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Turbulence Modifications in Ohm's Law and Tokamak Transport Phenomenology†

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Abstract

Electron momentum balance relations derived from the full Boltzmann equations are modified significantly by electrostatic turbulence. The resultant particle, energy and momentum transport is well-known to be "anomalous", namely, the fluxes and the gradients are not generally related by classical or neo-classical constitutive laws. A possible, self-consistent, phenomenological approach to the **turbulent** constitutive relations is outlined. It is compared and contrasted with a philosophically similar, earlier approach due to R.J. Bickerton (J.Phys. D , **11**, 1781, (1978)). In particular, the necessity for the inclusion of an anomalous thermal force coefficient and consequences for experiments (with illustrative applications to DITE

and JET discharges) will be discussed.

Experimentally measurable particle transport properties D and V_{pinch} are shown to be correlated to the anomalous thermal force coefficient and the anomalous resistivity tensor. Relations obtaining between these properties, the poloidal beta of the discharge, and the D/χ ratio under quasi-steady conditions will be given. These are discharges in which all the *external* sources of particles, momentum, energy and current are held constant over time-scales long compared with the measured confinement time-scales and sawtooth period. The relations in question depend only on the forms of the turbulent constitutive relations and not on the details of the possible instability mechanisms responsible. As such, they are directly testable (in principle) against experiment, independently of particular instability theories.

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I. Introduction: Ohm's Law/ Electron Momentum Balance

- The aim of tokamak physics is to determine the march of physically important variables like n, $T_{e,i}$, $\mathbf{B}$ etc, given the sources of particles, current, momentum and energy under reactor-relevant boundary conditions.

- Classical and neo- classical theories provide constitutive relations in the form of momentum and thermal diffusivities, resistivity etc. The omnipresent turbulence renders these irrelevant in many cases.

- Since a completely reliable and readily calculable theory of tokamak turbulence (spectra and consequent turbulent constitutive properties) is not yet available, it seems reasonable and useful to follow the well-established phenomenological approaches to turbulent constitutive relations familiar in engineering fluid mechanics and meteorology.

- Of course, such procedures are well-known in plasma physics. For example, in transport codes such as TRANSP, the heat flux is written in the form, $Q = -\chi_{effective} n.\frac{dT}{dr}$, where $\chi_{effective}$ is an empirically determined, phenomenological thermal diffusivity. Similar ansatzes are also often applied to particle and momentum fluxes.

- Here we are specifically concerned with turbulence effects in Ohm's law. Starting with the full electron kinetic equations including collisions and taking the velocity moment we get the electron momentum equation:

$$m_e n_e \frac{D\mathbf{v}_e}{Dt} = -\nabla p_e + e n_e(\mathbf{E} + \frac{\mathbf{v}_e \times \mathbf{B}}{c}) + \nabla.\mathbf{P}_e + \mathbf{R}_e \quad (1)$$

- To illustrate ideas, consider periodic cylinder geometry. Assuming mean flux surfaces exist (labelled by r) and averaging over θ and $\phi = z/R$ and the turbulence time-scales (typically 10-300 khz), we obtain the following poloidal component of the turbulence-averaged equation of momentum balance:

$$\langle \tilde{n}_e \tilde{E}_\theta \rangle = \frac{\langle n_e v_{er} \rangle \langle B_z \rangle}{c} + ... \tag{2}$$

where the dots denote all terms not explicitly written down.

- The equation of continuity averaged similarly becomes,

$$\frac{1}{r} \frac{\partial r \langle n_e v_{er} \rangle}{\partial r} = \langle S(\mathbf{r}, t) \rangle \tag{3}$$

- It follows that we may write,

$$\Gamma_r = \langle n_e v_{er} \rangle = \frac{c}{B_z} \left[\langle \tilde{n}_e \tilde{E}_\theta \rangle + ... \right] \tag{4}$$

The dots denote small corrections.

Recall that with isotropic resistivity η, the simple classical form of Ohm's law is, $\mathbf{E}+\frac{\mathbf{v}\times\mathbf{B}}{c} = \eta\mathbf{j}$. From the poloidal component on a flux surface (r=constant!) it is easily seen that,

$$\Gamma_r \equiv n_e v_r = \frac{cn_e}{B_z}(-\eta j_\theta) \tag{5}$$

Since pressure balance gives, $j_\theta = \frac{c}{B_z}\frac{dp}{dr} + j_z\frac{B_\theta}{B_z}$, the particle flux across the surfaces is expressed in terms of the pressure gradient and the toroidal current density and the fields. As is well-known, this expression simply leads to the *classical particle diffusivity* if the scalar resistivity η is taken to be $O(\eta_{Spitzer})$.

II. Ohm's Law Phenomenology: Bickerton Model

Confronted with this situation, Bickerton (J.Phys. D, **11**, 1781, (1978)) proposed that the turbulence driven friction forces arising in the averaged electron momentum balance equation could be modelled in **analogy** with classical friction forces using an anisotropic resistivity tensor thus:

$$\mathbf{E}+\frac{\mathbf{v}\times\mathbf{B}}{c}=\mathbf{F}_{turb}\equiv\eta_{||}\mathbf{j}_{||}+\eta_{\perp}\mathbf{j}_{\perp} \qquad (6)$$

where, in analogy with Prandtl-Boussinesq effective viscosity and $\chi_{effective}$, the friction forces $\mathbf{F}_{turb}$ arising from the turbulence (generally much larger in size compared with classical or even neo-classical forces which should also be included in general on the rhs of Eq.(6)) are parametrized in terms of the 'turbulent' constitutive quantities $\eta_{||}$ and $\eta_{\perp}$. The latter are to be regarded as anomalous coefficients, to be determined from experiment or from detailed turbulence models. The quantities occurring in Eq.(6) are to be understood as *mean* properties time-averaged (at least) over the relatively rapid turbulent fluctuations. This is consistent with the experimental fact that the time-scales of the turbulence are small compared with typical confinement times.

Taking the parallel component of Eq.(6) we have, $E_{\|} = \eta_{\|} j_{\|}$. Experiment shows that the total plasma resistance is consistent in order of magnitude with that obtained by taking $\eta_{\|}$ to be given by the Spitzer formula.

Defining $v_r \equiv \frac{\Gamma_r}{n_e}$, we clearly have the relation, $\frac{v_r B}{c} = -\eta_{\perp} j_{\perp} + E_{\perp} = -\frac{c\eta_{\perp}}{B}\frac{dp}{dr} - E_z \frac{B_\theta}{B}$. Use has been made of the transverse component of Bickerton's form of turbulent Ohm's law and the pressure balance equation. E_z is the externally applied "toroidal" field coming from the loop voltage.

A more instructive, equivalent form is,

$$\Gamma_r \equiv n_e v_r = -c^2 \eta_{\perp} \frac{p}{B^2}\left(1 + \frac{d \ln T}{d \ln n_e}\right)\frac{dn_e}{dr} - n_e \frac{cE_z}{B}\frac{B_\theta}{B} \qquad (7)$$

- Note that the temperature appearing in the formula is related to the plasma pressure by the relation, $p \equiv n_e T$.

In order that the particle flux be consistent with experimentally measured density and temperature profiles, it is necessary to take $\eta_\perp \simeq 100.\eta_\parallel$, as observed by Bickerton.

- It follows that Bickerton's model leads to the following expressions:

$$D_{Bickerton} = c^2 \eta_\perp \frac{p}{B^2}(1 + \frac{d\ln T}{d\ln n_e}) \tag{8}$$

$$V_{Bickerton} = \frac{cE_z}{B}\frac{B_\theta}{B} \tag{9}$$

$$j_z = \frac{E_z}{\eta_\parallel}(\frac{B_z}{B})^2 - \frac{c}{B}\frac{dp}{dr}(\frac{B_\theta}{B}) \tag{10}$$

- It is readily seen that the 'inward' pinch velocity $V_{Bickerton}$ is independent of $\eta_\perp$ and of $\eta_\parallel$ and smaller than the Ware pinch due to trapped particle banana drifts and much smaller than the experimental 'anomalous' inward pinch. For realistic values of $\eta_\perp$, the pinch term in Γ_r is negligible in this model.

- Observe that the toroidal current density is entirely independent of the choice of $\eta_\perp$ and is identical in form with the expression obtaining in classical theory using Eq.(6).

- In addition to the inductive current, there is also a 'bootstrap' current driven by the pressure gradient. This is of course much smaller than the former, typically $O(\beta j_{inductive})$. It is als smaller by t factor $\frac{B_\theta^2}{B^2}(\frac{R}{a})^{1/2}$ than the 'neoclassical' bootstrap current due to trapped particles.

- Other features of this model relating to plasma diamagnetism and poloidal betas attainable in the absence of particle sources will be discussed later. It is useful to remark that in regions where the particle source is negligible (there are JET and DITE discharges where this is a good approximation over the whole discharge) the Bickerton model predicts flat pressure profiles upon neglecting the small pinch term in steady state. It follows from the boundary conditions that the beta poloidal must be nearly zero (ie $O(\frac{\eta_\parallel}{\eta_\perp})$).

III. Ohm's Law Phenomenology: $\eta_\theta - \alpha$ Model

- A model, superficially very similar to Bickerton's was proposed by the authors (Haas and Thyagaraja (1986),(1989), (1990), Thyagaraja and Haas (1988)) and used to explain a host of experimental observations relating to density clamp, Hugill behaviour, flow-driven currents, impurity transport and features of L-H transitions.

- This model shares the phenomenological outlook and the general philosophy of the Bickerton model but is able to give a unified and coherent account of a large body of experimental data on many tokamaks. It also makes predictions which, in principle, can be experimentally tested. Its main function is to provide a concrete intermediate step between experimental data relating to particle and energy transport and turbulence theories by expressing the data compactly as turbulent constitutive properties.

- As before we begin with Ohm's law. However, in contrast to Bickerton, we parametrize the turbulent friction forces differently. Assuming the existence of equilibrium flux (ie. pressure surfaces) and tokamak geometry, we write the averaged Ohm's law *within* the surface as follows:

$$E_z \mathbf{e_z} + \frac{\mathbf{v} \times \mathbf{B}}{c} = \mathbf{F}_{turb} \equiv \boldsymbol{\eta} . \mathbf{j} - \frac{\alpha \chi \nabla p \times \mathbf{B}}{cp} \qquad (11)$$

where, $\boldsymbol{\eta}$ and α are respectively the anomalous resistivity tensor and thermal force coefficients. χ is the usual anomalous thermal diffusivity. As before, $\mathbf{v} \equiv \frac{\Gamma_r}{n_e} \mathbf{e_r}$.

- It follows from the vectorial character of the friction forces that on a flux surface they may be expressed as a general linear combination of $\mathbf{j}$ and $\nabla p \times \mathbf{B}$. However, since pressure balance actually relates these two vectors, the tensor $\boldsymbol{\eta}$ can be chosen without loss of generality to be diagonal with respect to some specified set of principal directions. Alternatively, the tensor resistivity can be taken to have non-trivial off-diagonal elements and α set to zero. We choose the former, but in contrast to Bickerton, the principal axes are taken to be the toroidal and poloidal directions.

- This choice amounts to recognizing the *global* toroidal symmetry for turbulence, invoking the spontaneous breakdown of the *local* gyro-angle symmetry in a plane perpendicular to the averaged **B** field direction at a point on a mean flux surface.

- Thus, $\boldsymbol{\eta} = \eta_z \mathbf{e_z e_z} + \eta_\theta \mathbf{e}_\theta \mathbf{e}_\theta$, where we take η_z to be of order $\eta_{Spitzer}$ whilst $\eta_\theta \simeq \eta_z (\frac{B_z}{B_\theta})^2$. This should be compared with Bickerton's assumption relating to $\frac{\eta_\perp}{\eta_\parallel}$. The consequences of these phenomenological constitutive assumptions will now be discussed.

IV. $\eta_\theta - \alpha$ Model: Transport Equations

The model is completed by adding the equations of continuity, pressure balance, Ampère's law and the energy equation. For illustrative purposes, the equations are given for the cylindrical model, although there is no difficulty in writing them down in toroidal geometry taking account of electron and ion temperatures separately. The ion toroidal and poloidal momentum equations and the radial component of Ohm's law together determine the flows in the system and the radial electric field, given the momentum sources and the anomalous viscous stress tensor. The model is not at present designed to calculate these plasma properties.

$$\frac{1}{r}\frac{d(rn_e v_r)}{dr} = S(r) \tag{12}$$

$$\frac{dp}{dr} = \frac{j_\theta B_z - j_z B_\theta}{c} \tag{13}$$

$$j_z = \frac{c}{4\pi}\frac{1}{r}\frac{d(rB_\theta)}{dr} \tag{14}$$

$$j_\theta = -\frac{c}{4\pi}\frac{dB_z}{dr} \tag{15}$$

$$\frac{1}{r}\frac{d}{dr}(r\chi\frac{dp}{dr}) + E_z j_z + P_0(r) = \frac{1}{r}\frac{d}{dr}(\frac{5}{2}rpv_r) \tag{16}$$

$$E_z + \frac{v_r B_\theta}{c} = \eta_z j_z - \frac{\alpha\chi}{cp}\frac{dp}{dr}B_\theta \tag{17}$$

$$-\frac{v_r B_z}{c} = \eta_\theta j_\theta + \frac{\alpha\chi}{cp}\frac{dp}{dr}B_z \tag{18}$$

The equations are written for macroscopically steady states assuming the knowledge of the particle source $S(r)$, the net auxiliary heating source $P_0(r)$, the externally applied toroidal magnetic field B_z and B_θ at $r = a$. The sources may also be functions of the plasma properties, but this dependence (if any) is assumed known and is not explicitly shown. The turbulent constitutive properties η_z, η_θ, α and χ are also assumed known, at least in principle.

Many interesting qualitative and quantitative conclusions can be drawn from the set of equations and constitutive assumptions presented (see, for example, Haas and Thyagaraja (1990)). Here, we look at some of the results which have recently been obtained and their relevance to JET, DITE and other experiments.

V. $\eta_\theta - \alpha$ Model: Results

The model gives the following expressions for the particle flux and the associated transport coefficients:

$$\Gamma_r = -D\frac{dn_e}{dr} - Vn_e \qquad (19)$$

$$D = \left(\frac{1}{1+(\frac{\eta_z B_z^2}{\eta_\theta B_\theta^2})}\frac{c^2\eta_z p}{B_\theta^2} + \alpha\chi\right)\frac{d\ln p}{d\ln n} \qquad (20)$$

$$V = \frac{cE_z}{B_\theta}\frac{1}{(1+(\frac{\eta_z B_z^2}{\eta_\theta B_\theta^2}))} \qquad (21)$$

It is immediately seen that gross stability requires the positivity of D, which in turn implies the inequality $\alpha > -(\frac{c^2\eta_z p}{\chi B_\theta^2})/(1+\frac{\eta_z B_z^2}{\eta_\theta B_\theta^2})$.

It is also apparent that the inward pinch velocity cannot exceed $\frac{cE_z}{B_\theta}$. It can, however be larger than the neo-classical pinch. Clearly, V does not explicitly depend upon α

The toroidal component of Ohm's law can be written in the form:

$$\frac{E_z}{\eta_z} = j_z \left(1 + \frac{\eta_\theta B_\theta^2}{\eta_z B_z^2}\right) + \frac{c}{B_\theta}\frac{dp}{dr}\left(\frac{\eta_\theta B_\theta^2}{\eta_z B_z^2}\right) \tag{22}$$

It is interesting to note that this form implies an effectively larger toroidal resistivity than η_z and also involves a 'bootstrap' current which can be larger than the usual neo-classical one driven by the pressure gradient. Note that this expression is entirely independent of the anomalous thermal force coefficient α.

An exact expression for the poloidal beta is also derivable from the transport equations:

$$\beta_p = 1 + \frac{8\pi}{a^2 B_\theta^2}\int_0^a \left(\frac{v_r B_z^2}{c^2 \eta_\theta} + \frac{\alpha \chi B_z^2}{c^2 \eta_\theta}\frac{d \ln p}{dr}\right) r^2 dr \tag{23}$$

A number of interesting conclusions can now be drawn:

- If $\alpha = 0$ (ie no thermal force), Eq.(23) shows that $\beta_p \geq 1$ in steady state given particle **sources**. If the source is negligible, the steady state value of β_p must be exactly unity, *irrespective of the heating power.* Thus, if no particle sources are supplied and the current is held fixed, the confinement must degrade like $\frac{1}{P}$. Physically, as the temperature of the discharge increases, the density "clamps" and the pressure essentially remains constant.

- If there is no particle source and no thermal force term, the following relation can be shown to hold:

$$\chi = \frac{c^2\eta_z}{4\pi}\left[1 + \frac{8\pi^2 Rq(r)}{r^2 B_z c I_p}\int_0^a \frac{r\,dr}{\eta_z}\int_0^r rP_0(r)dr\right] \quad (24)$$

$$I_p = 2\pi E_z \int_0^a \frac{r\,dr}{\eta_z} \quad (25)$$

- When $\alpha > 0$, the highest achievable steady-state poloidal beta in the absence of a particle source ($v_r = 0$) is always **less** than unity. DITE discharges provide examples of this: Shot #30955 had a current flat-top ($I_p = 96kA$) of 300msecs, with electron energy confinement time $\tau_{Ee} \simeq 9$ msecs. The line-average electron density is also nearly constant. The working gas is Helium with no sources or sinks-just recycling at the limiter. The toroidal field $B_z =$ 2 Tesla, $n_e = 10^{19}m^{-3}$,$T_{e0} \simeq 750$ ev, $a = 20$ cms, and $R = 80$ cms. The β_p is essentially constant throughout the flat-top with a value of $\beta_p = 0.25$. Using these values and our prescription for η_θ and $\chi \simeq 10^4 cm^2/sec$, the value of α can be estimated to be 0.13. This immediately gives values for D and V which, can be measured independently and compared with the predictions of the model. Conversely, in any experiment, a measurement of D and V can be used to determine η_θ and α and then used to **predict** the limiting poloidal beta for particle source-free steady conditions.

- The thermal force coefficient α can be negative so long as the total D remains positive. In this case, even in the absence of particle sources, the poloidal beta of the discharge can exceed unity.

- The model has been applied to a JET discharge which is not steady in the sense that the line-averaged density is ramped down by moving the plasma to contact the wall (to terminate the discharge without disruptions) over 10-15 secs. There is a surface sink of particles but no volume sink. Simple estimates show that the time derivative terms in the equations are negligible. Typical plasma parameters are: $B_z = 2$ Tesla, $I_p = 3$ MA, $V_{loop} = 0.6$ volts, with $\beta_p \simeq 0$. The model is consistent with experiment if $v_r = 0.1$

m/sec and $\alpha \simeq 0.01$.

- For low β_p experiments with no particle sources or sinks in the body of the plasma, the parameter α measures roughly speaking the ratio of the energy and particle confinement times. If it is positive, high poloidal betas can only be obtained by having strong particle sources in addition to a power source.

VI. Conclusions

- All theories of turbulence need to be compared with experimental transport studies to establish their limits of validity.

- This process requires that the experimental data have to be presented in terms of certain turbulent constitutive relations. The purpose of the present work was to highlight a possible general approach to the complete set of such properties, especially those concerned with particle transport and modifications which necessarily occur in Ohm's law due to the presence of electrostatic turbulence in tokamaks.

- The original motivation for this work were the results of Coppi and Sharky (1981) and an earlier phenomenological model of Bickerton. The model incorporates the usual anomalous thermal diffusivity χ, a toroidal resistivity η_z of the order of Spitzer (but ultimately determined from total plasma resistance), an anomalous *poloidal* resistivity η_θ (presumably due to electrostatic turbulence) of order $O(\eta_z \frac{B_z^2}{B_\theta^2})$ and an anomalous thermal force coefficient α which is a non-dimensional number of either sign satisfying rather general stability limits.

- Noting the fact that in tokamak turbulence both theory and experiment suggest that $k_\perp \gg k_\parallel$, it is possible to derive the result that the turbulent friction forces in Ohm's law satisfy $\frac{F_\theta}{F_z} = O(\frac{B_z^2}{B_\theta^2})$ (Richard Hazeltine, private communication). It should be possible to check this experimentally.

- By adopting the principle that tokamak turbulence respects the *global* toroidal symmetry whilst spontaneously breaking the *local* gyro-angle symmetry about the time-averaged field direction at a point, we have predicted relations obtaining between plasma particle transport coefficients, poloidal beta etc in a self-consistent manner including pressure balance.

- Just as the Bickerton model made well-defined predictions which could be compared with experiment, so does the present model. It has already been successful in explaining a number of hitherto apparently unrelated tokamak observations. More experimental and theoretical work to support its conclusions or disprove it would seem to be appropriate.

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