obs} = 0.55\,\omega_{ci} < \omega^{DCI} = 0.8\,\omega_{ci}$$

field energy $\frac{1}{2}\rho_k \phi_k$

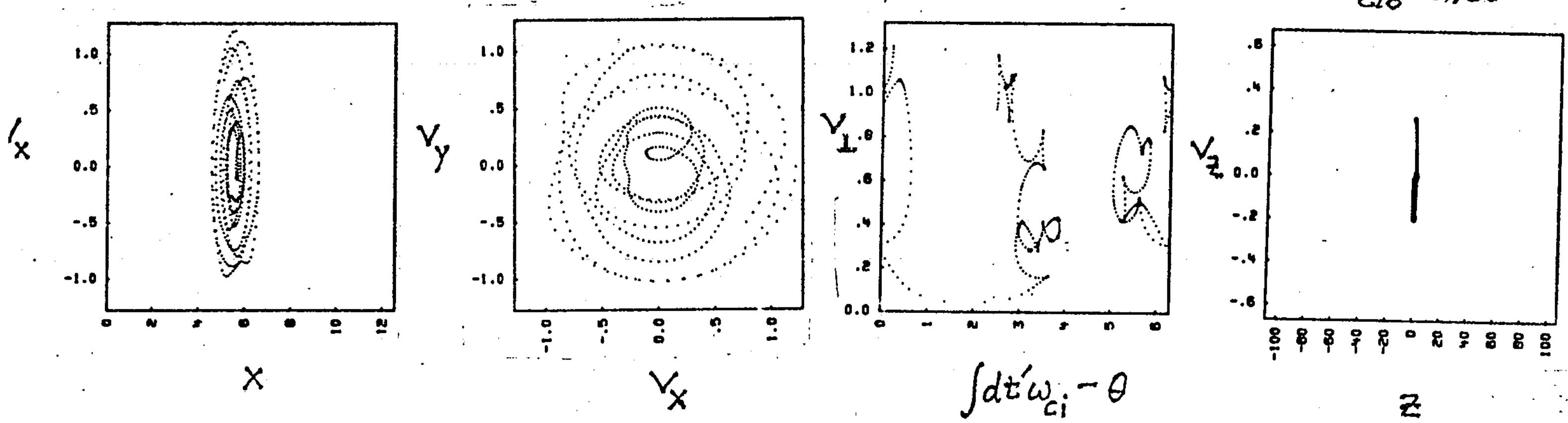


$\frac{1}{2} n_0 m_i v_{th}^{o\,2} = 475$



Wave Trapping and Stochasticity of Ion Orbits

- Ion orbits from simulation

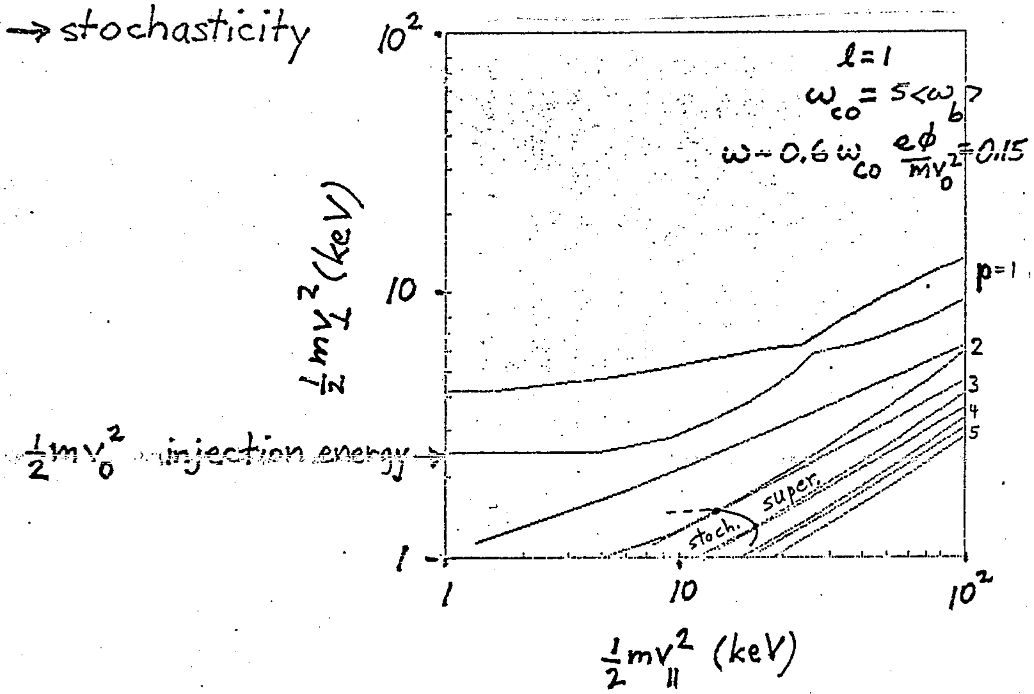


trapping: $\omega_{trap} = \frac{e\phi}{m v_\perp^2} k v_\perp J_1(k v_\perp / \omega_c) \gtrsim |\omega - \omega_c| \sim 0.2$ to $0.4\,\omega_c$

observe $e\phi / m v_0^2 = 0.15 - 0.2$, $v_0 \equiv e/m \equiv 1$

- Ion bounce resonances overlap: $\ell \omega_{co}(1 + W_\parallel / 2W_\perp) - 2p\omega_b(\mu) - \omega = 0$

$\rightarrow$ stochasticity



Secondary islands $2(2s+1)\,\omega_{trap} \approx \frac{\omega}{\ell}$ also overlap, $\frac{\langle \omega_{trap} \rangle}{\omega_{co}} \gtrsim \frac{1}{4}$

Scaling of Dielectric Response with m_e/m_i and $\omega_{ci}^2/\omega_{pi}^2$

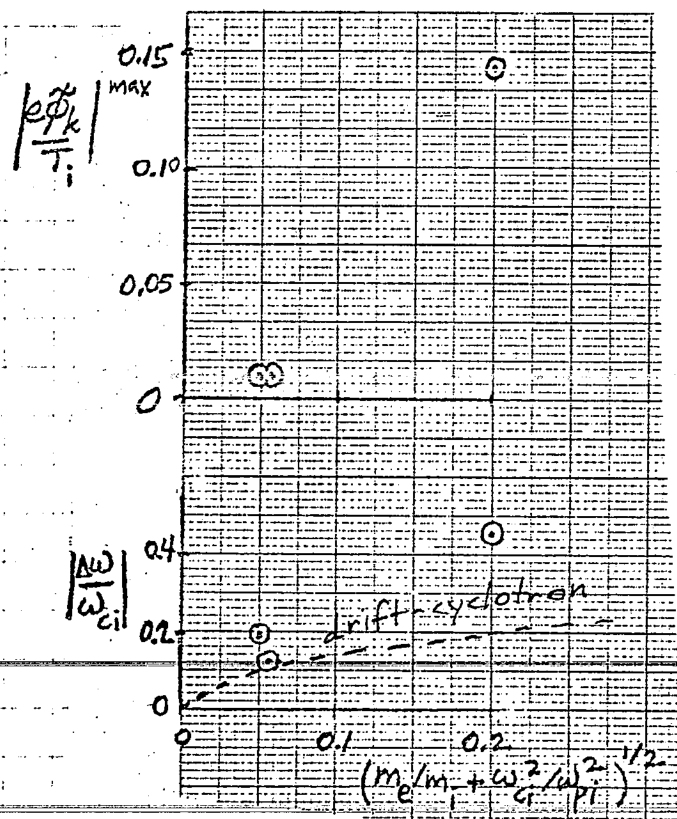
- $\epsilon(k,\omega) = 1 + \chi_e + \chi_i$ depends on m_e/m_i and $\omega_{ci}^2/\omega_{pi}^2$ via $(m_e/m_i + \omega_{ci}^2/\omega_{pi}^2)$

- Drift-cyclotron (Maxw. $f(v_\perp)$) dispersion near marginal stability:

$$1 - \omega/\omega_{ci} = (m_e/m_i + \omega_{ci}^2/\omega_{pi}^2)^{1/4} / (8\pi)^{1/4}$$

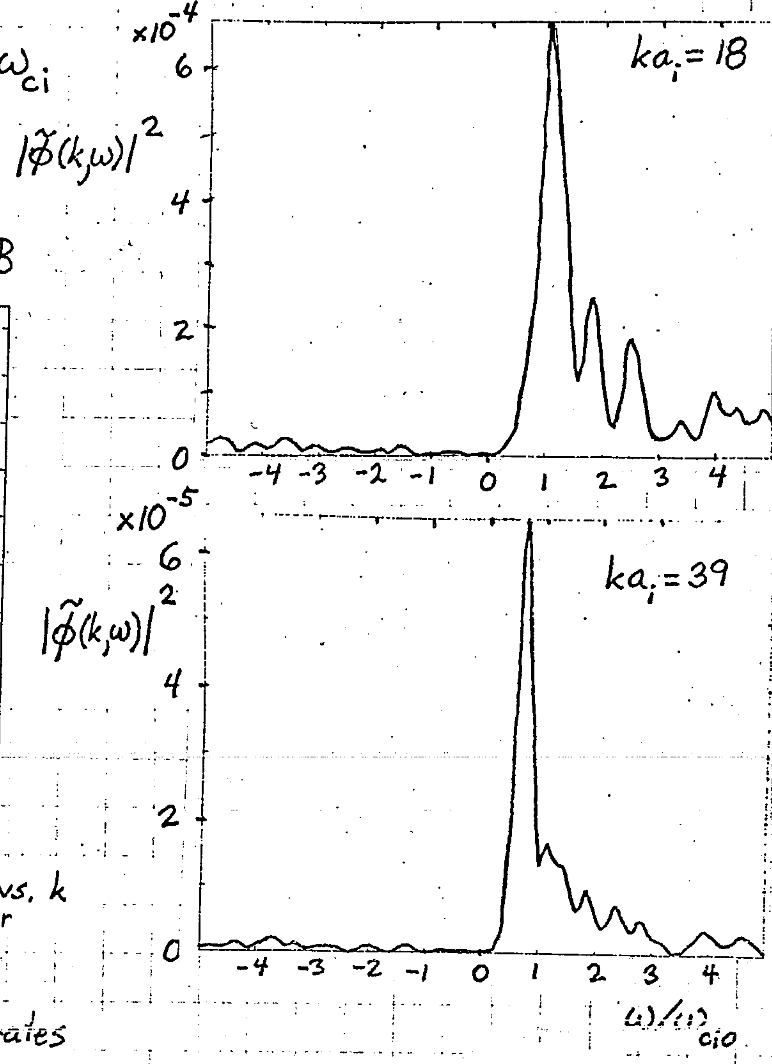
- As $m_e/m_i + \omega_{ci}^2/\omega_{pi}^2$ decreases, $\omega \to \omega_{ci}$ and wave amplitudes decrease.

- Wave amplitudes and frequencies vs. $(m_e/m_i + \omega_{ci}^2/\omega_{pi}^2)$ for most unstable mode:

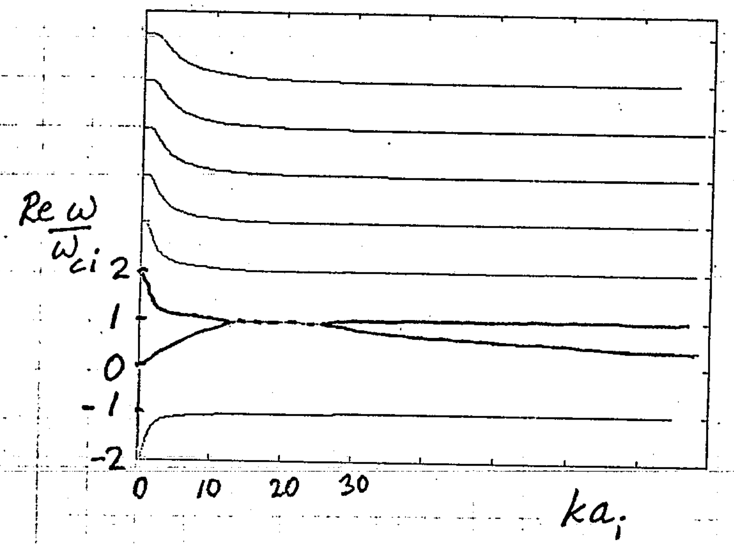


High Density Simulations Show Multi-Mode Turbulence

- Power spectra peaks near $\omega \sim \omega_{ci}$

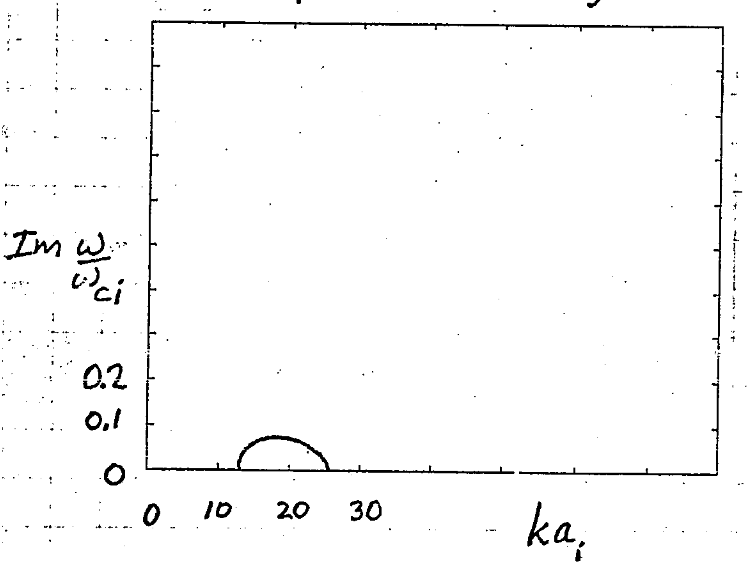


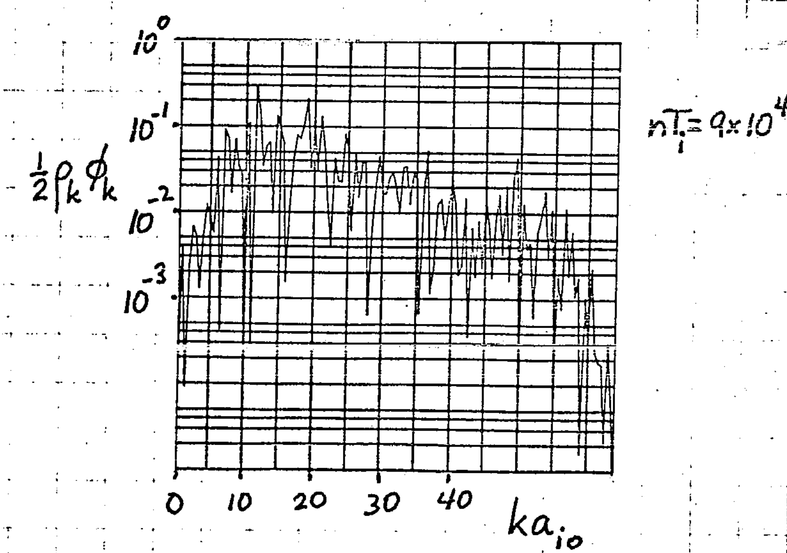
$f_i(v_\perp) = $ Maxw., local theory, uniform B



- Saturated $|\tilde{E}_k|^2$ vs. k tracks $Im\,\omega_{linear}$ vs. k

drift-cyclotron linear growth rates





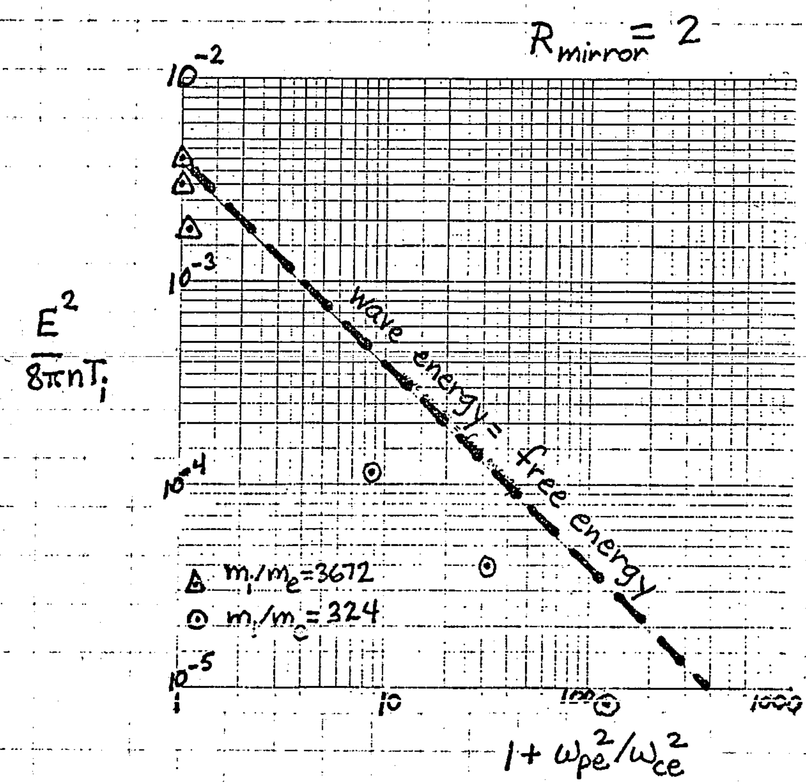
$\omega_{pi}^2/\omega_{ci}^2 = 10^4$ $m_i/m_e = 324$ $R_p/a_{io} = 10$ $\omega_{cio}/\bar{\omega}_{Bio} = 5$ $R_{mirror} = 2$

neutral-beam injected, no stream $\omega_{cio}\tau_{cx} = 150$ $\omega_{cio}\tau_{drag}^{1/e} = 500$

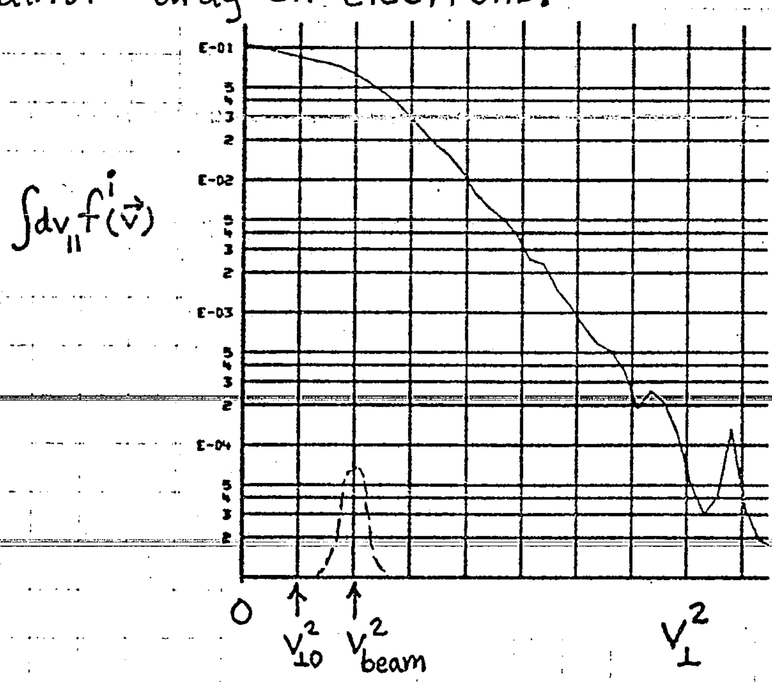
Free-Energy Bound on Wave Energy

- wave energy $= \int dk\, \omega \frac{\partial \epsilon}{\partial \omega} \frac{|E_k|^2}{8\pi} \approx 2\left(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\right) \frac{E^2}{8\pi}$

$$\leq n_0 \int d^3\vec{v}\, \frac{m_i}{2} v_\perp^2 \left(f^{\text{loss-cone}}_{\text{unstable}} - f^{\infty}_{\text{stable}}\right) = \text{free energy in } f_i(v_\perp)$$



- Difference free energy − wave energy > 0 heats ion tail, sustaining it against drag on electrons.



steady-state distribution function heated by fluctuations.

Axial Electron Dissipation Moderates DCLC Turbulence

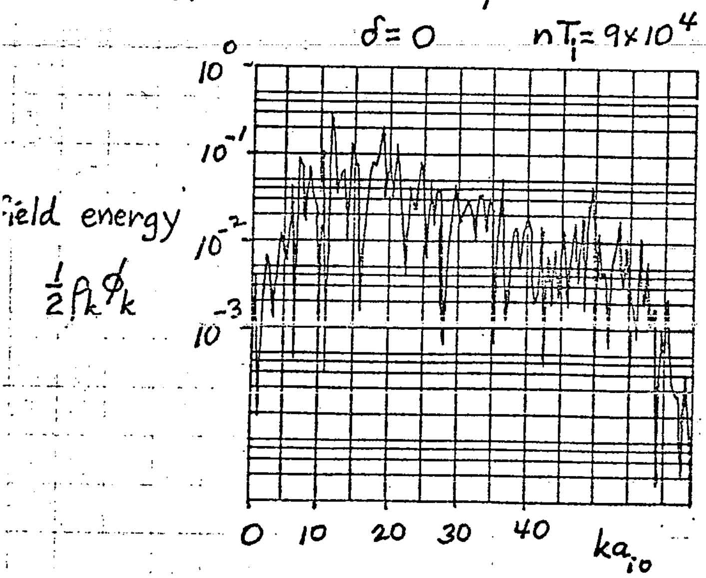
- Electron Landau damping in mirror throat justifies outgoing-wave boundary conditions axially, modeled by

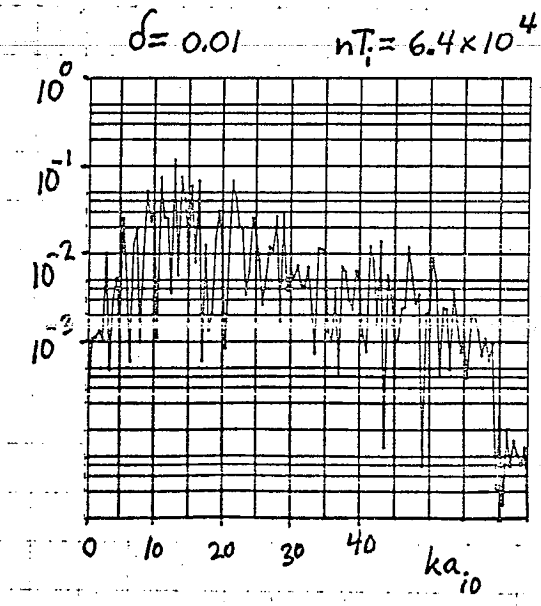
$$\epsilon(k,\omega) \rightarrow \epsilon(k,\omega)_{flute} + i\,\delta\,\omega_{pi}^2 \Delta t^2 \left(\frac{m_e}{m_i} + \frac{\omega_{ci}^2}{\omega_{pi}^2}\right) R\, k^2 a_i^2 \frac{\omega}{\omega_{ci}}$$

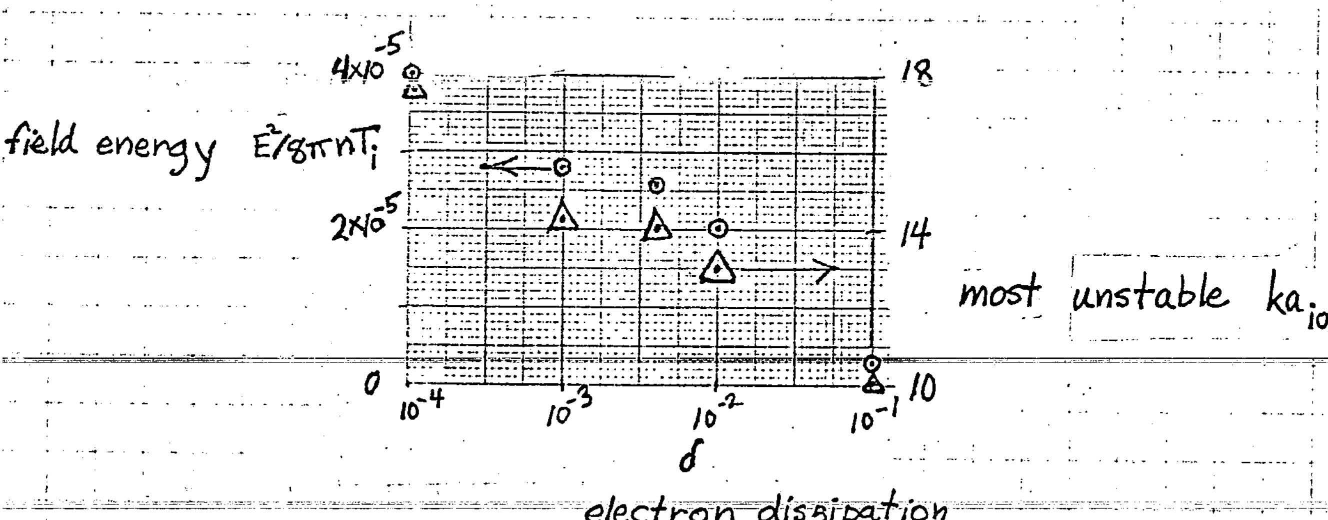
($k^2 a_i^2$ ← increases with k^2)

ref. H.L. Berk et al., Phys. Rev. Lett. 22, 876 (1969); D.E. Baldwin et al., IAEA (Madison, 1972) Vol. II, p. 735.

- Turbulence is reduced and shifted to longer wavelengths by electron dissipation.







CONCLUSIONS OF DCLC RESEARCH IN PROGRESS

- ION CYCLOTRON WAVE TURBULENCE TRAPS IONS AND STOCHASTICIZES ORBITS IN SIMULATIONS CONSISTENT WITH THEORY.
- NEUTRAL BEAM IS DIFFUSED IN VELOCITY AND $F_i(v_\perp)$ IS QUASILINEARLY FLATTENED AND STABILIZED BY WAVE TURBULENCE.
- FREE ENERGY IN $F_i(v_\perp)$ SETS UPPER BOUND ON WAVE ENERGY IN CLOSED AND OPEN SYSTEMS. SURPLUS FREE ENERGY IS USED BY WAVES TO SUSTAIN ION VELOCITY TAIL AGAINST ELECTRON DRAG.
- SCALINGS OF WAVE FREQUENCIES AND AMPLITUDES WITH $(m_e/m_i + \omega_{cio}^2/\omega_{pi}^2)$ HAVE BEEN INVESTIGATED AND POINT TOWARD $\omega \sim \omega_{cio}$ AND $|e\tilde{\phi}_k/T_i| \lesssim 10^{-2}$ OBSERVED IN 2XIIB.
- DISCRETE WAVENUMBER SPECTRUM OF DCLC TURBULENCE CAN EVOLVE TO A SINGLE-MODE STATE.
- AXIAL ELECTRON DISSIPATION MODERATES DCLC AND DRIFT-CYCLOTRON TURBULENCE.

DRIFT WAVE SOLITONS AND TURBULENCE

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Drift Wave Solitons and Turbulence

J. Meiss + W. Horton

I Taxonomy

II Coherent Structures in Turbulence

III Ideal Gas of Solitons

IV Further work

Taxonomy of Drift Wave Solitons

1) Fluid Theory, Electrostatic
2) No Shear
3) Slab Model

$$\frac{d}{dt}\left(\log n_0(x) + \frac{e\phi}{T_e(x)} \right) + \nabla \cdot \underset{\sim}{V}_i = 0$$

$$\frac{d}{dt} V_i = -c_s^2 \nabla\left(\frac{e\phi}{T}\right) + \underset{\sim}{V} \times \underset{\sim}{\Omega}_i$$

A) $k_y \rho_s \ll k_x \rho_s \sim \left(\rho_s / L_n\right)^{1/2}$

1) Treat radial structure $O(1)$
2) ∇T_e nonlinearity dominates

Follow Todoroki and Sanuki (1974)

$$\frac{e}{T}\phi(x,y,z,t) = \psi(y-ct,z,t)\, g(x)$$

$$\rho_s^2 \frac{\partial}{\partial x} n_0(x) \frac{\partial}{\partial x} g + n_0(x)\left[\frac{V_d(x)}{c} - \frac{1}{T_e(x)}\right] g = 0$$

↳ determines $g(x)$, c

obtain <u>Kadomtsev - Petviashvili Equation</u>

$$\frac{\partial}{\partial y}\left[\frac{\partial \psi}{\partial t} + \alpha \psi_{yyy} + \beta \psi \psi_y \right] + \gamma \psi_{zz} = 0$$

$$\alpha \simeq v_d \rho_s^2$$

$$\beta = \frac{1}{2} c \frac{\langle n_0 \frac{T_e'}{T^2} g^3(x) \rangle}{\langle n_0' g^2(x) \rangle} \sim v_d \left(\frac{L_n}{L_T}\right)$$

$$\gamma \simeq c_s^2 / v_d$$

Soliton

$$\psi = A \operatorname{sech}^2 \left(w \left(y + k_{\parallel} z - Vt \right) \right)$$

<u>Note</u>: 1) Stable to ion acoustic perturbations

2) propagates in y, z plane

B) $k_x\rho_s \sim \left(\rho_s/L_n\right)^{1/2} \ll (k_y \rho_s) \ll \left(\left(\rho_s/L_n\right)\right)^{1/4}$

1) Neglect radial structure $k_x < k_y$

2) ∇T nonlinearity dominates

Follow Petviashvili (1977)

$$\left(1 - \rho_s^2 \frac{\partial^2}{\partial y^2}\right)\phi_t + v_d \phi_y - v_d \frac{L_n}{L_T}\frac{e}{T_e}\phi\phi_y = 0$$

Solitary wave

$$\eta_e \frac{e\phi}{T_e} = -3\left(\frac{u}{v_d} - 1\right)\operatorname{sech}^2\left(\frac{1}{2}\left(1 - \frac{v_d}{u}\right)^{1/2}\frac{y - ut}{\rho_s}\right)$$

$$\boxed{u > v_d \quad \text{or} \quad u < 0}$$

2) Two-Dimensional Solitons $k_x \sim k_y$

For $(k\rho_s)^3 \gg \eta_e \rho_s/r_n$ keep convective nonlinearity $\underset{\sim}{E} \times \underset{\sim}{B}$

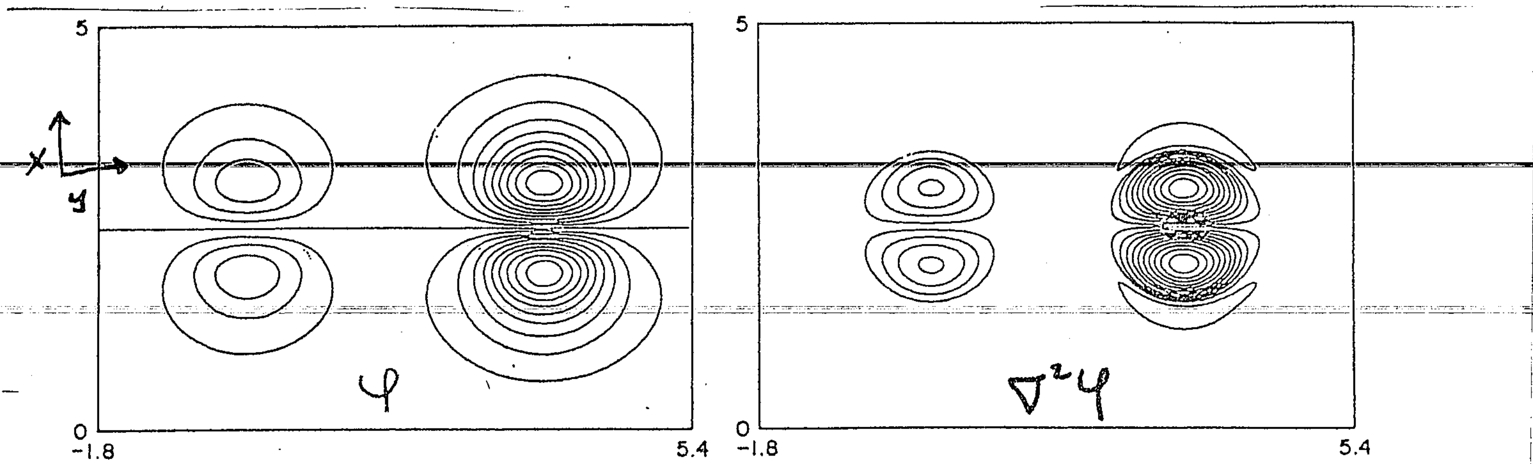
Follow Hasegawa & Mima (1978)

$$(1 - \rho_s^2 \nabla^2)\varphi_t + V_d \varphi_y - c_s \rho_s [\varphi, \rho_s^2 \nabla^2 \varphi] = 0$$

[derived by Charney, 1948]

Soliton Solution: [Stern, 1975; Flierl, 1980]

$$\varphi_s = \left\{ \begin{array}{ll} A K_1(\beta r/a) & r > a \\ B r/a + C J_1(\gamma r/a) & r < a \end{array} \right\} \times \cos\theta$$



Two parameters:

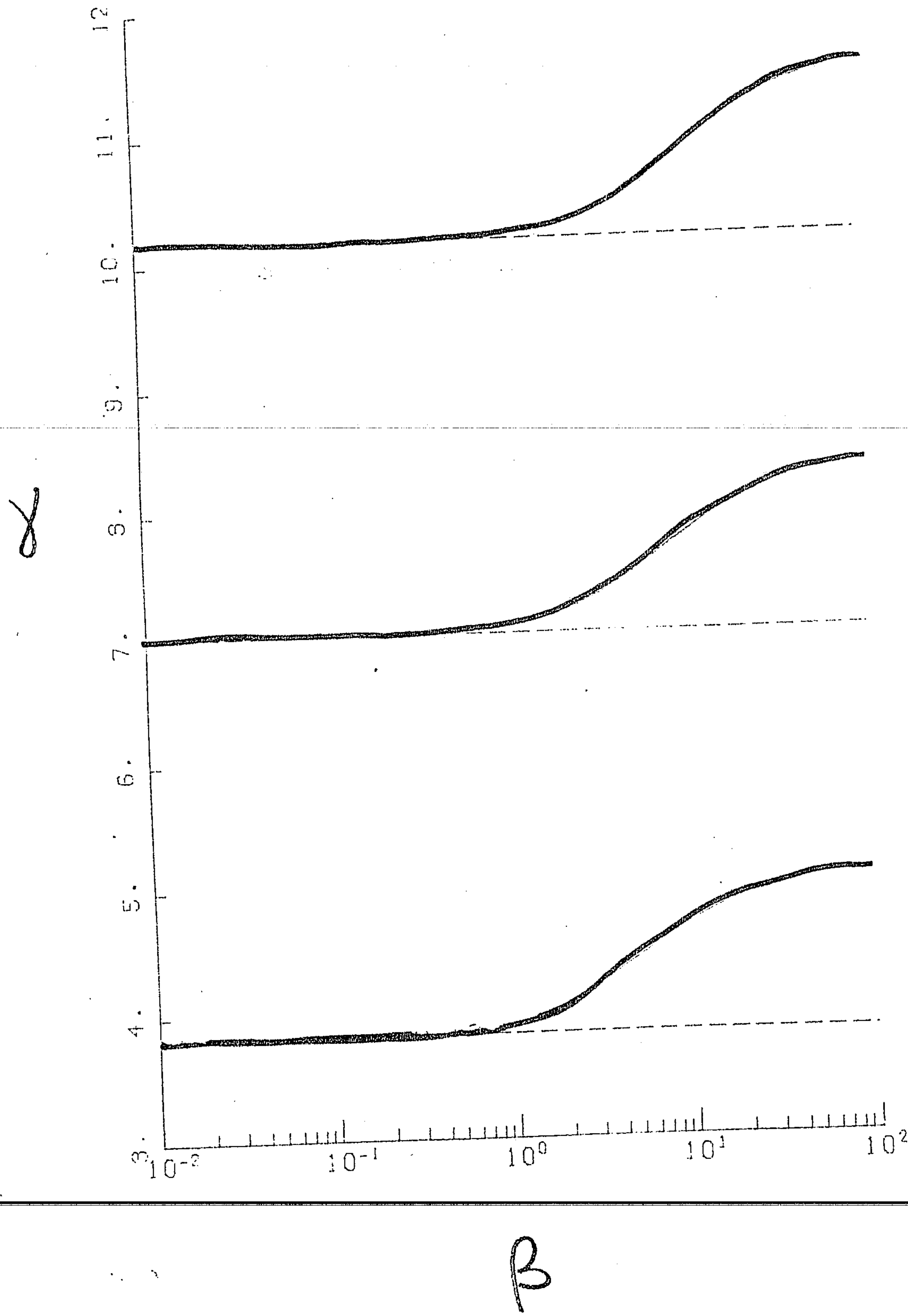
a = radius

c = speed

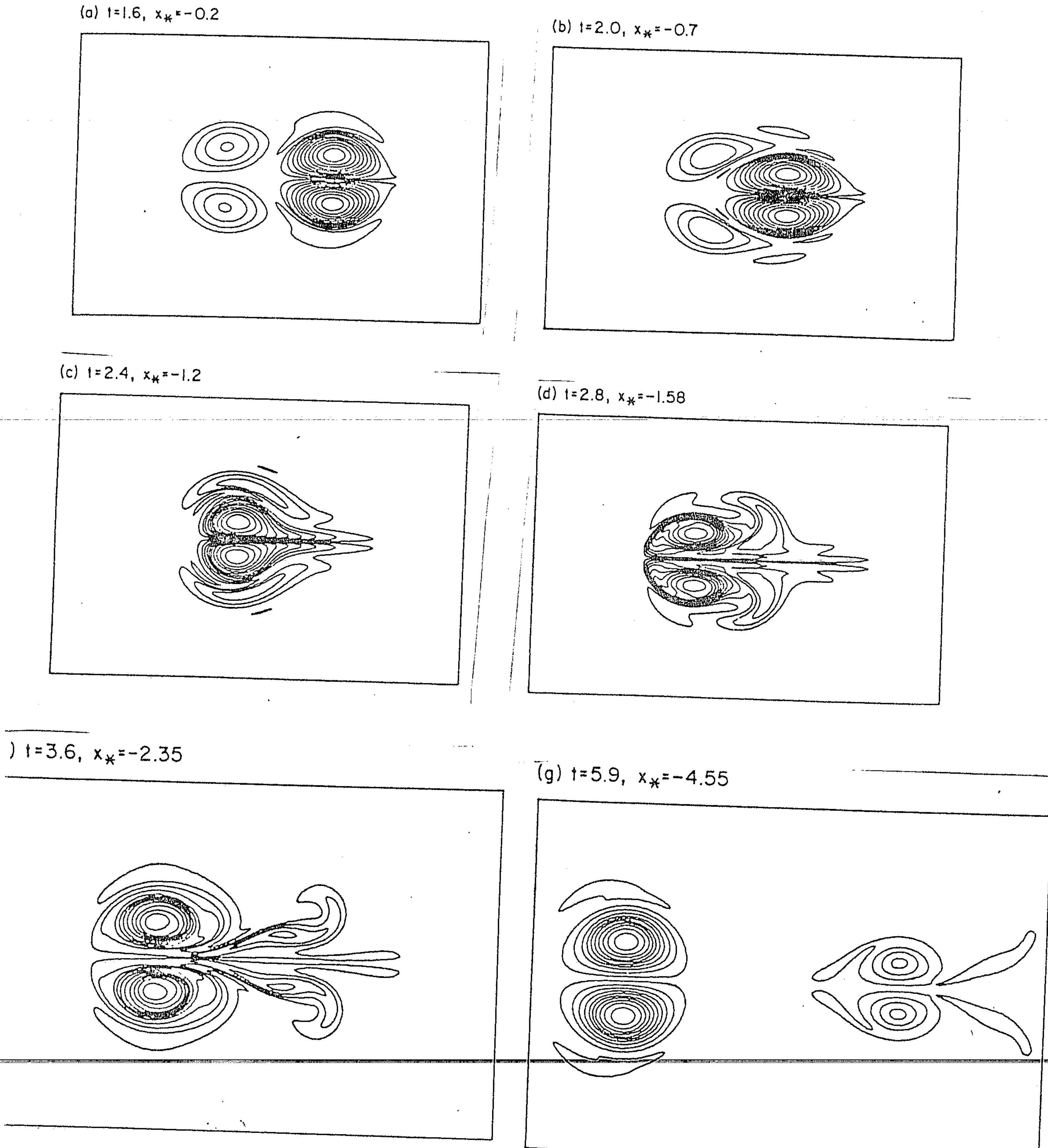
$\beta = a\sqrt{1 - v_d/c}$ = outer scale

$\gamma(\beta)$ determined by continuity at $r = a$

Note: $c < 0$ or $c > v_d$


γ
3.
4.
5.
6.
7.
8.
9.
10.
11.
12.
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
β

Collision: from McWilliams and Zabusky (81)



Nearly preserved, However sometimes destroyed in head on collisions.

Soliton Turbulence

Examples

1) Langmuir - Ion Sound Turbulence

Zakharov Equations

Pumped Turbulence:

Weak Turbulence ⇒ Modulational Instability ⇒

Soliton Formation ⇒ Soliton Collapse ⇒

Landau damping

2) Fluid Turbulence ⇒ Coherent Structures

a) Vortex rolls and Pairing in Shear Flows

Browand (1980)

b) Turbulent Spots or Bursts in Mixing Layer

Cantwell (1981)

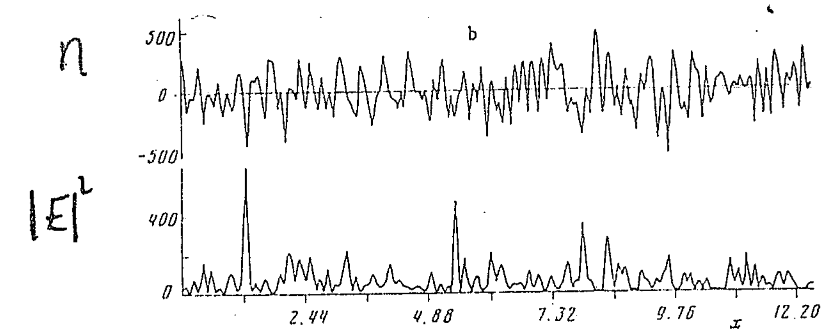
Strong Langmuir Turbulence

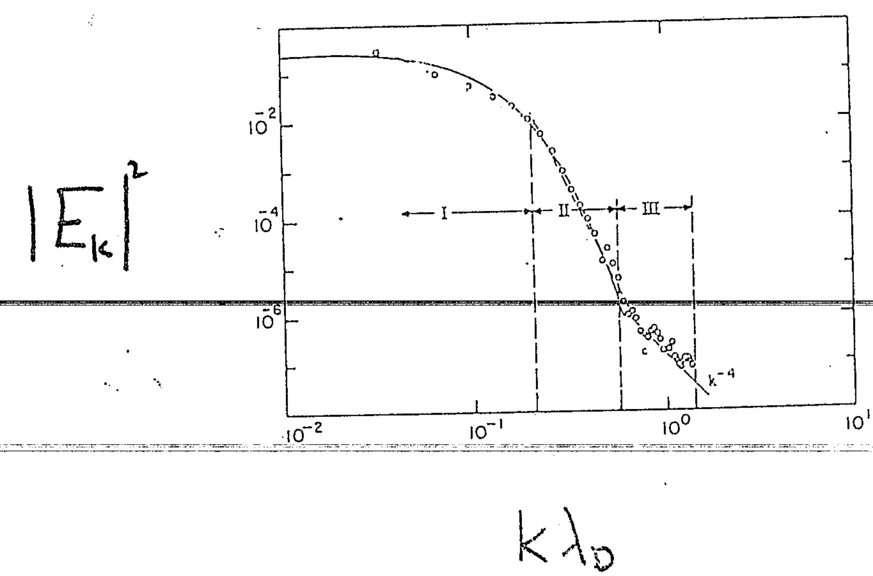
Pump: $E_0 e^{i\omega t}$

Dissipation: Landau damping

resonant wave-particle interaction

Large as $k\lambda_D \to 1$





Pereira et al (1977)

Ideal Gas Approximation

$$\varphi = \sum_{i=1}^{N_s} \varphi_s(y - y_i, u_i t)$$

Spectral Density

$$S(k,\omega) = \left[\left\langle \varphi(x+\xi, t+\tau)\, \varphi(x,t) \right\rangle \right]^{F.T.}$$

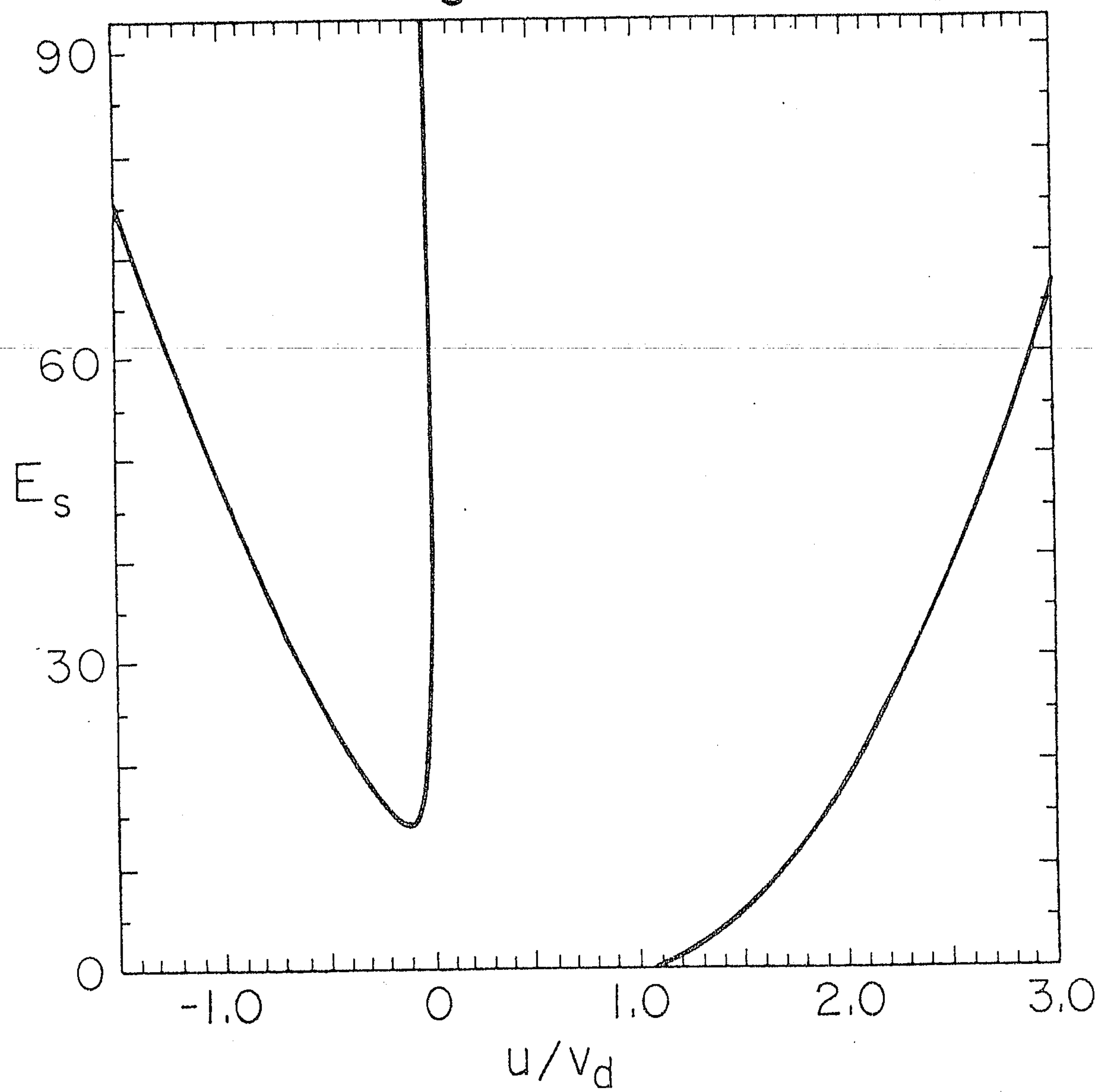
Example KdV or Petviashvili

$$S(k,\omega) = \frac{1}{L} f_s\left(\frac{\omega}{k}\right) \left[12\pi \rho_s \frac{\omega}{v_d} \operatorname{csch}\left(\pi k \rho_s \sqrt{\frac{\omega}{\omega - k v_d}} \right) \right]^2$$

$f_s(u)$ = Soliton distribution function for speed u

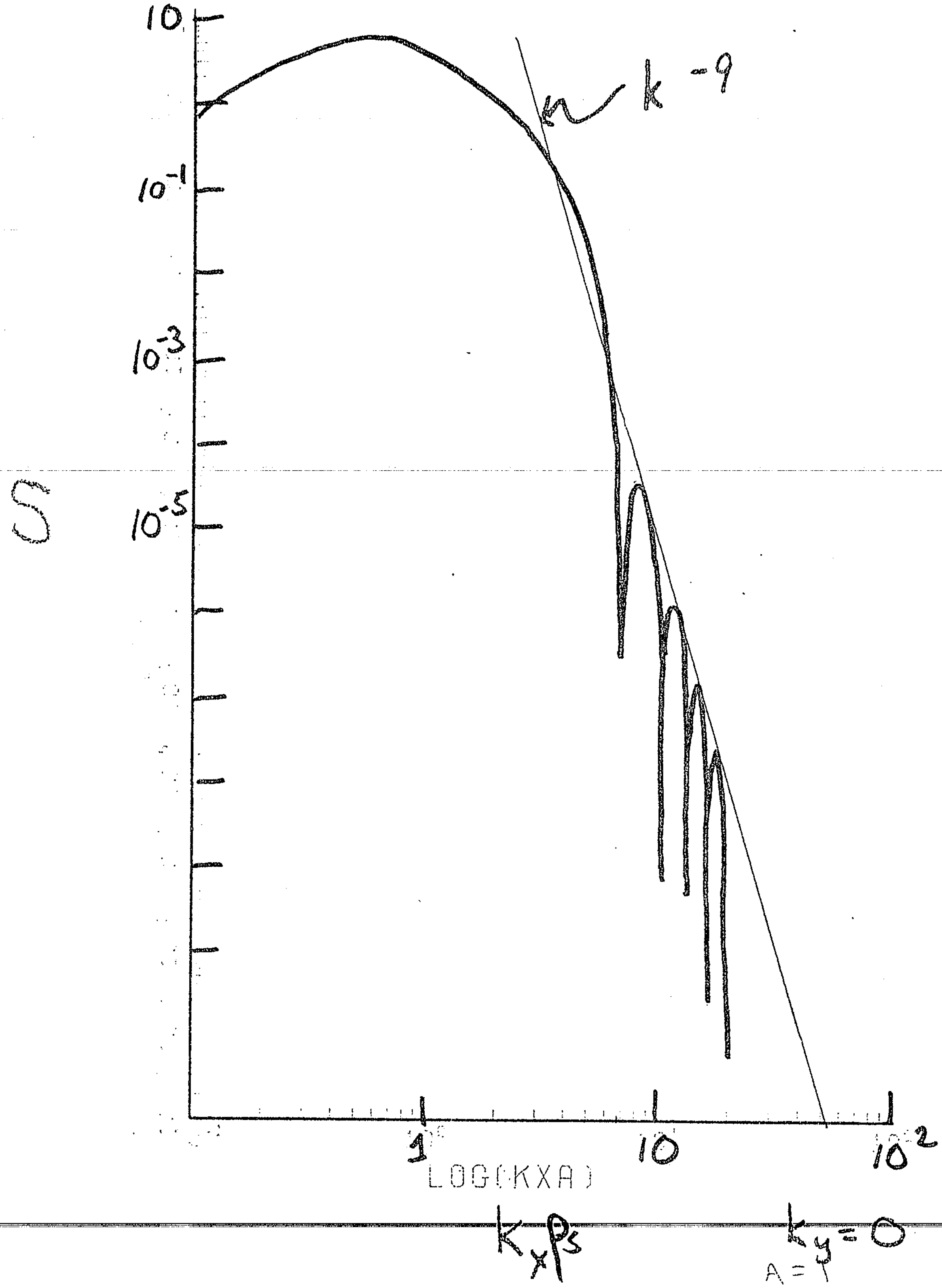
Soliton Energy

$$E = \int dx\,dy \left[\phi^2 + (\rho_s \nabla_\perp \phi)^2 \right]$$



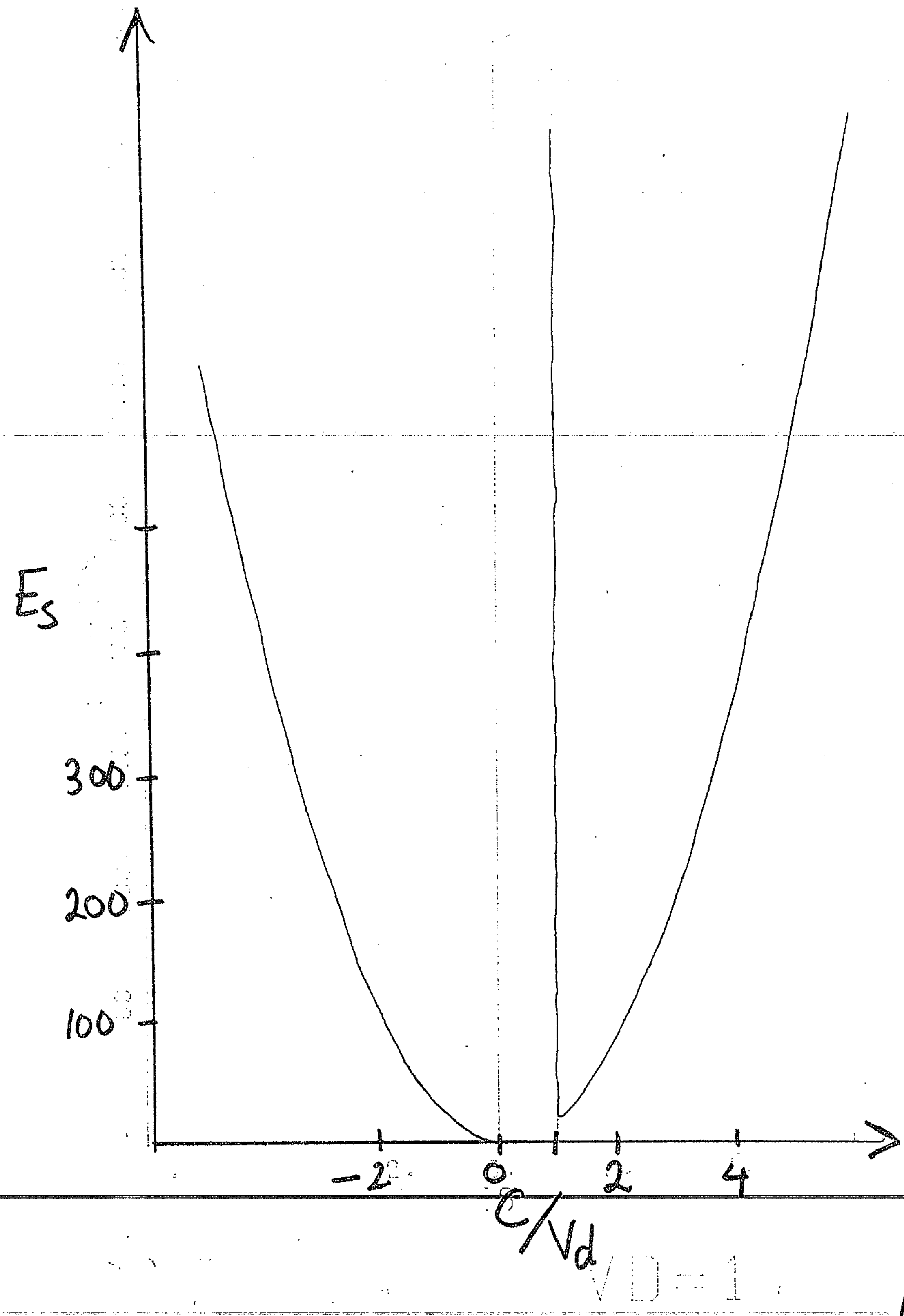
For Petviashvili Equation

Solitary Vortex Spectrum



A = 1
C = 1.5

Energy: Solitary Vortex



A = 1.0

Ideal Gas : Canonical Ensemble

1) $\mathcal{P} \propto e^{-\beta_s E_s}$; $f_s(u) = N_s P(u)$

2) Fix β_s by requiring correct $\langle E_s \rangle$

Total energy $= N_s \langle E_s \rangle = \langle \varphi^2 \rangle L/\rho_s$

3) Find N_s by inverse scattering transform

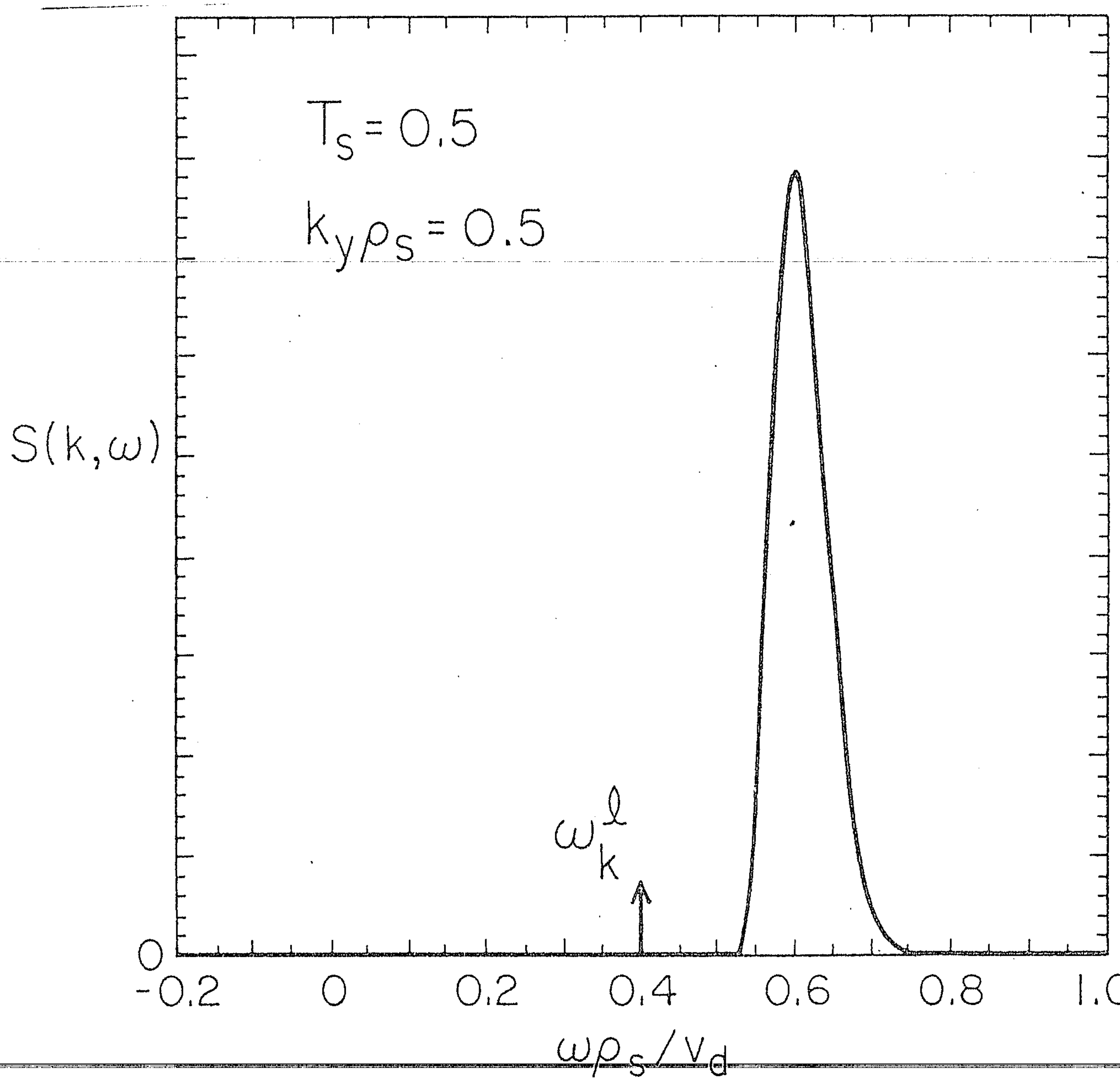
$$\langle N_s \rangle = 0.064 \frac{L}{\rho_s} \sqrt{\langle \varphi^2 \rangle}$$

valid for KdV only...

$\langle \varphi^2 \rangle = 0.01$

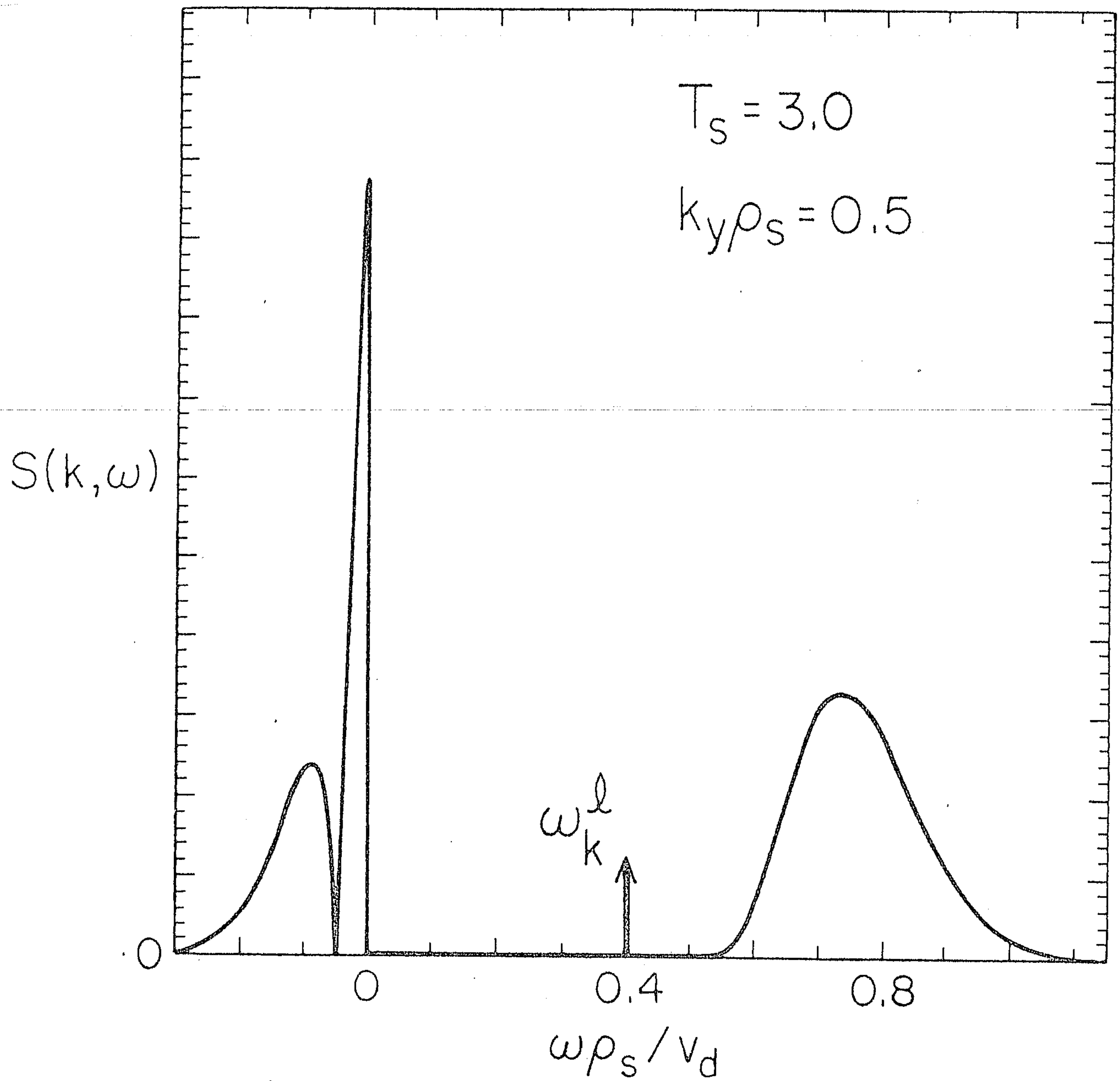
$\langle E_s \rangle = 15.6 (\langle \varphi^2 \rangle)^{3/4}$

$N_s / l = \frac{15.6}{\rho_s} (\langle \varphi^2 \rangle)^{1/4}$



$\frac{\omega_{peak}}{k V_d} \simeq 1.25 \qquad \frac{\Delta\omega}{k V_d} = 0.5$

$\langle \varphi^2 \rangle = 0.1$



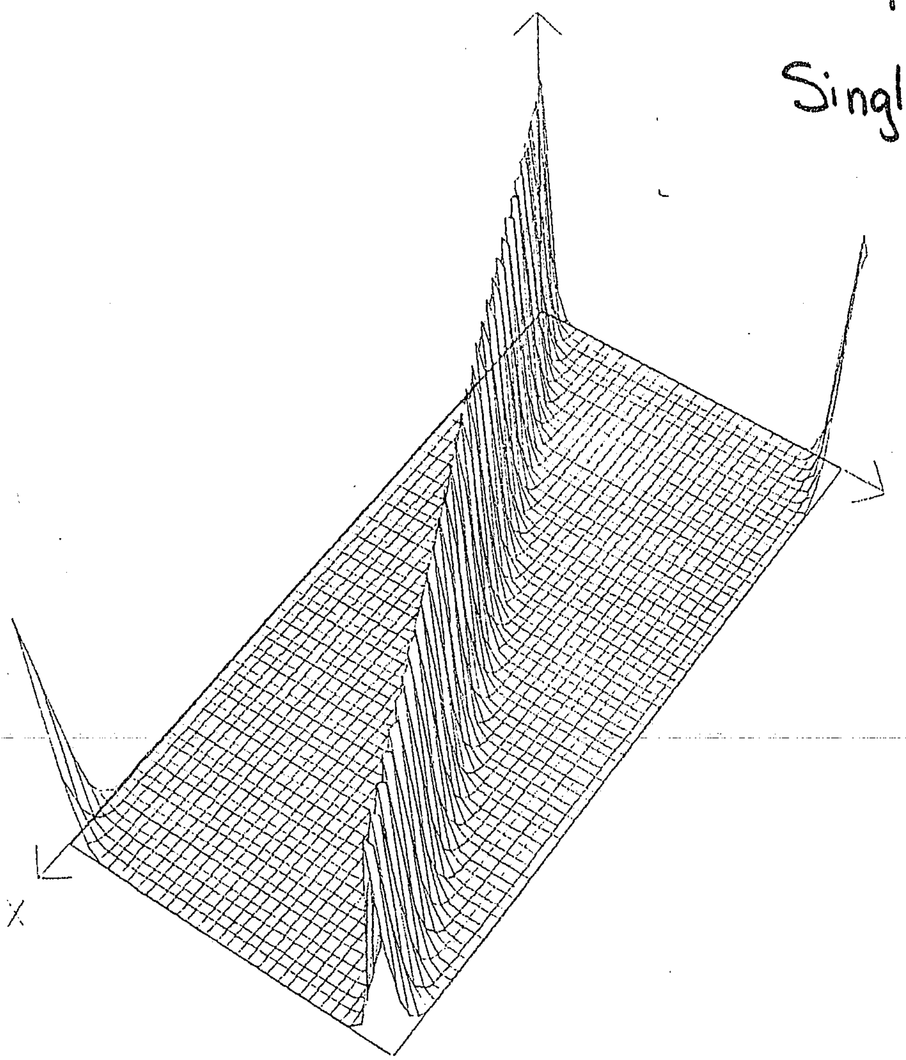
1) $\frac{\omega_{peak}}{k v_d} = 1.5$

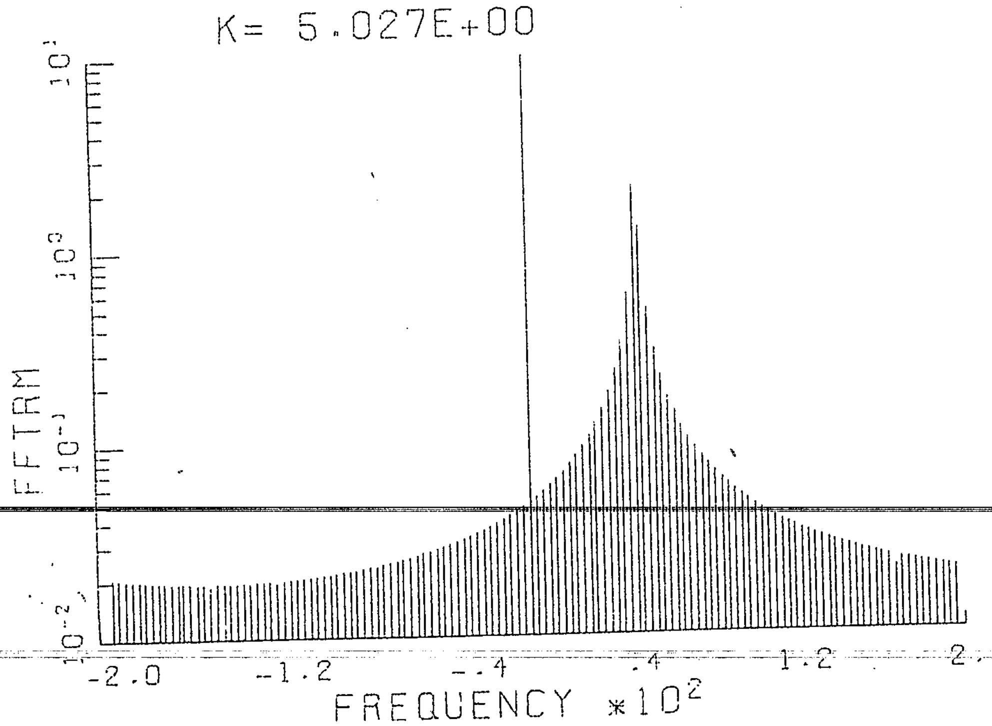
2) significant $\omega < 0$ fluctuations

3) $\frac{\Delta\omega}{k v_d} \sim 0.8$

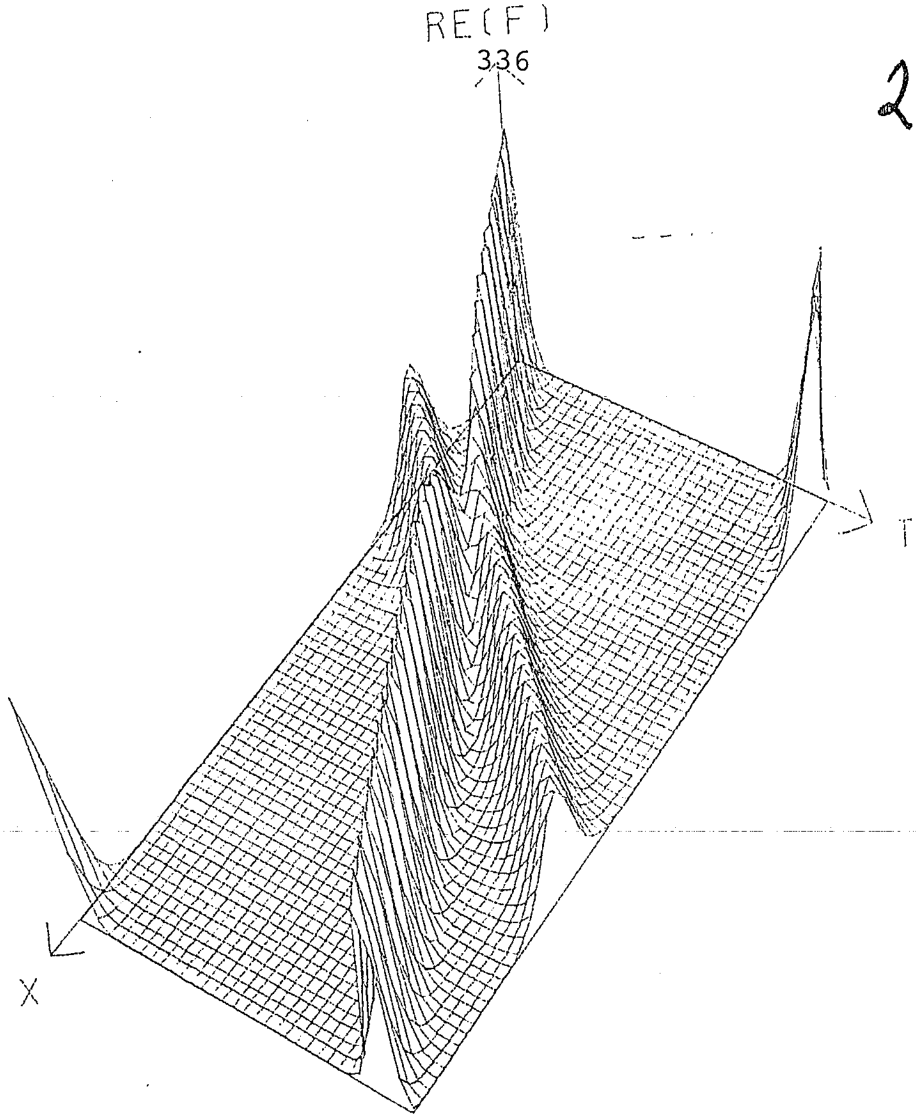
KdV Eq.
Single Soliton
V= 10

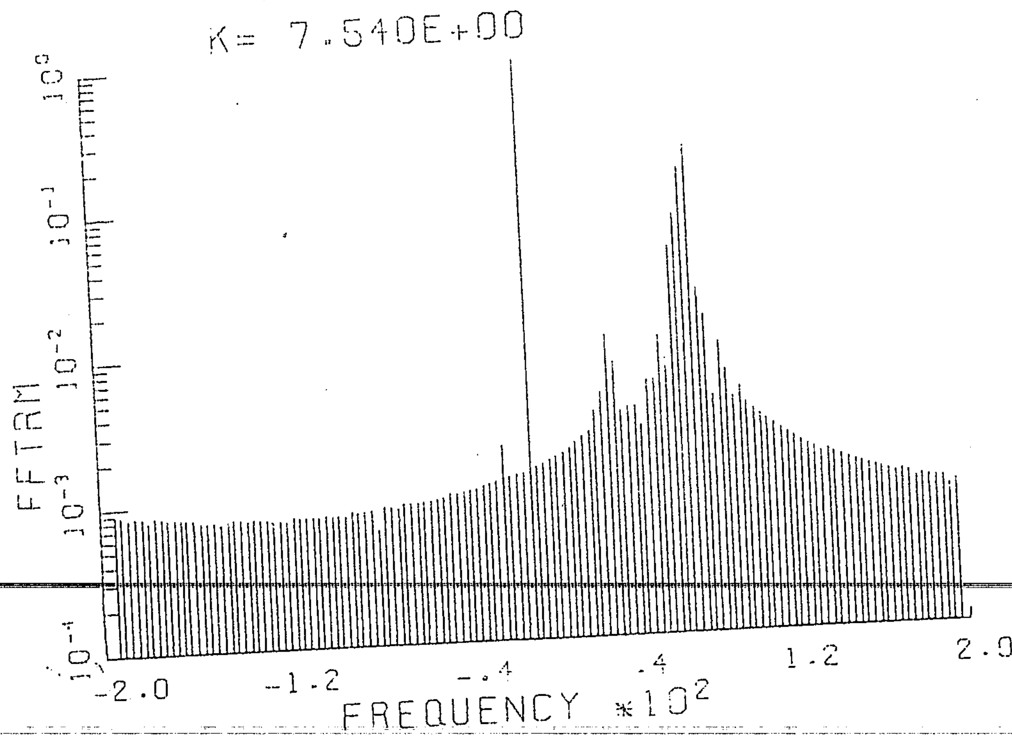
$X_{mn} = -10$
$X_{mx} = 10$
$t_{mx} = 2$



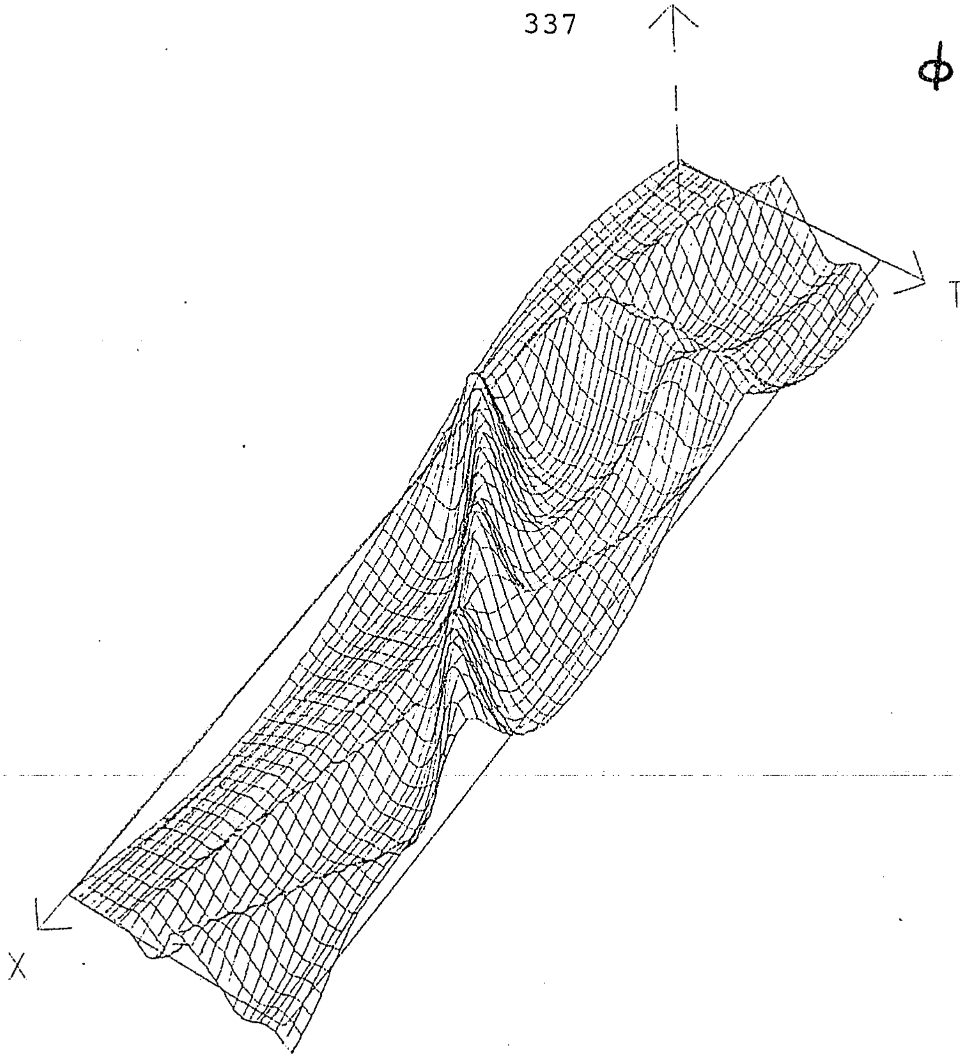


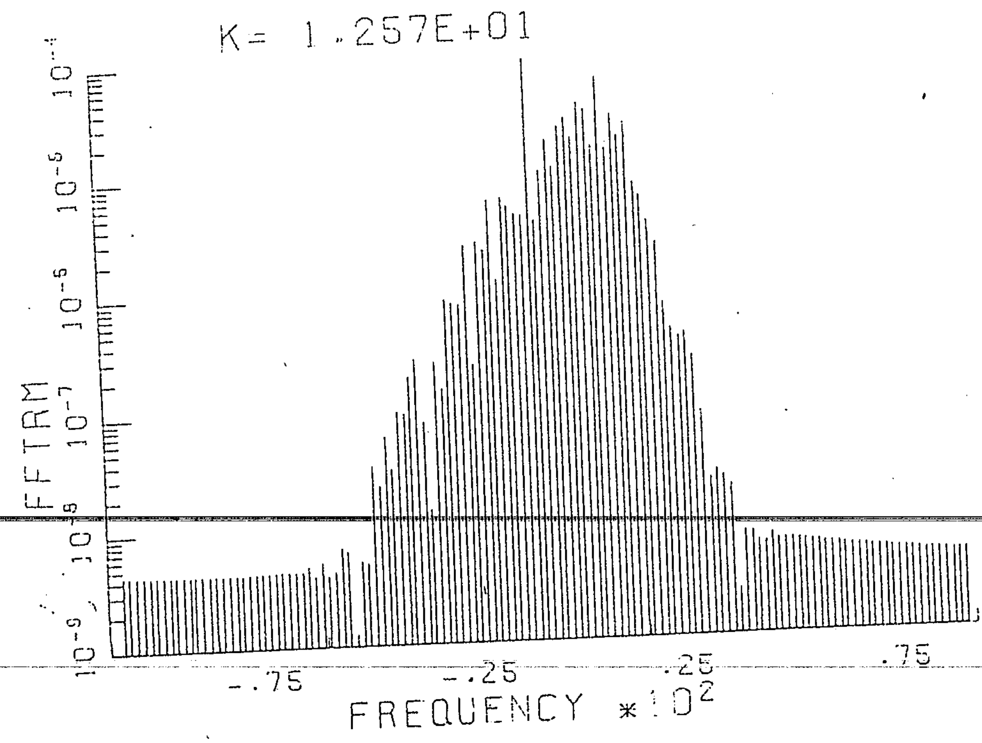
KdV
2 Soliton
$V_1 = 10$
$V_2 = 5$





KdV
$\phi = \sin(\pi x/10)$
2 solitons produced





Future Work

1) Effect of shear

seems straight forward for

$$k_y \rho_s \ll (\rho_s/L_n)^{1/2}$$

nonlinear ducting, reduced shear damping

2) Kinetic Effects

a) e^- : Follow Ott and Sudan for ion acoustic waves

b) ion : with P. Morrison

3) Couple Turbulence with Solitons

Follow a calculation of M.N.R. for water waves

NONLINEAR BEHAVIOR OF UNSTABLE TOROIDALLY INDUCED DRIFT MODES IN TOKAMAK GEOMETRY

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Non Linear interaction of Toroidicity Induced Modes

P. SIMILON

P. H. DIAMOND

Aknoledgments M. N. Rosenbluth

Introduction

Drift wave fluctuations in toroidal geometry:

- TI modes: localized in θ by ion magnetic drifts
- driven by electrons (e.g. trapped electrons)
- negligeable shear damping ($\neq$ slab)

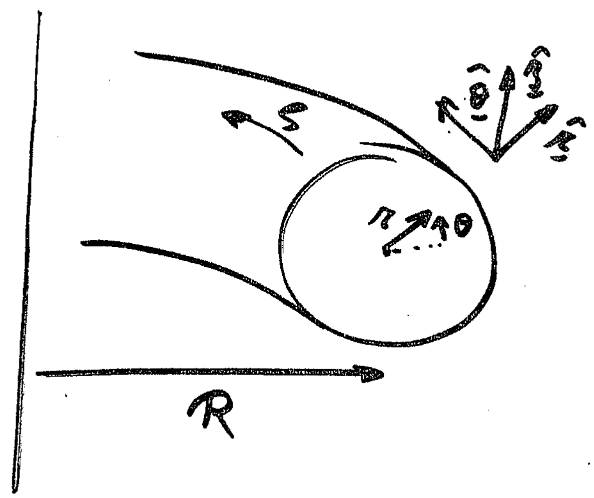
$\Rightarrow$ unstable

(Taylor Hastie Connor ; Chen Cheng)

* delicate structure of eigenmodes
  (shallow potential wells in WKB)
  $\Rightarrow$ susceptible to be modified by low levels of turbulence.

* Questions:
  - Transferts of Energy between modes ~~and effectiveness?~~
  - N.L. Effect on eigenmode structure ?
  - Level of turbulence for N.L stabilization?

Toroidal geometry (circular magnetic surfaces)



$$q(r) = \frac{r}{R}\,\frac{B_\zeta}{B_\theta}$$

$$\underline{B} = B\,\underline{n}$$

$$\underline{n} \simeq \hat{\zeta} + \hat{\theta}\,\frac{r}{Rq}$$

$$\hat{z} = \hat{\theta}\cos\theta + \hat{r}\sin\theta$$

Ballooning formalism

fluctuation = sum of "quasimodes" with eikonal representation

$$f(r,\theta,\zeta) = \sum_{n,m} \hat{f}_{n,m}(\theta, r)\ \exp -in\left[\zeta - q(r)\,\theta + S_m(r)\right]$$

"slow" θ dependence (non periodic in θ) — "global analysis" issue — constant along B-line — eikonal in r

NB: $\hat{f}_{n,m}$ not all independent :

requirements of f | - reality
| - periodicity in θ

$$\left[\text{e.g.}\quad S_m(r) = 2\pi m\, q(r)\right.$$

$$\left.\hat{f}_m(\theta) = \hat{f}_0(\theta - 2\pi m)\quad\right]$$

Define

$$\theta_k(r) \equiv \frac{\frac{dS}{dr}}{\frac{dq}{dr}} = \frac{dS}{dq}$$

$$\hat{s}(r) \equiv \frac{r}{q}\frac{dq}{dr}$$

$$k_\theta \equiv \frac{nq}{r}$$

$$\Rightarrow \nabla_r \phi = \frac{\partial \phi}{\partial r} \longrightarrow \left[\frac{\partial}{\partial r} + i k_\theta \hat{s}(\theta - \theta_k)\right]\hat{\phi}$$

$$\nabla_\theta \phi = \frac{1}{r}\frac{\partial \phi}{\partial \theta} \longrightarrow \left[\frac{1}{r}\frac{\partial}{\partial \theta} + i k_\theta\right]\hat{\phi}$$

$$\nabla_\zeta \phi = \frac{1}{R}\frac{\partial \phi}{\partial \zeta} \longrightarrow -i k_\theta \frac{r}{Rq}\hat{\phi}$$

$$\nabla_\parallel \phi = \underline{n} \cdot \underline{\nabla}\phi \longrightarrow \frac{1}{Rq}\frac{\partial}{\partial \theta}\hat{\phi}$$

$$\nabla_z \phi = \underline{z} \cdot \underline{\nabla}\phi \longrightarrow \approx i k_\theta\left[\cos\theta + \hat{s}(\theta - \theta_k)\sin\theta\right]\hat{\phi}$$

$$\nabla_\perp^2 \phi \longrightarrow \approx -k_\theta^2\left(1 + \hat{s}^2(\theta - \theta_k)^2\right)\hat{\phi}$$

N.L. Equations

Electrostatic $\underline{E} = -\underline{\nabla}\phi$

Quasi neutrality $n_e = n_i$ $(q_e = -q_i = -e)$

Electrons : linear response

$$\tilde{n}_e = n_0 (1 + \alpha) \frac{e}{T_e} \tilde{\Phi}$$

1 — adiabatic response; α — non adiabatic part (includes sources ...)

Ions : N.L. gyrokinetic equation $k_\perp \rho_i \sim 1$; $\omega \ll \Omega_i$

$$\tilde{f}_i = -\frac{q_i}{T_i} \tilde{\Phi} F_0 + \tilde{h}_i (v_\perp, v_\parallel, t, \underline{x})$$

adiabatic response; F_0 — Maxwellian; $\tilde{h}_i$ — non adiab. response; $\underline{x}$ — guiding center coordinate

$$\frac{\partial h_i}{\partial t} + v_\parallel \underline{n} \cdot \underline{\nabla} h_i - \frac{q_i}{T_i} \frac{\partial}{\partial t} \langle \Phi \rangle_\varphi F_0$$

$\langle\ \rangle_\varphi$ — gyroangle average (FLR)

$$+ \underline{v}_{D_i} \cdot \underline{\nabla} h_i + \frac{q_i}{T_i} \underline{v}_{*i} \cdot \underline{\nabla} \langle \Phi \rangle_\varphi F_0$$

magnetic drift; diamagnetic drifts

$$+ \langle \underline{v}_E \rangle_\varphi \cdot \underline{\nabla} \tilde{h}_i = 0$$

NL term

with $\underline{v}_{*i} = \frac{cT_i}{q_i B} \frac{1}{L_n} \hat{\underline{\theta}}$ $\quad L_T^{-1} = 0$

$\underline{v}_{D_i} = \frac{cT_i}{q_i B} \frac{m_i}{T_i} \left(\frac{1}{2} v_\perp^2 + v_\parallel^2\right) \frac{1}{R} \hat{\underline{z}}$ $\quad \hat{\underline{z}} = \hat{\underline{\theta}} \cos\theta + \hat{\underline{r}} \sin\theta$

Linearized equation in ballooning

$$\underbrace{\left[-i\omega + \frac{v_\parallel}{Rq}\frac{\partial}{\partial\theta} + i k_\theta v_{*i} \frac{L_n}{R}\frac{m_i}{T_i}\left(\tfrac{1}{2}v_\perp^2 + v_\parallel^2\right)\left(\cos\theta + \hat{s}(\theta-\theta_k)\sin\theta\right)\right]}_{\text{Linear particle propagator } \hat{G}^{-1}_{k_\theta \theta_k}} \hat{h}$$

$$= -i\left(\omega - k_\theta v_{*i}\right) J_0\left(\frac{v_\perp}{\Omega_i}k_\perp(\theta)\right) F_0(v) \frac{q_i}{T_i}\hat{\phi}(\theta)$$

F.L.R; Maxwellian

solution ⇒ TI modes

Non linear terms double sum of terms, such as

$$\langle \hat{\underline{v}}_{E\times B}\rangle''_{\varphi} \cdot \underline{\nabla}\hat{h}'$$

$$= -\frac{c}{B} k_\theta' k_\theta'' \hat{s}\left(\theta_k'' - \theta_k'\right) J_0\left(\frac{v_\perp}{\Omega_i}k_\perp''(\theta)\right)\hat{\phi}''\hat{h}'$$

$$+ \cdots$$

with eikonal such that

$$\left|\begin{array}{l} k_\theta = k_\theta' + k_\theta'' \qquad (\text{i.e. } n = n' + n'') \\ n\theta_k = n'\theta_k' + n''\theta_k'' \end{array}\right.$$

N.B.: vanishes if $\theta_k' = \theta_k''$, i.e. local $\underline{k}_\perp' \parallel \underline{k}_\perp''$

(Ref Chen)

Renormalized weak turbulence theory

Test mode in background similar modes (TI, SL, ...)

Compton scattering contribution (ε^3 in WTT)

ions resonate with beat fluctuation

$\Rightarrow$ mediates transfer of energy.

One gets (WTT or DIA)

a non linearly induced ion density fluctuation

(k_θ, θ_k fixed)

$$\frac{\hat{n}_i^{nl}}{n_0}(\theta) = \frac{q_i}{T_e}\int d\theta' \left\{ \left(\frac{1}{-i\omega}\right) \int dv \, F_0 \sum_{\substack{k_\theta' \\ \theta_k'}} \right.$$

non resonant $G \sim \frac{1}{-i\omega}$ ($\omega \gg \omega_{bi}, k_\parallel v_i$)

$$\left(\frac{c}{B}\right)^2 k_\theta^2 k_\theta'^2 s^2 \left(\theta_k - \theta_k'\right)^2 \; G_{\substack{k_\theta'' \\ \theta_k''}} (\theta, \theta')$$

resonant G : ion – beat fluctuation

$$J_0\left(\frac{v_\perp}{\Omega} k_\perp'(\theta)\right) J_0\left(\frac{v_\perp}{\Omega} k_\perp'(\theta')\right) J_0\left(\frac{v_\perp}{\Omega} k_\perp(\theta)\right) J_0\left(\frac{v_\perp}{\Omega} k_\perp(\theta')\right)$$

FLR

$$\left. \left(\frac{\omega_{*e}}{\omega} - \frac{\omega_{*e}'}{\omega'}\right) \langle \hat{\Phi}_{k_\theta'}(\theta') \, \hat{\Phi}^*_{k_\theta'}(\theta) \rangle \right\} \hat{\Phi}_{k_\theta}(\theta')$$

non resonant G

linear electrons + quasi-neutrality : $n_e = \hat{n}_i^{\,lin} + \hat{n}_i^{\,non.lin}$ — 8 —

$\Rightarrow$ <u>Non linear equation for $\hat{\Phi}$</u>

Here (simplicity) : $\frac{T_i}{T_e} \sim \frac{1}{4} \ll 1$

$J_0 \simeq 1$

$\theta_K = 0 \pmod{2\pi}$

(N.B. modes $\theta_K = \pi \pmod{2\pi}$: more stable)

$$\frac{\partial^2 \hat{\Phi}(\theta)}{\partial \theta^2} + Q(\theta)\, \hat{\Phi}(\theta) + \Delta Q\, \hat{\Phi} = 0$$

linear potential — non linear part, functional in $\langle \phi^2 \rangle$, non local in θ

$$Q(\theta) = -\omega^2 \frac{R^2 q^2}{c_s^2} \Big[(1+\alpha) + \rho_s^2 k_\theta^2 (1 + \hat{s}^2 \theta^2) - \frac{v_* k_\theta}{\omega} - \frac{v_D k_\theta}{\omega} (\cos\theta + \hat{s}\,\theta \sin\theta) \Big]$$

$$\Delta Q\, \hat{\Phi} = -i \frac{R^2 q^2}{c_s^2}\, \omega \int d\theta' \Big\{ \int d\underline{v}\, F_0 \sum_{\substack{k_\theta' \\ m}} \left(\frac{c}{B}\right)^2 k_\theta^2 k_\theta'^2 \hat{s}^2 \left(\frac{\omega_*}{\omega} - \frac{\omega_*'}{\omega''} \right) G_{\substack{k_\theta'' \\ \theta_K''}}(\theta, \theta') \, (2\pi m)^2 \langle \hat{\Phi}_{k_\theta'}(\theta' - 2\pi m)\, \hat{\Phi}^*_{k_\theta'}(\theta - 2\pi m) \rangle \Big\} \hat{\Phi}_{k}(\theta')$$

"turbulent collision operator"

$c_s^2 = \frac{T_e}{m_i}$

$v_* = v_{*e}$ $\qquad v_D = 2 \frac{L_n}{R} v_*$

Particle propagator $G(\theta,\theta')$ solution of

$$\left[-i\omega + \frac{v_\parallel}{Rq}\frac{\partial}{\partial\theta} + i\,k_\theta v_{T_i}\frac{L_n}{R}\left(\cos\theta + \hat{s}\,\theta\sin\theta\right)\frac{m_i}{T_i}\left(v_\parallel^2 + \frac{1}{2}v_\perp^2\right)\right]G$$

$\omega_D'(\cos\theta + \hat{s}\,\theta\sin\theta)$

$$= \delta(\theta-\theta')$$

$$G(\theta,\theta') \to 0 \quad \text{if} \left\{ \begin{array}{l} \theta \to \pm\infty \\ \operatorname{Im}\omega > 0 \end{array}\right.$$

$$\Rightarrow G(\theta,\theta') = H\left(\frac{\theta-\theta'}{v_\parallel}\right)\frac{Rq}{v_\parallel}\exp\left\{i\frac{Rq}{v_\parallel}\left\{\omega(\theta-\theta')\right.\right.$$

$$\left.\left. - \omega_D(\underline{v})\left[(\hat{s}+1)(\sin\theta - \sin\theta') - \hat{s}\left(\theta\cos\theta - \theta'\cos\theta'\right)\right]\right\}\right\}$$

complicated!
But:

$$\int d\theta' \int F_0\, d\underline{v} \quad G_\omega(\theta,\theta') \quad \langle |\phi|^2(\theta')\rangle$$

$$\simeq R(\omega) \quad \langle |\phi|^2(\theta)\rangle$$

resonance function

$\operatorname{Re}(R(\omega))$: (peaked at $\omega \simeq 0$, some width $\Delta\omega$

$$\left(\text{i.e. } R \simeq \frac{1}{-i\omega + \Delta\omega}\right)$$

The width $\Delta\omega$ is a sum of Doppler broadenings.

$$\Delta\omega = \Delta\omega_D + \Delta\omega_{\parallel} + \Delta\omega_{N.L.}$$

$\Delta\omega_D \downarrow v_{D_i}$, $\Delta\omega_{\parallel} \downarrow v_{\parallel_i}$, $\Delta\omega_{N.L.} \downarrow \underline{v}_{E\times B}$ (renormalization)

$$\Delta\omega_D \approx \omega_D(v_{Ti}) \; \Delta\left(\cos\theta + \hat{s}\,\theta\sin\theta\right)$$

average over mode structure

$$\Delta\omega_{\parallel} \approx \frac{v_i}{Rq}\,\frac{1}{\text{mode width in }\theta}$$

$$\Delta\omega_{NL} = \text{N.L. broadening}$$

<u>Typically</u>:

T I mode: $\Delta\omega_D \sim \Delta\omega_{\parallel} \sim \Delta\omega_{NL} \sim \left(\frac{T_i}{T_e}\right)^{1/2} \frac{L_n}{R}\,\omega_{*e}$

small

S.L. $\Delta\omega_{\parallel_{SL}} \sim \frac{v_i}{L_s} \gg \Delta\omega_{\parallel_{TI}} \sim \frac{v_i}{Rq}$

With propagator approximation : $\Delta Q(\theta,\theta')$ local in θ

$$\Delta Q(\theta) \simeq -i \frac{R^2 q^2}{S^2} v_* \, \omega \left(\frac{c}{B}\right)^2 \dots^2 \sum_{k_\theta'} k_\theta^2 k_\theta'^2$$

$$\left(\frac{k_\theta}{\omega} - \frac{k_\theta'}{\omega'}\right) R_{k_\theta'}(\omega - \omega')$$

$$\sum_{m=-\infty}^{\infty} (2\pi m)^2 \left< |\phi|^2_{k_\theta'}(\theta - 2\pi m) \right>$$

Remarks :

- interaction local in ω , i.e. $|\omega - \omega'| \lesssim \Delta\omega$
- θ dependence of ΔQ $\leftarrow$ θ dependence of $|\phi|^2_{k_\theta'}$
- $(2\pi)^2 \sim 40$: large factor
- if $|\phi|^2_{k_\theta'}$ localized within 2π : $m = \pm 1$ dominate
- sign of $\left(\frac{k_\theta}{\omega} - \frac{k_\theta'}{\omega'}\right)$ determines direction of energy transfer : from large k to small k.

Non linear analysis

must know all possible modes of fluctuation

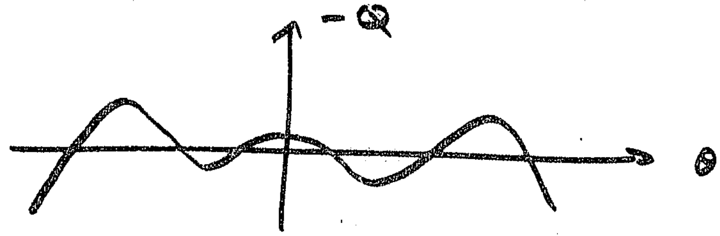
Ex: strong shear $\hat{s} = 1$ $q = 1$

$\varepsilon = .2 = \frac{L_n}{R}$

linear modes:

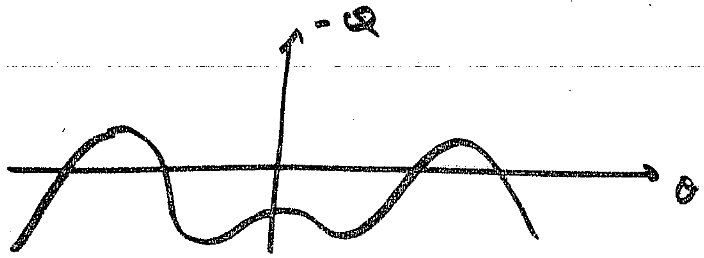
WTI:

destabilized by e. excitation



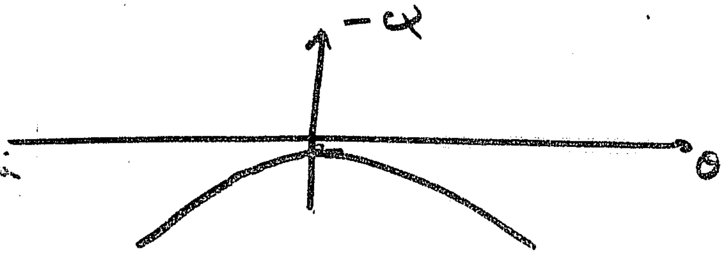
STI:

mostly stable ($\omega \gtrsim \omega_*$)



SL

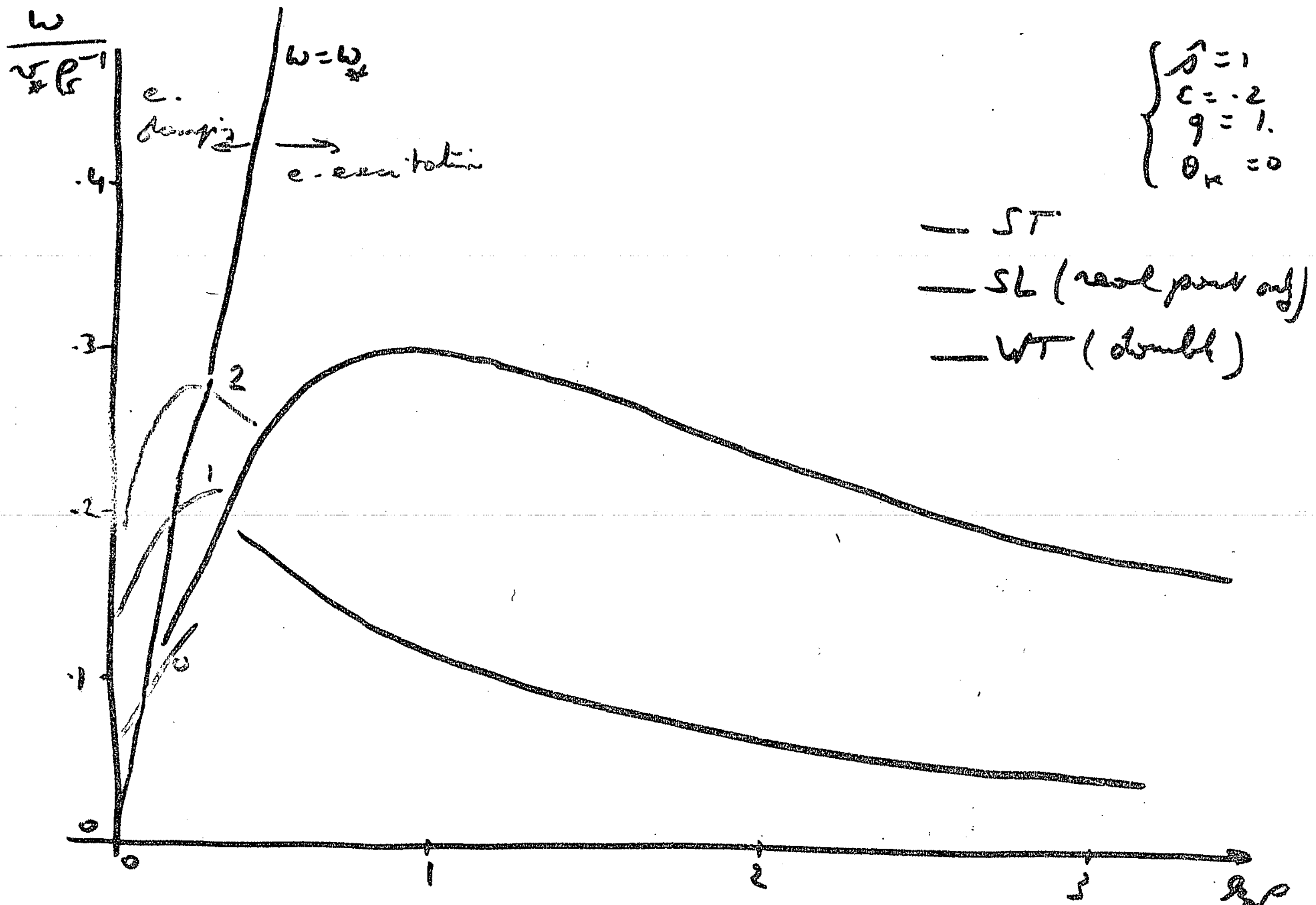
shear stabilization $\gamma \sim -\frac{L_n}{R}$

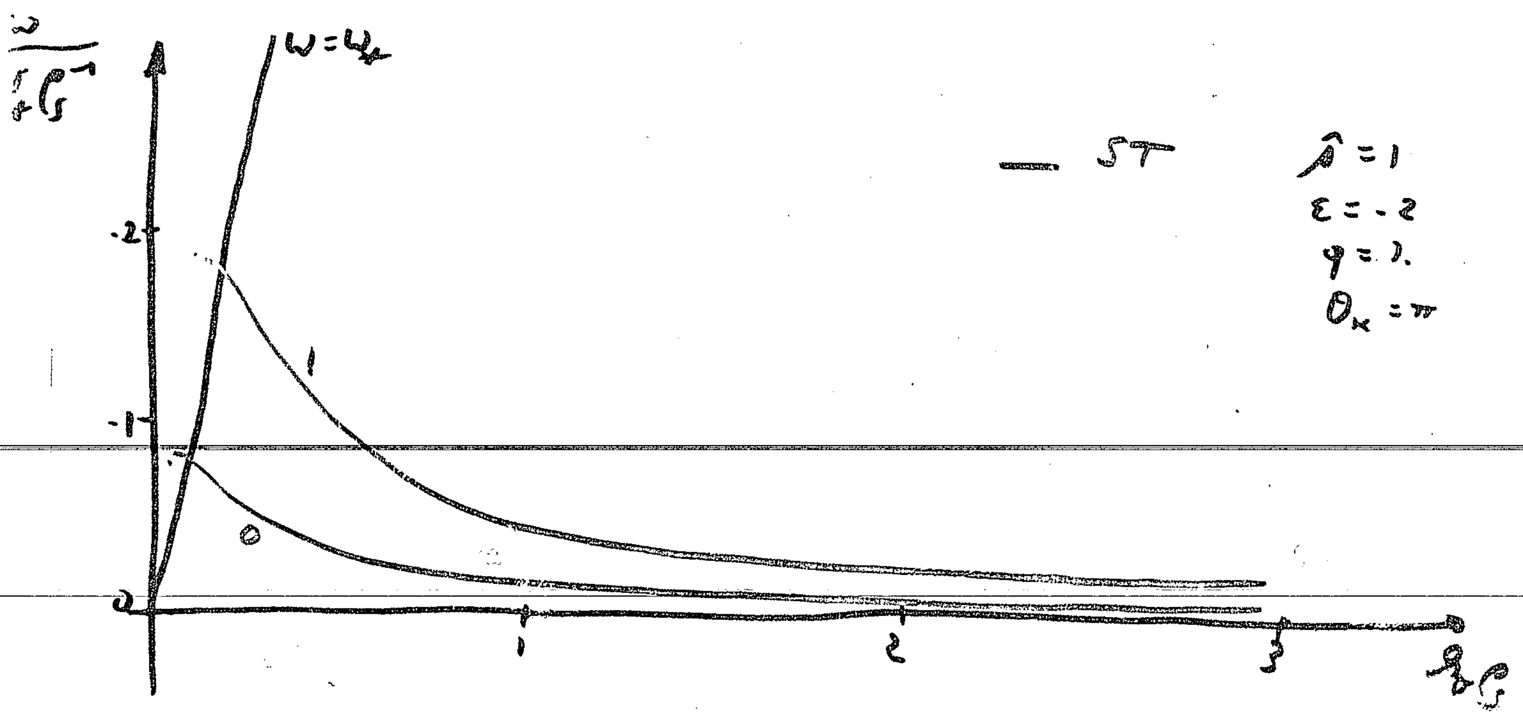


modes localized at $\theta_K = \pi$

more stable (favorable curvature region)

Linear dispersion relation (WKB)





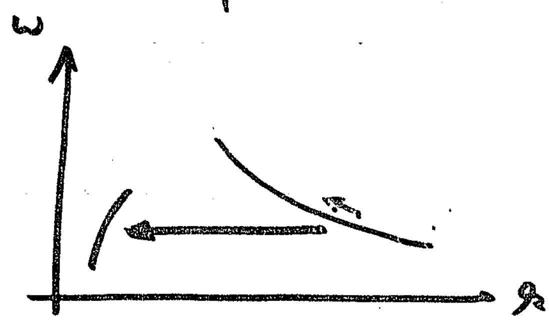
Possible coupling

when $|\omega - \omega'| < \Delta\omega$

WTI branch, driven by e

can couple

- to itself (close interaction) (inverse "cascade")
- to S.L.
- to (damped) STI (distant interaction)



Energy conservation

- Exact equation: N.L. terms conserve energy
  $\underline{v}_{E\times B} \cdot \underline{E} = 0$
- Renormalized equation for ϕ : idem (symmetric)
- sign of coupling coefficient ⇒ transfer from large k to small k

⇒ pure transfer $k \to k'$ if $k > k'$

Solution of equation

$$\frac{\partial^2}{\partial\theta^2}\hat{\phi} + Q\,\hat{\phi} + \Delta Q\,\hat{\phi} = 0$$

A: Perturbative solution

$$\Rightarrow \operatorname{Im}(\delta\omega)_{n.l.} = \left[\int d\theta \, \frac{\partial Q_0}{\partial\omega} \langle|\phi|^2(\theta)\rangle\right]^{-1}$$

$$\cdot \left\{\frac{R^2 q^2}{s^2} \, \omega_0 \left(\frac{c}{B}\right)^2 \rho^2 \sum_{k_\theta'} k_\theta^2 k_\theta'^2\right.$$

$$\left(\frac{\omega_*}{\omega} - \frac{\omega_*'}{\omega}\right) \operatorname{Re}\left(R_{k''}(\omega - \omega')\right)$$

$$\sum_m (2\pi m)^2 \int d\theta \, \langle|\phi_{k_\theta'}(\theta - 2\pi m)|^2\rangle \langle|\phi_{k_\theta}(\theta)|^2\rangle$$

$\Rightarrow$ growth rate:

$$\gamma = \gamma^{l}_{lead} + \gamma^{l}_{electron} + \gamma^{n.l.}_{Compton} + \gamma^{n.l.}_{shielding}$$

Find :

Distant interaction

- very efficient transfer
- significant stabilization for low levels of fluctuation (e.g. $\frac{\delta f}{f} \sim \frac{1}{20} \frac{\rho_i}{L_n}$)

Close interaction

- smaller than above

Coupling with S.L

here : heavily damped.

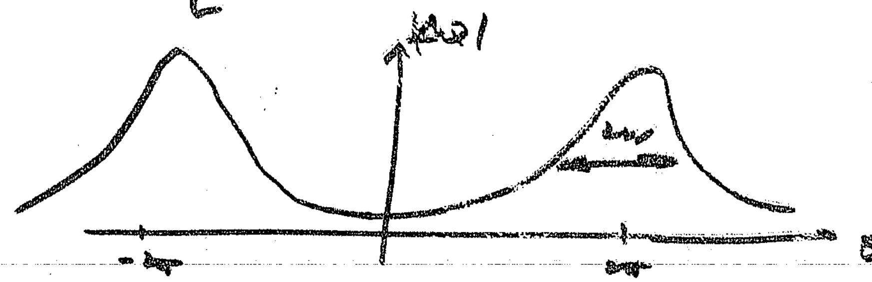
but if $\frac{R}{L_n}$ smaller : may be important

B. Non perturbative solution

Solve $\frac{\partial^2}{\partial\theta^2}\hat{\phi} + (Q + \Delta Q)\hat{\phi} = 0$

with model

$$\Delta Q \simeq i\,\mathcal{E}\left[e^{-\frac{1}{2}\left(\frac{\theta-2\pi}{w}\right)^2} + e^{-\frac{1}{2}\left(\frac{\theta+2\pi}{w}\right)^2}\right]$$



(structure of $\langle|\phi|^2\rangle$, shifted by $\pm 2\pi$)

⇒ distorsion of eigenmode structure

when $|\Delta Q| \gtrsim Q_s$: depth of well

(achieved for low fluctuation level)

NB: importance of $\mathrm{Re}(\Delta Q)$ for eigenmode structure

→ enhancement of convective damping

Conclusions

Results obtained so far show :

- NL interaction of TI modes causes very effective transfer of energy from large ℓ (unstable) to low ℓ (stable) (distant & close interaction)
- modes extended in θ are most affected
- low levels of fluctuation (fraction of $\frac{\rho_s}{L_n}$) appreciably stabilize TI modes and affect their mode structure
- sink of energy :
  electrons (modes $\omega > \omega_*$)
  ions (linear + n.l. landau; coupling to S.L.)
- NB: F.L.R. and non linear scatterings not expected to modify qualitatively the above conclusions (but reduction of flow)

Recent Observations on Microturbulence in PLT

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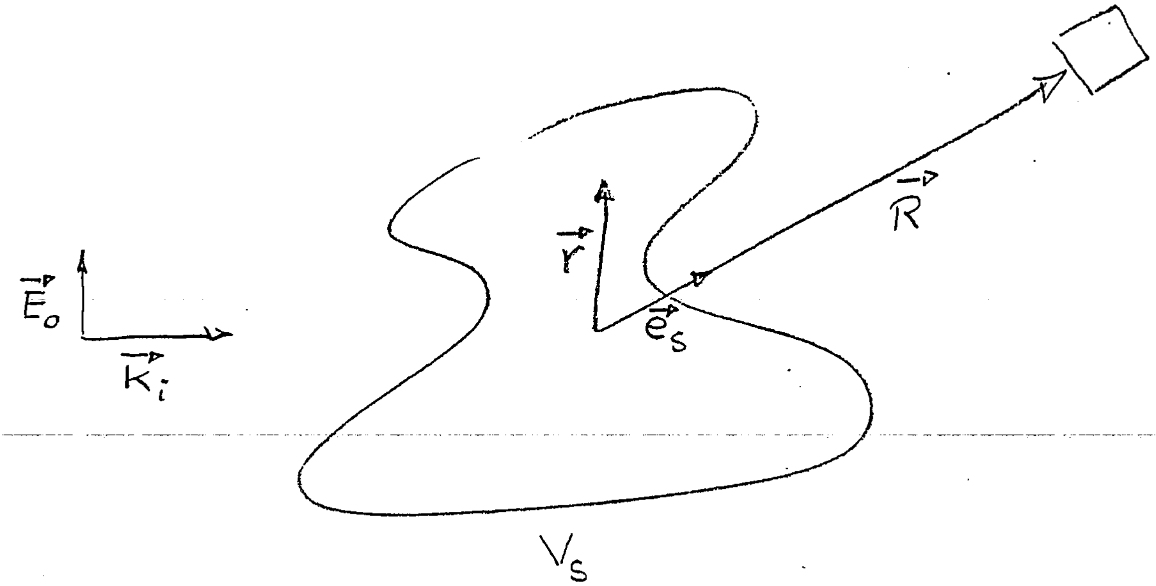
January 11, 1982



$$\vec{E}_i = \vec{E}_o \cos(\omega_i t - \vec{k}_i \cdot \vec{r})$$

$$E_s(\vec{R},t) = -\frac{r_e}{R} E_o \, \mathrm{Re} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \, \hat{n}(\vec{k}_s - \vec{k}_i, \omega) \exp\left(i(\omega_i + \omega)t\right)$$

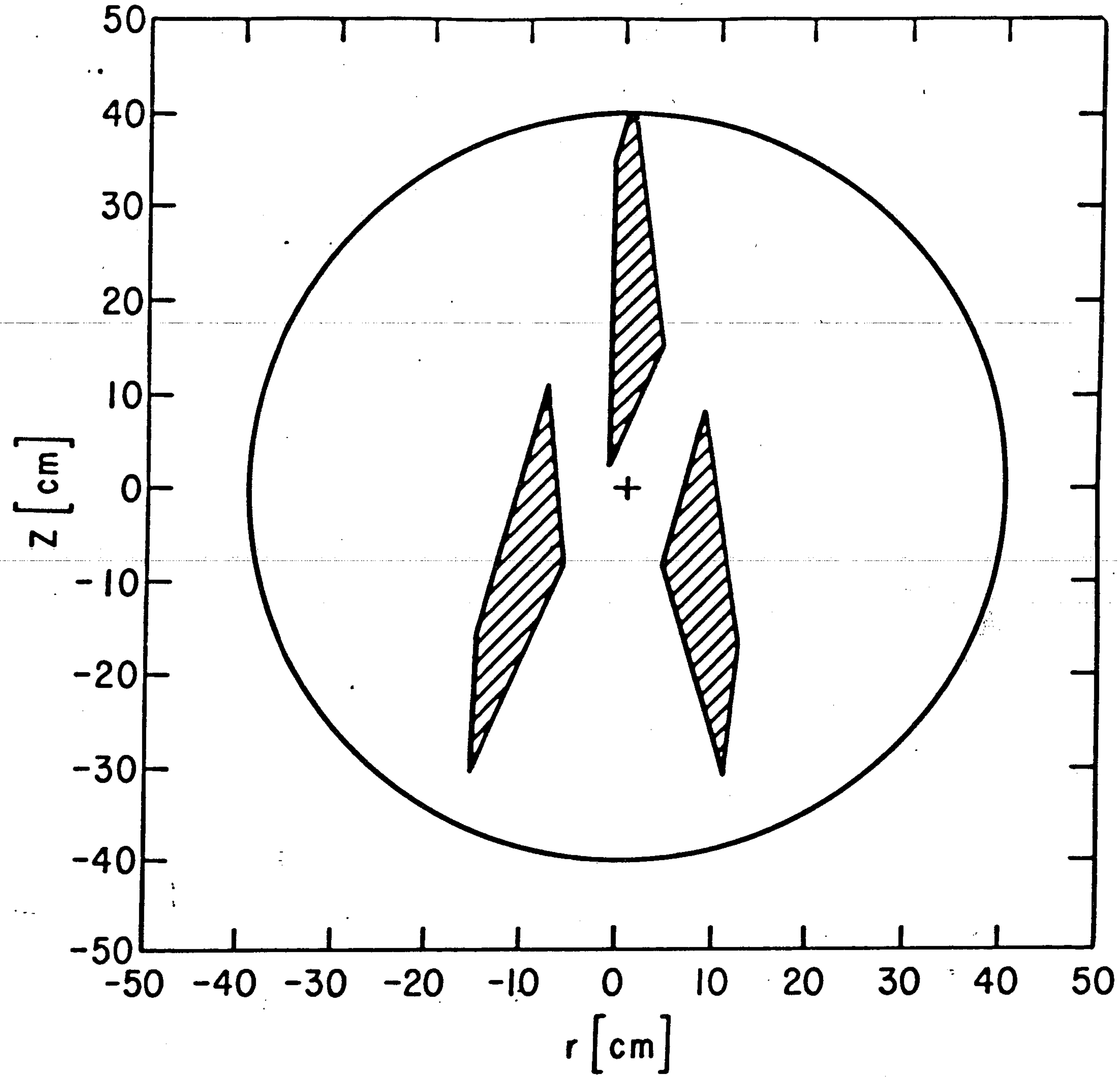
$$\hat{n}(\vec{k},\omega) = \int d\vec{k}' \, n(\vec{k}',\omega) \, K(\vec{k}' - \vec{k})$$

$$K(\vec{k}' - \vec{k}) = \frac{1}{(2\pi)^3} \int_{V_s} d\vec{r} \exp\left(i(\vec{k}' - \vec{k}) \cdot \vec{r}\right)$$

$$\bar{\omega} > \omega_*^e$$

$$k\rho_i \lesssim 1$$

$$|\tilde{n}|/\bar{n} \simeq 1/(kL_n)$$



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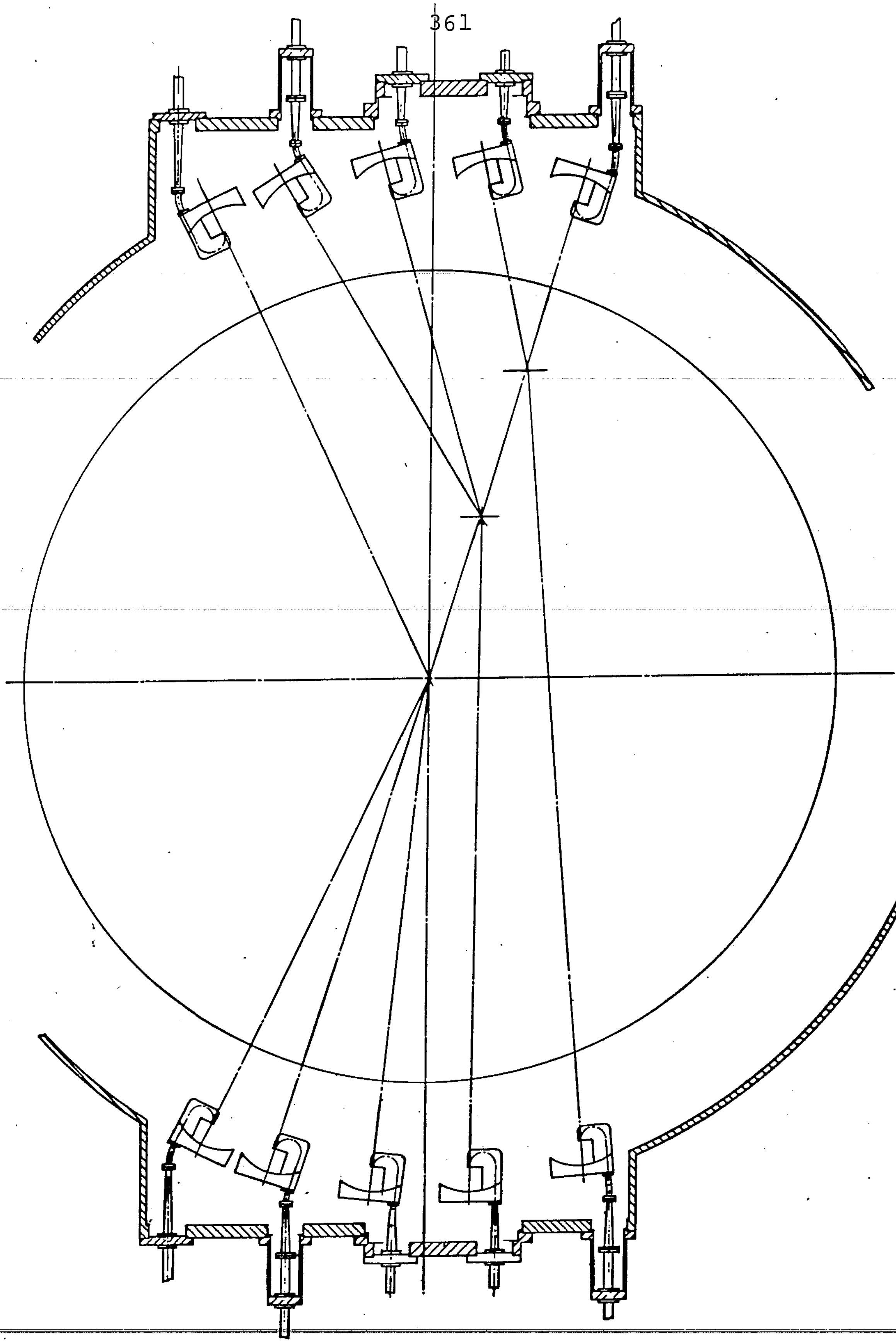

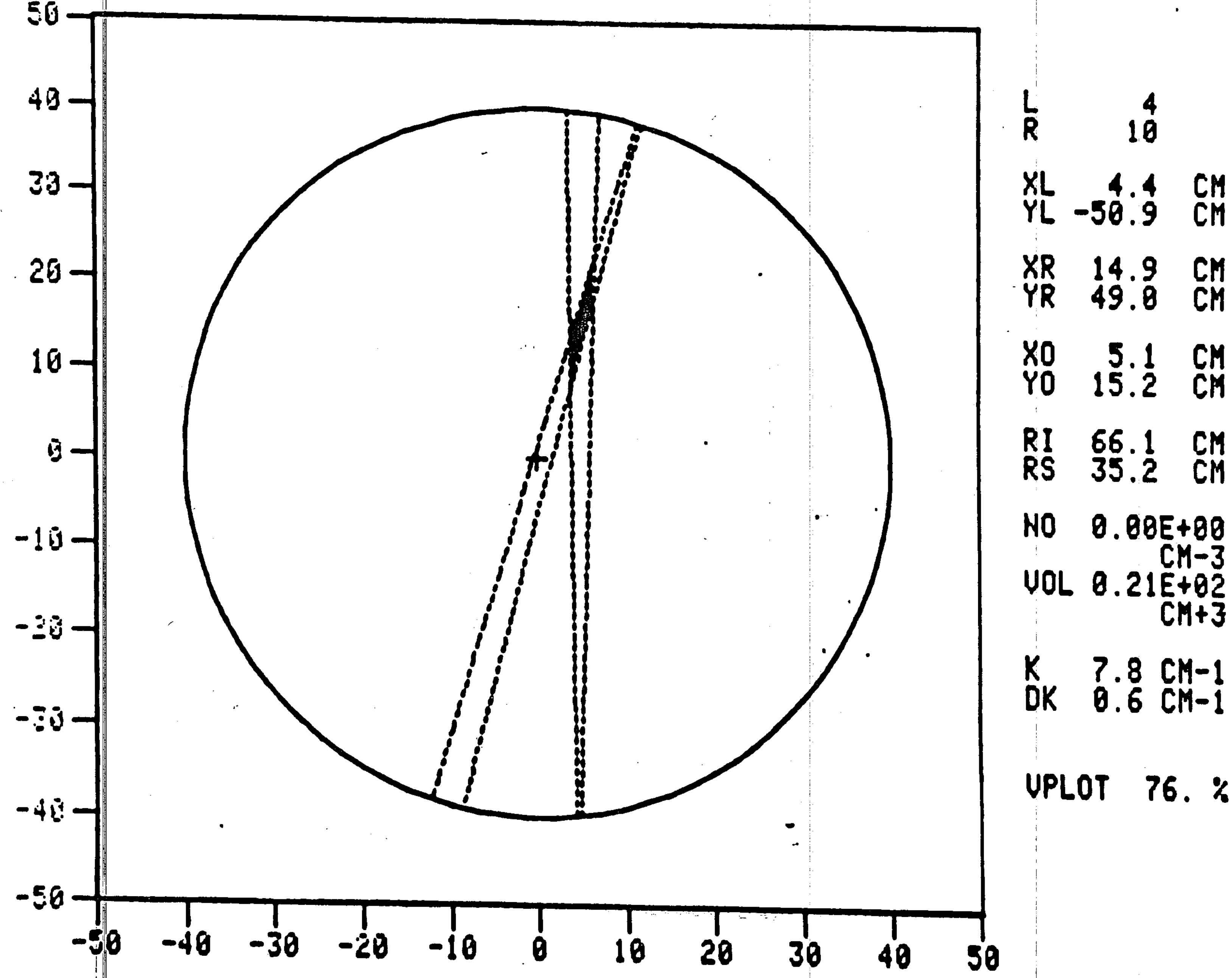
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YL -50.9 CM
XR 14.9 CM
YR 49.0 CM
XO 5.1 CM
YO 15.2 CM
RI 66.1 CM
RS 35.2 CM
NO 0.00E+00 CM-3
VOL 0.21E+02 CM+3
K 7.8 CM-1
DK 0.6 CM-1
VPLOT 76. %

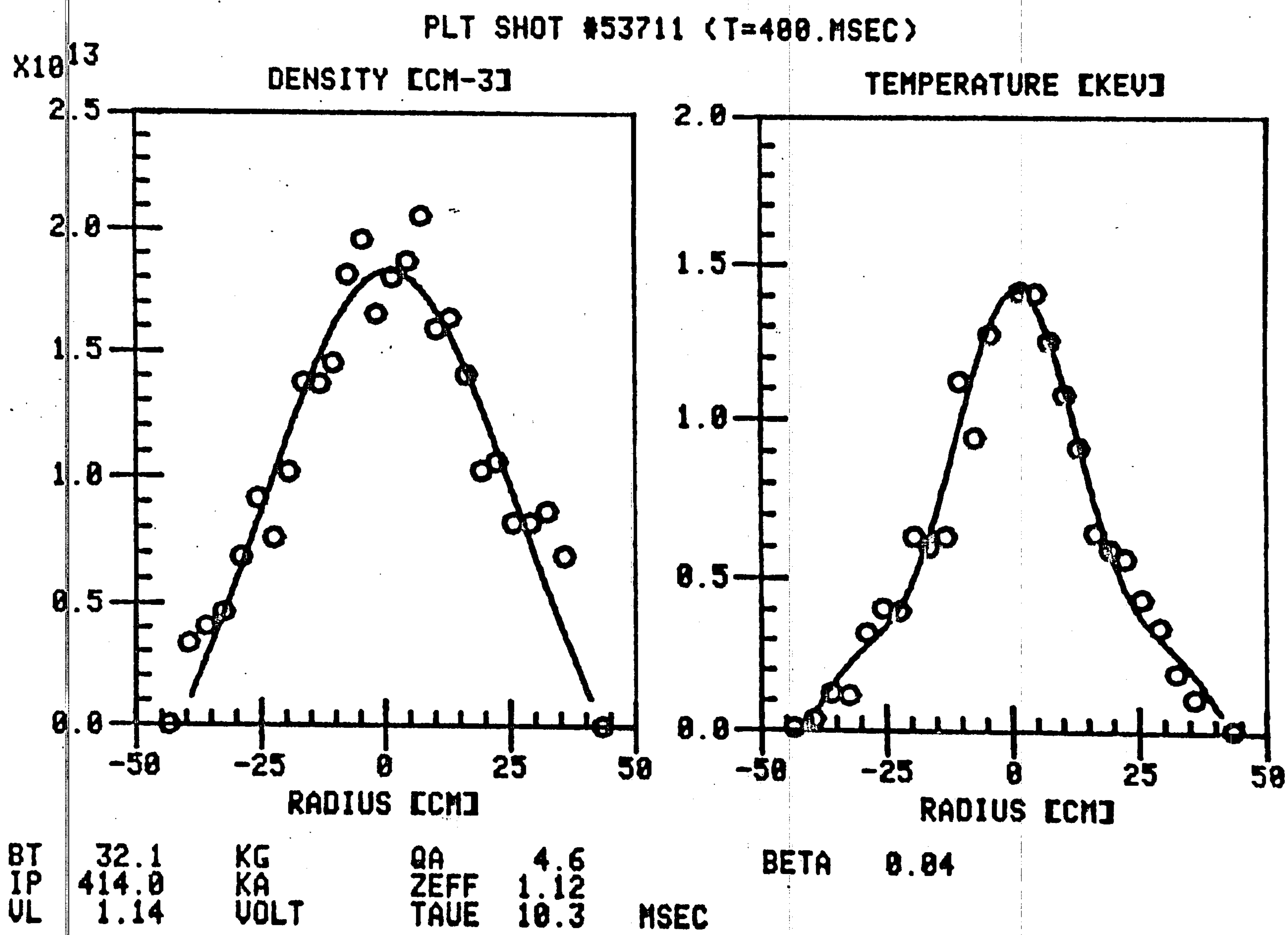


FIG. 7. ELECTRON DENSITY AND TEMPERATURE RADIAL PROFILES.

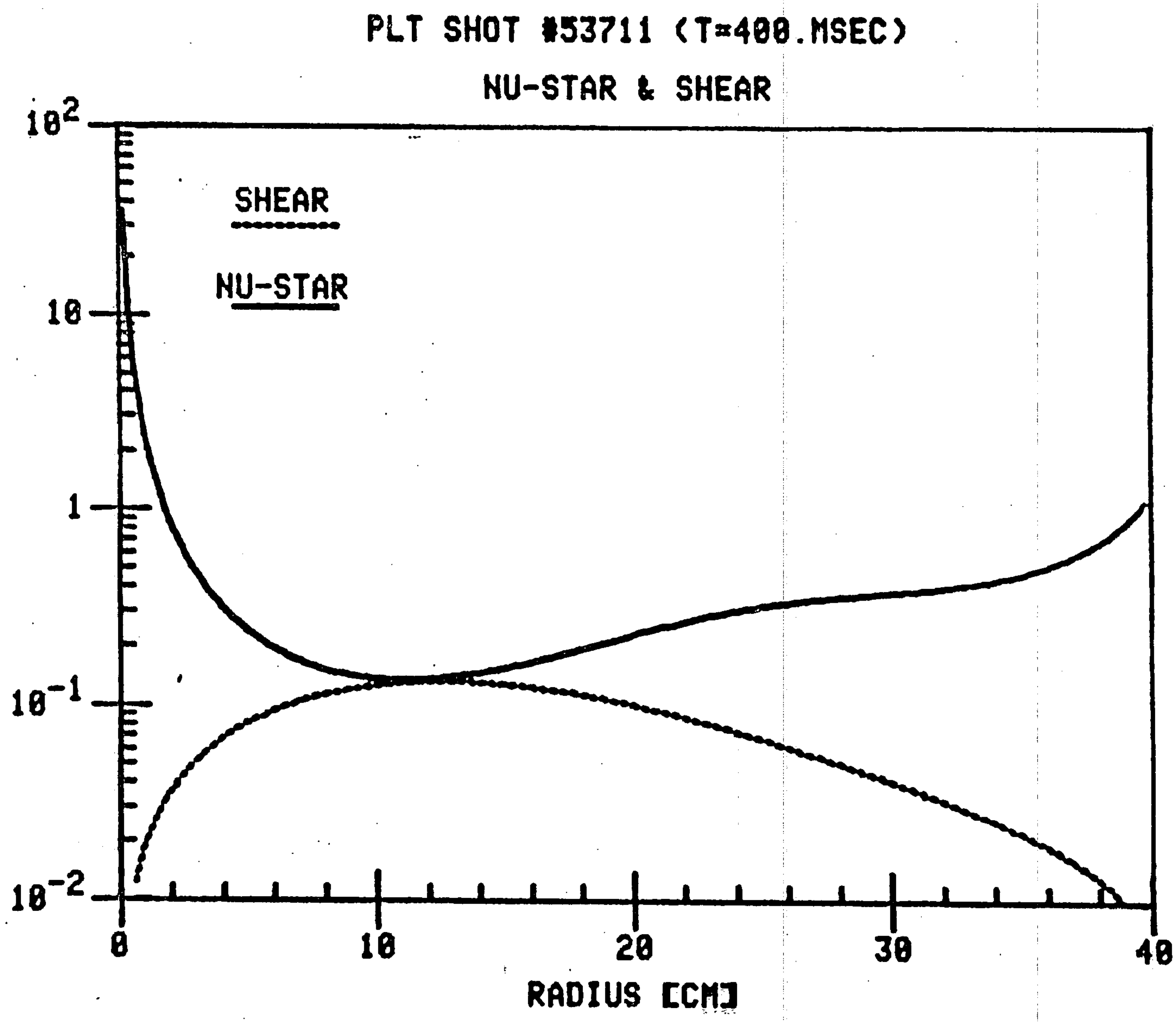


FIG. 8. RADIAL PROFILE OF SHEAR AND ν_*^e.

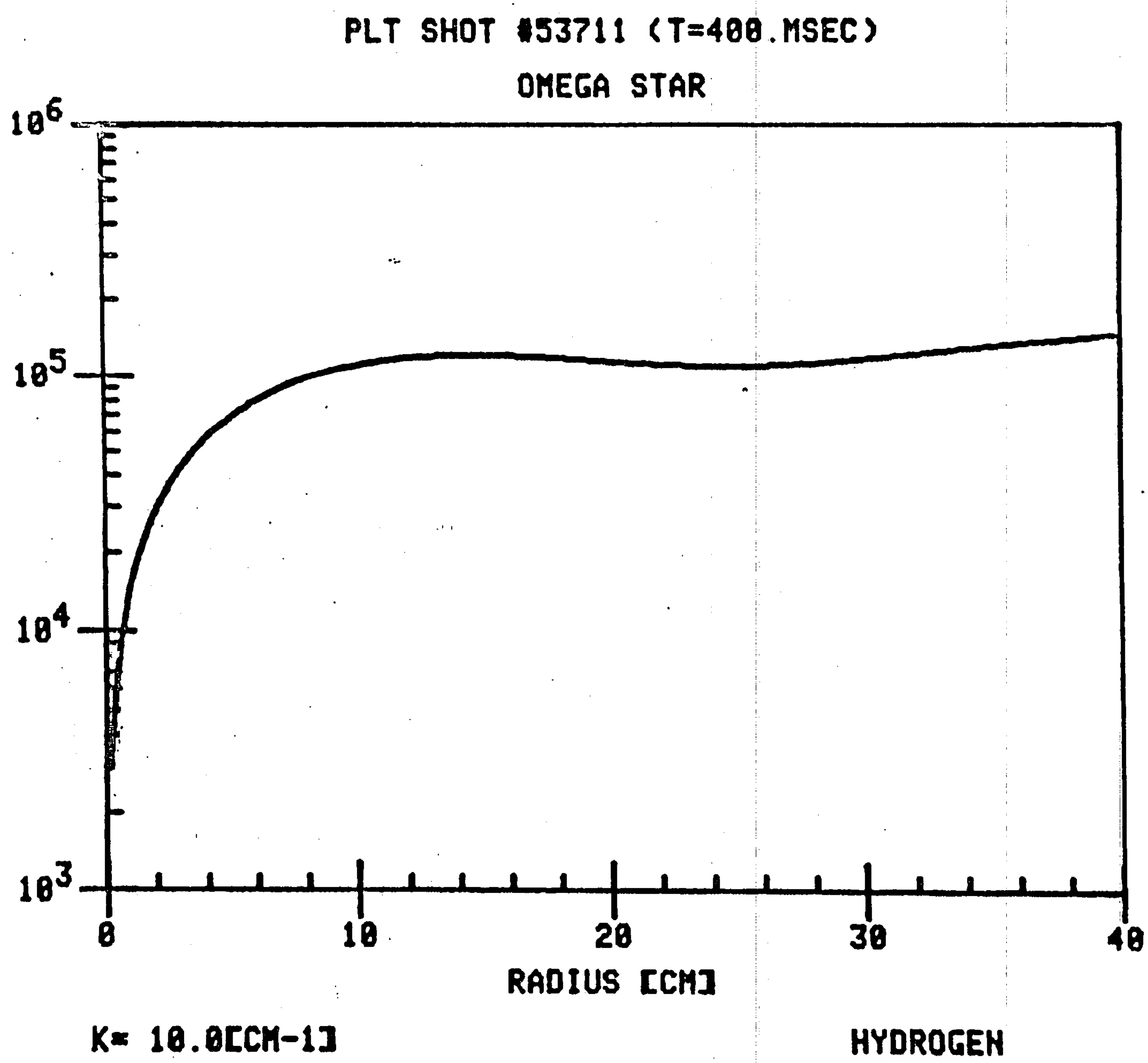


FIG. 9. RADIAL PROFILE OF $\omega_*^e/2\pi$ FOR $k_\perp = 10\,cm^{-1}$.

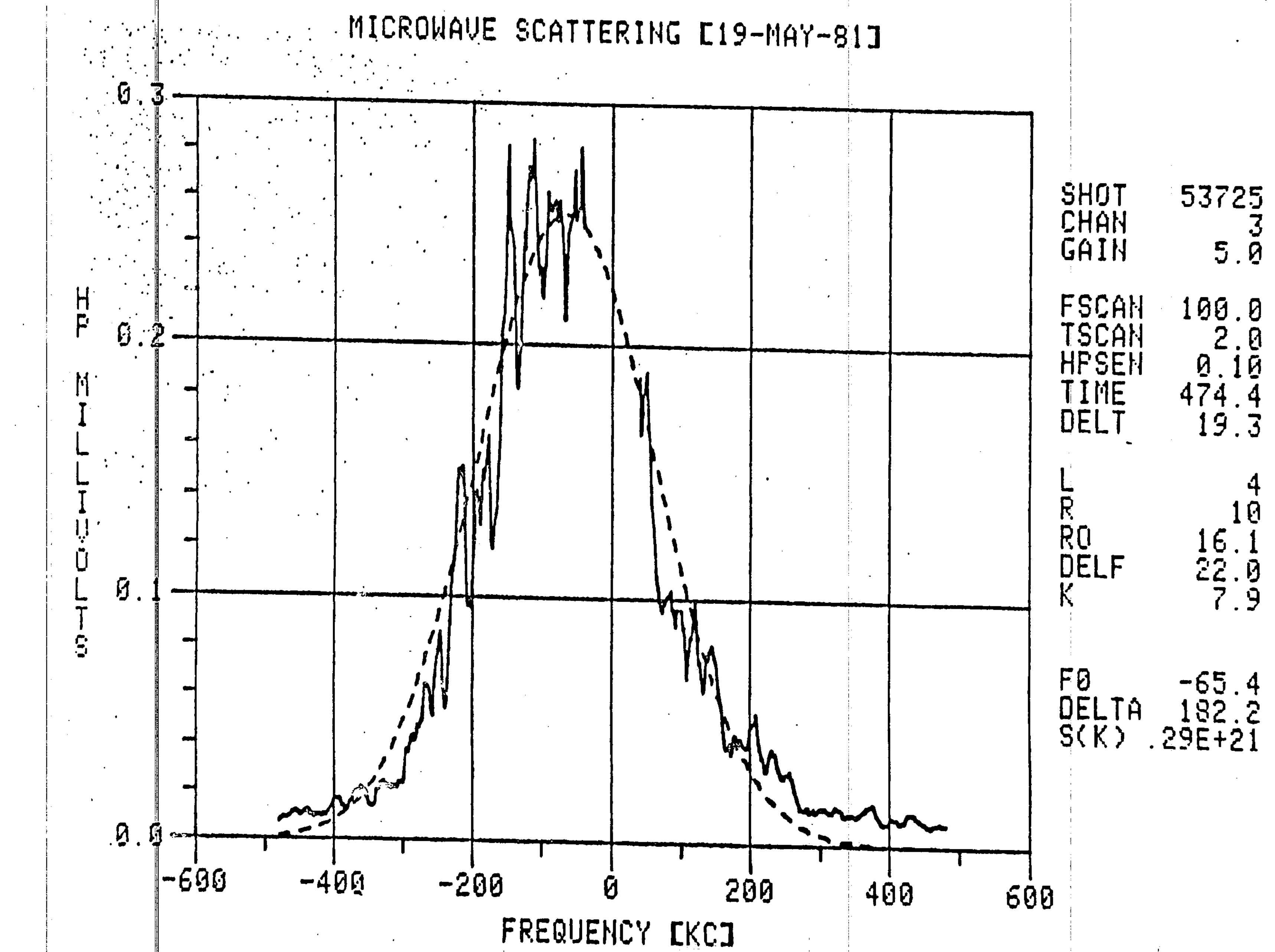
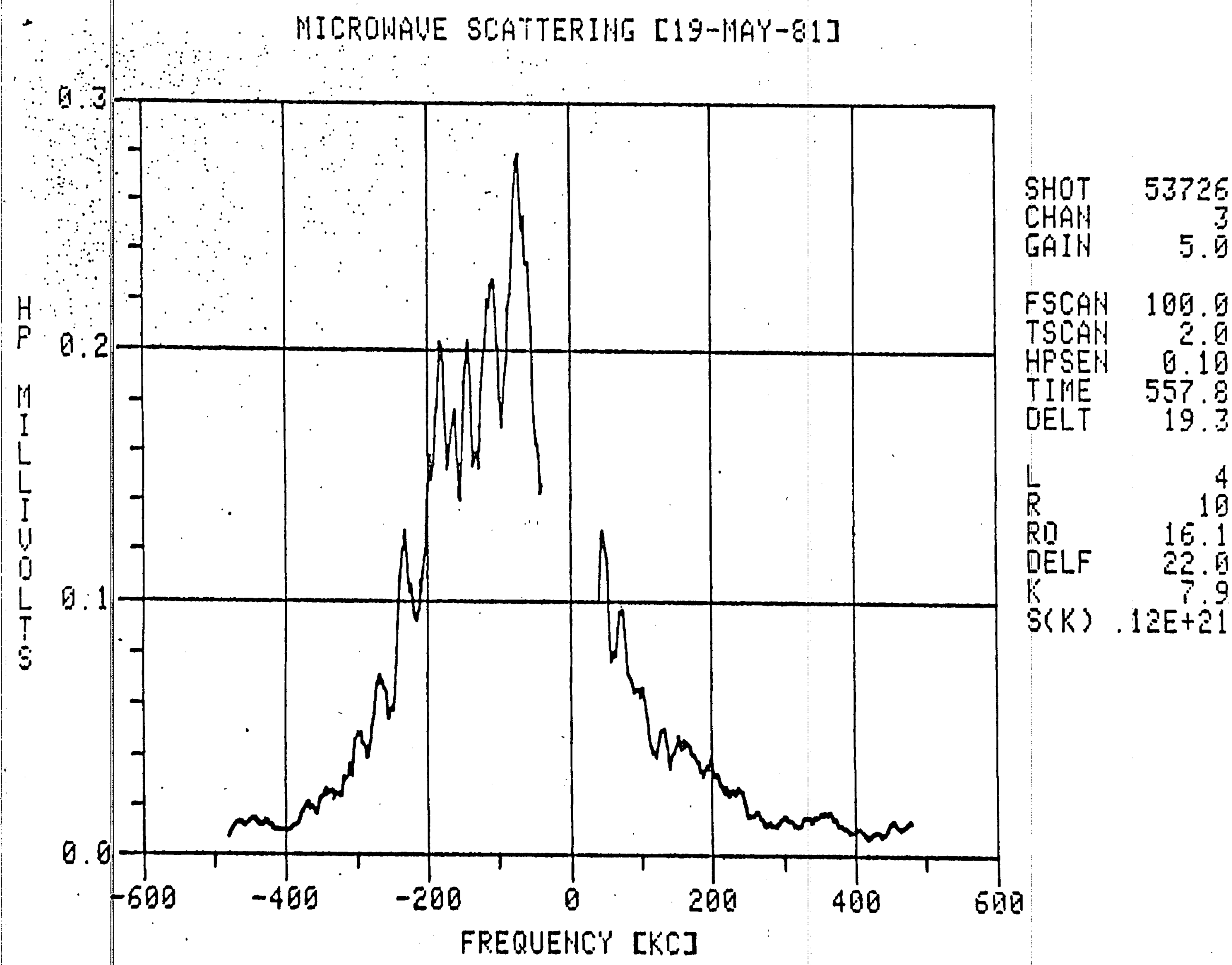
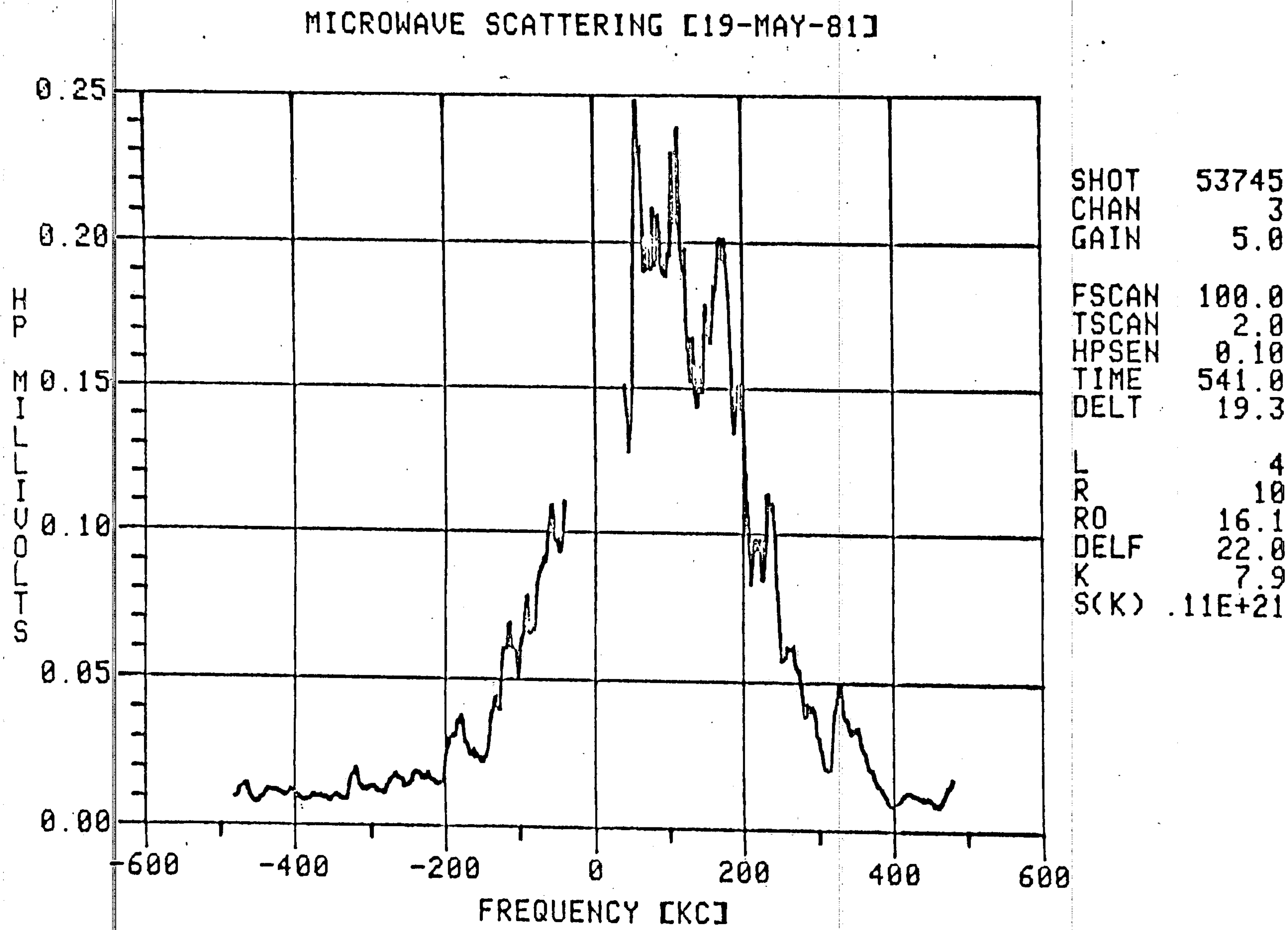


FIG. 11. FREQUENCY SPECTRUM OF FLUCTUATIONS WITH $k = 7\,cm^{-1} \pm 1\,cm^{-1}$ AT $R = 18$ cm $\pm$ 5 cm.



FREQUENCY SPECTRUM FOR K = 7.9 cm^{-1}, B_T = + 32 kG, $\bar{r}$ = 16 cm



Frequency Spectrum for $k = 7.9\ \mathrm{cm}^{-1}$, $B_T = -32$ kG, $\bar{r} = 16$ cm

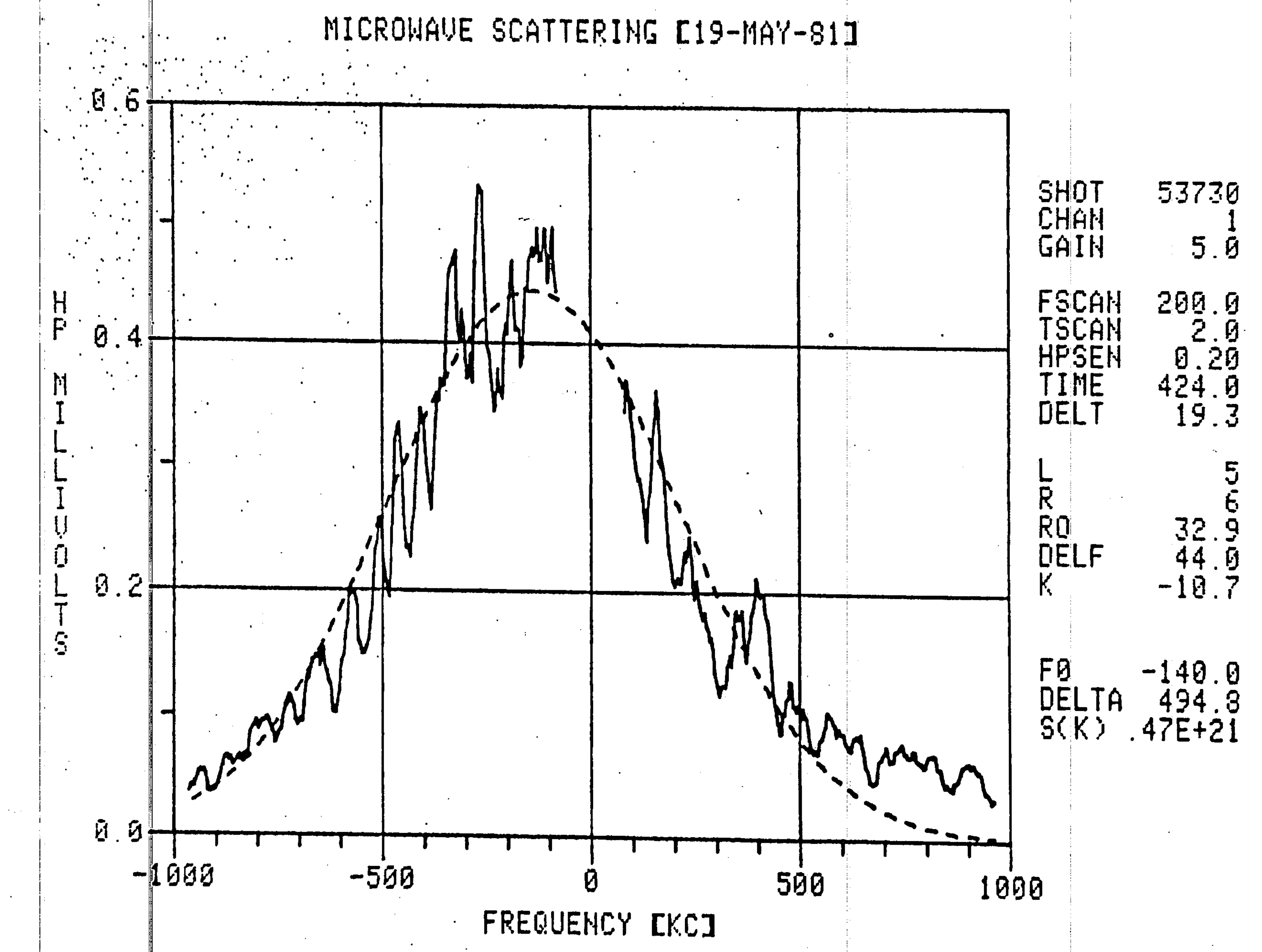


FIG. 12. FREQUENCY SPECTRUM OF FLUCTUATIONS WITH $k = 10.5\ cm^{-1} \pm 1\ cm^{-1}$ AT $R = 33\ cm \pm 4\ cm$.

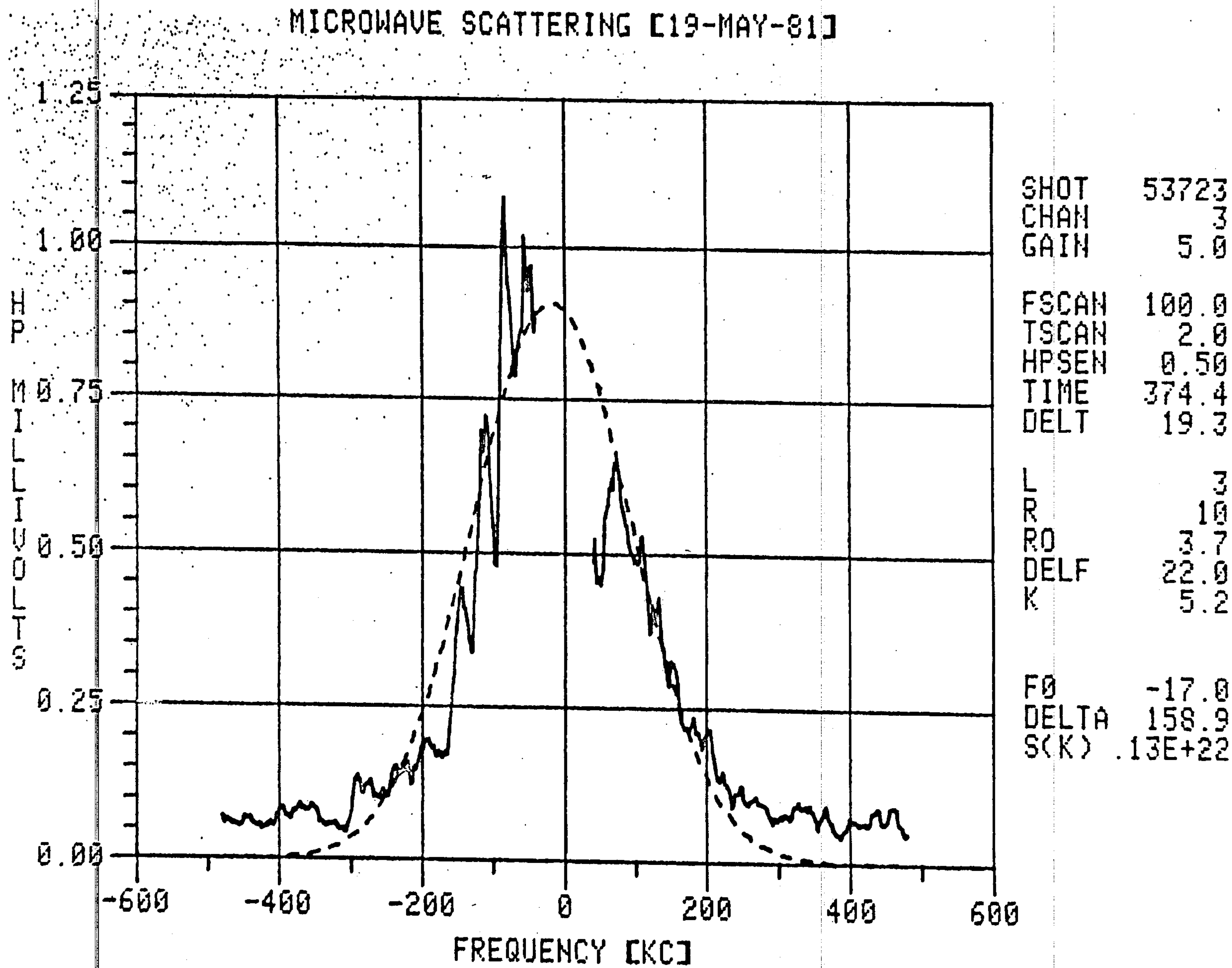


FIG. 10. FREQUENCY SPECTRUM OF FLUCTUATIONS WITH $k = 5\,cm^{-1} \pm 1\,cm^{-1}$ AT $R = 4\,cm \pm 4\,cm$.

#81X1175

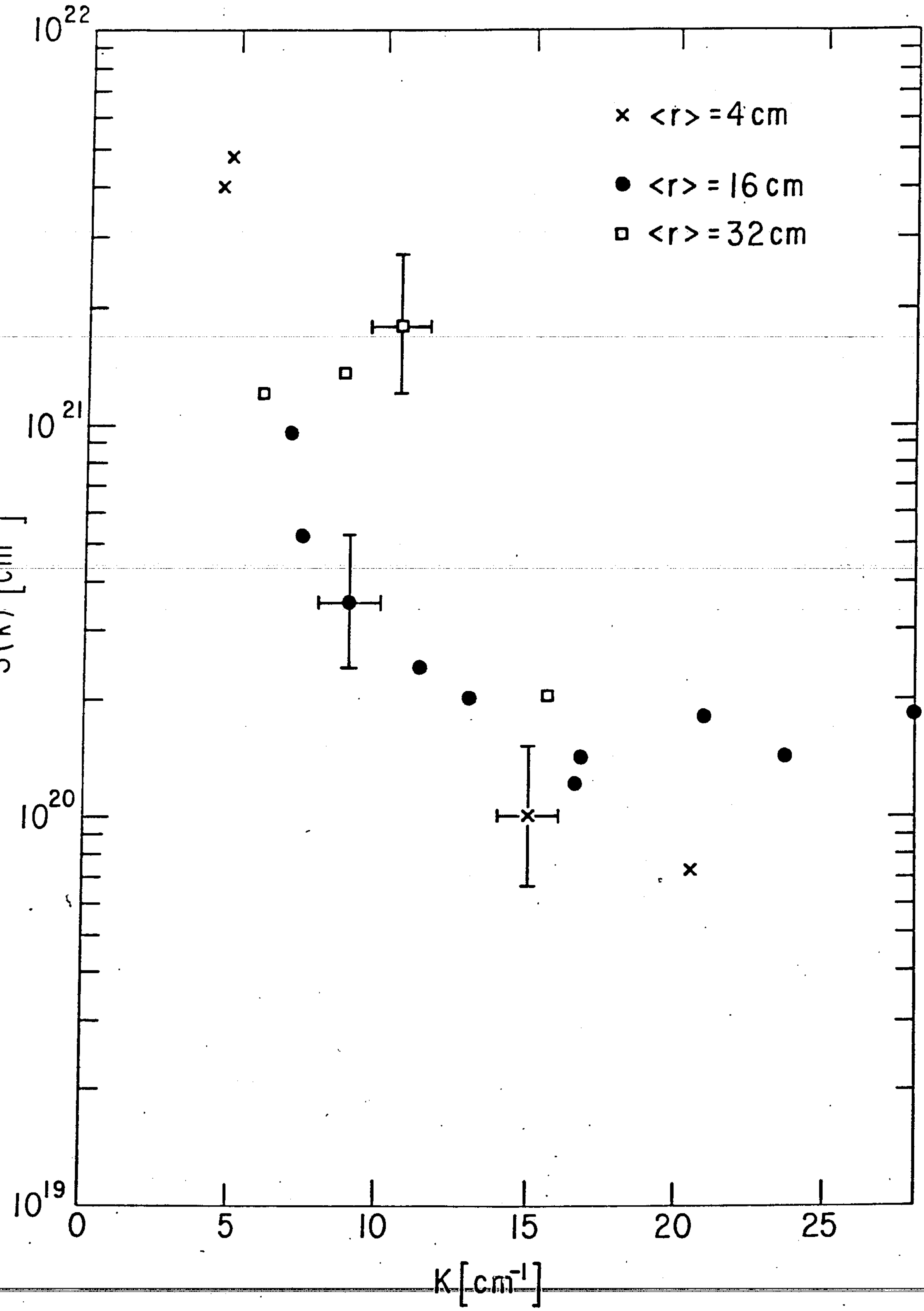


FIG. 13. SPECTRAL POWER DENSITY S(k).

#81X1177

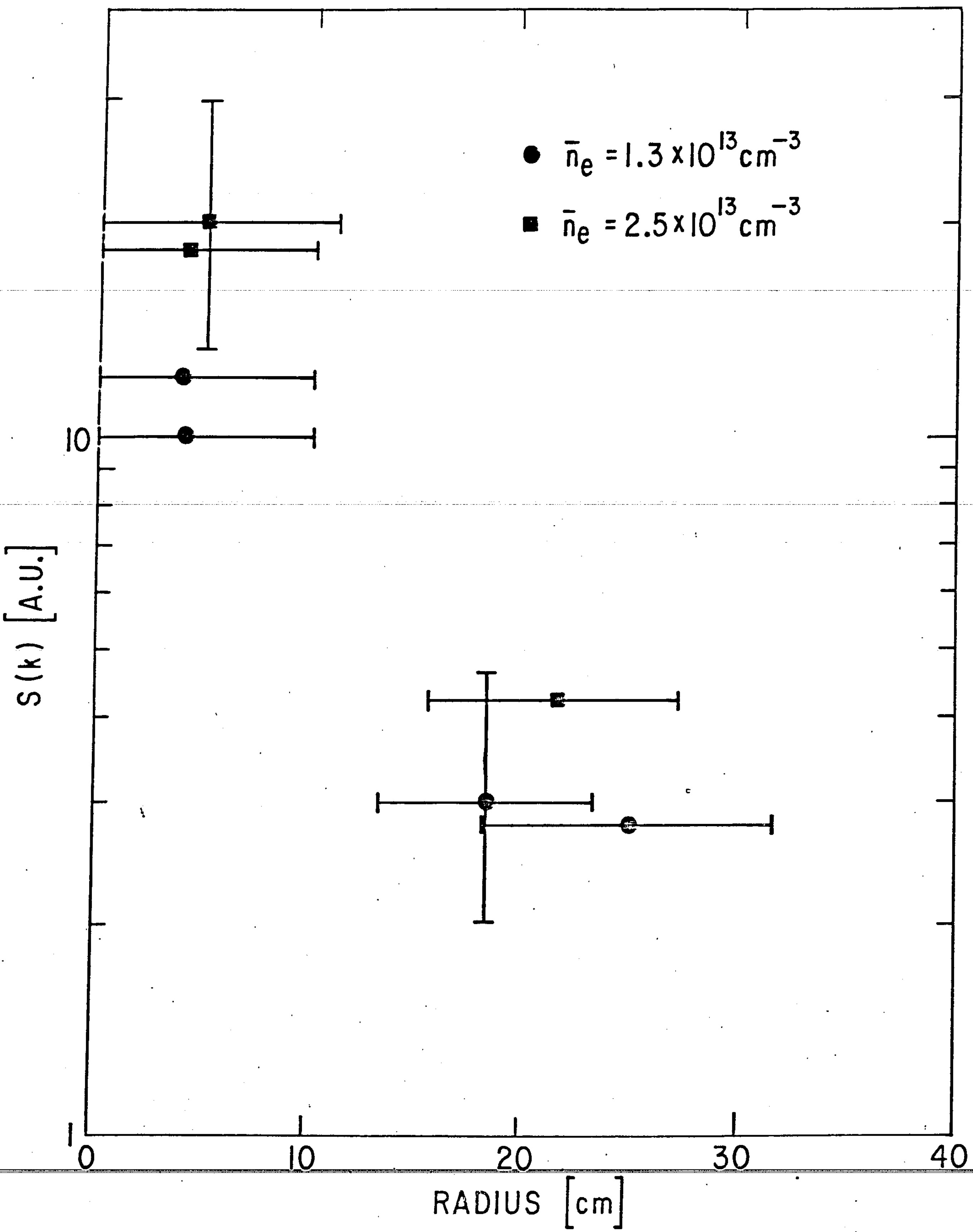


FIG. 14. SPECTRAL POWER DENSITY S(k) FOR $4.5\ cm^{-1} < k < 7\ cm^{-1}$.

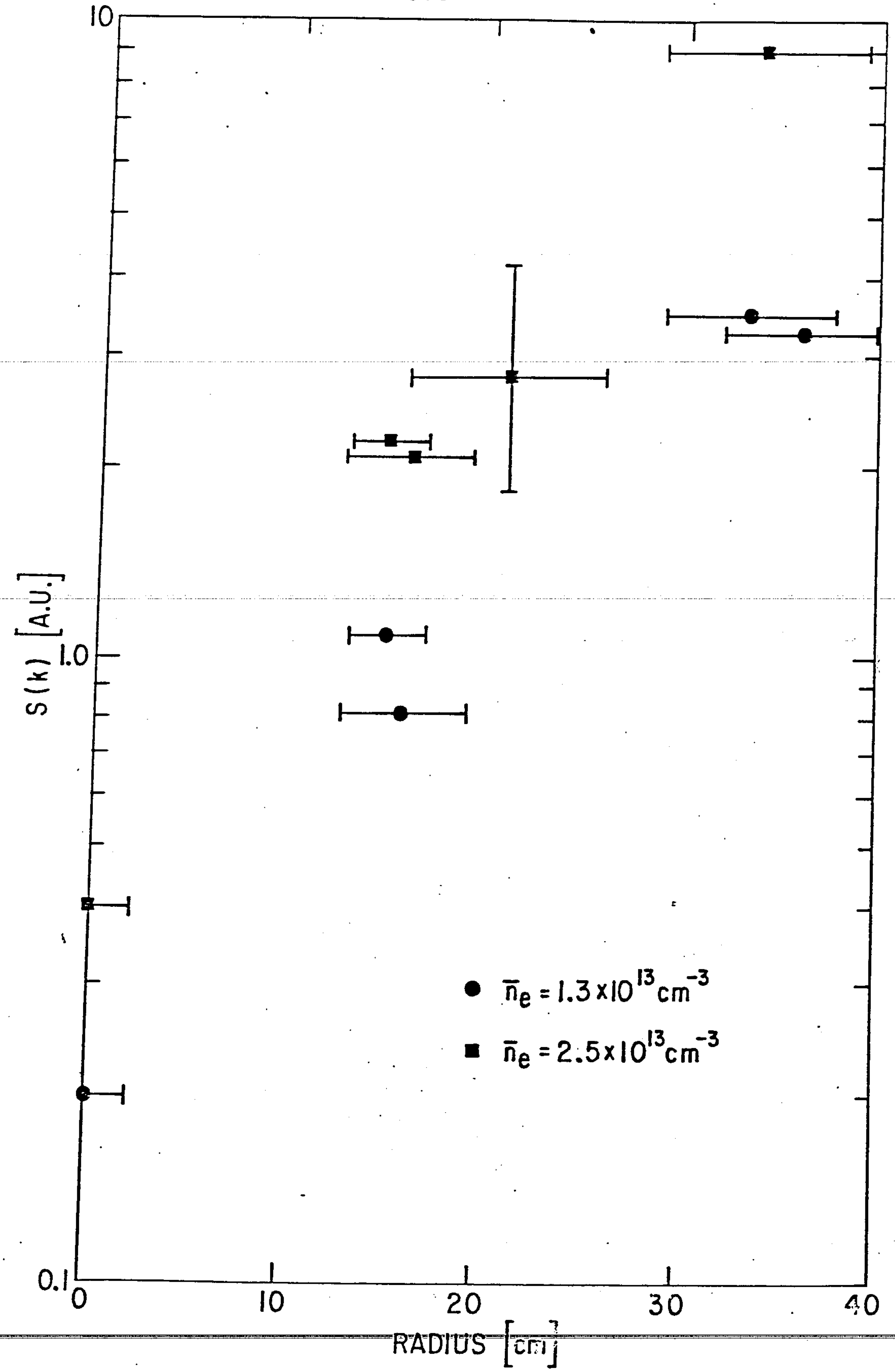


FIG. 15. SPECTRAL POWER DENSITY S(k) FOR $9\ cm^{-1} < k < 12\ cm^{-1}$.

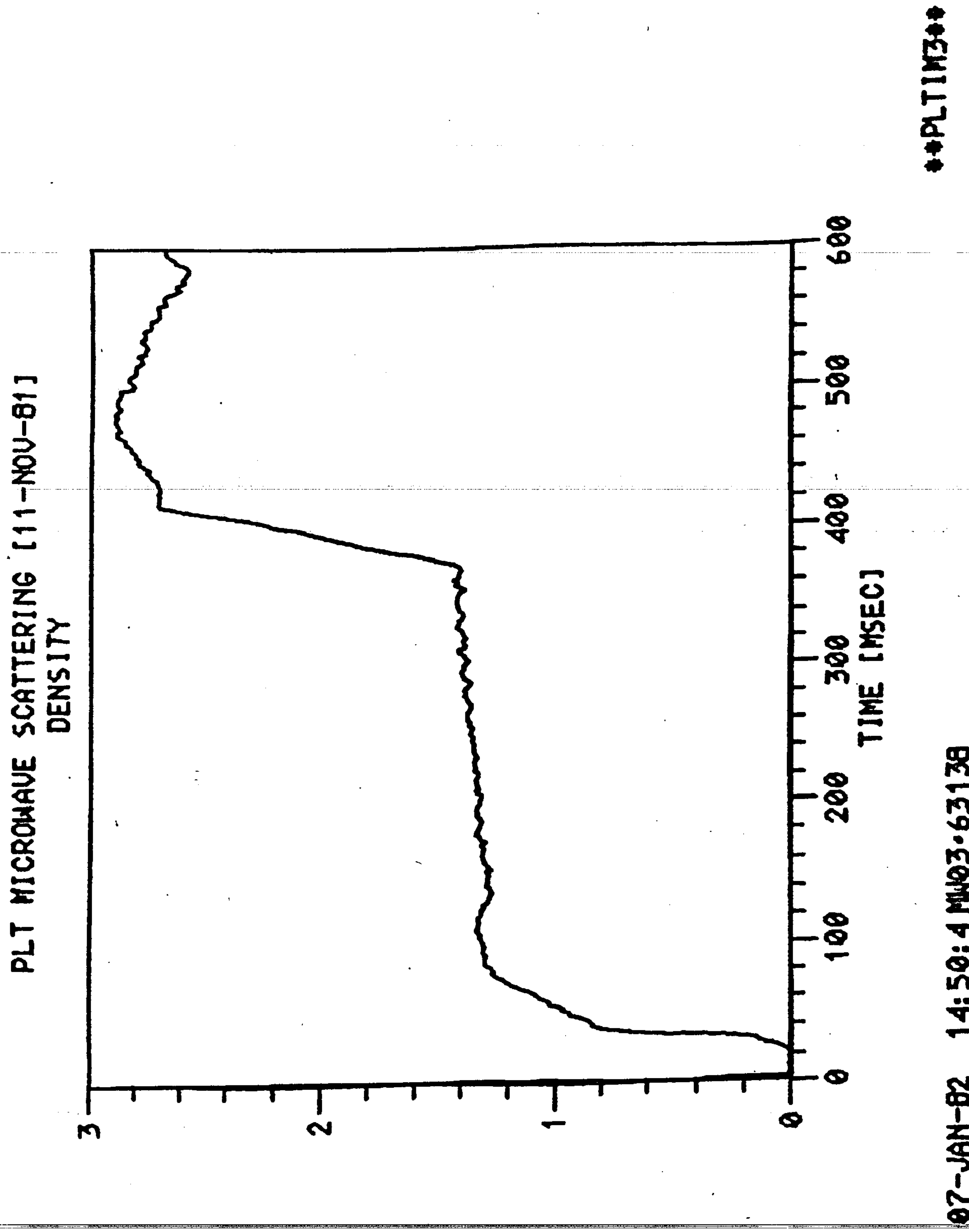
ASSUMPTIONS

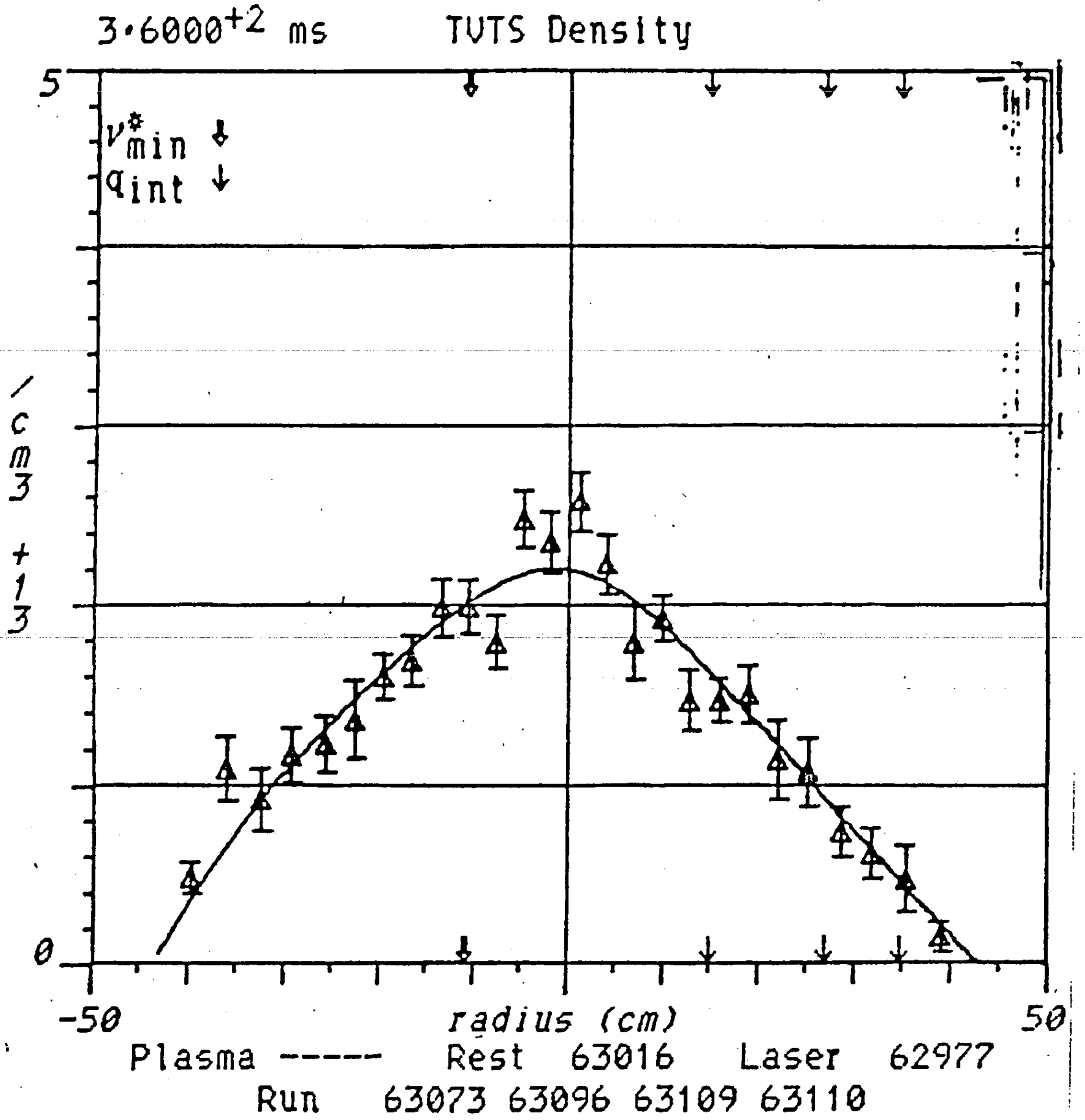
$$|k_{\parallel}| < \Delta k_{resol} < |k_{\perp}|$$

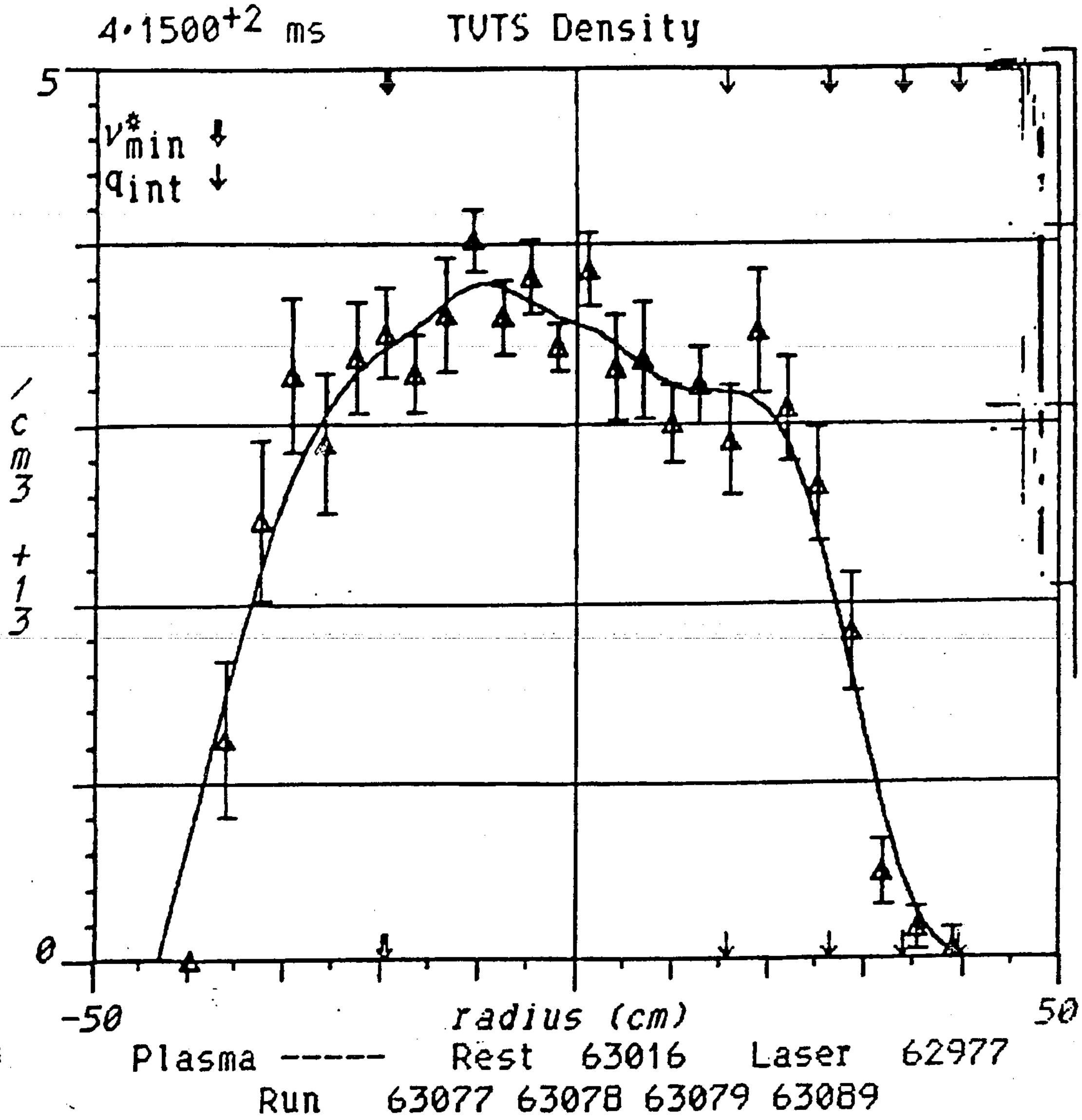
$$S(\bar{k}_{\perp}, k_{\parallel}) \simeq S(k_{\perp}, k_{\parallel})$$

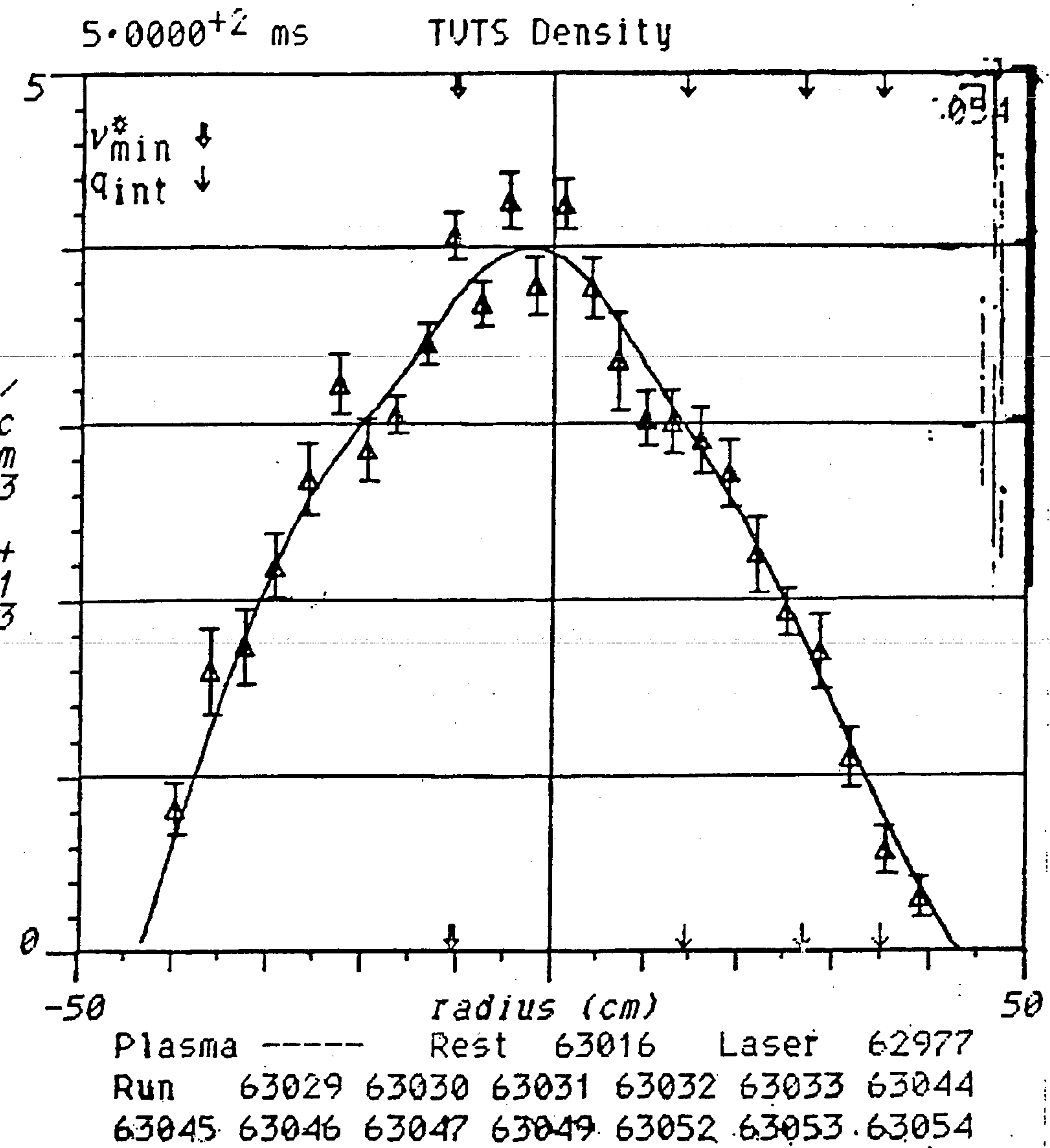
$$\frac{\langle|\tilde{n}|^2\rangle^{1/2}}{\bar{n}} = \begin{cases} (.5-1.0)\times 10^{-2} & \text{CENTER} \\ (2-4)\times 10^{-2} & \text{EDGE} \end{cases}$$

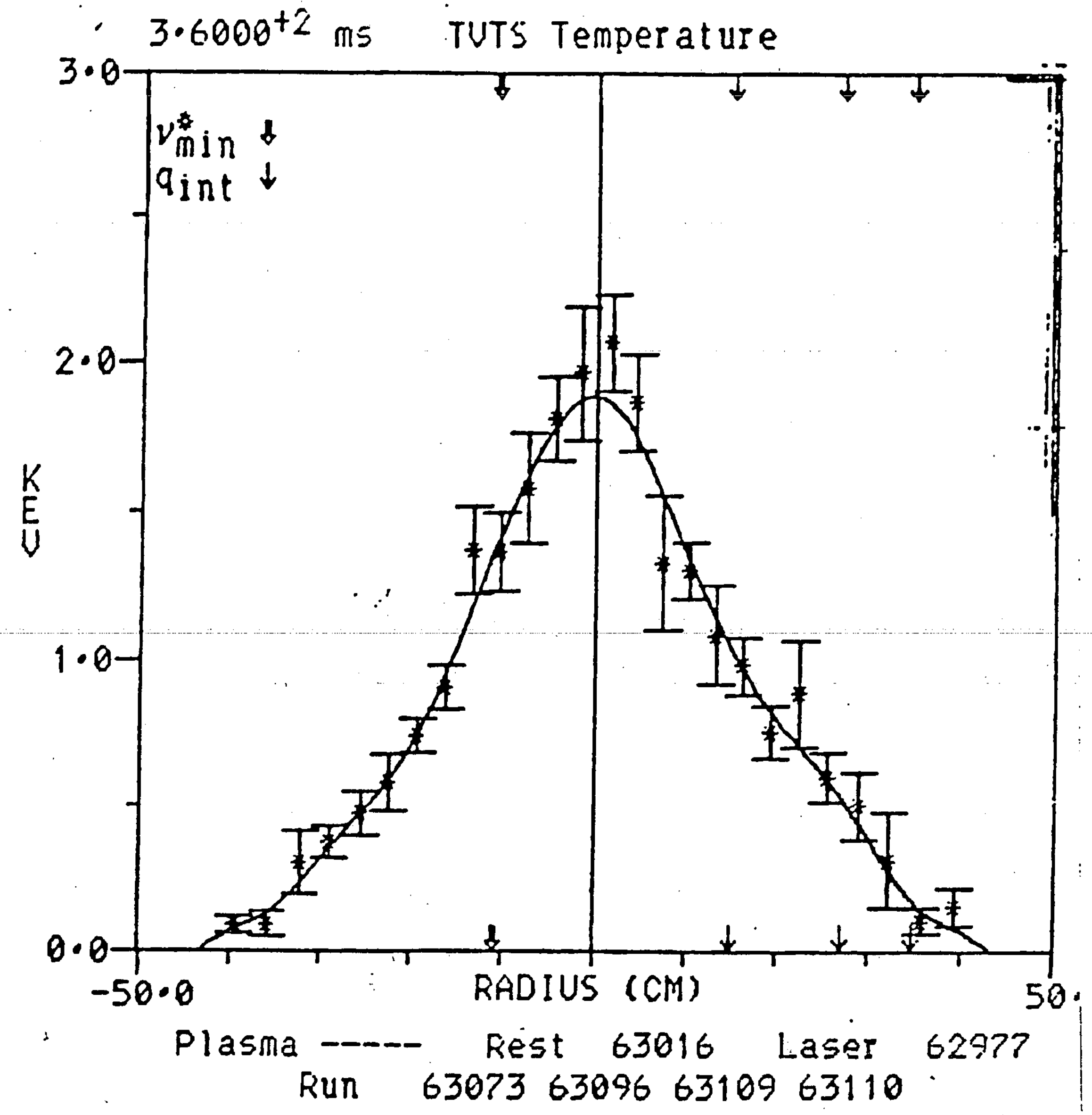
FOR $\bar{n} = 1.3\times 10^{13}\ \text{cm}^{-3}$

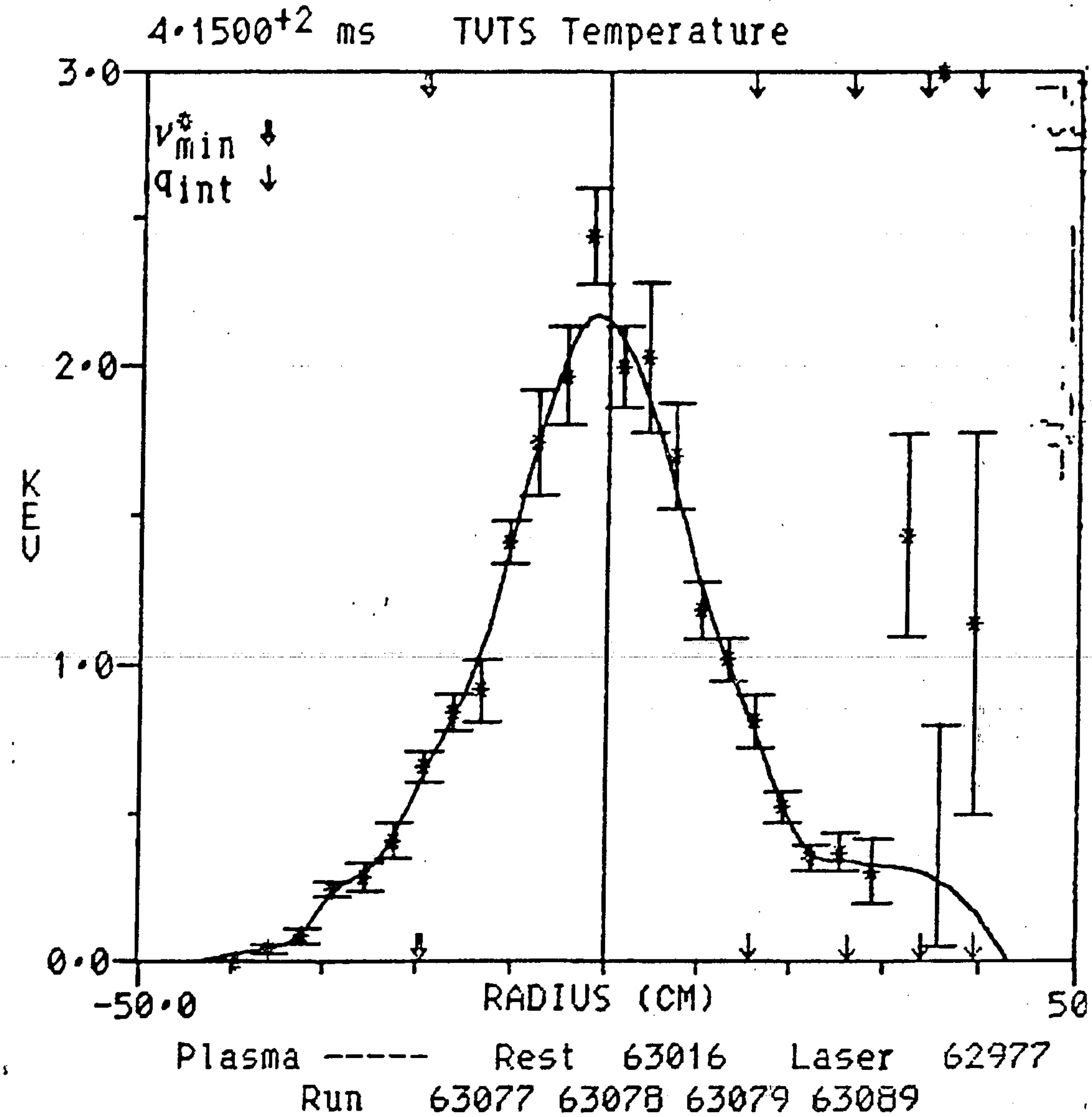

PLT MICROWAVE SCATTERING [11-NOV-81]
DENSITY
3
2
1
0
0
100
200
300
400
500
600
TIME [MSEC]
07-JAN-82 14:50:4 MW03.63138
PLTIM3


3·6000+2 ms
TVTS Density
ν^*_{min}
q_{int}
5
0
/cm3 +13
-50
50
radius (cm)
Plasma ----- Rest 63016 Laser 62977
Run 63073 63096 63109 63110

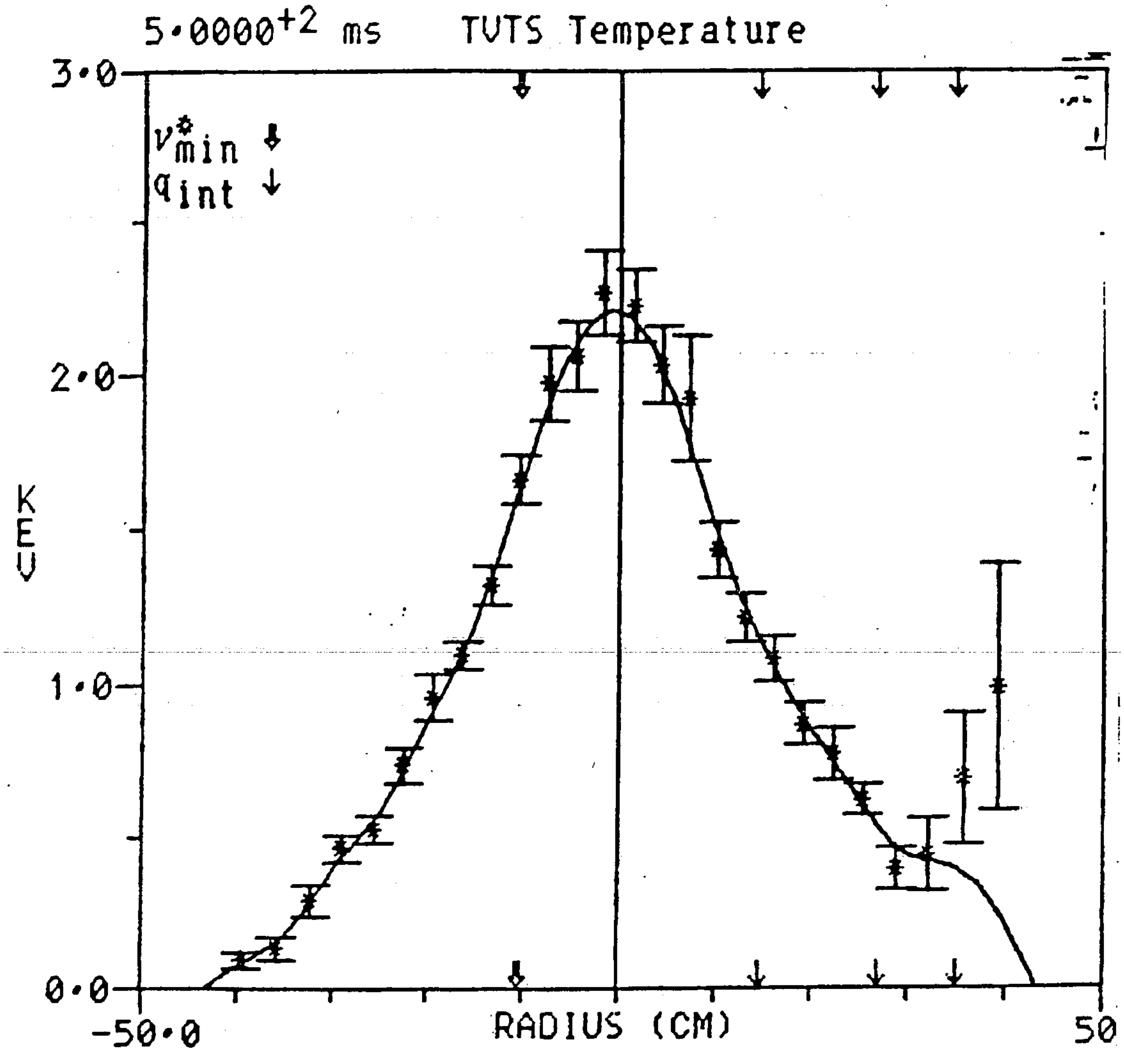

4·1500+2 ms
TVTS Density
ν^*_{min}
q_{int}
5
0
/cm3 +13
-50
radius (cm)
50
Plasma ----- Rest 63016 Laser 62977
Run 63077 63078 63079 63089


5·0000+2 ms
TVTS Density
5
ν*min
qint
/cm3 +13
0
-50
radius (cm)
50
Plasma -----
Rest 63016
Laser 62977
Run 63029 63030 63031 63032 63033 63044
63045 63046 63047 63049 63052 63053 63054

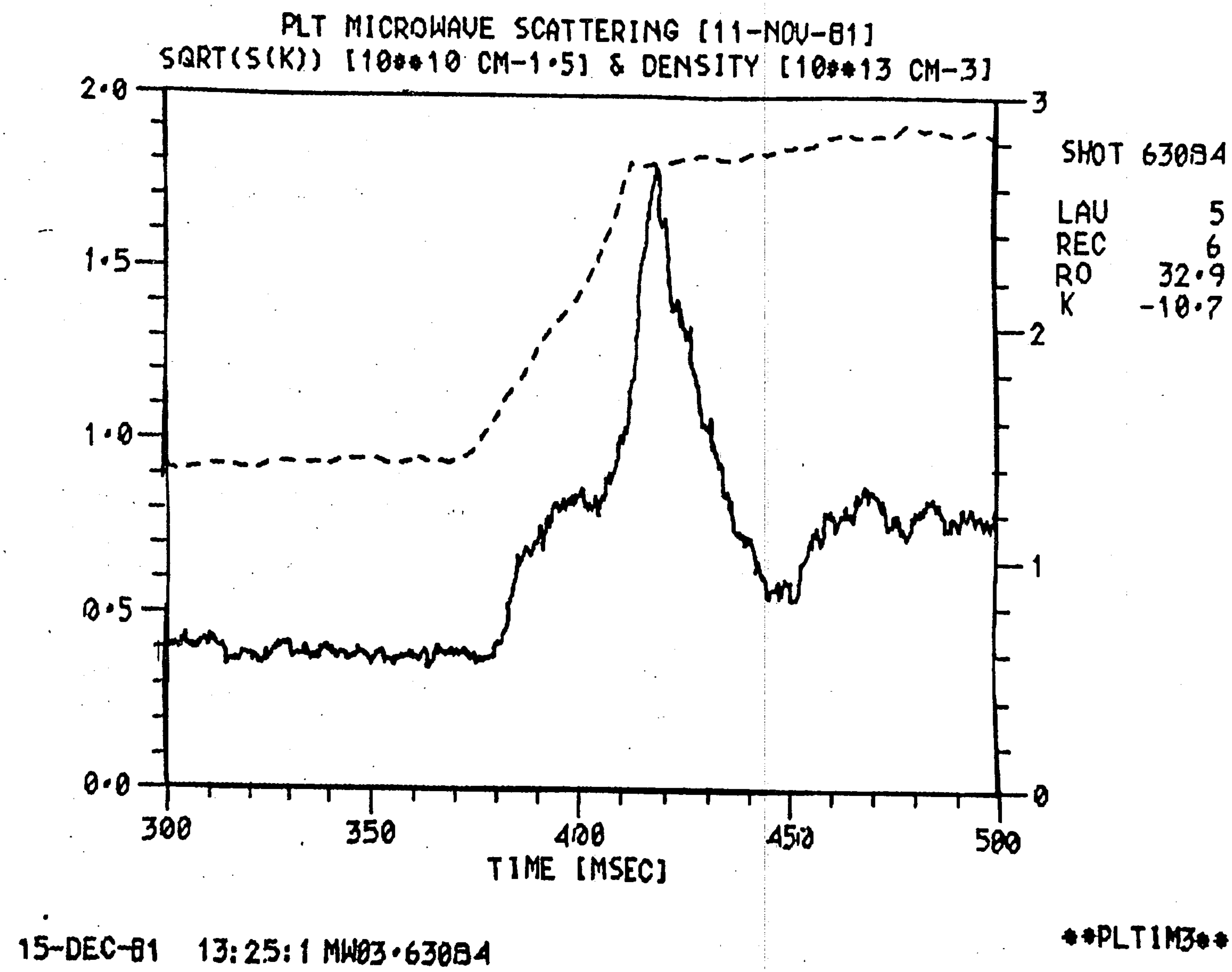

3·6000+2 ms TVTS Temperature
3·0
2·0
1·0
0·0
KEV
ν^*_{min}
q_{int}
-50·0
50·
RADIUS (CM)
Plasma ----- Rest 63016 Laser 62977
Run 63073 63096 63109 63110

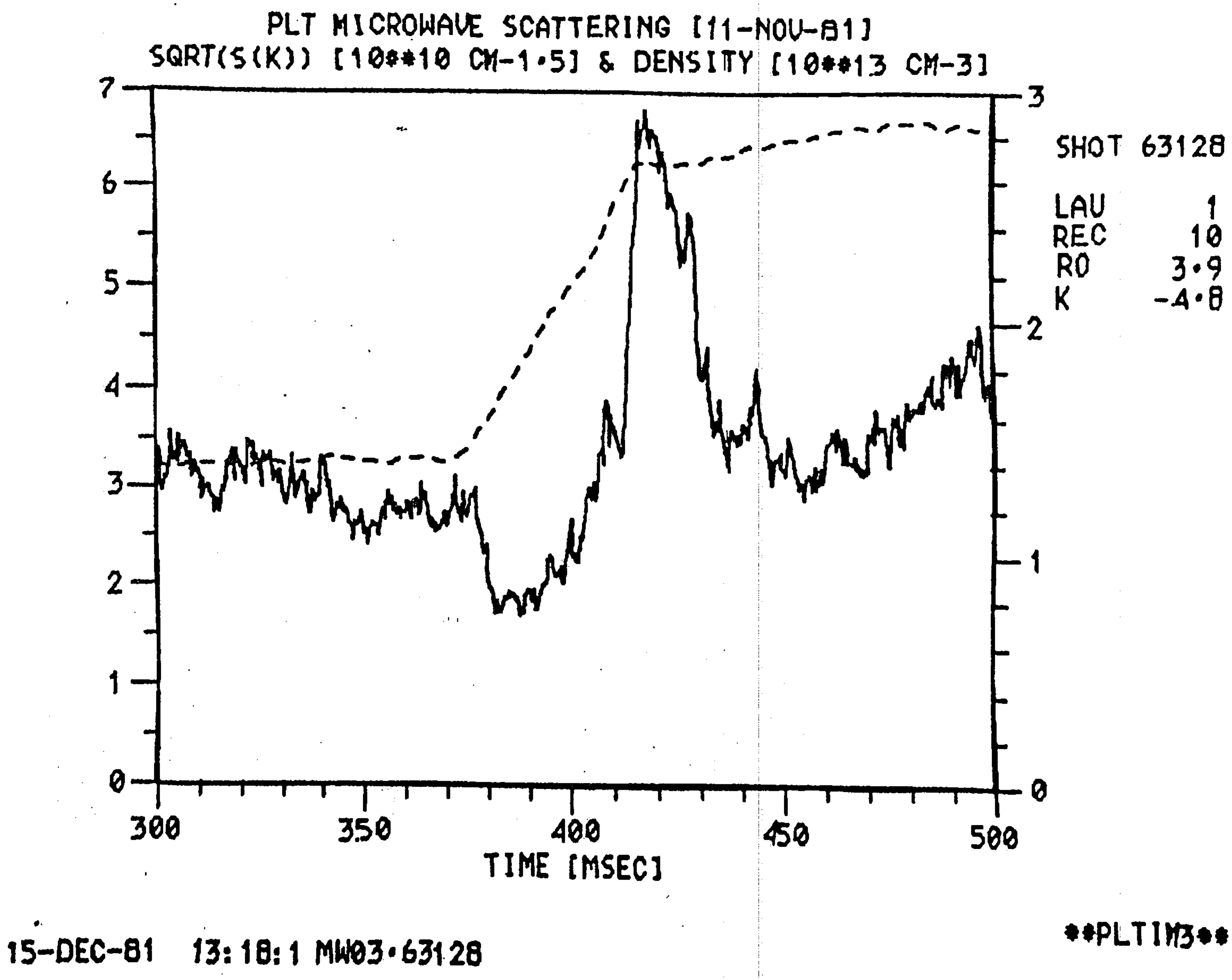


4·1500+2 ms TVTS Temperature
3·0
2·0
1·0
0·0
KEV
ν^*_{min}
q_{int}
-50·0
RADIUS (CM)
50
Plasma ----- Rest 63016 Laser 62977
Run 63077 63078 63079 63089



Plasma ----- Rest 63016 Laser 62977
Run 63029 63030 63031 63032 63033 63044
63045 63046 63047 63049 63052 63053 63054


PLT MICROWAVE SCATTERING [11-NOV-81]
SQRT(S(K)) [10**10 CM-1.5] & DENSITY [10**13 CM-3]
2.0
1.5
1.0
0.5
0.0
3
2
1
0
300
350
400
450
500
TIME [MSEC]
SHOT 63084
LAU 5
REC 6
RO 32.9
K -10.7
15-DEC-81 13:25:1 MW03.63084
PLTIM3


PLT MICROWAVE SCATTERING [11-NOV-81]
SQRT(S(K)) [10**10 CM-1.5] & DENSITY [10**13 CM-3]
7
6
5
4
3
2
1
0
3
2
1
0
300
350
400
450
500
TIME [MSEC]
SHOT 63128
LAU 1
REC 10
RO 3.9
K -4.8
15-DEC-81 13:18:1 MW03.63128
PLTIM3

CONCLUSION

WITH IMPROVED SPATIAL RESOLUTION WE FIND:

a) $\bar{\omega} > \omega_*^e$

b) $k\rho_i \lesssim 1$

c) FOR $\bar{n} = 1.3 \times 10^{13}$

- CENTER → $\frac{|\tilde{n}|}{\bar{n}} = (0.5-1) \cdot 10^{-2}$
- EDGE → $\frac{|\tilde{n}|}{\bar{n}} = (2-4) \cdot 10^{-2}$

d) $|\tilde{n}|/\bar{n}$ ↓ WHEN $\bar{n}$ ↑

BIFURCATION AND CHAOS IN COLLISIONAL DRIFT INSTABILITY

T. HATORI

NAGOYA UNIVERSITY

Birturcation and Chaos in Collisional Drift Instability

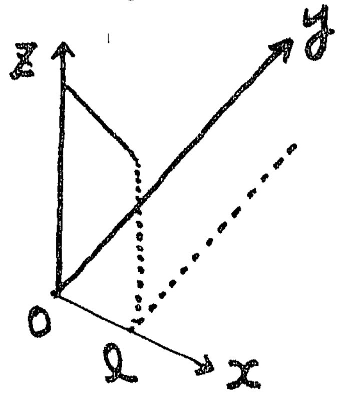
Inst. of Plasma Phys. Nagoya Univ. T. Hatori

By the drift wave turbulence, we usually imagine many unstable modes with different azimuthal wave numbers which couple to each other via nonlinearity.

On the other hand, after the discovery of strange attractor and indication that the presence of such strange attractor is a typical phenomena, fluid turbulence is proposed to be explained by the strange attractor.

Finite electical conductivity (dissipation) along $\vec{B}_0$ is essential to the drift instability, so that it is expected that there exist also the strange attractor in the case of the drift instability phenomena. We have demonstrated existence of strange attroctor in the collisional drift instability. We present that the caotic behavior is possible even for one unstable azimuthal mode. The interaction between the one unstable mode and the back ground density modification (real-space quasilinear process) is shown to be enough to generate 'turbulence' with a broad spectrum in time and space ($\parallel \nabla n_0$).

Geometry



$\vec{B} = B\hat{z}$, homogeneous

$N = N_0(1+\kappa x)$,

$\phi_1(0) = \phi_1(\ell) = 0$.

Basic two-fluid equations

Eq. of motion (el.), $m_e n_e \frac{d}{dt_e}\vec{v}_e = -\vec{\nabla}p_e - e n_e(\vec{E} + \frac{1}{c}\vec{v}_e \times \vec{B}) + \vec{R}$,

Eq. of motion (ion), $m_i n_i \frac{d}{dt_i}\vec{v}_i = -\vec{\nabla}p_i + e n_i(\vec{E} + \frac{1}{c}\vec{v}_i \times \vec{B}) - \vec{R} - \vec{\nabla}\cdot\overleftrightarrow{\Pi}$

Eq. of cont. (el.), $\frac{d}{dt_e} n_e + n_e \,\mathrm{div}\, \vec{v}_e = Q_e$,

Eq. of cont. (ion), $\frac{d}{dt_i} n_i + n_i \,\mathrm{div}\, \vec{v}_i = Q_i$,

where,

$$\vec{R} = -m_e n_e \nu_{ei}(0.51\,\vec{U}_\parallel + \vec{U}_\perp),$$

$$\vec{u} = \vec{v}_e - \vec{v}_i ,$$

$$\overleftrightarrow{\Pi} = \overleftrightarrow{\Pi}_S + \overleftrightarrow{\Pi}_{FLR}$$

= ion viscosity part

+ ion finite Larmor radius part.

For a more general case which includes the unperturbed temperature gradients and electric current along $\vec{B}$, and temperature perturbations, see

Phys. of Fluids. 21, 1127 (1978)

Assumptions

electrostatic mode	$\vec{E} = -\vec{\nabla}\phi$
quasi-neutrality	$n_e = n_i = n$
neglect the electron inertia	
small ion Larmor radius, a_i	$a_i^2 k_y^2,\ a_i^2 \frac{\partial^2}{\partial x^2} \ll 1$

Basic two equations for n and ϕ

$$\left(\frac{\partial}{\partial t} + \vec{U}_E\cdot\nabla\right)n + n D_z \nabla_z^2\left(\frac{e\phi}{T_e} - \ln n\right) - \nabla_\perp\cdot D_\perp \nabla_\perp n = Q_e,$$

$$\left(\frac{\partial}{\partial t} + \vec{U}_E\cdot\nabla\right)n - \nabla_\perp\cdot D_\perp \nabla_\perp n$$
$$+ \mathrm{div}\left\{\frac{n}{\Omega_i}\frac{\vec{B}}{B}\times\left[\frac{\partial}{\partial t} + (\vec{U}_E + \vec{U}_{d_i})\cdot\nabla\right]\vec{U}_E\right\}$$
$$+ \mathrm{div}\left\{\frac{1}{\Omega_i}\frac{\vec{B}}{B}\times\left[n\left(\frac{\partial}{\partial t} + \vec{U}_E\cdot\nabla\right)\vec{U}_{d_i} + \frac{1}{m_i}\nabla_\perp \overleftrightarrow{\Pi}_{FLR}(\vec{U}_\perp = \vec{U}_E)\right]\right\}$$
$$+ \mathrm{div}\left\{\frac{1}{m_i\Omega_i}\frac{\vec{B}}{B}\times\nabla\overleftrightarrow{\Pi}_s(\vec{U}_\perp = \vec{U}_E + \vec{U}_{d_i})\right\} = Q_i$$

where, $\vec{U}_E = (\vec{B}\times\vec{\nabla}\phi)(c/B^2)$: E×B drif velocity
$\vec{U}_{d_i} = (\vec{B}\times\vec{\nabla}n)(T_i/m_i\Omega_i)/(Bn)$: ion diamagnetic velocity

Decompose n and ϕ into $\begin{cases} n = N + \tilde{n} \\ \phi = \tilde{\phi} \end{cases}$

Separate nonlinear terms from linear one to get, in matrix form,

$$L\,U = S$$

where $\begin{cases} L : \text{linear operator} \\ U = \begin{pmatrix} \tilde{n}/N, \\ e\tilde{\Phi}/T_e \end{pmatrix} \\ S : \text{nonlinear terms} \end{cases}$

Perturbation scheme [J. Phys. Soc. Japan, **42**, 1010 (1977)]

Ordering : Weakly dissipative $\varkappa$-expansion

smallness parameter $\varepsilon \equiv |\varkappa|/\ell \ll 1$.

$k_y \ell \sim 1$.

$$\begin{cases} \dfrac{\tilde{n}}{N} \sim \dfrac{e\tilde{\phi}}{T_e} \sim \varepsilon \\ \dfrac{\omega}{\Omega_i} \sim \dfrac{\omega_*}{\Omega_i} \sim \varepsilon \end{cases}$$

$$\begin{cases} \dfrac{\gamma_L}{\Omega_i} \sim \dfrac{\nu_{ii}}{\Omega_i} \sim \dfrac{k_y^2 D_\perp}{\Omega_i} \sim \varepsilon^2 \\ \dfrac{\Omega_i}{k_z^2 D_z} \sim \dfrac{\Omega_i}{k_z^2 \dfrac{\sigma_z T_e}{e^2}} \sim \varepsilon^2 \end{cases}$$

We have another small parameter, $b \equiv k_\perp^2 a_i^2$. Our calculation retains the 1st-order quantities in b, so we have tacitly assumed,

$$\varepsilon \gg b$$

to guarantee the ε^3-order calculation meaningful.

Expansion

$$L = L^{(0)} + \varepsilon L^{(1)} + \varepsilon^2 L^{(2)} + \varepsilon^3 L^{(3)} + \cdots$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial y_0} + \varepsilon \frac{\partial}{\partial y_1} + \cdots$$

$$\frac{\partial}{\partial t} = \varepsilon \frac{\partial}{\partial t_1} + \varepsilon^2 \left(\frac{\partial}{\partial t_2} - V_g \frac{\partial}{\partial y_1} \right) + \cdots$$

$$U = \varepsilon U^{(1)} + \varepsilon^2 U^{(2)} + \varepsilon^3 U^{(3)} + \cdots$$

$$S = \varepsilon^2 S^{(2)} + \varepsilon^3 S^{(3)} + \cdots$$

$O(\varepsilon)$: $L^{(0)}U^{(1)} = \begin{pmatrix} -D_z\frac{\partial^2}{\partial z^2}, & D_z\frac{\partial^2}{\partial z^2} \\ 0 & 0 \end{pmatrix}\begin{pmatrix} n^{(1)}/N \\ e\varphi^{(1)}/T_e \end{pmatrix} = 0$

For $k_z \neq 0$ component, we have the Maxwell-Boltzmann rel.

assumption :

Even for the $k_z=0$ component (convective cell), the Maxwell-Boltzmann relation holds.

Therefore, we may write,

$$\frac{n^{(1)}}{N} = \frac{e\varphi^{(1)}}{T_e} \ (\equiv f)$$

$$= h(x, y_1, t_2) + \sum_{k_z} g(x, y_1, t_2)e^{2ik_z z} + \text{c.c.}$$

$$+ \sum_{k_z, k_y} [f_+(x, y_1, t_2)e^{ik_z z} + f_-(x, y_1, t_2)e^{-ik_z z}]e^{-i\omega t_1 + ik_y y_0} + \text{c.c.}$$

+ higher harmonics.

$O(\varepsilon^2)$: $L^{(0)}U^{(2)} + L^{(1)}U^{(1)} = S^{(2)}$

Explicitly, $-D_z\frac{\partial^2}{\partial z^2}\left(\frac{n^{(2)}}{N} - \frac{e\varphi^{(2)}}{T_e}\right) + \left(\frac{\partial}{\partial t_1} + V_*\frac{\partial}{\partial y_0}\right)f = -D_z\frac{\partial^2}{\partial z^2}\left(\frac{f^2}{2}\right)$,

and, $\left\{\frac{\partial}{\partial t_1}\left[1 - \frac{1}{2}\left(1 + \frac{T_e}{T_i}\right)a_i^2\Delta_{\perp 0}\right] + V_*\frac{\partial}{\partial y_0}\right\}f$

$$= \frac{1}{4}\frac{T_e}{T_i}\left(1 + \frac{T_e}{T_i}\right)a_i^2\Omega_i\left(-\frac{\partial f}{\partial y_0}\frac{\partial}{\partial x} + \frac{\partial f}{\partial x}\frac{\partial}{\partial y_0}\right)\Delta_{\perp 0}f$$

[Hasegawa-Mima eq.]

right-hand side: polarization drift.

assumption : Coupling coefficient of H-M equation is small enough not to generate higher harmonics other than h, g and $f_\pm$.

The H-M eq. determines just a characteristic frequency ω (real) for the drift wave, so we have to proceed to the $O(\varepsilon^3)$ to determine h, g and $f_\pm$

$\underline{O(\varepsilon^3)}$: $L^{(0)}U^{(3)} + L^{(1)}U^{(2)} + L^{(2)}U^{(1)} = S^{(3)}$

The electron part :

$$-D_z \frac{\partial^2}{\partial z^2}\left[\frac{n^{(3)}}{N} - \frac{e\varphi^{(3)}}{T_e}\right] + \left[\frac{\partial}{\partial t_2} + (-V_g + V_*)\frac{\partial}{\partial y_1} - \kappa x V_* \frac{\partial}{\partial y_0} - D_\perp \Delta_\perp\right] f$$

$$= -\left(V_* \frac{\partial}{\partial y_0} - \kappa x D_z \frac{\partial^2}{\partial z^2}\right)\left[\frac{f^2}{2} + \frac{n^{(2)}}{N} - \frac{e\varphi^{(2)}}{T}\right] + \frac{T_e}{m_e \Omega_e} \frac{\vec{B}}{B} \times \vec{\nabla}_\perp f \cdot \vec{\nabla}_\perp \left[\frac{n^{(2)}}{N} - \frac{e\varphi^{(2)}}{T_e}\right]$$

$$-\left(\frac{\partial}{\partial t_1} + V_* \frac{\partial}{\partial y_0}\right)\frac{n^{(2)}}{N} + D_z \frac{\partial^2}{\partial z^2}\left[\frac{f^3}{3} - f\frac{n^{(2)}}{N}\right],$$

The ion part :

$$\left[1 - \frac{1}{2}\left(1+\frac{T_e}{T_i}\right)a_i^2 \Delta_{\perp 0}\right]\frac{\partial}{\partial t_2} f + \left\{-V_g\left[1 - \frac{1}{2}\left(1+\frac{T_e}{T_i}\right)a_i^2 \Delta_{\perp 0}\right] + V_* - \left(1+\frac{T_e}{T_i}\right)a_i^2 \frac{\partial^2}{\partial t_1 \partial y_0}\right\}\frac{\partial f}{\partial y_1}$$

$$+\left\{-\left[\kappa x V_* \frac{\partial}{\partial y_0} - \frac{1}{2} a_i^2 \kappa V_* \frac{\partial^2}{\partial x \partial y_0} + \left(1+\frac{T_e}{T_i}\right)\frac{1}{2}\kappa a_i^2 \frac{\partial^2}{\partial t_1 \partial x}\right] - D_\perp \Delta_{\perp 0} - \frac{3}{10}\left(1+\frac{T_e}{T_i}\right)\nu_{ii} \frac{a_i^2}{2}\Delta_{\perp 0}\right\} f$$

$$= \left(V_* \frac{\partial}{\partial y_0} - \frac{\partial}{\partial t_1}\frac{T_e}{T_i}\frac{a_i^2}{2}\Delta_{\perp 0}\right)\left[\frac{n^{(2)}}{N} - \frac{e\varphi^{(2)}}{T_e}\right] - \left\{\frac{\partial}{\partial t_1}\left[1 - \frac{1}{2}\left(1+\frac{T_e}{T_i}\right)a_i^2 \Delta_{\perp 0}\right] + V_* \frac{\partial}{\partial y_0}\right\}\frac{n^{(2)}}{N} + S_i^{(3)}.$$

From the compatibility conditions of above equations, the $\underline{\text{real-space quasilinear}}$ system :

$$\frac{\partial}{\partial t_2} h = D_\perp \frac{\partial^2}{\partial x^2} h + 2\sum \frac{\omega_* \delta_k}{\kappa} \frac{\partial}{\partial x}\left[|f_+|^2 + |f_-|^2\right],$$

$$\frac{\partial}{\partial t_2} g = D_\perp \frac{\partial^2}{\partial x^2} g + 2\sum \frac{\omega_* \delta_k}{\kappa} \frac{\partial}{\partial x}(f_+ f_-^*),$$

$$\left(\frac{\partial}{\partial t_2} - \Delta V_g \frac{\partial}{\partial y_1}\right) f_+ = \left[\gamma_k \left(1 + \frac{1}{\kappa}\frac{\partial h}{\partial x}\right) + \hat{O} + i\delta\omega\right] f_+ + \left[\frac{1}{\kappa}\omega_* \delta_k \frac{\partial g}{\partial x} + i\delta\omega'\right] f_-$$

$$\left(\frac{\partial}{\partial t_2} - \Delta V_g \frac{\partial}{\partial y_1}\right) f_- = \left[\gamma_k \left(1 + \frac{1}{\kappa}\frac{\partial h}{\partial x}\right) + \hat{O} + i\delta\omega\right] f_- + \left[\frac{1}{\kappa}\omega_* \delta_k \frac{\partial g^*}{\partial x} + i\delta\omega'^*\right] f_+,$$

where,

$$\hat{O} = D_\perp \Delta_{\perp 0} - \frac{3}{10}\left(\frac{T_e}{T_i} + 1\right)\nu_{ii}\left(\frac{1}{2}a_i^2 \Delta_{\perp 0}\right)^2,$$

$$\gamma_k = \omega_* \delta_k, \qquad \delta_k = (\omega_* - \omega)/k_z^2 D_z,$$

$$\Delta V_g = \left(1 + \frac{T_e}{T_i}\right)\left(-\frac{1}{2}V_* a_i^2 \Delta_{\perp 0} + a_i^2 \omega k_y\right)$$

$$\delta\omega = \kappa x \omega_* [1 + O(b)], \qquad \delta\omega' = \delta\omega \cdot b$$

(P, P) truncation (P. 392 ~ P. 12)
(16, 16)

Table I. Characteristics of unstable modes

η		$\eta_4=0.10$ ↓	$\eta_3=0.16$ ↓	$\eta_2=0.28$ ↓	$\eta_1=0.56$ ↓	$\eta_c=1.40$ ↓
Linearly unstable modes	F_1, F_2, F_3, F_4, F_5,	F_1, F_2, F_3, F_4	F_1, F_2, F_3	F_1, F_2	F_1	
Transition η		$\eta_a=0.14$		$\eta_{os}=0.29$ $\eta'_{os}=0.47$		
Nonlinear behavior	Aperiodic amplitnde oscillation		Periodic amplitnde oscillation	Saturation		Stable equilibrium

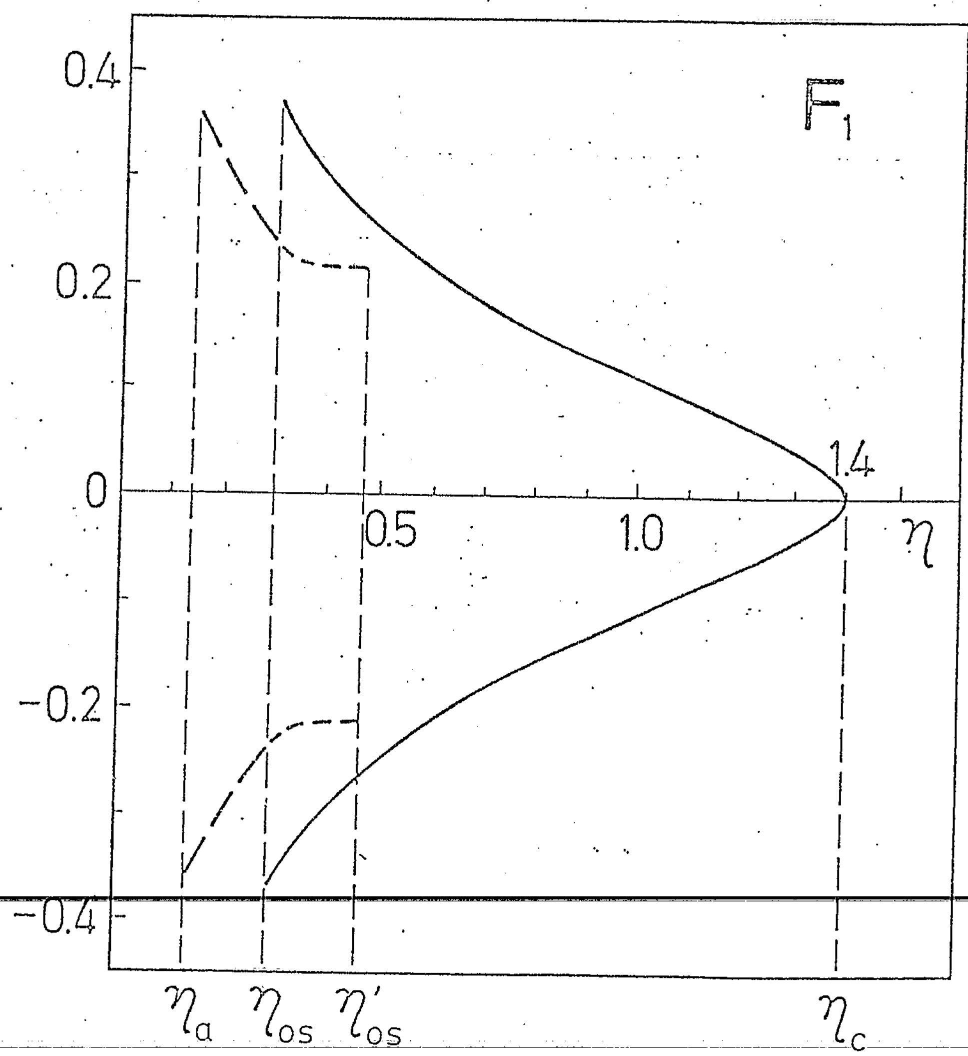


Fig. 1 The final behavior of F_1

Numerical analysis

Model equation $[\, g,\ f_- \longrightarrow 0 \,]$

$$\frac{\partial}{\partial \tau} H = \alpha \frac{\partial^2}{\partial \xi^2} H + 2 \frac{\partial}{\partial \xi} (F \Delta F) \quad : \quad \text{classical diff + wave-enhanced diff.}$$

$$\frac{\partial}{\partial \tau} F = -\left(1 - \frac{\partial H}{\partial \xi}\right) \Delta F - \eta \Delta^2 F \quad : \quad \text{growth + damping}$$

where dimensionless quantities are,

$\tau = \gamma_L t_2 (\pi / l k_y)^2$, $k = k_y \frac{\pi}{l}$

$\xi = \pi x / l$, $H = \frac{\pi}{\kappa l} h$

$\alpha = k_y^2 D_\perp / \gamma_L$, $F = \frac{\pi}{\kappa l} f_+$

$\eta = \frac{3}{40} \left(1 + \frac{T_e}{T_i}\right) \nu_{ii} \left(\frac{\pi}{l} a_i\right)^4 k^2 / \gamma_L$

$\Delta = \frac{\partial^2}{\partial \xi^2} - k^2$

Parameters : α, k fixed

η : external parameter

Boundary cond. : $F(\xi = 0, \tau) = F(\xi = \pi, \tau) = 0$

$H(\xi = 0, \tau) = H(\xi = \pi, \tau) = 0$

Fourier components F_p, H_p

$$F(\xi, \tau) = \sum_{p=1}^{\infty} F_p(\tau) \sin p\xi$$

$$H(\xi, \tau) = \sum_{p=1}^{\infty} H_p(\tau) \sin p\xi$$

Truncation scheme

(p, p) truncation , $(F_1 \cdots F_p ; H_1 \cdots H_p)$ [J. Phys. Soc. Jpn. 45, 998 '78

(p, 2p) truncation , $(F_1 \cdots F_p , H_1 \cdots H_{2p})$ [J. Phys. Soc. Jpn. 47, 1659 '79

(P, P)

5

Time development for the case $\eta = 0.5$

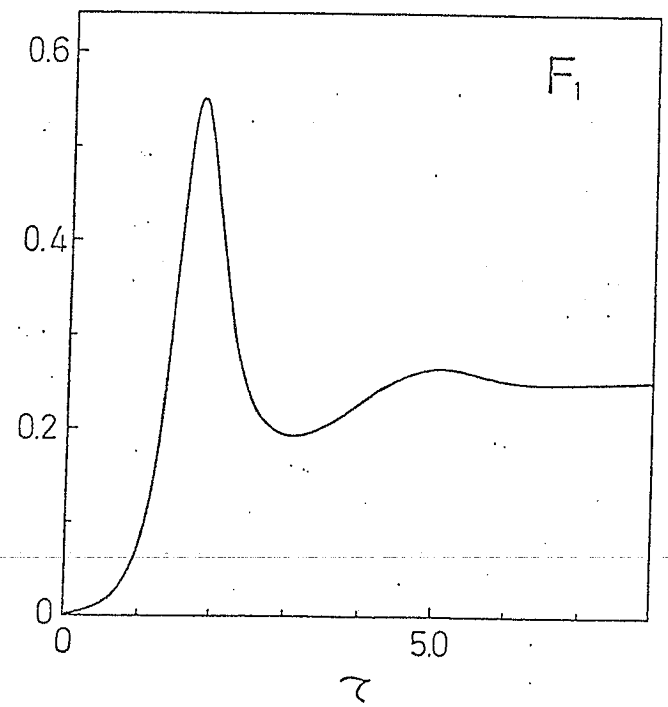


Fig. 2 (a)

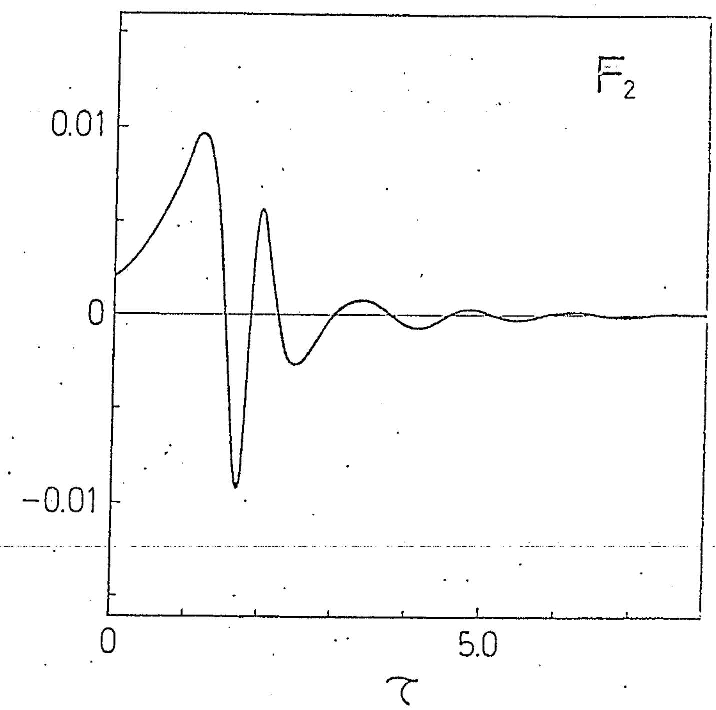
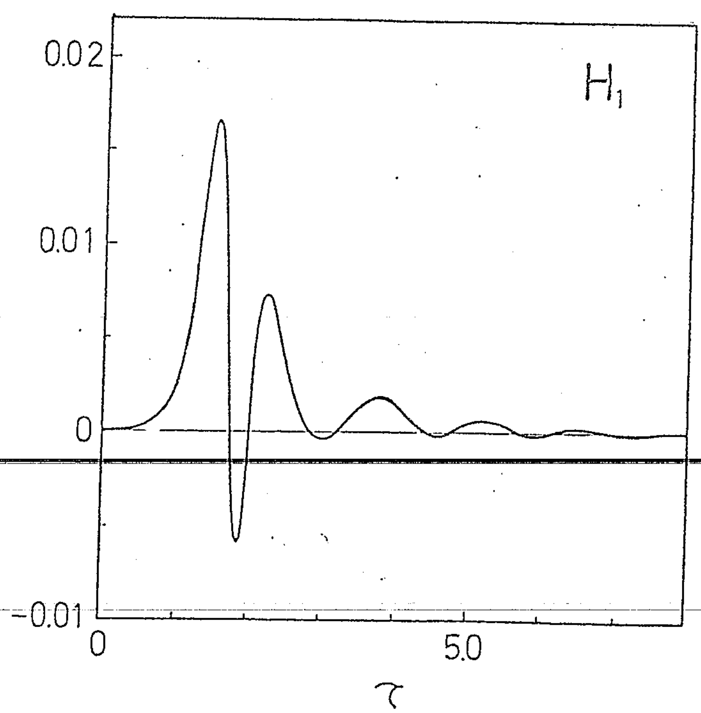
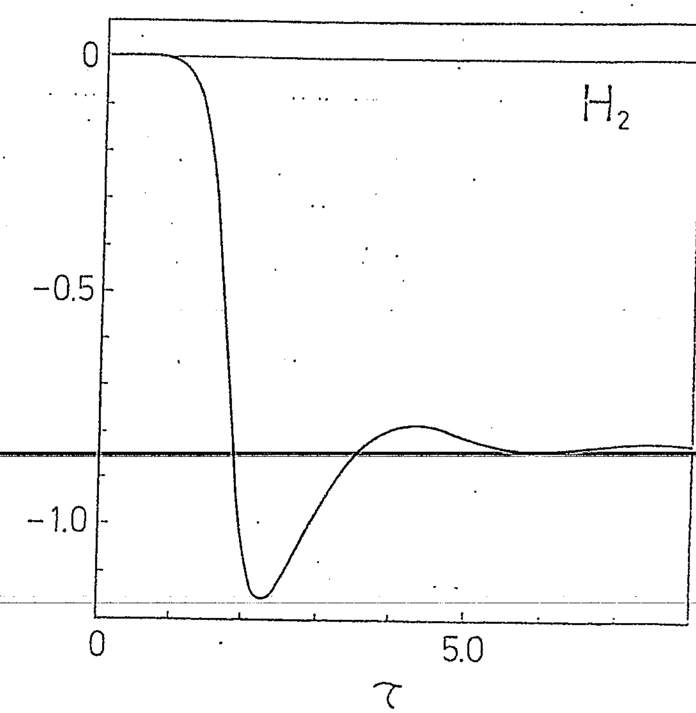


Fig. 2 (b)





6

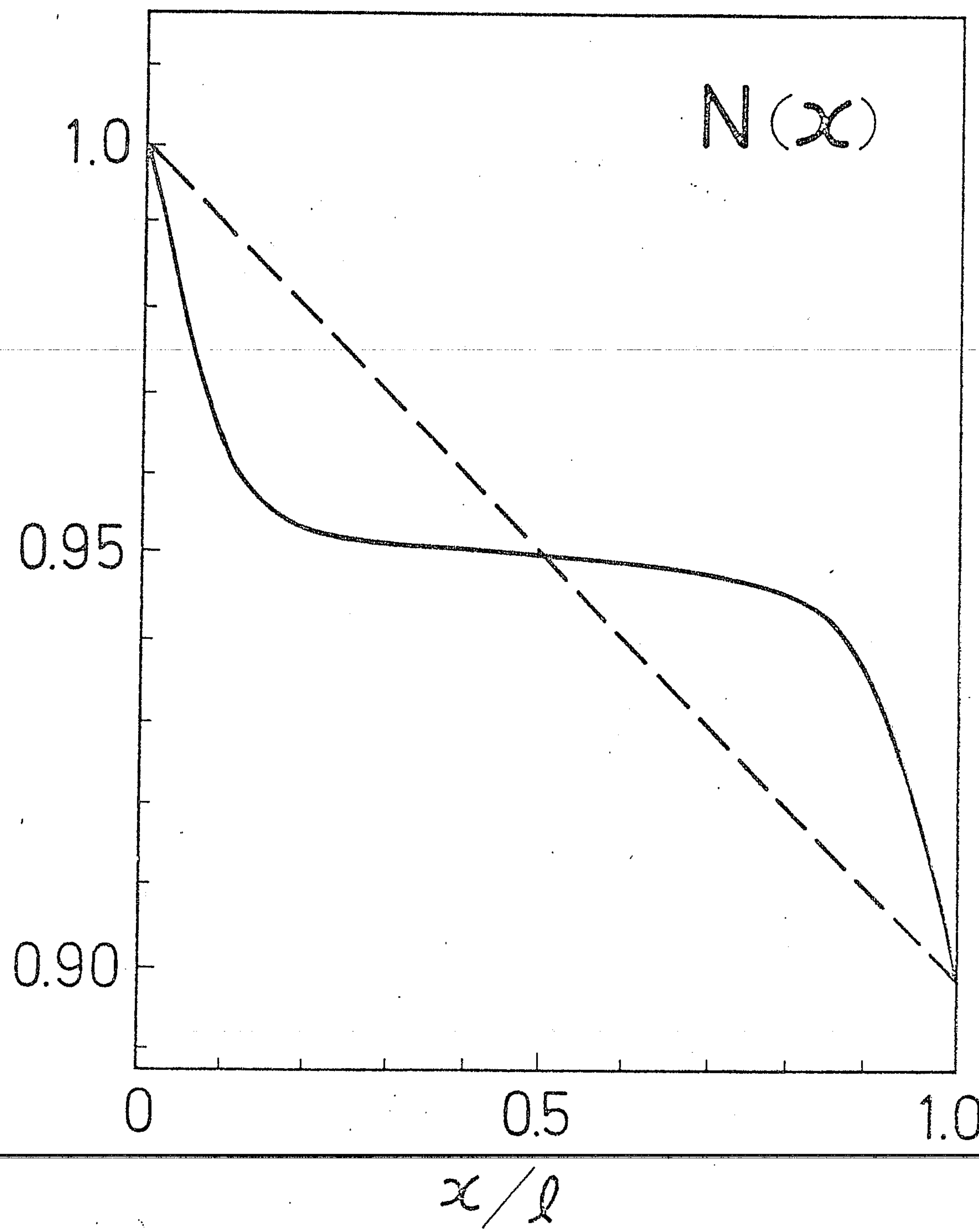


FIG.4

Time development for the case $\eta=0.2$

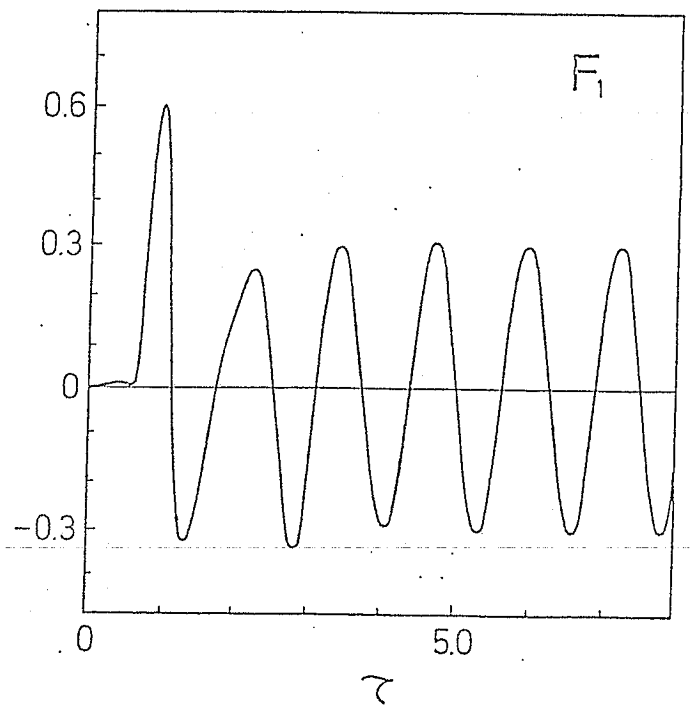


Fig. 6 (a)

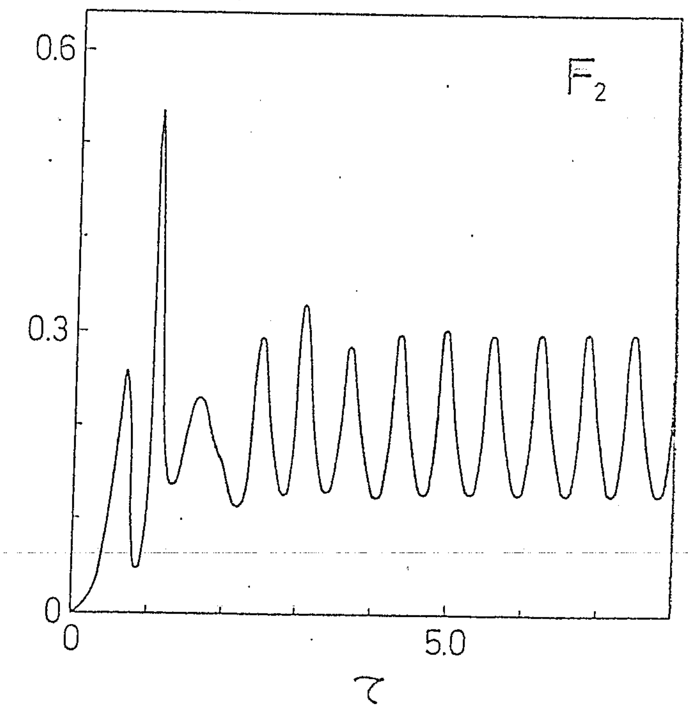
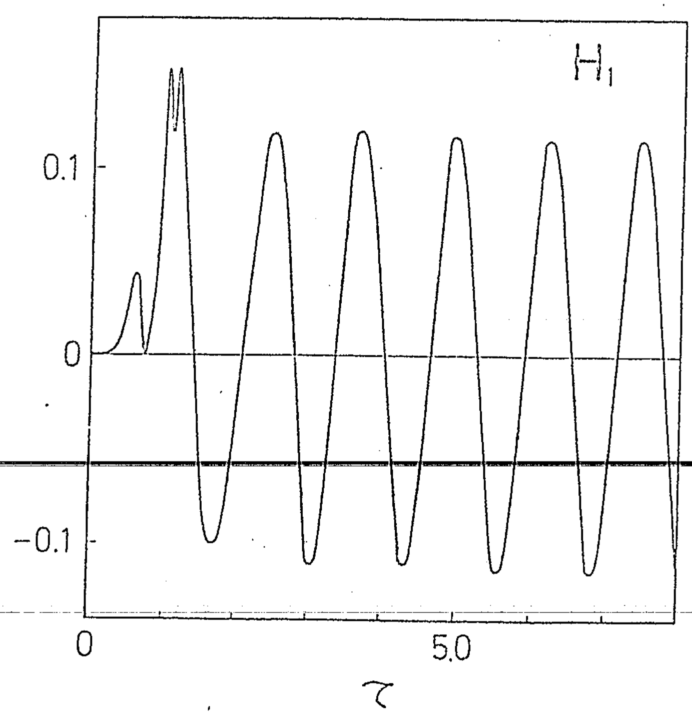
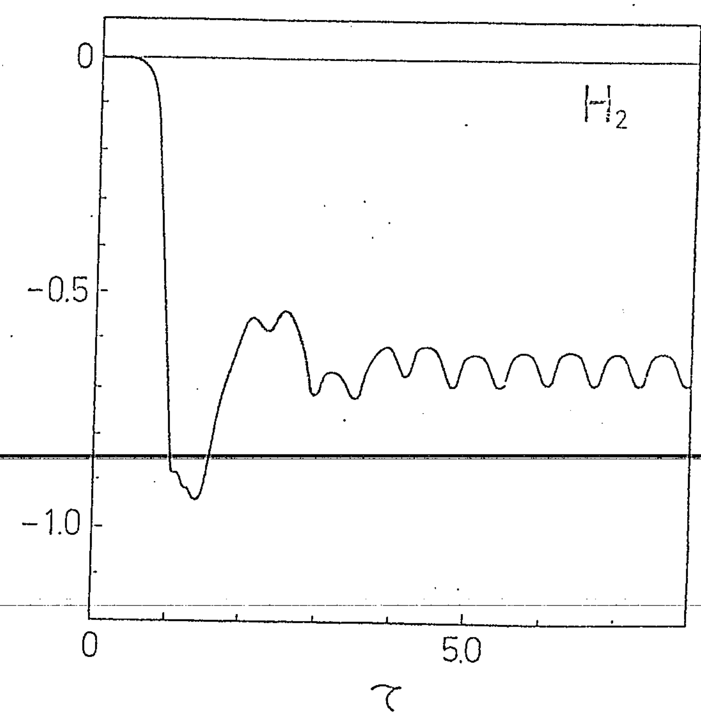


Fig. 6 (b)





Time development of solution in the H_2-F_1 plane

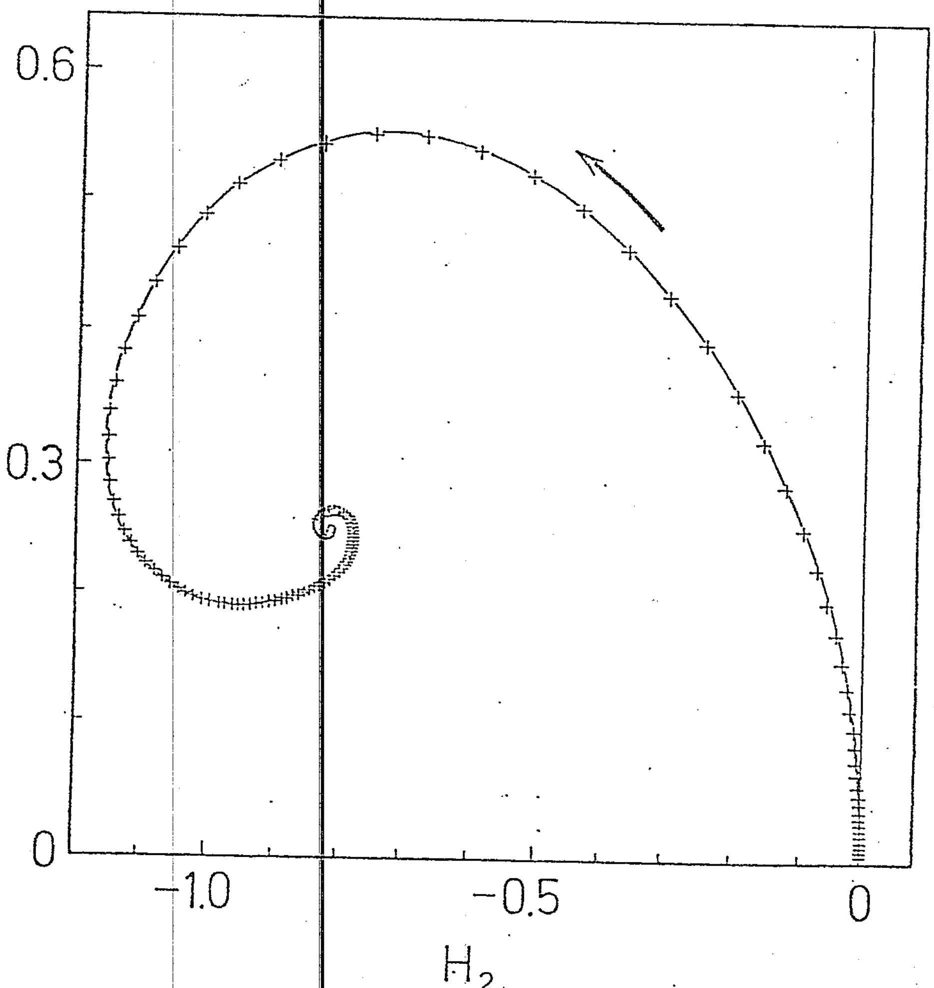


Fig. 5 (a) Saturation for the case $\eta = 0.5$

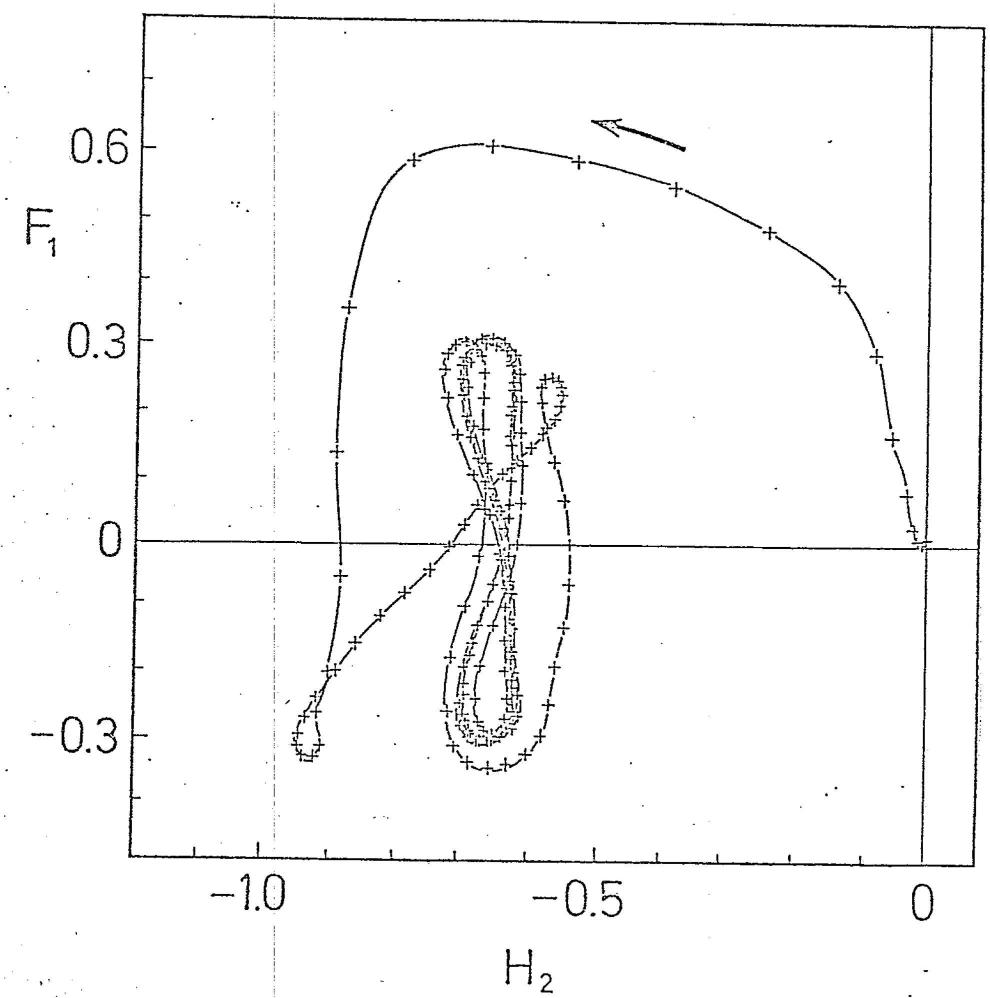


Fig. 5 (b) The 8-figure oscillation (limit cycle) for the case $\eta = 0.2$

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Aperiodic amplitude oscillation for the case $\eta = 0.12$

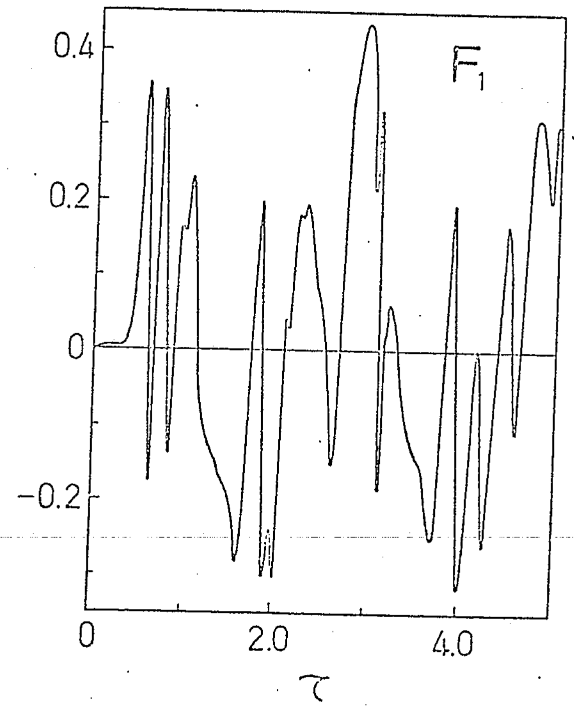


Fig. 8 (a)

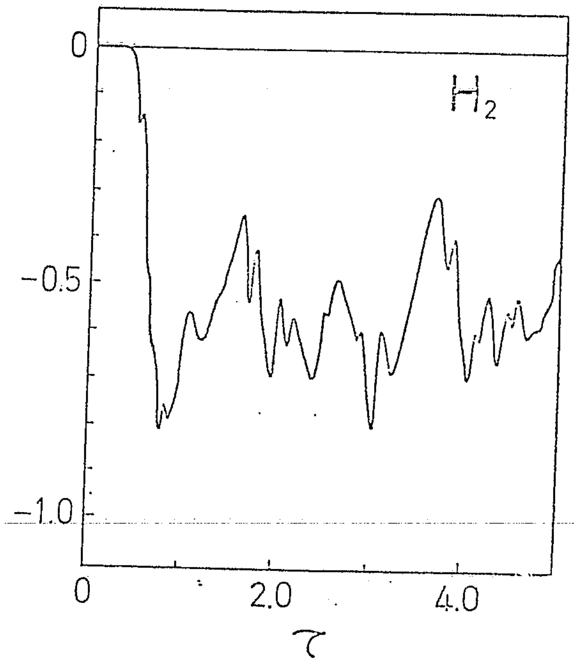
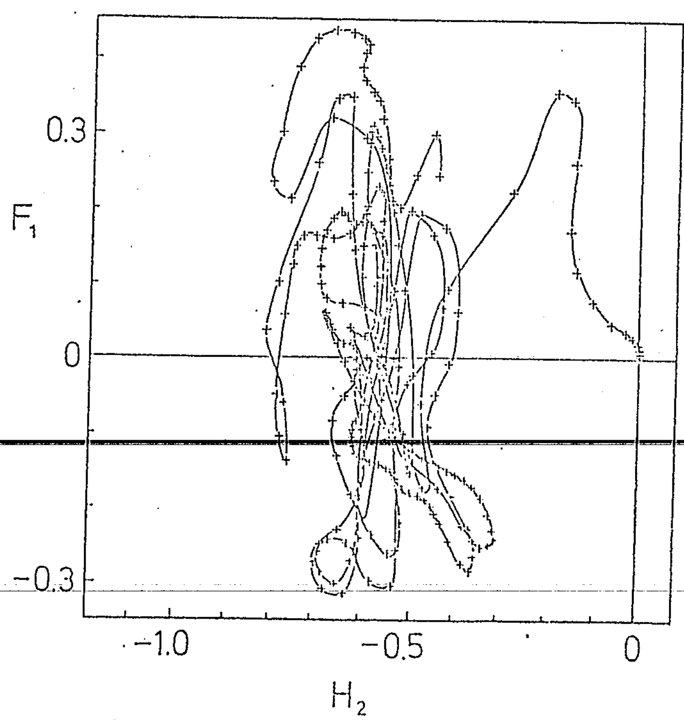


Fig. 8 (b)



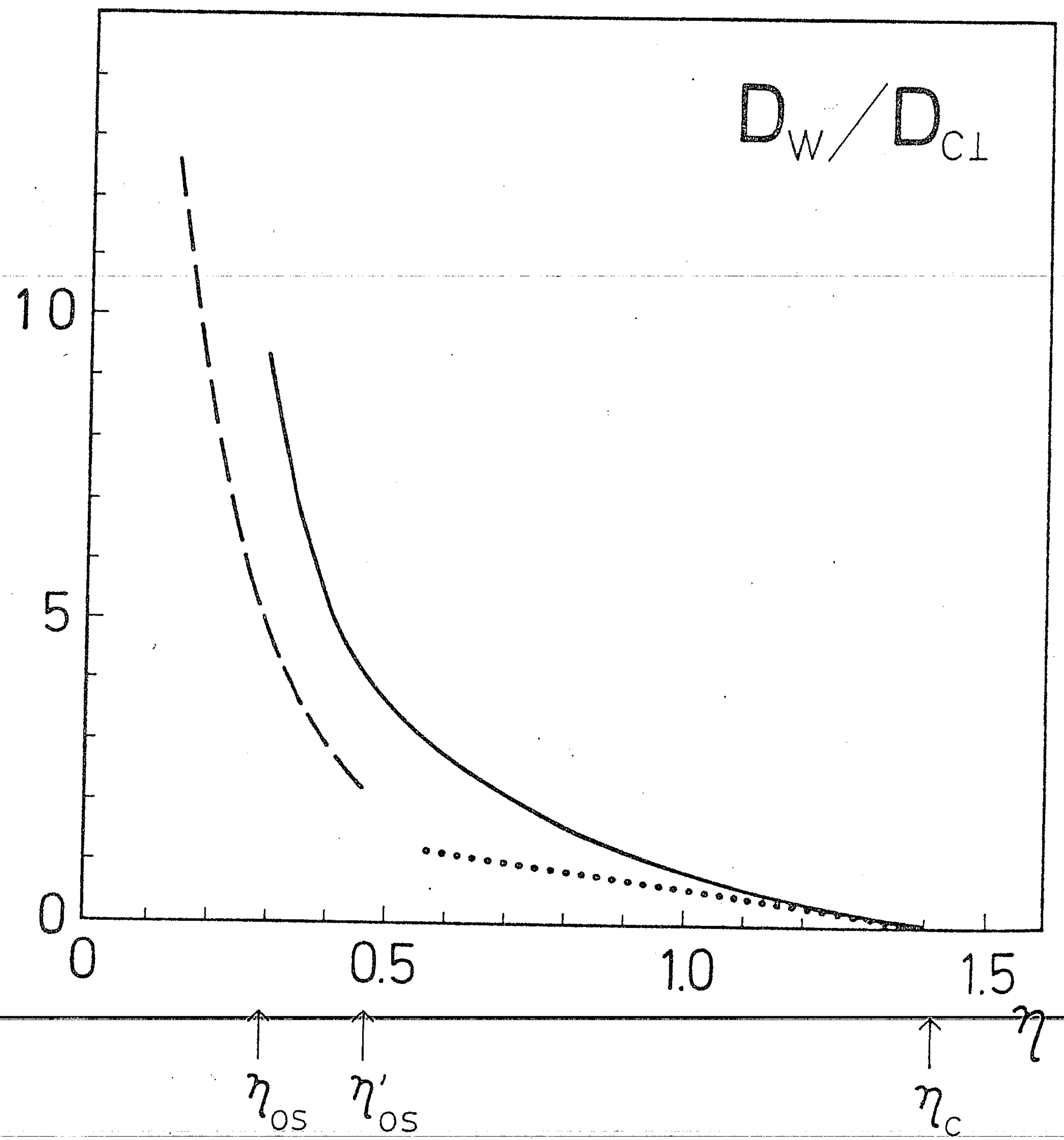


FIG. 9

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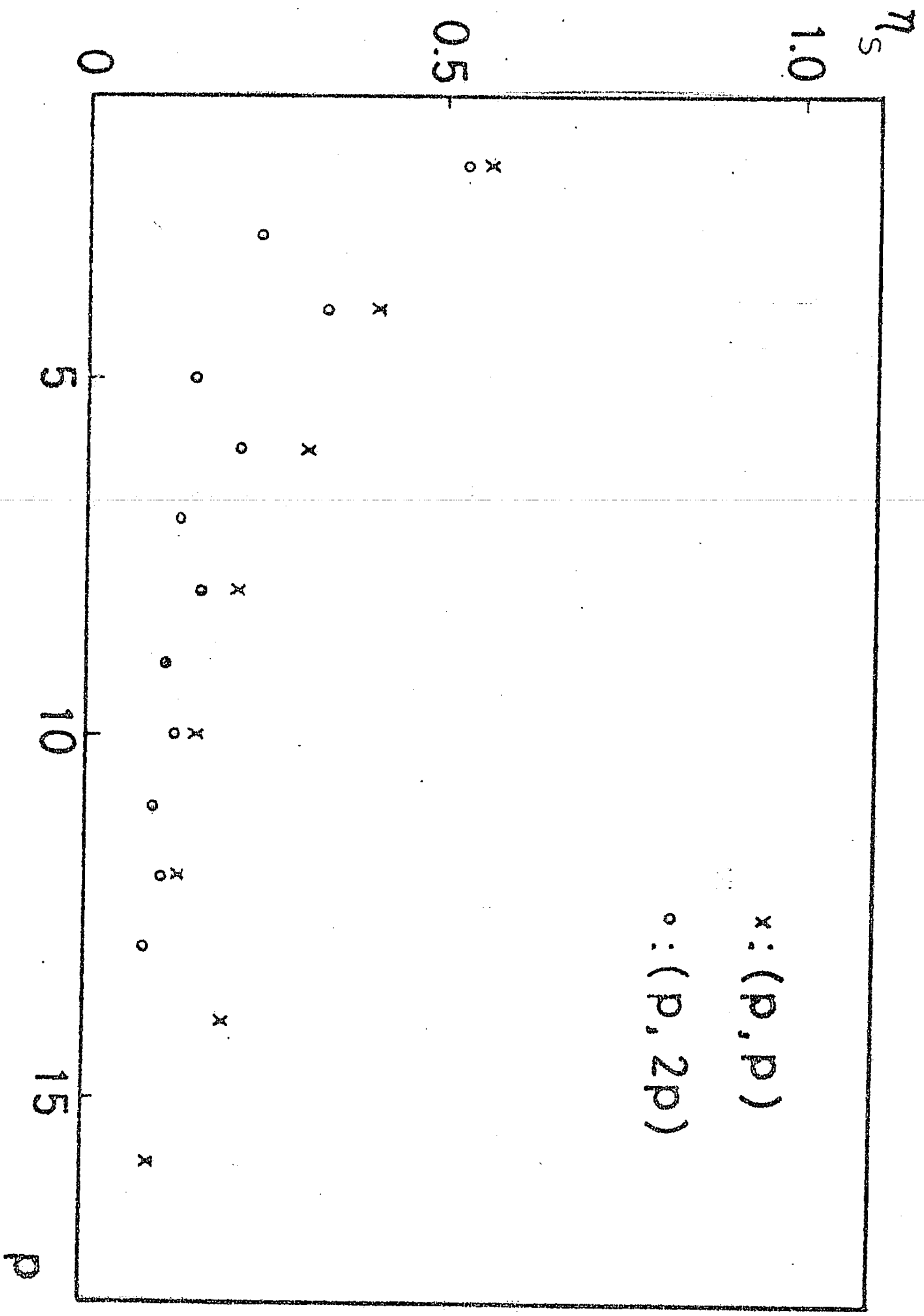


Fig. 2

(P, 2P) 19

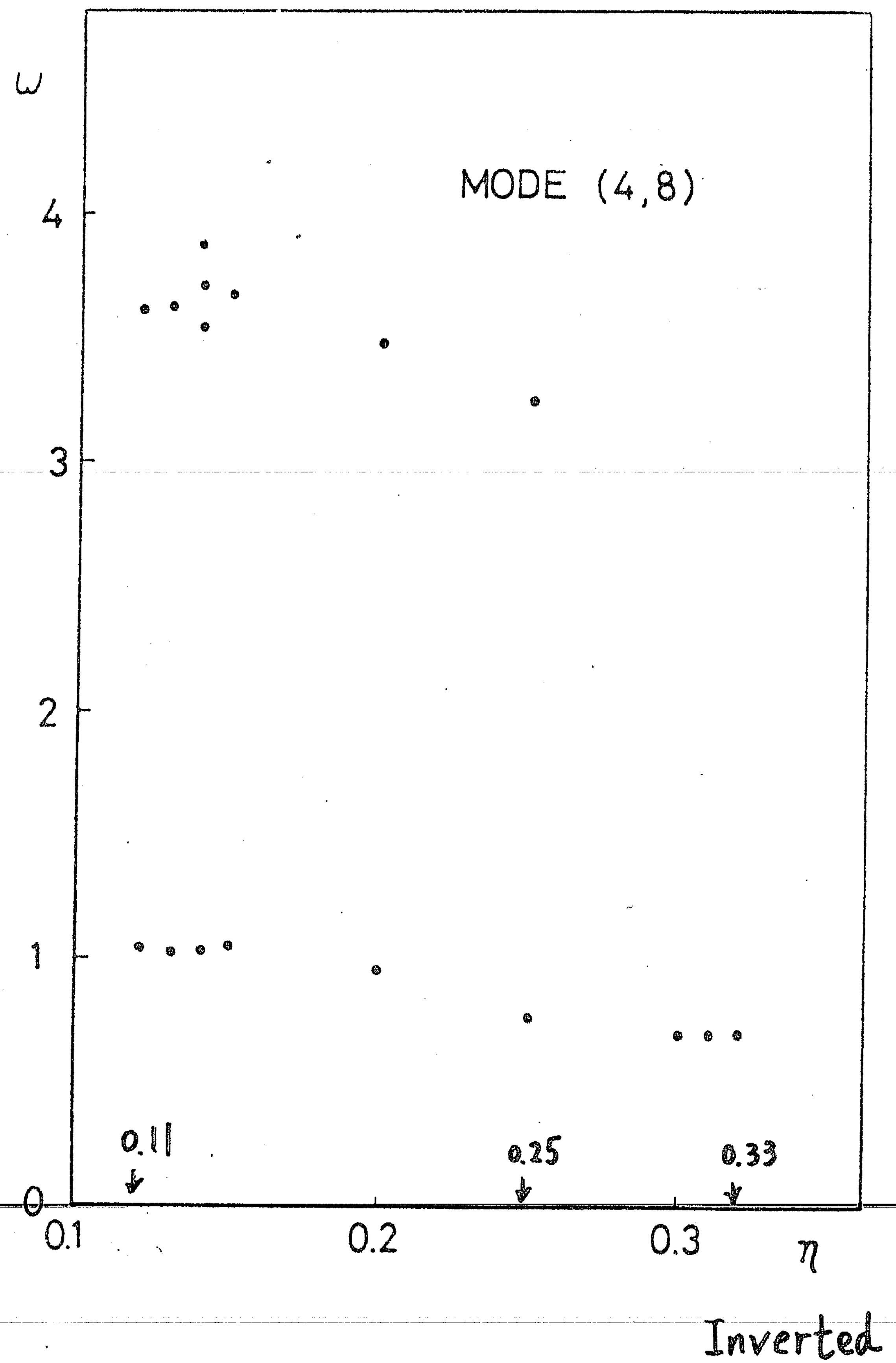


Fig. 6

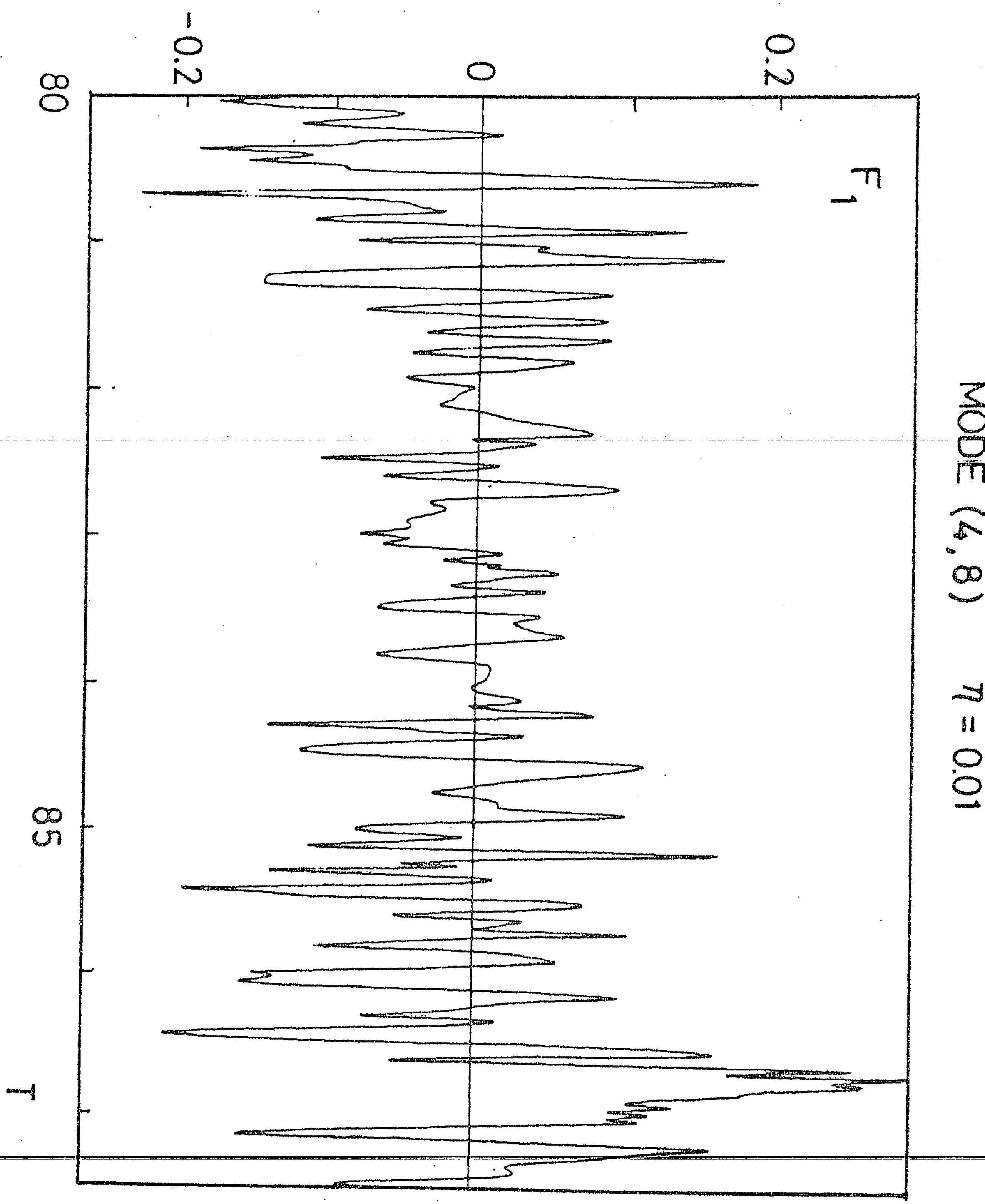


Fig. 8

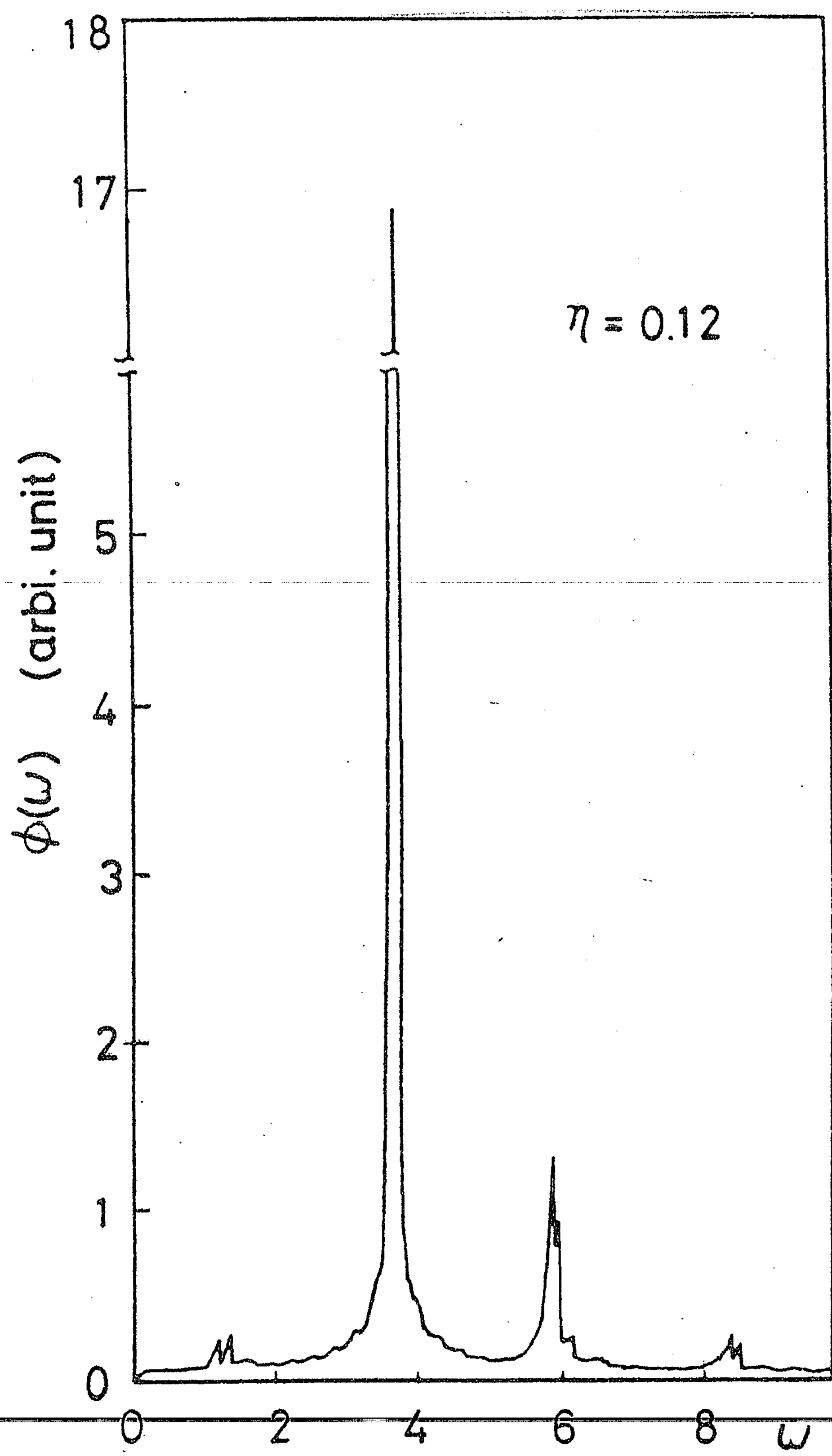


Fig. 7a

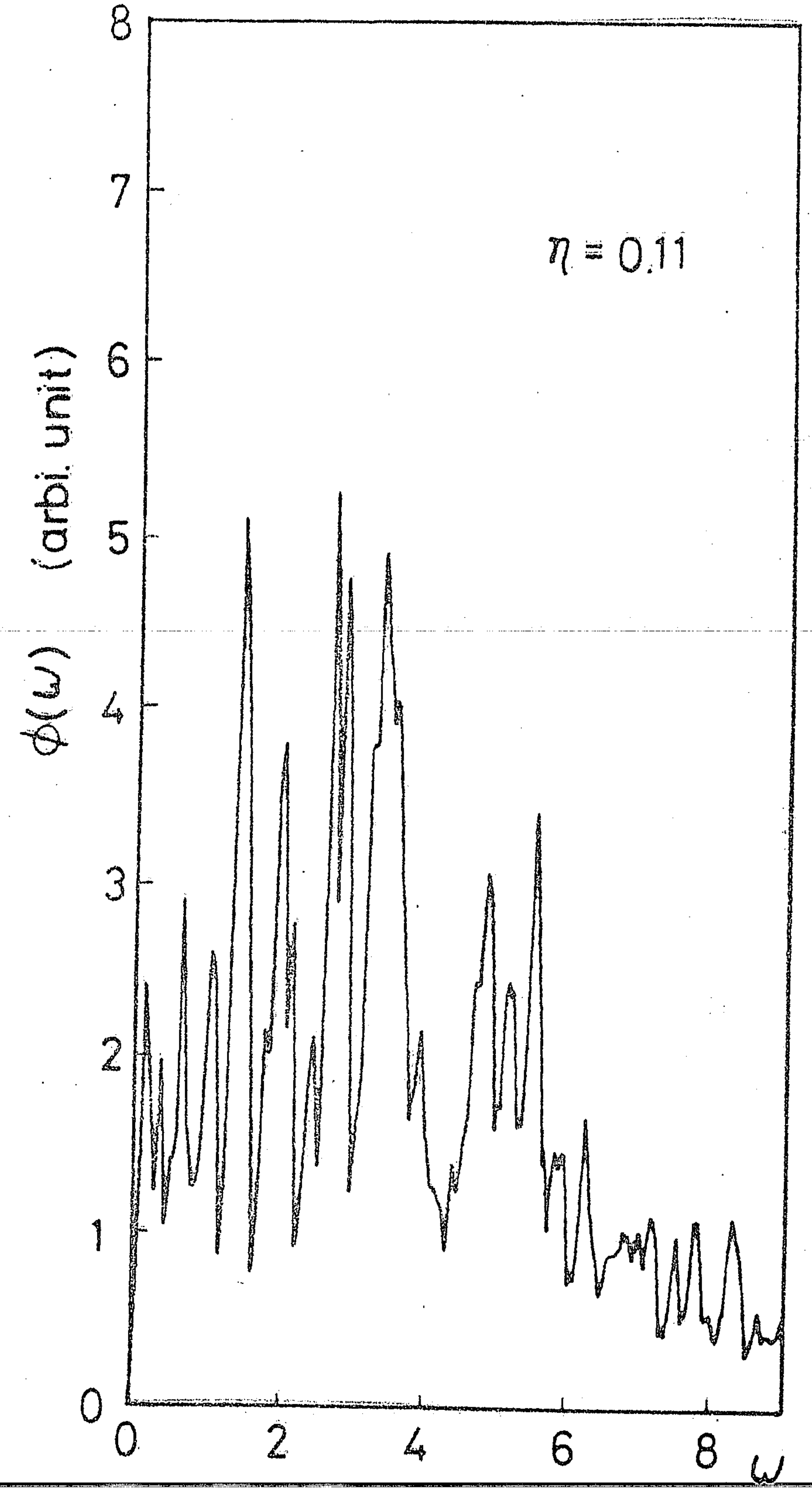
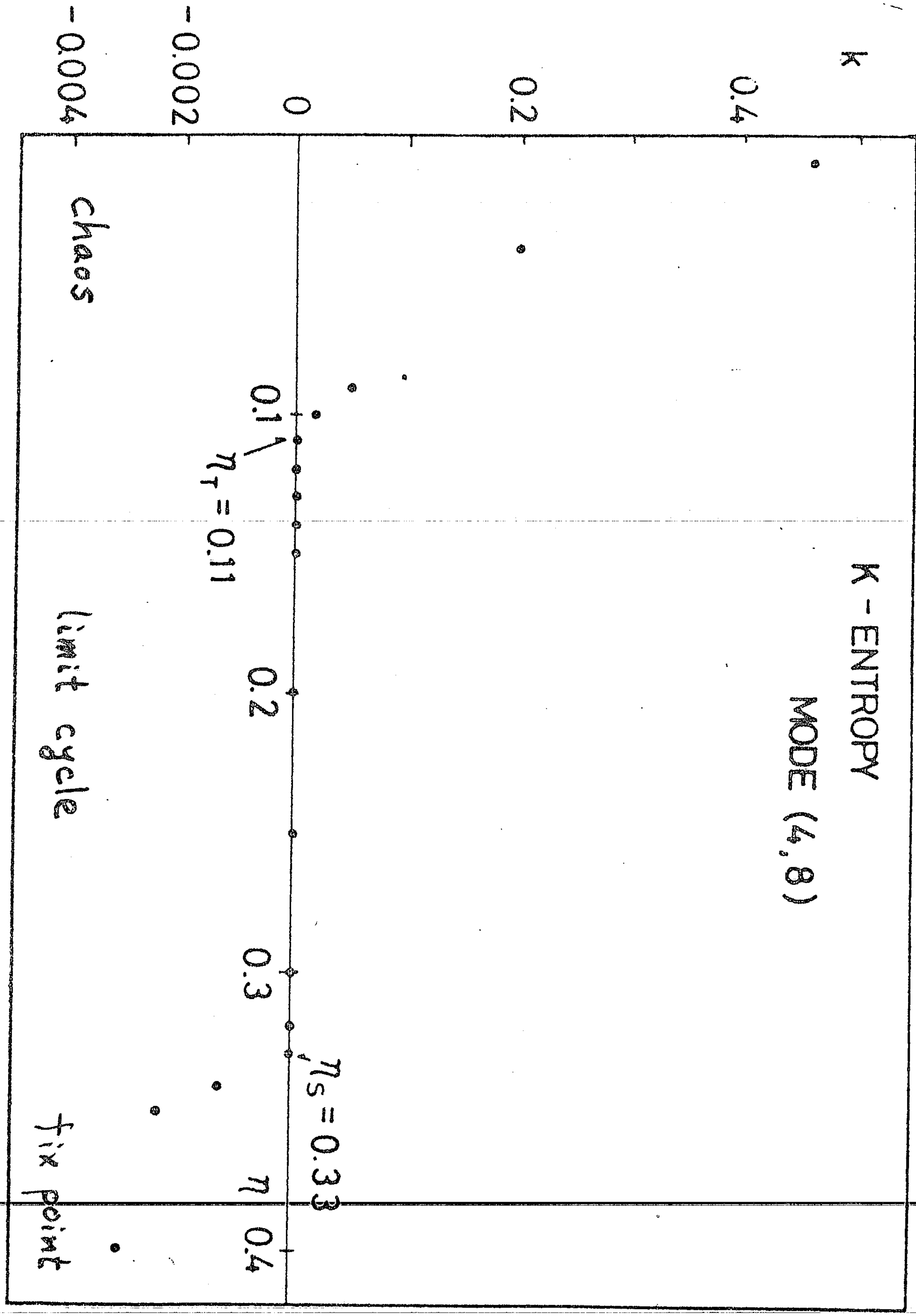


Fig. 7b


K - ENTROPY
MODE (4,8)
k
0.4
0.2
0
-0.002
-0.004
0.1
$\eta_T = 0.11$
0.2
0.3
$\eta_S = 0.33$
η
0.4
chaos
limit cycle
fix point

	p-p	p-2p
1		
2	Inverted	Normal
3	I	N
4	N	I
5	I	N
6	I	N
7	I	N
8	I	N
9	I	N
	⋮	⋮
16	I	N

Summary

(1) A systematic perturbation scheme is given for the collisional drift instability
— weakley dissipative κ- expansion.

(2) Existence of chaos is demonstrated.

(3) "Phase transition" of transport coefficient.

(4) Problem of reduction scheme to a finite degree of freedom from a continuous system.

- As the degree of freedom increases, the critical viscosity decreases.
- Type of bifurcation depends on p_{max}.
- Type of bifurcation depends on a truncation scheme [(p, p) or (p, 2p)].

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Successive Bifurcations ~~due~~ to Chaotic state in the Nonlinear Evolution of Collisional Drift Wave

THEORY OF TWO-POINT CORRELATION FOR TRAPPED ELECTRONS AND THE FREQUENCY SPECTRUM OF DRIFT WAVE TURBULENCE IN TOKAMAKS

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Theory of Two-Point Correlation for Trapped Electrons and the Frequency Spectrum of Drift Wave Turbulence in Tokamaks.

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T. Boutros-Ghali, P. L. Similon.

1.)

Motivation

i.) 2-Fold goal of D.W.T. studies $\langle$ D / spectrum

ii.) Current studies in kinetic theory $\Leftrightarrow$

Renormalized Coherent Response (1 Pt. ; $\omega = \omega(k)$).

Results:

1) Attractive : D, $N(k)$

2) Unattractive : Fails on $N(\omega)$, $\Delta\omega_k = 0$
Sensitivity to eigenmode structure
γ (?)

$\Rightarrow$

iii.) Formulate Renorm. Kinetic Theory of Spectrum $\langle \delta g \, \delta g \rangle \Leftrightarrow \langle \tilde{\Phi}^2 \rangle$

1) Admits spectrum constituents other than waves; $\Delta\omega_k \neq 0$ (Incoherent flucutustions)

2) Re-assess γ_k. ; 3) Unify Theory D, $N(k)$, $N(\omega)$, γ_k.

iv.) Specifics

1) Theory of 2 Pt. Correlation for Trapped Electrons

~~2) Renorm. of (Equal Time) Correlation Eqn. (2Pt. DIA)~~

$\rightarrow$ 3) Noise Mech. $\Leftrightarrow$ Mode Coupling in <u>wave</u> - <u>particle</u> <u>resonant</u> system.

Outline of Program

i.) Nonlinear Eqn. for Bounce Averaged Dist. – Ballooning Representation

ii.) Constr. and Renorm. of 2 Pt. Corr. Eqn $\Leftrightarrow$ Trapped Electrons

iii.) (Approx.) Calculation of Correlation Fctn.

iv.) Derive Integral Eqn. for Spectrum Structure

v.) Mechanism for Free Energy Extraction $\Leftrightarrow$ 2 Pt. Theory.

vi.) (Approx. Soln.) Spectrum Eqn. $\Rightarrow \Delta\omega_k$, γ_k.

vii.) Results: Analytical Expression $\Delta\omega_k$; $\underline{\Delta\omega_k \sim \omega_k}$

viii.) Saturation Criterion $\Leftrightarrow$ Impact Incoherent noise on Compton scattering

ix.) Determination $N(k)$, D $\Leftrightarrow$ Schematics

x.) Comment on Stability Pblm.

3.)

Basic Nonlinear Eqn.

i.) Non-Lin. Elec. DKE – Ballooning Rep. { Chen, Frieman; Swartz, et al.

$$\begin{cases} -i(\omega-\omega_D(\eta))\hat{g}_{\substack{n\\ \omega}} + \frac{V_\parallel}{Rq}\frac{\partial}{\partial\eta}\hat{g}_{\substack{n\\ \omega}} + N_{\substack{n\\ \omega}} = i\frac{|e|}{T}(\omega-\omega_{*e})\langle f\rangle\hat{\phi}_{\substack{n\\ \omega}}(\eta) \\ N_{\substack{n\\ \omega}} = \frac{c}{B_0}\sum_{n',\omega'}\sum_{m'}\left\{k_\theta k_\theta'\hat{s}(2\pi m')e^{2\pi i n' q m'}\hat{\phi}_{\substack{n'\\ -\omega'}}(\eta+2\pi m')\,g_{\substack{n+n'\\ \omega+\omega'}}\right\} \end{cases}$$

ii.) B. F. $\rightarrow$ { n (good quant. #) parametrizes fluct.; $k_r' = -\frac{n'q}{r}(\eta-\eta_{0k})$; $k_\theta' = \frac{nq}{r}$

iii.) $\eta_{0k} = 0$ (outside torus)

constrains { $k_r' = 2\pi m'\hat{s}k_\theta'$; $\phi_{\substack{-n'\\ -\omega'}}(\eta+2\pi m')$

iv.) $\omega_b > \omega, \omega_{NL} \Leftrightarrow \frac{V_\parallel}{Rq}\frac{\partial}{\partial\eta}\hat{g}^{0}_{\substack{n\\ \omega}} = 0$

Energy Scattering Out!

$$-i(\omega-\bar{\omega}_D\epsilon)\hat{g}_{\substack{n\\ \omega}} + \bar{N}_{\substack{n\\ \omega}} = i\frac{|e|}{T}(\omega-\omega_{*e})\langle f\rangle\bar{\phi}_{\substack{n\\ \omega}}$$

$$\bar{N}_{\substack{n\\ \omega}} = \frac{c}{B_0}\sum_{n',\omega'}\sum_{m'}k_\theta k_\theta'\hat{s}(2\pi m')e^{2\pi i n' q m}\,\overline{\Phi_{n'}(\eta+2m'\pi)}\,g_{n+n'}$$

$g_{n'} \equiv$ Bounce Avg. quasimode distribution Fctn.
(η independent)

Construction of (Equal Time) 2 Pt. Correlation Eqn.

i) $\hat{g} = \sum_n e^{in\phi}\hat{g}_n$, $\quad \hat{\omega}_{b} = \omega_{*}/n$; $\quad \bar{E}_\phi = -\frac{\partial}{\partial\phi}\bar{\phi}$.

$$\frac{\partial}{\partial t}\hat{g} + \hat{\omega}_b \epsilon \frac{\partial}{\partial \phi} g - \frac{c}{B_0} \nabla_\perp [\bar{\phi}] \times \hat{n} \cdot \nabla_\perp' [\hat{g}] =$$

$$-\frac{|e|}{T}\left\{ \langle f \rangle \frac{\partial}{\partial t}\bar{\phi} + \hat{\omega}_{*e} \bar{E}_\phi \right\}$$

$$-\frac{c}{B_0} \nabla_\perp [\bar{\phi}] \times \hat{n} \cdot \nabla_\perp [\hat{g}] = \sum_n e^{in\phi}\left(-\frac{c}{B_0} \sum_{n',\omega} \sum_{m'} (2\pi m' \hat{S}) k_\theta k_\theta' \, e^{2\pi i n q m'} \bar{\phi}_{-n', -\omega'}(n + 2\pi m) \hat{g}_{n+n', \omega+\omega'} \right).$$

$\Downarrow$

(all conventional properties)

$\Rightarrow$

i.) $\underset{(T)}{\frac{\partial}{\partial t}\langle g(1) g(2)\rangle} + \overset{(B)}{\hat{\omega}_b\left(\epsilon_1 \frac{\partial}{\partial \phi_1} + \epsilon_2 \frac{\partial}{\partial\phi_2}\right)\langle g(1) g(2)\rangle}$ (T)

$$-\frac{c}{B_0}\left\langle \nabla_{\perp 1}[\bar{\phi}(1)] \times \hat{n} \cdot \nabla_\perp [g(1)] \hat{g}(2) \right\rangle - \frac{c}{B_0}\left\langle \nabla_{\perp 2}[\bar{\phi}(2)] \times \hat{n} \cdot \nabla_{\perp 2}[g(2)] g(1) \right\rangle$$

(S) $\qquad$ (S)

$$= -\frac{|e|}{T}\left\{ \langle f(1)\rangle \langle g(2) \frac{\partial}{\partial t}\bar{\phi}(1)\rangle + \hat{\omega}_{*} \langle f(1)\rangle \langle g(2) \bar{E}_\phi(1)\rangle \right\} + (1 \Leftrightarrow 2)$$

ii.) B $\Leftrightarrow$ Ballistic Term ; $\epsilon \, \partial/\partial\phi$.

T $\Leftrightarrow$ Triplet : $\langle (\bar{E}_\perp(1) \times \hat{n} - \bar{E}_\perp(2) \times \hat{n}) \cdot \nabla_\perp \hat{g}(1) \hat{g}(2) \rangle$

$T \Rightarrow 0$ as $1 \Leftrightarrow 2$

S $\Leftrightarrow$ "Source" : <u>Finite</u> as $1 \Leftrightarrow 2$ { Gradient $\Rightarrow$ Free Energy.

(microscale)

5.)

Renormalization of 2 pt. Eqn.

i) Closure of Triplet ; ($t_{slow} \to \infty$)

$\begin{cases} \phi_\pm = \phi_1 \pm \phi_2 \\ \text{Avg. on } \phi_+ . \end{cases}$

$$T = \sum_B \sum_{\substack{n' \\ \omega'}} e^{in(\phi_1-\phi_2)} \Big\{ \Big\langle k_\perp' [\bar{\Phi}^{(1)}_{-n', -\omega'}] \times \hat{n} \cdot k_\perp [\hat{g}^{(2)}_{\substack{n+n' \\ \omega+\omega'}}(1)] \hat{g}_{\substack{-n \\ -\omega}}(2) \Big\rangle$$

$$- e^{in'(\phi_2-\phi_1)} \Big\langle k_\perp' [\bar{\Phi}^{(2)}_{-n', -\omega'}] \times \hat{n} \cdot k_\perp [\hat{g}_{\substack{-n \\ -\omega}}(1)] \hat{g}^{(2)}_{\substack{n+n' \\ \omega+\omega'}} \Big\rangle \Big\} + \cancel{(\;)}$$

To close:

$$\begin{cases} \hat{g}^{(2)}_{\substack{n+n' \\ \omega+\omega'}} = L_{\substack{n+n' \\ \omega+\omega'}} \dfrac{c}{B_0} \sum_{m'} (2\pi m') k_\theta k_\theta' \hat{s}\, e^{-2\pi i n_2' m'} \Big\{ \bar{\phi}_{\substack{n' \\ \omega'}}(n+2\pi m') \; \hat{g}_{\substack{n \\ \omega}} - \hat{g}_{\substack{n' \\ \omega'}}(\;) \; \bar{\phi}_{\substack{n \\ \omega}} \Big\} \\ L^{-1}_{\substack{n+n' \\ \omega+\omega'}} = \left[-i(\omega+\omega') + i(\bar{\omega}_0 + \bar{\omega}_0')\epsilon\right] \end{cases}$$

(Irrev. ⇔ wave/particle resonance; contrast hydro ⇔ μ_{NL}).

Consider 1) Resonant $\hat{g}$ ⇒

$$L_{\substack{n+n' \\ \omega+\omega'}} \to L_{\substack{n' \\ \omega'}}$$

[Reson → Particle Scat. Terms]

2) Retain terms so that $T \to T_{op} \langle \hat{g}(1) \hat{g}(2) \rangle$.

⇒

$$\begin{cases} T = -\dfrac{c^2}{B_0^2} \sum_{\substack{n \\ \omega}} \sum_{\substack{n' \\ \omega'}} \sum_m (2\pi m)^2 k_\theta'^2 k_\theta^2 \hat{s}^2 e^{in(\phi_-)} \Big\{ L_{\substack{n' \\ \omega'}}(1) \langle \bar{\phi}(1)^2 \rangle_{\substack{n' \\ \omega'}} \langle \hat{g}(1) \hat{g}(2) \rangle_{\substack{n \\ \omega}} \\ \quad - L^{(2)}_{\substack{n' \\ \omega'}} e^{2\pi i m \, r/\Delta} e^{in'\phi_-} \langle \bar{\phi}(1) \bar{\phi}(2) \rangle_{\substack{n' \\ \omega'}} \langle \hat{g}(1) \hat{g}(2) \rangle_{\substack{n \\ \omega}} \Big\} \end{cases}$$

$$\begin{cases} = -\left(D^{(1,1)} \dfrac{q^2}{r^2} \dfrac{\partial^2}{\partial \phi_1^2} \langle \hat{g}(1) \hat{g}(2) \rangle + D^{(1,2)} \dfrac{q^2}{r^2} \dfrac{\partial^2}{\partial \phi_1 \partial \phi_2} \langle \hat{g}(1) \hat{g}(2) \rangle \right) \\ \qquad \downarrow \qquad\qquad\qquad\qquad\qquad \downarrow \\ \quad (1\text{ pt.}) \qquad\qquad\qquad (\text{cross operator}) \Leftrightarrow \text{M.C.} \end{cases}$$

6)

Source Term, $S_{1,2}$

$$S_{1,2} = -\frac{|e|}{T}\langle f(1)\rangle\left\{\langle \tilde{g}(2)\frac{\partial}{\partial t}\bar{\phi}\rangle + \omega_{*e}\langle \tilde{g}(2)\bar{E}_\phi(1)\rangle\right\}$$

$$\Rightarrow \quad \tilde{g} = g^c_{\omega} + \tilde{g}^{n}_{\omega} \quad ; \quad g^c = -\frac{|e|}{T}\langle f\rangle\frac{(\omega-\omega_{*e})}{\omega-\bar{\omega}_D t}\bar{\phi}$$

$\tilde{g} \Leftrightarrow$ incoherent

$$S_{1,2} = C(1,2)\langle f(1)\rangle\langle f(2)\rangle + \bar{F}(1,2)\langle f(1)\rangle$$

$$\underset{\text{coherent}}{C(1,2)} = \sum_{n,\omega} e^{in\phi} L^{n}_{\omega}\left(\frac{|e|}{T_e}\right)^2(\omega-\omega_{*0})^2\langle\bar{\phi}(1)\bar{\phi}(2)\rangle^{n}_{\omega}$$

$$\underset{\text{incoherent}}{\bar{F}(1,2)} = \sum_{n,\omega} e^{in\phi}\frac{i|e|}{T}(\omega-\omega_{*e})\langle\tilde{g}(2)\bar{\phi}(1)\rangle^{n}_{\omega}$$

$$\frac{\partial}{\partial t}f^2 = 0 \quad \Rightarrow \quad \frac{\partial}{\partial t}\langle \delta f^2\rangle\Big|_{x_-\to 0} = -\frac{\partial}{\partial t}\langle f\rangle^2$$

$$\partial\langle f\rangle/\partial t = D\langle f\rangle'' - \bar{F}\langle f\rangle'$$

$$C(1,2) \Leftrightarrow D \quad ; \quad \bar{F}(1,2) \Leftrightarrow \bar{F}.$$

7.)

Structure of Renorm. 2 PT. Equation

$\bar{V} = \epsilon_T V_+$
$y_- = r\phi_- / q$

i.) $\frac{\partial}{\partial t}\langle \tilde{g}(1)\tilde{g}(2)\rangle + \bar{V}\,\epsilon_- \frac{\partial}{\partial y_-}\langle \tilde{g}(1)\tilde{g}(2)\rangle - D_- \frac{\partial^2}{\partial y_-^2}\langle \tilde{g}(1)\tilde{g}(2)\rangle$

$= C(1,2)\langle f(1)\rangle\langle f(2)\rangle + \bar{F}(1,2)\langle f(1)\rangle$

$D_- = \left[2D - D^{1,2} - D^{2,1}\right] \underset{x_- \to 0}{\to} 0$ ✓ $\qquad D = D^{1,2}|_0$

$D^{12} = \frac{c^2}{B_0^2}\sum_{n',\omega'} e^{ik y_- - k_\theta^2 \hat{s}^2}\, L_{n',\omega'} \sum_n (2\pi n)^2 e^{2\pi i n \frac{r}{\Delta}} \langle \tilde{\phi}(1)\tilde{\phi}(2)\rangle_{n',\omega'}$

(k ignored)

ii.) $\langle \tilde{g}(1)\tilde{g}(2)\rangle$ determined by
1) input from source (L_n^{-1})
2) decay by a) relative magnetic drift
b) relative $\underset{\sim}{E}\times\underset{\sim}{B}$ diffn.

iii.) Scales of Collective Correlation (Clump)

$\phi_- < 1/k_\theta$; $\quad r_- < \Delta = 1/k_\theta \hat{s}$ (Ballooning Corr.)

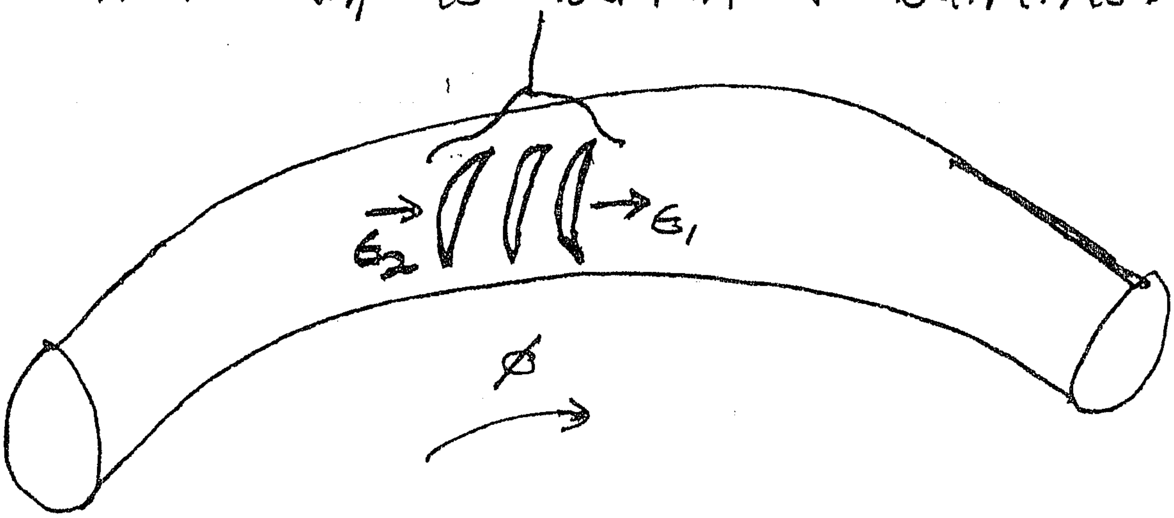
$\epsilon_- < \Delta\epsilon_T \qquad \Delta\epsilon_T \equiv$ Energy Resonance Width

defined: $\Delta\epsilon_T\, \bar{\omega}_D\, \tau_c = 1$; $\quad \tau_c = 1/k_\theta^2 D$.

$\phi_- < 1/k_\theta$, $r_- < \Delta$, $\epsilon_- < \Delta\epsilon_T$, $\tau_{cor} > \tau_c \Rightarrow$ Particles "clumped."

iv.) Key is B.F. representation $\underset{\sim}{E}\times\underset{\sim}{B}$ diffusion

T. P. Clump as "bunch of bananas."



$$\varepsilon_- < \Delta\varepsilon_T$$

$$\phi_- < 1/n$$

$$\Gamma_- < \Delta.$$

Calculation of Correlation Function

$$\langle \tilde{g}(1)\tilde{g}(2)\rangle = \tau_{CL}(y_-, \epsilon_-, r_-)\, S_{1,2}$$

(i.) τ_{CL} from:
$$\left(\frac{\partial}{\partial t} + \bar{V}\epsilon_-\frac{\partial}{\partial y_-} - D\frac{\partial^2}{\partial y_-^2}\right) G = 0.$$

(ii.) Moments $\Rightarrow$
$$\frac{\partial}{\partial t}\langle y_-^2\rangle = \langle D_-\rangle + 2\bar{V}\langle \epsilon_- y_-\rangle$$
$$= k_\theta^2 D\langle y_-^2\rangle + k_\theta^2 D\bar{s}^2\langle r_-^2\rangle + 2\bar{V}\langle \epsilon_- y_-\rangle$$

$$\frac{\partial}{\partial t}\langle \epsilon_- y_-\rangle = \bar{V}\langle \epsilon_-^2\rangle$$

$\Rightarrow$
$$\frac{\partial^2}{\partial t^2}\langle y_-^2\rangle - k_\theta^2 D\frac{\partial}{\partial t}\langle y_-^2\rangle = 2\bar{V}^2\epsilon_-^2$$

$$\frac{\partial}{\partial t}\langle r_-^2\rangle = 0$$

("Fast" radial diffn in ϕ diffn.)

I.C.'s:
$$\begin{cases}\langle y_-^2\rangle = y_-^2 \\ \frac{\partial}{\partial t}\langle y_-^2\rangle = k_\theta^2 D y_-^2 + \bar{s}^2 k_\theta^2 D\, r_-^2 + 2\bar{V}\epsilon_- y_-\end{cases}$$

(iii.) τ_{CL} from $k_\theta^2\langle y_-^2(\tau_{CL})\rangle = 1$
$$\tau_{CL} = \tau_0 \ln\left[1/k_\theta^2\left(y_-^2 + 2\bar{V}\epsilon_- y_-\tau_0 + 2\bar{V}^2\epsilon_-^2\tau_0^2 + \bar{s}^2 r_-^2\right)\right]$$

Here, no coupling streaming and diffn.

9.)

Incoherent Potential Spectrum

i.) $t \to \infty$; follow TPM to develop fluctuation theory

1 time eqn. as I.C. for 2 time $\Rightarrow$

a) $\langle \tilde{g}\tilde{g} \rangle_{k,\omega} = 2\, \mathrm{Re}\, L_{k,\omega} \langle \tilde{g}\tilde{g} \rangle_k \simeq 2\pi\, \delta(\omega - \bar{\omega}_D \epsilon) \langle \tilde{g}\tilde{g} \rangle_k$

↳ T.P. ballistic prop.

b.) "Dressed Clump" Picture $\Rightarrow$

$n_i - n_e^c = \tilde{n}_e \Rightarrow L \frac{|e|}{T} \tilde{\phi} = \tilde{n}_e$

$\Rightarrow \tilde{\phi} = L^{-1} \frac{T}{|e|} \tilde{n}_e = L^{-1} \tilde{\phi}$;

ii.) $\tilde{g} = g^c + \tilde{g}$, seek $\langle \tilde{g}^2 \rangle$

$\langle \tilde{g}\tilde{g} \rangle = \langle \tilde{g}\tilde{g} \rangle - \langle g^c g^c \rangle - 2 \langle g^c \tilde{g} \rangle$

$= (\mathcal{J}_{CL} - \mathcal{J}_C) \{ C(1,2) \langle f(1) f(2) \rangle + F(1,2) \langle f(1) \rangle \}$

$\mathcal{J}_{CL}$: ∞ at $x \to 0$, $\mathcal{J}_C$: finite

iii.)
$$\boxed{\langle \tilde{\phi}^2 \rangle_{k,\omega} = \frac{A(k)}{\epsilon_T^2 V_T^2}\, u \left[C(u) \left(F(u) \right)^2 + F(u) \left(F(u) \right) \right]}$$

$u = |\omega / \omega_D|$

$A = \frac{(2\pi)^2}{|e| k^2 |k|} \left(1 - J_0\left(\frac{2k}{k_D} \right) \right)$

$\Downarrow$

Integral Eqn. for $\langle \tilde{\phi}^2 \rangle$!

$\Leftrightarrow$ Need examine L^{-1} (1), Source str. (2)

D) $L^{-1}_{k,\omega}$ & Relation $\langle \bar{\phi}^2 \rangle_{k,\omega}$ to $\langle \tilde{\phi}^2 \rangle_{k,\omega}$

i.) $L_{k,\omega} \Phi_{k,\omega}(\eta) = 0$ — Lowest Order B.F eigenmode eqn.

$\Rightarrow$

$$L^{-1}_{k,\omega} A = \int d\eta' \, G_{k,\omega}(\eta,\eta') A(\eta')$$

$$G_{k,\omega}(\eta,\eta') = \sum_\alpha \psi_\alpha(\eta)\psi_\alpha(\eta') / N_\alpha \, d_\alpha(k,\omega)$$

ψ_α ↓ wave Fctn. ; d_α ↓ dispersion fctn.

$$d = \left(\frac{\omega_*}{\omega} - 1 - k^2\rho_s^2\right) + d_{IM}.$$

ii.) Here $L \Phi_{k,\omega} = \tilde{\phi}_{k,\omega}$

↳ from bounce-averaged dynamics
⇒ no η structure in source.

$\Rightarrow$

$$\bar{\phi}_{k,\omega} = \sum_\alpha \bar{\psi}_\alpha \int_{-\infty}^{+\infty} \frac{\psi_\alpha(\eta')\tilde{\phi}_{k,\omega}}{N_\alpha d_\alpha(k,\omega)}$$

1 α dominant.

$$= \frac{S_{k,\omega}}{d(k,\omega)} \tilde{\phi}_{k,\omega} \quad ; \quad S = \bar{\psi}_\alpha \int_{-\infty}^{+\infty} \frac{d\eta' \cdot \psi_\alpha(\eta')}{N_\alpha}.$$

iii.) $S \Leftrightarrow$

Bounce Avgd. Structure Function ~ Real

$S_{PB} \sim \sqrt{L_n/L_s}$; $S_{TI} \sim 1/2$

↳ mode extended along field line

$S, d \rightarrow$ Linear Theory Info.

11.)

2) Source Term ⟷ Free Energy Extraction Mech.

i.) $S(u) = C(u)\,(F(u))^2 + \bar{F}(u)\,(F(u))$

$$S(u) = \sum_{n',\omega'} \left[2 L_{n',\omega'} \frac{(\omega' - \omega_{*e}')\, S_{n',\omega'}' F(u)}{|d(n',\omega')|^2} \left\{ (\omega' - \omega_{*e}')\, S_{n',\omega'}' \frac{F(u)}{2} \langle \tilde{\phi}^2 \rangle_{n',\omega'} + d_{IM}(n',\omega') \frac{T_e}{|e|} \langle \tilde{g}(u)\, \tilde{\phi} \rangle_{n'} \right\} \right]$$

ii.) $S(u) > 0 \;\Leftrightarrow\; d_{IM}(k',\omega') < 0 \;\Rightarrow\; \gamma_{k'} < 0$

$\Rightarrow$ Modes damped (> saturation), to absorb driving of noise source

iii.)
$$\begin{cases} d_{IM}(k',\omega') = -S_{k',\omega'}' (\omega' - \omega_{*e}') \sqrt{u'} \dfrac{F(u')}{|\bar{\omega}_D'|} + d_{IM}^{ION}(k',\omega') \\ \langle \tilde{\phi}^2 \rangle_{k',\omega'} = 2 \dfrac{T_e}{|e|} \dfrac{\sqrt{u'}}{|\bar{\omega}_D'|} \langle \tilde{g}(u')\, \tilde{\phi} \rangle_{k'} \end{cases}$$

d_{IM}^{ION} ↓ (stabilizing).

$$\Rightarrow S(u) = \sum_{k',\omega'} 2 \left[\frac{(\omega' - \omega_{*e}')\, S_{k',\omega'}' F(u)\, R(\omega' - \bar{\omega}_D' u)}{|d(k',\omega')|^2} \right] \left\{ \left((\omega' - \omega_{*e}')\, S_{k',\omega'}' \frac{T_e}{|e|} \frac{\sqrt{u'}}{|\bar{\omega}_D'|} \right) \leftarrow \right.$$

$$\left. \left(F(u) \langle \tilde{g}(u') \tilde{\phi} \rangle_{k'} - F(u') \langle \tilde{g}(u) \tilde{\phi} \rangle_{k'} \right) + \frac{T_e}{|e|} \langle \tilde{g}(u)\, \tilde{\phi} \rangle_{k'}\, d_{IM}^{ion}(k',\omega') \right\}$$

(In the last line the $\tilde{g}(u')$ and $\tilde{g}(u)$ terms in the first bracket are struck through by hand.)

a) Like-particle pieces of diffn. and drag cancel, a la TPM, $d\langle f \rangle/dt$.

b) Cons. energy + mom. in 1D u, u' interaction $\Rightarrow$
$u, u' \to u', u$ $\Rightarrow$ initial state = final state
$\Rightarrow$ no source contribution.

v.) $$S(u) = -\sum_{k',\omega'} 2\frac{T_e}{|e|}\left[(\omega-\omega_{*e})\, S_{k',\omega'}\, \frac{\delta(\omega'-\bar{\omega}_D' u)}{|d(k',\omega')|^2} \langle \tilde{g}\phi \rangle_H |d_{IM}^{ion}|\right]$$

Note: 1) $S_{\underline{Electron}} \sim \langle \omega - \omega_{*e} \rangle (-|d_{IM}^{ion}|)$

⇒ Ion dissipation in electron source via (inter-species) drag.

2) By comparison,

$S_{\underline{ION}} \sim \langle \omega - \omega_{*i} \rangle\, d_{IM}^{Elec.}$

3) Drag induced enhanced accessibility of electron free energy via d_{IM}^{ion}

⇔ General result for electron D.W.

4) d_{IM}^{ion} stabilizes system, ⇒ S_E large

Key: Like-particle term cancellation uncovers large source in ion dissipation.

[d_{IM}^{Ion} : shear damping, NLLD, etc.]

12.)

v.) $$S(u) = -\sum_{k',\omega'} 2\frac{T_e}{|e|}\left[(\omega-\omega_{*e})\, S_{k',\omega'} \frac{\delta(\omega'-\bar{\omega}_D' u)}{|\epsilon(k',\omega')|^2} \langle \tilde{g}\phi\rangle_H |d_{IM}^{ion}|\right]$$

Note: 1) $S_{\underline{Electron}} \sim \langle \omega-\omega_{*e}\rangle(-|d_{IM}^{ion}|)$

$\Rightarrow$ Ion dissipation in electron source via (inter-species) drag.

2) By comparison

$$S_{ION} \sim \langle \omega-\omega_{*i}\rangle\, d_{IM}^{Elec.}$$

3) Drag induced enhanced accessibility of electron free energy via d_{IM}^{ion}

$\Leftrightarrow$ General result for electron D.W.

4) d_{IM}^{ion} stabilizes system, $\Rightarrow S_E$ large

Key: Like-particle term cancellation uncovers large source in ion dissipation.

[d_{IM}^{Ion} : shear damping, NLLD, etc]

Determination $\Delta\omega_k$

i.) Spectrum Eqn. : (up to $N(u)$)

$$\langle \tilde{g}(u)\tilde{\phi}\rangle_k = 2\bar{A}(k)\frac{\sqrt{u}}{\epsilon_T}(1-\epsilon_T u)\int \frac{dk'/k' \langle \tilde{g}(u)\tilde{\phi}\rangle_{k'} \, |d_{IM}(k', \bar{\omega}_0' u)|}{|d(k', \bar{\omega}_0' u)|^2}$$

ii.) Basic Picture:

$$\langle \tilde{\phi}^2\rangle_{k,\omega} \cong \frac{|S_{k,\omega}|^2 \langle \tilde{\phi}^2\rangle_{k,\omega}}{(\omega-\omega_k)^2\left|\frac{\partial d_0}{\partial \omega_k}\right|^2 + |d_{IM}(k,\omega_k)|^2}$$

$\langle \tilde{\phi}^2\rangle_{k,\omega} = 0 \Rightarrow d(k,\omega) = 0$
$\omega = \omega_k;\ d_{IM}(k,\omega_k) = 0$

$\langle \tilde{\phi}^2\rangle_{k,\omega} \neq 0 \Rightarrow d_{IM}(k,\omega) < 0$

Constituents of Spectrum
1) T.E. clumps; noise source
2) Over-saturated modes.

$\Rightarrow$
iii.) Turbulent State $\Leftrightarrow$ Broadened (by over-damp) Collective Resonances.

Resonances at ω_k; with $\Delta\omega_k = |d_{IM}(k,\omega_k)| \Big/ \left|\frac{\partial d}{\partial \omega}\right|$
(under-est.)

$\Rightarrow$
iv.) Need determine $|d_{IM}(k,\omega_k)|$. $\Rightarrow \Delta\omega_k$

:) Lorentzian Model

$$\langle \tilde{g}(u)\tilde{\phi}\rangle_k = 2\bar{A}(k)\frac{\sqrt{u}}{\epsilon_T}(1-\epsilon_T u)F(u)\int \frac{dk'/k'/S_{k'}\ \langle \tilde{g}(u)\tilde{\phi}\rangle_{k'}\ |d_{IM}(k',\omega_0' u)|}{\left[\left((k-k_r'(u))\left|\frac{\partial d}{\partial k}\right|\right)^2 + |d_{IM}(k,k_r(u))|^2\right]}$$

Numerator sampled at $k_r(u) \Rightarrow$

$$|d_{IM}(k,\omega_k)| = C(k,\omega_k)\,|d_{IM}^{ion}(k,\omega_k)|$$

$$C(k,\omega_k) = 2\sqrt{\pi}\,A(k\rho_s)\frac{S}{\epsilon_T}\exp\left[-\omega_k/\bar{\omega}_D\right]\sqrt{\frac{\omega_k}{\omega_{ke}}}\left(1-\frac{\omega_k}{\bar{\omega}_{ke}}\right)$$

$$d_{IM}(k,\omega_k) = d_{IM}^{T.E.} - |d_{IM}^{ion}| < 0 \quad \text{(Ampl. in } d^{ion}\text{)}$$

$\Rightarrow$

$$d_{IM}^{ion}(k,\omega_k) = d_{IM}^{T.E.}/\left[1-C(k,\omega_k)\right] \rightarrow \begin{cases}\text{Saturation}\\ \text{Condition}\end{cases}$$

$$\begin{cases} |d_{IM}(k,\omega_k)| = C(k,\omega_k)\,d_{IM}^{T.E.}/\left[1-C(k,\omega_k)\right] \\ \Delta\omega_k = C(k,\omega_k)\,\gamma_k^{TE}/\left[1-C(k,\omega_k)\right] \end{cases}$$

Eliminate C $\Rightarrow$ $\gamma_{ion}^{NL} = \Delta\omega_k + \left(\gamma_L^{T.E} - \gamma_{ion}^{L}\right)$

~~$[C \gtrsim 1 \Leftrightarrow$ Taylor Exp. Fails$]$~~

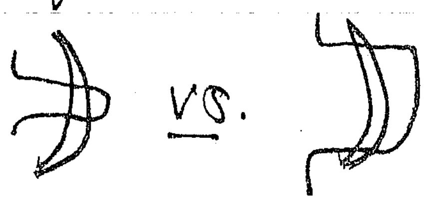
Results

i.) $\Delta \omega_k = C(k, \omega_k)\, \gamma_k^{T.E.} \Big/ \left[1 - C(k, \omega_k)\right]$

$$C(k, \omega_k) = 2\sqrt{\pi}\, A(k\rho_s) \sum_{\epsilon_T} S \exp\left[-\omega_k/\omega_D\right] \sqrt{\frac{\omega_k}{\omega_{*e}}} \left(1 - \frac{\omega_k}{\omega_{*e}}\right)$$

$\Delta\omega_k$ depends on: a.) $\frac{\omega_k}{\bar{\omega}_D} \to \epsilon_T$ and ω_k

Extent shielding → response in η. b.) S



ii.) Behavior:

$\frac{\Delta \omega_k}{|\omega_k|} \sim k^2\rho_s^2 \qquad k\rho_s \to 0$

$\frac{\Delta \omega_k}{|\omega_k|} \sim \text{flat} \qquad k\rho_s \simeq 1$

$\frac{\Delta \omega_k}{|\omega_k|} \sim 1/(k\rho_s)^6 \,, \qquad k\rho_s \to \infty$

~~iii.) Enhanced Growth $\quad d_{IM}^{ion} = d_{IM}^{T.E}/(1-C)$~~

$\Rightarrow \gamma_k^{T.E} = d_{IM}^{T.E}/(1-C)$

v.) Quantitative Results

$k\rho_s \equiv 1$

$$\frac{\Delta\omega_k}{|\omega_k|} = \left\{\frac{\pi}{2}\frac{S_k^2}{\epsilon_T^2}\exp[-1/\epsilon_T]\right\} \Big/ \left\{1 - \sqrt{2\pi}\frac{S_k}{\epsilon_T}\exp[-1/2\epsilon_T]\right\}$$

$$\frac{\gamma_k}{|\omega_k|} = \left\{\left(\sqrt{\frac{\pi}{2}}\frac{S}{\epsilon_T}\right)\exp[-1/2\epsilon_T]\right\} \Big/ \left\{1 - \sqrt{2\pi}\frac{S_k}{\epsilon_T}\exp[-1/2\epsilon_T]\right\}$$

$\epsilon_T \equiv 3.5$ here.

a.) P.-B. $\rightarrow S_k \sim \sqrt{L_n/L_s}$ $\qquad \frac{L_s}{L_n} \sim 10$

$$\frac{\Delta\omega_k}{|\omega_k|} \simeq .2$$

$$\frac{\gamma^{T.E}}{\omega_k} \sim .3$$

$$\frac{\gamma^{T.E.}}{\gamma_L} \sim 2.1$$

Also, for shear damping $\frac{\gamma^{SD}}{\omega_k} = \frac{L_n}{L_s}(1 + k^2\rho_s^2)$

~~$\gamma_k^{TE} - \gamma_k^{SD} > 0$, even for $\gamma_k^{T.E} - \gamma_k^{SD} < 0$~~

"Instability" persists for shears s/t linearly stable.

17.)

v.) T. I. $\Rightarrow \delta'_h \sim 1/2$

$\frac{\Delta\omega_h}{|\omega_h|} \simeq 1.1$; $\qquad \frac{\gamma^{T.E.}}{\omega_H} \simeq 1.2$

$\frac{\gamma^{T.E.}}{\gamma_L^{T.E.}} \simeq 5.9$

ic.) Note: $\delta' \sim O(1) \Leftrightarrow \Delta\omega_h \sim \omega_h$

n, x reciprocity $\Rightarrow \delta' \sim O(1) \Leftrightarrow \Delta x \sim \Delta \sim 1/k_\theta \delta'$

$\hookrightarrow \sim \frac{m_\theta}{r}$

$\delta \sim O(1) \Rightarrow$ Favorable to $\begin{cases} \Delta\omega_h \sim \omega_h \\ k_x \sim 1/\Delta \sim k_\theta \\ \text{(isotropy)} \end{cases}$

18.)

Saturation Condition (Interplay $\Delta\omega_k$; 1pt. Th.)

i.) $d_{IM}^{ion}(k, \omega_k) = d_{IM}^{T.E.} / \left[1 - G(k, \omega_k)\right]$

Balance
- (Clump) enhanced trapped electron growth
- Linear + nonlinear ion damping

ii.) Generic Form N.L. ion damping (renorm. Compton scattering ; example) (D+R).

$$d_{IM}^{Ion, NL} = \sum_{k'} \underline{\underline{\sum_{\omega'}}} \langle \Phi^2 \rangle_{k', \omega'} K(k, k', \omega, \omega', L^{ion}_{k+k', \omega+\omega'})$$

{ (ω now independent degree of freedom
Contrast 1pt. theory (W.T.T.) ;

$$\left(d_{IM}^{Ion, NL}\right)_{1pt.} = \sum_{k'} \langle \hat{\Phi}^2 \rangle_{k'} K(k, k', \omega_k, \omega_{k'}; L^{ion}_{k+k'})$$

iii.) $\langle \hat{\Phi}^2 \rangle_{k',\omega'} = \dfrac{N(u) \langle \tilde{\phi}^2 \rangle_{k',\omega'}}{|d(\omega', k')|^2} \simeq \dfrac{N(u)\, \Delta\omega_{k'}}{(\omega-\omega_{k'})^2 + (\Delta\omega_{k'})^2}$

$\Delta\omega_{k'} \to 0 \Rightarrow$

$$\langle \hat{\Phi}^2 \rangle_{k',\omega'} \simeq N(u)\, \delta(\omega-\omega_{k'})$$

$N(u) \downarrow N(k) \Leftrightarrow$ (W.T.T. occupation #.)

Full Spectrum:

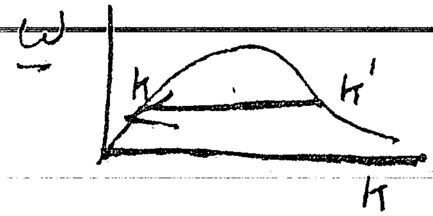
$$\langle \hat{\Phi}^2 \rangle_{k,\omega} = \frac{N(k)\ \Delta\omega_k}{\left[(\omega-\omega_k)^2 + (\Delta\omega_k)^2\right]}$$

iv.) $N(k) \Leftrightarrow$ Nonlinear $\left\{ \begin{matrix} \text{wave-particle} \\ \text{wave-wave} \end{matrix} \right\}$ scattering $\downarrow$ 1 pt. theory (WTT)

$\Rightarrow$ W.T.T. (1 pt.) processes energize k regions where clump noise small $\Rightarrow$ Impact $N(k)$ independently.

i.e.

2D $\quad \gamma_k^{NL} = -(1+k^2\rho_s^2)^{-1} \sum_{k'} \frac{c^2}{B^2} (k \cdot k' \times \hat{z})^2 |\Phi_{k'}|^2 \delta(\omega_k + \omega_{k'}) \left[k^2 - k'^2\right]$



$\Rightarrow$ Impacts $N(k)$ unless $\Delta\omega_k \gg \omega_k$.

(RGI?)

(Schematic)

v.) I.C.S. & $\Delta x \sim \Delta$; T.W. at O (s/kb)

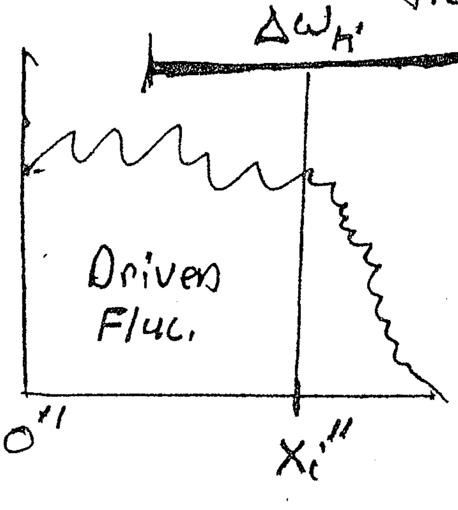
$$d^{ion,NL}_{IM\,\underset{\omega}{k}} = \sum_{\substack{m',n'\\ \omega'}} (k_\theta'^2 - k_\theta^2)\, \rho_s^2 \langle \hat{\Phi}^2 \rangle_{\underset{\omega'}{k'}} |\alpha x'|^2 \int \frac{dv_\parallel \langle f \rangle}{\omega + \omega' - k_\parallel' v_\parallel + i D_{k''}}$$

$$\langle \hat{\Phi}^2 \rangle_{\underset{\omega}{k}} = \left\{ N(k) H(x'/\Delta_k)\, \Delta\omega_k \right\} \Big/ \left\{ (\omega - \omega_k)^2 + (\Delta\omega_k)^2 \right\}$$

$\Rightarrow$

$$d^{ion,NL}_{IM\,\underset{\omega}{k}} \sim \int \frac{dk_\theta' N(k_\theta') (k_\theta'^2 - k_\theta^2) \rho_s^2\, i\, |\alpha\Delta|^2}{\left[\omega_k + \omega_k' + i\Delta\omega_k + i \left|\frac{k' v_\parallel}{L_s}\right| \Delta + i D_{k''} \right]}$$

$i\Delta\omega_k \to$ freq. spread; $i\left|\frac{k' v_\parallel}{L_s}\right|\Delta \to \Delta x'$ spread



$$x_i'' = \frac{(\omega_k + \omega_k') L_s}{|k v_{Th}|}$$

a.) Collective RB $\Leftrightarrow$ Particle R.B ($\gamma_{NL} \Leftrightarrow \Delta\omega$, sum!)

b.) Irrev. $\Rightarrow \tau \sim 1/\Delta\omega_k$

c.) Reduced sensitivity (Δx)

d.) Enhancement; NLLD

$d_{IM} \sim \nu_{eff}^2 \frac{\partial N}{\partial k_\theta}$

(2 species)

$\Delta\omega \sim \omega$; $\underline{\underline{\text{Enh.}}} \sim O\left(\frac{\Delta\omega}{\omega_{ti}}\right)^2 \sim \frac{L_s}{L_n}$

at fixed Φ_0

$\omega_{ti}^2 \Rightarrow \Delta\omega^2 \sim \omega^2\, L_s^2/L_n^2$

ii.) Spectrum integrations yield:

$$\frac{\gamma_{L,k}^{TE}}{[1-C(k,\omega_k)]} - \gamma_{L,k}^{Ion} = \int dk' N(k') \overline{K}(k, k', \Delta\omega_{k'}, \Delta_{k'})$$

⇒ Determines $N(k)$. ($N(k)$ broad in W.T.T.)

$$\Rightarrow \begin{cases} \langle \overline{\Phi}^2 \rangle_{k,\omega} = N(k) \left| S_{k,\omega} \right|^2_{\eta + 2\pi m} \Delta\omega_k / \left[(\omega - \omega_k)^2 + \Delta\omega_k^2 \right] \\ D_{TE}^R = \sum_{k',\omega'} \frac{c^3}{B_0^2} k_\theta'^2 \langle \overline{\Phi}^2 \rangle_{k',\omega'} \, \pi \delta(\omega' - \overline{\omega}_D' \epsilon) \end{cases}$$

[Eigenmode structure modification ⇒ $S_{k,\omega} = S_{k,\omega}[\Phi^2]$].

Have addressed: $\Delta\omega_k$, γ_k → calculated

$N(k)$, $D_{T.E.}^R$ } Reduced to "Familiar" Calculation

~~Linked $\Delta\omega_k$ ⇔ Poloidal isotropy.~~

ii.) T.I. ⇔ Branches couple (Similon / Diamond)

P.B. ⇔ $\Delta\omega_k \simeq .2$, follows Swartz, et al.

What Does "Unstable" Mean?

i.) Linearly; $\gamma_{L,k}^{T.E} > \gamma_{L,k}^{Ion}$

ii.) Linear (1 Pt.) Theory omitts effects which enhance free energy accessibility.

$\Rightarrow$

iii.) In context of this calculation, claim:

"Unstable" = Steady state with $d_{IM}^{Ion,NL} \neq 0$ predicted.

$\Leftrightarrow$

$$\frac{\gamma_k^E}{[1-C(k,\omega_k)]} > \gamma_{L,k}^{Ion}.$$

iv.) MIT group finds $(V_d/V_T)_{crit} < (V_d/V_T)_{Linear}$ for ion acoustic (2 species; analytics and simulation)

$\Rightarrow$

~~v.) Universal Mode in Slab!?~~

SOME ASPECTS OF DENSITY FLUCTUATIONS IN PDX AND ALCATOR C[†*]

BY

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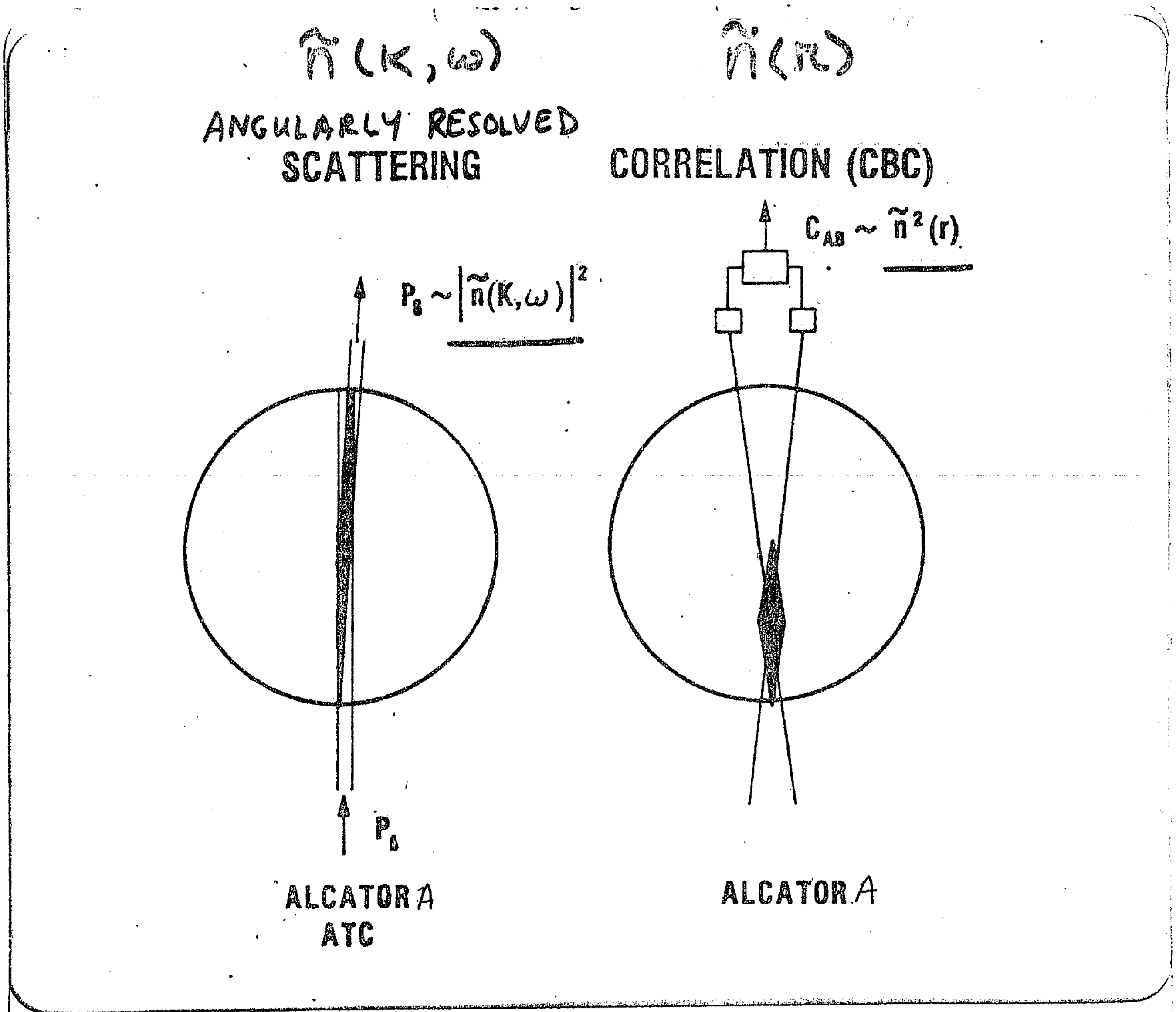
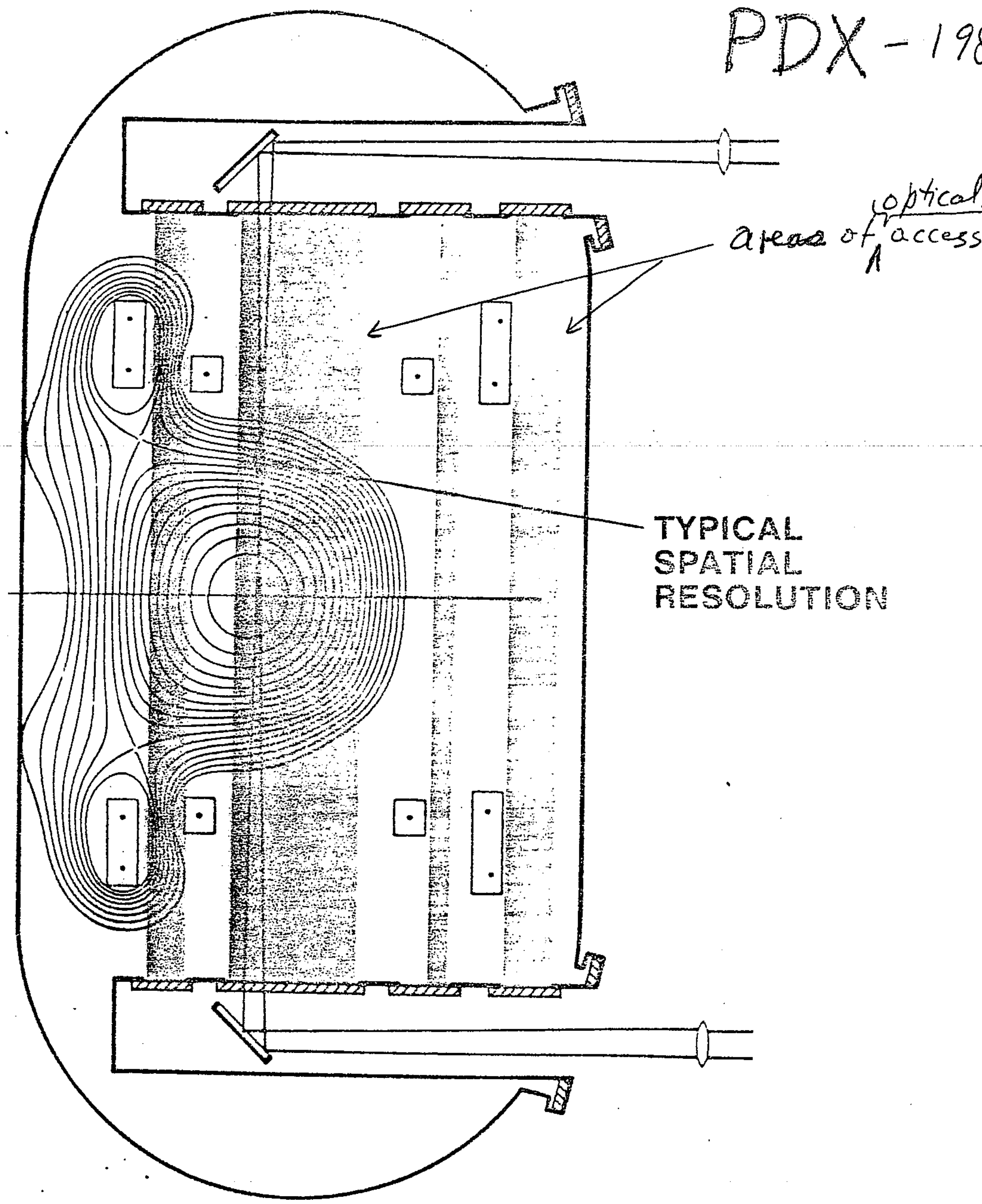
†THE PDX AND ALCATOR C PROJECTS ARE SUPPORTED BY THE U.S. DEPT. OF ENERGY

EXPERIMENTS ARE NOW IN PROGRESS ON PDX AND ALCATOR C TO STUDY THE SPECTRA AND SPATIAL DISTRIBUTION OF DENSITY FLUCTUATIONS. THE EXPERIMENTS UTILIZE BOTH SMALL-ANGLE CO_2 LASER SCATTERING AND CROSSED-BEAM CORRELATION TECHNIQUES. THE CURRENT STATE OF THESE RESULTS WAS REVIEWED AT THE CONFERENCE. HOWEVER, SINCE THE RESULTS ARE STILL PRELIMINARY, WE FELT THAT THE INCLUSION OF SPECIFIC DATA IN THE CONFERENCE PROCEEDINGS WAS INAPPROPRIATE. WE HAVE INCLUDED A SUMMARY OF WHAT PRESENTLY APPEAR TO BE THE MOST IMPORTANT RESULTS OF THESE EXPERIMENTS.

The techniques used:



PDX - 1981



SUMMARY

ALCATOR C

- $\tilde{n}/n \sim 0.5$ AT EDGE
  - ⇒ SIGNIFICANT EFFECTS ON LOWER HYBRID WAVES
- $\Delta\omega$ + $\bar{K}$ BROADER THAN Alcator A
- $\Delta\omega \propto K^2$

PDX

- $\tilde{n}$ PEAKS AT EDGE; it is likely that the EDGE FLUCT. BALLOON
- "SPIKEY" FLUCTUATIONS DURING INTENSE HEATING
  - (1) ARE NEAR PLASMA CENTER
  - (2) $\ell_c \gtrsim 5$ CM (SO PROBABLY MACROSCOPIC)

ALCATOR C & PDX

CORRELATION RESULTS ⇒

PUZZLING INTERNAL STRUCTURE

- FLUCT. NEAR $r = 0$ (or Low Level of fluct.)
- ZEROS NEAR $r/a \sim 0.5$

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KINETIC THEORY OF BALLOONING MODE

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Motivations and Status.

1 Can plasmas achieve stable high-β equilibria?

2 Can they be tamely confined?

→ 1. High-n Ballooning mode
— one candidate to cause Critical β.

· localizes to bad curvature region
· short-wave length, $k_{\parallel} \ll k_{\perp}$

→ 2. Anomalous transport becomes large?
deterioration of confinement?
disruption?
· magnetic field perturbation involved.

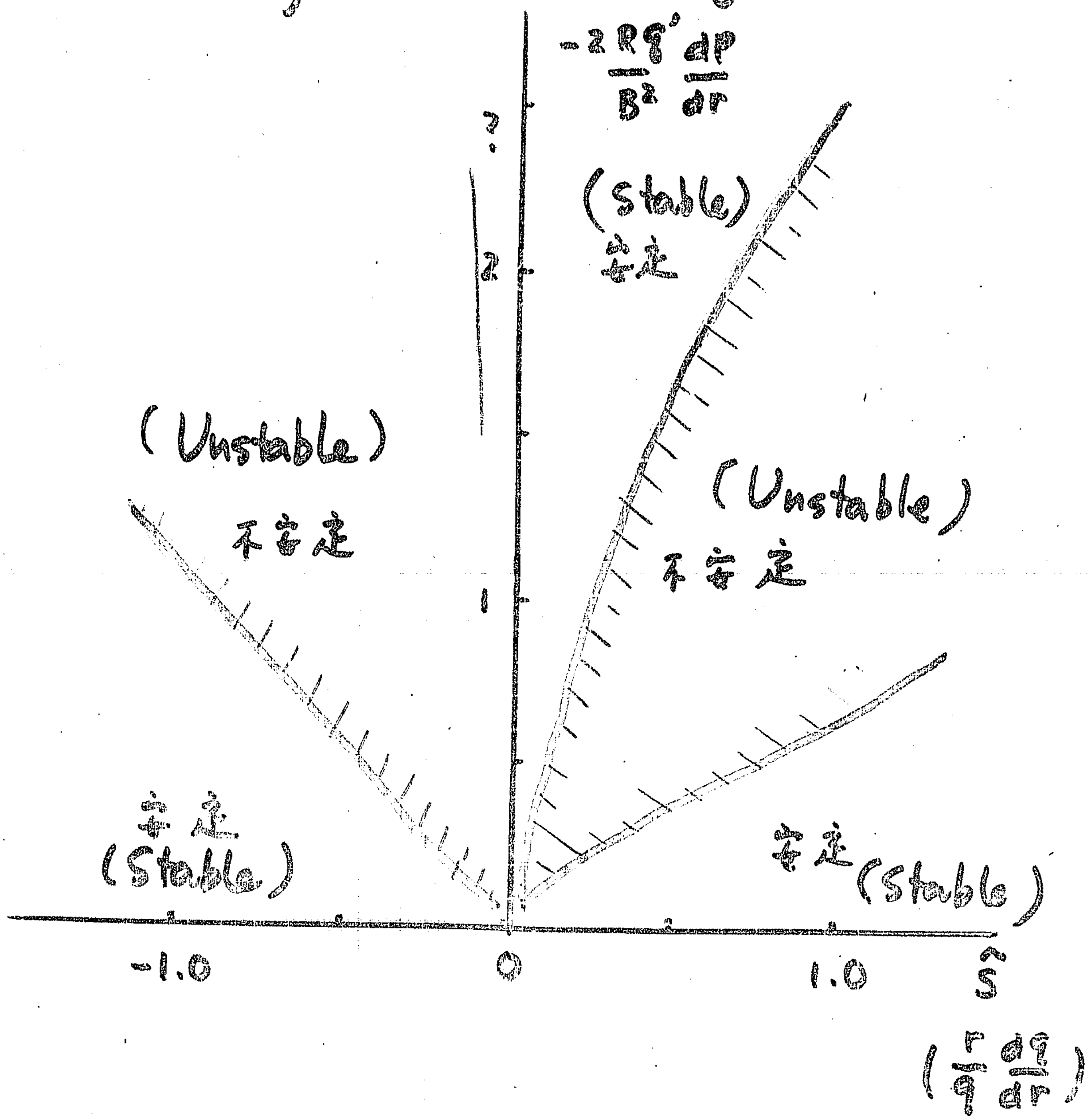
1' Critical - β MHD theory
· $\beta_{c1} < \beta < \beta_{c2}$ second stability region

· finite-gyro stabilization } same order

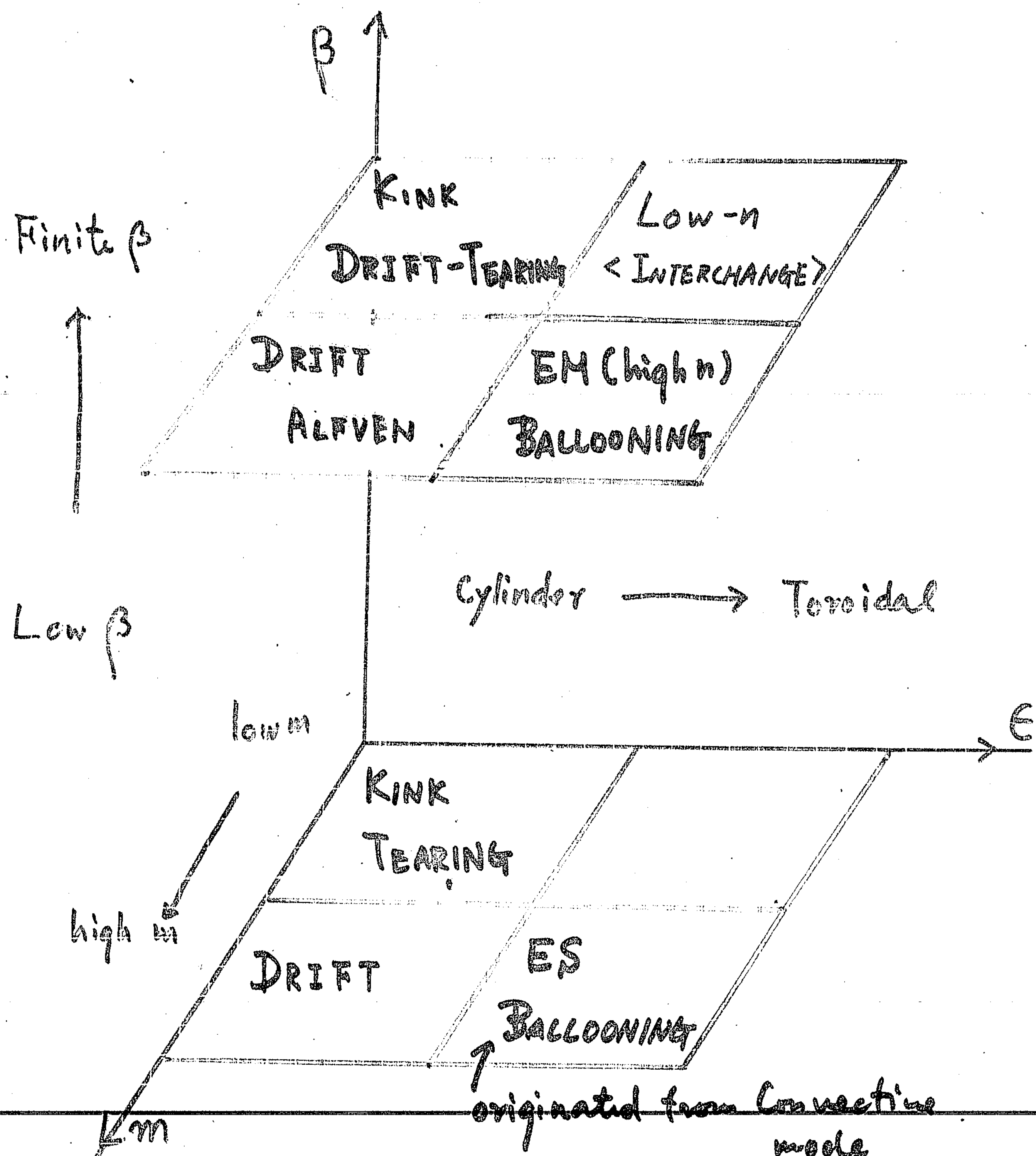
2' $E_{\parallel}$: coupling to drift Kinetic Treatment
$\tilde{B}_{r}$: modestructure across r_s
finite-β

MHD 理論による不安定領域

Theory Unstable region



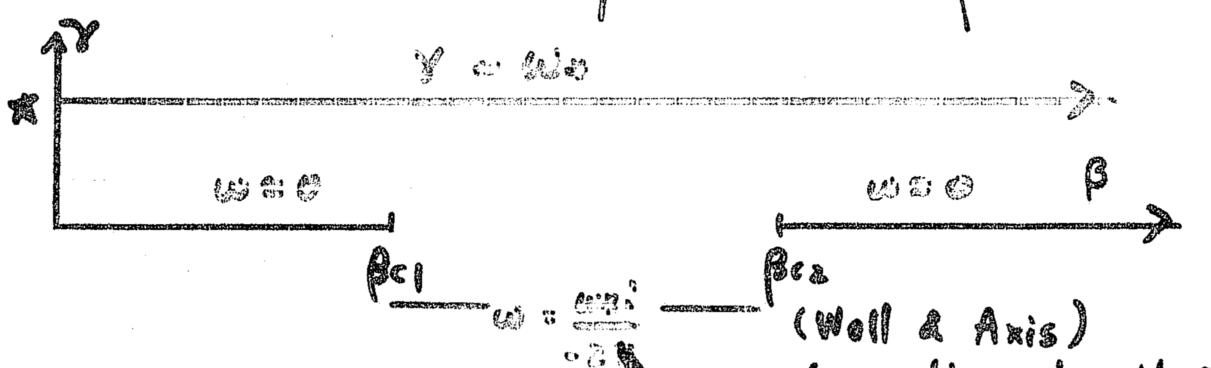
CLASSIFICATION OF LOW FREQUENCY INSTABILITIES



Results

- High-n Ballooning mode identified.
  — circular —
  - kinetic treatment ρ_i, full Zs (e & i) ..... MHD
  - coupling to drift branch ..... Slab
  - finite-β coupling. ..... ES & EM
  - magnetic shear profile. ..... $J(r)$ — ★
  - magnetic well & axis shift ..... Configuration & high-β.

- Unstable modes regardless of β value



- Magnetic island width has Max around $\beta \sim m_e/m_i$
  - odd-ϕ
    < not most unstable >

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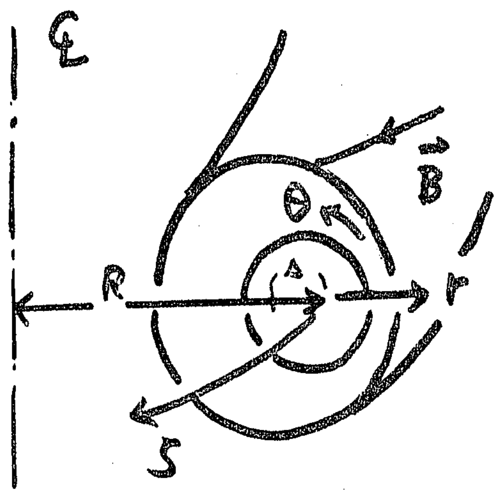
Model



$$\frac{\nabla B}{B} = -\frac{1}{R}(\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}) + \frac{\overline{\varpi}}{R}\hat{r} \;\ldots\text{ well}$$

$$B_p(r,\theta) = \bar{B}_p(r)\left\{1 - \left(\frac{d\Delta}{dr} + \frac{r}{R}\right)\cos\theta\right\}$$

Magnetic Axis Shift

Center of magnetic surface

$$\rho = R + \Delta(r)$$

New Coordinates (r, η, ζ)

$$\theta = \eta + \Lambda\sin\eta \quad , \quad \Lambda = -\frac{d\Delta}{dr} + \frac{r}{R}$$

Magnetic Field Line is "Straight";

$$d\eta/d\zeta = 1/q(r)$$

Unperturbed Orbit

$$v = v_\parallel \frac{B}{B} + \frac{cM(v_\perp^2 + v_\parallel^2)}{2 q_j B}\frac{B \times \nabla B}{B^2} + \text{gyromotion}$$

$\nabla p \; \nabla B$

$$\frac{d\eta'}{d\zeta'} = \frac{d\zeta'}{d\zeta'}\left\{\frac{1}{q} - \frac{1}{q^2}\frac{dq}{dr}(r'-r)\right\} - \frac{v_\perp^2 + v_\parallel^2}{r v q}\frac{cT}{2 q_j B R} \times$$

$$\left\{\cos\eta' - \Lambda + \frac{d\Lambda}{dr} + \Lambda\cos 2\eta' - \overline{\varpi}\right\}$$

Equilibrium Distribution Function

$$f_0(r,v) = \frac{n_0}{(2\pi v_T^2)^{3/2}} \exp\left\{ -\frac{1}{L_n}\frac{M_j c}{q_j B_p}\left(\frac{r q_j B_p}{M_j c} - \frac{v_\varphi}{R}\right) - \sum_\alpha \frac{v_\alpha v^\alpha}{2 v_T^2} \right\}$$

Assumptions :

- Tokamak Ordering $|B_\theta|^2 \ll |B_\zeta|^2$
- Neglect Trapped Particles
- $v_\parallel$: constant
- Incompressible Perturbation , $\tilde{B}_\zeta \ll \tilde{B}_r, \tilde{B}_\theta$
- Collisionless

Vlasov-Maxwell Equations : $\tilde{f} = (\tilde{E}, \tilde{B})$

Basic Equations

$$\tilde{n}_e = \tilde{n}_i \quad , \quad \nabla\times\tilde{B} = \frac{4\pi}{c}\tilde{J}$$

Fourier Component

$$\tilde{\Phi}(\mathbf{r},t) = \sum_m \Phi_m(r)\, e^{im\eta - in\zeta - i\omega t}$$

$\psi : rB_r$

$\phi : rE_\eta$

$x = (r - r_s)/\rho_i$

$$
\begin{aligned}
&(1+\frac{\Lambda^2}{2})\frac{d^2}{dx^2}\psi_m - b(1+\frac{5}{2}\Lambda^2)\psi_m \\
&+ \Lambda(\frac{d^2}{dx^2} - 2k\rho_i\frac{d}{dx} + b)\psi_{m-1} + \Lambda(\frac{d^2}{dx^2} + 2k\rho_i\frac{d}{dx} + b)\psi_{m+1} \\
&+ \frac{\Lambda^2}{4}(\frac{d^2}{dx^2} - 4k\rho_i\frac{d}{dx} + 3b)\psi_{m-2} + \frac{\Lambda^2}{4}(\frac{d^2}{dx^2} + 4k\rho_i\frac{d}{dx} + 3b)\psi_{m+2} \\
&= -\beta_i\frac{M_i}{M_e}\frac{\omega}{k_{/\!/}c}P(\phi_m + \frac{\omega}{k_{/\!/}c}\psi_m) \\
&\quad - \beta_i\varepsilon\frac{M_i}{M_e}\frac{\omega_*}{k_{/\!/}c}R\{(1-\frac{1}{k\rho_i}\frac{d}{dx})\phi_{m-1} + (1+\frac{1}{k\rho_i}\frac{d}{dx})\phi_{m+1} \\
&\quad + \Lambda(1-\frac{1}{k\rho_i}\frac{d}{dx})\phi_{m-2} + \Lambda(1+\frac{1}{k\rho_i}\frac{d}{dx})\phi_{m+2} - (2\Lambda+U)\phi_m\} \qquad (1)
\end{aligned}
$$

and

$$
\begin{aligned}
&(1+\frac{\Lambda^2}{2})\frac{d^2}{dx^2}\phi_m - b(1+\frac{5}{2}\Lambda^2)\phi_m \\
&+ \Lambda(\frac{d^2}{dx^2} - 2k\rho_i\frac{d}{dx} + b)\phi_{m-1} + \Lambda(\frac{d^2}{dx^2} + 2k\rho_i\frac{d}{dx} + b)\phi_{m+1} \\
&+ \frac{\Lambda^2}{4}(\frac{d^2}{dx^2} - 4k\rho_i\frac{d}{dx} + 3b)\phi_{m-2} + \frac{\Lambda^2}{4}(\frac{d^2}{dx^2} + 4k\rho_i\frac{d}{dx} + 3b)\phi_{m+2} \\
&= \frac{-1}{\xi_i Z(\xi_i)}\frac{\omega}{\omega\tau+\omega_*}\Big(P(\phi_m + \frac{\omega}{k_{/\!/}c}\psi_m) \\
&\quad + \frac{\varepsilon\omega_*}{\omega}\{R + \frac{(1+\tau)\omega_*}{\tau\omega}\}\{(1-\frac{1}{k\rho_i}\frac{d}{dx})\phi_{m-1} + (1+\frac{1}{k\rho_i}\frac{d}{dx})\phi_{m+1} \\
&\quad + \Lambda(1-\frac{1}{k\rho_i}\frac{d}{dx})\phi_{m-2} + \Lambda(1+\frac{1}{k\rho_i}\frac{d}{dx})\phi_{m+2} - (2\Lambda+U)\phi_m\}\Big) \qquad (2)
\end{aligned}
$$

where

$$P = \frac{\omega_* - \omega}{2\omega}Z'(\xi_e) - \frac{\omega\tau+\omega_*}{2\omega}Z'(\xi_i)$$

$$R = \frac{\omega_* - \omega}{2\omega}\{Z'(\xi_e) - \frac{\xi_e}{2}Z''(\xi_e)\} + \frac{\omega\tau+\omega_*}{2\omega\tau}\{Z'(\xi_i) - \frac{\xi_i}{2}Z''(\xi_i)\}$$

MHD Limit

$$\phi(r,t) = e^{-in\zeta - i\omega t} \sum_m e^{im\theta} \int \hat{\Phi}(\theta) e^{-i(m-nq)\theta} d\theta$$

Ballooning Representation

$$\Big[\frac{d}{d\theta} \{1+(s\theta - 2\Lambda \sin\theta)^2\} \frac{d}{d\theta} + \beta q^2 \frac{R}{L_n} \{\cos\theta + (s\theta - 2\Lambda \sin\theta)\sin\theta\}$$

$$+ \frac{q^2 R^2}{V_A^2} \omega (\omega + \frac{\omega_*}{\tau}) \{1+(s\theta - 2\Lambda\sin\theta)^2\} \Big] \hat{\Phi}(\theta) = 0$$

FGT ordering $\epsilon < \Lambda < 1$

$$\Lambda(r) = -\frac{r}{R} \Big\{ \frac{8\pi P}{B_p^2} - \frac{16\pi}{r^2 B_p^2} \int_0^r r\, dr\, p \Big\} \simeq \frac{1}{2} \beta q^2 \frac{R}{L_n}$$

$$\omega (\omega + \frac{\omega_*}{\tau}) = -\gamma_{MHD}^2$$

1. MHD Approximation : unstable if $\beta_{c1} < \beta < \beta$
2. FGR Correction : $\omega(\omega + \omega_*/\tau) = -\gamma_{MHD}^2$
3. Full kinetic effect : kinetic mode

Expanding parameters

- $\nabla^2 \rho_i^2 \ll 1$
- $\epsilon \ll 1$
- $1/n \to 0$

High-n Approximation

Invariant for Transformation
$(x, m) \to (x+\Delta, m+1)$

$$\Delta = \frac{1}{k\rho_i s}$$

$$\phi_{m+j}(x) = \phi_m(x - j\Delta)$$

$$\psi_{m+j}(x) = \psi_m(x - j\Delta)$$

Parity

$$\begin{cases} \phi(-x) = \phi(x) \\ \psi(-x) = -\psi(x) \end{cases} \qquad \begin{cases} \phi(-x) = -\phi(x) \\ \psi(-x) = \psi(x) \end{cases}$$

$$(1 + \frac{\Lambda^2}{2})\frac{d^2}{dx^2}\,\psi(x) - b(1 + \frac{5}{2}\,\Lambda^2)\psi_{(x)}$$

$$+ \Lambda(\frac{d^2}{dx^2} - 2k\rho_i\,\frac{d}{dx} + b)\,\psi(x-\delta) + \Lambda(\frac{d^2}{dx^2} + 2k\rho_i\,\frac{d}{dx} + b)\psi(x-\delta)$$

$$+ \frac{\Lambda^2}{4}(\frac{d^2}{dx^2} - 4k\rho_i\,\frac{d}{dx} + 3b)\psi(x+2\delta) + \frac{\Lambda^2}{4}(\frac{d^2}{dx^2} + 4k\rho_i\,\frac{d}{dx} + 3b)\psi(x-2\delta)$$

$$= -\beta_i\,\frac{M_i}{M_e}\,\frac{\omega}{k_{\shortparallel}c}\,P\{\phi(x) + \frac{\omega}{k_{\shortparallel}c}\,\psi(x)\}$$

$$- \varepsilon\beta_i\,\frac{M_i}{M_e}\,\frac{\omega_*}{k_{\shortparallel}c}\,R\{(1 - \frac{1}{k\rho_i}\,\frac{d}{dx})\phi(x+\delta) + (1 + \frac{1}{k\rho_i}\,\frac{d}{dx})\phi(x-\delta)$$

$$+ \Lambda(1 - \frac{1}{k\rho_i}\,\frac{d}{dx})\phi(x+2\delta) + \Lambda(1 + \frac{1}{k\rho_i}\,\frac{d}{dx})\phi(x-2\delta)$$

$$- (2\Lambda+U)\phi(x)\} \qquad (11)$$

and

$$(1 + \frac{\Lambda^2}{2})\frac{d^2}{dx^2}\,\phi(x) - b(1 + \frac{5}{2}\,\Lambda^2)\phi(x)$$

$$+ \Lambda(\frac{d^2}{dx^2} - 2k\rho_i\,\frac{d}{dx} + b)\phi(x+\delta) + \Lambda(\frac{d^2}{dx^2} + 2k\rho_i\,\frac{d}{dx} + b)\phi(x-\delta)$$

$$+ \frac{\Lambda^2}{4}(\frac{d^2}{dx^2} - 4k\rho_i\,\frac{d}{dx} + 3b)\phi(x-2\delta) + \frac{\Lambda^2}{4}(\frac{d^2}{dx^2} + 4k\rho_i\,\frac{d}{dx} + 3b)\phi(x-2\delta)$$

$$= \frac{-1}{\xi_i Z(\xi_i)}\,\frac{\omega}{\omega\tau+\omega_*}\,[P\{\phi(x) + \frac{\omega}{k_{\shortparallel}c}\,\psi(x)\}$$

$$+ \frac{\varepsilon\omega_*}{\omega}\,\{R + \frac{(1+\tau)\omega_*}{\tau\omega}\}\{(1 - \frac{1}{k\rho_i}\,\frac{d}{dx})\phi(x+\delta) + (1 + \frac{1}{k\rho_i}\,\frac{d}{dx})\phi(x-\delta)$$

$$+ \Lambda(1 - \frac{1}{k\rho_i}\,\frac{d}{dx})\phi(x+2\delta) + \Lambda(1 + \frac{1}{k\rho_i}\,\frac{d}{dx})\phi(x-2\delta)$$

$$- (2\Lambda+U)\phi(x)]. \qquad (12)$$

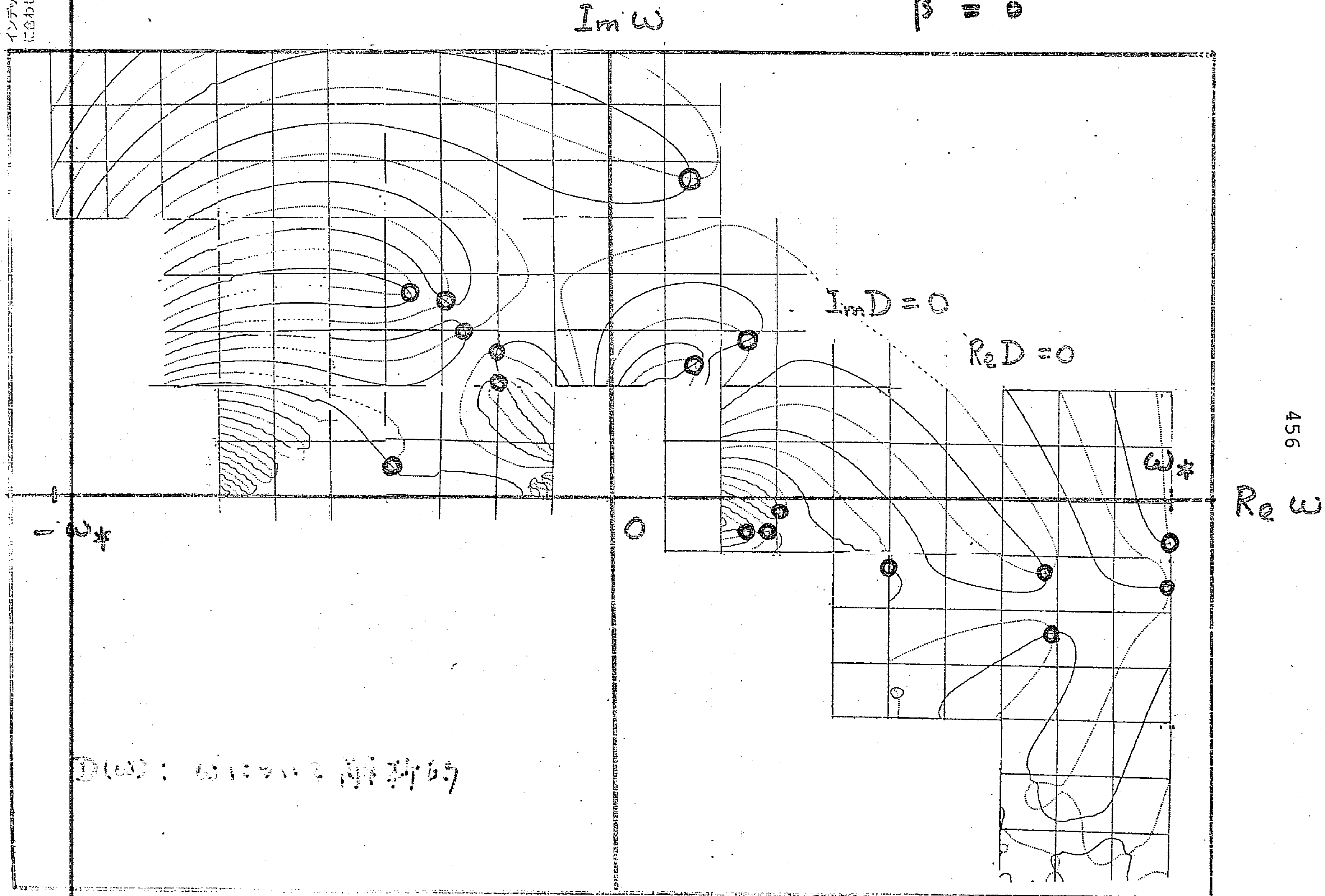
D(ω) 等高線

$s = 1$

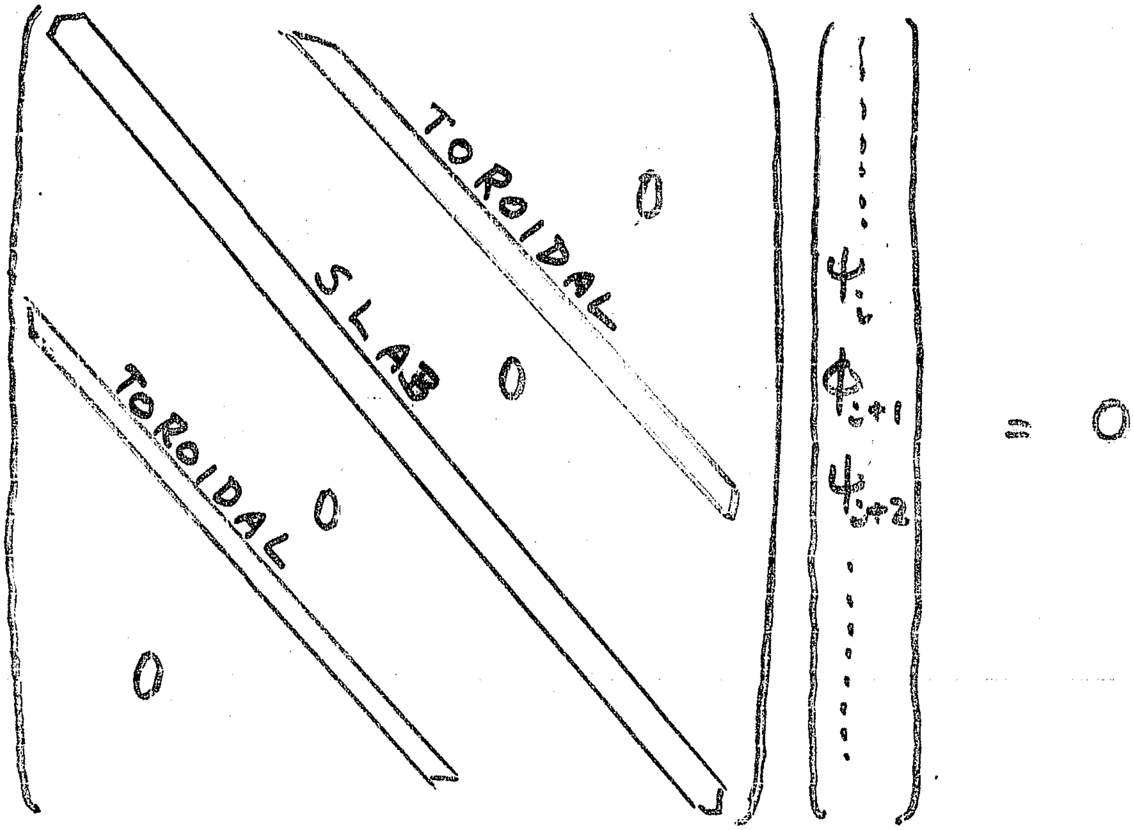
$\epsilon = 0.1$

$\varkappa L_s = 32.$

$\beta = 0$



$$\overleftrightarrow{A}\,\vec{X} = 0$$



$$D(\omega) \equiv \det \overleftrightarrow{A} = 0$$

$$\rightarrow \{\omega,\ \phi(x),\ \psi(x)\}$$

Contour Method

+

Newton Method

Renumbering of Matrix

Good Convergence, Accuracy & Efficiency

Fig. 2

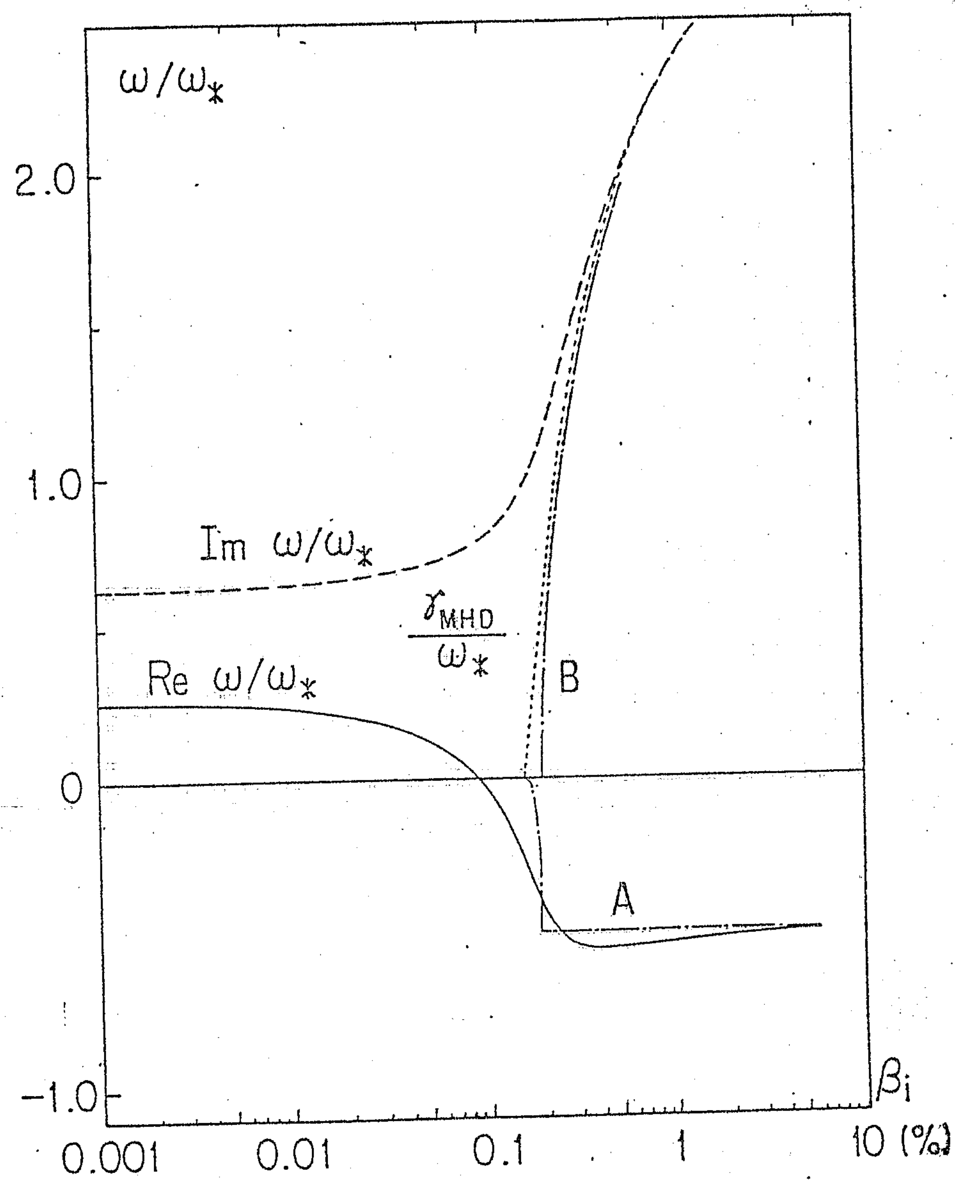


Fig. 2 The eigenvalue of the most unstable even-ϕ mode as a function of β_i. Other parameters are: $k\rho_i = 0.2$, $\epsilon = 0.1$, $s = 1$, $q = 3.2$, $T_e = T_i$, $M_i/M_e = 1836$. The dotted line shows the growth rate of MHD ballooning mode. The dotted and dashed lines denote ω_*-corrected MHD eigenvalue, ω_{FGR}. A for Re ω_{FGR}/ω_* and B for Im ω_{FGR}/ω_*. The magnetic well and shift of the axis is not incorporated. Smooth transition from ES ballooning mode to EM ballooning mode occurs around $\beta \sim \beta_c$(MHD).

Fig. 3b

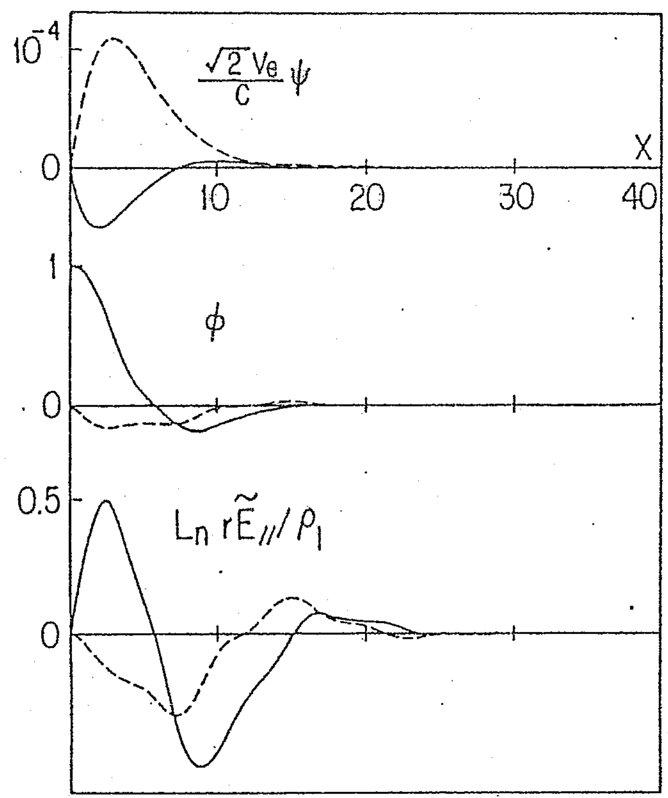


Fig. 3a

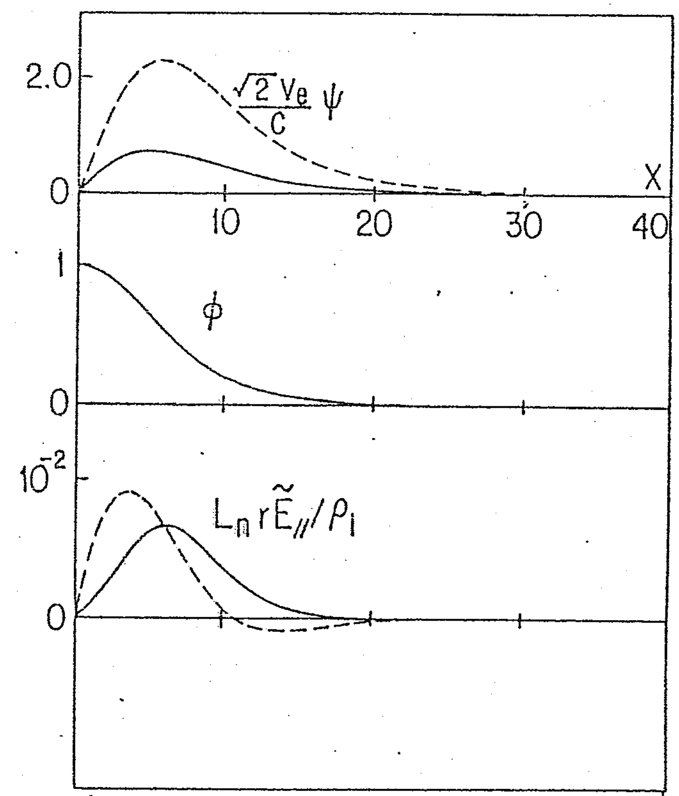


Fig. 3 Eigenmode structure of even-ϕ EM ballooning mode (a) and ES ballooning mode (b). Parameters are the same as in Fig. 2 and $\beta_i = 1$ % (a) and 10^{-5} % (b). When β increases ψ increases while $\tilde{E}_{\parallel}$ decreases. The eigenmode approaches to the MHD ballooning mode.

Fig. 4

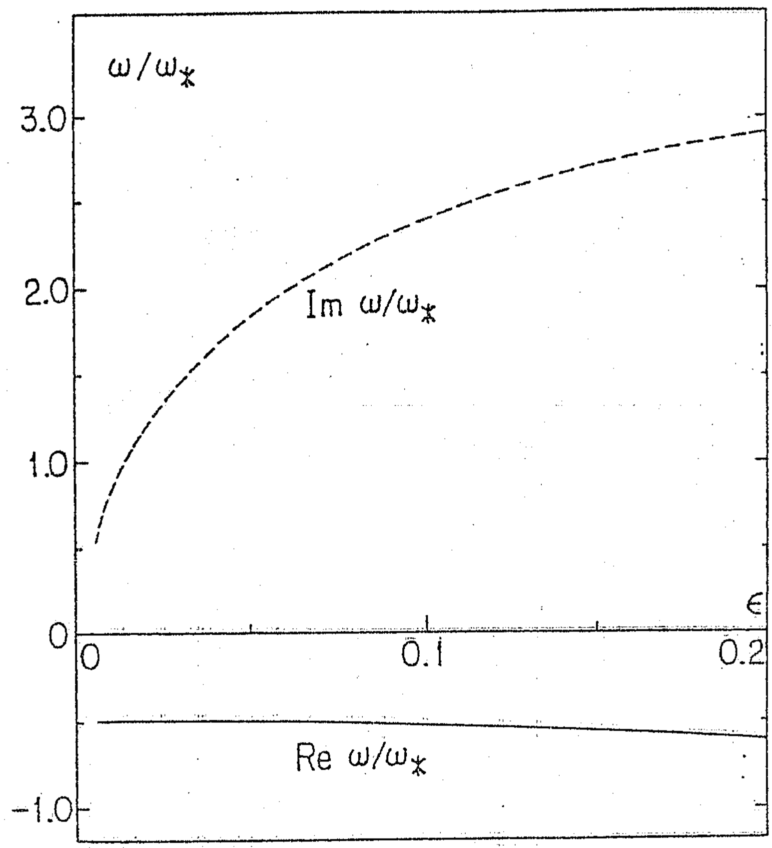


Fig. 4 ϵ dependence of ω of even-ϕ mode. Parameters are $\beta_i = 10^{-2}$ and standard values. In varying ε the condition $\beta > \beta_c$ is satisfied and parameters are in MHD unstable region. Re $\omega/\omega_* = -1/2$ holds.

Fig. 5

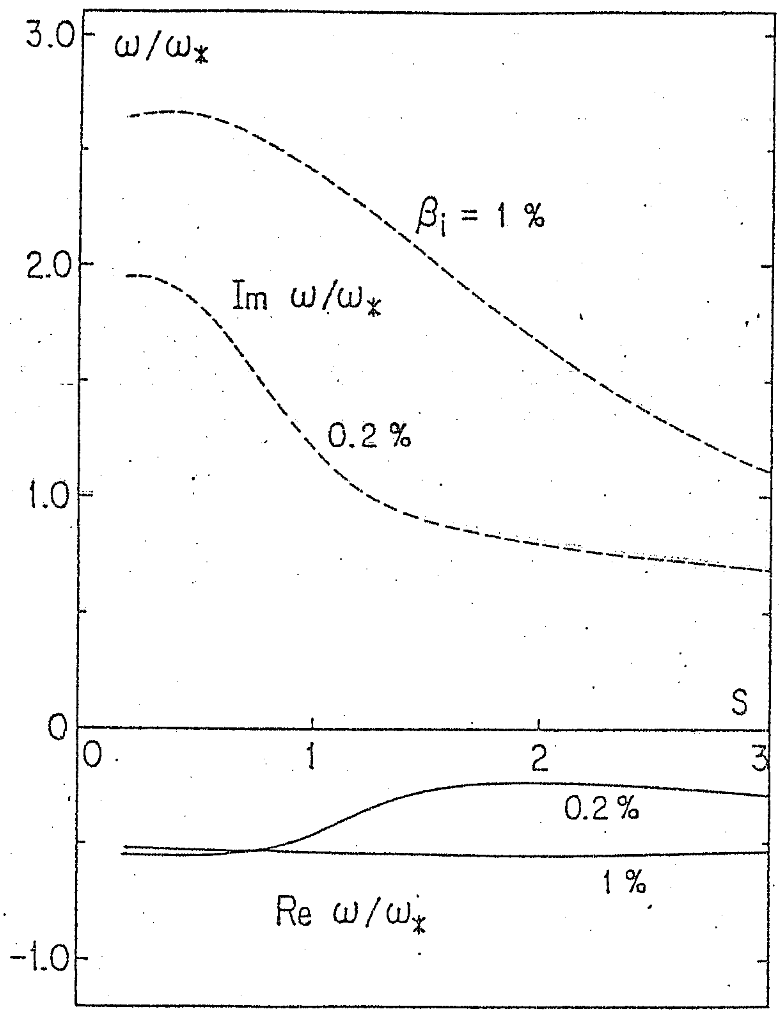


Fig. 5 The shear parameter dependence. Two cases, $\beta_i = 1$ % and 0.2 % are shown for standard parameters. For $\beta_i = 0.2$ %, MHD ballooning mode becomes stable if $s > s_c \sim 1.5$. Due to kinetic effects the instability remains even if $s > s_c$.

Fig. 6

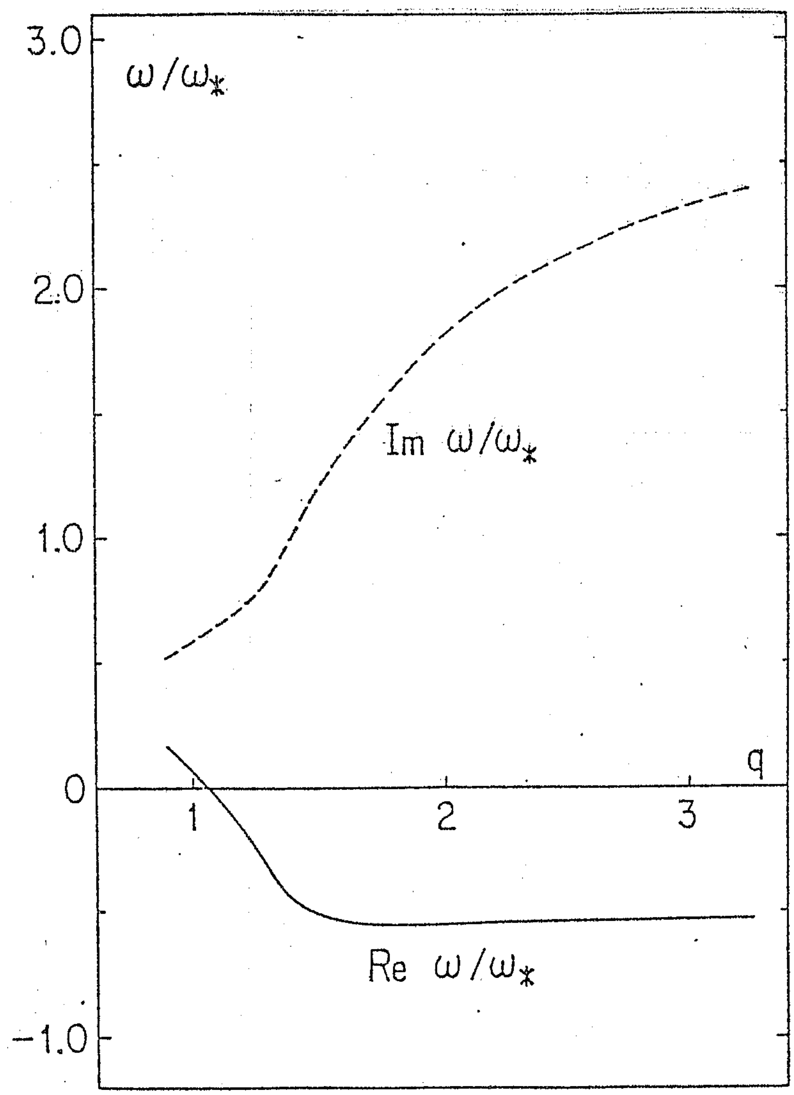


Fig. 6. The q dependence. Parameters are standard values and β_i = 1 %. Reducing q the MHD theory predicts the stability. The mode changes to the ES-ballooning-mode like and is still unstable.

Fig. 7

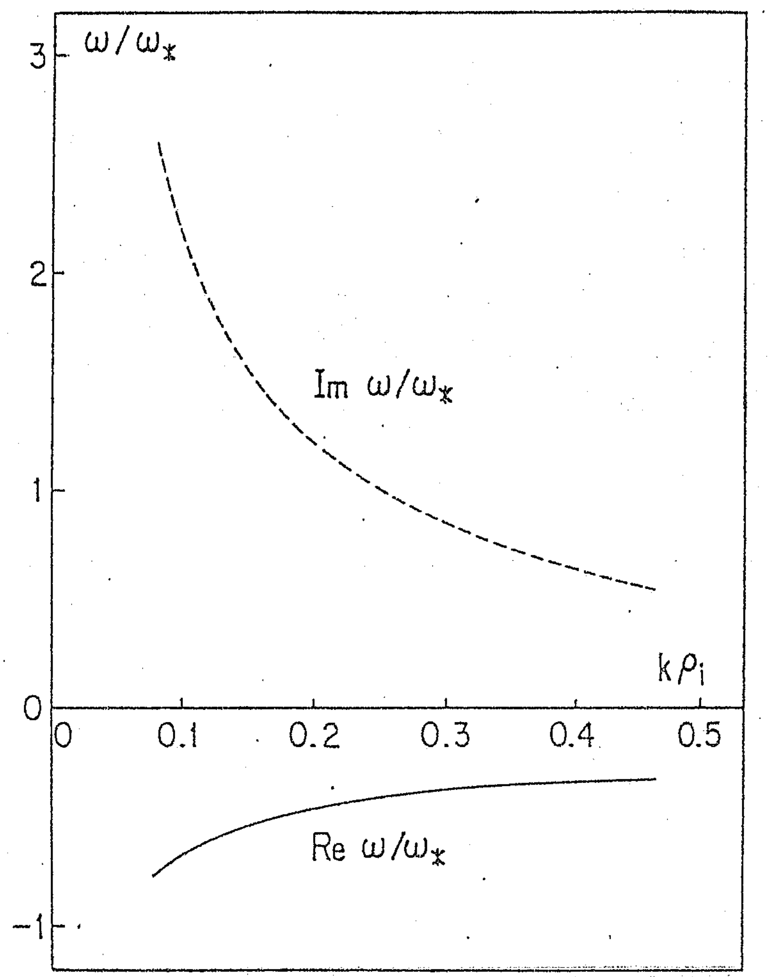


Fig. 7 $k\rho_i$ dependence. Parameters are standard values. The eigenvalue of the larger $k\rho_i$ mode ($k\rho_i \gtrsim 0.3$) is close to that of ES limit.

Fig. 8

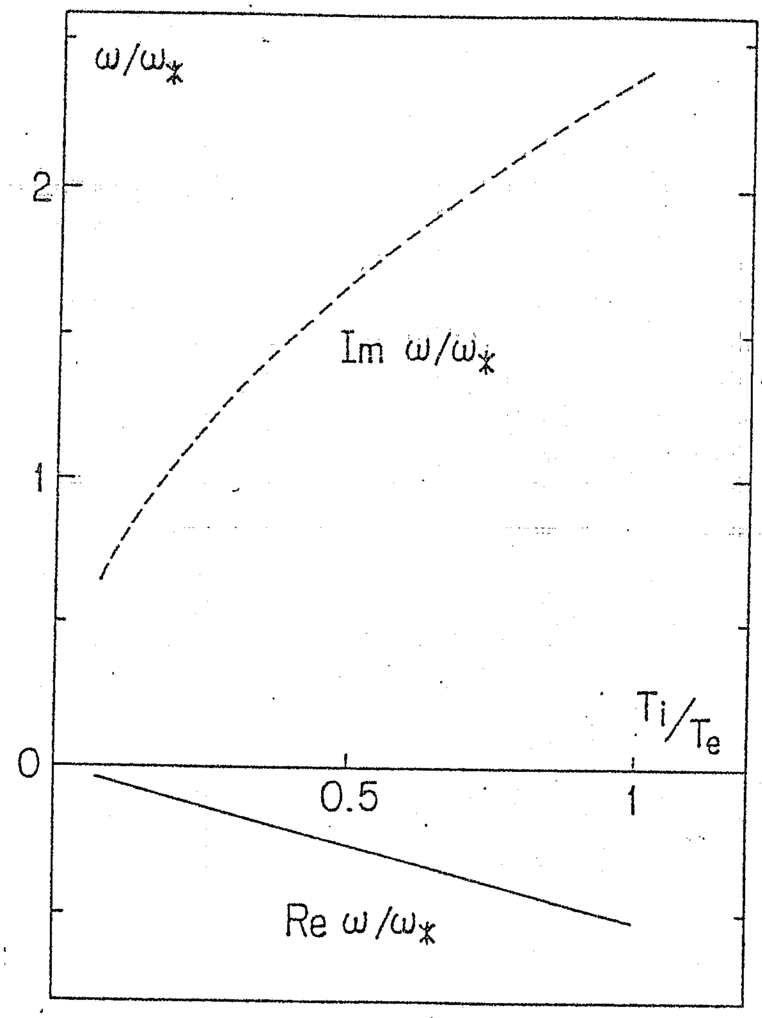


Fig. 8 τ dependence for standard parameters. The growth rate is large if $\tau \sim O(1)$. The real part ω is approximately $-\omega_*/2\tau$ and Eq. (23) shows a good approximation.

Fig. 9

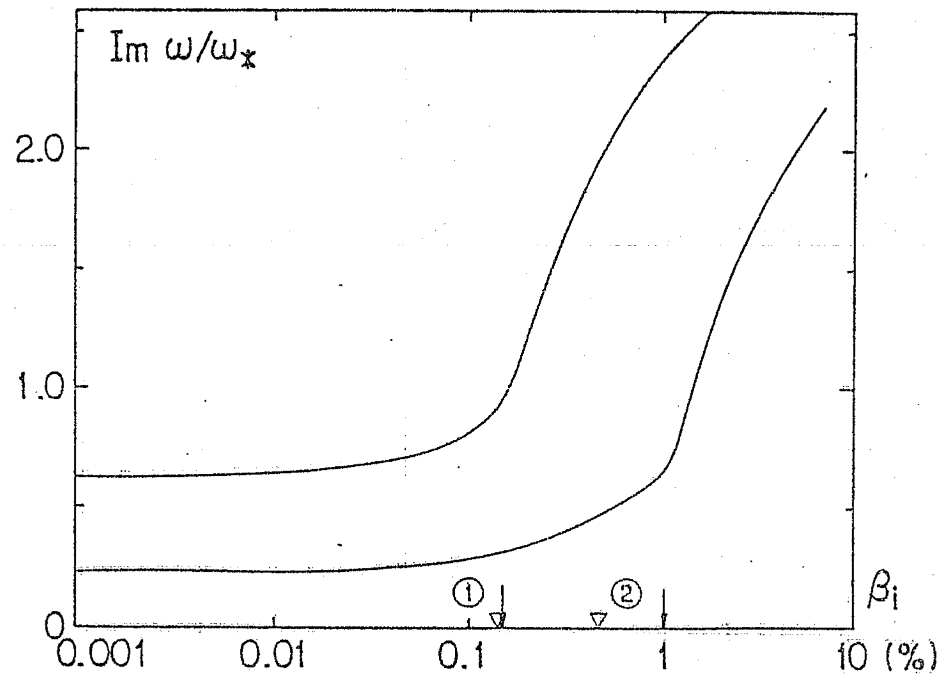


Fig. 9 The growth rates of the fundamental and 2nd even-ϕ eigen modes. The second mode has the smaller growth rate. The marks ∇ on the abscissa donete the critical β-values predicted by the MHD theory for the fundamental (1) and second (2) modes. The arrows show the FGR-corrected critical β-value where $\gamma_{MHD} = \omega_*/2$ holds.

Fig. 10

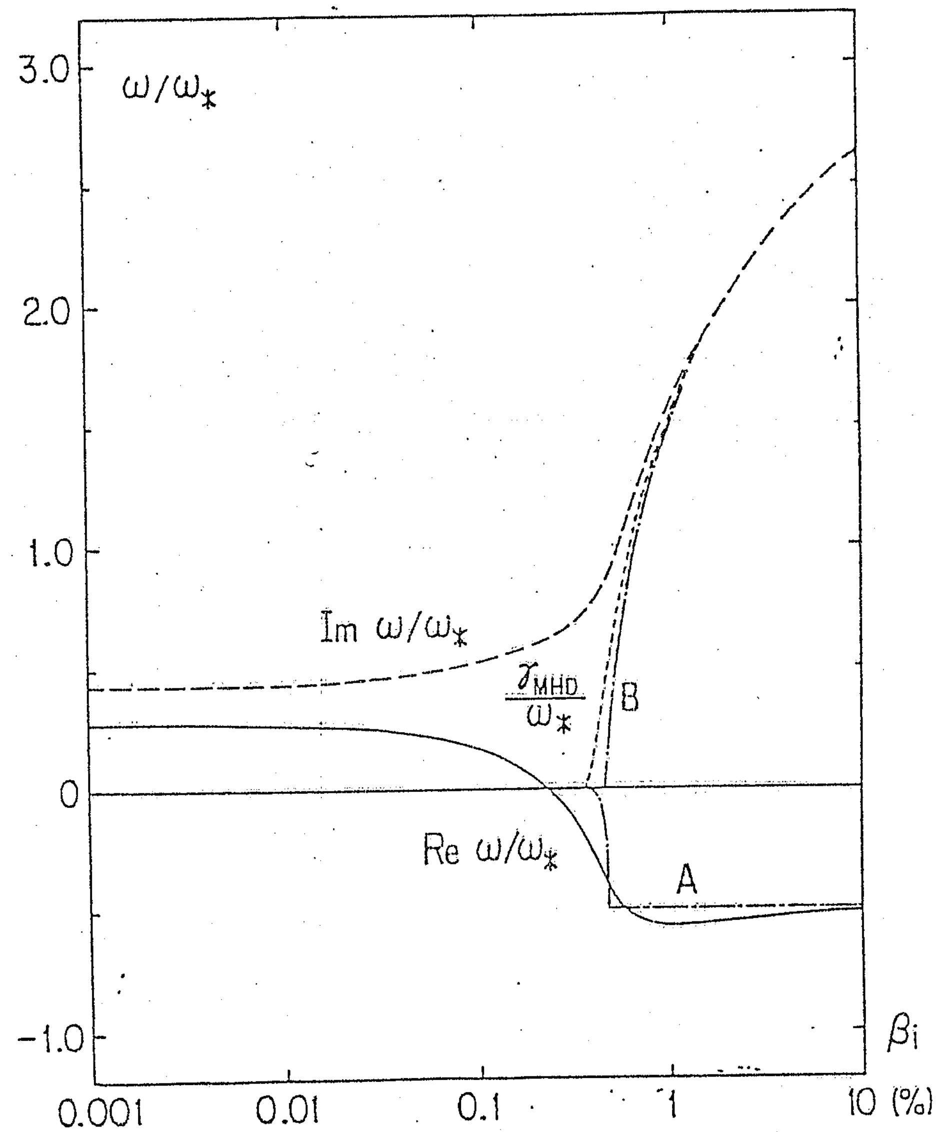


Fig. 10 The eigenvalue of the most unstable odd-ϕ mode for the parameters of Fig. 2. Compare to the results in Fig. 2. MHD result and the ω_*-corrected eigenvalue, ω_{FGR}, are also shown. A for Re ω_{FGR}/ω_* and B for Im ω_{FGR}/ω_*.

Fig. 11b

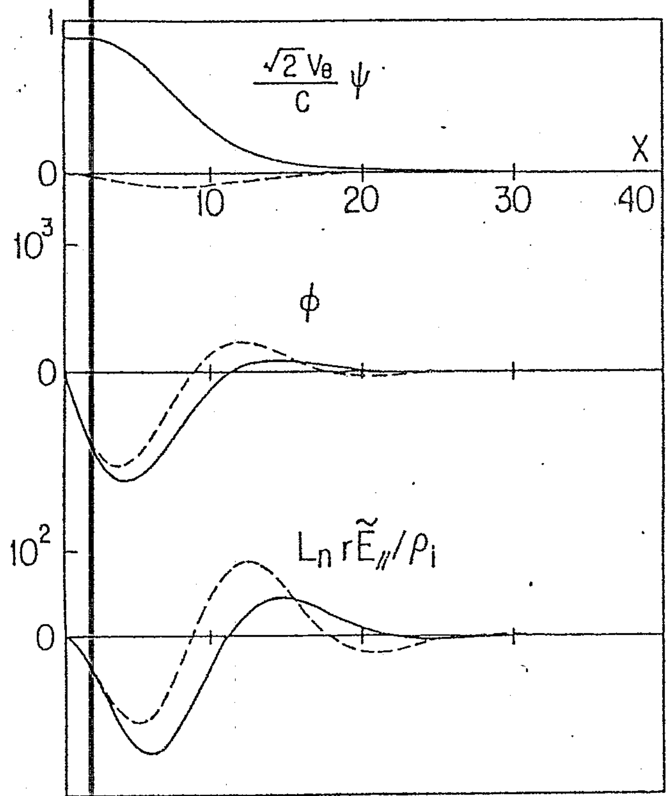


Fig. 11a

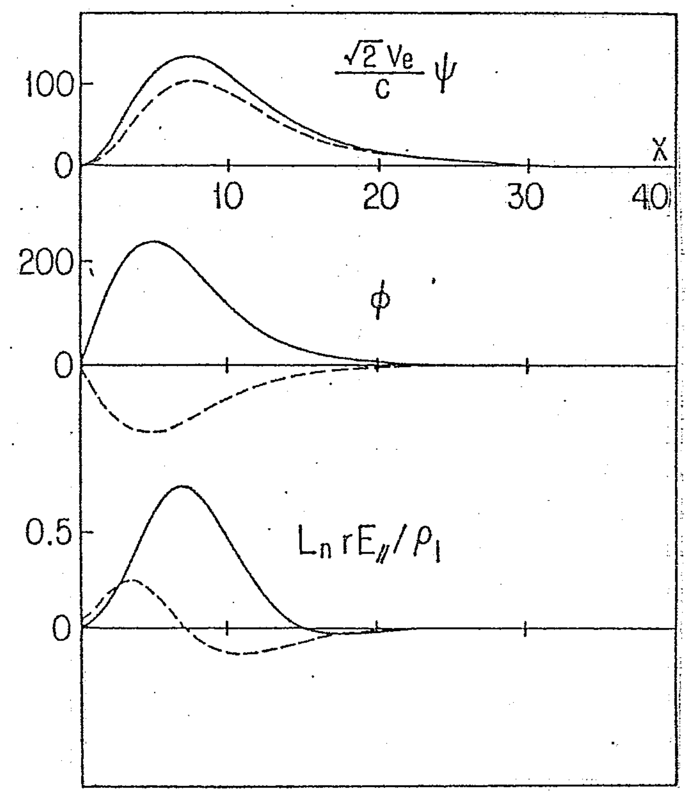


Fig. 11 Eigenmode structures of odd-ϕ EM ballooning mode (a) and ES ballooning mode (b). Parameters are the same as in Fig. 2 and $\beta = 1$ % (a) and 10^{-5} % (b). The normalization is taken such that $\sqrt{2}v_e\psi(0)/c = 1$ holds. Note that the radial structure of ψ of EM ballooning mode (a) becomes hollow, $\psi(0)$ is small and $|E_{\parallel}|$ remains to be small.

Fig. 12

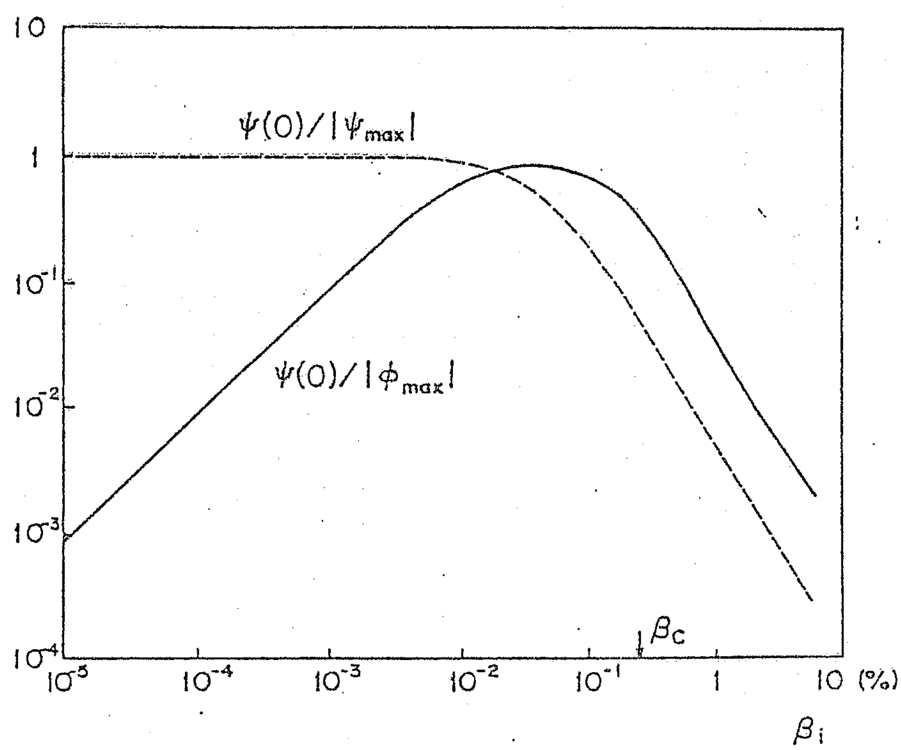


Fig. 12 The relative magnitute of $|\psi|$ and $|\phi|$ for the odd-ϕ mode. The solid line shows $\sqrt{2}v_e\psi(0)/c|\phi_{max}|$. As β_i approaches to M_e/M_i it increases, and then it saturates. For $\beta > \beta_c$ it starts to decrease.

Fig. 13

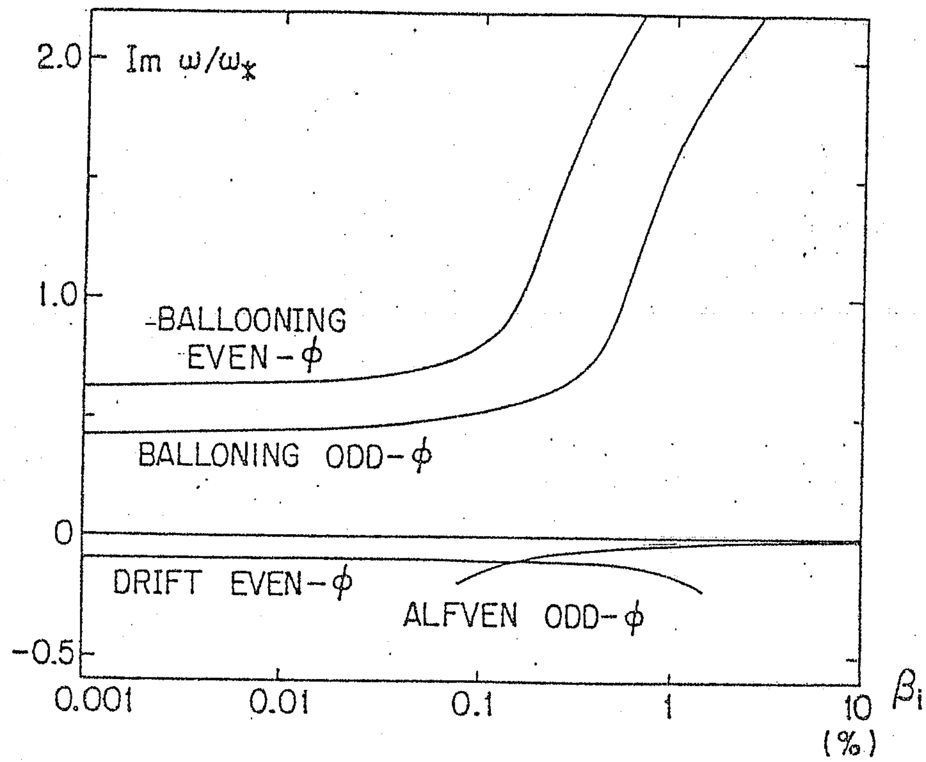


Fig. 13 Growth rates of ballooning modes of even-ϕ and odd-ϕ are compared to those of drift mode and drift-Alfvén modes. The most unstable mode in finite-β toroidal plasma in the ballooning mode.

Fig. 14b

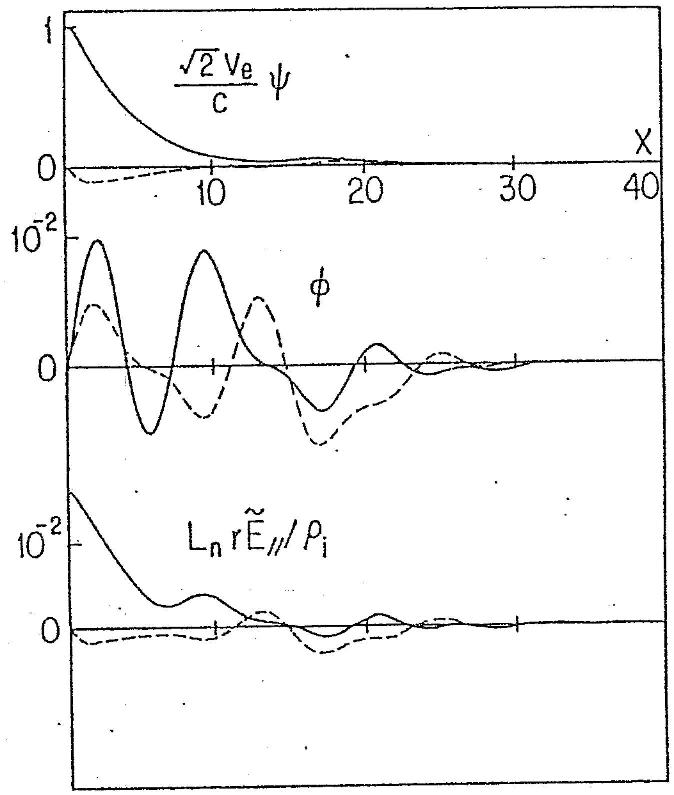


Fig. 14a

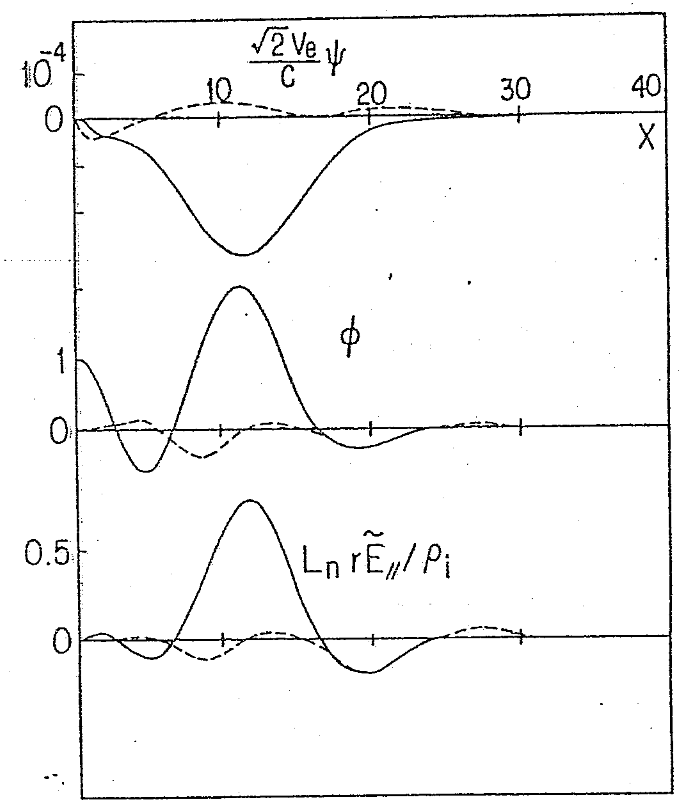


Fig. 14c

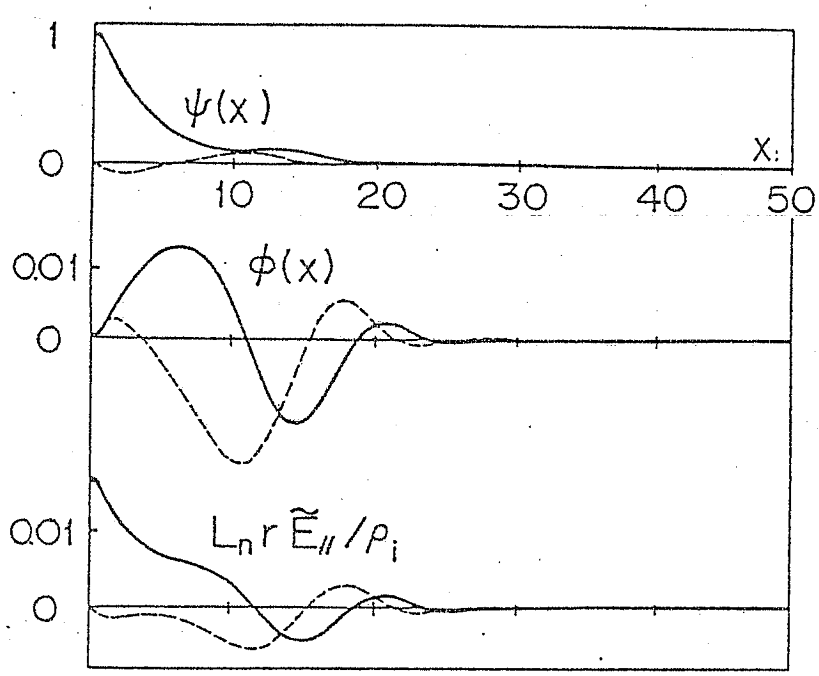


Fig. 14 The eigenmode structures of drift mode (a) and drift-Alfvén mode (b) in toroidal plasma. Parameters are standard except $\beta_i = 10^{-5}$ (a) and $\beta_i = 1$ % (b). (c) shows the slab drift-Alfvén mode for $k\rho_i = 0.2$, $\beta_i = 1$ %, $L_s/L_n = 32$, which is the same as (b) but no toroidal coupling. Toroidality has a small effect on the drift-Alfvén mode.

Fig. 15

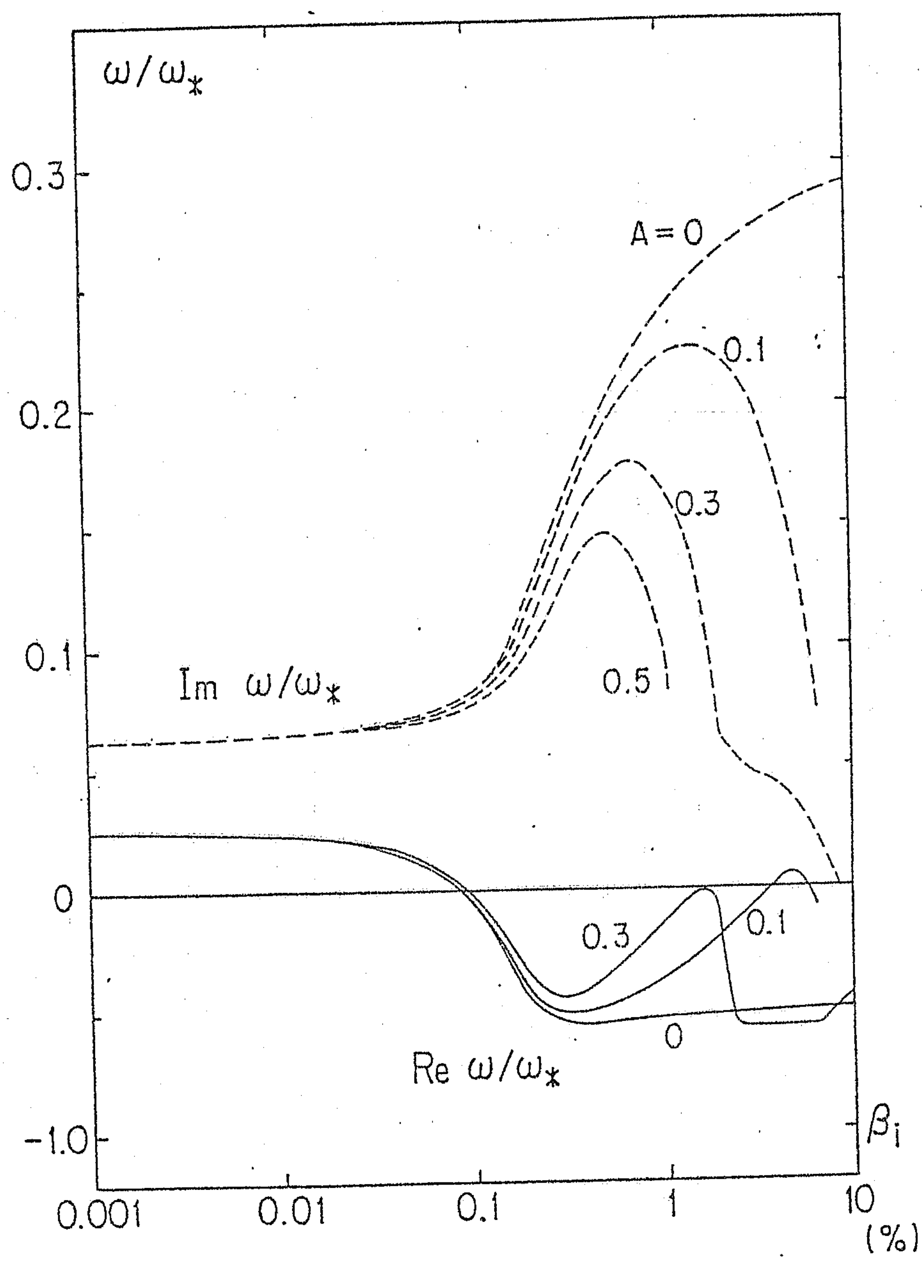


Fig. 15 The effect of the magnetic well is studied. Eigenvalue of even-ϕ mode for the standard parameters are shown. Model magnetic well is introduced as $U = A\ \beta q^2/2\epsilon$. "The second stability region" seems to appear for higher-β values but instability remains.

Fig. 16

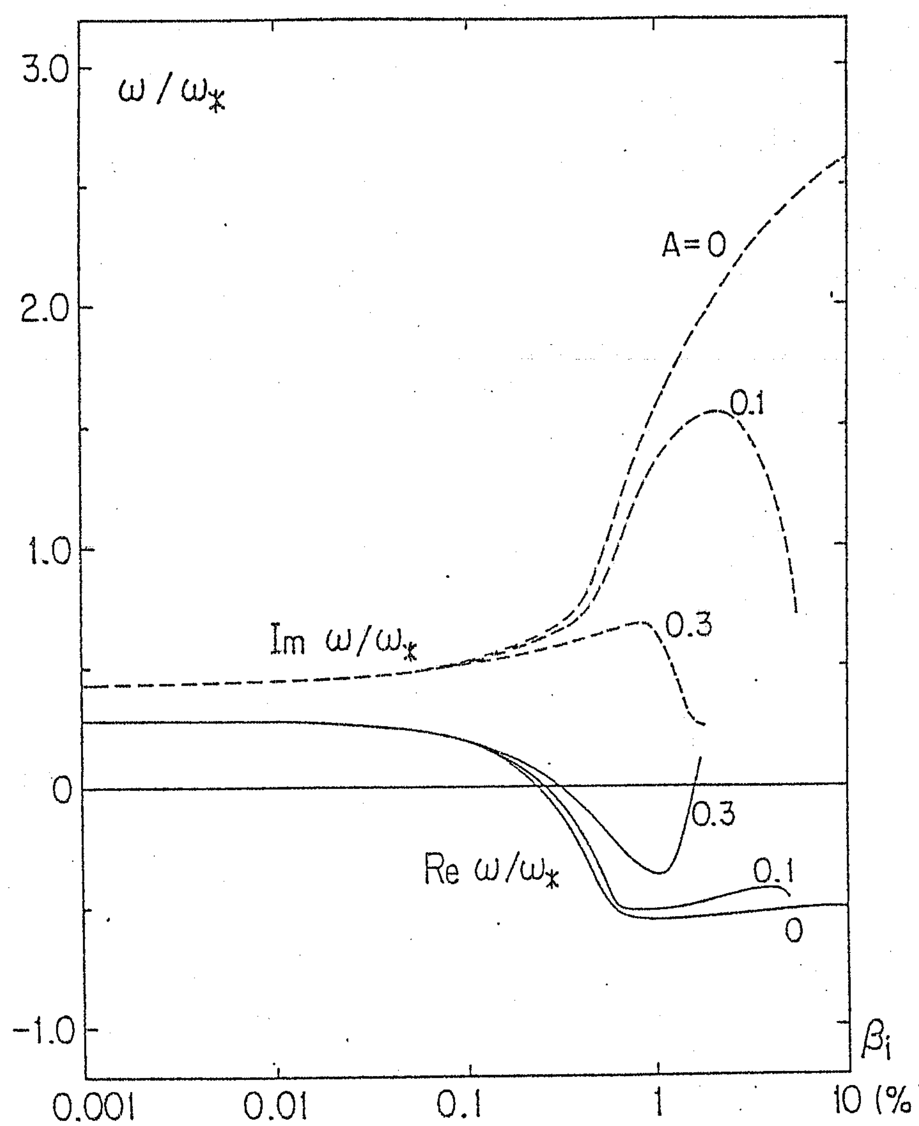


Fig. 16 The effect of the magnetic well on the odd-ϕ mode. Parameters are the same as Fig.15. The odd-ϕ mode is much more affected than even-ϕ mode. (See the value A).

Fig. 17

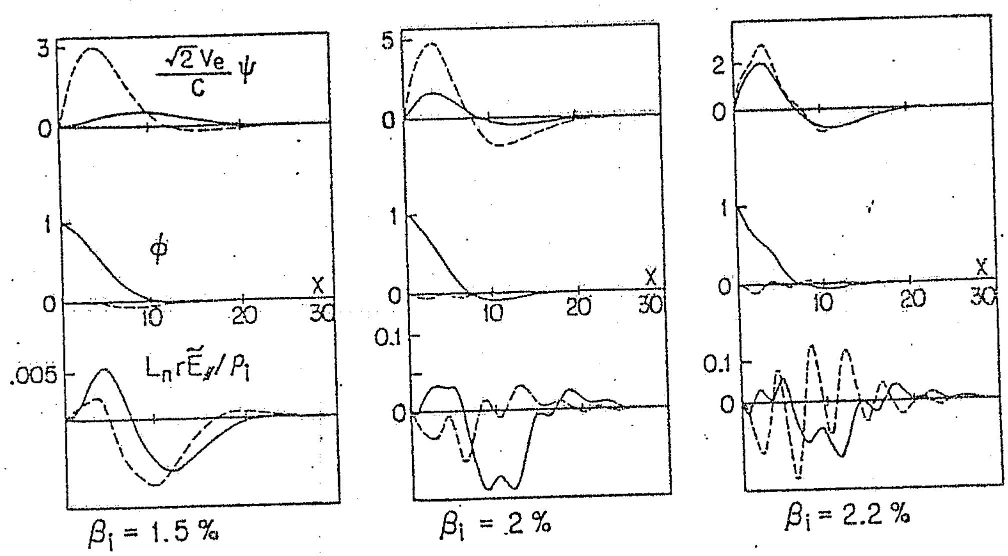


Fig. 17 The eigenvalue of even-ϕ mode in the presence of the magnetic well effect. Parameters are the same as Fig.15. As β increases the corrugations in $E_{\parallel}$ of the radial mode structure become noticiable.

Fig. 18

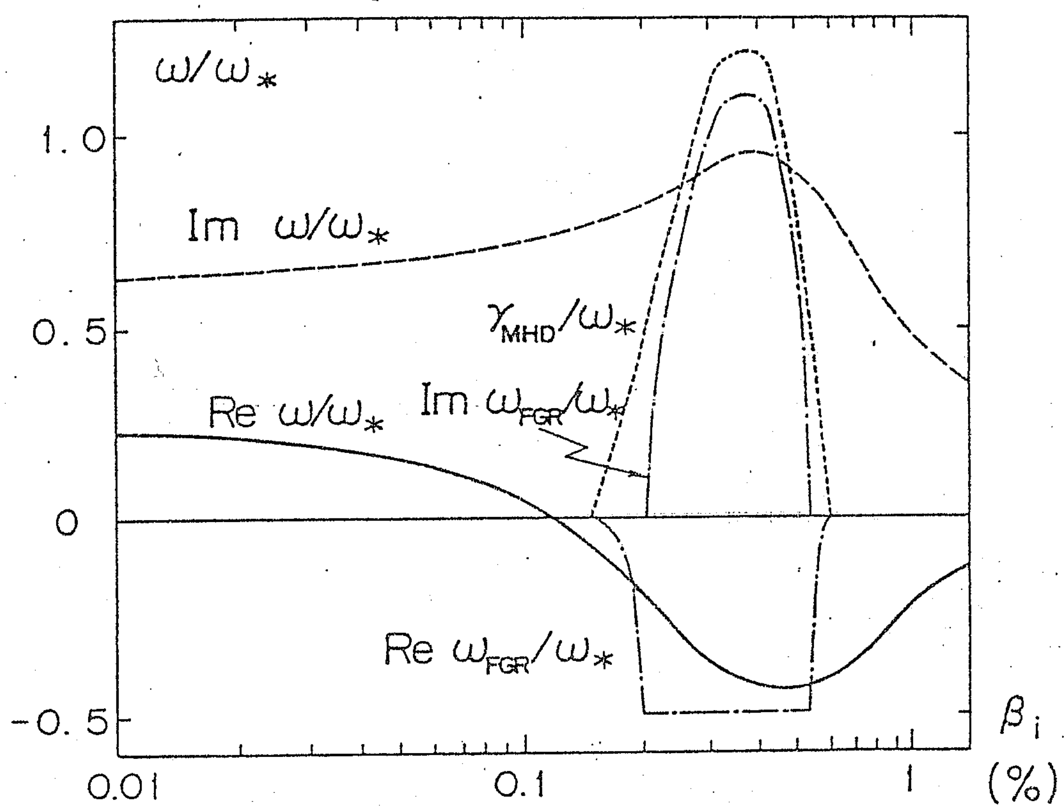


Fig. 18 The effect of the shift of magnetic axis. The eigenvalue, ω, of even-ϕ mode for the parameters of Fig. 2. MHD theory predicts the second stability but the kinetic mode remains to be unstable in both higher-β and lower-β regions. Peak of ω/ω_* is observed in MHD unstable region.

Fig. 19a

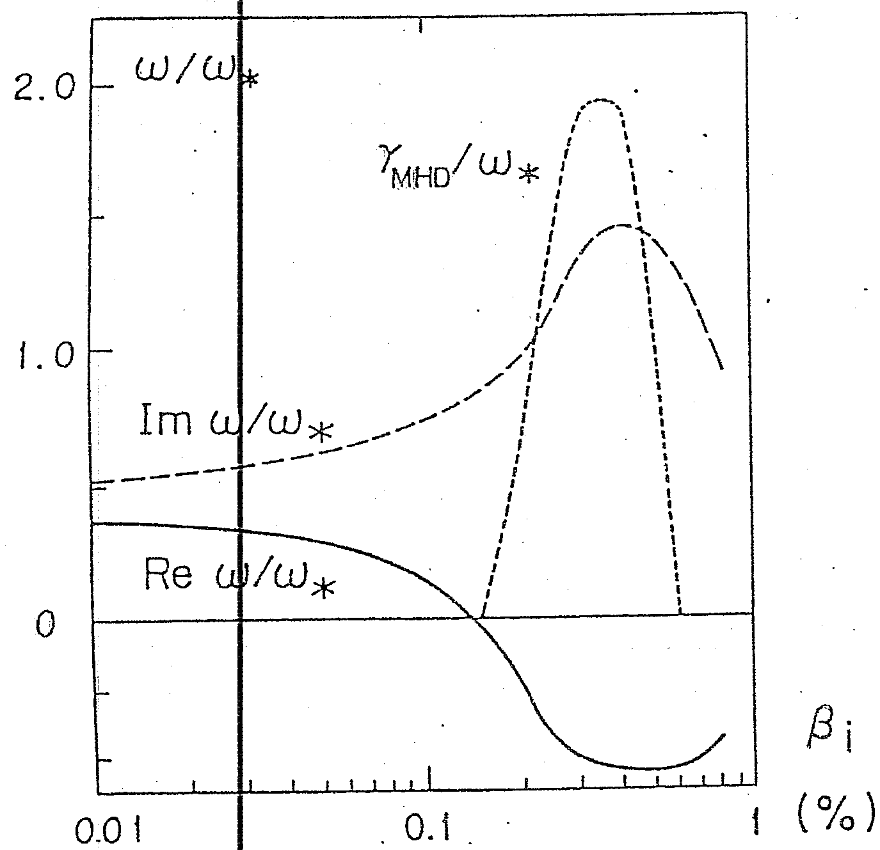


Fig. 19b

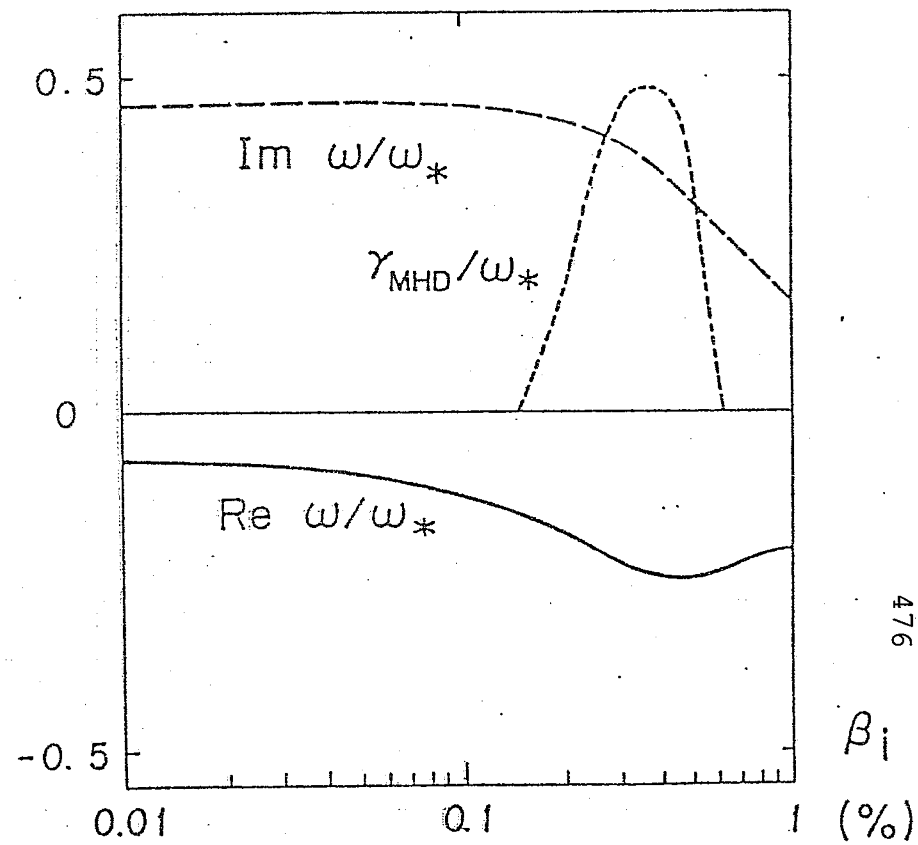


Fig. 19 The eigenvalue in the presence of shift of axis for other values $k\rho_i$. $k\rho_i = 0.125$ (a) and $k\rho_i = 0.5$ (b). For larger wave length mode, the hump of Im ω/ω_* is sharp. For the case $k\rho_i = 0.5$, the condition $\gamma_{MHD} < \omega_*/2$ is always satisfied for any β value and no apparent increase of Im ω/ω_* by β value is observed.

Fig. 20

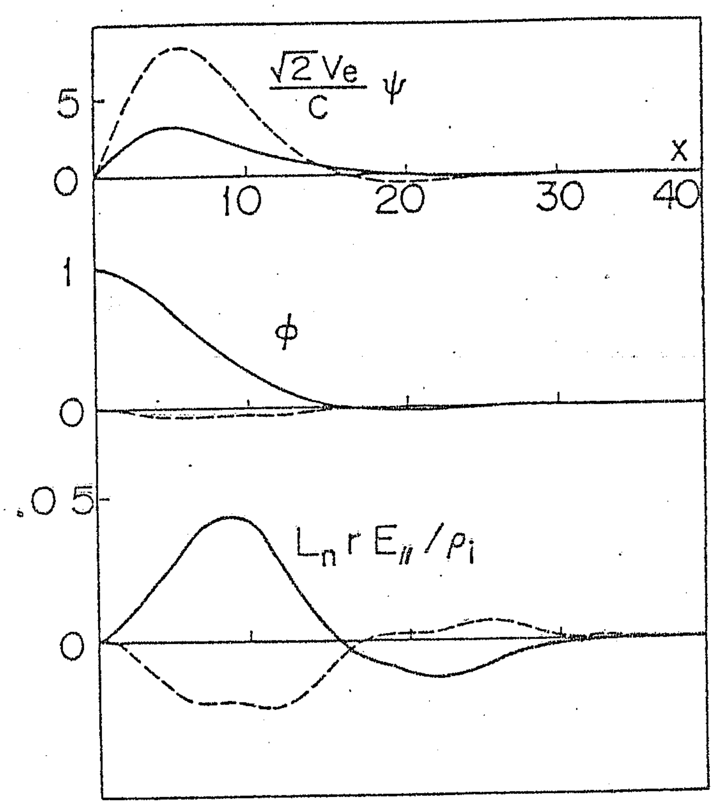


Fig. 20 The eigenmode structure for β_i = 1 %. Other parameters are the same as those in Fig.18. The MHD theory predicts stability but this mode is residually unstable $(\omega,\gamma) = (-0.225, 0.732)\omega_*$.

Fig. 21
タックインデックス貼付位
（インデックス下端を貼付位置ライン
に合わせて貼ってください。

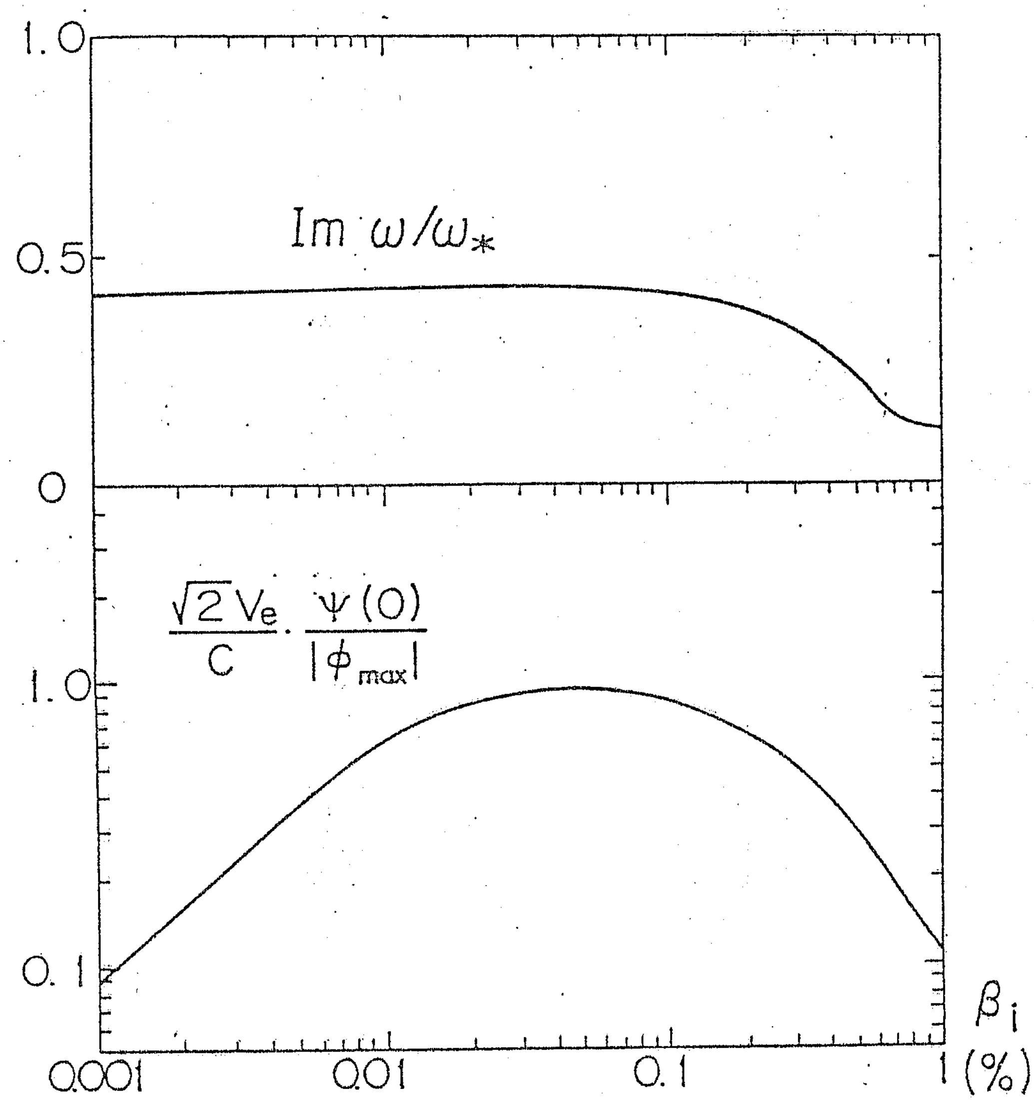


Fig. 21 Growth rate (top) and $\psi(0)/|\phi_{max}|$ (bottom) of odd-ϕ mode. Due to the shift of the magnetic axis, Im ω/ω_* decreases for higher β values; no apparent peak is observed. Magnetic perturbation has the maximum when β_i is around M_e/M_i.

Fig. 22

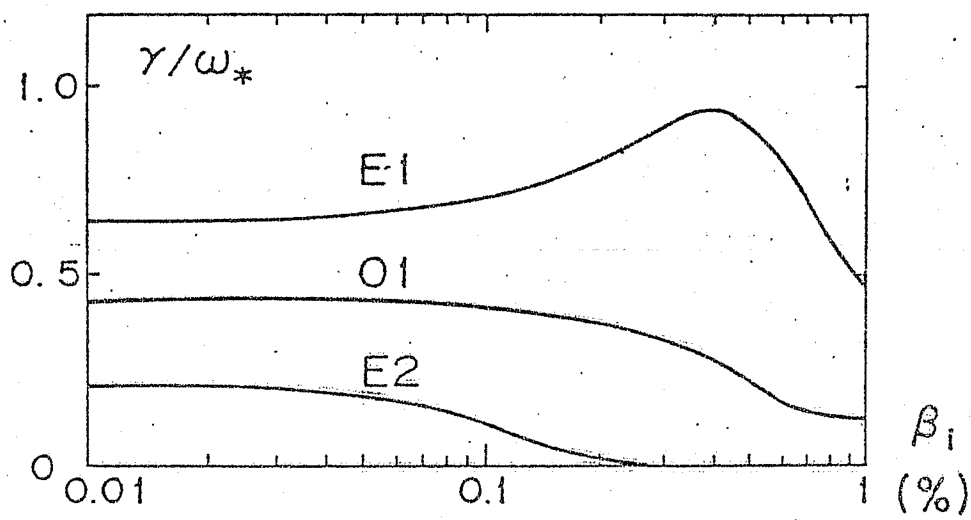
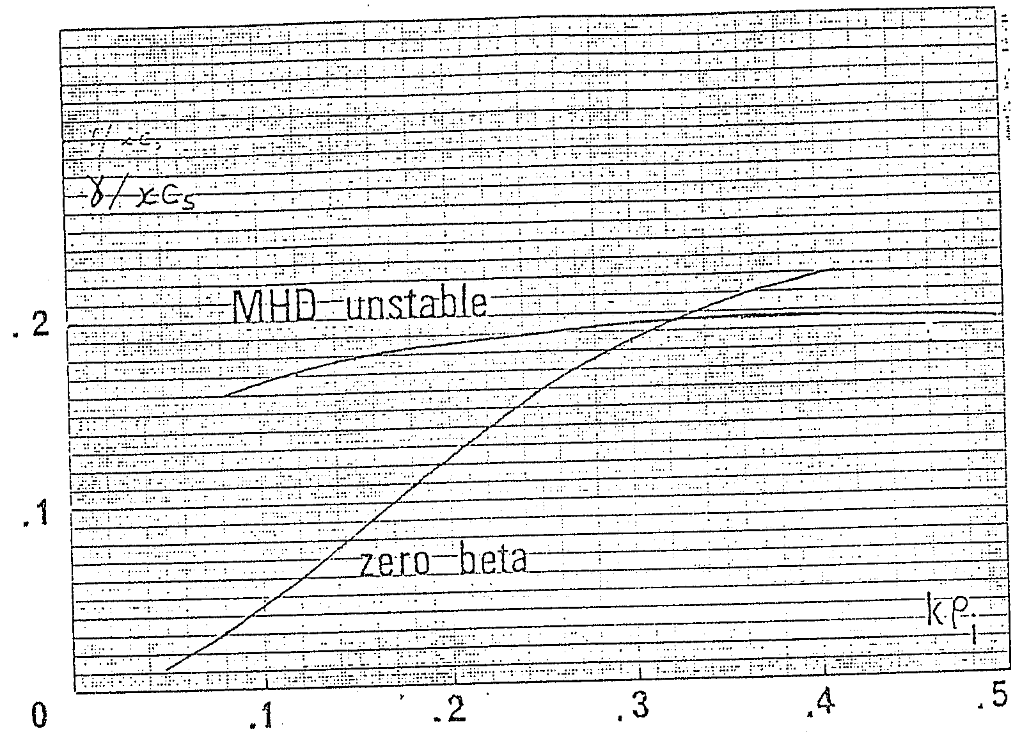
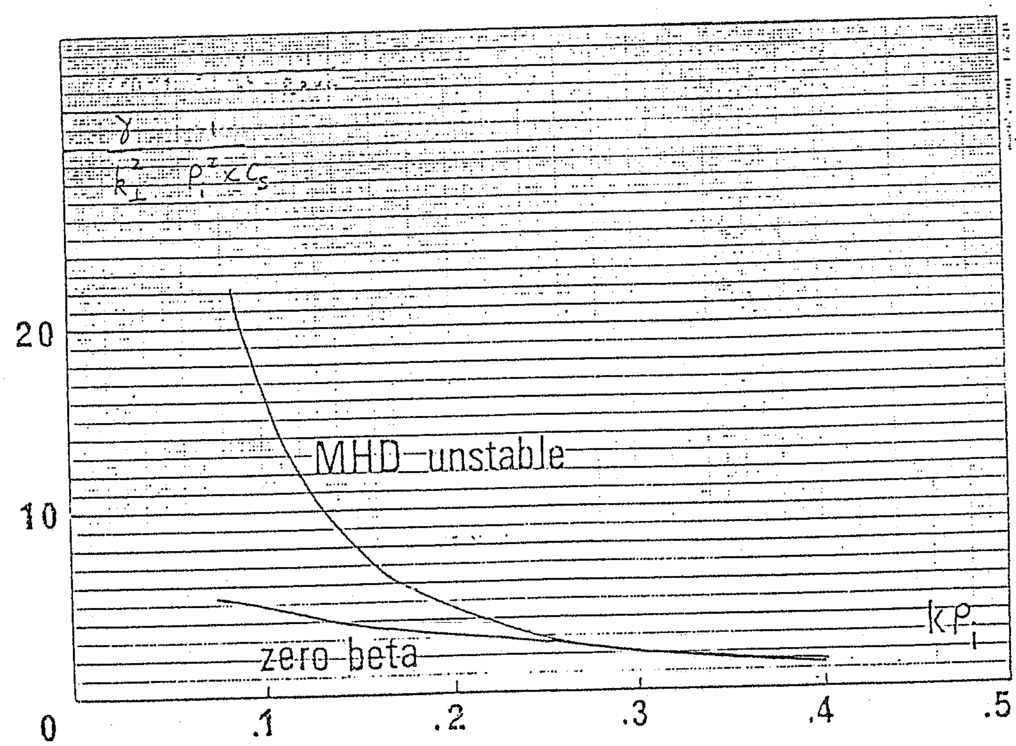
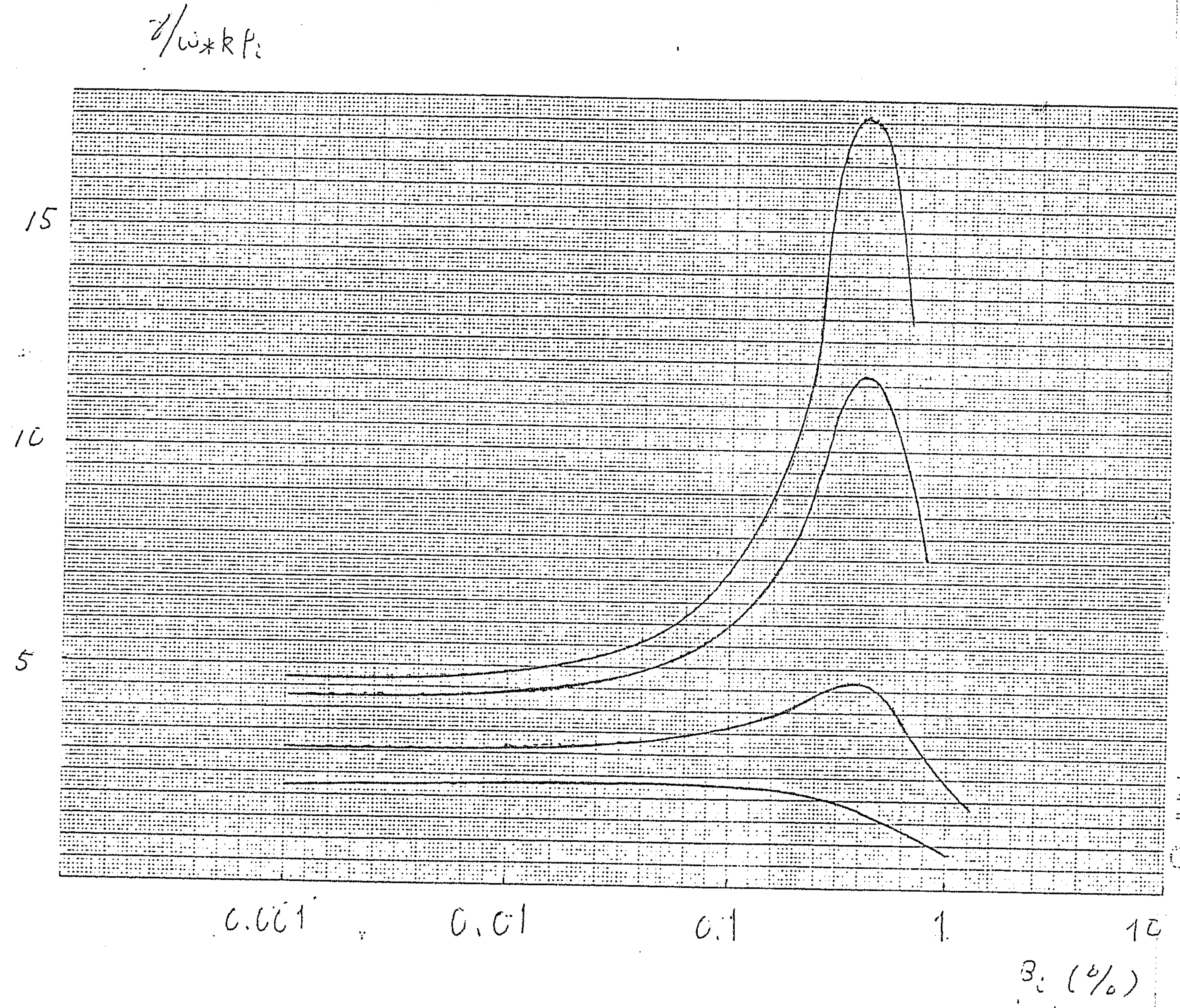


Fig. 22 Growth rate normalized to ω_* for the parameters of Fig.18. The fundamental (E1) and second (E2) even-ϕ modes and the fundamental odd-ϕ mode (O1). The stabilization by the shift of the magnetic surface is strong and the higher mode is finally stabilized for high β-values. Compare to Fig. 9.


γ/ xc_s
MHD unstable
zero beta
kρ_i
.2
.1
0
.1
.2
.3
.4
.5


γ/k_⊥²ρ_i² x c_s
MHD unstable
zero beta
kρ_i
20
10
0
.1
.2
.3
.4
.5


γ/ω*kρi
15
10
5
0.001
0.01
0.1
1
10
βi (%)

Summary and Implications

- Identification of high-n Ballooning Mode

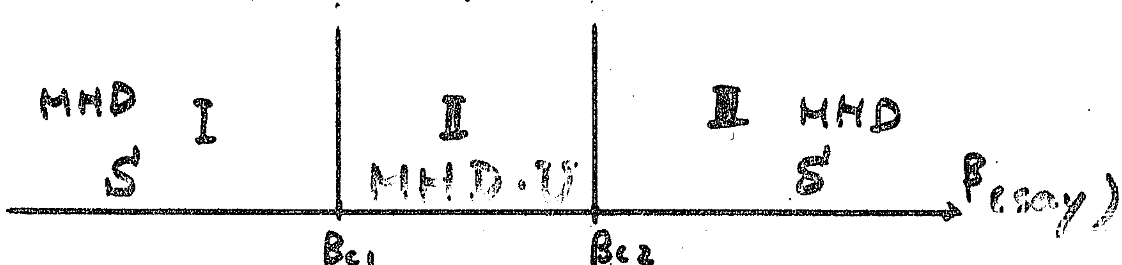
  < consistent kinetic treatment >
  ρ_i, full Z, $E_{\parallel}$,

- Recovered Previously obtained branches.

  < Mode Classification >
  - slab – toroidal
  - ES – EM
  - low-β – high-β
  - Low m – High m
  - Resonant – Off-resonant

- Obtained modestructures

  even ϕ most unstable
  odd ϕ magnetic island formation.



MHD v.s. (Residual) Kinetic

I $\gamma \sim \omega_*$, $\omega \sim 0$

II $\gamma_{MHD} \gg \omega_*$ $\gamma \sim \gamma_{MHD}$
$\omega_* \gg \gamma_{MHD}$ $\gamma \sim \omega_*$, $\Big\}$ $\omega \sim \dfrac{\omega_*}{2\tau}$

III $\gamma \sim \omega_*$, $\omega \sim 0$

• Possibility to Cross the unstable region.
to ~~reach~~ 2nd stability. one.

★ Stability

↔ Staying Probability in good curvature region
(number density)
(connection length)

★ Anomalous Transport ↔ may not be so enhanced.

· magnetic island width : small
< γ . modestructure around r_s >
↔ max when $\beta \sim m_e/m_i$

· how much the free energy of the plasma
is able to transfered to the wave
(hence) to thermalize magnetic field line.
↔ related to exchange rate balance
not only the growth rate.

ELECTROMAGNETIC KINETIC TOROIDAL EIGENMODES FOR GENERAL MHD EQUILIBRIA

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AND M.S. CHANCE

PRINCETON PLASMA PHYSICS LABORATORY

L1

Electromagnetic Kinetic Toroidal Eigenmodes for General MHD Equilibria

G. Rewoldt, W. M. Tang, and M. S. Chance

Plasma Physics Laboratory, Princeton University

Refereed Report PPPL-1830
Report PPPL-1769 for coll. operator

2

Motivation

- Calculation of tokamak linear eigenmodes and eigenfrequencies in as accurate and complete a manner as is feasible, for application to real experiments.

Features

(1) General numerical MHD equilibrium [from PEST-I or equivalent equilibrium code].

(2) Fully electromagnetic [Φ, $A_{\parallel}$, $A_{\perp}$].

(3) General form of ion response, valid for $|\omega| \lesssim \bar{\omega}_{bi}, \bar{\omega}_{ti}$, as

13

well as for $|\omega| > \bar{\omega}_{ti}$, and arbitrary $k_\perp \rho_i$, and $k_\perp \rho_{bi}$.

(4) Multiple ion species, with equilibrium distribution function for beam species which is non-Maxwellian in energy and anisotropic in pitch angle.

(5) Ballooning formalism, for $n \gg 1$. [Can compute "$1/n$ correction term" and radial envelope of eigenfunction].

(6) Energy and pitch-angle

dependent Krook collision operator, which reproduces results of Lorentz operator for two limits $|\omega| \ll \nu_{eff}$ and $|\omega| \gg \nu_{eff}$, for $\nu_i^* < 1$.

(7) Extension to $|\omega| \sim \ell \Omega_i$.

Equations

$\vec{B} = B_0 g(\psi) \nabla\xi \times \nabla\psi \qquad [0 \le \psi \le 1]$

$\xi \equiv \zeta - q(\psi)(\theta - \theta_0)$

Jacobian $\mathcal{J} \propto R^2$ [PEST-I coord.]

Ballooning representation:

$$\Phi(\psi, \theta, \zeta, t) = \exp[-i\omega t + in\zeta] \cdot \sum_{p=-\infty}^{\infty} \hat{\phi}(\theta - 2\pi p) \exp[-inq(\psi)(\theta - 2\pi p)]$$

15

likewise for $A_\parallel$, $A_\perp$, f, and

$$h \equiv f - e\Phi \frac{\partial F}{\partial E} - \frac{e}{B}\frac{\partial F}{\partial \mu}\Big\{(\Phi - \frac{v_\parallel}{c}A_\parallel)$$

$$\cdot\left[\exp(i\vec{k}_\perp\cdot\vec{v}_\perp/\Omega) - J_0(k_\perp v_\perp/\Omega)\right.$$

$$\left. + i\frac{v_\perp}{c}A_\perp J_1(k_\perp v_\perp/\Omega)\right\}$$

gyrokinetic equation:

$$\frac{v_\parallel}{JB}\frac{\partial}{\partial\theta}\hat{h}_\sigma - i[\omega + i\nu_{\text{ef}} - \omega_d(\theta)]\hat{h}_\sigma(\theta)$$

$$= -i\frac{e}{T}F[\omega - \omega_*^T]\{[\hat{\phi}(\theta) - \frac{v_\parallel}{c}\hat{A}_\parallel]J_0(\theta)$$

$$- i\frac{v_\perp}{c}A_\perp J_1(\theta)\}$$

$$J_\ell(\theta) \equiv J_\ell\left(\frac{n v_\perp}{\Omega}|\nabla\xi|\right),$$

$$|\nabla\xi| \equiv \Big[\left(\frac{1}{R^2} + q^2|\nabla\theta|^2\right) + \left(\frac{dq}{d\psi}\right)^2(\theta-\theta_0)^2|\nabla\psi|^2$$

$$+ 2q\frac{dq}{d\psi}(\theta-\theta_0)\nabla\psi\cdot\nabla\theta\Big]^{1/2}.$$

Non-Maxwellian Species:

$$F(\omega-\omega_*^T) \longrightarrow \left[\omega\left(-T\frac{\partial F}{\partial E}\right)+\frac{ncT}{eB_0q}\frac{\partial}{\partial\psi}F\right]$$

circulating particles:

$$\hat{h}_\sigma(\theta)=\frac{e}{T}F(\omega-\omega_*^T)\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dK$$

$$\frac{\exp[iK\tilde{\theta}(\theta)+i\sigma W_d(\theta)]}{\omega+i\nu_f-K\omega_t}$$

$$\cdot\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\theta'\frac{|\omega_t|B\mathcal{J}}{|v_\parallel|}\exp[-iK\tilde{\theta}(\theta')-i\sigma W_d(\theta')]$$

$$\left\{\left[\hat{\phi}(\theta')-\frac{v_\parallel}{c}\hat{A}_\parallel(\theta')\right]J_0(\theta')-i\frac{v_\perp}{c}A_\perp(\theta')J_1(\theta')\right\}$$

where

$$\tilde{\theta}(\theta)\equiv|\omega_t|\int_0^\theta d\theta''\,B\mathcal{J}/|v_\parallel|,$$

$$W_d(\theta)\equiv-\int_0^\theta d\theta''\,\omega_d(\theta'')B\mathcal{J}/|v_\parallel|.$$

7

trapped particles (turning points θ_1, θ_2):

$$\left[\hat{h}_+(\theta) \pm \hat{h}_-(\theta)\right] = \frac{e}{T} F(\omega - \omega_*^T) \sum_{p=-\infty}^{\infty}$$

$$\left[e^{ip\tilde{\theta}(\theta)} \left(\frac{\Psi_+^p \exp[i w_d(\theta)]}{\omega + i\nu_f - \omega_d^{(0)} - p\omega_b} \pm \right.\right.$$

$$\left. \frac{\Psi_-^p \exp[-i w_d(\theta)]}{\omega + i\nu_f - \omega_d^{(0)} + p\omega_b} \right) - \frac{(-1)^p}{\cos \pi\Gamma}$$

$$\begin{Bmatrix} i \sin \\ \cos \end{Bmatrix} [\Gamma \tilde{\theta}(\theta) + w_d(\theta)]$$

$$\left. \cdot \left(\frac{\Psi_+^p}{\omega + i\nu_f - \omega_d^{(0)} - p\omega_b} - \frac{\Psi_-^p}{\omega + i\nu_f - \omega_d^{(0)} + p\omega_b} \right) \right]$$

where

$$\Psi_\sigma^p \equiv \frac{1}{2\pi} \int_{\theta_1}^{\theta_2} d\theta \frac{\omega_b B \mathcal{J}}{|v_\parallel|} \exp[-ip\tilde{\theta}(\theta) - i\sigma w_d(\theta)]$$

$$\left\{ \left[\hat{\phi}(\theta) - \frac{v_\parallel}{c} A_\parallel(\theta) \right] J_0(\theta) - i \frac{v_\perp}{c} A_\perp(\theta) J_1(\theta) \right\},$$

18

$$\tilde{\theta}(\theta) \equiv \omega_b \int_{\theta_1}^{\theta} d\theta'' B\mathcal{J}/|v_\parallel| - \pi$$

$$\omega_d^{(0)} \equiv \frac{1}{\tau_b} \int_{\theta_1}^{\theta_2} d\theta'' \omega_d(\theta'') B\mathcal{J}/|v_\parallel|$$

$$W_d(\theta) \equiv -\int_{\theta_1}^{\theta} d\theta'' \left[\omega_d(\theta'') - \omega_d^{(0)}\right] B\mathcal{J}/|v_\parallel|$$

$$\Gamma \equiv \left[\omega + i\nu_f - \omega_d^{(0)}\right]/\omega_b$$

Mode Equations:

quasi-neutrality:

$$0 = \sum_j e_j \int d^3v \Big[\hat{h}_0 J_0 + e_j \frac{\partial F_j}{\partial E} \Phi + \frac{e_j}{B} \frac{\partial F_j}{\partial \mu} \Big\{ \Big(\hat{\Phi} - \frac{v_\parallel}{c} \hat{A}_\parallel\Big)(1 - J_0^2) + i \frac{v_\perp}{c} \hat{A}_\perp J_0 J_1 \Big\}$$

+ parallel Ampere's law

+ perpendicular Ampere's law

9

$$\int d^2 v \equiv \sqrt{2}\,\pi \int_0^\infty dE \sqrt{E} \int_0^{h(\theta)} d\Lambda \{h(\theta)\sqrt{1-\Lambda/h(\theta)}\}^{-1}$$

$$h(\theta) \equiv B_0/B(\theta)\,;\ \Lambda \equiv \mu B_0/E$$

Solution Method (Ritz):

- $\hat{\phi}(\theta) = \sum_{\ell=0}^{L-1} \hat{\phi}_\ell h_\ell(\theta)/\sqrt{g}$

[likewise for $\hat{A}_\parallel(\theta)$, $\hat{A}_\perp(\theta)$]

- Operate with

$$\frac{T_e}{e^2 n_e} \int_{-\infty}^{\infty} d\theta \sqrt{g}\, h_{\ell'}(\theta)$$

- $3L \times 3L$ Matrix equation: $\mathbb{A}\begin{pmatrix}\hat{\phi}_0\\ \vdots \\ \hat{A}_{\perp}\end{pmatrix} = \begin{pmatrix}0\\ \vdots\end{pmatrix}$

- eigenvalues $\lambda_i(\omega)$ ($i = 1, 2, \ldots, 3L$)

- eigenvalue condition on ω is

$\lambda_i(\omega) = 0$ for some chosen i

- standard root-finding method for ω

Results

I. Artificial FCT Sequence

$T_j, n_j \propto \sqrt{\beta}$;

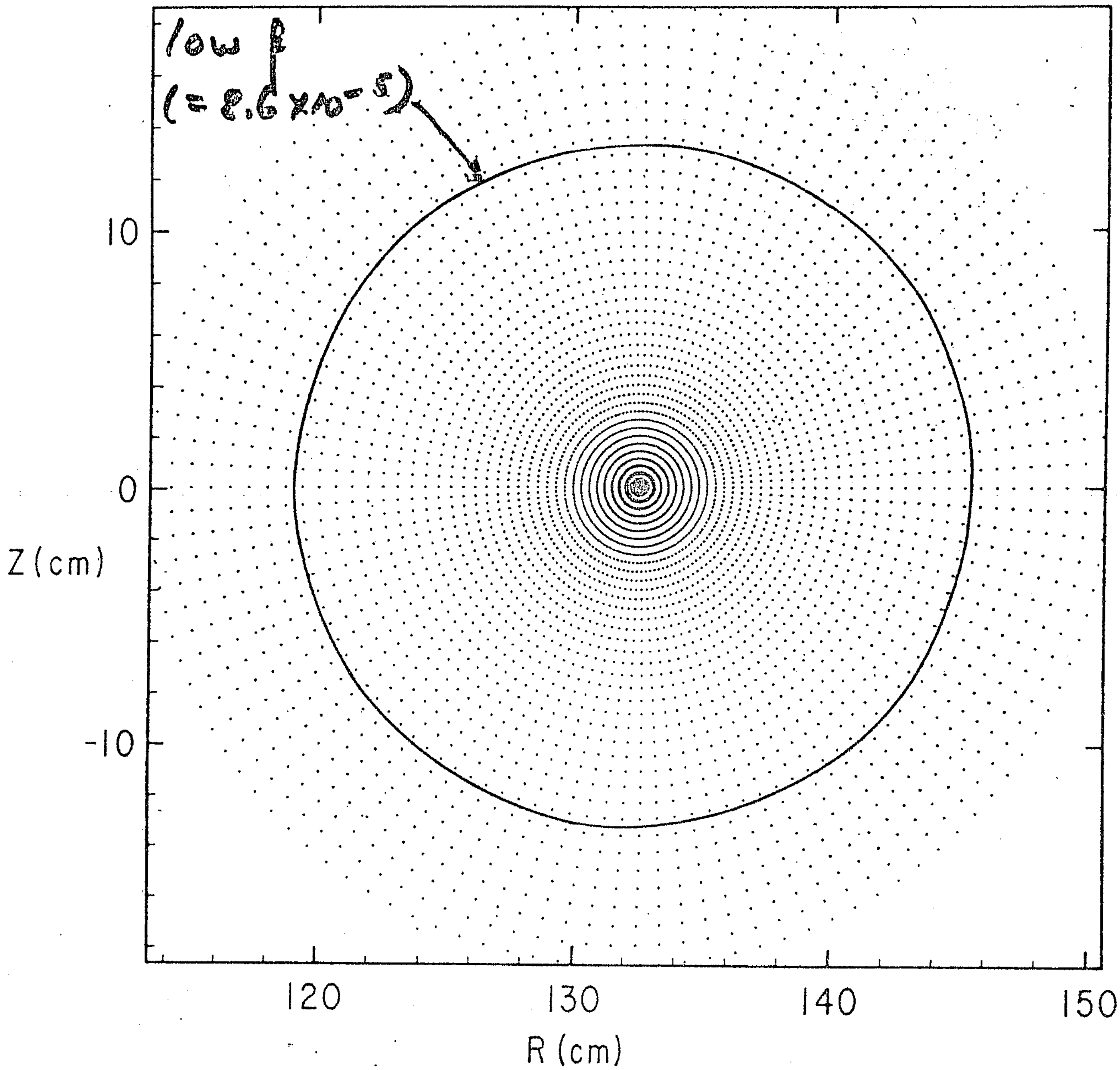
b_0, n fixed

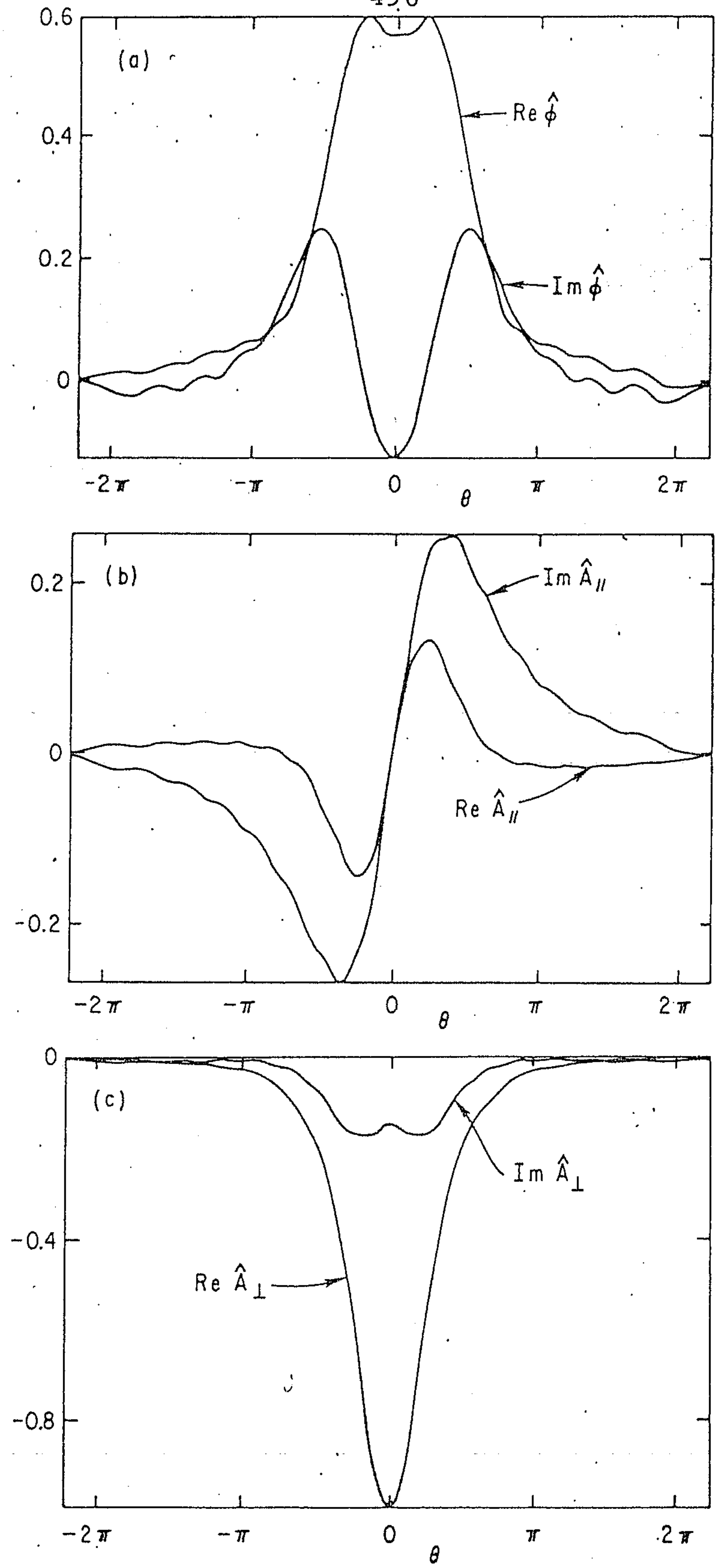
II. D-ness variation

III. Ellipticity variation

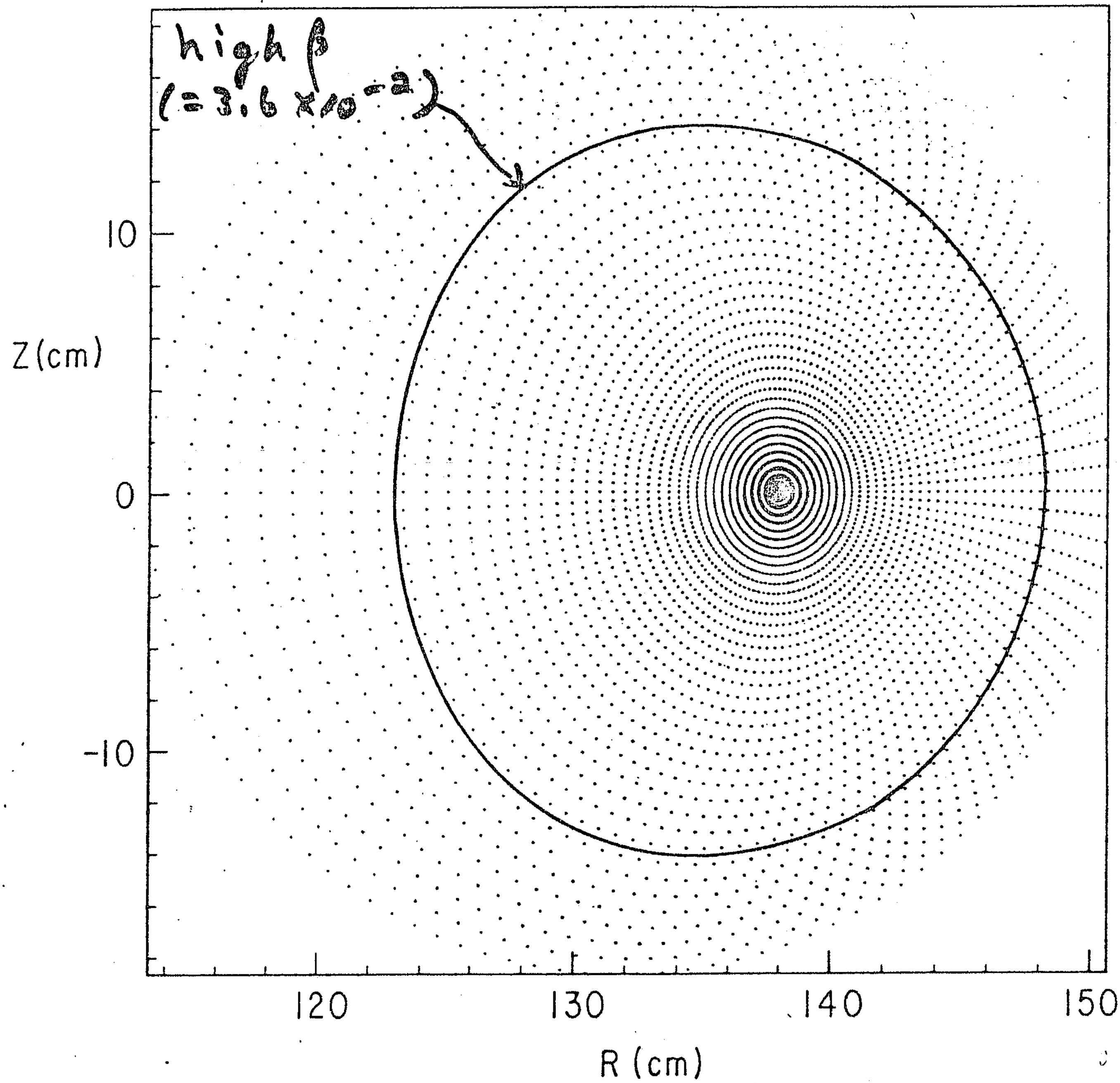
IV. PDX (Shot 35666)

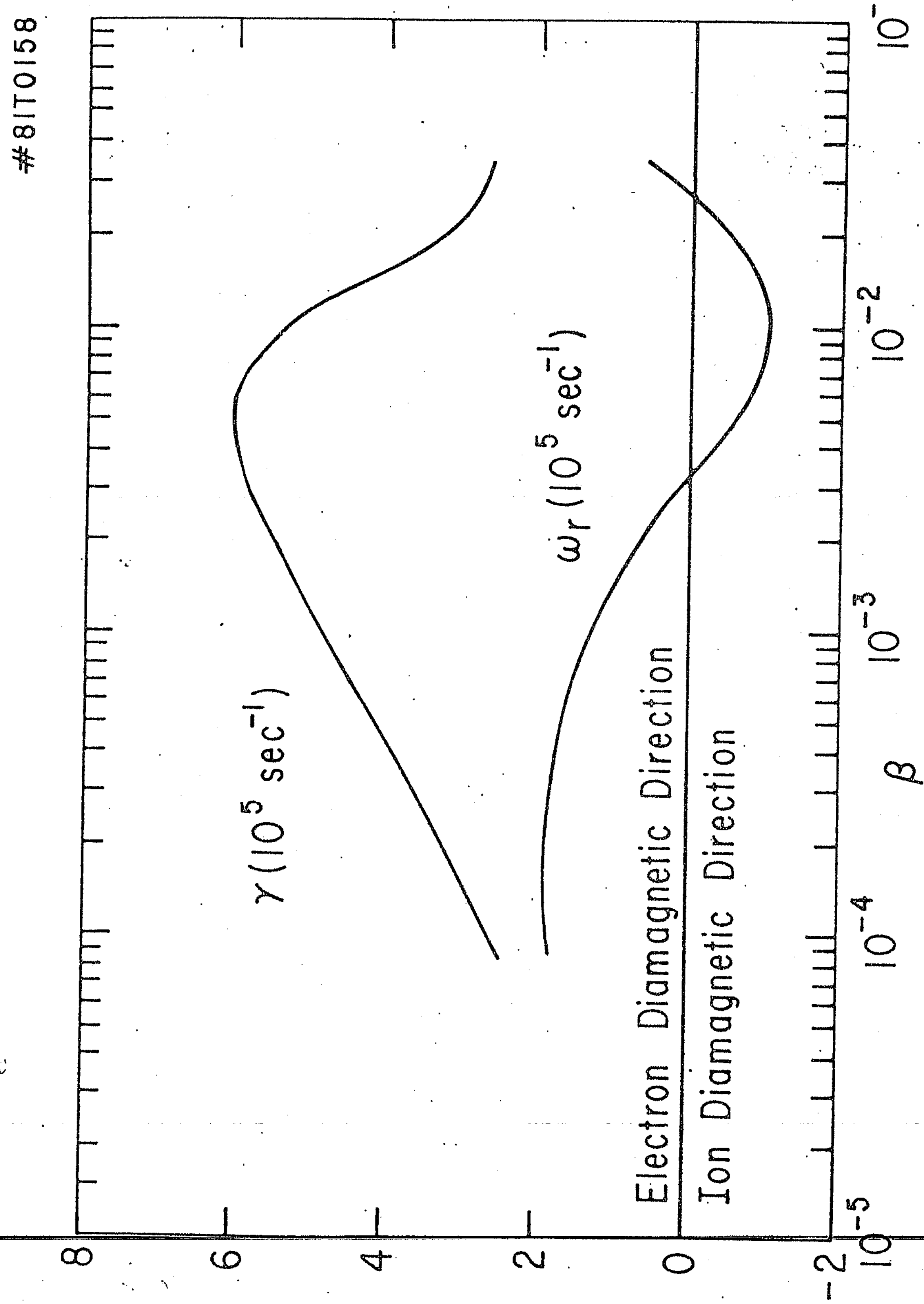
#81 T0150

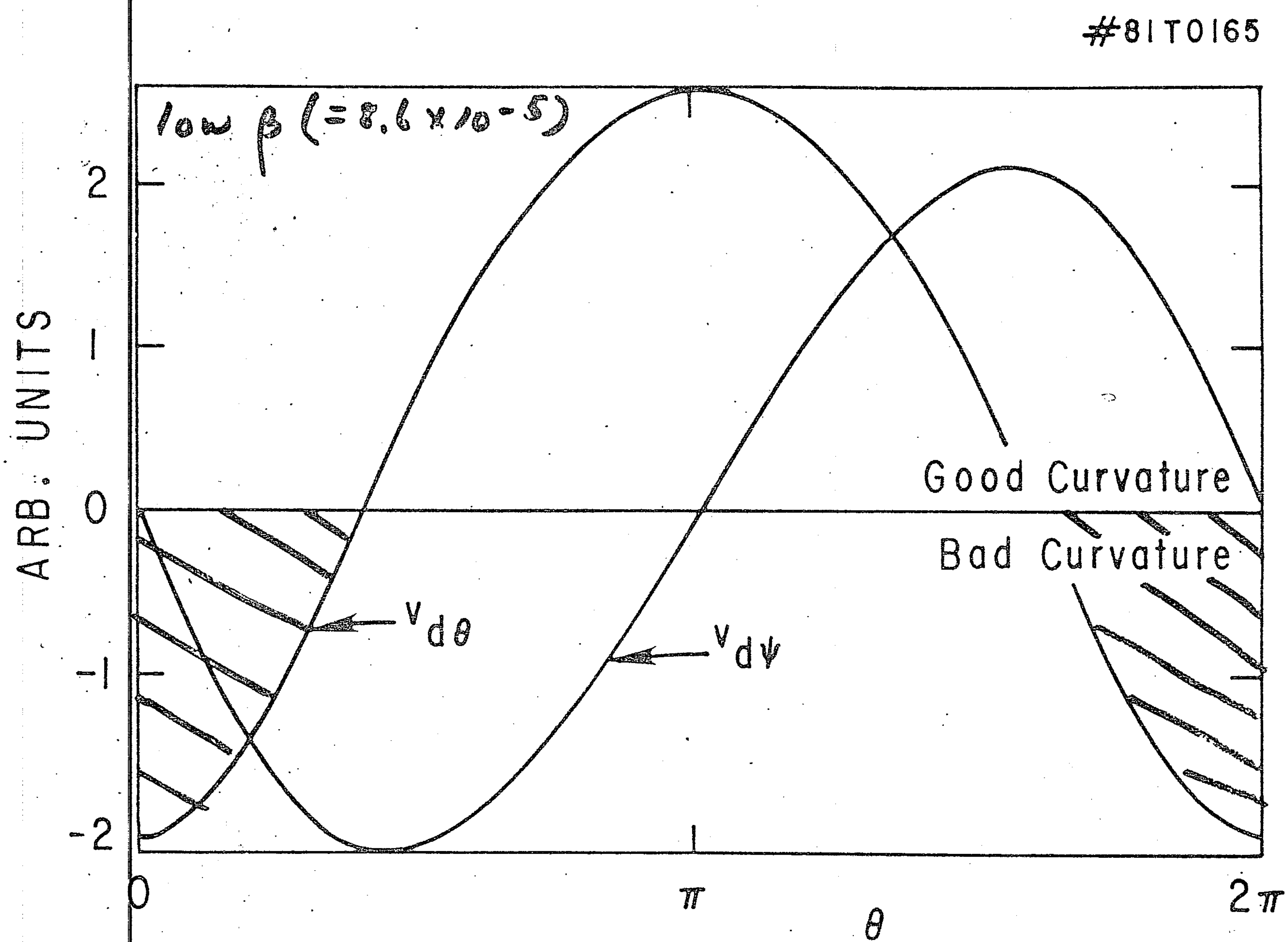


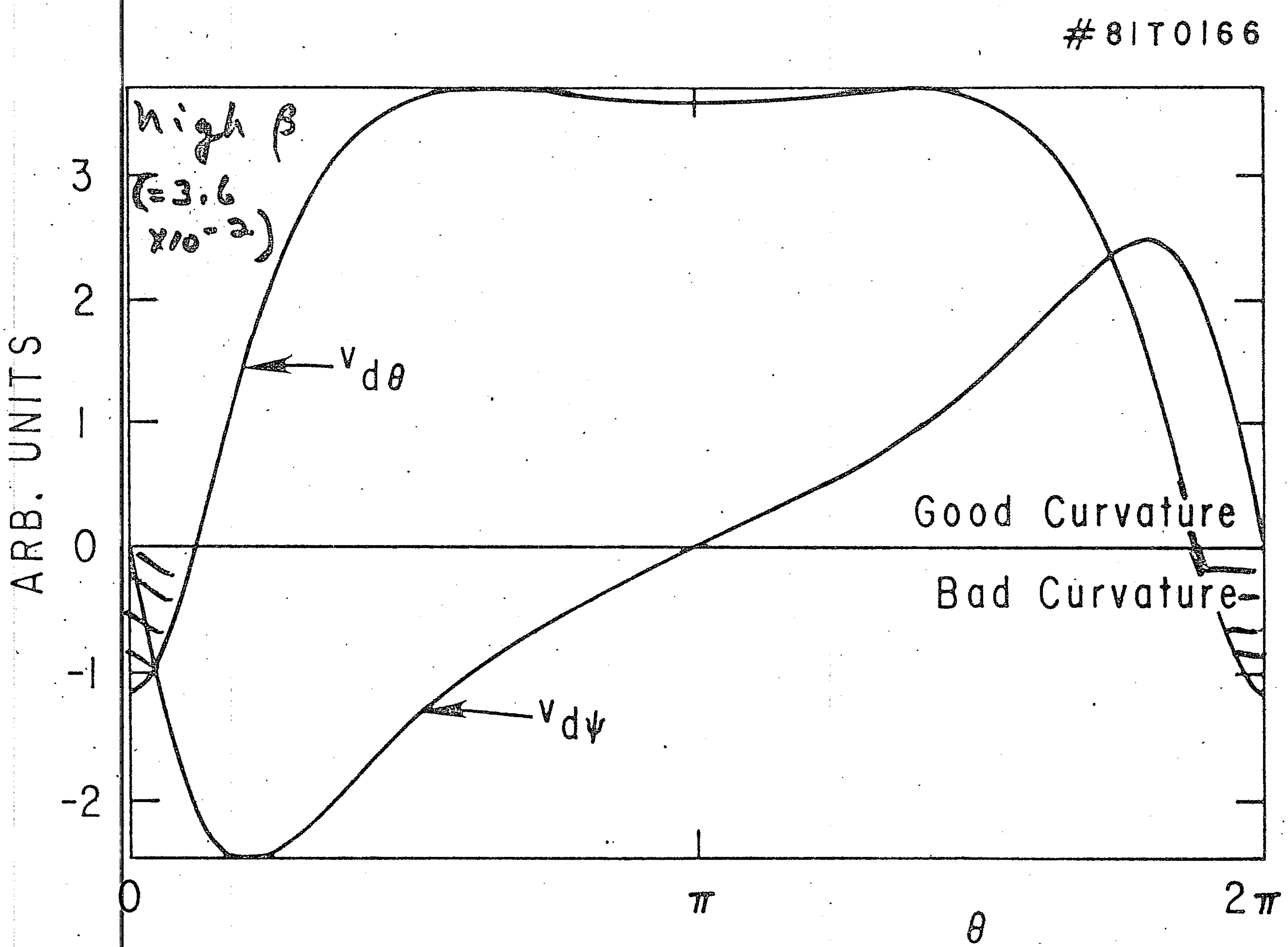

0.6
0.4
0.2
0
(a)
Re $\hat{\phi}$
Im $\hat{\phi}$
−2π
−π
0
π
2π
θ
0.2
0
−0.2
(b)
Im $\hat{A}_{\parallel}$
Re $\hat{A}_{\parallel}$
−2π
−π
0
π
2π
θ
0
−0.4
−0.8
(c)
Im $\hat{A}_{\perp}$
Re $\hat{A}_{\perp}$
−2π
−π
0
π
2π
θ

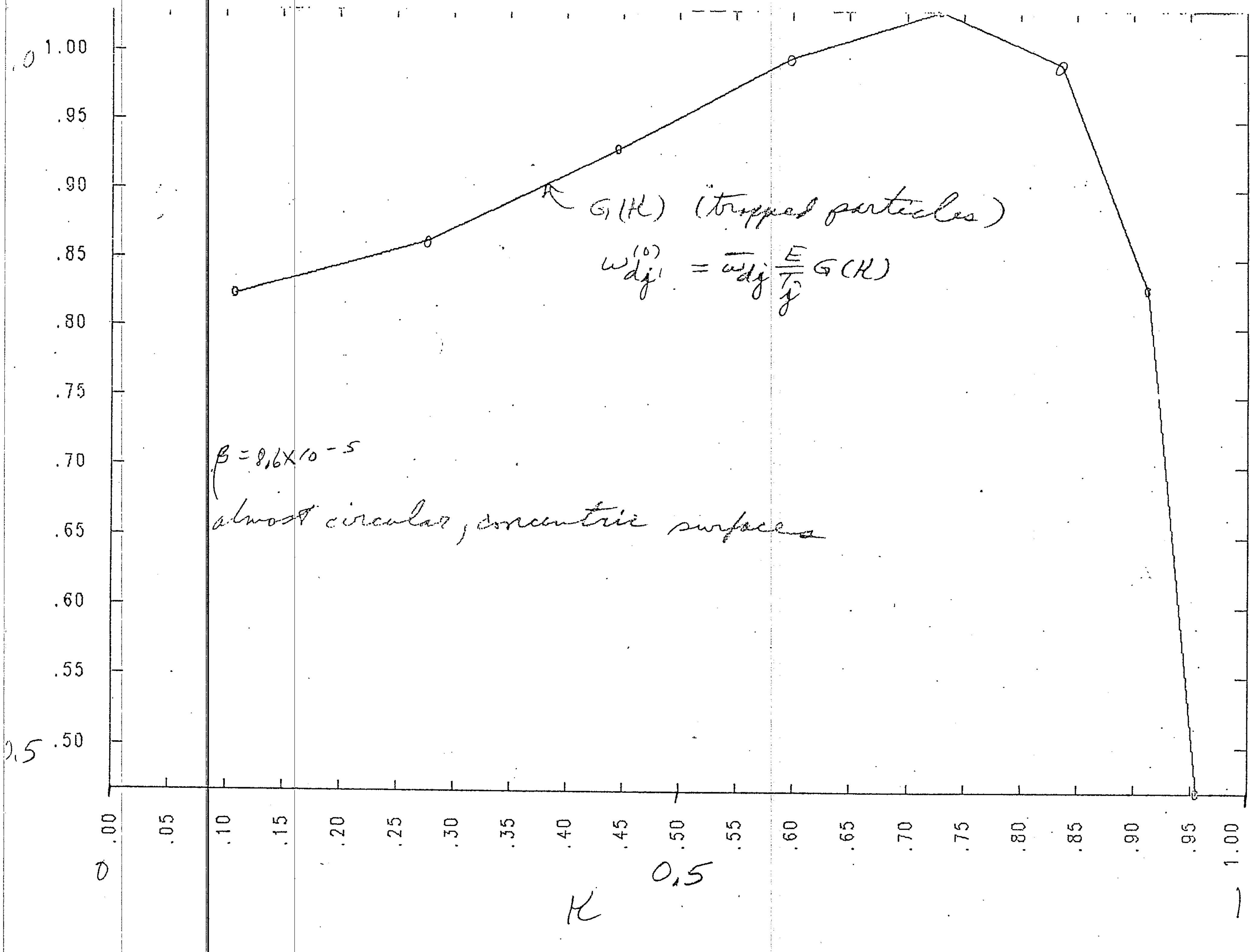
#81T0167



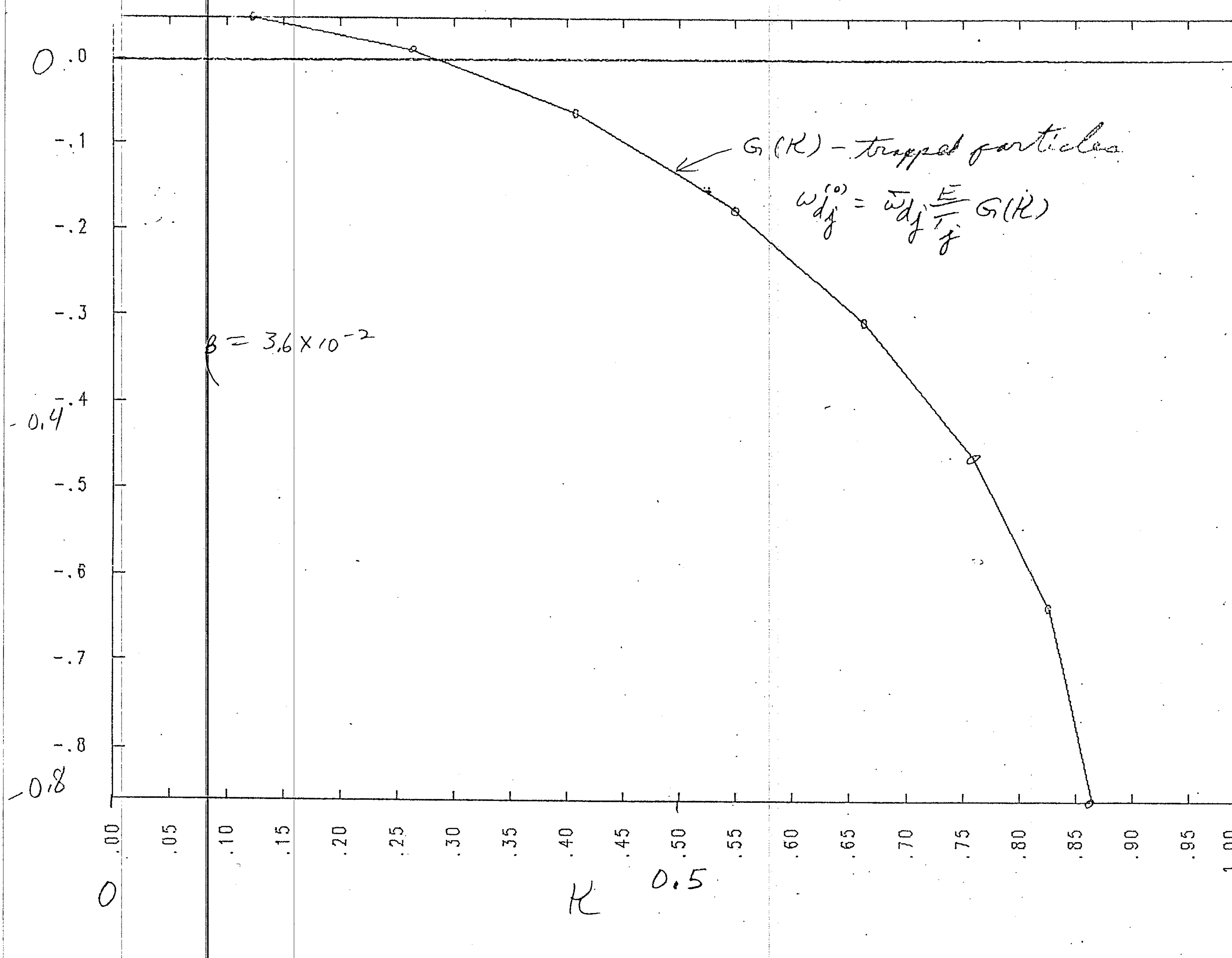

#81T0158
γ (10^5 sec^-1)
ω_r (10^5 sec^-1)
Electron Diamagnetic Direction
Ion Diamagnetic Direction
β
8
6
4
2
0
-2
10^-5
10^-4
10^-3
10^-2
10^-1

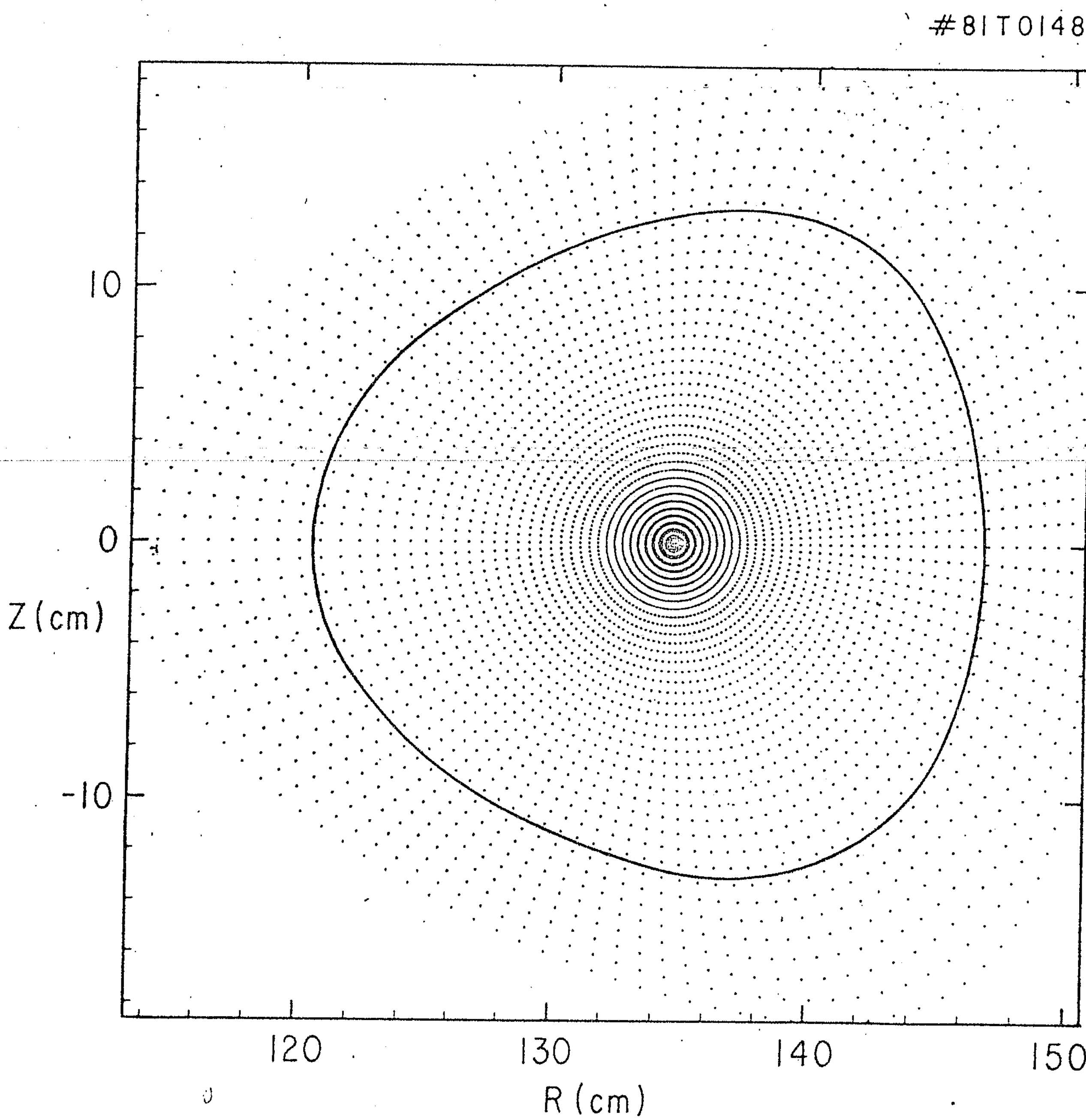

#81T0165
low β (= 8.6 x 10-5)
Good Curvature
Bad Curvature
$v_{d\theta}$
$v_{d\psi}$
2
1
0
-1
-2
ARB. UNITS
0
π
2π
θ

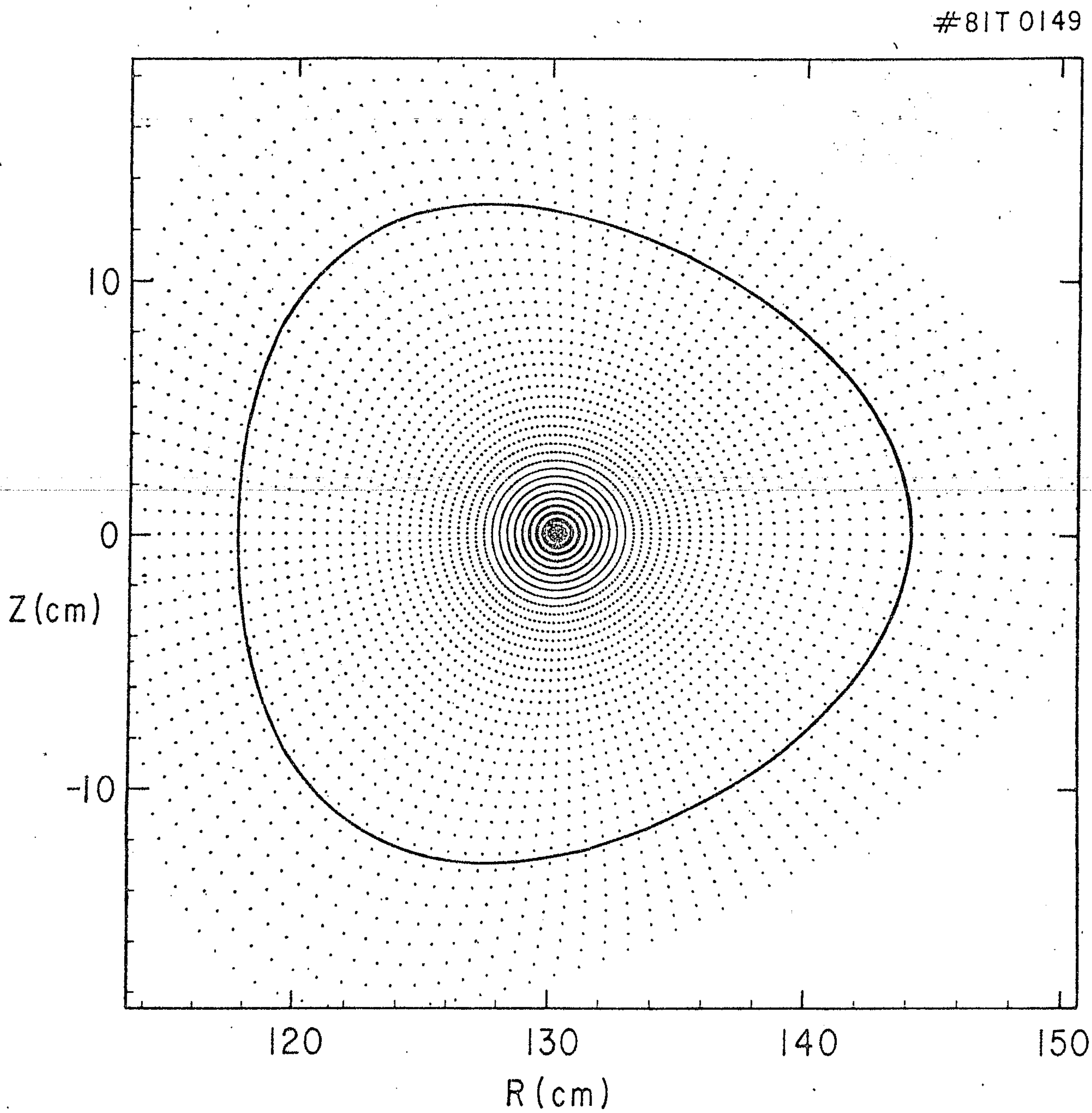

#81T0166
high β
(=3.6
x10^-2)
3
2
1
0
-1
-2
ARB. UNITS
$v_{d\theta}$
$v_{d\psi}$
Good Curvature
Bad Curvature
0
π
2π
θ

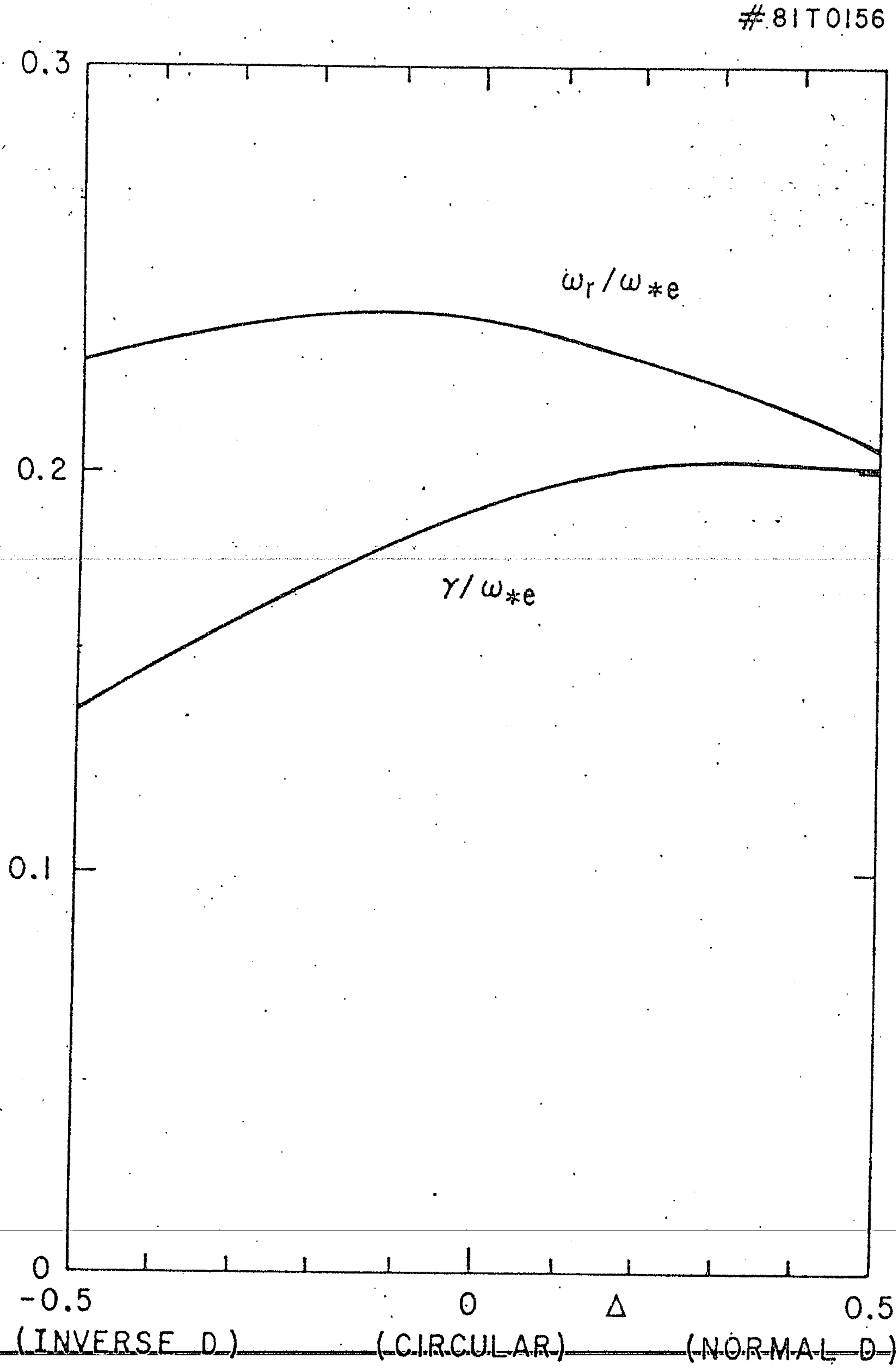


1.00
.95
.90
.85
.80
.75
.70
.65
.60
.55
.50
.00
.05
.10
.15
.20
.25
.30
.35
.40
.45
.50
.55
.60
.65
.70
.75
.80
.85
.90
.95
1.00
G(K) (trapped particles)
$\omega_{dj}^{(0)} = \bar{\omega}_{dj} \frac{E}{T_j} G(K)$
$\beta = 8.6 \times 10^{-5}$
almost circular, concentric surfaces
0
0.5
K

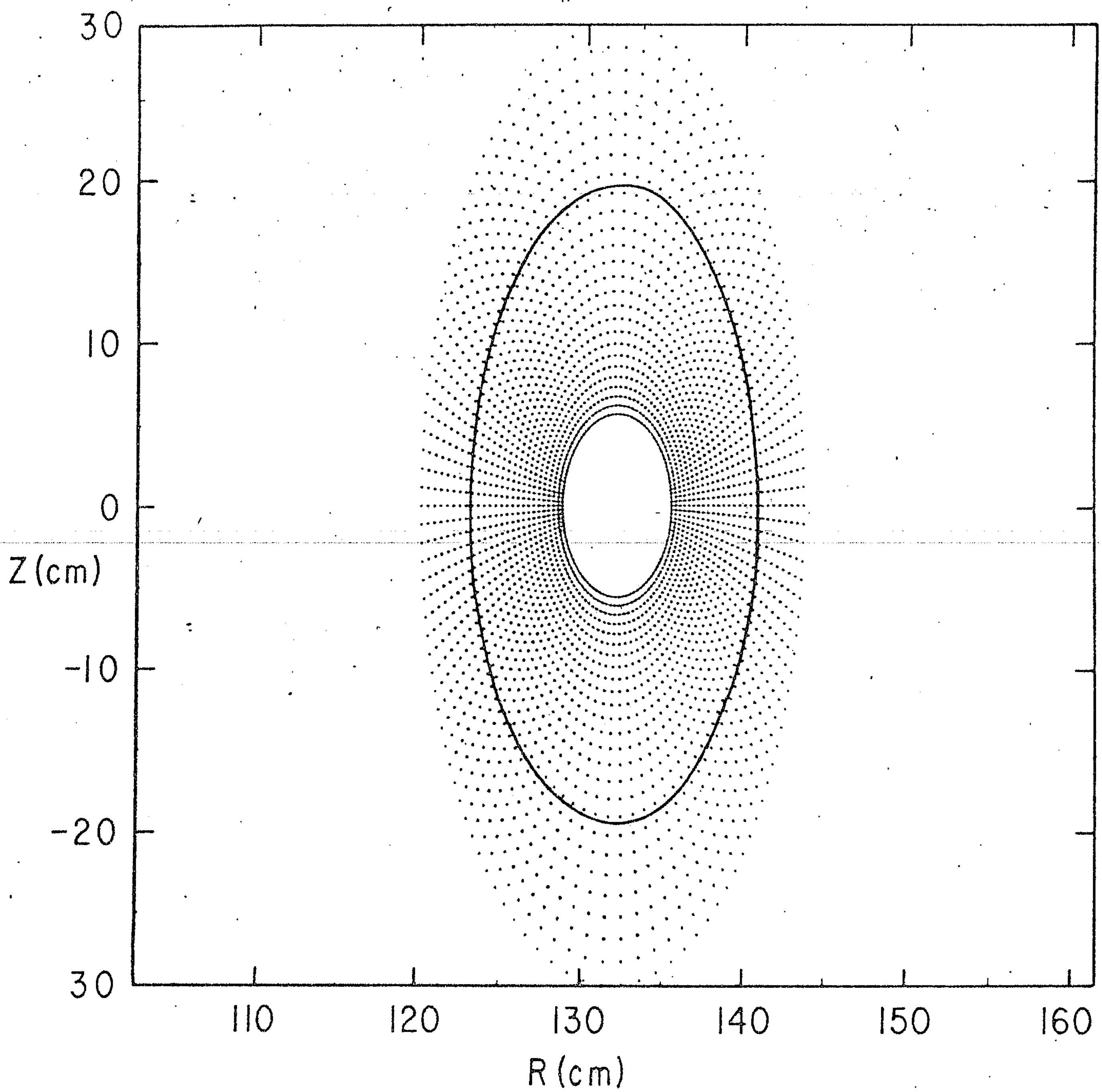

G(K) – trapped particles
$\omega_{dj}^{(0)} = \bar{\omega}_{dj} \frac{E}{T_j} G(K)$
$\beta = 3.6 \times 10^{-2}$
0 .0
-.1
-.2
-.3
-0.4 -.4
-.5
-.6
-.7
-.8
-0.8
.00
.05
.10
.15
.20
.25
.30
.35
.40
.45
.50
.55
.60
.65
.70
.75
.80
.85
.90
.95
1.00
0
0.5
1
K


#81T0148
10
0
-10
Z (cm)
120
130
140
150
R (cm)

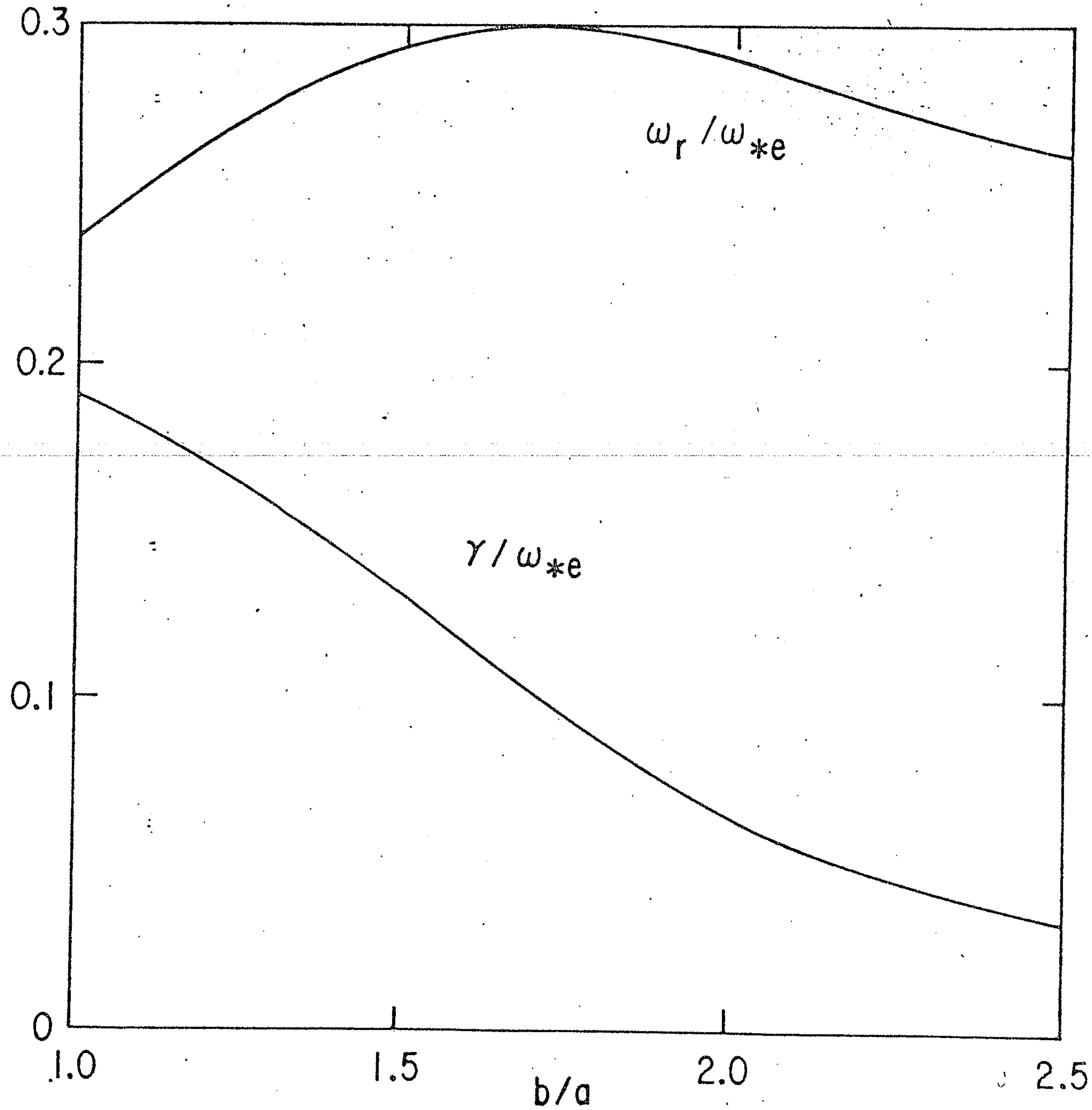

#81T0149
10
0
-10
Z(cm)
120
130
140
150
R(cm)


#81T0156
0.3
0.2
0.1
0
ω_r/ω_{*e}
γ/ω_{*e}
-0.5
0
0.5
Δ
(INVERSE D)
(CIRCULAR)
(NORMAL D)

#81T0182



#81T0157



PDX

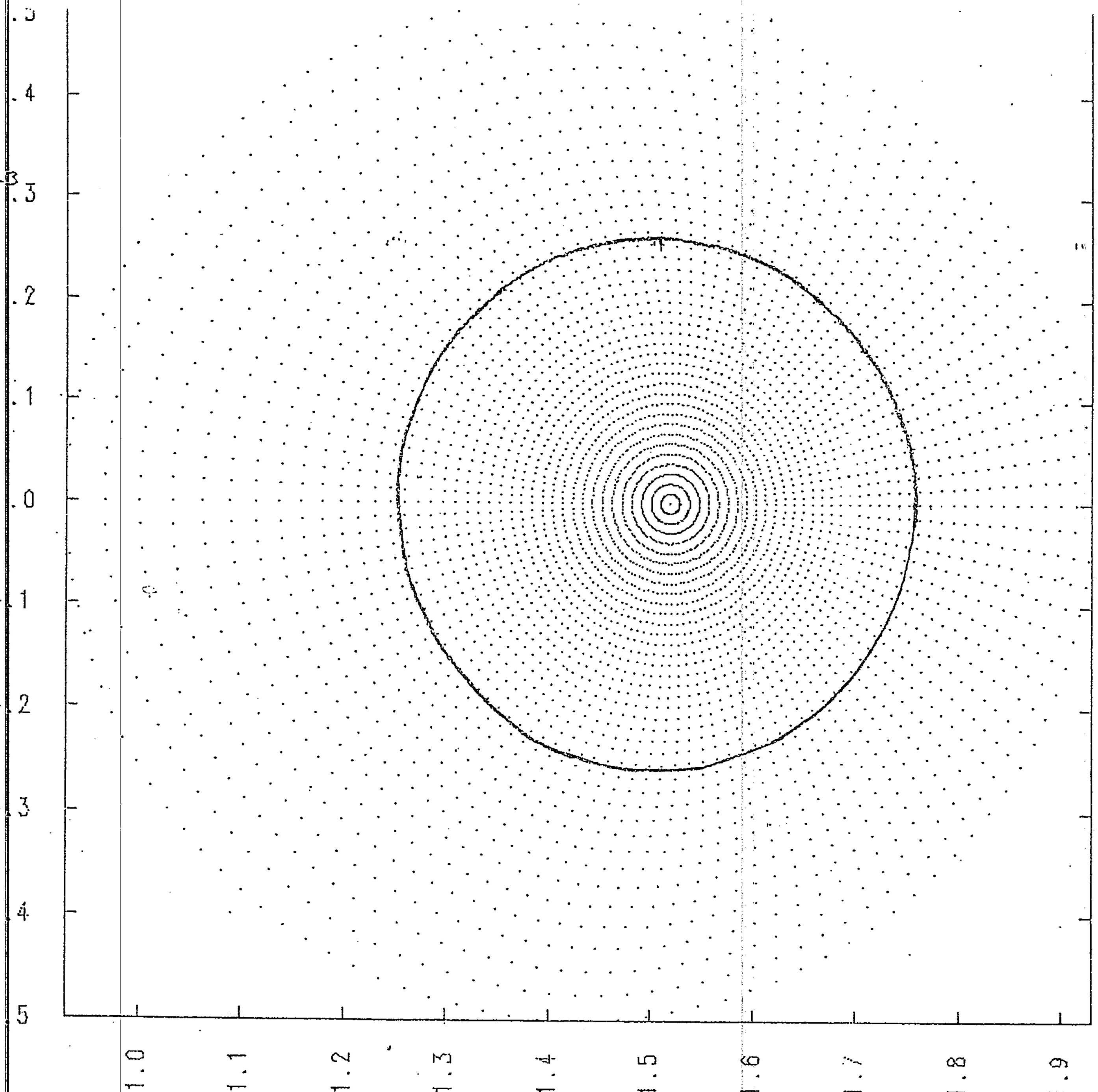
parameters at $r = 0$:

$n_e = 5.65 \times 10^{13}\ cm.^{-3}$

$T_e = 2.64$ keV

$T_i = 5.81$ keV

$\beta_0 = 3.9\ \%$



parameters at $r = 25$ cm.:

$n_e = 4.27 \times 10^{13}\ cm.^{-3}$

$T_e = 1.56$ keV

$T_i = 2.31$ keV

$\beta = 1.3\ \%$

$q = 1.37$

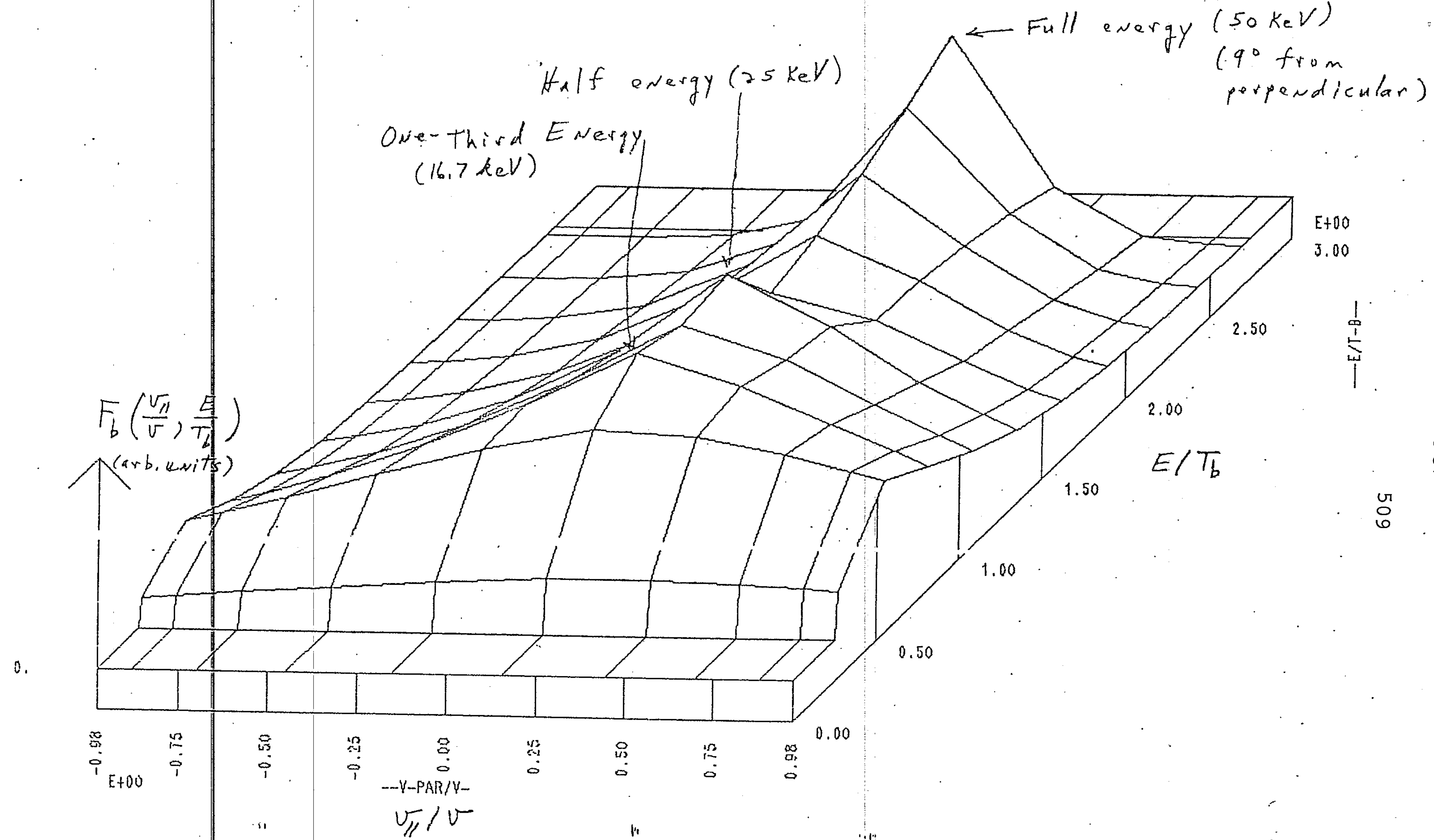
$q'r/q = 1.17$

$\eta_e = 4.29$

$\eta_i = 10.2$

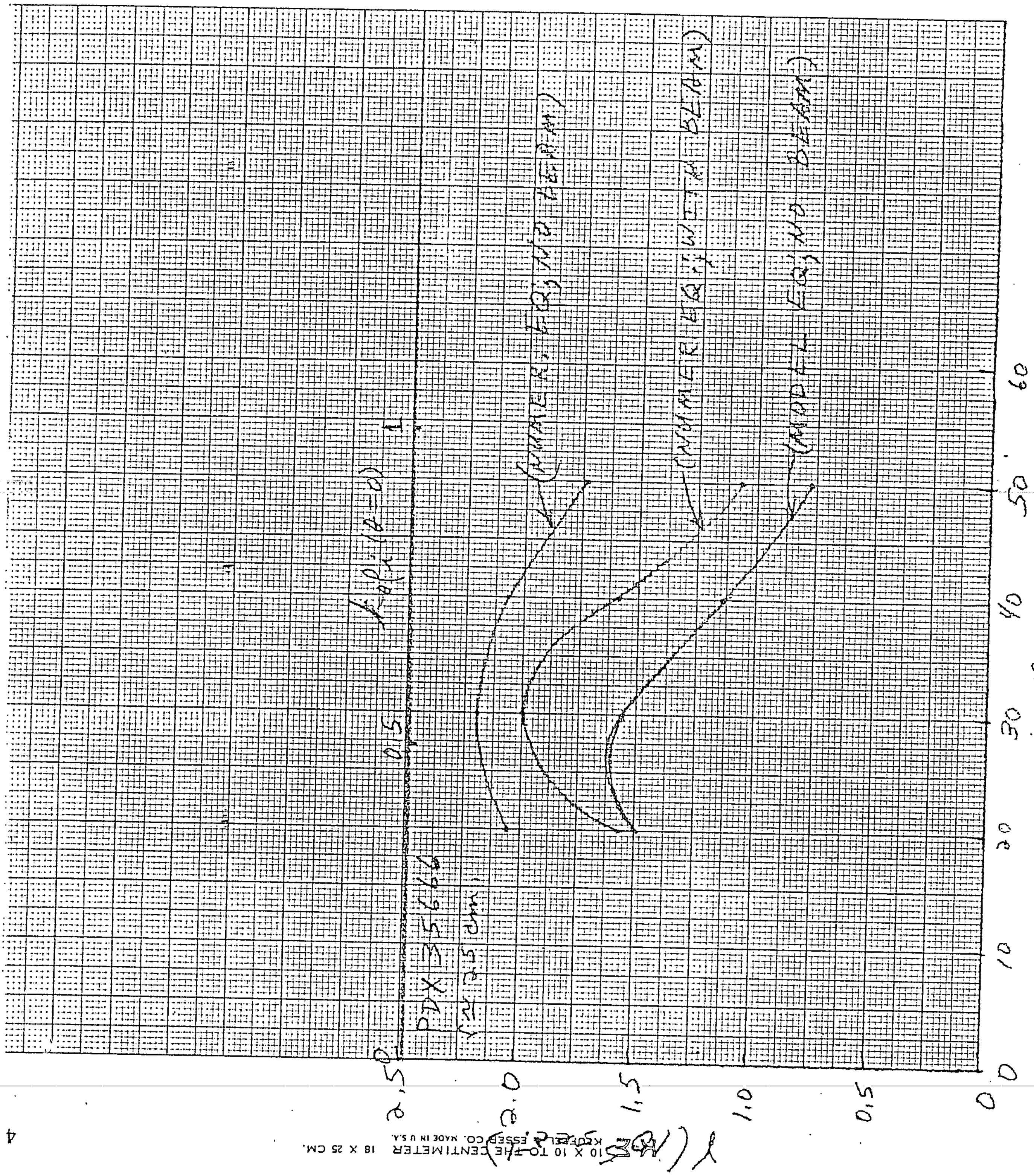
$\frac{n_b}{n_e} = 0.076$

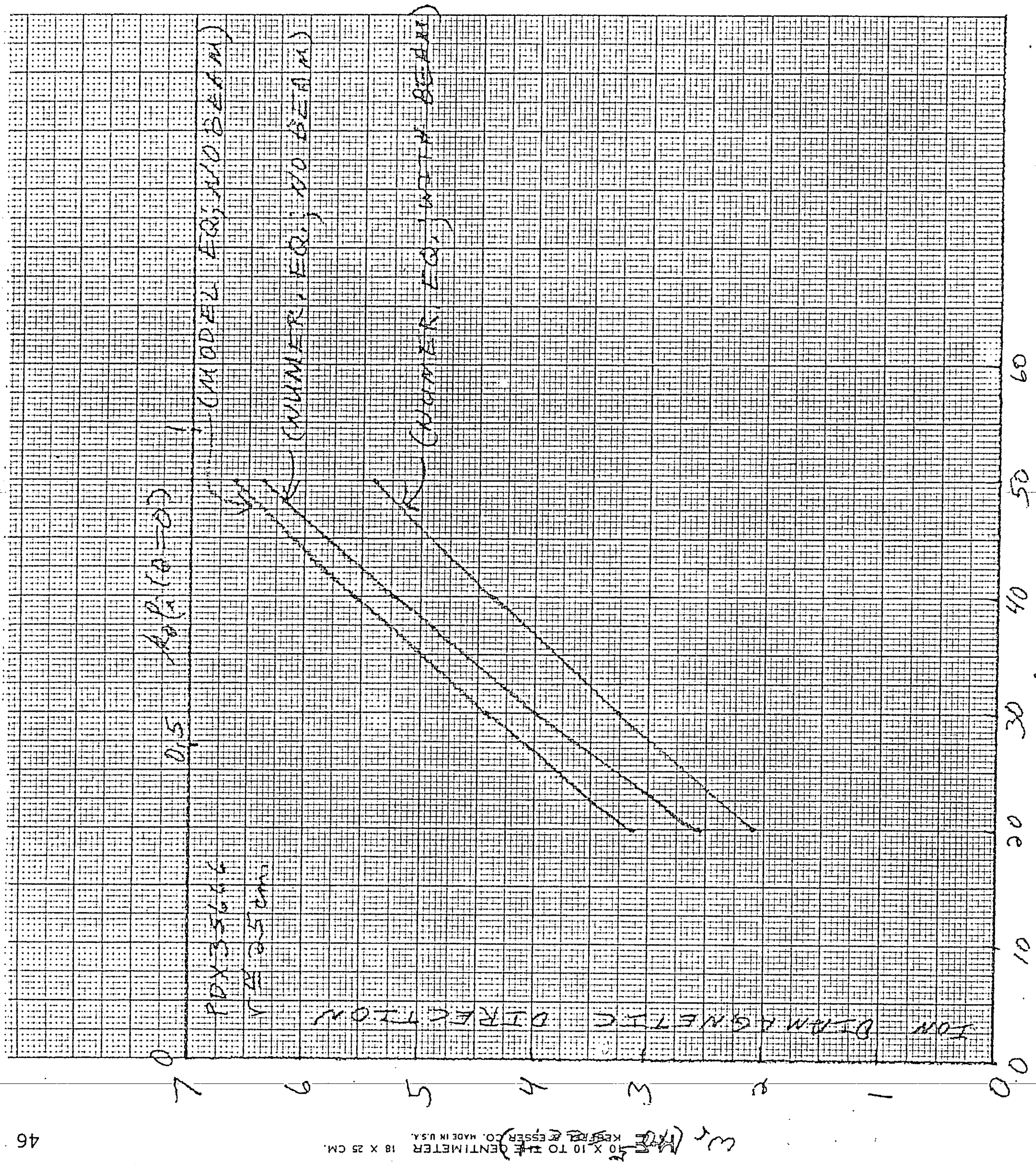
Fig. 1



PDX beam particle distribution function; r ≅ 25 cm.

Fig. 2


(NUMER. EQ; NO BEAM)
(NUMER. EQ; WITH BEAM)
(MODEL EQ; NO BEAM)
PDX 35666
(r ≈ 25 cm)
η
0
10
20
30
40
50
60
0
0.5
1.0
1.5
2.0
2.5


(MODEL EQ.; NO BEAM)
(NUMER. EQ.; NO BEAM)
(NUMER. EQ.; WITH BEAM)
FOR DIAMAGNETIC DIRECTION
n
0
10
20
30
40
50
60
1
2
3
4
5
6
7
10 X 10 TO THE CENTIMETER 18 X 25 CM.
KEUFFEL & ESSER CO. MADE IN U.S.A.
46

PARTICLE SIMULATION OF THE DRIFT WAVE INCLUDING THE ELECTROMAGNETIC EFFECT

H. ABE AND S. HAGIHARA

KYOTO UNIVERSITY

Particle Simulation of the Drift Wave Including the Electromagnetic Effect

H. Abe and S. Hagihara

This work is a part of the master thesis of Hagihara, who is now in a company, Sumitomo Denko Inc..

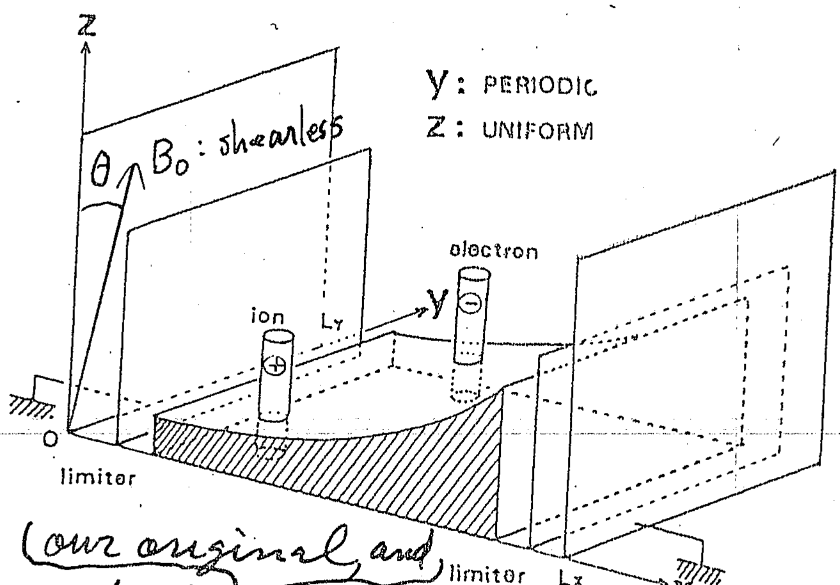
just obtained the very preliminary results, including check of the code, which has been newly developed.

limited to the simplest case.

compared the results between the electrostatic and the magnetostatic ones

↳ This reference is almost the same as that of the work done by PPPL People, except for the simulation parameters.

$2\frac{1}{2}$ Magnetostatic Slab Model with Conducting Walls



This code is (our original and) more exact and a little more time saving than the conventional code.

Equation of Motion:

Solved the full dynamics for ion and electron

Maxwell Equation :

$$\mathrm{rot}\,\mathbb{E} = -\frac{\partial \mathbb{B}}{\partial t}$$

$$\mathrm{rot}\,\mathbb{B} = \mu_0 j + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$= \mu_0 j^T + \frac{1}{c^2}\frac{\partial E^T}{\partial t} \longrightarrow \text{neglect}$$

$$+ \mu_0 j^L + \frac{1}{c^2}\frac{\partial E^L}{\partial t}$$

, where $\mathrm{div}\,\mathbb{E}^T = 0$ and $\mathrm{rot}\,\mathbb{E}^L = 0$

Then,

$$\left.\begin{aligned} &\mu_0 j^L + \frac{1}{c^2}\frac{\partial E^L}{\partial t} = 0 \quad : \text{Electrostatic} \\ &\mathrm{rot}\,\mathbb{B} = \mu_0 j^T \end{aligned}\right\} \text{Magnetostatic model}$$

{E1, E2 : Electrostatic
{M1, M2 : Magnetostatic

Simulation Parameters

Run Name	E1	E2	M1	M2	Unit
$L_x \times L_y$	64 x 32	—	—	—	$\Delta \times \Delta$
$\left(\frac{\omega_{pe\,min}}{\omega_{ce}}\right)^2$	0.10	—	—	—	
$\left(\frac{\omega_{pe\,max}}{\omega_{ce}}\right)^2$	1.00	—	—	—	
$\left(\frac{\omega_{pe\,ave}}{\omega_{ce}}\right)^2$	0.39	—	—	—	
$\kappa = \frac{1}{n}\frac{dn}{dx}$	1/27.8	—	—	—	Δ^{-1}
m_i/m_e	25	—	—	—	
v_{Te}	3.20	—	—	—	$\bar{\omega}_{pe}\Delta$
Initial Temp.	$T_{eo} = 2T_{io}$	—	—	—	
$N_e = N_i$	32,768	—	—	—	
$\lvert B_0 \rvert$	1.6	—	—	—	$\frac{\bar{\omega}_{pe} m_e}{e}$
θ	1.5	11.4	1.5	13.2	
c	X	X	7.2	7.2	$\bar{\omega}_{pe}\Delta$
$\bar{C}_A$	X	X	4.6	4.6	$\bar{\omega}_{pe}\Delta$
$\bar{\beta}$	X	X	4.8	4.8	%

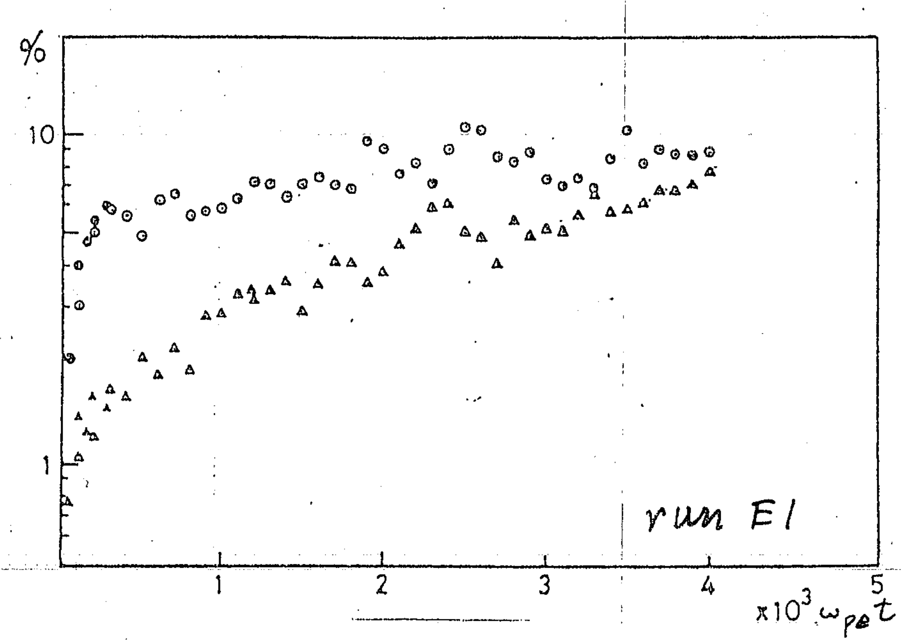
$$\bar{\beta} = 2\,\frac{\bar{\omega}_{pe}^2}{\omega_{ce}^2}\,\frac{v_{te}^2}{c^2}\left(1 + \frac{T_i}{T_e}\right)$$

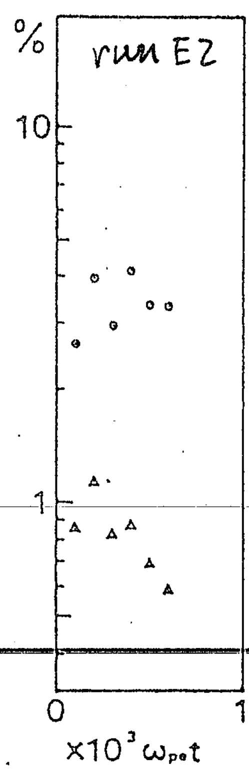
$$\bar{C}_A^2 = c^2\,\frac{\omega_{ci}^2}{\bar{\omega}_{pi}^2}$$

— : same

X : meanless

II - 2

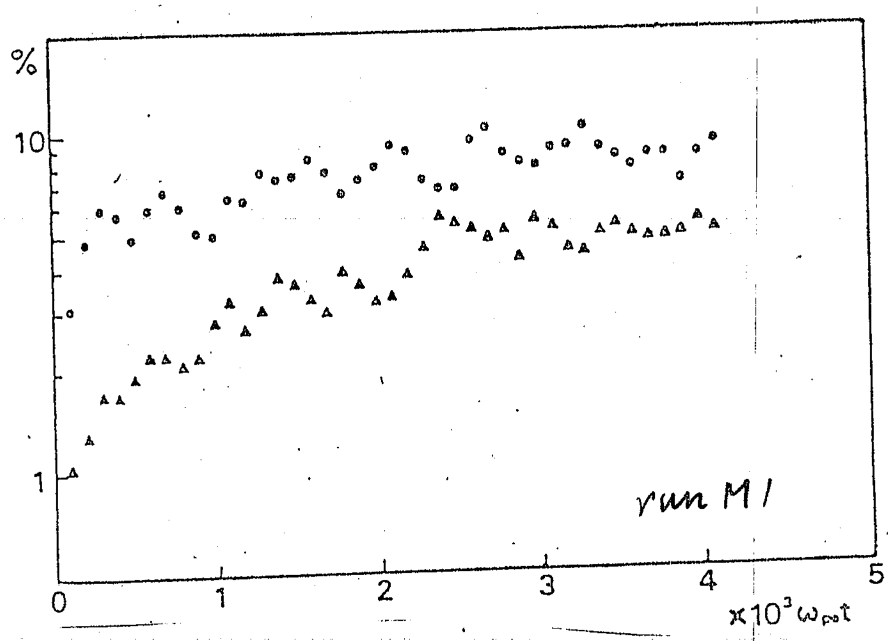




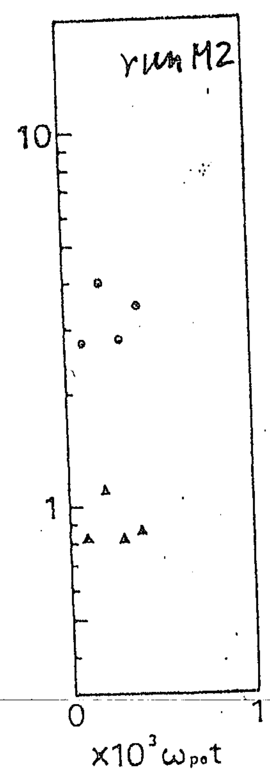
II - 10

$$\circ = \left\langle \frac{\delta n(k_y=1,t)}{n_0} \right\rangle_{\text{over } x \text{ and } t}$$

$$\triangle \propto \left\langle \frac{e\tilde{\phi}}{T_e} \right\rangle_{\text{over } x \text{ and } t}.$$



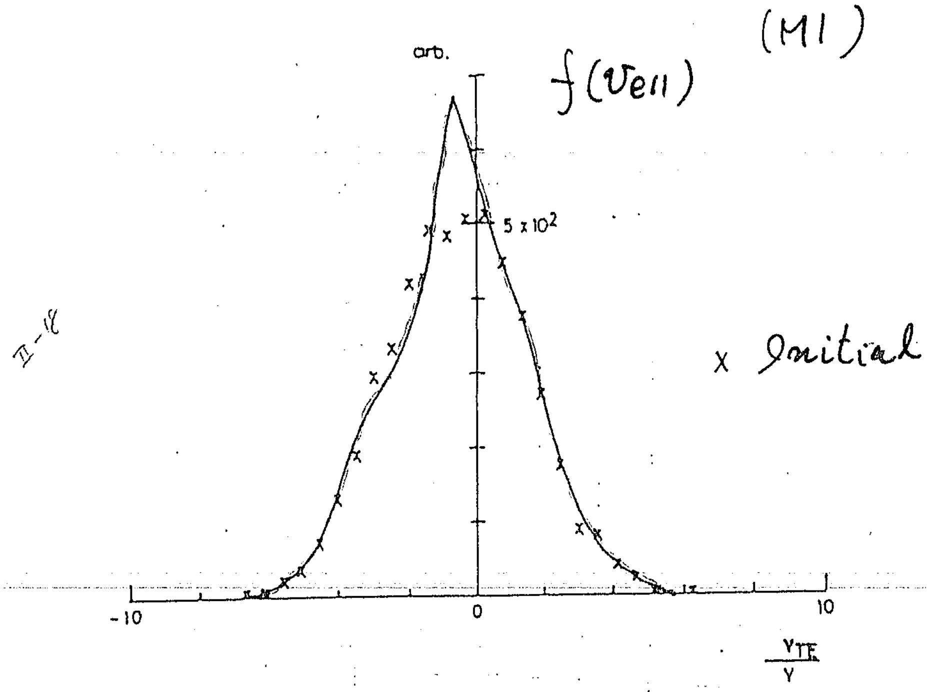
Ⅱ-11

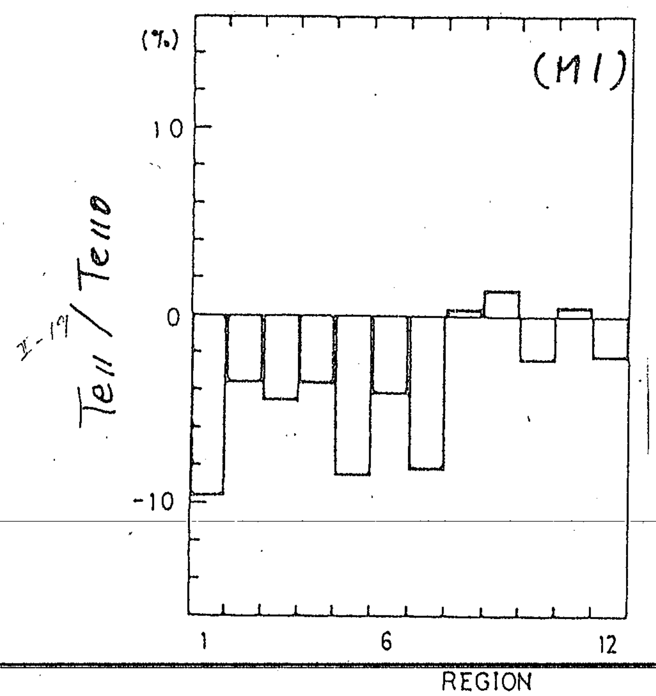


Ⅱ-20

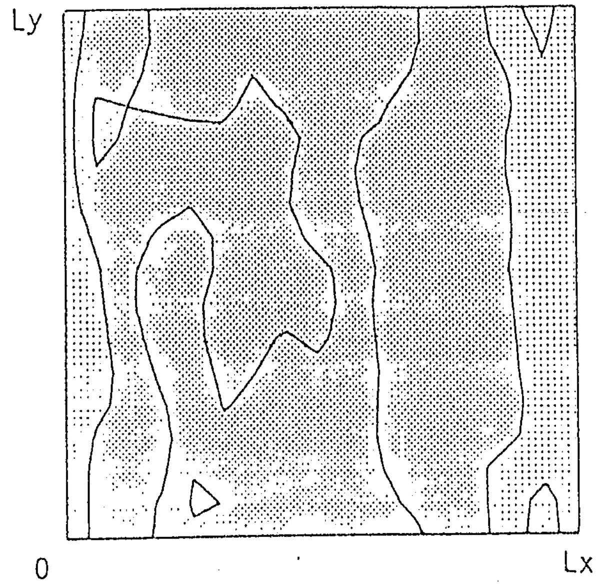
$$\circ = \left\langle \frac{\delta n(k_y=1, t)}{n_0} \right\rangle_{\text{over } x \text{ and } t}$$

$$\triangle \propto \left\langle \frac{e\tilde{\Phi}}{T_e} \right\rangle_{\text{over } x \text{ and } t}$$

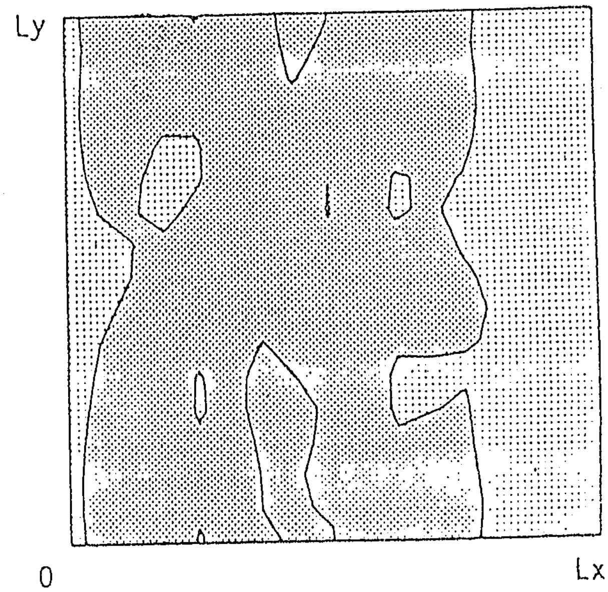

(M1)
arb.
$f(v_{e\parallel})$
5 x 10²
x Initial
-10
0
10
$\frac{v_{TE}}{v}$
II-18


(%)
(M1)
10
0
-10
Te∥ / Te∥0
II-19
1
6
12
REGION

Contour of Equal $\tilde{\phi}$ (M1)

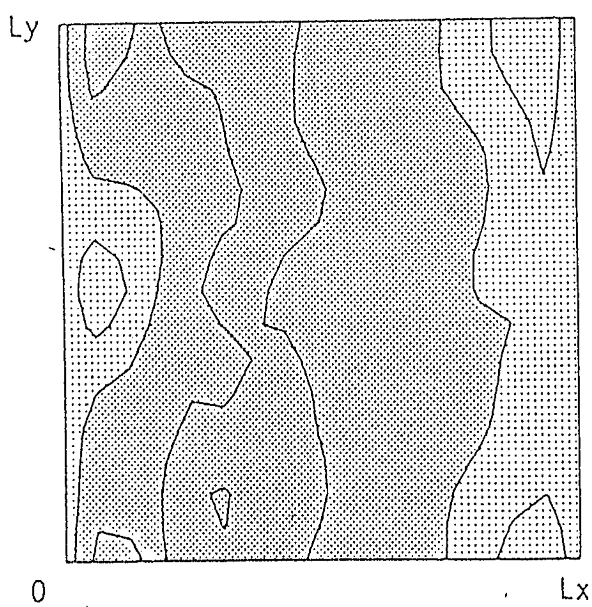


(a) 100 $\omega_{pe}t$

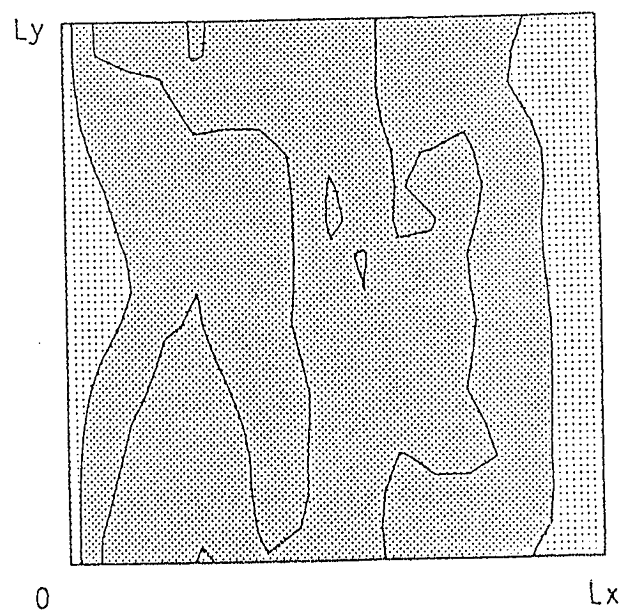


(b) 200 $\omega_{pe}t$

II-15

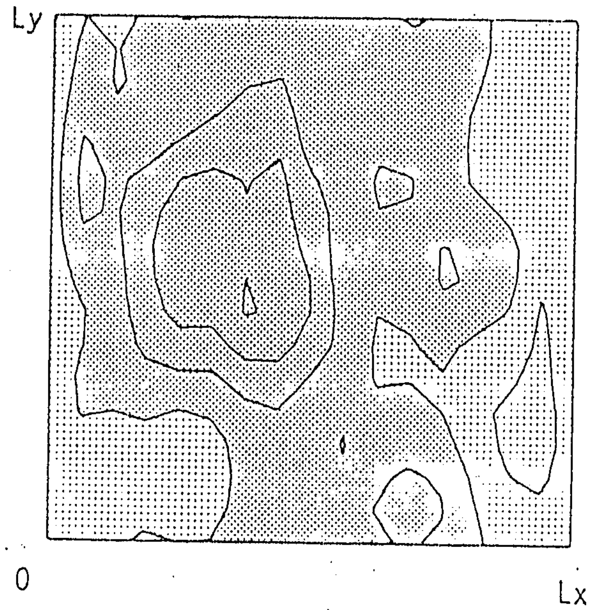


(c) 300 $\omega_{pe}t$

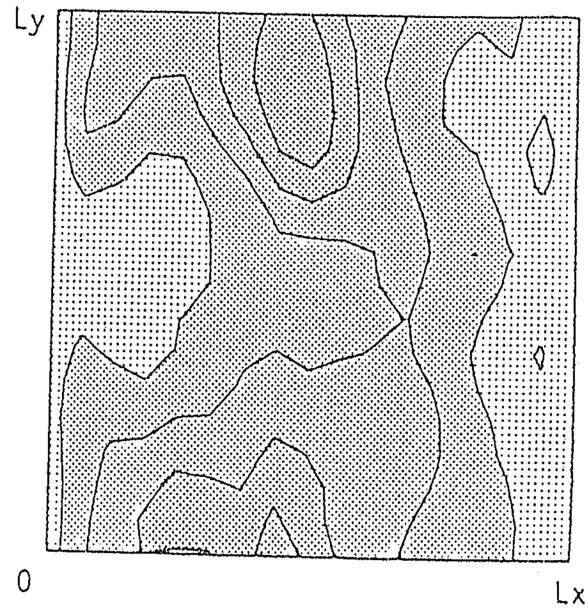


(d) 400 $\omega_{pe}t$

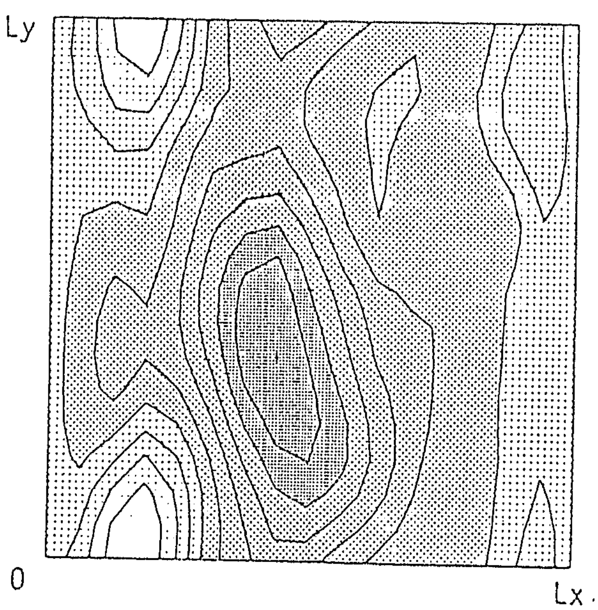
Contour of Equal Φ (MI)



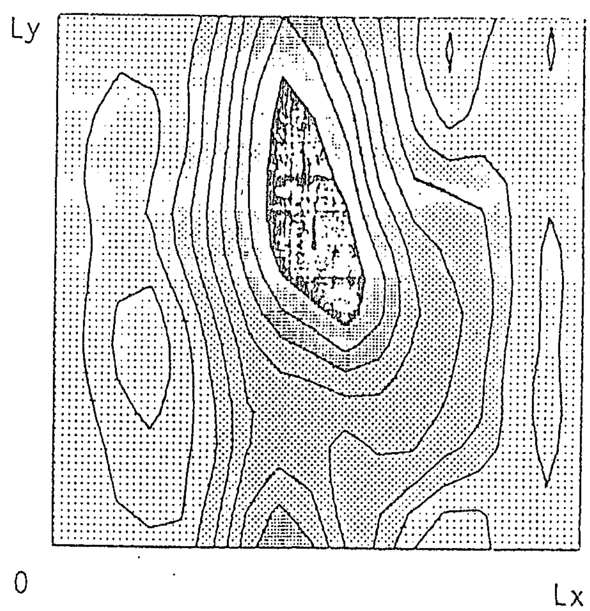
(e) 1000 $\omega_{pe}t$



(f) 2000 $\omega_{pe}t$



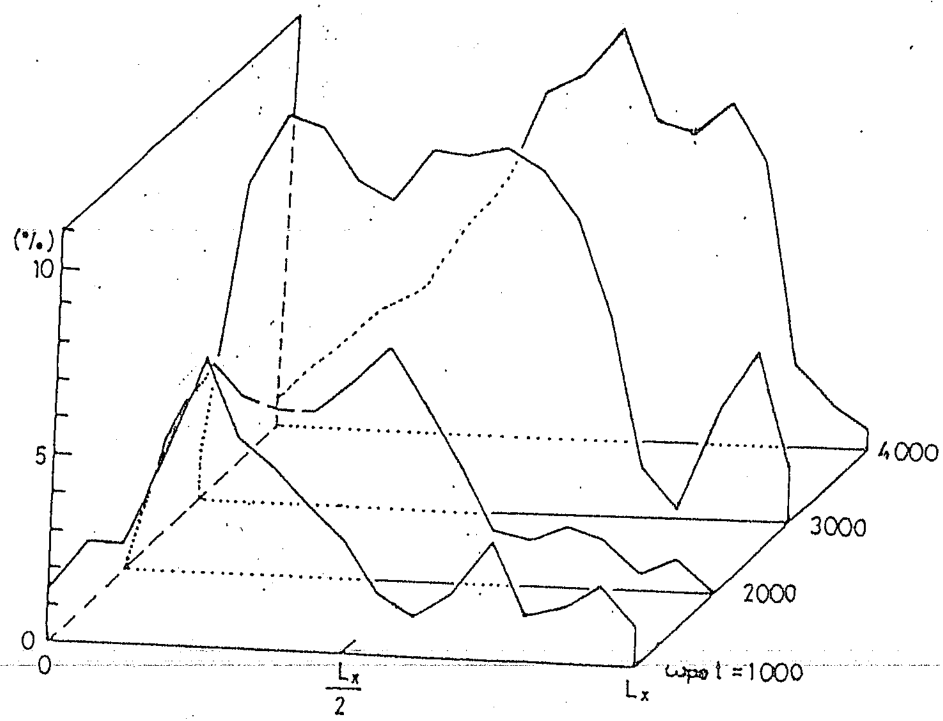
(g) 3000 $\omega_{pe}t$



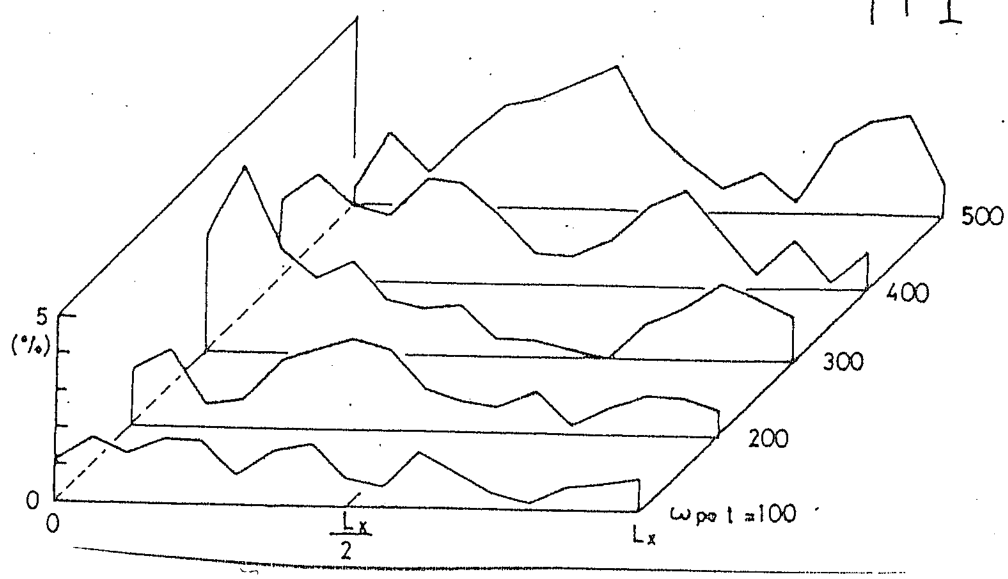
(h) 4000 $\omega_{pe}t$

$e\tilde{\phi}/T_e$ vs. x

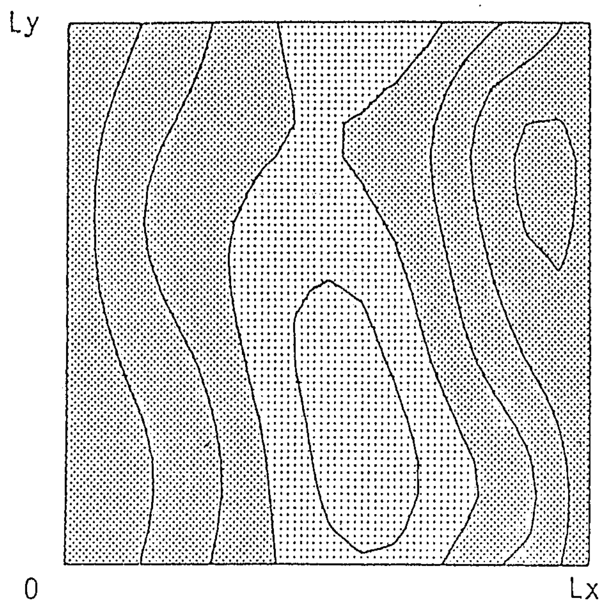
II-14



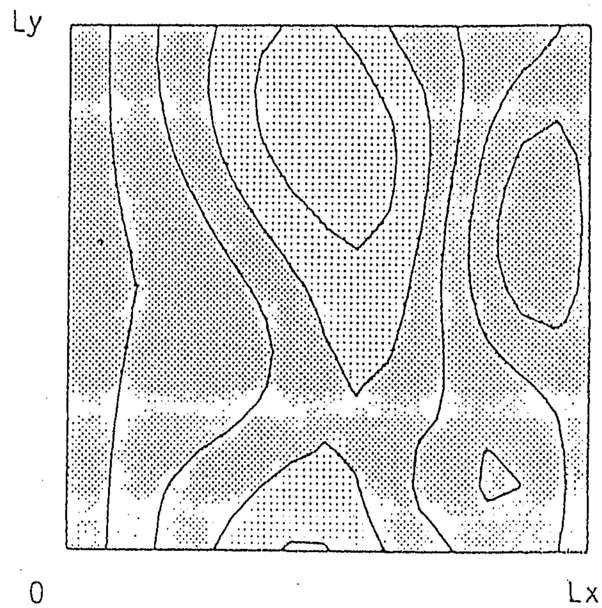
M 1



Contour of Equal $\tilde{A}_z$ (M1)

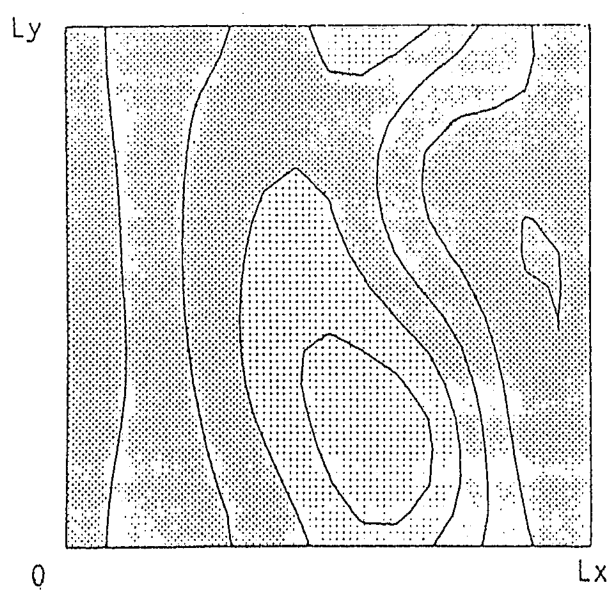


(a) 100 $\omega_{pe}t$

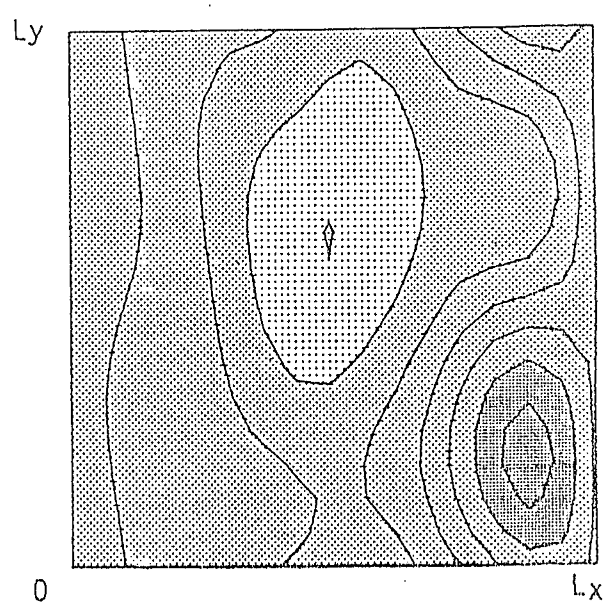


(b) 200 $\omega_{pe}t$

II-16

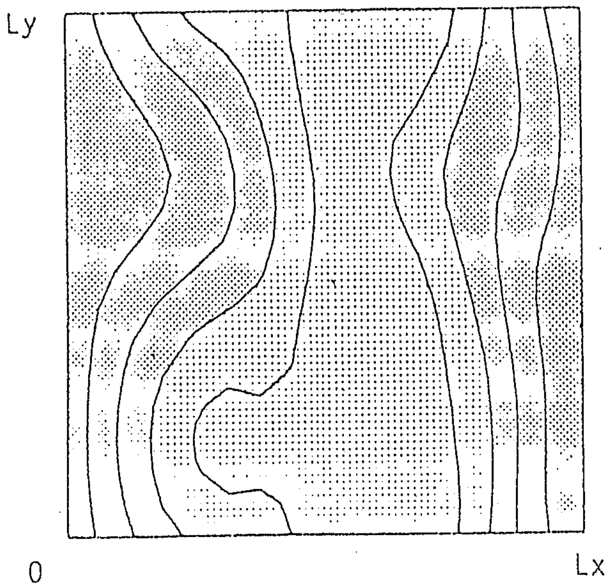


(c) 300 $\omega_{pe}t$

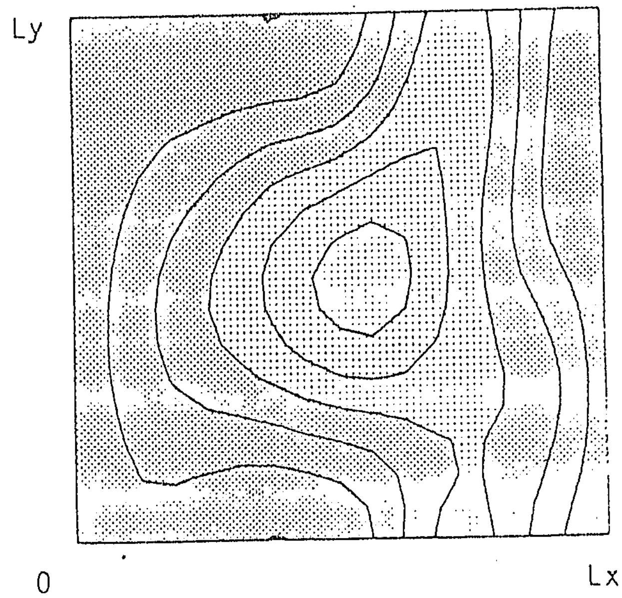


(d) 400 $\omega_{pe}t$

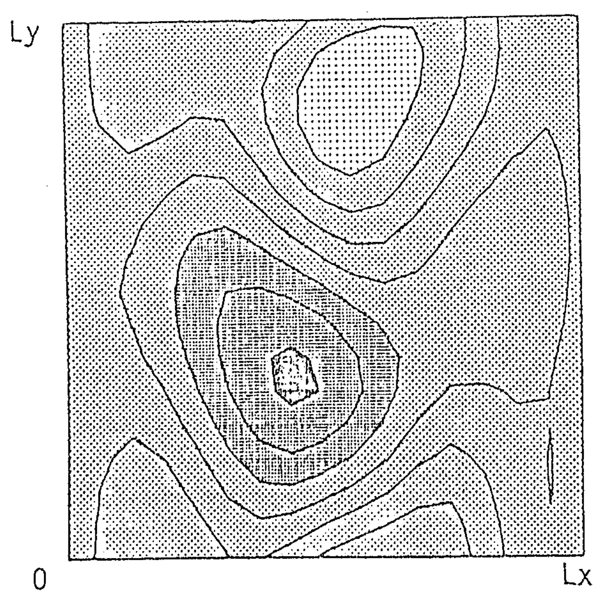
Contour of Equal $\tilde{A}_z$ (M1)



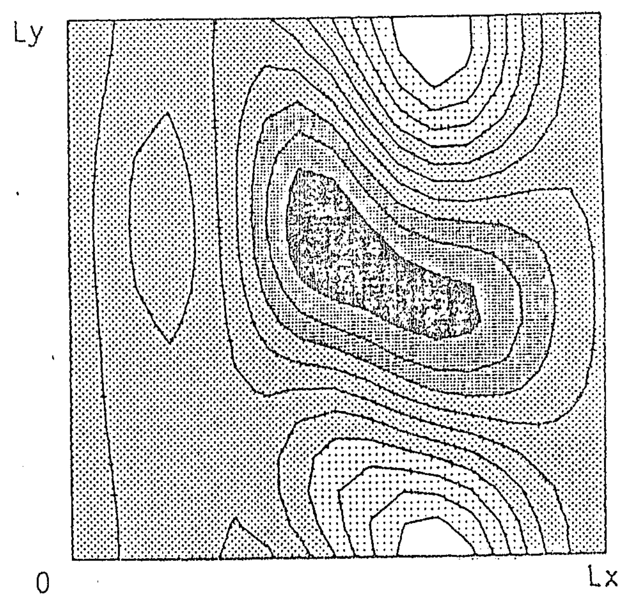
(e) 1000 $\omega_{pe}t$



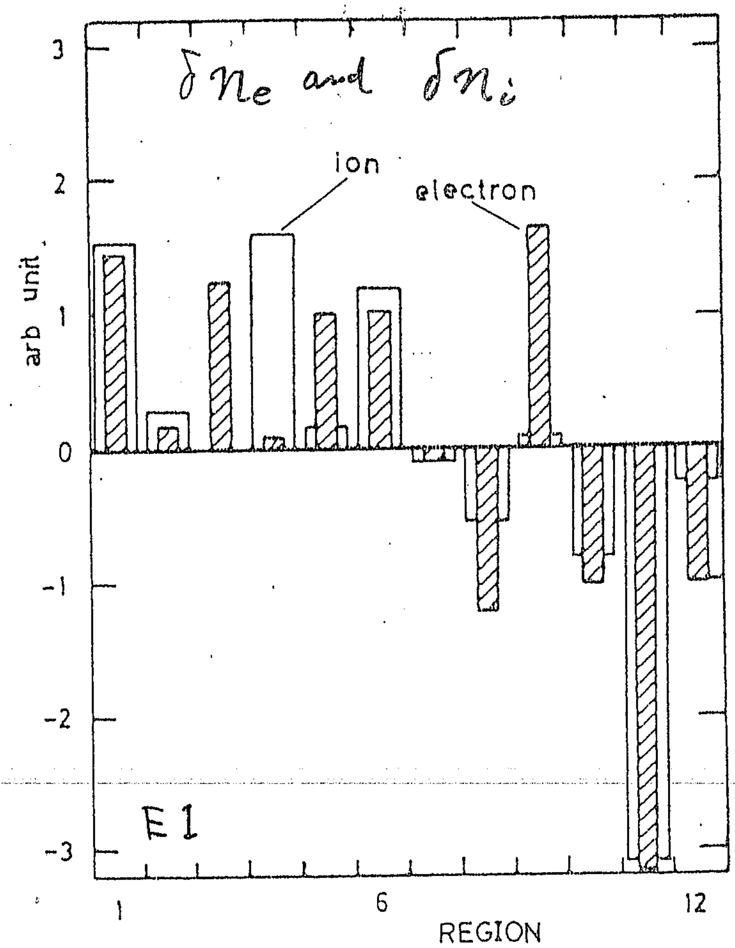
(f) 2000 $\omega_{pe}t$



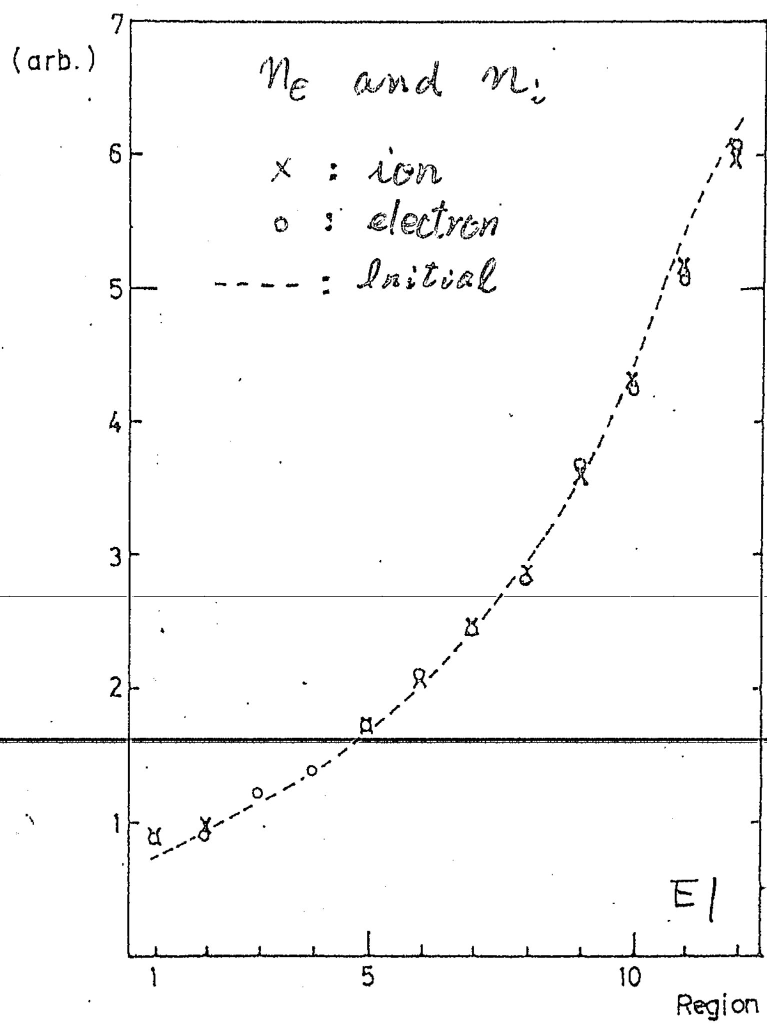
(g) 3000 $\omega_{pe}t$



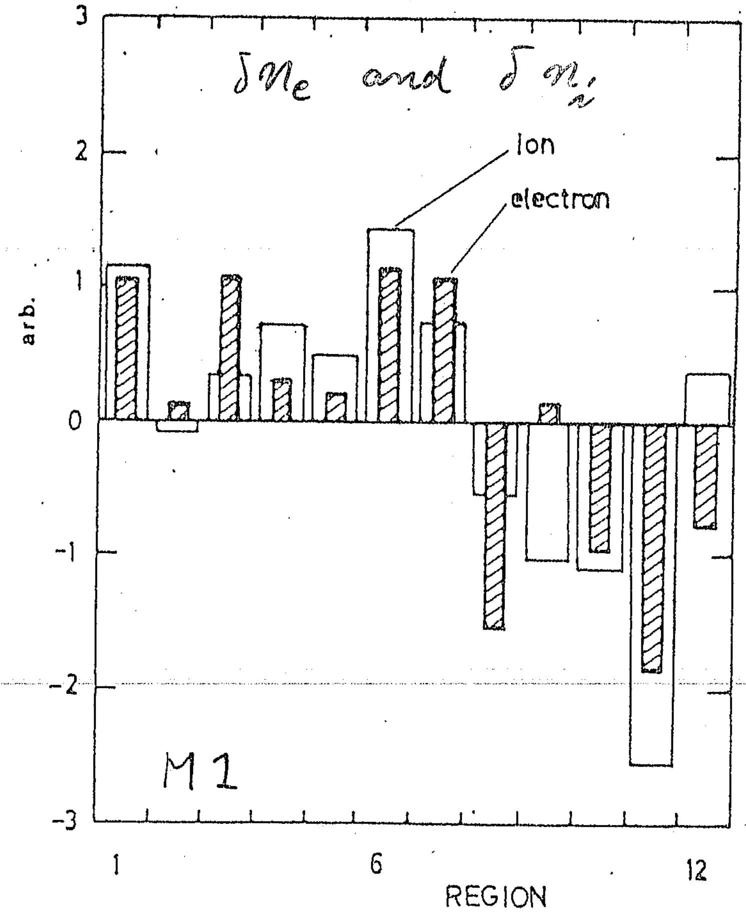
(h) 4000 $\omega_{pe}t$


δne and δni
ion
electron
arb unit
E1
REGION
3
2
1
0
-1
-2
-3
1
6
12

II-6


(arb.)
ne and ni
x : ion
o : electron
---- : Initial
E1
Region
7
6
5
4
3
2
1
1
5
10

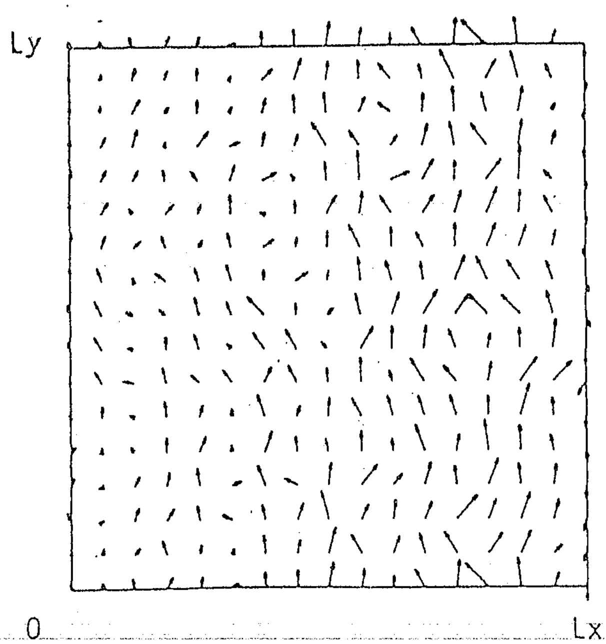
II-7



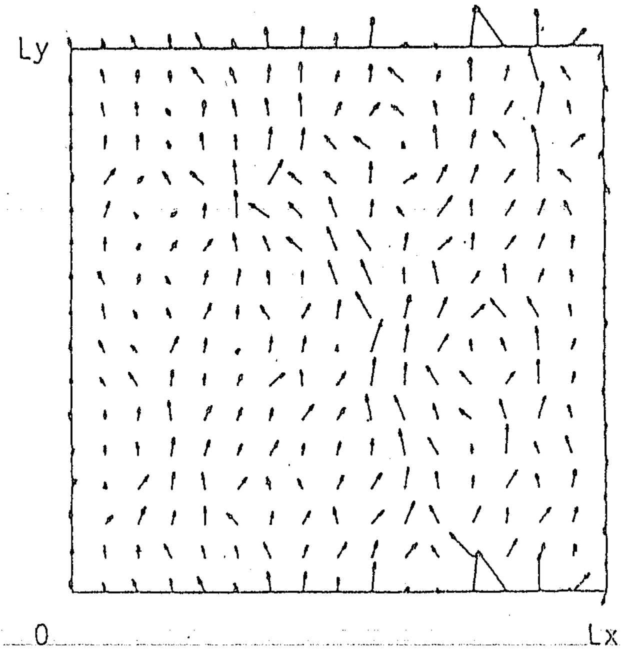
II-29

perpendicular flow of Ion (MI)

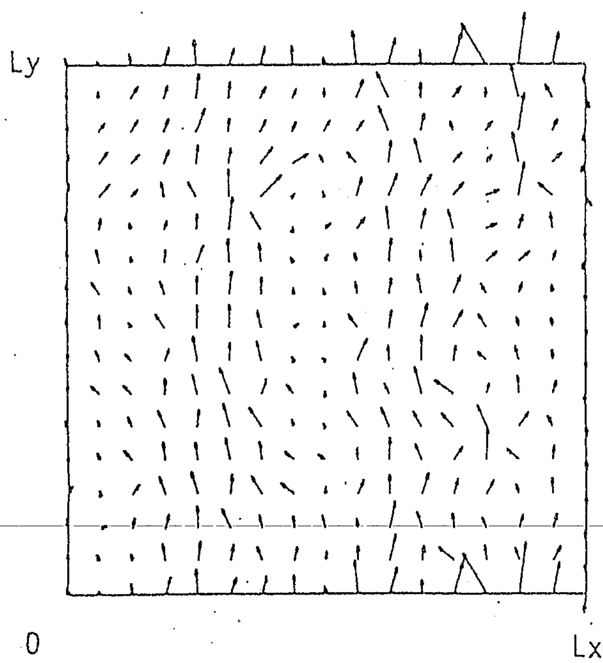
II-17



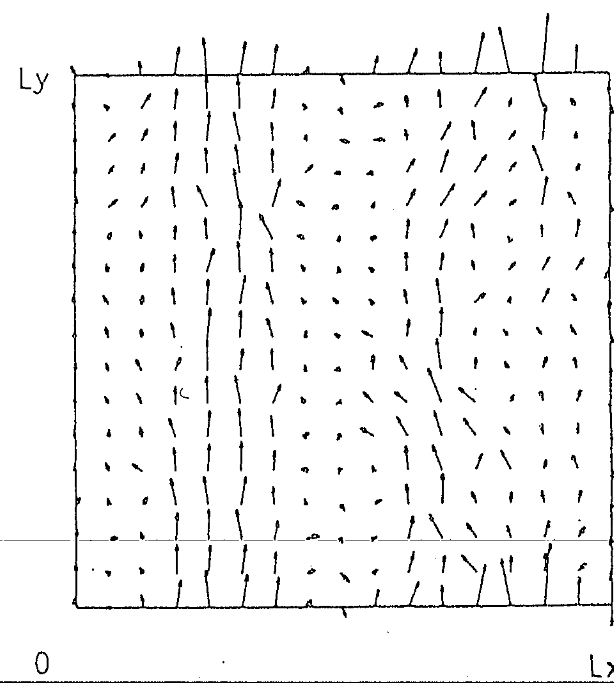
(e) 1000 $\omega_{pe}t$



(f) 2000 $\omega_{pe}t$

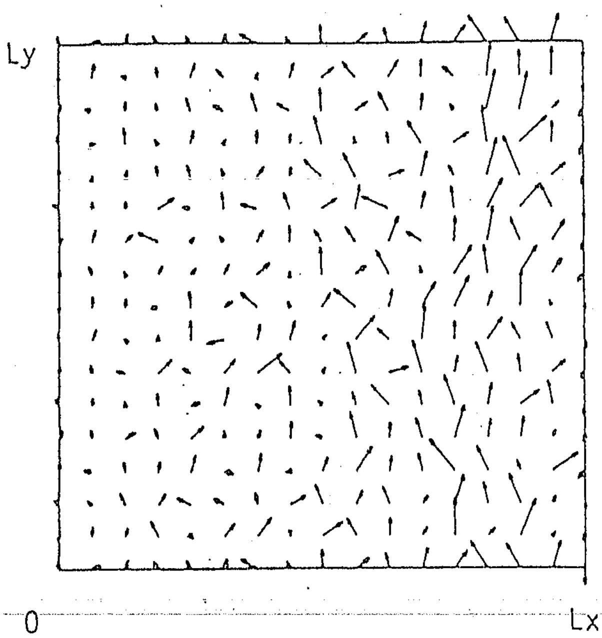


(g) 3000 $\omega_{pe}t$

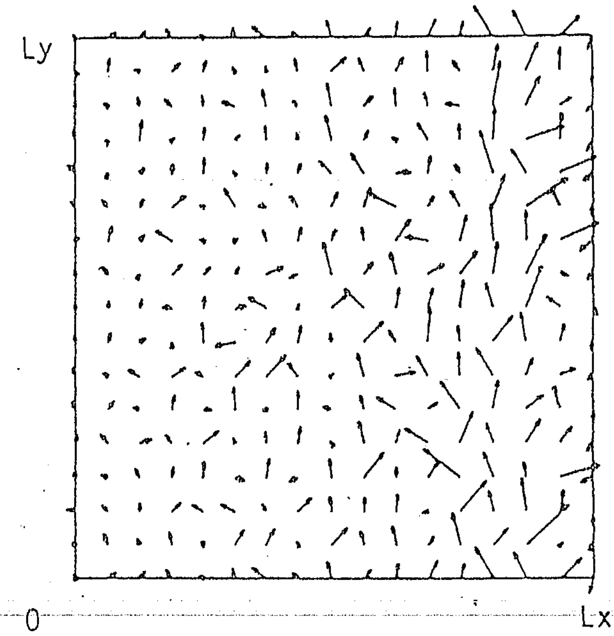


(h) 4000 $\omega_{pe}t$

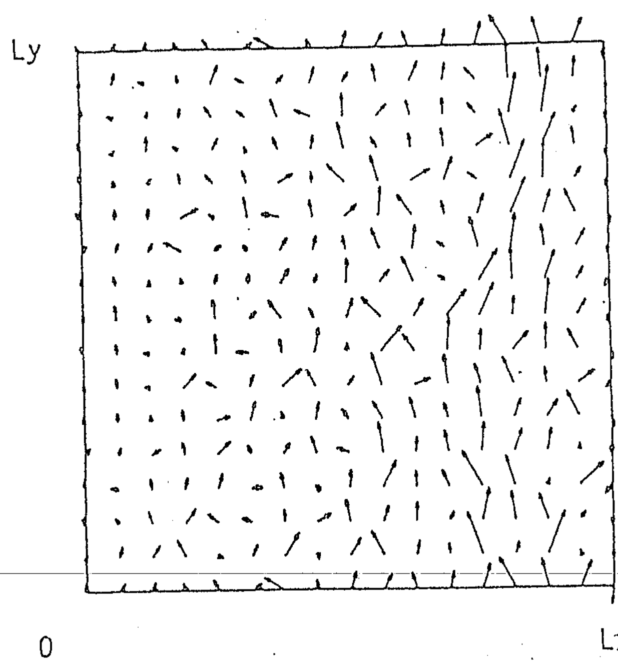
Perpendicular flow of Ion (M1)



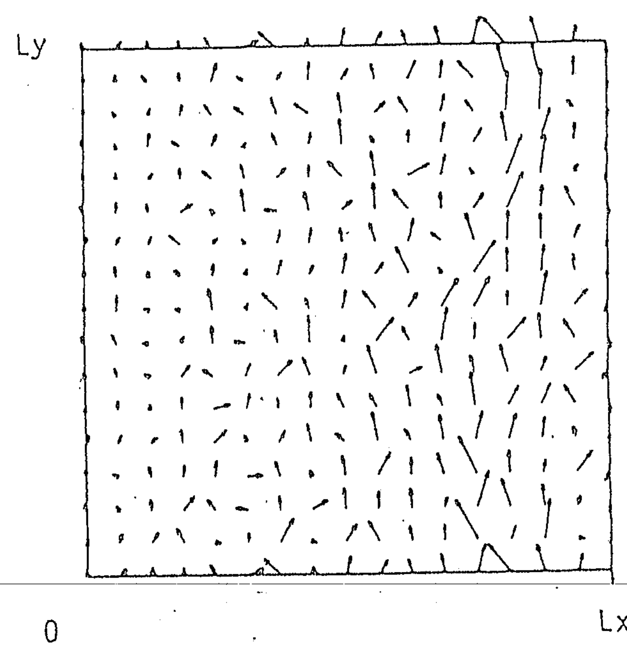
(a) 100 $\omega_{pe}t$



(b) 200 $\omega_{pe}t$

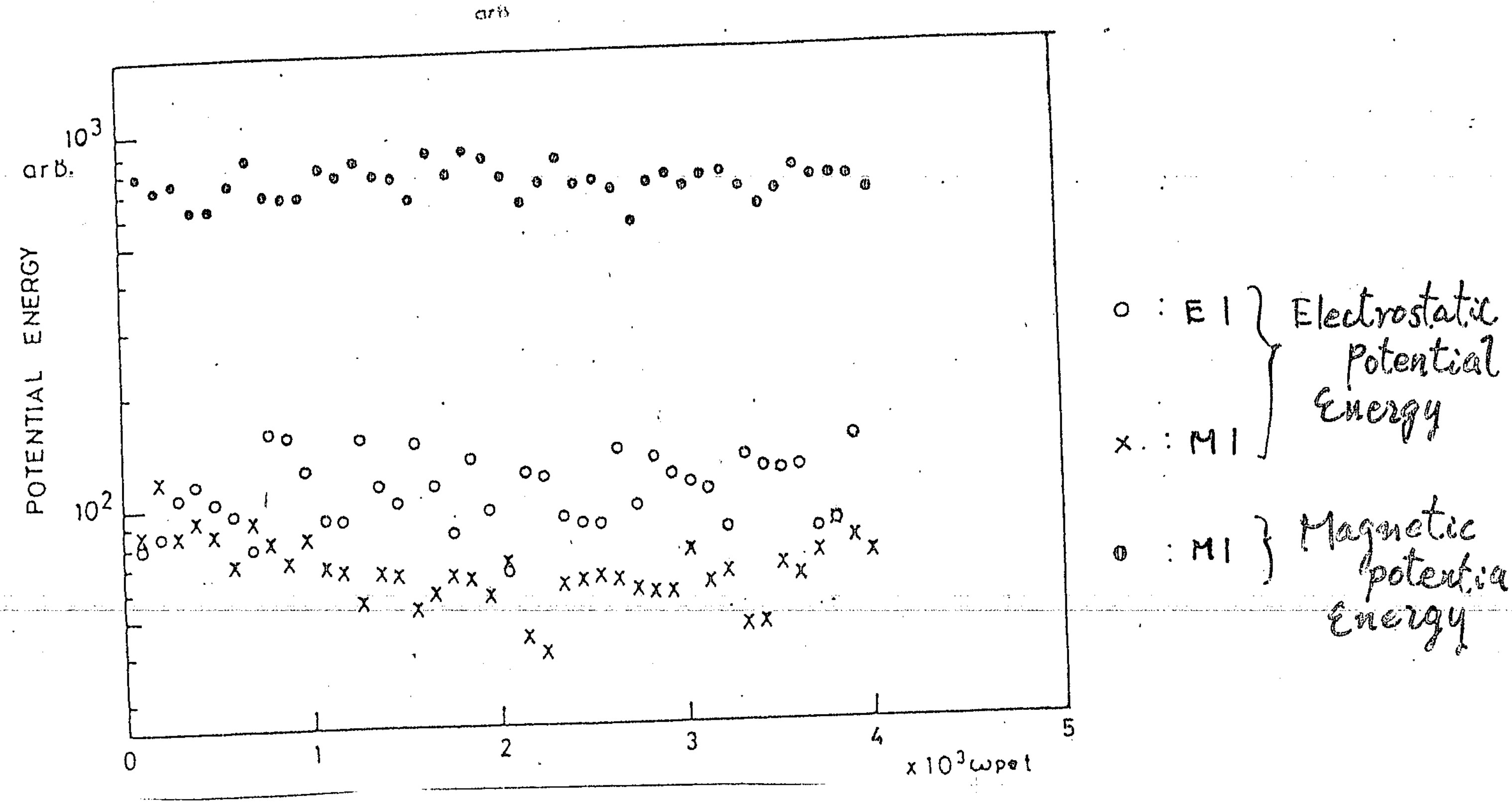


(c) 300 $\omega_{pe}t$



(d) 400 $\omega_{pe}t$

II-12


arb.
10^3
10^2
POTENTIAL ENERGY
0
1
2
3
4
5
x 10^3 ωpet
o : E1
x : M1
● : M1
Electrostatic Potential Energy
Magnetic potential Energy

Linear Dispersion Relation
(Normal Mode Analysis)

for E1 and E2

$$e^{-s} I_0(s) \frac{\omega + \omega_*}{k_z v_i} Y\left(\frac{\omega}{k_z v_i}\right) + \frac{\omega - \omega_*}{k_z v_e} Y\left(\frac{\omega}{k_z v_e}\right) = 2$$

: Independent of Density

for M1 and M2

$$\varepsilon_{\parallel} \left\{ 1 - \left(\frac{\omega}{c k_{\parallel}}\right)^2 \varepsilon_{\perp} \right\} + \left(\frac{k_{\perp}}{k_{\parallel}}\right)^2 \varepsilon_{\parallel} = 0$$

where

$$\varepsilon_{\parallel} = \sum_{\alpha} (k_{\parallel} d)^{-2} \left(1 - \frac{\omega_n}{\omega}\right) \left\{ 1 + i\sqrt{\pi}\, x\, W(x) I_0(z) e^{-z} \right\}$$

$$\varepsilon_{\perp} = 1 + \sum_{\alpha} (k d)^{-1} \left(1 - \frac{\omega_n}{\omega}\right)\left(1 - I_0 e^{-z}\right)$$

$$x = \frac{\omega}{|k_{\parallel}| v_t}, \quad z = \frac{k_{\perp}^2 T}{m \omega_{ce}^2}, \quad \omega_n = \frac{k_y \kappa T}{m \omega_c}$$

$$d = \sqrt{\frac{T}{m \omega_c}}$$

Conclusion:

1. Although this is quite preliminary result, we cannot see any change in the linear stage between the $\beta = 0$ case and $\beta = 5\%$ case.
2. In the nonlinear stage,

$$|\tilde{E}^L|^2_{\beta=0} > |\tilde{E}^L|^2_{\beta=5\%}$$

Problems:

1. Enlarge of the system size
2. increase of the mass ratio
3. treat the 3-dimensional case

→ may need larger and faster computers
or more sophisticated model
⇓
depends on strategy of research and developing speed of the computers
⇓
good ideas are needed.

4. need to treat exact eigenvalue problems for the prediction of the linear stage in the case of the finite β.

ANOMALOUS ION THERMAL CONDUCTIVITY

W. HORTON
UNIVERSITY OF TEXAS AT AUSTIN

U.S.-JAPAN WORKSHOP ON
DRIFT-WAVE TURBULENCE
JANUARY 11-15, 1982

TOROIDAL STABILITY

Two Point Eigenvalue Problem

Toroidicity	Gradient	Transform	Shear
$\varepsilon_n = \dfrac{r_n}{R}$	$\eta_i = \dfrac{d\ln T_i}{d\ln n}$	$q = \dfrac{rB_\phi}{RB_\theta}$	$\xi \equiv s = \dfrac{rq'}{q}$

$$\frac{d^2f}{dy^2} + \frac{\omega^2 q^2}{\varepsilon_n^2}\left[\frac{\omega - k}{\omega + k(1+\eta_i)} + \frac{2\varepsilon_n k}{\omega} K(y) + k^2 + k^2\xi^2 y^2\right] f(y) = 0$$

Boundary Values $\quad f(0) = 0$ or $f'(0) = 0$

& $\quad f' = i\,\dfrac{k\xi q\omega}{\varepsilon_n}\, y f(y)$ @ $y_2 \to \infty$

$$K(y) = \cos y + \xi y \sin y$$

$$\omega = \omega(k, \eta_i, \varepsilon_n, q, \xi)$$

↖ complex function

$\omega(k, \eta_i, \varepsilon_n, q, \xi)$

ANALYTIC LIMITS

(i) $\xi \ll q$ Mathieu Eq (Toroidal)

(ii) $\xi \gg q$ or $S' = \frac{\xi \varepsilon_n}{q} \gg \varepsilon_n$ (Slab)

(iii) $\xi \sim q$ Strong Coupling [$\exp(-\frac{1}{2}\sigma \delta y^2)$]

(iv) $\omega = \omega_{*e}(1+\lambda)$ Sturm-Liouville for toroidal effect λ on Drift W...

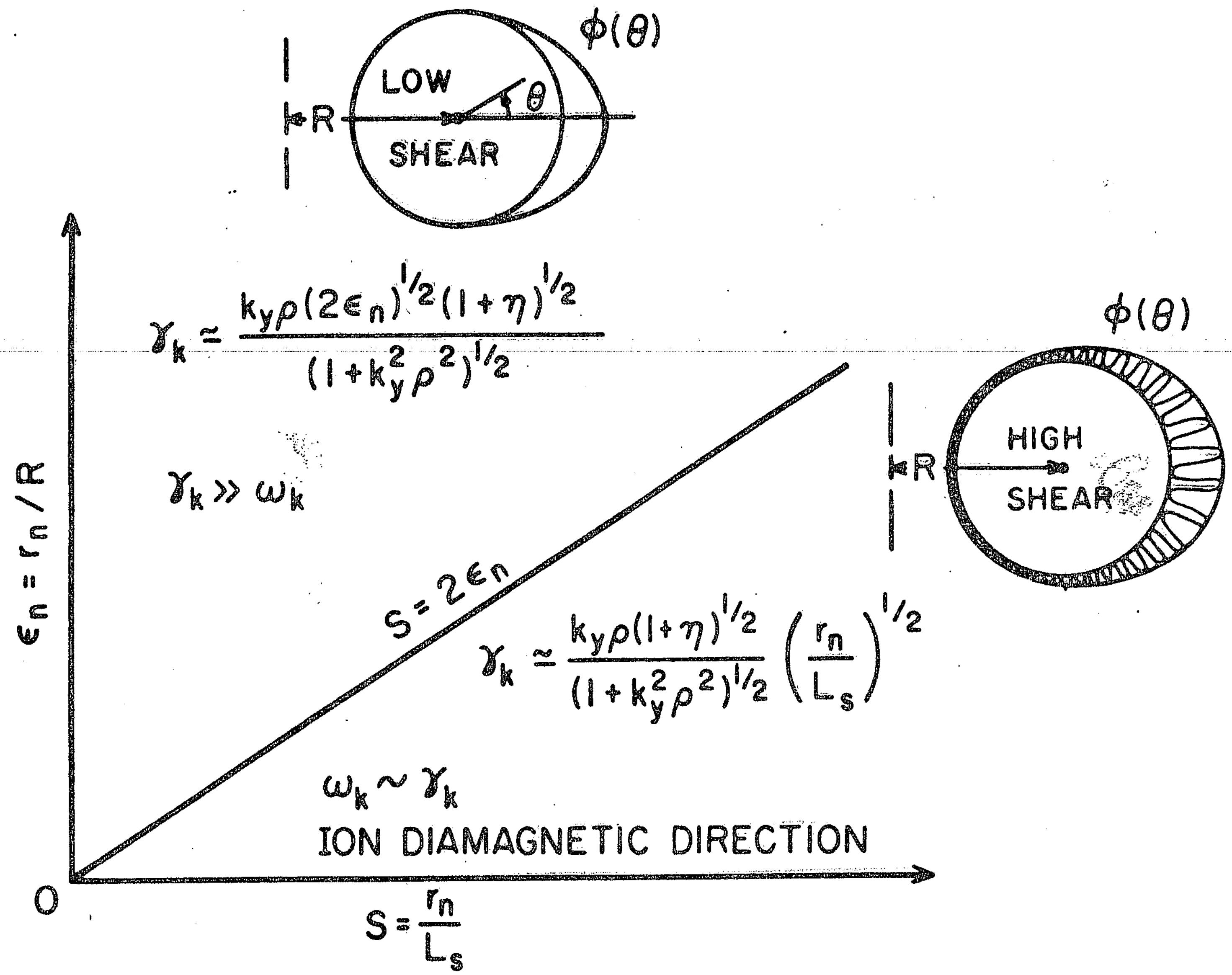
Refs. for Methods

1. Taylor, Berchtesgaden IAEA '78
2. Hastie, Hesketh, Taylor, Nucl. Fus. '79
3. Choi and Horton, P.F. '80
4. ~~Chen and Cheng, P.F. '80~~

etc.

Ion Pressure Gradient Toroidal Modes

W. Horton, D.I. Choi, P.W. Terry, W.M. Tang

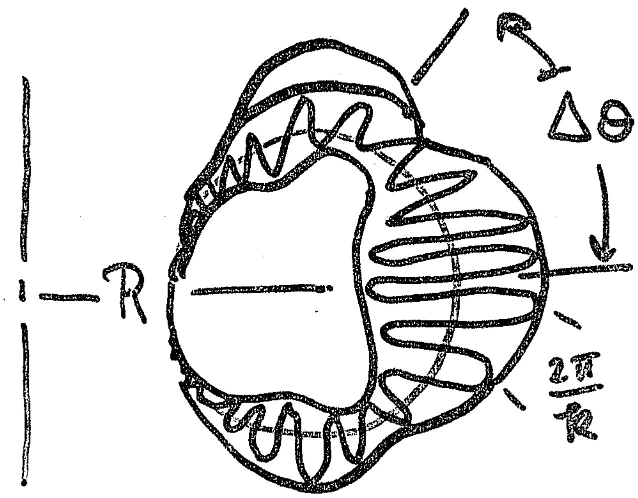


Coppi, Rosenbluth, Sagdeev

Phys. Fluids 10, 582(1967)

Nonlinear Simulation Poster 6Q3 with Biskamp

CHARACTERISTICS OF MODES



$$\overline{\Delta\theta} = \frac{(2\varepsilon_n)^{1/4}}{k\, \xi^{1/2}\, q^{1/2} (1+\eta)^{1/2}} \quad 1/4$$

$$\langle k_r^2 \rangle^{1/2} = k_\theta\, \xi\, \overline{\Delta\theta}$$

$$= \frac{(2\varepsilon_n)^{1/4}\, \xi^{1/2}}{q^{1/2} (1+\eta_i)^{1/4}}$$

$$\langle k_n^2 r_n^2 \rangle^{1/2} = \frac{k\, \xi^{1/2}\, \varepsilon_n^{3/4} (1+\eta)^{1/4}}{q^{1/2} (1+k^2)^{1/2}}$$

Check Fluid Conditions Satisfied

e.g. Consider $\varepsilon_n = 1/4$, $\eta = 1$, $q = 3$, $\xi = 1$

@ $k = 0.6$ $\quad \overline{\Delta\theta} = 40°$ $\quad \overline{k_n r_n} = 0.1$

$\overline{k_r \rho} = 0.4$ $\quad \gamma / \overline{k}_n c_s = 2.9$

Maximum γ From Kinetic Theory from Ref. 4 with Bessel fnc. and $(\omega-\omega_D)^{-1}$ propagator

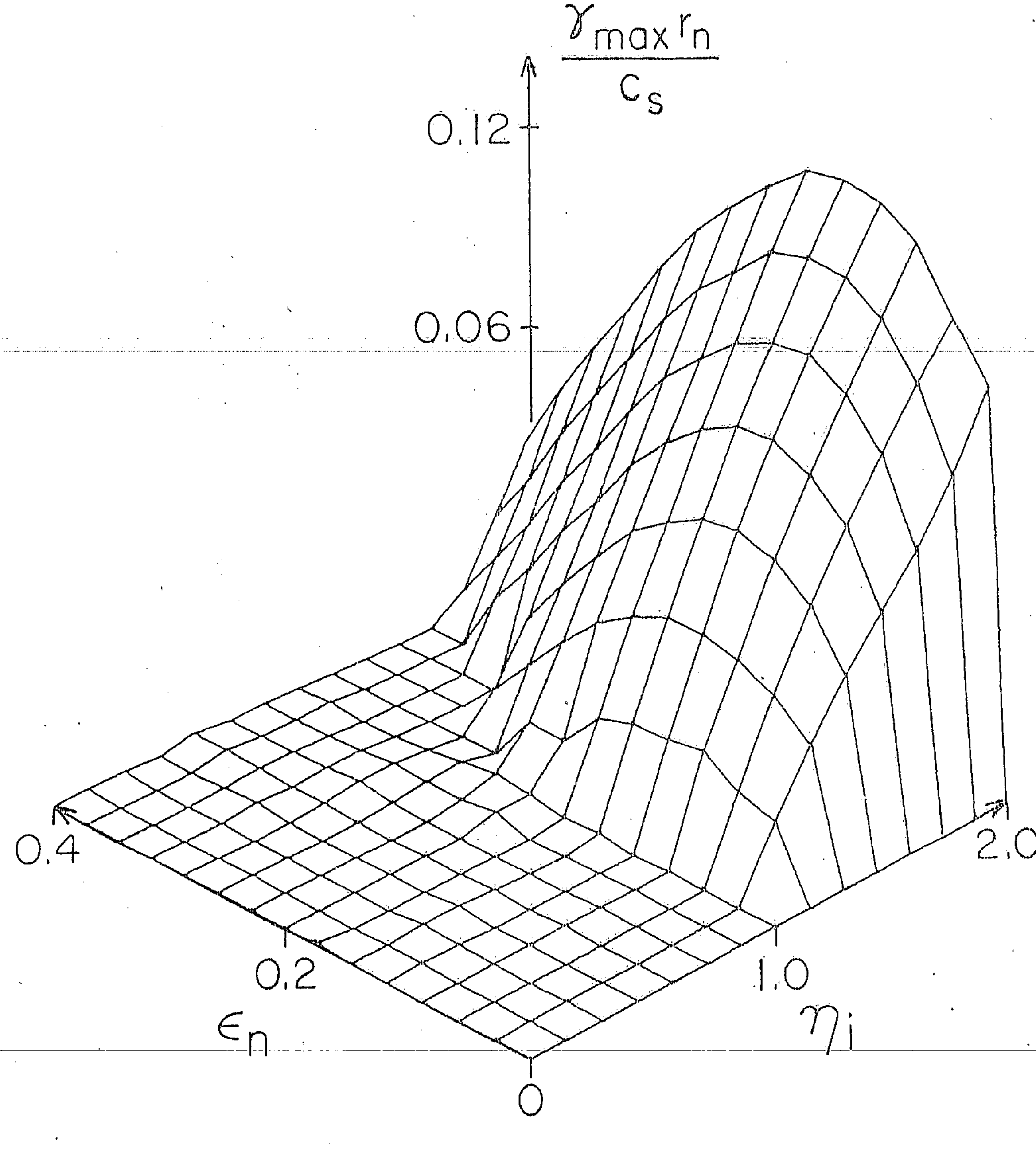


Fig. 6

$3D \rightarrow 2D$ Ballooning to Outside

Drops θ, ξ dependence

$$\hat{k}_n v_k \simeq 2k\varepsilon_n(\xi-\tfrac{1}{2})y^2(\varphi_k+\delta p_k) + k^3\xi^2(1+\eta)y^2\varphi_k + ik^2\xi^2 y^2 \frac{\partial \varphi_k}{\partial t}$$

2D NONLINEAR MODEL

$$(1+k^2)\frac{d\varphi_{\vec{k}}}{dt} = -iku_k\varphi_{\vec{k}} + 2ik\varepsilon_n\delta p_{\vec{k}}$$

$$\frac{d}{dt}\delta p_{\vec{k}} = -ik(1+\eta)\varphi_{\vec{k}} + \sum_{\vec{k}_1+\vec{k}_2=\vec{k}} (\vec{k}_1\times\vec{k}_2)_{\parallel}\,\varphi_{\vec{k}_1}\delta p_{\vec{k}_2}$$

Analog of Vlasov Turbulence

$$\varphi_{\vec{k}} \rightarrow \varphi_{\vec{k}}$$

$$\delta p_{\vec{k}} \rightarrow \delta f_{\vec{k}}$$

$$1+\eta \rightarrow \frac{\partial F_0}{\partial v}$$

ELIMINATE LINEAR EQUATION

$$\mathcal{E}_k(\omega)\,\varphi_k + \sum_{\substack{k_1+k_2\\=k}}{}' \mathcal{E}^{(2)}_k(k_1,k_2)\,\varphi_{k_1}\varphi_{k_2} = 0$$

$$\mathcal{E}_k(\omega) = \omega^2(1+k^2) - \omega k u_k + k^2\gamma_0^2$$

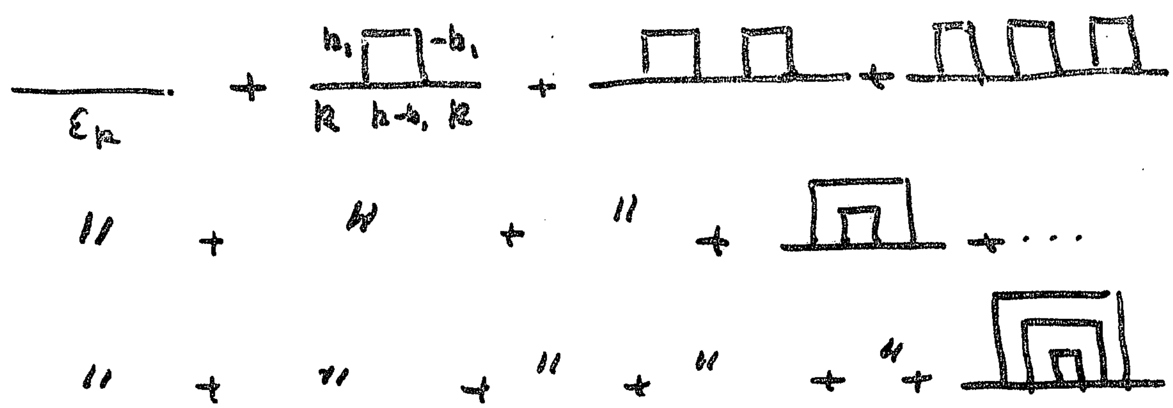
$$\mathcal{E}^{(2)}_k(k_1,k_2) = \frac{ik}{2}(\vec{k}_1\times\vec{k}_2)_\parallel\left[u_{k_2}-u_{k_1}-\frac{\omega_2}{k_2}+\frac{\omega_1}{k_1}\right]$$

ITERATION and SUMMATION

SR

DR

FR



$$\{\mathcal{E}^{n}_k\} \longrightarrow \mathcal{E}^{nl}_k = \mathcal{E}_k(\omega) - \sum_{\vec{k}_1}{}' \int \frac{d\omega_1}{2\pi}\,\frac{\mathcal{E}^{2}(k_1,k-k_1)\,\mathcal{E}^{2}(-k_1,k)}{\mathcal{E}^{nl}(k-k_1)}\, I_0$$

Phys. Reports 49, 273 (1979)

LINEARLY UNSTABLE → Marginally Stable

I_k $\xrightarrow{\gamma t}$

$\varepsilon_k(\omega) I_k = 0$ $\qquad$ $\varepsilon_k^{ne}(\omega, I) I = 0$

Reductions

- Simply Renormalized Series
- $\omega_1 \simeq \omega(\underline{k}_1)$ in Interaction terms
- Frozen k_x Spectrum fnc. k_x^2

3 moments of k_x Spectrum

$$I(k) = \int_{-\infty}^{+\infty} I(k, k_x^2)\, dk_x$$

$$\langle k_x^2 \rangle I(k) = \int_{-\infty}^{+\infty} k_x^2\, I(k, k_x^2)\, dk_x$$

$\langle k_x^2 \rangle$ = linear theory

Exact Constants of Motion

$$\langle \delta p \rangle = \langle \varphi \rangle = 0$$

$$\langle \varphi^2 \rangle - \frac{2\varepsilon_n}{1+\eta} \langle \delta p^2 \rangle = \text{const.} \simeq 0$$

Approximate Results

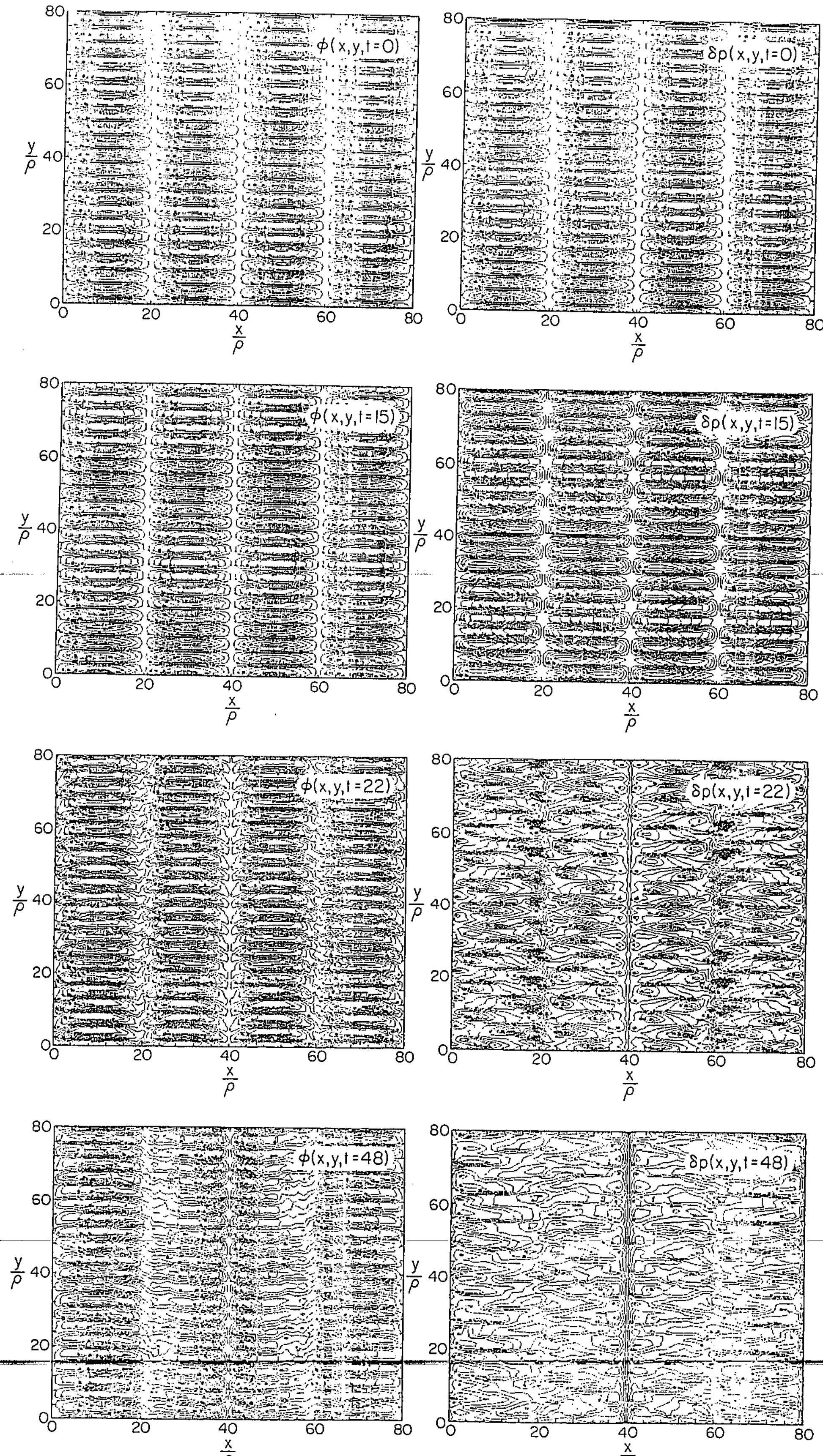
$$\langle \varphi^2 \rangle = \int_0^\infty I(k)\,dk = \frac{\gamma_0^2}{2\langle k_x^2 \rangle}$$

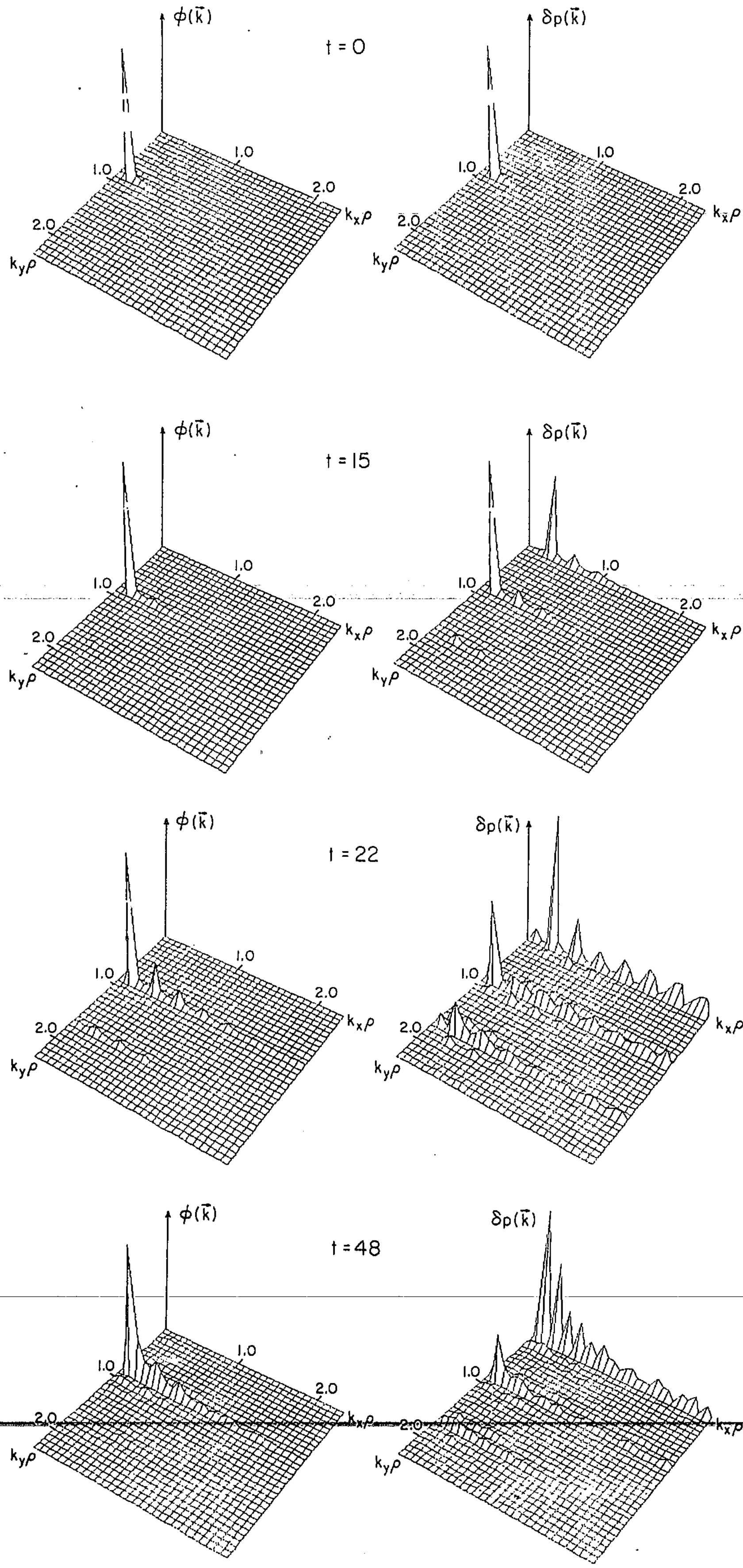
$$\langle \delta p^2 \rangle = \frac{(1+\eta)^2}{2\langle k_x^2 \rangle}$$

$\Rightarrow$

$$\langle k_x^2 \rangle^{1/2} \delta p_{rms} \sim \frac{1}{\sqrt{2}} \left| \frac{dP_0}{dx} \right|$$

= Mixing-Length ~~Est~~ Formula for Saturation Level


φ(x,y,t=0)
δp(x,y,t=0)
φ(x,y,t=15)
δp(x,y,t=15)
φ(x,y,t=22)
δp(x,y,t=22)
φ(x,y,t=48)
δp(x,y,t=48)
y/ρ
x/ρ
0
20
40
60
80


t = 0
t = 15
t = 22
t = 48
$\phi(\vec{k})$
$\delta p(\vec{k})$
1.0
2.0
$k_x\rho$
$k_y\rho$

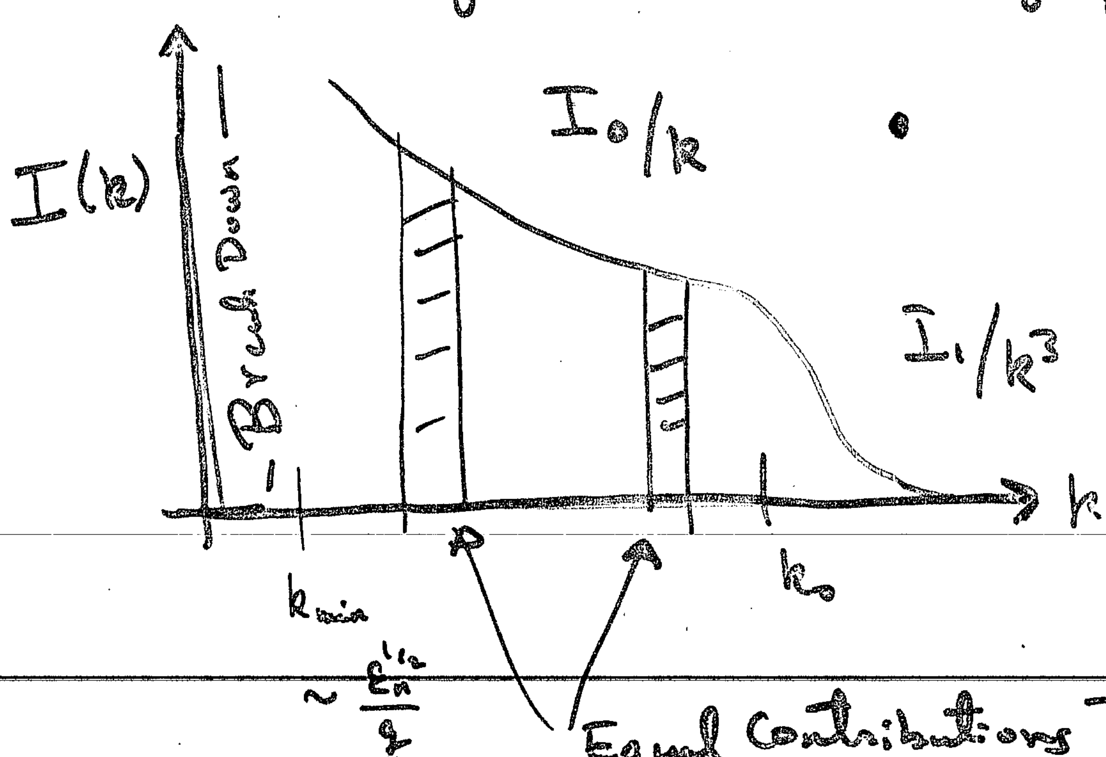
QUASILINEAR TRANSPORT

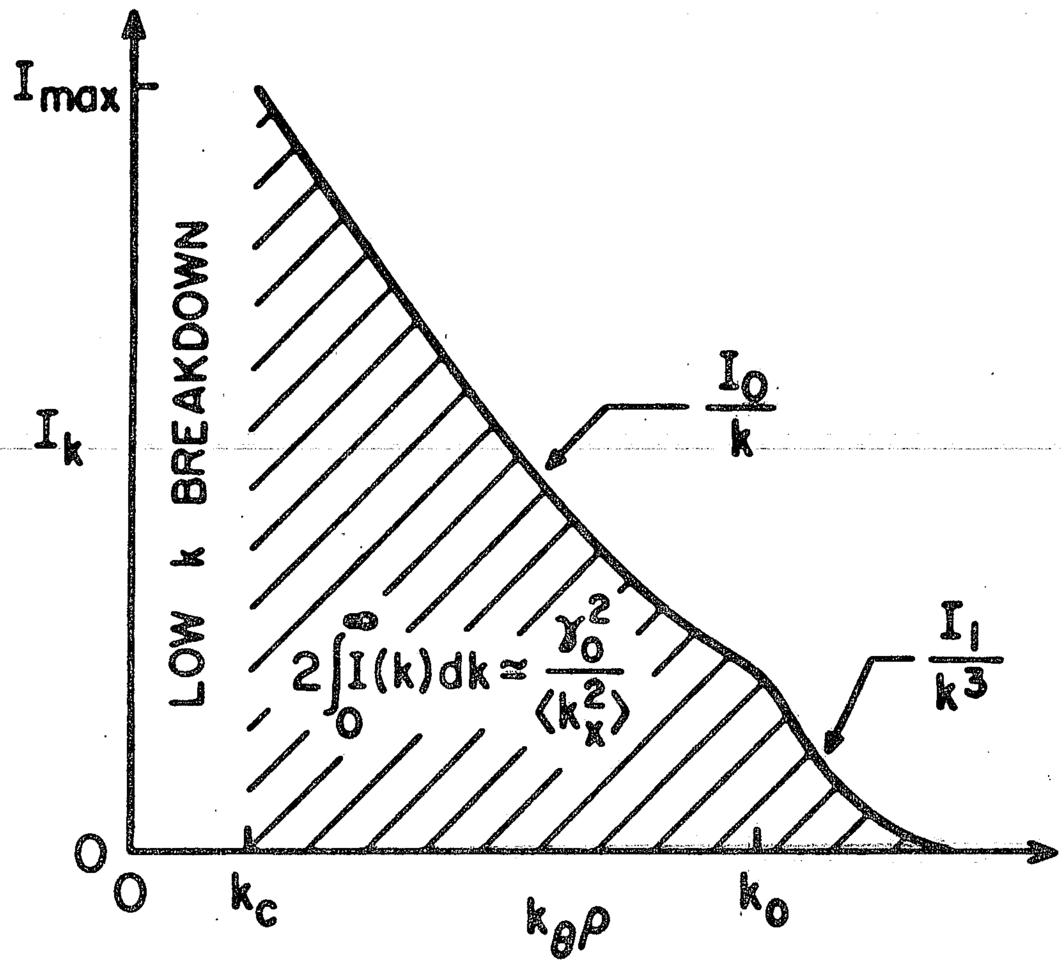
$$\Gamma = \langle n v_{Ex} \rangle = \frac{\rho}{r_n}\frac{cT}{eB}\sum_k ik\varphi_k^* n_k = 0$$

$$Q = \langle p v_{Ex} \rangle = \frac{\rho}{r_n}\frac{cT}{eB}\sum_k' ik\,\varphi_k^*\,\delta p_k$$

$$= -\frac{\rho}{r_n}\frac{cT}{eB\gamma_0}\hat{\chi}_i\frac{dp_0}{dx}$$

$$\hat{\chi}_i = \int_0^\infty k\,I(k)\,dk = \int_0^\infty dk_x \int_0^\infty k\,I(k_x^2,k)\,dk$$





I max
I k
LOW k BREAKDOWN
$\frac{I_0}{k}$
$\frac{I_1}{k^3}$
$2\int_0^\infty I(k)\,dk \approx \frac{\gamma_0^2}{\langle k_x^2 \rangle}$
O
O
k c
k θ ρ
k o

SUMMARY OF RESULTS

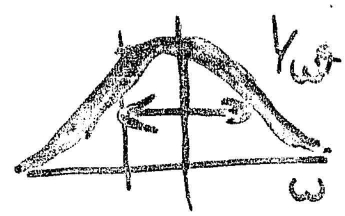
WAVENUMBER SPECTRUM

$$I(k) = \int_0^\infty I(k_x^2, k)\, dk_x \cong \frac{(2\mathcal{E}_n)(1+\eta)}{\langle k_x^2 \rangle \Delta} \begin{cases} 1/k \\ 1/k^3 \end{cases}$$

or Formulas in Appendix

FREQUENCY SPECTRUM

given $\vec{k}$ $\quad I_{\vec{k}}(\omega) \sim \dfrac{S(\vec{k})}{|\mathcal{E}^{na}_{\vec{k}}(\omega)|^2} \approx$



RMS Fluctuation Levels ([redacted])

$$\langle \varphi^2 \rangle = \frac{2\mathcal{E}_n(1+\eta)}{\langle k_x^2 \rangle} \qquad \langle \delta p^2 \rangle = \frac{(1+\eta)^2}{\langle k_x^2 \rangle}$$

in Normalized Units

ANOMALOUS ION THERMAL COND.

$$\Gamma \approx 0 \qquad Q = -\frac{\rho}{r_n}\frac{cT}{eB}\hat{\chi}\frac{dp_0}{dx}$$

ANOMALOUS ION THERMAL COND.

$$\chi_i^A = c_1 \frac{\rho}{r_n} \frac{cT}{eB} \frac{q(1+\eta)^{1/2}}{\xi} \qquad \left(c_1 \lesssim \frac{1}{3} - \frac{1}{2}\right)$$

Scales with B, n, T, q as Neocl. Plateau

$$\chi_i^{nc} = 2.6 \frac{\rho}{r_n} \frac{cT}{eB} q \, \varepsilon_n$$

ENHANCEMENT

$$\approx \frac{c_1 (1+\eta_i)^{1/2}}{2.6 \, \xi \, \varepsilon_n} \sim \text{a few}$$

Properties (1) Plateau-like conductivity into the Banana regime

(2) Associated Fluctuations (particular with ~~localization to outside of Torus and small~~ azimuthal phase velocities

ESTIMATE TFR B4 (Nucl. Fusion 19, 211, 1979)

$I = 3[illegible]$ kA $B = 5[illegible]$ kg $V_R = 1.??$ [illegible]

$r = 10$ cm $T_e(T_i) = 1.2$ kev $n_e(r) = 7 \times 10^{13}\ cm^{-3}$

$r_n = 18$ cm $r_T = 6$ cm $\eta = 3$ $\rho = .05$ cm

diffusion unit $\frac{\rho}{r_n}\frac{cT}{eB} = 2.3 \times 10^3\ cm^2/s$

$$\chi_e = \frac{\rho}{r_n}(\rho c_s)\left(\frac{L_s}{r_n}\right) = 2.4 \times 10^4\ cm^2/s$$

$$\tau_E^e = \frac{a^2}{\chi_e} = 17\ ms \Bigg/ \tau_E = \frac{a^2}{\ }$$

cp:

$$\tau_E = \frac{a^2}{10^{18}/n_e} = \frac{400\ cm^2}{1.4 \times 10^4\ cm^2/s} = 29\ ms$$

$$\chi_i = (1+\eta)^{1/2}(q/s+1)\ 2.3 \times 10^3\ cm^2/s = 9 \times 10^3\ cm^2/s$$

$$\tau_{Ei} = \frac{400\ cm^2}{9 \times 10^3} = 44\ ms$$

cp $\chi^{nc}_{i\,pe} = 1.6 \times 10^3$ and $\tau_{Ei} = 250\ ms$

$$\chi^{nc}_{pl,i} < \chi_i < \chi_e$$

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1. W. Horton, D. I. Choi, and W. M. Tang, Phys. Fluids 24, 1077 (1981).

2. B. Coppi, M. N. Rosenbluth, R. Z. Sagdeev, Phys. Fluids 10, 582 (1967).

3. W. Horton, R. D. Estes, and D. Biskamp, Plasma Phys. 22, 663 (1980).

4. P. W. Terry, W. C. Anderson, and W. Horton, Institute for Fusion Studies Report #36 (1981).

5. D. Brock and W. Horton, Institute for Fusion Studies Report #32 (1981). (to appear in Plasma Phys. 1982)

SUMMARIES AND DISCUSSION

T. SATO	HIGH FREQUENCY DRIFT TURBULENCE
A. HASEGAWA	FLUID PICTURE OF TURBULENCE
F.W. PERKINS	REMARKS ON SCALING
P. SIMILON	KINETIC THEORY BALLOONING MODE TURBULENCE
E. MAZZACATO	EXPERIMENTS FLUCTUATION MEASUREMENTS
R. HATAKEYAMA	LOW FREQUENCY MAGNETIC INSTABILITY
R. WALTZ	WAVE-WAVE COUPLING, TRANSPORT AND STOCHASTICITY
W. LEE	DRIFT WAVE SIMULATIONS
T. TANIUTI	DRIFT WAVE SOLITONS AND CONVECTIVE CELLS
M.N. ROSENBLUTH	SUMMARY AND CONCLUDING REMARKS

HIGH FREQUENCY DRIFT TURBULENCE

T. SATO

High Frequency Drift Modes

3 talks (Gladd, Sato, Nevins)

$k\rho_i \sim 1$	$k\rho_e \sim 1$	$k\rho_e \sim 1$
$\omega \ll \omega_{ci}$	$\omega \sim \omega_{ci}$	$\omega \gg \omega_{ci}$

$\rho_i/L_N = (m_e/m_i)^{1/2}, \; (m_e/m_i)^{1/4}, \; 1$

universal

Tokamaks	Mirrors	
Stellarators		θ - pinches
EBT core		EBT boundary
		Space

LHD (Gladd)

- Key parameters:

  $\frac{\nabla n}{n}$, $\frac{\nabla B}{B}$, Shear, Curvature,

  Hot, Cold plasma, ∇T, Collisions etc

- Energy sources:

  Drift energy, Magnetic energy

- Saturation mechanisms:

  Current reduction

  QL plateau formation on f_i

  Resonance broadening

  Ion trapping, Electron trapping

- Nonlinear effects:

  Anomalous resistivity, transport, heating

- Problems:

  How does the LHD mode produce electromagnetic fluctuations near a null magnetic field?

2-D Simulation of LHD (Sato)

- Early stage

LHD inst → Anomalous resistivity

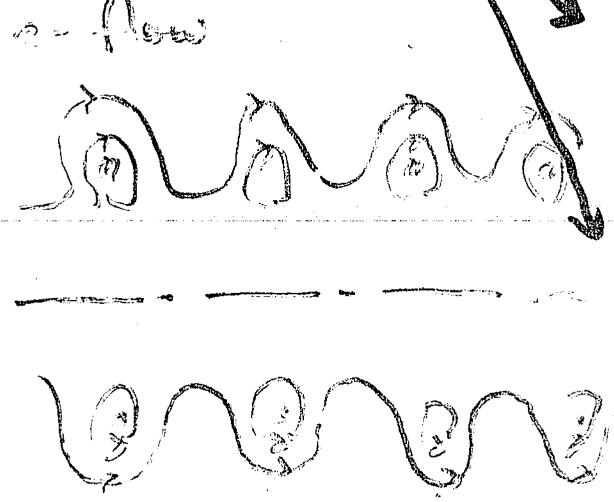
$$\eta_a \approx \frac{\omega_{LH}}{\omega_{pe}^2}\left(\frac{V_{di}}{v_{thi}}\right)^{1.9}$$

→ Anomalous transport

$$\langle v_x \rangle \approx .12\, \omega_{LH} L_B$$

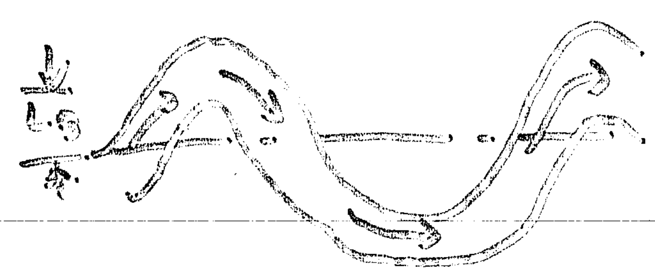
→ Electron heating

$$\Delta(nT_e) \approx \Delta\left(\frac{B^2}{8\pi}\right)$$



- Later stage

nonlinear electromagnetic mode



$$\leftarrow \frac{c}{\omega_{pe} L_B} = 0.5 \quad (m_i/m_e = 185)$$

$$? \leftarrow \frac{c}{\omega_{pe} L_B} = 0.05 \quad (m_i/m_e = 1850)$$

(Blackhill)

Simulation of Drift-Cone Turbulence

(Nevins)

- Particle simulations of steady state DCLC turbulence with realistic sources and sinks (but with compressed time scales)

- Useful tool for studying nonlinear theories of DCLC turbulence

- Allows detailed study of nonlinear processes, e.g. ion orbit modification

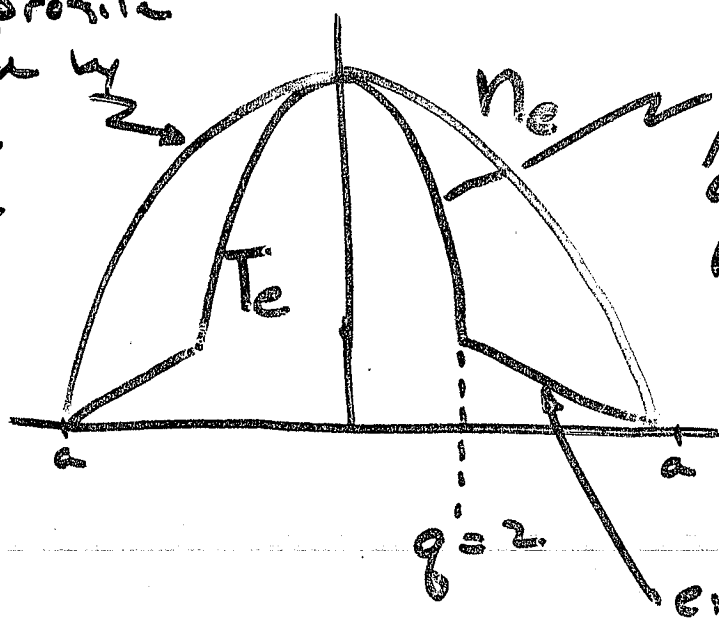
Small amplitude → Nonlinear frequency shift

Large amplitude → Ion trapping and/or stochastic diffusion

REMARKS ON SCALING

F.W. PERKINS

Transport in Tokamaks



$$\frac{\partial T}{\partial r} = \frac{Q}{\kappa}$$

1. Drift wave turbulence thermal transport applies only between $q = 1$ and $q = 2$ surfaces in a tokamak.

2. Discharges with high values of q_L are effectively constricted

$$q = q_L \, r^2/a^2$$

$$r^2 = a^2 \frac{q}{q_L} = q \frac{2RI}{cB_z}$$

3. Two classes of thermal conductivity are needed: magnetic flutter and drift wave

4. Drift Wave turbulence thermal transport dominated by "random walk" of trapped electrons

- trapped electrons undergo radial excursions in fine-scale $\vec{E}\times\vec{B}$ drifts associated with microinstabilities

- use neoclassical diffusion approach:

$$\Delta x = \text{step size} = \min \begin{cases} \left\langle \frac{k_\perp \phi c}{B\omega} \right\rangle \\ \left\langle \frac{k_\perp \phi c}{B} \right\rangle \frac{1}{\nu_{eff}} \end{cases}$$

- ν_{eff} must be compared with $\langle \omega \rangle$, not with ω_{be}

- characteristic step size can be determined experimentally:

$$\langle \Delta x^2 \rangle = \left\langle \left(\frac{\tilde{n}}{n}\right)^2 \right\rangle L_n^2$$

5. $$D = (\Delta x)^2 \langle \nu \rangle \left(\frac{r}{R}\right)^{1/2}$$

↑ fraction of trapped particles.

Scaling of Diffusion Coefficients.

Collisionless Limit

$$D = \left\langle \frac{k_\theta \phi c}{B \omega} \right\rangle^2 \langle \omega \rangle \left(\frac{r}{R}\right)^{1/2}$$

$$= \left\langle \left(\frac{e\phi}{T}\right)^2 \right\rangle L_n^2 \left\langle \frac{k_\theta T c}{e B L_n} \right\rangle \left(\frac{r}{R}\right)^{1/2}$$

$$= \left\langle \frac{1}{k_\perp} \right\rangle \frac{Tc}{eBL_n} \left(\frac{r}{R}\right)^{1/2} = \frac{\rho_i T c}{eBL_n} \left(\frac{r}{R}\right)^{1/2}$$

$$\propto \frac{T^{3/2}}{B^2 a}$$

Used: $\left\langle \left(\frac{e\phi}{T_e}\right)^2 \right\rangle = \left\langle \frac{1}{k_\theta^2} \right\rangle \frac{1}{L_n^2}$

Collisional limit $\quad \nu_{eff} > \langle \omega \rangle$

$$D = \left\langle \frac{k_\theta \phi c}{B \nu_{eff}} \right\rangle^2 \nu = \frac{Tc}{eB} \frac{\rho_i}{L_n} \left(\frac{V_i}{L_n \nu_{eff}}\right)$$

$$\propto \frac{T^{7/2}}{n a^2 B^2}$$

Results for B scaling Exps.

$I =$ constant

$r^2 \propto \frac{1}{B}$ ($\delta = 2$ surface shrinks as B increases)

Power Balance

$$\frac{nT\, r^2}{\tau_E} = \left(\frac{I}{r^2}\right)^2 \eta\, r^2 \propto \frac{1}{r^2 T^{3/2}} = \frac{B}{T^{3/2}}$$

$$\frac{1}{\tau_E} = \frac{D}{r^2} = DB$$

$$\therefore\ nTD = \frac{B}{T^{3/2}}$$

Collisionless Scaling

$$D = T^{3/2} B^{-2}$$

$$T = B^{3/4} n^{-1/4}$$

$$\tau_E = B^{-1/8} n^{3/8}$$

Collisional Scaling

$$D = T^{7/2} B^{-2} n^{-1}$$

$$T = B^{1/2}$$

$$\tau_E = n B^{-3/4}$$

Observations:

- Drift Wave Turbulence Transport Theory is close to meaningful, tractable results.

- Scalings of D can be obtained without solving complete problem.

- Scaling interactions in actual experiments not intuitive
  - Temperature, channel size are crucial.

- Different (probably magnetic flutter) model needed to explain exterior high conductivity region.

KINETIC THEORY BALLOONING MODE TURBULENCE

P. SIMILON

Similon

Non Linear Ballooning formalism

- eikonal representation isolates fast dependence of the fluctuations across field and the slow dependence along field

  ($k_\perp \sim \rho_i^{-1}$; $k_\parallel \sim \frac{1}{Rq}$)

  in toroidal geometry $\exp -in\left[\varphi - q\theta + S(r)\right]$

- Application | in — MHD
  \ kinetic theory

  for — electrostatic modes
  \ elm. modes

  in — linear theory
  \ non linear theories

At this workshop :
many presentations
in linear theory : reviewed and discussed
by W. Tang , S. Itoh
and G. Rewoldt

The formalism : important in N.L. theories

since
- simplification of linear operator and particle propagator (ordinary differential operator)
- simplification of the N.L terms (eikonal form)

L. Chen :

presented a N.L. gyrokinetic equation
(F.L.R $k_\perp \rho_i \sim 1$; $\omega \ll \Omega_i$)

and its form in the ballooning formalism.

N.L term : the $\langle \tilde{\underline{v}}_{E\times B} \rangle_\varphi \cdot \underline{\nabla}\, \bar{h}$

takes the form

$$- \frac{c}{B} k_\theta' k_\theta'' \hat{} \; J_0\left(\frac{k_\theta' v_\perp}{\Omega_i}(1+s^2\theta^2)^{1/2}\right)$$

$$2\pi m \; \hat{\Phi}'(\theta - 2\pi m)\, \hat{h}''(\theta)$$

K. Swartz :

Renormalization of N.L. bounce averaged equation for trapped electrons.

For S.L. mode, balance of linear + N.L. electron excitation, and ion Compton Scattering + shear damping

$\Rightarrow$ spectral distribution & transport.

Reduction of N.L. term by $\left(\frac{L_n}{L_s}\right)$

due to sampling of the spread of eigenfunction by bounce motion of partic

P. Similon

Uses ion N.L. gyrokinetic equation to get Renormalized weak turbulence theory of ion Compton Scattering for T I mode. Shows that distant & close interaction of fluctuations causes a large N.L. transfer of energy at low levels of turbulence, from large k to small k. Importance of distortion of eigenmodes, and non linear stabilization.

P. Diamond

Considers, using ballooning formalism, the two point correlation for T. E. fluctuations, and implications of frequency spectrum predicted by clump theory on drift wave turbulence.

- analytic calculation gives $\frac{\Delta\omega}{\omega} \sim 1$
- enhanced growth of modes $\gamma^{nl} \sim \frac{\gamma}{1-C}$
- describes the impact of $\Delta\omega$ on N.L. wave particle interaction.

W. Horton

Examines Ion pressure gradient driven drift modes in the turbulent regime.

From the "simply renormalized" theory deduces the N.L. dispersion relation.

The resulting spectral distribution provides ^anomalous^ ion thermal conductivity χ_i which scales with n, T and B as the neoclassical plateau formula.

DRIFT WAVE SIMULATIONS

W. LEE

Lee

Low Frequency Particle Code Simulation

1. Importance

2. Recent Activities

3 Difficulties & Improvements

	Numerical	Physical
Time Step	Implicit Methods	Gyrokinetic
Profile Modificatio	Particle Recycling	Multiple spatial scale
Grid Size	Higher order Interpolation	?

4. Cooperation between theorists & particle pushers

5. Gyrokinetic Codes

2½ D slab < ES, MS

3 D slab < ES, MS

3D toroidal

6. ~~Saturation Mechanisms~~

Nonlinear $\underline{E} \times \underline{B}$

Nonlinear polarization

DRIFT WAVE SOLITONS AND CONVECTIVE CELLS

T. TANIUTI

Solitons & Convective Cell.

(A) Soliton

(1) J. D. Meiss & W. Horton

Drift Wave Turbulence

~ Soliton gas. : The Gibbs' ensemble for the ideal gas limit of the N_s (>> 1) soliton system.

(Model field : The Petviashvili equation for the temperature gradient drift wave).

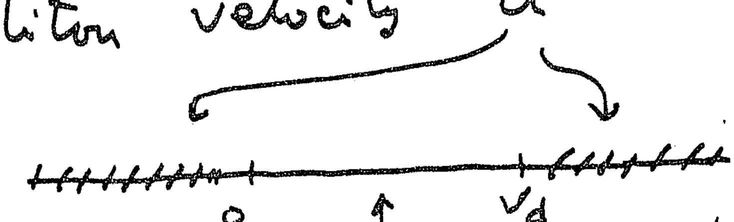
conclusion :

the fluctuation spectrum has a broad frequency spectrum for fixed k. The width of the frequency spectrum $\Delta\omega \sim \sqrt{\langle\varphi^2\rangle}$. For a given k component, the peak frequency ω_p of the spectrum $> kV_d$

($\omega_p/\omega_p^l \sim 5$ can be given easily)

Crucial point

the soliton velocity u



↑ linear phase velocity

(A) (2). Makino, Kamimura, Taniuti

Vortex Solitary waves of H.M.

Eq. : Two Solitary waves exhibit behaviours of Solitons in mutual interaction. (Movie)

Further numerical and analytical investigations will be interesting so that (1) and (2) could be incorporated.

(B) Convective-cell Mode.

Chen-Okuda's numerical simulation

⟹ Convective cell and Anomalous transport even in the case <u>with</u> <u>shear</u>.

But. H.M. Eq. excludes. Convective Cell Mode

In Collisional Drift Wave Turbulence with weak Dissipation, it was shown ($T_i = 0$)

without shear, the excitation of the convective-cell mode is well-understood.

Modified H.M + 2D Navier Stokes coupled turbulence.

With Weak Shear:

$x \sim \rho_s$, $t \sim \omega_{ci}^{-1} \epsilon^{-1}$, $n_0 \sim e^{-\epsilon x}$,

$\varphi \sim \epsilon$ $\delta n / n \sim \epsilon$

$B_y(x)/B_0 \sim \epsilon^{3/2}$, $k_z \sim \epsilon^{\frac{3}{2}}$

Only one mode rational surface (at $x = 0$). ($\Rightarrow k_z = 0$)

H. M. Eq is valid. for

$$x \gg x_c \sim \sqrt{\frac{\nu}{\epsilon^2} / \langle k_y^2 \rangle}$$

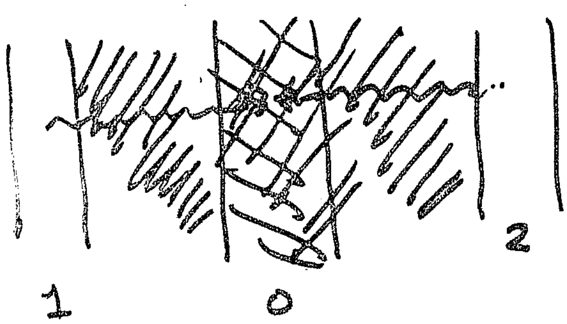
5

where $\nu = \frac{1}{\omega_{ci}} \frac{m}{M} \nu_{ei}$.

(Note. ion Landau Damping is not considered)

lural. Mode Rational

urfaces, $k_z \neq 0$.



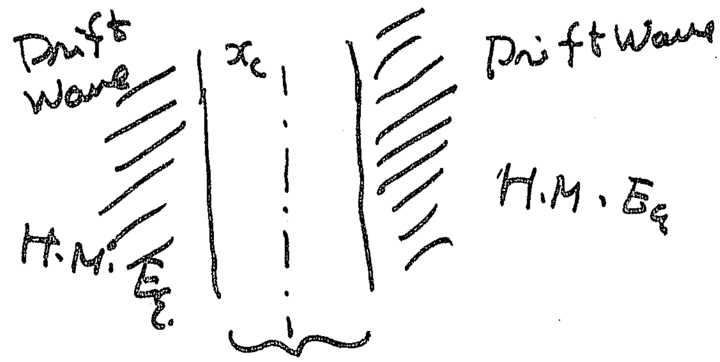
↓

D_1-D_2 interaction

⟹ convection cell

at $x = 0$.

is possible.



↑ convection cell mode region where the Boltzman Distribution is not valid, 2 D. Navier Stokes is ok.

<u>No excitation of Convection Cell</u>

SUMMARY AND CONCLUDING REMARKS

M.N. ROSENBLUTH

Conclusions

I Drift type modes ~~are~~ responsible for transport. (Correct character, level, etc.)

II Serious non-linear calculations are now beginning, but

III No predictive capability as yet. Collisionless, electrostatic turbulence won't predict $D \sim \frac{1}{n}$. Are collisional ($\nu_{eff} > \omega_*$) and β effects sufficient? $\frac{1}{a^2}$ scaling, if true, is nearly impossible to understand.

$$\underline{\text{Expt}} \quad D \sim \frac{1}{n}$$

$$\underline{\text{Theory}} \quad D \sim \frac{T}{B}\frac{\rho_s}{a} \sim \frac{T^{3/2}}{B^2 a}$$

IV Present models for $\Delta\omega$

a) Minimum entropy model requires $\Delta\omega$, $\vec{\nabla} \times \vec{V}$.
b) Doppler shift associated with large eddies.
c) $\gamma_{Non\ Linear}$ associated with mode-coupling.
d) ~~Clump~~ ballistics lead to spread.
e) Solitons.

V Destabilization mechanisms

a) collisions

b) curvature (drifts)

c) electron non-linearity

d) clump enhancement

VI Saturation Mechanisms ~ β effects

a) large electron non-linearity (res. broadening)

b) Wave scattering (γ_{NL}) related to

c) Ion Compton scattering

VII Ingredients for a successful calculation?

a) Ballooning representation (toroidal effects)

b) Moderate turbulence – DIA w. Δω, Ion Compton scattering

c) Clumps Convective cells

d) Collisions, finite β,

e) Simulation

VIII Problems

a) Too complicated? Experiments!

b) Is weak turbulence good enough? Ballooning modes a good representation?

c) Another ingredient? Strong turbulence effects (solitons)

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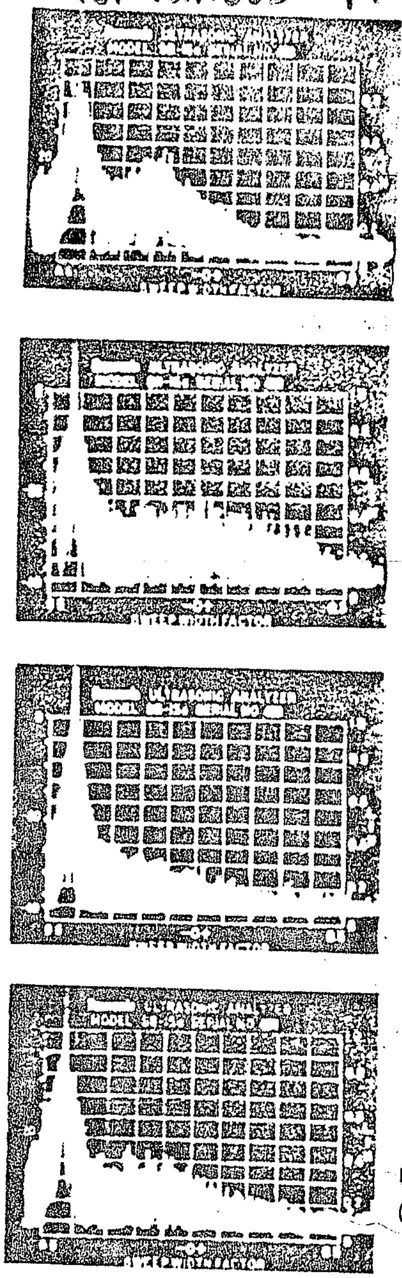
DRIFT WAVE TURBULENCE
JANUARY 11-15, 1982
AUSTIN, TEXAS

FM-1 continued

$S(k, \omega)$ for various $k\rho_i$



$k_\theta\rho_i = .13$

$k_\phi \rho_i = 0.13$
($k_\phi = 2\, k_0 \cos 75°$)

$k\rho_i = .25$

$k_\phi \rho_i = 0.25$
($k_\phi = 2\, k_0 \cos 60°$)

$k\rho_i = .36$

$k_\phi \rho_i = 0.36$
($k_\phi = 2\, k_0 \cos 45°$)

$k_r\rho_i = .97$

$k_\chi \rho_i = 0.97$
($k_\chi = 2\, k_0 \cos 15°$)

10 kHz/div

FIG. 4. The frequency power spectrum observed by microwave scattering in the high shear configuration in a hydrogen plasma. $\delta f \approx 2$ kHz.

Broad [illegible] peak [illegible]

big peak [illegible]

* they see stabilization
(transition from $\Delta f \sim f$ to $\Delta f \ll f$)

Scaling as $\frac{a}{L_s} \propto \left(\frac{m_e}{m_i}\right)^{1/3}$

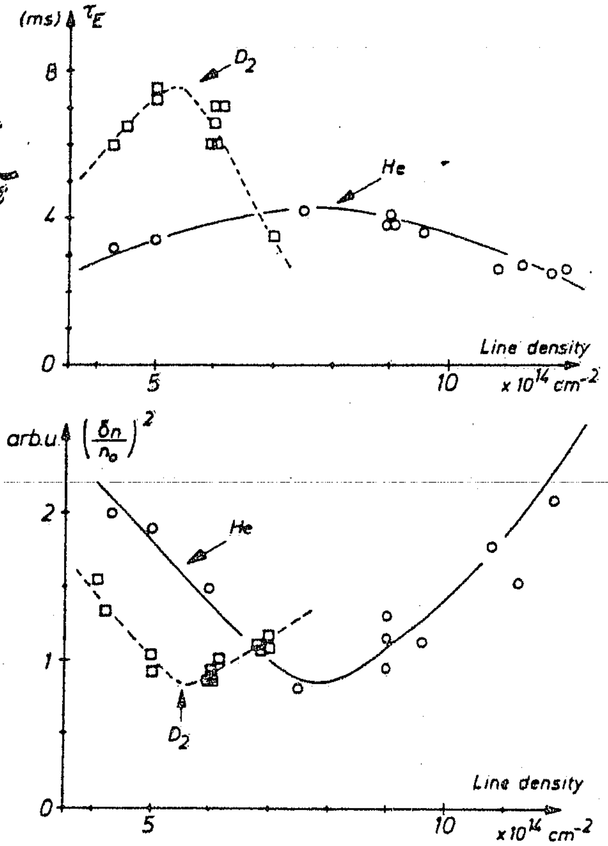
as in Perlstein-Berk

Wendelstein VII-A   "Garching Stellarator"

Meyer + Mahn PRL **46** (1981) 1206 – CO_2 scattering

Saw close relation between fluctuation level + energy confinement time

τ_E



$\frac{\delta n_k}{n}$

$\int n\,dl \rightarrow$

← τ_E increases, then decreases with $\bar{n}$

← correlates with density fluctuation level $\frac{\delta n_k}{n}$ for $k = 125\,cm^{-1}$

(So this is fluctuation level for $k = 125\,cm^{-1}$ ($k\rho_s \sim 7$) integrated over all ω – not total $\tilde{n}/n$)

* Spectrum seen is broad: slight peak at $f \sim 50$–100 kHz, $\Delta f \sim 100$ kHz

W VII A Parameters: $R = 200$ $a = 16$ $B_0 \leq 35$ kG

$T_0 \sim 300$–500 eV $I \sim 20$ kA

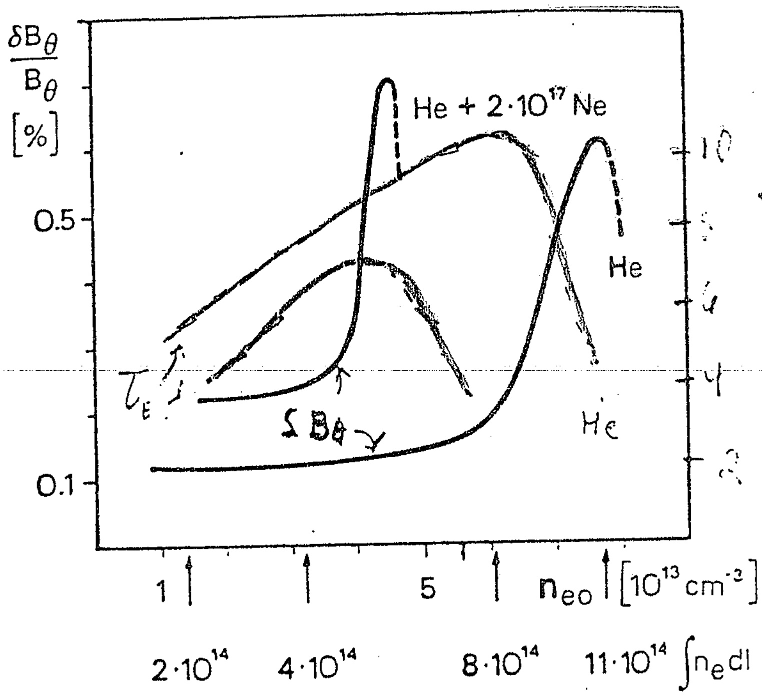
helical windings $l = 2, m = 2$ shearless ext. transform $\iota = .23$]

53

Wendelstein VII A – Poster APS-NYC – WVIIA-Team

Also see deterioration of confinement at high $\bar{n}$ correlated with MHD activity – (2,1) + (3,2)

(tearing modes, deformation & ergodization)



τ_E msec

in low density regime, find

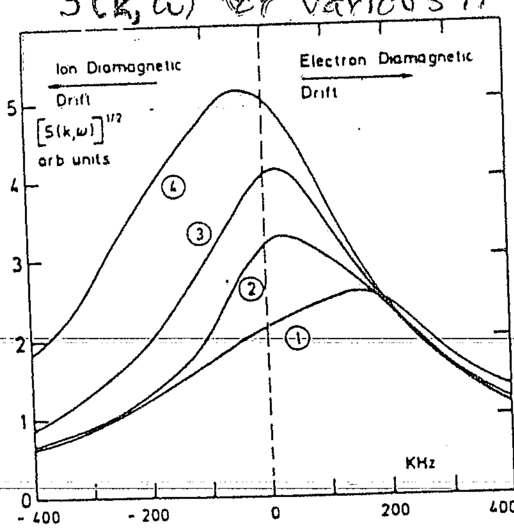
$n\tau_E \simeq \frac{3\times10^{18}}{T_e^{2/3}}$ cm^{-3}sec (T_e in eV)

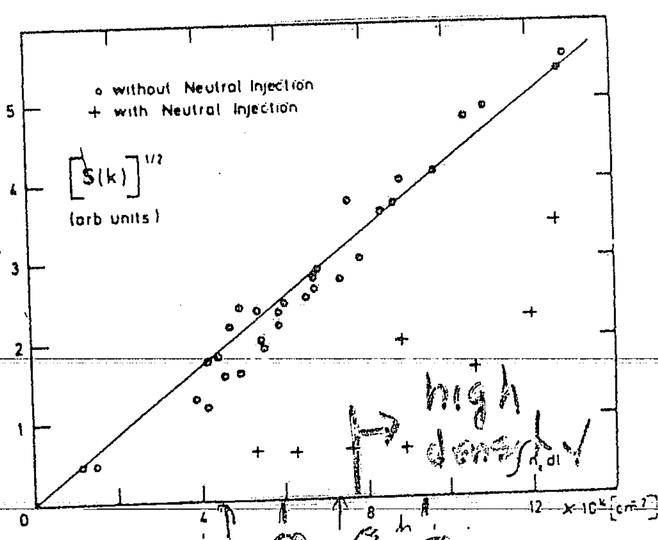
(lower than INTOR

Microwave scattering (WVIIA-Team IAEA-CN-38/H-1) $k_\perp \simeq 6$-25 cm^{-1} $k\rho_s \sim 1$) showed no correlation of $\delta\tilde{n}_k$ and τ_E

S(k, ω) for various $\bar{n}$

S(k, ω)





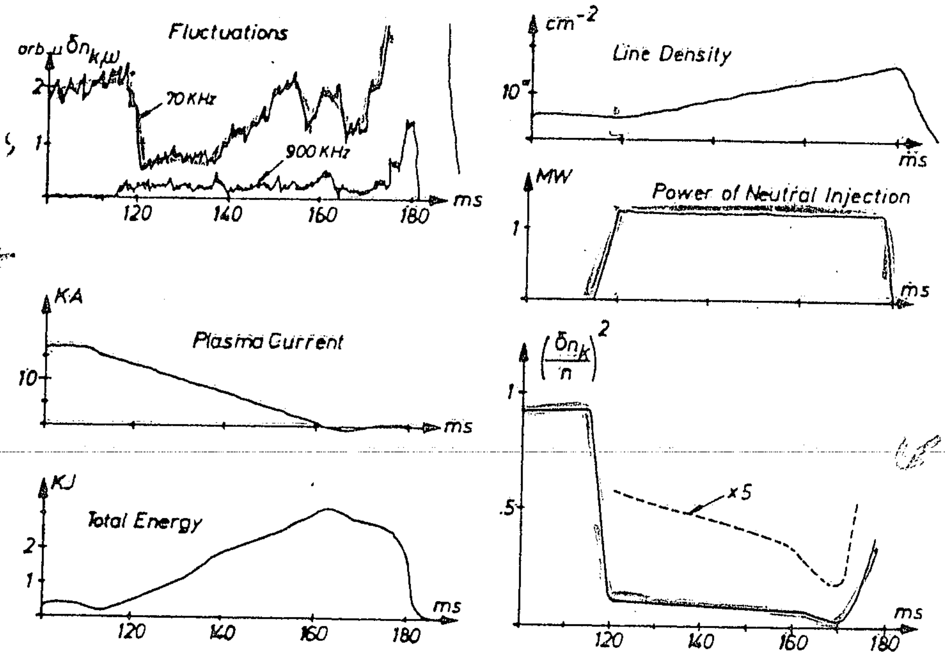
① ② ③ ④

density $\int ndl$

Wendelstein VII A - Neutral Beam injection

- Sharp drop in $\tilde{n}_k/n$ + change in spectra within 1 msec of start of injection - faster than profiles change
- seen in both CO_2 + μwave spectra
- Confinement is better during beam injection

low freq. fluctuations drop ~ 70 kHz



Here, fig. from Meyer + Hahn CO_2

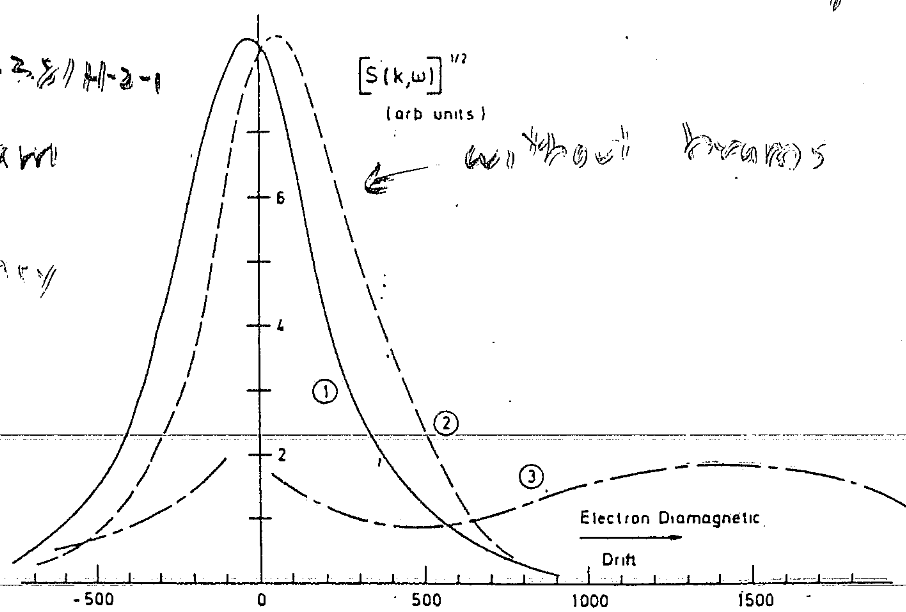
level drops by 10

Meyer + Hahn suggest that since fluctuations change before any global parameters or profiles change, subsequent improved confinement is due to lower $\tilde{n}$ level

μwave scattering spectra change during beam injection

IAEA-CN-38/H-3-1
WVIIA Team

low frequency fluctuation level drops



a)

Fluctuations & Transport have also been studied in

RFPIs

Zeta - Robinson & Rusbridge Plasma Phys 11 (1969) 73 & refs therein

Rusbridge Plasma Phys 22 (1980) 331

ZT-40 Jacobson Appl. Phys. Lett 39 (1981) 795

Plasma Phys 23 (1981) 927

EBT's Roth, Krawczonek, Powers, Hong & Kim PRL 40 (1978) 1195

Linear

Boisser Nuc. Fus 18 (1978) 867

Carter, Edwards, Mossack, Rusbridge, Hastie Plasma Phys 23 (1981) 819

Hendel, Chu, Politzer P.F. 11 (1968) 2426

Other Multipoles - Yoshikawa Review Nuc Fus 13 (1973) 432

Culham Levitron IAEA Brussels

Conclusions

1. Ohmic Tokomaks have broadband, turbulent spectra, $\Delta\omega > \omega$ for frequencies $\omega \lessapprox \omega_*$

2. Similar spectra seen in other experiments under a wide variety of conditions

3. Not clear what this low frequency turbulence is - "drift-wave"? "convective cell"? "MHD"? etc.

4. Probably turbulence is related to transport

Is mode coupling important for fluctuation levels $\tilde{n}/n$ measured in tokamaks?

DEFINE $\left.\frac{\tilde{n}}{n}\right|_{crit.}$ - turbulence level such that

nonlinear modecoupling term (n.linear $\tilde{E}\times\tilde{B}$) equal to linear (ω^*) term for $k\rho_s = 1$

then, from Hasegawa Mima eqn.

$$\left.\frac{\tilde{n}}{n}\right|_{crit} \approx \frac{\rho_s}{4L_n} \sim \frac{\rho_s(T_e(0))}{2a}$$

Tokamak	$\left.\frac{\tilde{n}}{n}\right\|_{crit}$	$\left.\frac{\tilde{n}}{n}\right\|_{measured}$	Ratio $\frac{\tilde{n}_m}{\tilde{n}_c}$ (if $\gtrsim 1$ mode coupling impt.)
PLT	10^{-3}	.005-.01	4-8
Alcator-A (low density)	4×10^{-3}	$.07\pm.03$	17
ATC { μ	$4\text{-}5\times10^{-3}$	.005-.01	1
ATC { ω^2		.03	6
Macrotor	8×10^{-3}	.1	12
Microtor	5×10^{-3}	~.04	8
TFR $\bar{n} = 5\times10^{13}$	2×10^{-3}	5×10^{-3}	2.5
TFR $\bar{n} = 10^{14}\,cm^{-3}$		2×10^{-3}	1

In all cases, mode coupling is important

MICROINSTABILITY, ENTROPY-PRODUCTION AND PLASMA CONFINEMENT

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Jan. 1982

Microinstability, Entropy-production and Plasma Confinement

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A Non-equilibrium thermodynamic approach

Motivations and Status

Anomalous plasma transport

...... explain Scaling Laws ?

Previous. traditional analyses

- Force - Balance equilibrium

↓

- Microinstability analysis

↓

- given profiles, calculate fluxes (induced)

↓

< Saturation level., spectrum analyses >.

quasilinear, non-linear (turbulence weak, strong etc.)

↓

Γ, Q, J $\rightarrow \tau_p$, τ_E ??

These are depending on geometric factors!

↓

- How can we determine geometric factors such as , χ, χ_T, ...

Confined Plasma

- Non-thermal equilibrium (inhomogeneities)
  κ, κ_p ....
- Steady-state with
  External Supplies

(Anomalous) Loss →

← Input Source { particle, ohmic, neutral beam, RF heating, Current Drive etc.

← balance. →

Steady-State

Losses induced by fluctuations

balance with External Source

- Internal fluctuation level, spectrum

should be determined by External Source Condition!

☆ Similar.

- Pseudo-classical, NeoBohm Yoshikawa
- Profile Consistency , B. Coppi,

a method to determine plasma losses

in Non-equilibrium steady-state w external supply

"Ansatz"

"relevant state has the minimum entropy production rate (irreversible), under the conditions imposed to the system"

$\vec{\Gamma}, \vec{Q}, \vec{J}$ ···(flow) ← induced by fluctuations, I_k

↓

$\dot{S}$ < Gibbs' relation
local. quasi-static change >

↓

$\left.\frac{dS}{dt}\right)_{irr}$ $(\kappa, \kappa_T, \vec{\Gamma}, \vec{Q}\ I_k)$

∘ Variational Principle < Wave-Kinetic Eq. >

· Conditional Minimum (density, energy · conservations)

↓

Determine, $\bar{\Gamma}, \bar{Q}, \kappa, \kappa_T, \int I_k$ ···
simultaneously.

Basic Equations and Induced Fluxes

$$m_j n_j \frac{d\vec{V}_j}{dt} - \frac{q_j}{c}\vec{\Gamma}_j \times \vec{B} = q_j \langle \tilde{n}_j \tilde{\vec{E}}_j + \frac{1}{c}\tilde{\vec{\Gamma}}_j \times \tilde{\vec{B}} \rangle + n_j q_j \vec{E} - \nabla p_j \, , \quad (1)$$

$$\frac{\partial}{\partial t}(3n_j T_j) + \nabla\cdot\vec{Q}_j + 2p_j \nabla\cdot\vec{V}_j + \nabla p_j \cdot \vec{V}_j = q_j \langle \tilde{\vec{\Gamma}}_j \cdot \tilde{\vec{E}} \rangle + \dot{P}_j , \quad (2)$$

where $\vec{\Gamma} = n\vec{V}$, $Q_{ijk} = \frac{1}{2}\int m_j (v_i - V_i)(v_j - V_j)(v_k - V_k) f d\vec{v}$ is the heat conduction, $Q_k = 3T\Gamma_k + \sum_i Q_{kii}$ is the total heat flux

$$\Gamma_r = \frac{c}{B}\langle (\tilde{n}\tilde{\vec{E}} + \frac{1}{c}\tilde{\vec{\Gamma}} \times \tilde{\vec{B}})_\theta \rangle \quad (3)$$

$$\sum_i Q_{rii} = \sum_i \frac{c}{B} [\langle (\tilde{\vec{E}} + \frac{\vec{V}\times\tilde{\vec{B}}}{c})_\theta \tilde{p}_{ii} \rangle + 2\langle (\tilde{\vec{E}} + \frac{\vec{V}\times\tilde{\vec{B}}}{c})_i \tilde{p}_{i\theta} \rangle - \frac{B}{nc} p_{ii}\Gamma_r$$

$$- \frac{1}{n} p_{i\theta} \langle (\tilde{n}\tilde{\vec{E}} + \frac{\tilde{\vec{\Gamma}}\times\tilde{\vec{B}}}{c})_i \rangle + \frac{1}{c}\langle \tilde{B}_r \tilde{Q}_{zii} - \tilde{B}_z \tilde{Q}_{rii} \rangle] . \quad (4)$$

for cylindrical plasma, κ, κ_T, u, EM fluctuations (low freq.)

☆ $$\Gamma_r = \sum_k [\frac{\omega - k_{\parallel} u}{\omega} - \frac{\omega_{*j}}{\omega}\{1 - \frac{\kappa_T}{\kappa}(1-2\xi^2)\}] R_k \, , \quad (3')$$

☆ $$Q_r = \sum_k [(\frac{\omega - k_{\parallel} u}{\omega} - \frac{\omega_{*j}}{\omega}\{1 - \frac{\kappa_T}{2\kappa}(1-2\xi^2)\})\{m(\frac{\omega}{k_{\parallel}} - u)^2 + 2T\} - 2T\frac{\omega_*}{\omega}\frac{\kappa_T}{\kappa}] R_k \quad (4')$$

and

$$R_k = \frac{nq_j c}{Bk_\theta T} \, \mathrm{Im}\left(\frac{\omega}{\sqrt{2}|k_{\parallel}|v_T} Z(\xi)\right) |\tilde{E}_\theta + \frac{\omega}{k_{\parallel} c}\tilde{B}_r|_k^2 , \qquad k \equiv (\vec{k}, \omega)$$

where $\kappa_T = -\nabla T/T$, $\omega_* = -\kappa k_\theta cT/q_j B$, $\xi = (\omega - k_{\parallel} u)/\sqrt{2}|k_{\parallel}|v_T$, $v_T^2 = T/m$

Induced Flows v.s. Force acting on Plasma

$$\begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{pmatrix} = \begin{pmatrix} -K_1, & \frac{1}{2}K_1, & K_2, & K_4 \\ \frac{1}{2}K_1, & -\frac{5}{4}K_1, & -\frac{1}{2}K_2, & \frac{1}{2}K_4 \\ K_2, & -\frac{1}{2}K_2, & -K_3, & K_5 \\ 0, & 0, & 0, & T/\eta_c \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} \tag{14}$$

where the "flow" $(J_1 \cdots J_4)$ is $(\Gamma_r, J_{sr}, \dot{W}, TJ_{0\parallel})$ and the "force" $(X_1 \cdots X_4)$ is $(\kappa, \kappa_T, 1/T, E_\parallel)$, $\dot{W} \equiv q\langle \overleftrightarrow{\Gamma} \cdot \overleftrightarrow{E} \rangle$ is the energy exchange with the wave, $\bar{\xi} \equiv \omega/\sqrt{2}|k_\parallel| v_T$,

$$K_1 = n \int dk \ \mathrm{Im}\bar{\xi} Z \frac{I_k c^2}{B^2 \omega} \tag{15-1}$$

$$K_2 = n \int dk \ \mathrm{Im}\bar{\xi} Z \frac{e I_k c^2}{B k_\theta}, \tag{15-2}$$

$$K_3 = n \int dk \ \mathrm{Im}\bar{\xi} Z \frac{e^2 \omega}{k_\theta^2} I_k c^2, \tag{15-3}$$

$$K_4 = n \int dk \ \mathrm{Im}\bar{\xi} Z \frac{k_\parallel c^2 I_k}{\eta_c \omega k_\theta B T}, \tag{15-4}$$

$$K_5 = n \int dk \ \mathrm{Im}\bar{\xi} Z \frac{e k_\parallel c^2 I_k}{\eta_c k_\theta^2 T}, \tag{15-5}$$

$I_k = |\tilde{E}_\theta + \omega \tilde{B}_r / k_\parallel c|^2_k$ and η_c is the classical resistivity*).

Definition of Entropy production rate

Gibbs' relation . quasi-static change

$$\frac{d}{dt}U = T\frac{d}{dt}s - p\frac{d}{dt}\left(\frac{1}{\rho}\right) + \sum_k \mu_k \frac{d}{dt}\left(\frac{\rho_k}{\rho}\right) \qquad (5)$$

Entropy balance eq.

$$\rho\frac{ds}{dt} + \nabla\cdot\vec{J}_s = \left.\frac{dS}{dt}\right)_{irr} \qquad (6)$$

Irreversible Entropy production rate (i & e)

$$\left.\frac{dS_j}{dt}\right)_{irr} = \frac{1}{T_j}\{-T_j\vec{\Gamma}_j\cdot\frac{\nabla p_j}{p_j} - \vec{J}_s\cdot\nabla T_j + q_j\langle\tilde{\vec{\Gamma}}_j\cdot\tilde{\vec{E}}\rangle + \dot{P}_j - \frac{5}{2}T_jS_{pj}\} \qquad (7)$$

Wave Entropy

$$S_k = (N_k+1)\,ln(N_k+1) - N_k\,ln(N_k) \qquad S_w = \sum_k S_k\,. \qquad (9)$$

Total Entropy (local)

$$\left.\frac{dS}{dt}\right)_{irr} = \left.\frac{dS_e}{dt}\right)_{irr} + \left.\frac{dS_i}{dt}\right)_{irr} + \left.\frac{dS_w}{dt}\right)_{irr} \qquad (10)$$

Principle of Minimum Entropy Production Rate

$$\sigma \equiv \int d\vec{r} \left.\frac{dS}{dt}\right)_{irr} \quad (11)$$

Constraints

$$\nabla\cdot\vec{\Gamma}_j = S_{pj} , \quad (12)$$

$$\nabla\cdot\vec{Q}_j + 2p_j\nabla\cdot\vec{V}_j + \nabla p_j\cdot\vec{V}_j = q_j\langle\vec{\tilde{\Gamma}}_j\cdot\vec{\tilde{E}}\rangle + \dot{P}_j . \quad (13)$$

Steady state, wave energy conservation

$$\sum_j q_j\langle\tilde{\Gamma}_j\cdot\tilde{E}\rangle + \nabla\cdot(\tilde{E}\times\tilde{B}) = 0 \quad (8)$$

A model example for electrons (point)

$$L \equiv \frac{dS_e}{dt}\Big]_{irr} = -(X_1^2 + \frac{3}{2}X_2^2)K_1 + 2K_2X_1X_3 - K_3X_3^2 + (\tau \quad - \frac{5}{2})S_p. \tag{17}$$

Conditions of the particle and energy balances in a steady state are rewritten as

$$\frac{1}{R}\{-K_1X_1 + \frac{1}{2}K_1X_2 + K_2X_3\} = S_p \tag{18}$$

and

$$2K_1(X_2 - \frac{1}{R})X_1 + K_2(\frac{2}{R} - \frac{3}{2}X_2 - X_1)X_3 + K_3X_3^2 = \tau S_p, \tag{19}$$

$$\dot{P} = \tau S_p, \quad \nabla X_3 = -X_2 X_3, \quad \frac{1}{r}\frac{\partial}{\partial r}r \sim \frac{1}{R}$$

introduce Lagrange's indeterminate coefficients λ_{1_0} and λ_{2_0}, and obtain the functional $\hat{L}$ as

$$\hat{L} = -K_1(X_1^2 + \frac{3}{2}X_2^2) - K_3X_3^2 + 2K_2X_1X_3 - \lambda_1(-K_1X_1 + \frac{1}{2}K_1X_2 + K_2X_3 - RS_p)$$

$$- \lambda_2\{2K_1(X_2 - \frac{1}{R})X_1 + K_2(\frac{2}{R} - \frac{3}{2}X_2 - X_1)X_3 + K_3X_3^2 - \tau S_p\}. \tag{20}$$

Find solutions

$$\star\ \delta\hat{L}/\delta X_i = 0 \qquad (i=1,2,3) \tag{23}$$

and

$$\star\ \delta\hat{L}/\delta K_i = 0 \qquad (i=1,2) \ . \tag{24}$$

Remark Ⓐ $\circ\ \delta\hat{L}/\delta K_i$ $\circ\ \delta K_i/\delta I_k$ → $\delta\hat{L}/\delta I_k \doteq 0$

Ⓑ $\{X_1, X_2, K_1, K_2 X_3, K_3 X_3^2\}$ indep.

X_1, X_2 → (Table) determined ! x. Kr.

K_1 → dissipation rate

$$n\int dk\ \mathrm{Im}\bar{\xi}Z\ \frac{c^2 I_k}{B^2\omega R^2} = C\ S_p \tag{25}$$

⟶ inelastic collision freq. (e ↔ wave)

$n\,\mathrm{I}_m\xi Z \propto n_{r}(I_k/\omega_k \text{ or } N_k)$, $\propto \langle \tilde{n}\,\hat{E}^3\rangle$)

Wave fluctuation of Finite Amplitude is dictated by Source condition!

Solutions for various values of τ

τ	$(R\kappa, R\kappa_T)_1$	$(R\kappa, R\kappa_T)_2$	$(R\kappa, R\kappa_T)_3$	$(R\kappa, R\kappa_T)_4$
0.1		(3.0, -0.87)	(-1.7, 3.1)	
0.5	(0.43, 1.34)	(3.0, -0.87)	(-0.87, 1.8)	(0., 0.83)
1.0	(0.63, 1.67)	(2.9, -0.88)		
2.0	(0.85, 2.06)	(2.8, -0.89)	(1.3, -1.6)	(-2.3, -0.47)
3.0	(1.0, 2.35)	(2.7, -0.9)	(3.1, -4.4)	(-2.6, -0.51)

Wave Spectrum

$$K_1 = n\int dk\ \mathrm{Im}\bar{\xi}Z\ \frac{I_k c^2}{B^2\omega}$$

$$K_2 = n\int dk\ \mathrm{Im}\bar{\xi}Z\ \frac{eI_k c^2}{Bk_\theta},$$

$$K_3 = n\int dk\ \mathrm{Im}\bar{\xi}Z\ \frac{e^2\omega}{k_\theta^2} I_k c^2,$$

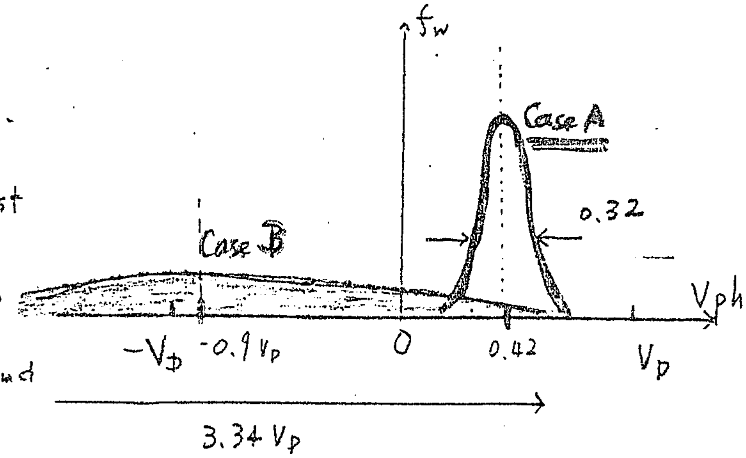
	$R\kappa$	$R\kappa_T$	K_1/R^2S_p	K_2X_3/RS_p	$K_3X_3^2/S_p$	$\Gamma . Q$	> out
Ⓐ	.85	2.06	1.67	.71	.34	$\Gamma > 0$ $Q < 0$	
Ⓑ	1.29	-1.63	-.332	.30	-.93	$\Gamma, Q, > 0$	

$$K_2X_3/K_1 = \left\langle \frac{\omega}{k_\theta} \Big/ \frac{cT}{eBR} \right\rangle$$

1st

$$K_3X_3^2/K_1 = \left\langle \left(\frac{\omega}{k_\theta} \Big/ \frac{cT}{eBR}\right)^2 \right\rangle$$

2nd



rough estimation of saturation (ES)

$$\int dk_\perp d\omega\,(k_\theta/\omega)\,I_k \sim \left(\int dk_\perp I_k\right)/\left(\overline{\omega/k_\theta}\right), \quad \mathrm{Im}\,\xi\,\tilde{z} \sim O(1)$$

$$\left|\tilde{E}_\theta + \frac{\omega}{k_\parallel c}\tilde{B}_r\right|^2 \sim C' \frac{S_p\,\kappa^2\,T^2\,N}{n\ \ \omega_* e^2} \qquad (26)$$

$$\left(\frac{e\phi}{T}\right)^2 \sim \frac{\kappa^2}{k_\perp^2}\left(\frac{C' S_p N}{n\omega_*}\right) \qquad (27)$$

Summary and Discussions.

- A method to analyze the plasma confinement is proposed.

  * Minimum entropy production rate
  < in non-equilibrium steady-state, Loss induced by fluctuations balances with external source >

  Variational ; Conditional minimum
  ↓
  particle, energy conservations.

$\left.\frac{d\dot{S}}{dt}\right)_{irr}$ min ⟶ κ, κ_T, Γ_v, Q_v, $\int I_k$ $\int V_p I_k$ $\int V_p^2 I_k$ determined simultaneously.

; plasma loss ← Source condition
; plasma profile
; saturation ← Source
; Spectrum Information $\langle V_{ph} \rangle$, $\langle V_{ph}^2 \rangle$

- Without Source

$$S_p = \tau S_p = E_{\parallel} = 0$$

$$\left.\frac{dS}{dt}\right)_{irr} \geq 0$$ for any low-freq. EM mode.

- A model example
  - electron, energy exchange with wave
  - $x_1, x_2, K_1, K_2 x_3, K_3 x_3^2$ obtained.
  - dissipation rate K_1 is balancing with S_p.
  - expected spectrum profile obtained.

- Different Input Source scheme.
  < RF, NB, Ohmic, Thermonuclear etc >
- Wave-Kinetic (Non linear, propagator ....)
  ~~within weak turbulence (o.k.)~~
- randomization, thermalization of B (mag.)

- $\delta\sigma = 0$; with constraints!

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MEASUREMENTS OF TOKAMAK EDGE FLUCTUATIONS AND TRANSPORT

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CALIFORNIA INSTITUTE OF TECHNOLOGY

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MEASUREMENTS OF TOKAMAK EDGE FLUCTUATIONS & TRANSPORT

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} CIT & { R. J. TAYLOR
UCLA

topics

- Significance & Properties of EDGE
- Particle diffusion measurements
- SPECTRUM OF EDGE FLUCTUATIONS
- $\tilde{E}_p \times B_T$ INDUCED Transport (just $\tilde{E}_p$)

[MAGNETIC FLUCTUATIONS: UCLA Articles + M. HEDEMANN Thesis, CIT]

Significance of EDGE

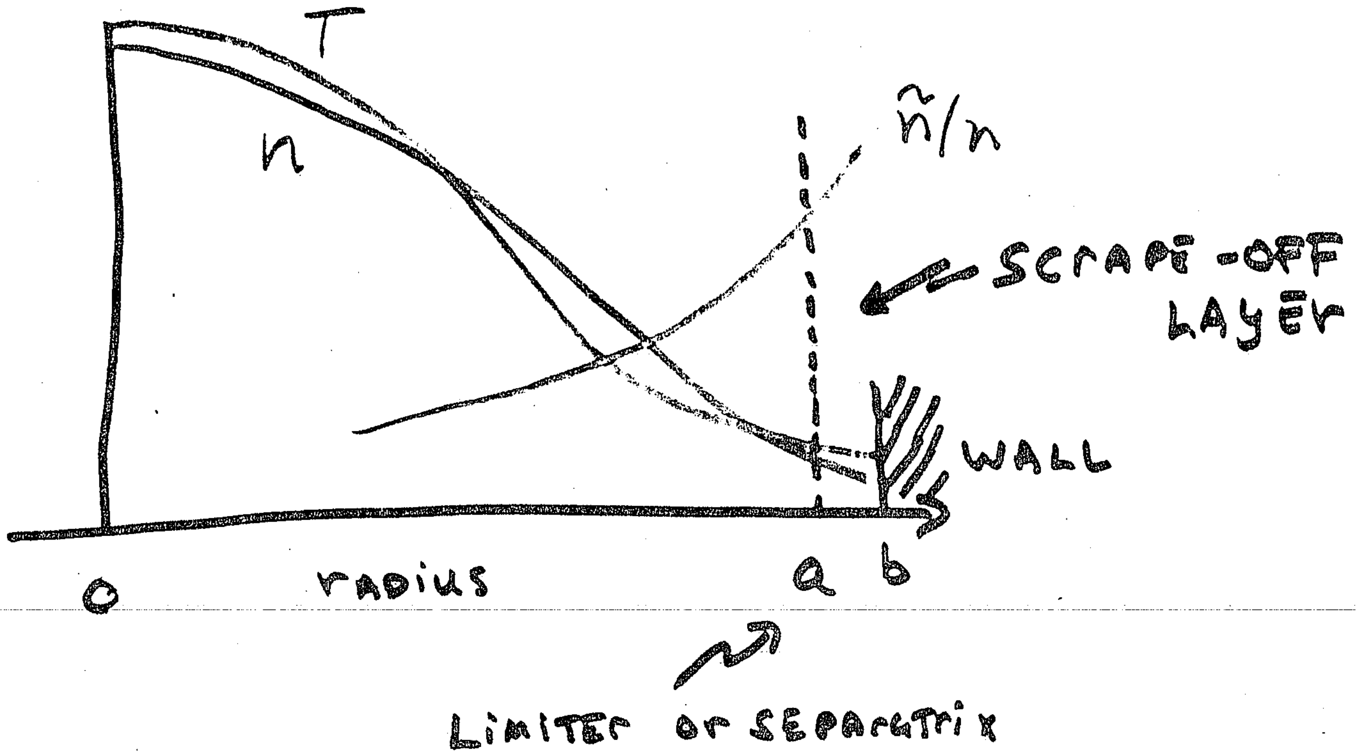
- EXAMPLE of strongly turbulent SYSTEM

- many tokamak EDGE MEASUREMENTS give ~ similar results [UCLA, CIT, ISX-B, PDX, Alcator, ATC, DITE, JFT-II, DIVA ...]

- important for reactors:
  - heat + particle removal (large D, χ good)
  - impurity release + shielding
  - divertor design
  - WAVE propagation

ALSO – relatively EASY to STUDY with probes

What is 77 "EDGE"



EDGE ~ region in contact w/ wall

- $r/a \gtrsim .8$ (?)
- $n \simeq .1\, n_0 \sim 10^{11}\,cm^{-3} - 10^{12}\,cm^{-3}$
- $T_e \simeq 5-30$ eV

How far does influence of wall (separatrix) extend inward past scrape-off layer?

[Is noise at edge heard in interior?]

4a

EDGE PROPERTIES

What makes EDGE TURBulent?

	EDGE	CIT CENTER	HOT CENTER
n	$\sim 10^{12}$	$\sim 10^{13}$	$\sim 10^{14}$
T	20	100	1000

- drift parameter $\xi \propto T/n$ not UNUSUAL
- Collisionality $\propto n/T^2$ relatively high
- density relatively low (β low also)
- possible atomic physics ($n_0 \gtrsim 10^{11}$ (?))
- relatively large ∇n (not ∇T)
- OPEN FIELD LINES (?)
- LARGE Diffusion: $D_\perp \sim D_{Bohm}$ (Alcator $\propto \frac{1}{n}$?)

(5)

PARTICLE DIFFUSION

RELATIVELY EASY TO MEASURE IN LIMITER SHADOW (SCRAPE-OFF LAYER):

- DISTANCE TO LIMITER ALONG B IS "L" ($L \sim 4\pi R$)
- FLOW SPEED $\sim C_s \sim \bar{v}_i$
- measure $n(r)$ in shadow region

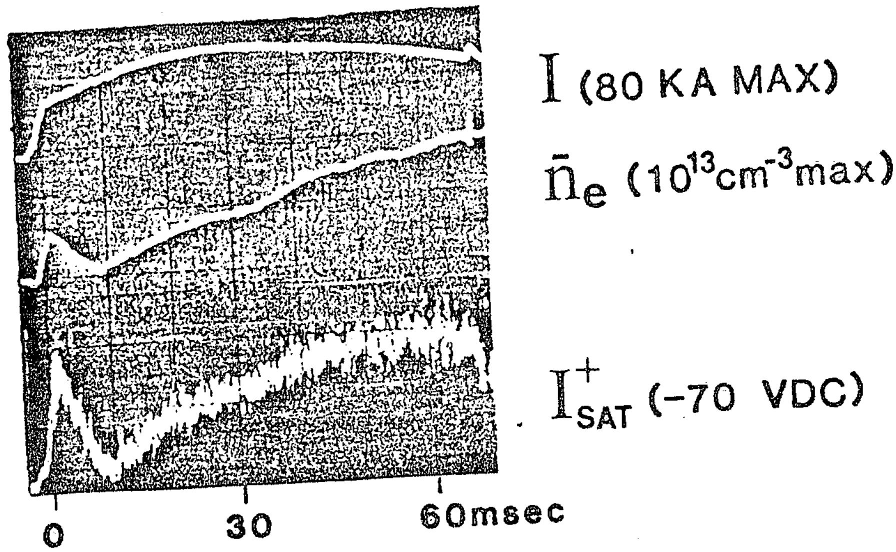
USUALLY $n(r) \propto e^{-r/\lambda_\perp}$

$$\therefore \quad D_\perp \approx \lambda_\perp^2 / (L/C_s)$$

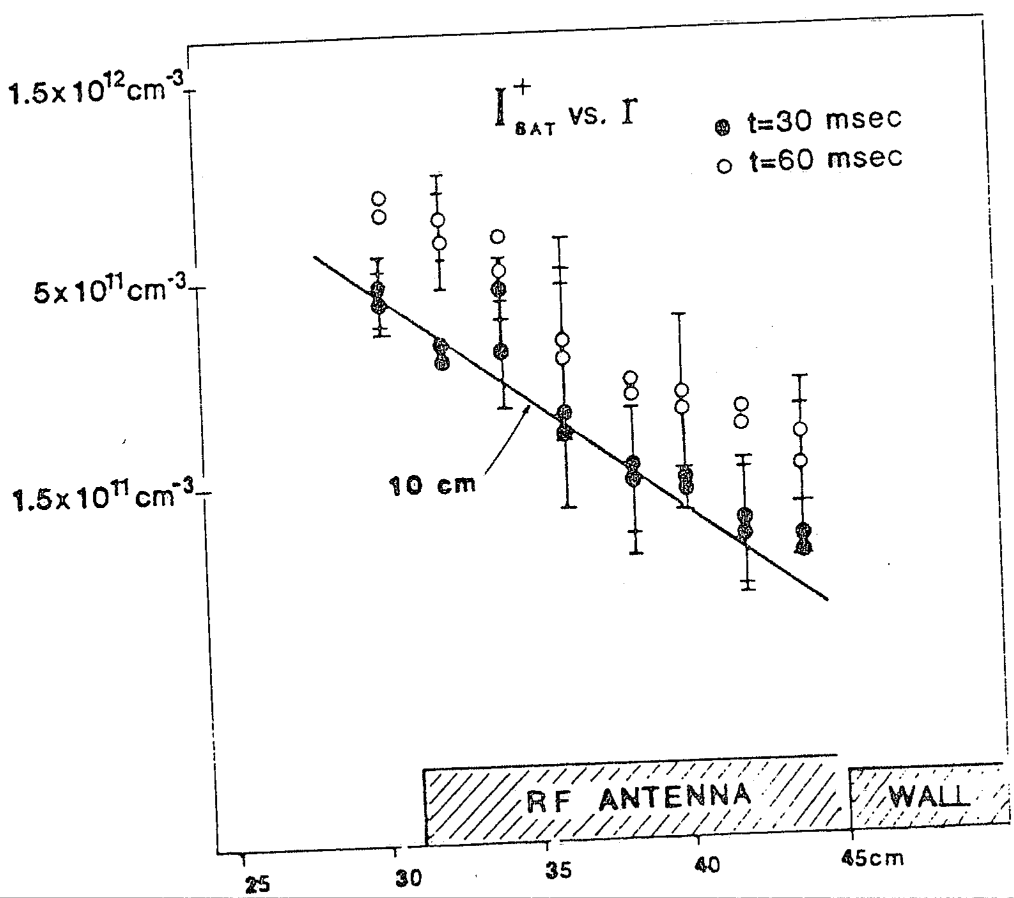
[ASSUME NO IONIZATION, recomb., reflection]

(6)

MACROTOR (2.5kg)



120° away from LIMITER



$\lambda_\perp \sim 10$cm

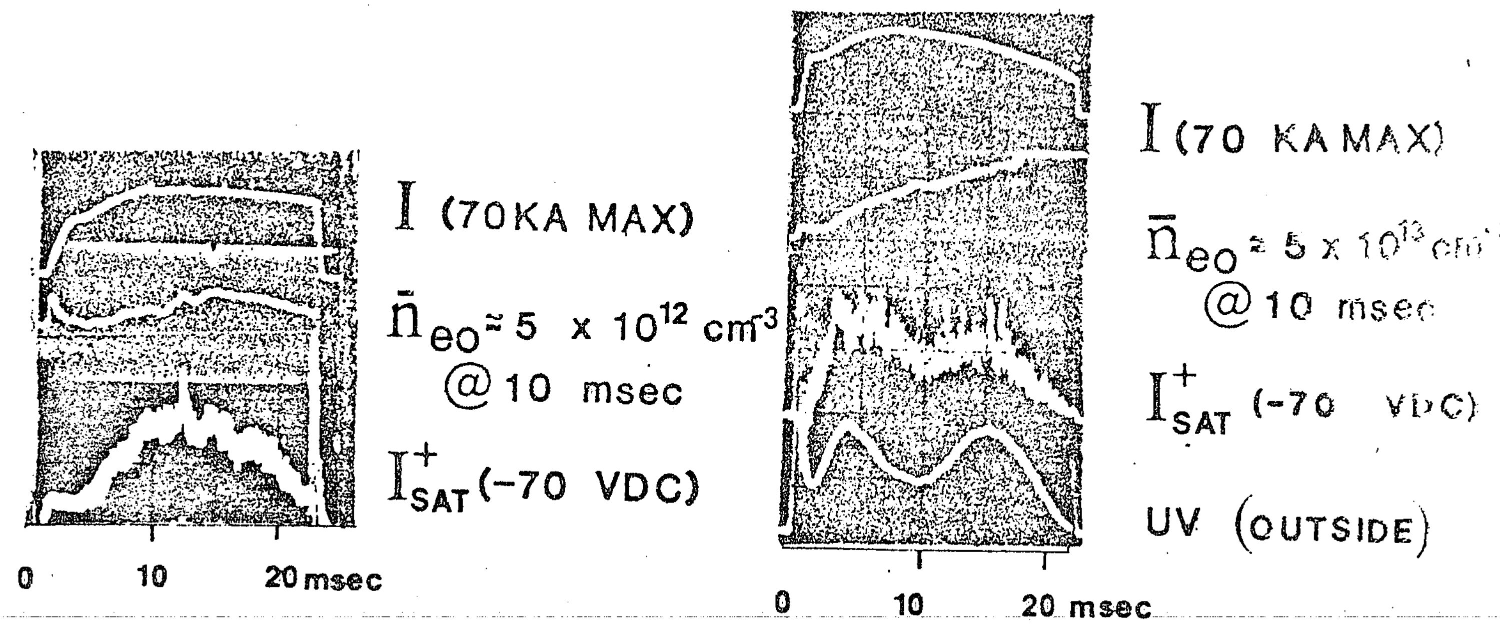
$D_\perp \sim 10^5$ cm^2/sec

$D_\perp \sim 2 D_{Bohm}$

Fig. 5b. Radial profile of I_{SAT}^+ 120° toroidally from a grounded RF antenna which acts as a rail limiter.

MICROTOR (20kg)

LOW DENSITY HIGH DENSITY



$\lambda_\perp \sim 1$ cm $\Rightarrow 5 \times 10^3$ cm^2/sec $\sim .5\, D_{Bohm}$

~ 60° away from Limiter

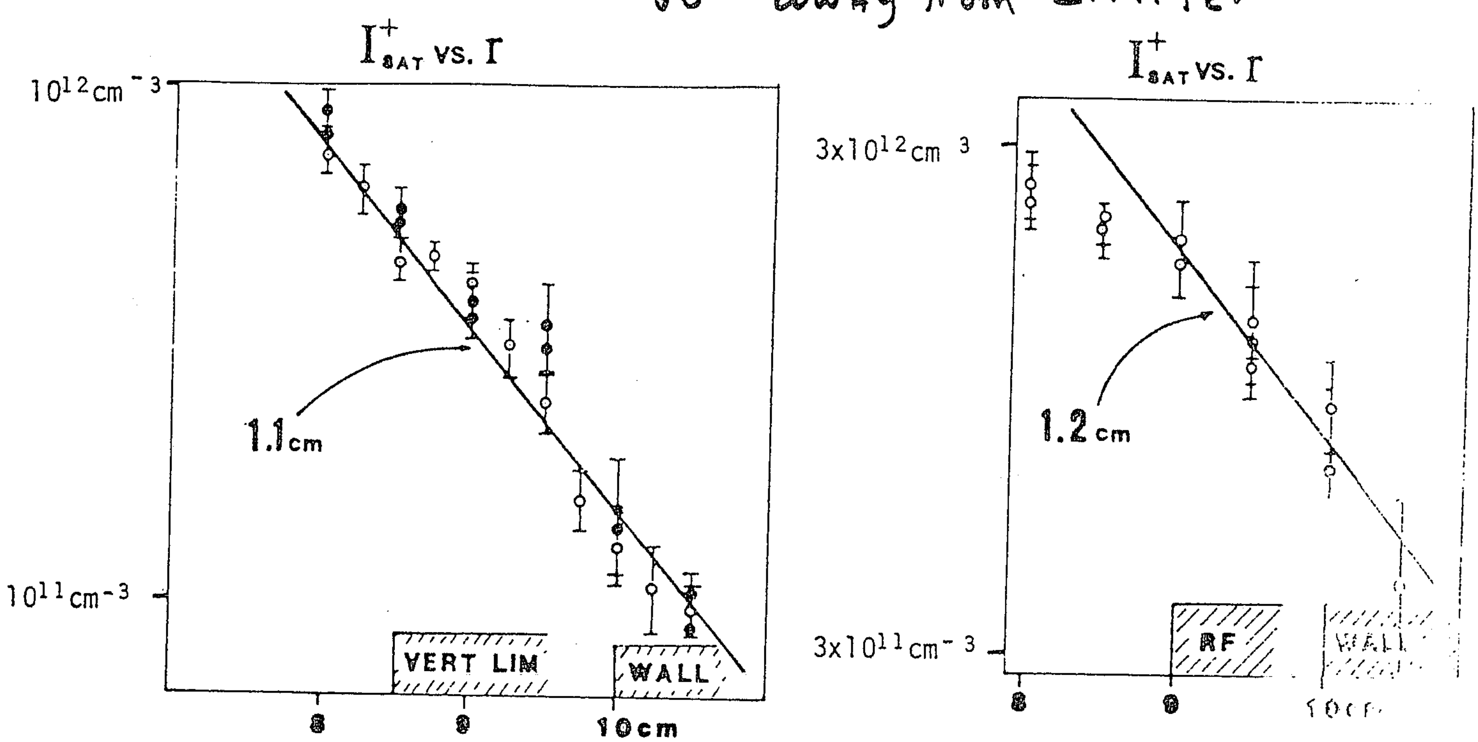
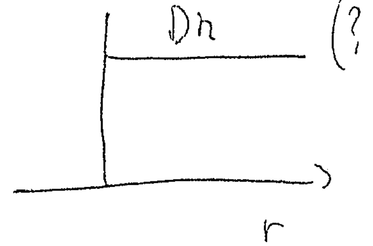


Fig. 6. Radial profiles of I_{SAT}^+ for low (left) and high (right) density Microtor discharges. The two types of points at left show the reproducibility from one day to the next. A "T" shaped langmuir probe was used for this data (the calculated n axis is only approximate).

Cause of $D_\perp$?



MAGNETIC: $\lambda_B < \left(\frac{\tilde{B}_r}{B_T}\right) L \ll \lambda_\perp$

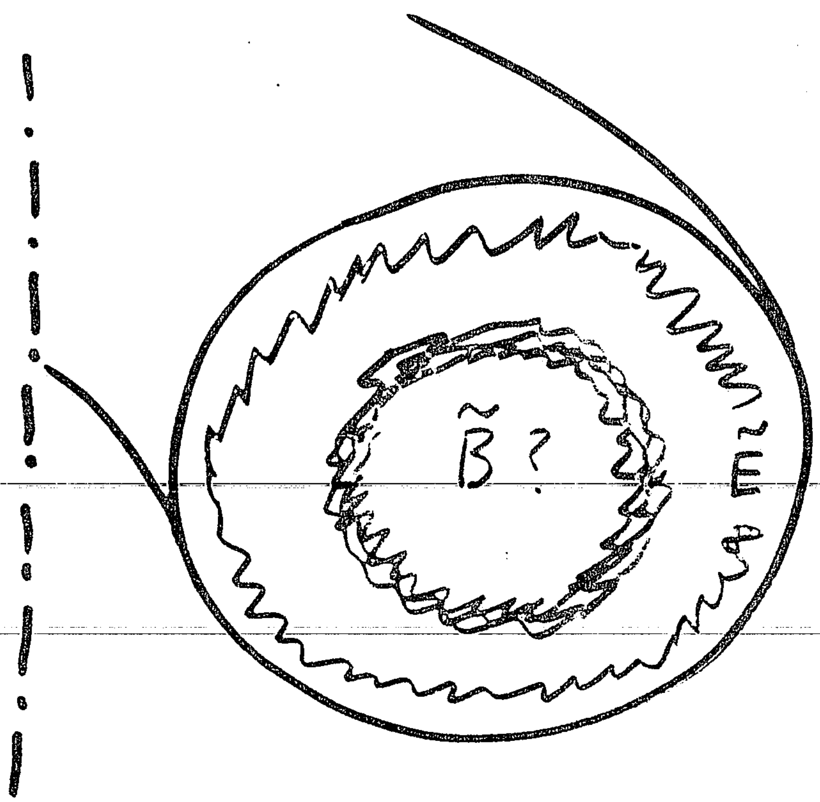
10^{-5} macrotor

probably negligible at Edge

Electric: $\tilde{n}/n \sim .1 \to .3 \Rightarrow D_{Bohm}$ (?)

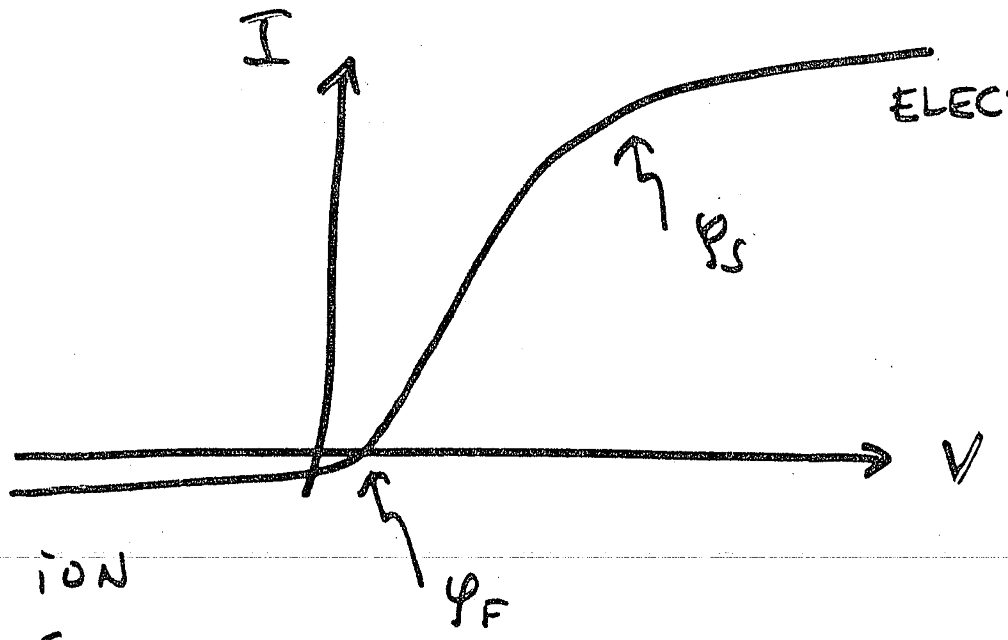
probably does cause EDGE Diffusion

possible model



(99)

FLUCTUATION MEASUREMENTS



LANGMUIR PROBE CHARACTERISTIC

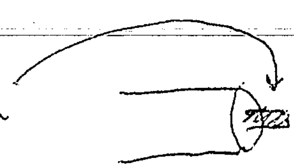
roughly:

$$\begin{cases} \varphi_f = \varphi_s - 3.6\,kT_e \\ I_i = ne\sqrt{T_e/m_i}\,A_i \\ I_e = ne\sqrt{T_e/m_e}\,A_e \end{cases}$$

($\tilde{T} = 0$)

n.b.
- possible probe PERTURBATIONS (seems ok)
- difficult to INTERPRET EXACTLY (e.g. possible $\tilde{T}$?)

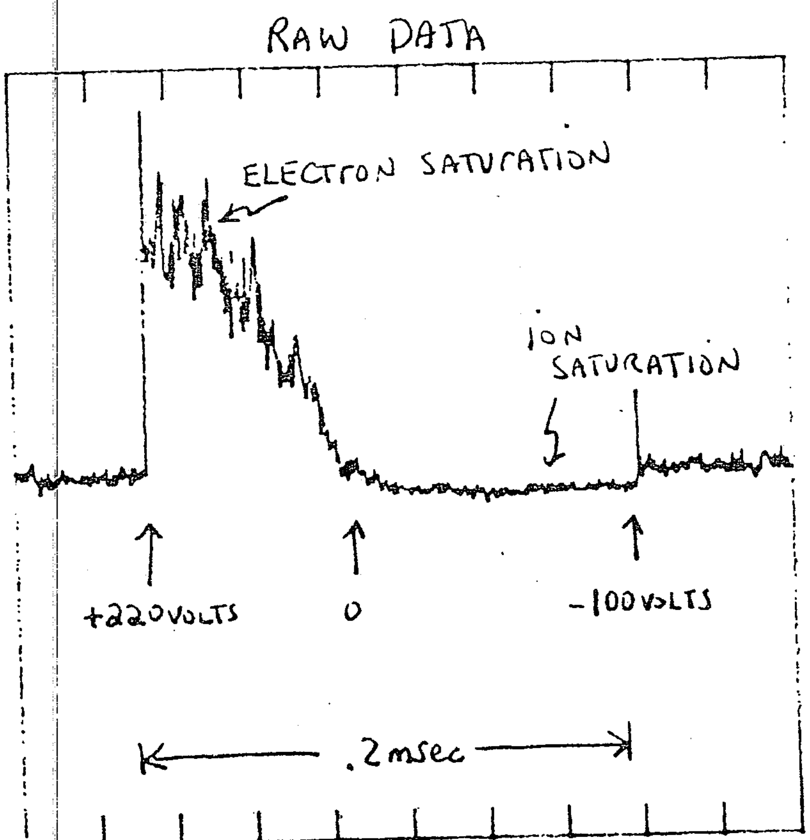
probe tip .1cm x .3cm

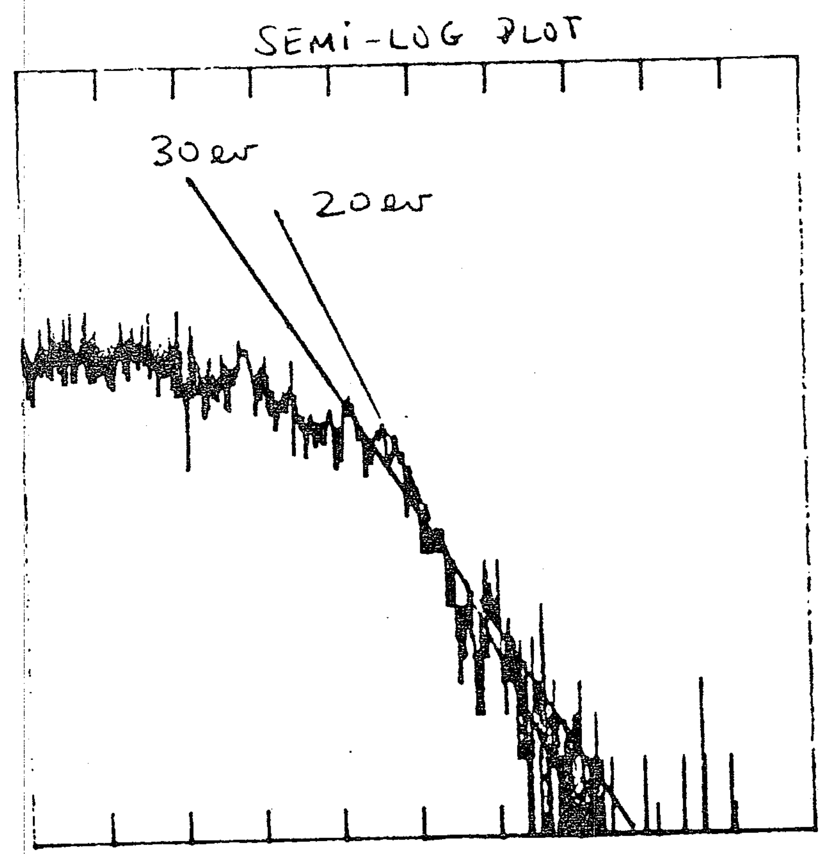


(10) CALTECH L14

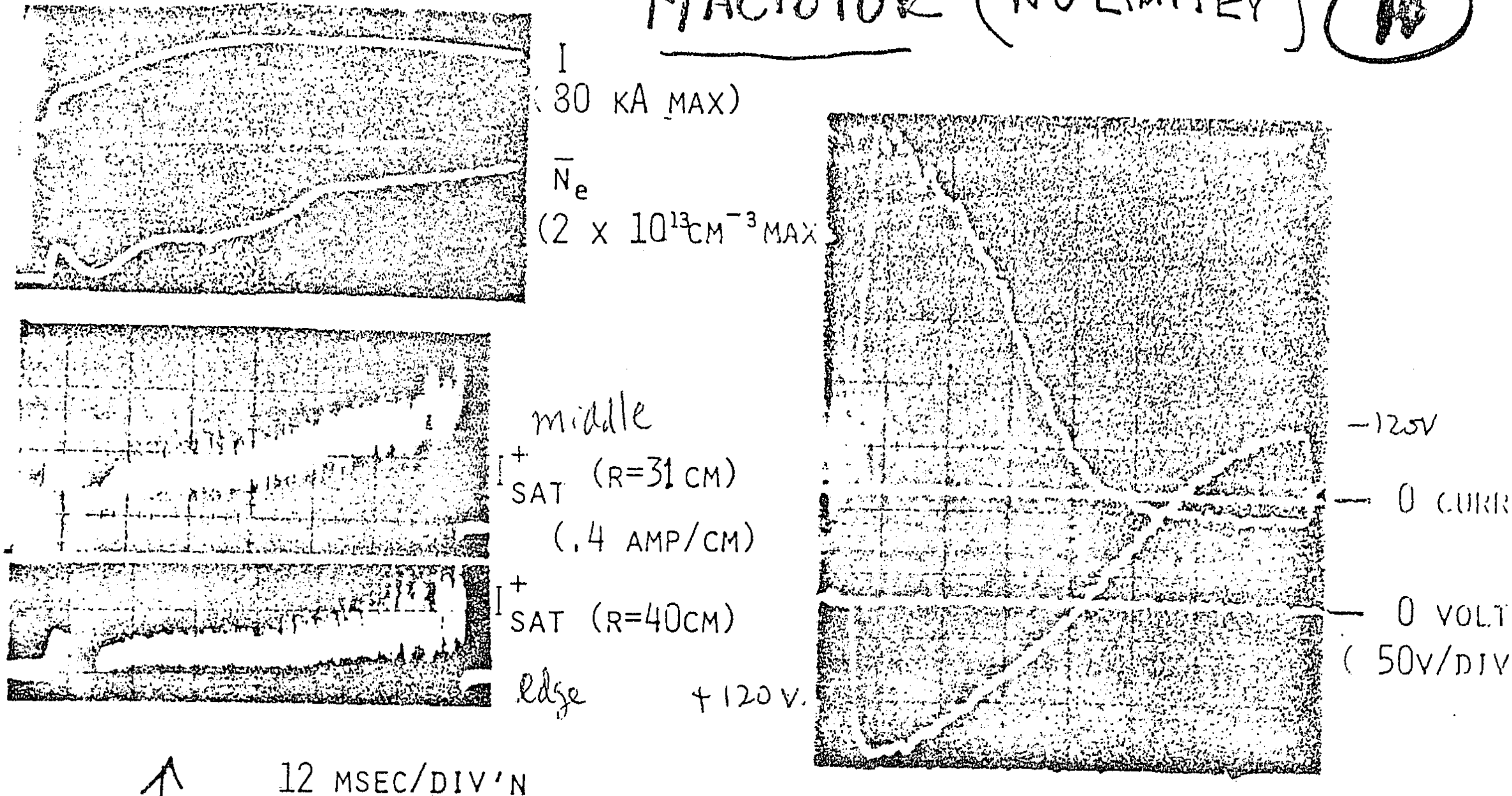
LANGMUIR PROBE SWEEP

SPACE POTENTIAL ~ +100V; $T_e \sim 25$ eV ; $e\tilde{\varphi}/kT \sim .4$





MACROTOR (NO LIMITER) 16



FLUCTUATIONS WITH NO LIMITER ≈ SAME AS THOSE IN LIMITER SHADOW.

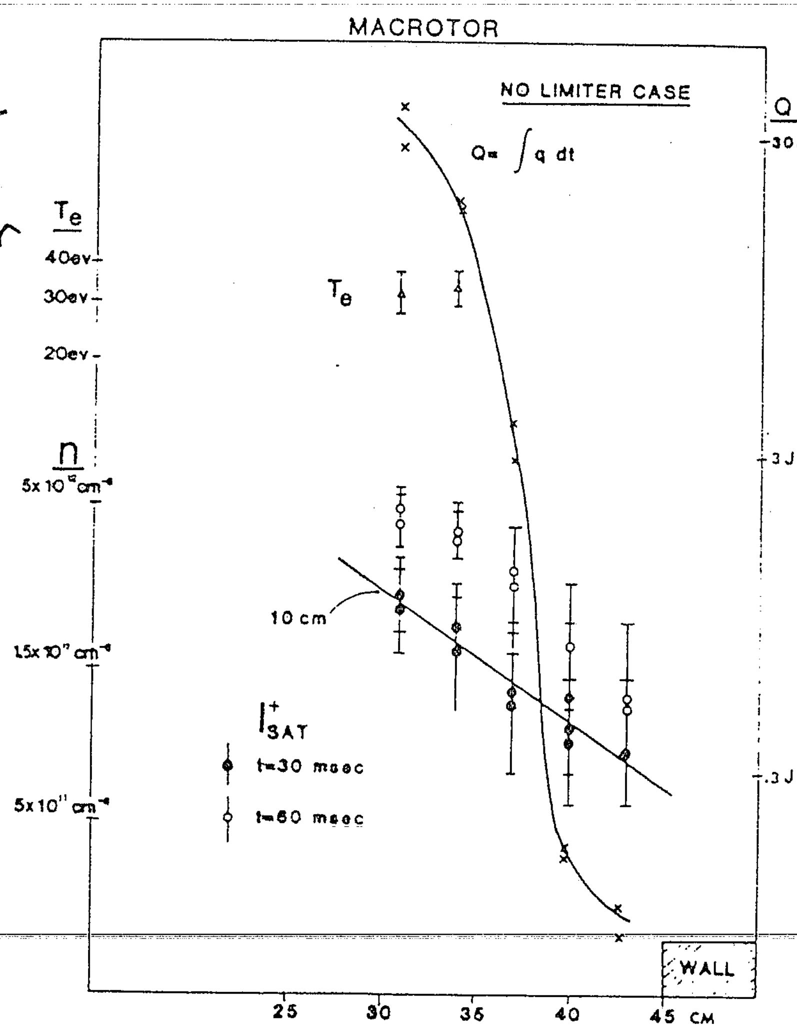


Fig. 9. $I_{SAT}{}^+$, T_e, and Q for a limiterless Macrotor discharge. The profile of $I_{SAT}{}^+$ is similar to that in the limiter cases of Figs. 4 and 5. The Langmuir probe trace at top was made at r=31 cm.

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FLUCTUATION SPECTRA

$\tilde{\varphi}_F$ IN CALTECH TOKAMAK EDGE:

1) vs. TIME during shot
2) vs. Adjacent TIMES
3) vs. radius in Edge region
4) with/without gas puff (vs. n)
5) dirty/clean (vs. Z)
6) with strong MHD mode

$\tilde{n}_e$ vs. DENSITY IN MACROTOR

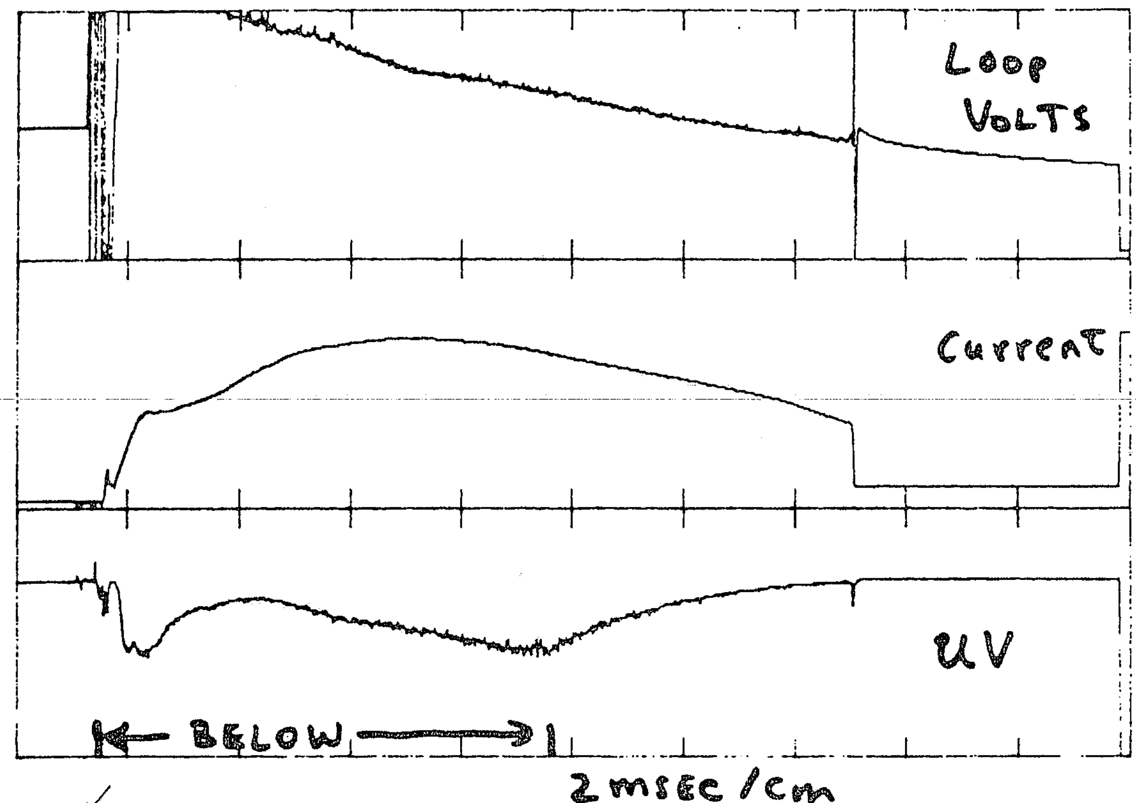
[if $\tilde{T} \cong 0$ ∴ $\tilde{\varphi}_f = \tilde{\varphi}_s$]

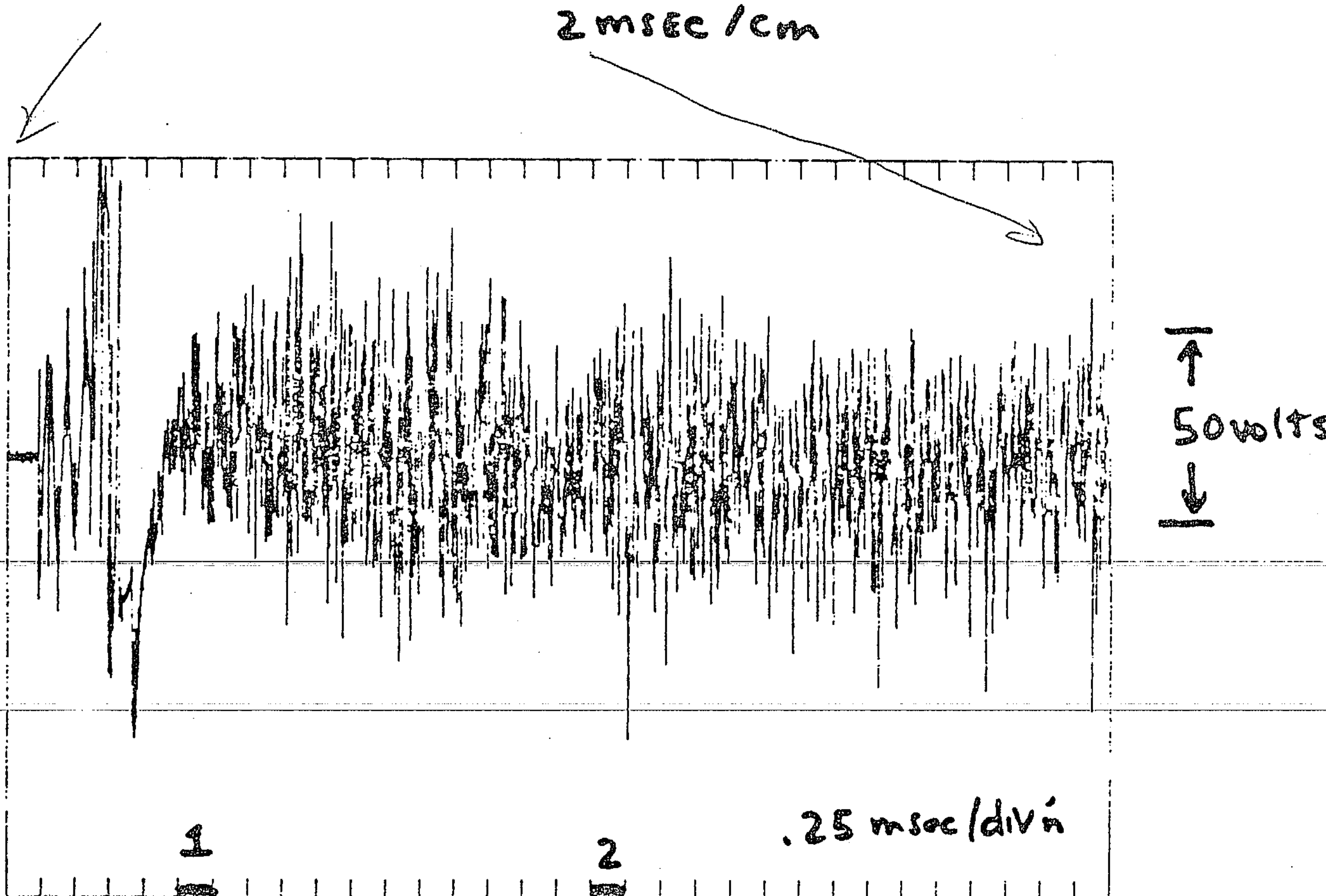
$\tilde{\varphi}_F$ vs. Time

[r/a = .9
CIT TOK]

(13)

EARLY TIMES





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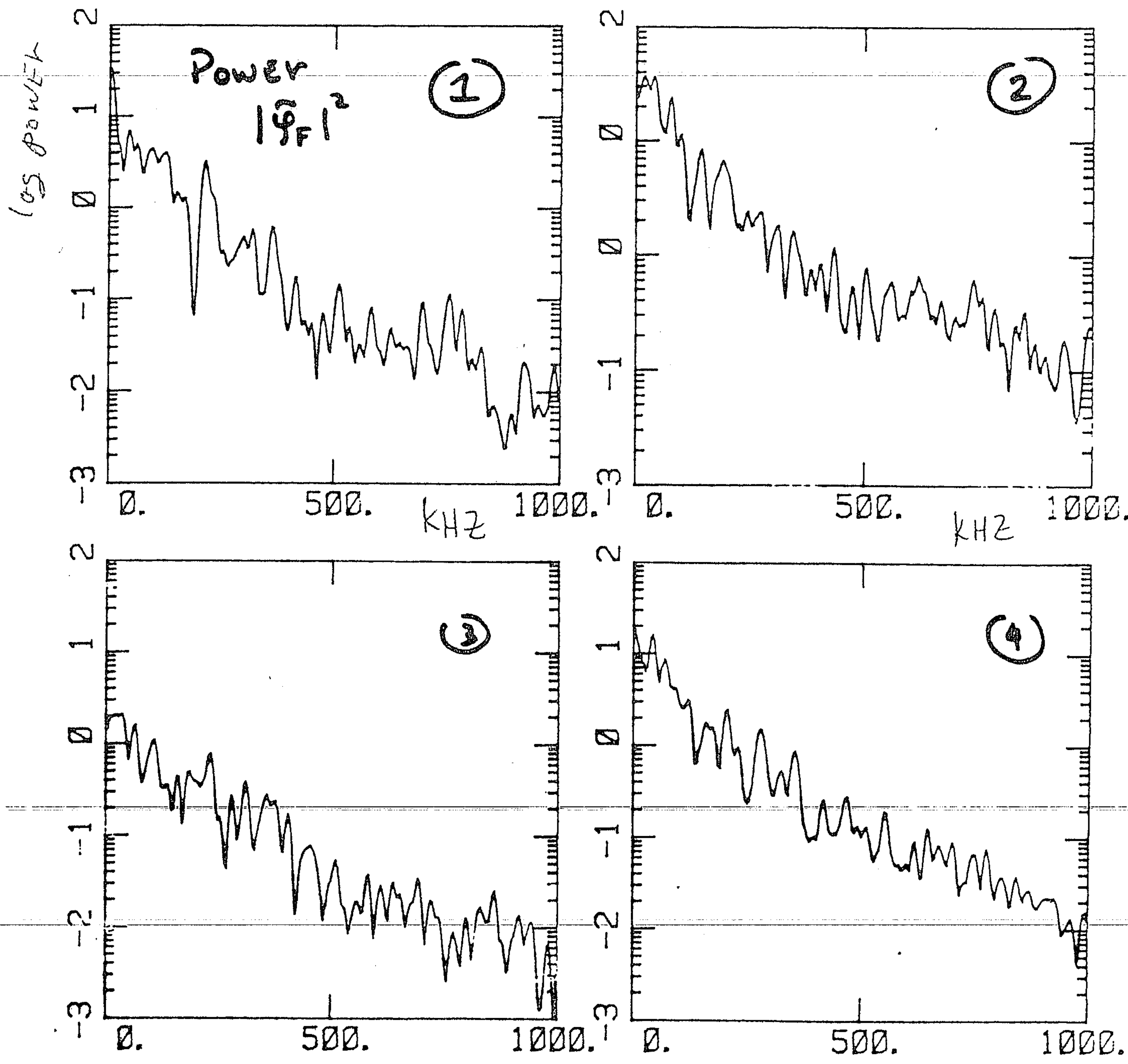
$\tilde{\varphi}_F$ SPECTRA VS. TIME

$r/a = .9$ (1.5 cm in)

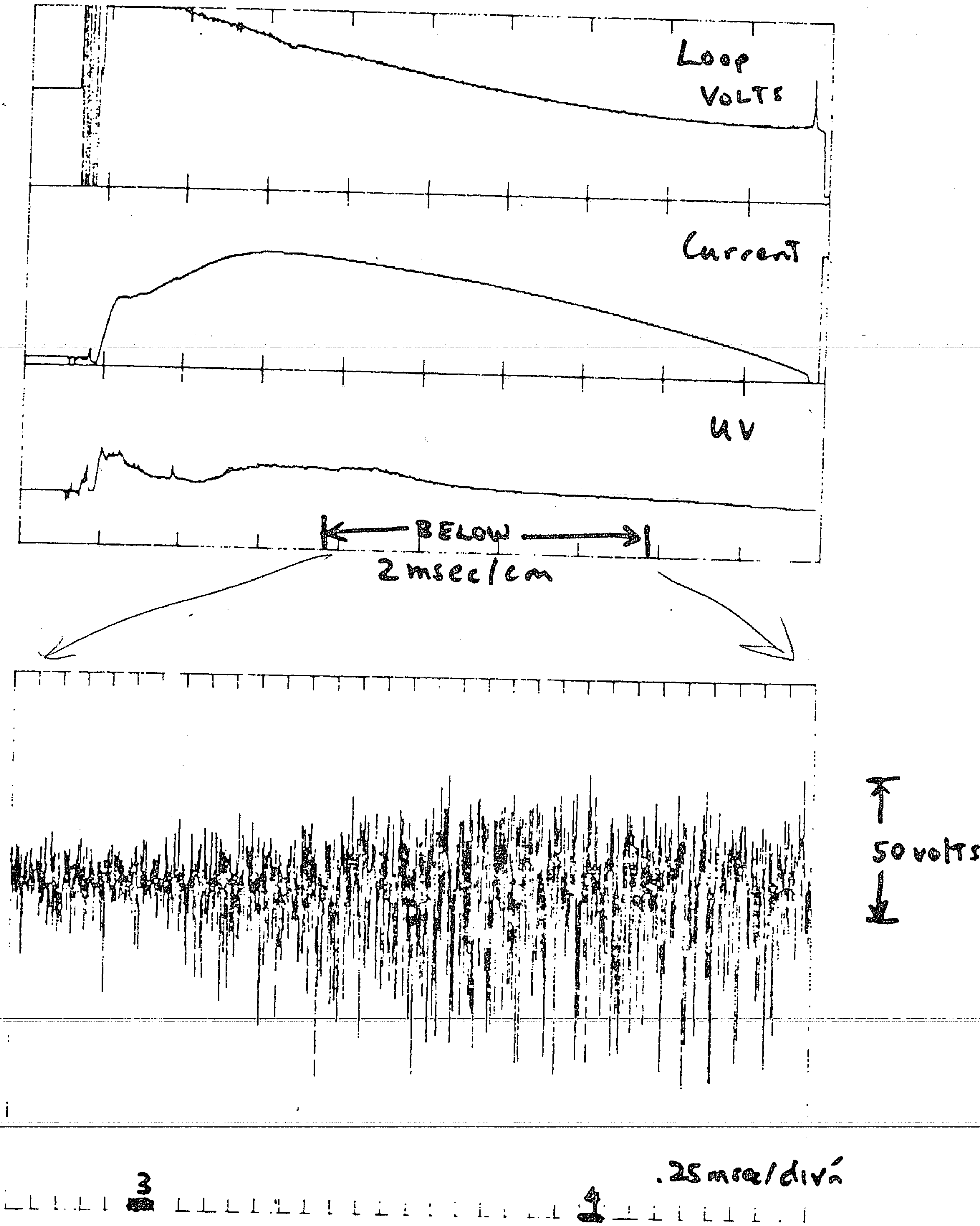
1) current rise
2) high I, low n (before puff)
3) high I, high n (puff)
4) low I, low n



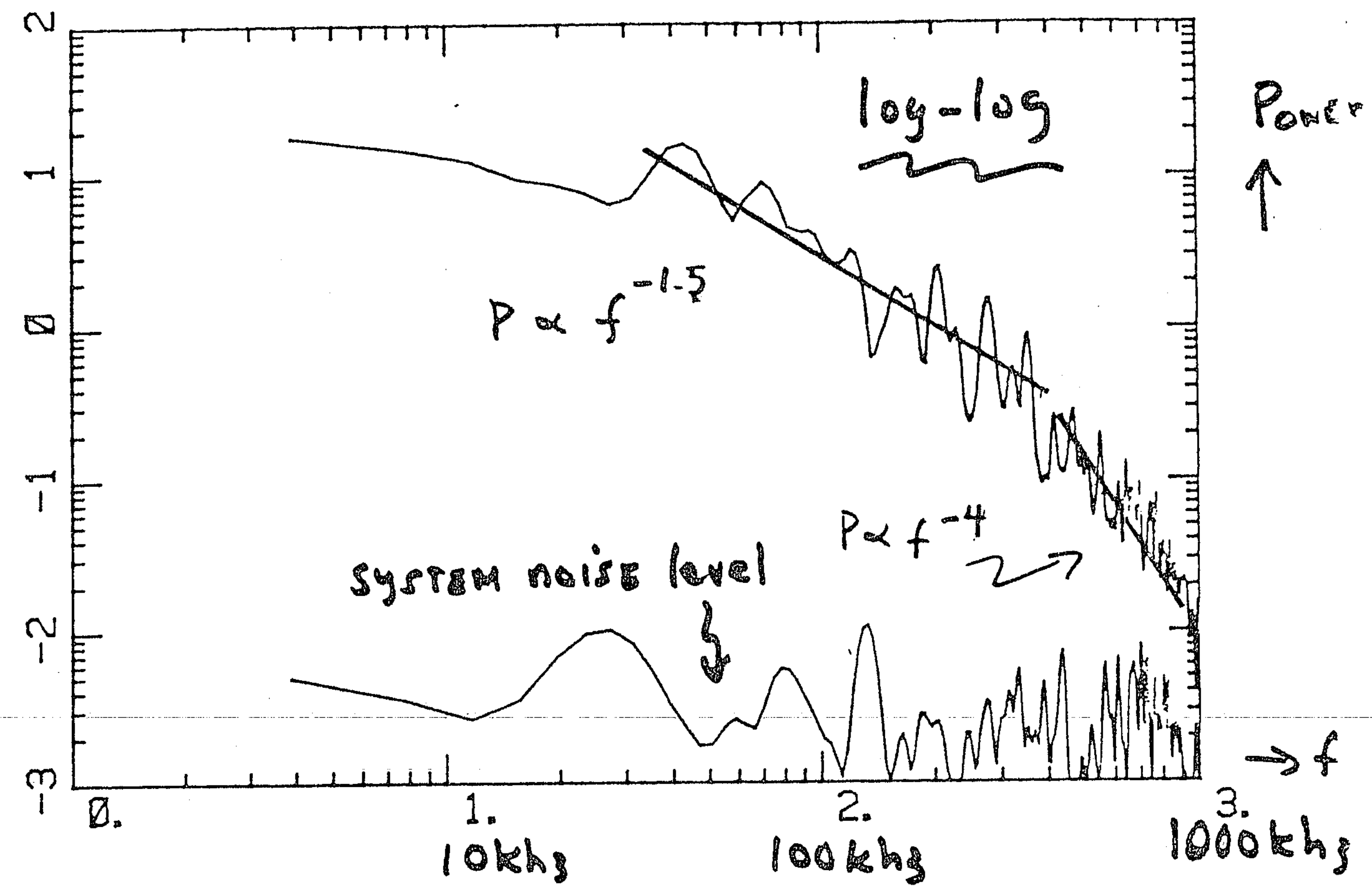
ρ_F vs. TIME

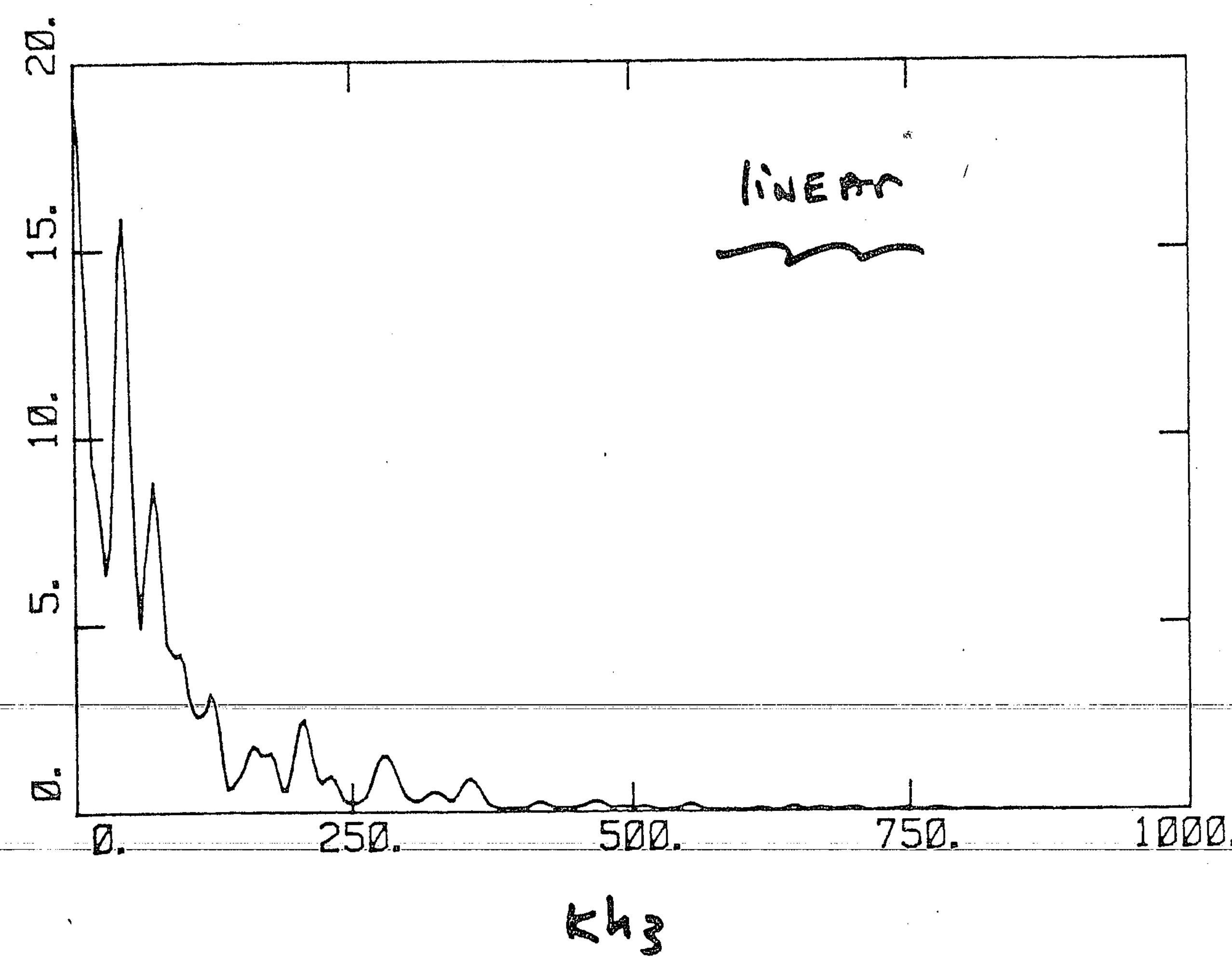
[r/a = .9
CIT TOK] (15)

LATE TIMES

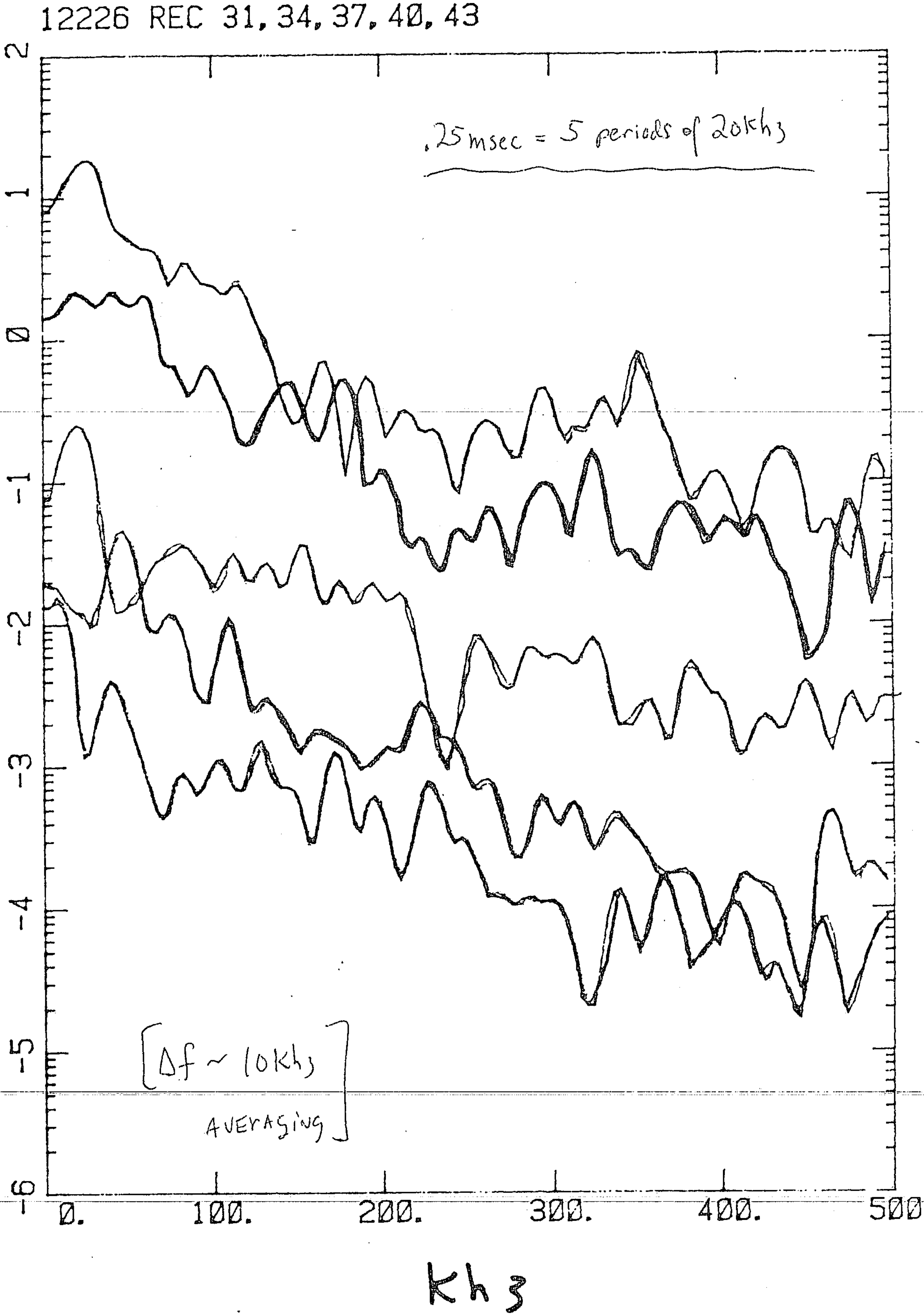


DIFFERENT 90 PRESENTATION of (4)





$|\tilde{\varphi}_F|^2$ for ADJACENT .25 msec INTERVALS (17)



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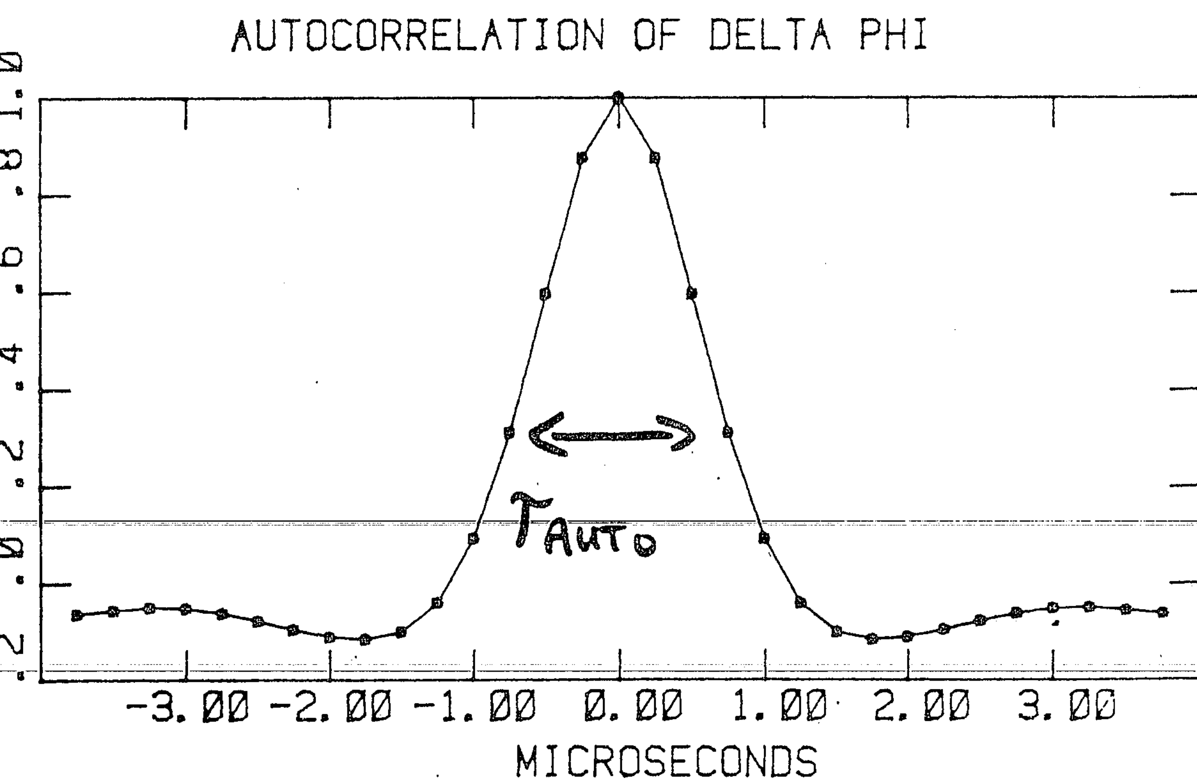
AUTOCORRELATION of $\Delta\varphi$

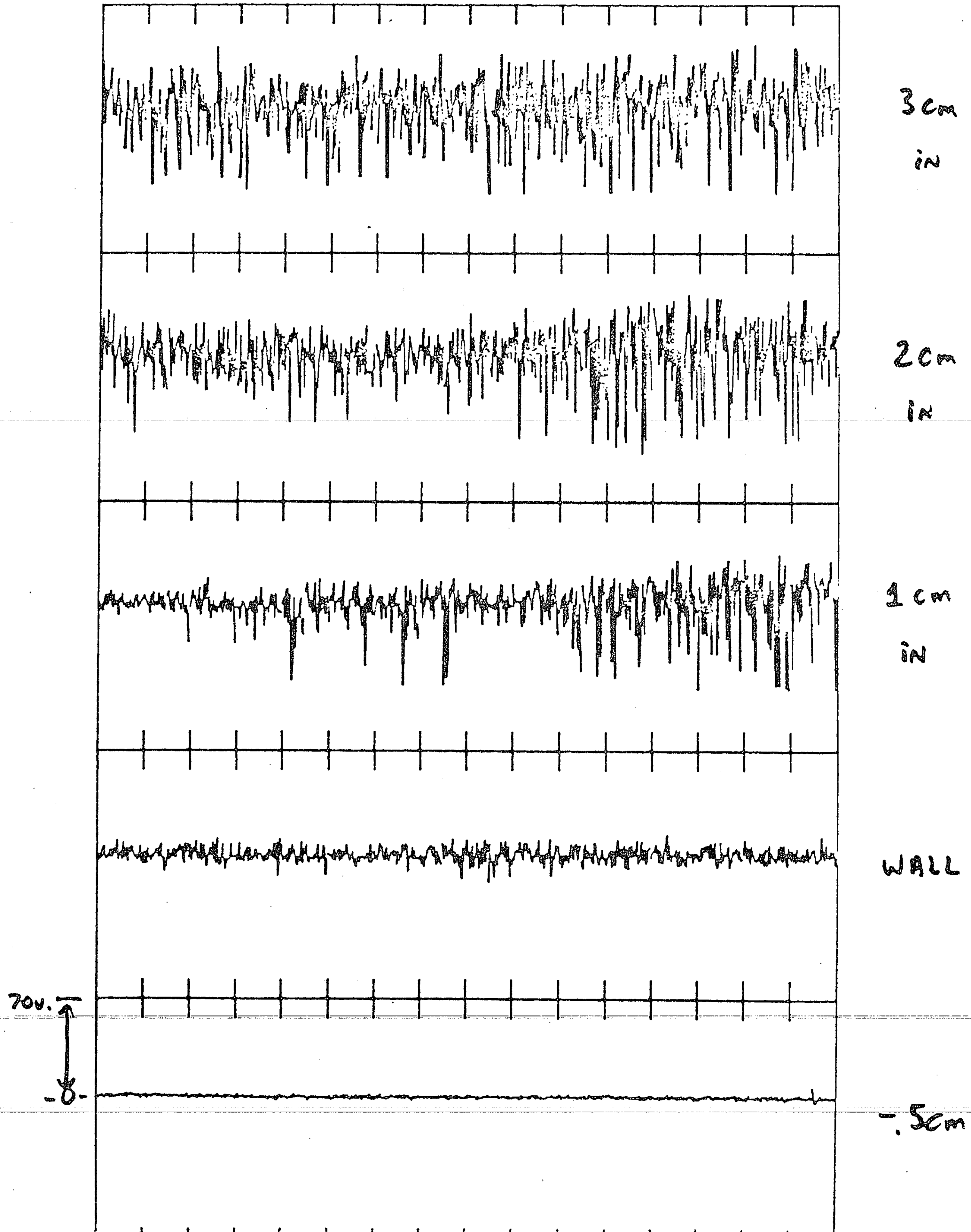
$$C(\tau) = \frac{\frac{1}{T}\int_0^T \Delta\tilde{\varphi}(t)\,\Delta\tilde{\varphi}(t-\tau)\,dt}{\frac{1}{T}\int_0^T (\Delta\varphi(t))^2\,dt}$$

HERE $\tau_{AUTO} = C(0) \approx 2\,\mu SEC$

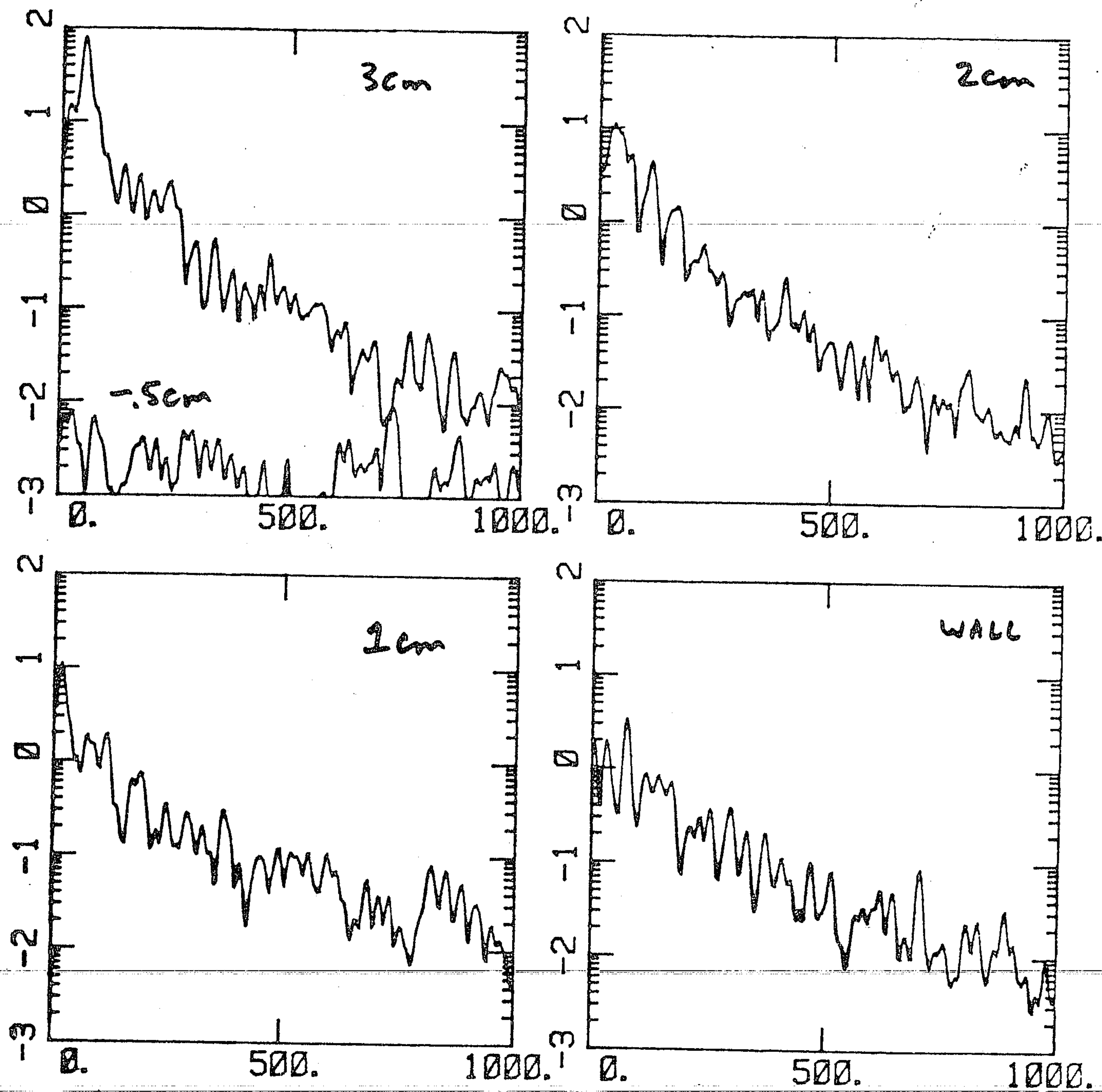


$\tilde{\varphi}_{IF}$ vs. RADIUS near Edge

(19)



$\tilde{\varphi}_F$ SPECTRA vs. r



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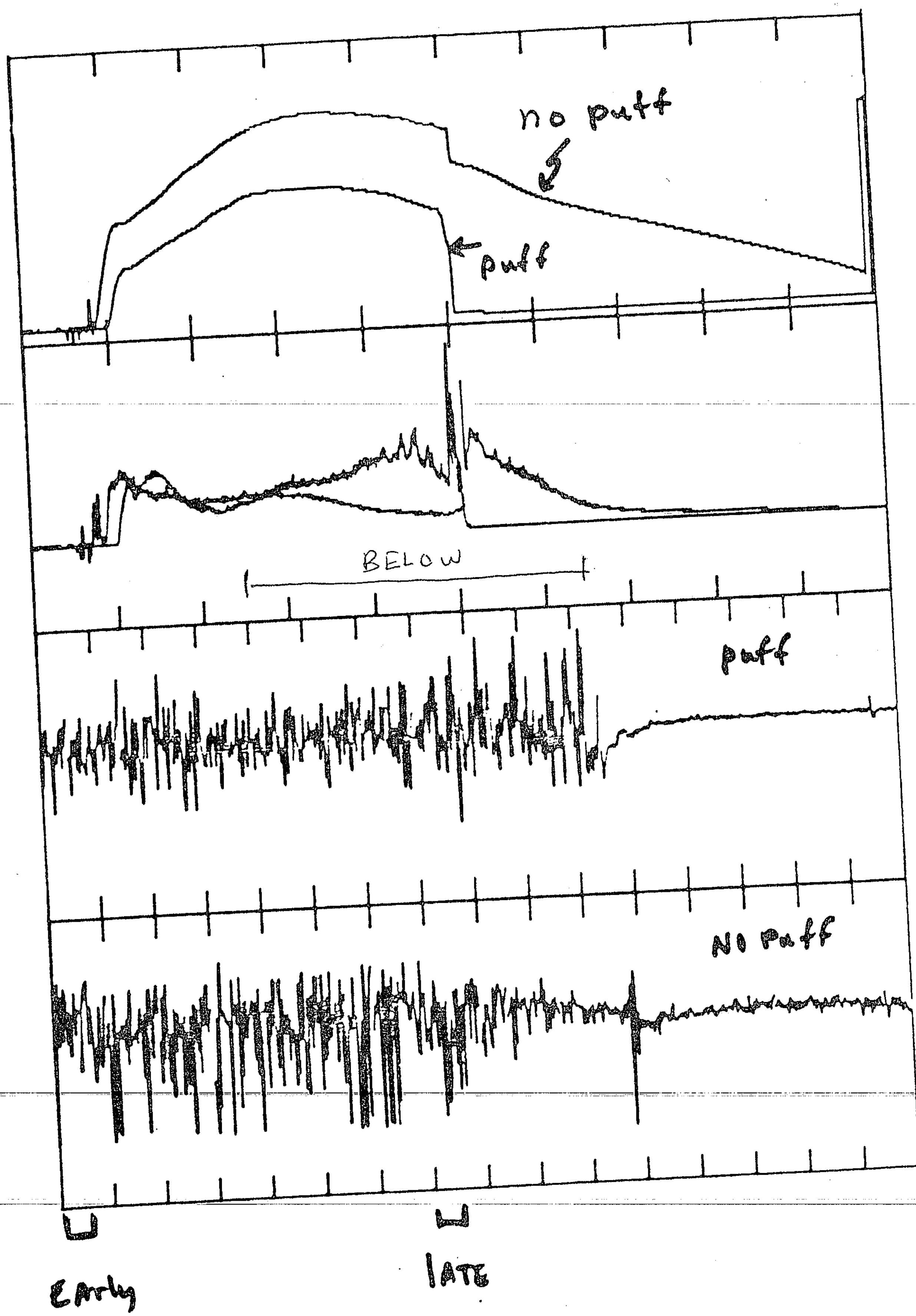
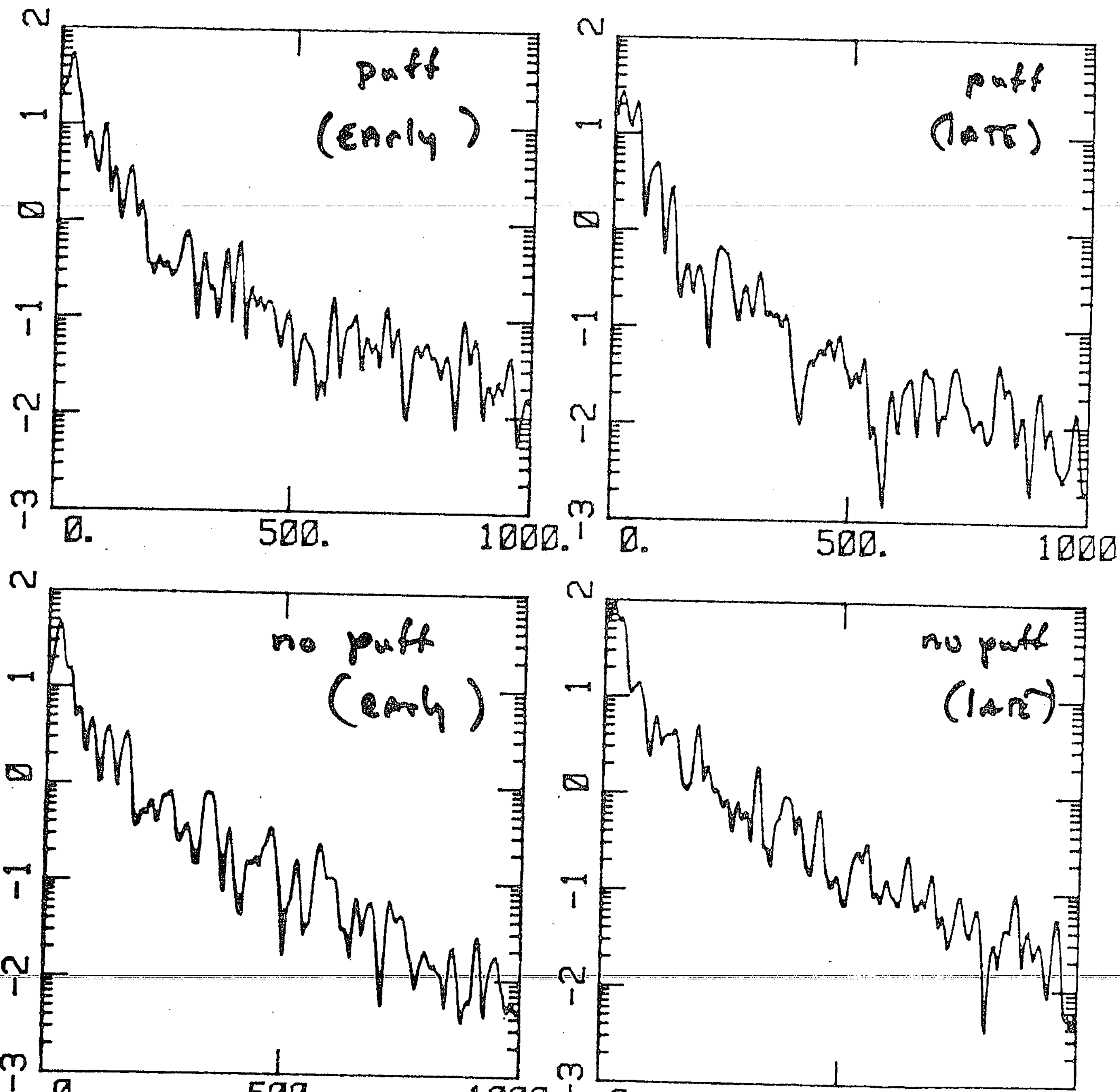
$\tilde{\varphi}_f$ w/wo GAS Puff



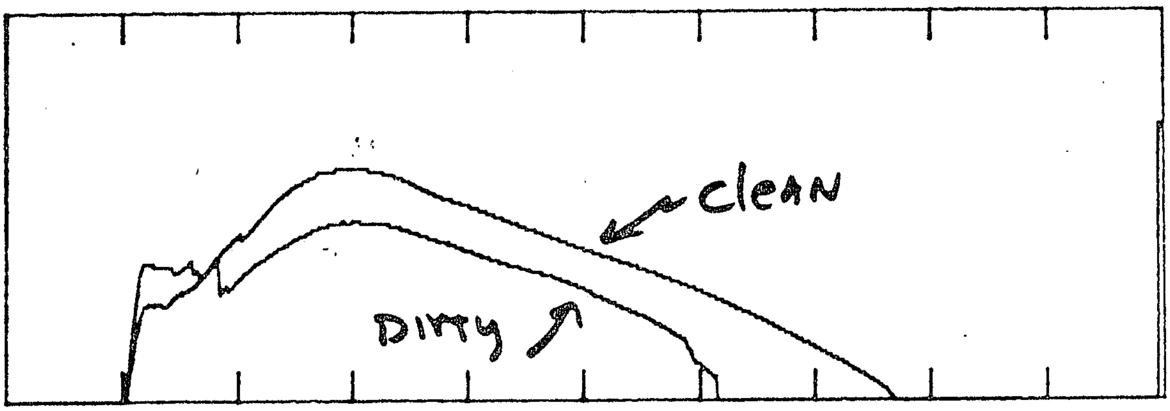
$\tilde{\varphi}_F$ Spectra w/wo Puff

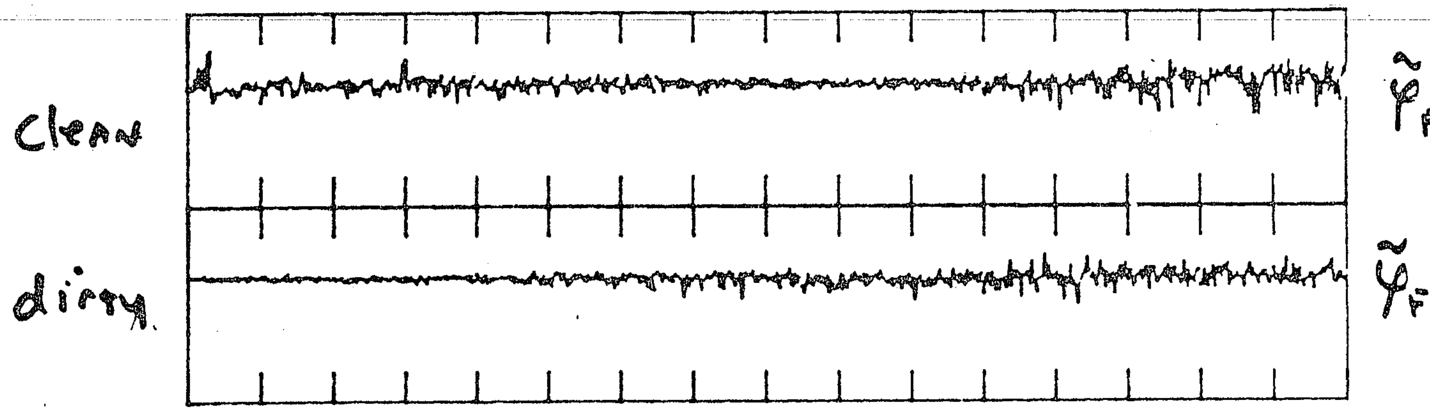
Difference in $\bar{n}_\sim$ x5

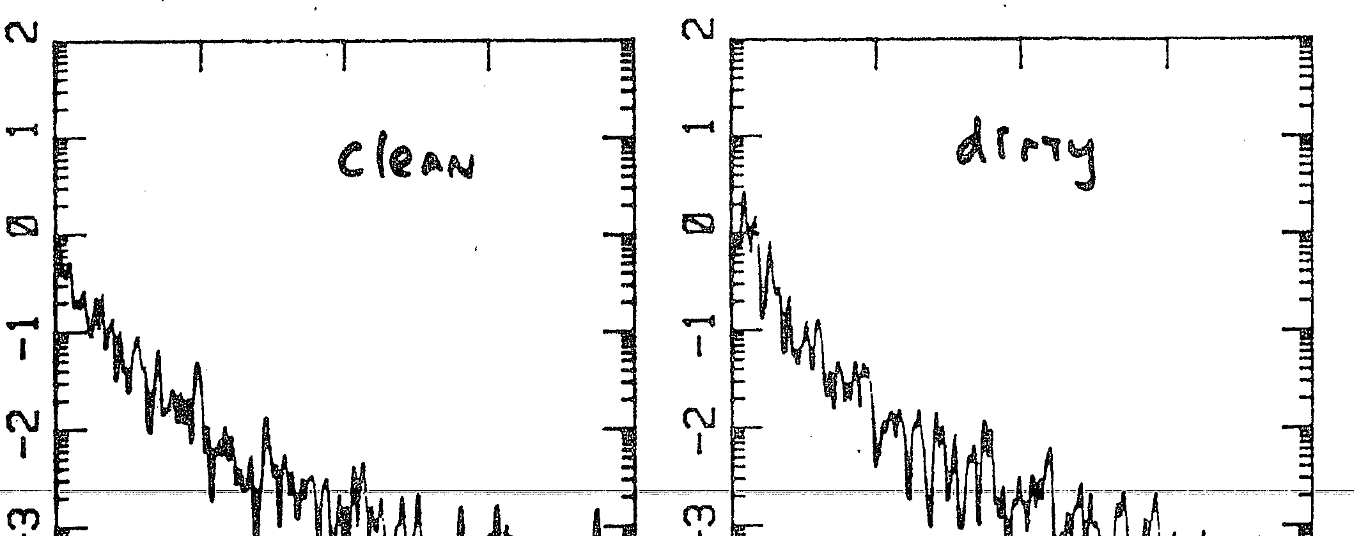


$\tilde{\varphi}_F$ dirty/clean

(23)



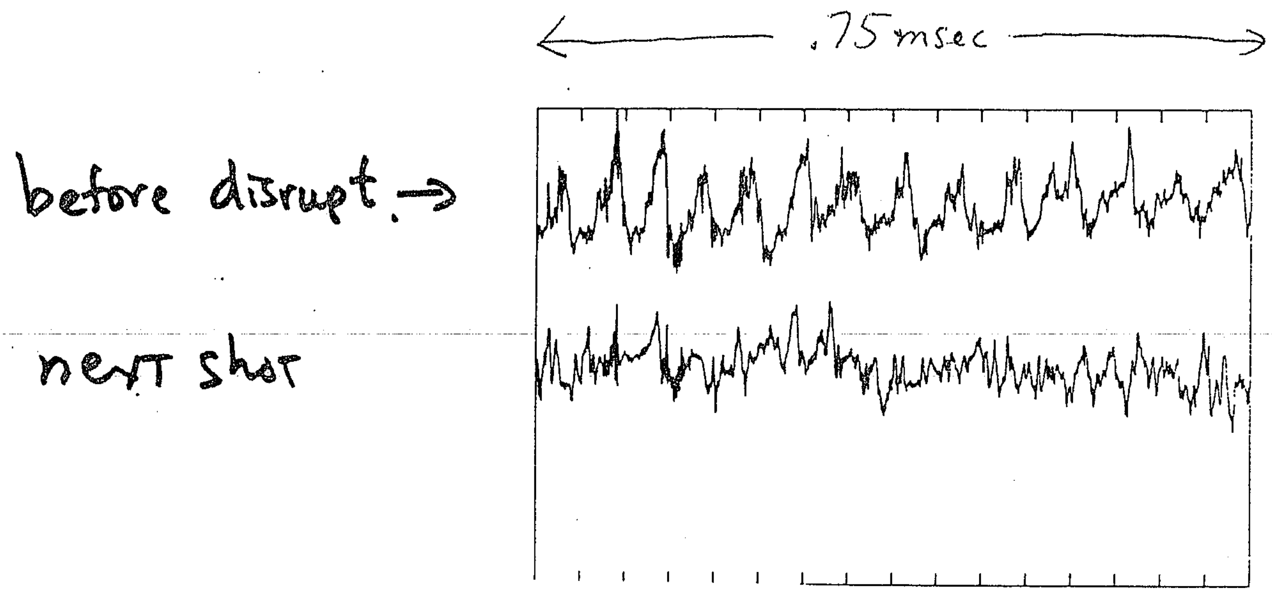


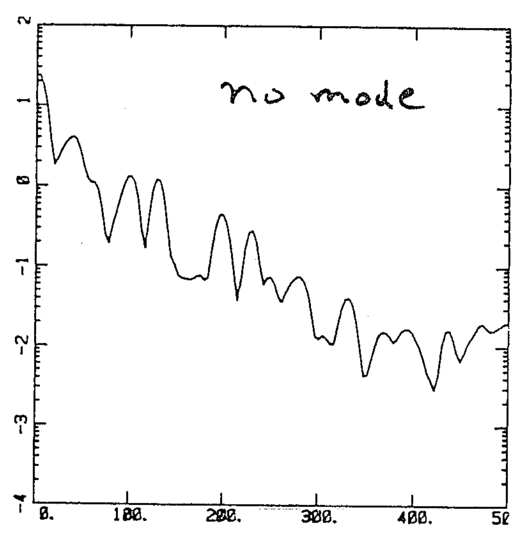


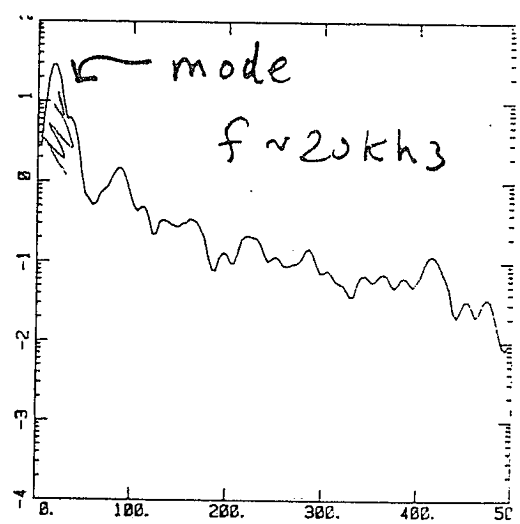
(24)

COHERENT $\tilde{\varphi}_f$ w/ MHD MODE

ONLY CASE of single mode







(25)

MACROTOR

UCLA report
PPG - ~~548~~ 463

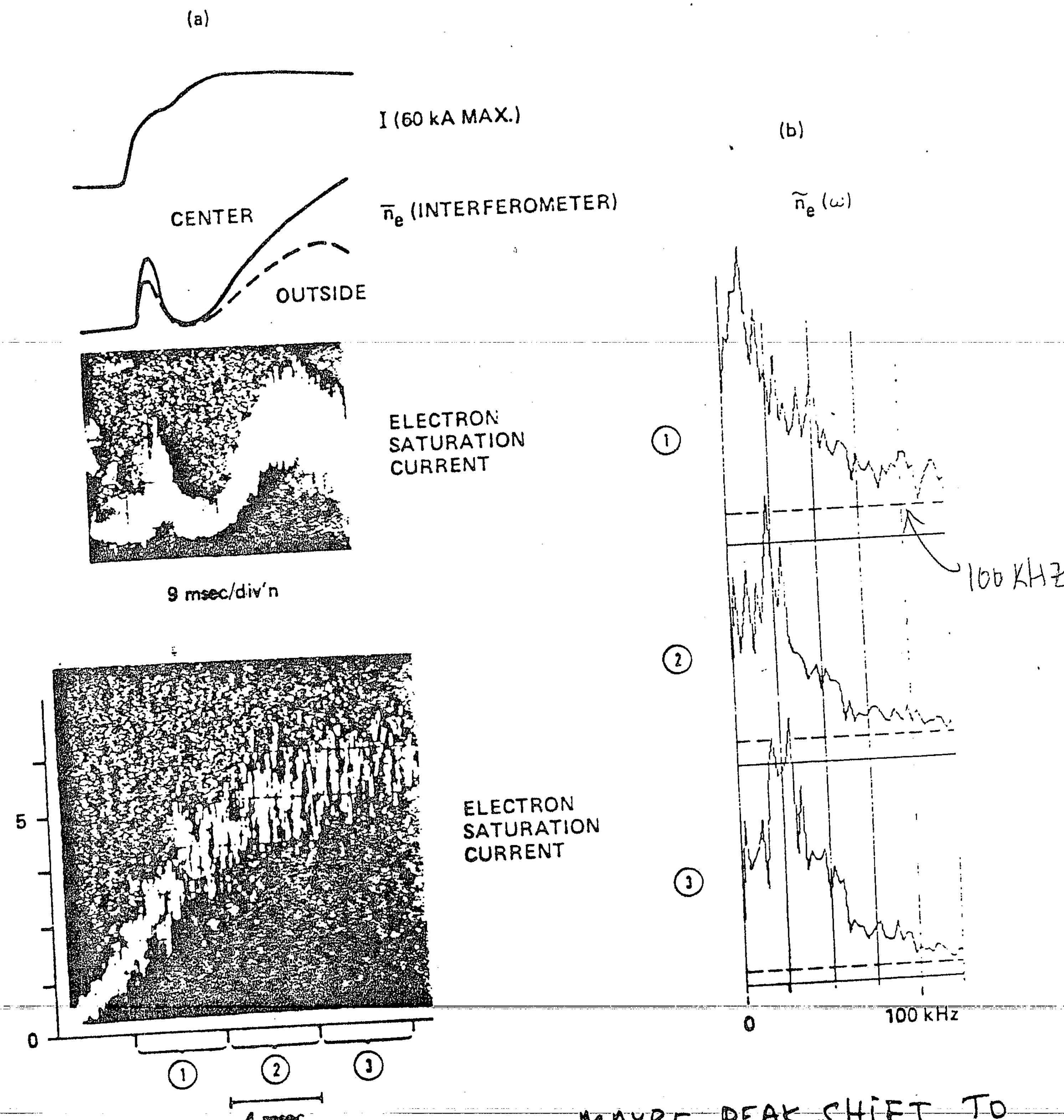


FIGURE 11.

MAYBE PEAK SHIFT TO Higher freq. w/ n

Double-Probe Measurements

- $(\tilde{\varphi}, \tilde{\varphi})$, $(\tilde{n}, \tilde{n})$ correlation lengths (UCLA)
- $\tilde{E}_p = \Delta\tilde{\varphi}_f / \Delta X_{pol}$ (assuming $\tilde{T}/T \ll \tilde{\varphi}_s/\varphi_s$)
- better $\tilde{n}$ from floating double-probe
- $(\tilde{n}, \tilde{\varphi})$ Transport (?) $\tilde{E}_r \sim \tilde{E}_p$

Triple-Probe

$(\tilde{E}_p, \tilde{n}) \rightarrow$ radial flux $\Gamma_r = \langle \tilde{n}, \tilde{v}_r \rangle$

assuming $\vec{E} \times B$ drifts