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Abstract

It is shown that ion transport in the banana regime in tokamaks is reduced in the presence of a strong shear in the radial electric field E_r, as is often observed in the edge region. For simplicity, the ordering with $\rho_{pi}|d \ln E_r/dr| \gg 1$ but $c|E_r|/B_p v_{ti} < 1$ is adopted. Here, ρ_{pi} is the ion poloidal gyroradius, B_p is the poloidal magnetic field strength, v_{ti} is the ion thermal speed, and c is the speed of light. A kinetic transport theory similar to those for bumpy tori and stellarators is developed to show that the ion thermal conductivity χ_i is reduced by a factor of roughly $S^{-3/2}$, where $S = 1 - (\rho_{pi} d \ln E_r/dr)(cE_r/B_p v_{ti})$. The result reflects more than simple orbit squeezing: the fraction of trapped ions is also modified by S.

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I. Introduction

In the edge region of a tokamak, especially during H-mode [1] operation, the values of the radial electric field E_r and its radial gradient dE_r/dr are larger than those allowed by the standard neoclassical theory [2, 3, 4]. The results of the standard neoclassical theory are therefore not applicable. For example, when $M_p = |cE_r/B_p v_{ti}| \sim 1$, it is possible, on theoretical grounds, to have a shock [5, 6, 7, 8, 9]. Here, c is the speed of light, B_p is the poloidal magnetic field strength, and v_{ti} is the ion thermal speed. It also has been demonstrated both experimentally and theoretically that the nonlinearity of plasma viscosity in M_p becomes important when $M_p \simeq 1$ [10]. Furthermore, it is well known that when $S = 1 - \rho_{pi} M_p \, dE_r/|E_r| dr > 1$, the size of the ion particle orbit also deviates from the standard neoclassical theory [11, 12]. It is therefore necessary to reexamine the relevant ion transport theory to understand edge plasma transport phenomena.

Several effects on ion transport can be expected for plasmas with $M_p \sim 1$ and $|S| > 1$. In the case of $0 < M_p < 1$, the variations of plasma density and temperature in a magnetic surface associated with poloidal rotation enhance the ion transport. However, when $1 < M_p < B/B_p$, ion transport will be improved mainly because of the dramatic decrease in plasma viscosity [13]. For plasmas with $|S| > 1$, a reduction of ion transport from squeezing of the ion orbits is expected.

In this paper, we intend to demonstrate the effect of strong shear in E_r on ion transport. To isolate this effect, we adopt the ordering that $|S| > 1$ but $M_p < 1$, so that we can neglect the variations of plasma density and temperature in the magnetic surface associated with poloidal rotation. We find that the ion thermal conductivity χ_i is reduced by a factor of $|S|^{-3/2}$. The result can be understood from the random walk argument together with the decrease in the size of the trapped ion orbits and the increase in the number of trapped ions by a factor of $\sqrt{|S|}$.

The paper is organized as follows. In Sec. II, we derive the linearized drift kinetic equa-

tion. The variation of the poloidal angular velocity needed in solving the linearized drift kinetic equation is calculated in Sec. III. In Sec. IV, we solve the linearized drift kinetic equation to obtain the particle distribution function. The ion thermal conductivity is derived in Sec. V. The procedures employed in Secs. III-V are very similar to those in the kinetic theory of bumpy tori [14]. Concluding remarks are given in Sec. VI.

II. Linearized Drift Kinetic Equation

Besides adopting the ordering that $|S| > 1$ but $M_p < 1$, we further assume $\left\langle BV_{\|}\right\rangle / \left\langle B^2\right\rangle^{1/2} v_{ti} \ll 1$, where $V_{\|}$ is the ion parallel flow velocity. The angular brackets denote flux surface average. This assumption is motivated by the experimental observations that $V_{\|}/v_{ti}$ is indeed small in the edge region of a tokamak. We also neglect the Ohmic inductive electric field, which is irrelevant in calculating χ_i. With these orderings and assumptions, the drift kinetic equation for ions can be written as

$$\left(v_{\|}\hat{n} + \mathbf{V}_E\right) \cdot \nabla\theta \frac{\partial f}{\partial \theta} + \mathbf{v}_d \cdot \nabla\psi \frac{\partial f}{\partial \psi} = C(f) \ , \tag{1}$$

where f is the particle distribution function, $\hat{n} = \mathbf{B}/|\mathbf{B}|$ is the unit vector in the direction of magnetic field $\mathbf{B}$, $v_{\|}$ is the parallel (to $\mathbf{B}$) ion speed, and $\mathbf{V}_E = c\mathbf{E} \times \mathbf{B}/B^2$ is the $\mathbf{E} \times \mathbf{B}$ drift velocity, with $\mathbf{E} = -\nabla\Phi$ the electric field and Φ the electrostatic potential. Because $M_p < 1$ and $\left\langle BV_{\|}\right\rangle / \left\langle B^2\right\rangle^{1/2} v_{ti} \ll 1$, $\Phi = \Phi(\psi)$ is a function only of the flux ψ. We adopt standard tokamak flux coordinates here with $\mathbf{B} = I\nabla\zeta + \nabla\zeta \times \nabla\psi$. The poloidal and toroidal angles are denoted by θ and ζ, respectively. Note that $I = R^2\mathbf{B} \cdot \nabla\zeta$. The radial guiding center drift is given by

$$\mathbf{v}_d \cdot \nabla\psi = I v_{\|}\hat{n} \cdot \nabla \left(\frac{v_{\|}}{\Omega}\right) \ ,$$

where $\Omega = eB/Mc$ is the ion gyrofrequency, with M the ion mass. The independent variables in Eq. (1) are (ψ, θ, E, μ), where μ is the magnetic moment, $E = v^2/2 + e\Phi/M$, and v is the particle speed.

Our assumption of weak plasma flow is appropriate to the annulus, commonly observed near the plasma edge in tokamak experiments, where the electrostatic potential is close to its peak. Thus, $\mathbf{V}_E$ is retained in Eq. (1) not because it is large (in fact $\mathbf{V}_E \cdot \nabla\theta \ll v_\parallel \hat{n} \cdot \nabla\theta$), but because its derivative $dV_E/d\psi \propto d^2\Phi/d\psi^2$ is large.

Assuming $\nu \sim (v_\parallel \hat{n} + \mathbf{V}_E) \cdot \nabla\theta \gg \mathbf{v}_d \cdot \nabla\psi/\psi_e$, we obtain from Eq. (1), to the lowest order in $(\mathbf{v}_d \cdot \nabla\psi/\psi_e\nu)$,

$$\left(v_\parallel \hat{n} + \mathbf{V}_E\right) \cdot \nabla\theta \, \frac{\partial f_0}{\partial\theta} = C(f_0) \ , \tag{2}$$

where ψ_e is the equilibrium scale length in ψ for the equilibrium distribution f_0. A solution to Eq. (2) is a Maxwellian distribution

$$f_0 = f_M(\psi) \ . \tag{3}$$

The next-order linearized equation is then

$$\left(v_\parallel \hat{n} + \mathbf{V}_E\right) \cdot \nabla\theta \, \frac{\partial f_1}{\partial\theta} + \mathbf{v}_d \cdot \nabla\psi \, \frac{\partial f_1}{\partial\psi} + \mathbf{v}_d \cdot \nabla\psi \, \frac{\partial f_0}{\partial\psi} = C(f_1) \ , \tag{4}$$

where f_1 is the perturbed particle distribution. Here the term involving $\partial f_1/\partial\psi$ may appear to be a higher order. That it is in fact comparable to the other terms in Eq. (4), can be understood as follows. We note that $\partial f_1/\partial\psi \sim f_1/\psi_p$, with ψ_p the typical size of a particle orbit in ψ, and $\partial f_0/\partial\psi \sim f_0/\psi_e$. Because $f_1 \sim (\psi_p/\psi_e) f_0$, we conclude that $\partial f_1/\partial\psi \sim \partial f_0/\partial\psi$. The sharp radial variation of f_1 distinguishes the edge region (more specifically, the potential-peak region) of a tokamak plasma, making conventional kinetic analysis inappropriate. The point is that when the poloidal drift changes significantly over a nominal orbit width, the collisionless orbits become rather different from conventional tokamak bananas. Indeed, orbits in the edge region have much in common with those in bumpy torus geometry, which explains the resemblance between the following analysis and that appropriate to bumpy torus transport.

III. Variation of Poloidal Angular Velocity over Particle Trajectory

In order to solve Eq. (4) it is necessary to express the poloidal angular velocity,

$$\omega = (v_{\|}\hat{n} + \mathbf{V}_E) \cdot \nabla\theta = \left(v_{\|} + \frac{cI\Phi'}{B}\right) \hat{n} \cdot \nabla\theta \ , \tag{5}$$

in terms of θ and constants of the motion. To this end we use the guiding center energy,

$$E = \left(\frac{1}{2}\right) v_{\|}^2 + \frac{e\phi(\psi)}{M} + \mu B(\theta) \ , \tag{6}$$

and the canonical angular momentum,

$$P = \psi - \frac{I v_{\|}}{\Omega} \ . \tag{7}$$

Note that the approximation $R^2\mathbf{v} \cdot \nabla\zeta \approx (I/B)v_{\|}$ depends upon small flow velocity as discussed previously. It is not hard to show that

$$\omega = \left(\frac{I}{\Omega}\right) \hat{n} \cdot \nabla\theta \left(\frac{\partial P/\partial\psi}{\partial P/\partial E}\right) \ . \tag{8}$$

The potential in (6) is expanded in a Taylor series,

$$\Phi(\psi) = \Phi_0 + \Phi_0'(\psi - \psi_0) + \left(\frac{1}{2}\right) \Phi_0''(\psi - \psi_0)^2 \tag{9}$$

where we abbreviate

$$\Phi_0 = \Phi(\psi_0) \ , \quad \Phi_0' = \left.\frac{d\Phi}{d\psi}\right|_{\psi_0} \ , \quad \Phi_0'' = \left.\frac{d^2\Phi}{d\psi^2}\right|_{\psi_0} \ .$$

The parameter ψ_0 labels a reference flux surface in the vicinity of which we study transport. Equation (9) is a local assumption, requiring that near its peak the function $\Phi(\psi)$ be approximately parabolic; it does not assume that Φ varies slowly. It is sometimes convenient to represent the energy and magnetic moment by means of the invariant

$$v_{\|0} = v_{\|}(\psi_0, \theta_0) = \{2\,[E - e\Phi_0/M - \mu B(\psi_0, \theta_0)]\}^{1/2} \tag{10}$$

where θ_0 is a convenient reference angle, chosen to insure its general accessibility. In this regard we note that for large $|S|$ trapped particles reside on the inside (smaller major radius side) of the torus, so that the appropriate choice is $\theta_0 = \pi$ (for $|S| > 1$).

The guiding center trajectories

$$\Delta\psi = \psi(\theta, P, E, \mu) - \psi_0 \tag{11}$$

are determined by the three constants of the motion, and derived from (6), (7) and (9). The results may be found in, for example, Ref. 11. Here we need only the poloidal frequency; from (5) and (7),

$$\omega = \hat{n} \cdot \nabla\theta \left\{ \left(\frac{h_0}{h}\right) v_{\|0} + hV_{E_0} + \left[\frac{1}{h} + h(S-1)\right] \left(\frac{\Omega_0}{I}\right) \Delta\psi \right\}$$

where the magnetic field is expressed as $B = B_0/h(\psi, \theta)$, where B_0 is the field on the magnetic axis, and $V_{E_0} \equiv cI\Phi_0'/B_0$. After eliminating $\Delta\psi$ using (6), (7), and (9), we find that

$$\omega = \hat{n}\cdot\nabla\theta \left\{ \left(V_{E_0} + \frac{h_0}{v_{\|0}} h\right)^2 - [1 + h^2(S-1)] \left(\frac{h_0}{h} - 1\right) \left[2\left(E - \frac{e\Phi_0}{M}\right)(h_0/h + 1) - \frac{2\mu B_0}{h}\right] \right\}^{1/2} \tag{12}$$

where $h_0 = h(\psi_0, \theta_0)$.

Equation (12) is valid for arbitrary S; in particular one can verify that for $S \to 1 (\theta_0 \to 0)$ and $V_{E_0} \to 0$, it reproduces the conventional expression for poloidal motion, proportional to $v_{\|}$. At this point however it is convenient to simplify (12) by approximation for large aspect ratio and large $|S|$. Thus we assume

$$\varepsilon \ll 1 \,, \qquad \frac{1}{|S|} \ll 1 \,, \tag{13}$$

while allowing $\varepsilon S \sim 1$. Then $h = R/R_0 = 1 + \varepsilon \cos\theta$ and we find the lowest order form,

$$\omega = \pm\hat{\omega} \left[1 + k \sin^2\left(\frac{\theta}{2}\right)\right]^{1/2} \,, \tag{14}$$

with

$$\widehat{\omega} = |\widehat{n} \cdot \nabla\theta| \left\{ (V_{E_0} + v_{\|0})^2 + 4S\varepsilon \left(v_{\|0}^2 + \mu B_0 \right) \right\}^{1/2} , \tag{15}$$

$$k = 4|S|\varepsilon \frac{(v_{\|0}^2 + \mu B_0)}{[(V_{E_0} + v_{\|0})^2 + 4S\varepsilon(v_{\|0}^2 + \mu B_0)]} . \tag{16}$$

It can be seen that $k \to \infty$ as $\widehat{\omega} \to 0$; this limit corresponds to deeply trapped particles. The trapped-passing transition occurs at $k = 1$, or

$$(V_{E_0} + v_{\|0})^2 = 8\varepsilon|S|(v_{\|0}^2 + \mu B_0) \quad (\text{at } k = 1) . \tag{17}$$

Since the distribution function is nearly isotropic, (17) implies that the fraction of trapped particles, f_t, is proportional to $(\varepsilon|S|)^{1/2}$:

$$f_t \sim (\varepsilon|S|)^{1/2} . \tag{18}$$

The analysis leading to (15) allows f_t to be of order one; indeed it might exceed unity near the potential peak in some experiments. However, in the kinetic theory that follows, we shall assume $f_t \ll 1$, as might occur for very large aspect ratio. The point is that analytical treatment of the Coulomb collision operator is extremely awkward unless either f_t or $1 - f_t$ is considered small, and the former assumption seems somewhat more realistic. Of course we cannot expect quantitative accuracy from an analysis based on $f_t \ll 1$.

The physical point is that orbit squeezing is inherently accompanied by enhanced trapping. This can be seen, without large aspect ratio approximation, by noticing that the large-$|S|$ limit of (12) requires all particles to reside near $\theta = \pi$. The ultimate effect is to make ion heat transport at low collisionality depend strongly on S. We demonstrate this conclusion in Sec. V, after computing the distribution function for squeezed orbits.

IV. Solution to Linearized Drift Kinetic Equation

Employing the constant of motion P, we can rewrite Eq. (4) with independent variables (P, θ, E, μ) to obtain

$$-\frac{I}{\Omega}\frac{\widehat{n} \cdot \nabla\theta}{\partial P/\partial E}\frac{\partial P}{\partial \psi}\frac{\partial f_1}{\partial \theta}\bigg|_{P,E,\mu} + \frac{I}{\Omega}\frac{\widehat{n} \cdot \nabla\theta}{\partial P/\partial E}\frac{\partial P}{\partial \theta}\frac{\partial f_0}{\partial \psi} = C(f_1) . \tag{19}$$

We now perform a subsidiary ordering in terms of the expansion parameter $(\nu_i/S\epsilon)/(v_{ti}\sqrt{S\epsilon}/Rq)$ $\ll 1$, as is appropriate in the banana regime, to obtain the lowest-order equation,

$$\frac{I}{\Omega}\frac{\hat{n}\cdot\nabla\theta}{\partial P/\partial E}\frac{\partial P}{\partial\psi}\left(\frac{\partial f_1^0}{\partial\theta}\bigg|_{P,E,\mu}\right)-\frac{I}{\Omega}\frac{\hat{n}\cdot\nabla\theta}{\partial P/\partial E}\frac{\partial P}{\partial\theta}\frac{\partial f_0}{\partial\psi}=0\ , \tag{20}$$

and the next-order equation,

$$-\frac{I}{\Omega}\frac{\hat{n}\cdot\nabla\theta}{\partial P/\partial E}\frac{\partial P}{\partial\psi}\left(\frac{\partial f_1^1}{\partial\theta}\bigg|_{P,E,\mu}\right)=C\left(f_1^0\right)\ , \tag{21}$$

where the superscripts in f_1 indicate the subsidiary ordering. Note that the definition of the conventional ion collisionality ν_{*i} is also modified by the orbit squeezing factor S to $\nu_{*i}\equiv\nu_i Rq/\left[v_{ti}(S\epsilon)^{3/2}\right]$.

The solution to Eq. (20) is simply

$$f_1^0=-\psi\frac{\partial f_0}{\partial\psi}+g \tag{22}$$

where $g=g(P,E,\mu)$ is an integration constant to be determined from Eq. (21).

To proceed further, we need an explicit form for the collision operator $C(f)$. We adopt the pitch angle scattering operator

$$C(f)=\nu_D\frac{v_\parallel}{B}\frac{\partial}{\partial\mu}\mu v_\parallel\frac{\partial f}{\partial\mu}\ , \tag{23}$$

where ν_D is the deflection frequency defined in Ref. 15, and the magnetic moment $\mu=v_\perp^2/2B$ with $v_\perp$ the perpendicular (to $\mathbf{B}$) speed of the ion [15]. The independent variables in Eq. (23) are (E,μ). Changing variables from (E,μ) to (E,ω), using $\frac{\partial\psi}{\partial\omega}\cong\frac{I}{(\hat{n}\cdot\nabla\theta)S\Omega}$, and neglecting the $\partial f/\partial\omega$ term for $\sqrt{\epsilon|S|}\ll 1$, we obtain

$$C(f)=\nu_D\frac{v^2}{2R^2q^2}\frac{\partial^2 f}{\partial\omega^2}\ . \tag{24}$$

We have also neglected a term of the order of $M_p<1$ in obtaining Eq. (24).

Our use of simple pitch-angle scattering can be justified by a conventional neoclassical argument based on the small fraction of trapped particles. The theory of transport in the

plasma core shows that Eq. (24) applies after one has appended to f a term corresponding to a shifted Maxwellian, the size of the shift (mean parallel flow) being ultimately determined by momentum conservation. Somewhat different considerations pertain in the edge region, where parallel flows are observed to be very small, apparently because of atomic physical processes. Equation (24) then pertains for small f_t provided some (implicit) damping process annihilates parallel flow. In this case momentum conservation ultimately to determines the radial electric field.

Because k is the parameter that characterizes the trapped and circulating particles, it is more convenient to use (E, k) as the independent variables for $C(f)$ to obtain

$$C(f) = \nu_D \frac{v^2}{2R^2q^2} \frac{2k\omega}{\widehat{\omega}^2} \frac{\partial}{\partial k} \left(\frac{2k\omega}{\widehat{\omega}^2} \frac{\partial f}{\partial k} \right) . \tag{25}$$

The boundary conditions for f_1^1 in Eq. (21) are

$$f_1^{1+}(\theta = 0) = f_1^{1+}(\theta = 2\pi) ,$$

$$f_1^{1-}(\theta = 0) = f_1^{1-}(\theta = 2\pi) , \tag{26}$$

for $0 < k < 1$, and

$$f_1^{1+}(\theta = \theta_t) = f_1^{1-}(\theta = \theta_t) ,$$

$$f_1^{1+}(\theta = -\theta_t) = f_1^{1-}(\theta = -\theta_t) , \tag{27}$$

for $1 < k < \infty$, where the superscripts "+" and "−" indicate the sign of ω and θ_t is the turning point of the trapped ions. With these boundary conditions, we can annihilate the left side of Eq. (21), by bounce averaging for both trapped and circulating ions, to obtain an equation for g,

$$k \frac{\partial}{\partial k} \left(\frac{k}{\widehat{\omega}^2} \langle \omega \rangle_\theta \frac{\partial g}{\partial k} \right) = 0 , \tag{28}$$

where

$$\langle \omega \rangle_\theta = \frac{1}{2\pi} \int_0^{2\pi} \omega \, d\theta \tag{29}$$

for $0 < k < 1$, and

$$\langle\omega\rangle_\theta = \frac{1}{2\pi}\int_{-\theta_t}^{\theta_t} |\omega| d\theta \tag{30}$$

for $1 < k < \infty$.

The solution to Eq. (28) is

$$\frac{\partial g}{\partial k} = \frac{C\widehat{\omega}^2}{k\,\langle\omega\rangle_\theta}\,, \tag{31}$$

where C is an integration constant. For circulating particles, we find, from Eqs. (22) and (31),

$$\frac{\partial f_1^0}{\partial \omega} = -\frac{I}{S\Omega\widehat{n}\cdot\nabla\theta}\frac{\partial f_0}{\partial\psi} + 2C\frac{\omega}{\langle\omega\rangle_\theta}\,. \tag{32}$$

Because $\partial f_1^0/\partial\omega$ is localized in the trapped particle phase space, and $\lim_{k\to 0}\langle\omega\rangle_\theta = \omega$, we choose $C = \frac{I}{2(\widehat{n}\cdot\nabla\theta)S\omega}\frac{\partial f_0}{\partial\psi}$ to obtain

$$\frac{\partial f_1^0}{\partial \omega} = -\frac{I}{S\Omega\widehat{n}\cdot\nabla\theta}\frac{\partial f_0}{\partial\psi}\left(1 - \frac{\omega}{\langle\omega\rangle_\theta}\right) \tag{33}$$

for $0 < k < 1$. For trapped particles, $\partial g/\partial k$ is even in the sign of ω. In order for the even part of $\partial g/\partial k$ to be continuous across the $k = 1$ boundary, we choose $C = 0$ for $1 < k < \infty$ because the even part of $\partial g/\partial k$ vanishes for circulating particles. We conclude that

$$\frac{\partial f_1^0}{\partial \omega} = -\frac{I}{S\Omega\widehat{n}\cdot\nabla\theta}\frac{\partial f_0}{\partial\psi}\left[1 - H(1-k)\frac{\omega}{\langle\omega\rangle_\theta}\right]\,, \tag{34}$$

where H is a step function with $H(x) = 1$ for $x > 0$ and $H(x) = 0$ for $x < 0$. The discontinuity in $\partial f_1^0/\partial\omega$ at $k = 1$ can be removed by including the collisional boundary layer effect. However, for the purpose of calculating thermal conductivity, the correction associated with the boundary layer is higher order and can be neglected. As shown in Sec. V, we only need to know $\partial f_1^0/\partial\omega$ to calculate χ_i.

V. Ion Thermal Conductivity

The flux-surface-averaged radial particle flux Γ and heat flux q can be calculated from the definition

$$\Gamma_j = \frac{1}{\Delta\psi}\int_{\psi_1}^{\psi_2} d\psi \int_0^{2\pi} \frac{d\theta}{2\pi} \int d^3v\, (\mathbf{v}_d \cdot \nabla\psi)\, \Xi_j\, f_1 \ , \tag{35}$$

where T is the ion temperature; for $j = 1$, $\Gamma_1 = \Gamma$ and $\Xi_1 = 1$, and for $j = 2$, $\Gamma_2 = q/T$ and $\Xi_2 = v^2/v_{ti}^2 - 5/2$. The radial average in Eq. (35) is carried out over a distance $\psi_p \ll \Delta\psi = \psi_2 - \psi_1 \ll \psi_e$. Employing Eq. (19) to express $(\mathbf{v}_d \cdot \nabla\psi)$ in Eq. (35), we find

$$\Gamma_j = \frac{1}{\Delta\psi}\int_{\psi_1}^{\psi_2} d\psi \int_0^{2\pi} \frac{d\theta}{2\pi} \int d^3v\, f_1 \left[C(f_1) + \frac{I}{\Omega}\frac{\partial P/\partial\psi}{\partial P/\partial E}\,\hat{n}\cdot\nabla\theta\,\frac{\partial f_1}{\partial\theta}\right]\left(\frac{\partial f_0}{\partial\psi}\right)^{-1}\Xi_j \ . \tag{36}$$

It is straightforward to show that the terms involving $\partial f_1/\partial\theta$ vanish after the operation $\int d\psi \int d\theta \int d^3v$. Then we have

$$\Gamma_j = \int_0^{2\pi} \frac{d\theta}{2\pi} \int d^3v\, f_1 C(f_1) \left(\frac{\partial f_0}{\partial\psi}\right)^{-1} \Xi_j \ . \tag{37}$$

Because the integrand in Eq. (37) varies over a distance of the order of $\psi_e \gg \Delta\psi$, we approximate $(\Delta\psi)^{-1}\int_{\psi_1}^{\psi_2}\psi \simeq 1$ in obtaining Eq. (37). Substituting Eq. (24) into Eq. (37), and integrating by parts with respect to ω, we obtain

$$\Gamma_j = \int_0^{2\pi} \frac{d\theta}{2\pi}\, 4\pi \int dE\, Rq\cdot\nu_D\, \frac{v^2}{2R^2q^2}\left(\frac{\partial f_0}{\partial\psi}\right)^{-1} \int d\omega \left(\frac{\partial f_1}{\partial\omega}\right)^2 \Xi_j \ . \tag{38}$$

With $\partial f_1^0/\partial\omega$ given in Eq. (34), we find

$$\Gamma_j = -\frac{\mathrm{Im}}{\sqrt{2\pi}}\, N\, \frac{\sqrt{\epsilon}}{S^{3/2}}\rho_{pi}^2\nu_t \int_0^\infty dy\, \frac{\nu_D}{\nu_t} y^{3/2} e^{-y}\, \Xi_j \left[\frac{p'}{p} + \frac{e\Phi'}{T} + \left(y - \frac{5}{2}\right)\frac{T'}{T}\right] \ , \tag{39}$$

where $\nu_t = \nu_{ii}$ is the ion collision frequency defined in Ref. 15, $y = v^2/v_{ti}^2$, $p = NT$, ρ_{pi} is the ion poloidal gyroradius, and N is the plasma density. The constant $\mathrm{Im} = \int_0^\infty \left(dk/k^{3/2}\right)\left(\langle\hat{\omega}/|\omega|\rangle_\theta - \hat{\omega}H(1-k)/\langle|\omega|\rangle_\theta\right)$ is calculated in Ref. 14 and is $\mathrm{Im} = 2(0.69) \simeq 1.4$.

The ion particle flux calculated from Eq. (39) is about a factor of $\sqrt{M/M_e}$ larger than the electron flux. (Here, M_e is the electron mass.) To maintain ambipolarity, a radial electric field, which is proportional to Φ', develops. To the lowest order of $\sqrt{M_e/M}$, we have

$$\frac{p'}{p} + \frac{e\Phi'}{T} = -\frac{\mu_2}{\mu_1}\frac{T'}{T} , \tag{40}$$

where

$$\mu_1 = \int_0^\infty dy \, \frac{\nu_D}{\nu_t} y^{3/2} e^{-y} , \tag{41}$$

and

$$\mu_2 = \int_0^\infty dy \, \frac{\nu_D}{\nu_t} y^{3/2} \left(y - \frac{5}{2}\right) e^{-y} . \tag{42}$$

We note that the procedure employed here to determine Φ' is equivalent to that in determining poloidal flow velocity in the conventional neoclassical theory because we have assumed $\left\langle BV_{\|} \right\rangle / \left\langle B^2 \right\rangle^{1/2} v_{ti} = 0$. Equation (40) makes $\mathbf{V}_E$ comparable to the diagmagnetic drift, and therefore

$$\mathbf{V}_E \sim (\rho_{pi}/a) v_{ti} ,$$

consistent with our neglect of finite-M_p terms. Here a is the minor radius. The large gradient of $\mathbf{V}_E$ is also consistent with Eq. (40) because the latter is a local relation, valid near the potential peak. Of course, the physics causing the peak in $\Phi(\psi)$, presumably related to orbit losses, is not included in our analysis.

Substituting Eq. (40) into Eq. (39), we find

$$\frac{q}{T\,|\nabla\psi|} = -\frac{\mathrm{Im}}{\sqrt{2\pi}} N \frac{\sqrt{\epsilon}}{S^{3/2}} \rho_{pi}^2 \, \nu_t \left(\mu_3 - \frac{\mu_2^2}{\mu_1}\right) \frac{T'}{T} \, |\nabla\psi| , \tag{43}$$

where

$$\mu_3 = \int_0^\infty dy \frac{\nu_D}{\nu_t} y^{3/2} \left(y - 5/2\right)^2 e^{-y} . \tag{44}$$

From Eq. (43), we find that the ion thermal conductivity is a factor of $S^{-3/2}$ smaller than that in the conventional neoclassical theory [16, 17].

VI. Concluding Remarks

The result in Eq. (43) can be understood from the random walk argument. Because of the strong shear in E_r, the size of the trapped ion orbit is reduced by a factor of $\sqrt{S}$, i.e.,

$$(\Delta r)^2 \simeq \frac{\left(\sqrt{\epsilon}\rho_{pi}\right)^2}{S} , \tag{45}$$

and the fraction of the number of trapped ions is increased by the same amount, as shown in Eq. (18). We then have

$$\chi_i \simeq f_t(\Delta r)^2 \frac{\nu_t}{S\epsilon} \simeq \sqrt{\epsilon}\frac{\nu_t \rho_{pi}^2}{S^{3/2}} . \tag{46}$$

From the results presented here and elsewhere, it is clear that the conventional neoclassical theory is not applicable in the edge region of a tokamak where large values of both S and M_p are observed. Therefore, one should not compare experimental results with those of conventional neoclassical theory.

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