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A widely used method of plasma stability analysis uses the Rigid Displacement Model (RDM) of plasma behaviour. In the RDM it is assumed that the plasma displacement is small and usually plasma inertia effects are neglected. In addition, it is considered that no changes in plasma shape, plasma current, and plasma current profile take place throughout the plasma motion. The massless- filament approximation (instantaneous force-balance) accurately reproduces the unstable root of the passive stabilization problem. Then, on the basis that the instantaneous force-balance approximation is correct in the passive stabilization analysis, the massless approximation is utilized also in the study of the plasma vertical stabilization by active feedback. It is shown here that the RDM (without mass effects included) does not provide correct stability results for a tokamak configuration (plasma column, passive conductors, and feedback control coils). Therefore, it is concluded that inertia effects have to be retained in the RDM system of equations. It is shown analytically and numerically that stability diagrams with and without plasma-mass corrections differ significantly. When inertia effects are included, the stability region is more restricted than obtained in the massless approximation .

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Introduction

Elongated plasmas in tokamak configurations are unstable to axisymmetric vertical displacements. The vacuum vessel and passive conductors can stabilize the plasma motion in the short time scale. For stabilization of the plasma motion in the long time scale an active feedback control system is required. Different models [1 – 7] are used to study the plasma vertical stability. A widely used model is the Rigid Displacement Model (RDM).

In the RDM it is assumed that the plasma displacement is small and the plasma current is constant during the plasma motion. It is considered that no change in plasma shape and current profile takes place throughout the plasma motion. In several previous works [4 – 6] it has been demonstrated that the massles filament approximation (instantaneous force balance) accurately reproduces the unstable root in the passive stabilization problem. Using this simlification, the problem has been extended [4 – 6] to include an active coil, intended to provide plasma stability on a longer time scale.

The motivation of this paper is to show that massless approximation (no inertia effects) does not provide correct stability diagrams for a tokamak configuration (composed of plasma, passive conductors and feedback control coils). The plasma mass has to be retained in the RDM system of equations to obtain accurate results. It is shown analitically and numerically that the stability diagrams obtained with mass and without inertia effects are different. In the massless approximation the stability region obtained is wider than obtained with inertia effects retained.

The plasma rigid displacement model

An axisymmetric vertical plasma displacement can be described by a system of equations which includes: force balance equation, circuit equations and voltage control equation. The plasma is modelled by a single rigid filament. The plasma motion equation in the RDM is

$$m\ddot{z} = -\frac{\partial B_r}{\partial z}\frac{2\pi R_p}{c}I_p z - \sum_{j=1}^{2} B_j I_p \frac{2\pi R_p}{c} I_j$$

where m is the plasma mass; c is the speed of light; R_p is major radius of the plasma filament; B_r is the external radial magnetic field on the plasma filament; B_j is radial magnetic field produced by induced and active unit currents on the plasma filament; I_p and I_j are the currents in the plasma and the external (pssive and active) filaments, respectively; z is the plasma displacement; and, $\ddot{z}$ is the plasma acceleration. We define the quantities

$$
\begin{aligned}
S_b &= -\frac{\partial B_r}{\partial z}\frac{2\pi R_p}{c}I_p \\
S_j &= -B_j I_p \frac{2\pi R_p}{c} \\
S_j^M &= I_p \frac{\partial M_{pj}}{\partial z}
\end{aligned}
$$

and the index 1 for the active coils and the index 2 for the passive conductors. Thus the equations for plasma motion and the circuits can be written as

$$
\begin{aligned}
& m\ddot{z} = S_b z + \sum_{j=1}^{2} S_j I_j \,, \\
& L_1 \dot{I}_1 + M\dot{I}_2 + R_1 I_1 + S_1^M \dot{z} = V \,, \qquad \text{and} \\
& L_2 \dot{I}_2 + M\dot{I}_1 + R_2 I_2 + S_2^M \dot{z} = 0 \,,
\end{aligned}
$$

where L_j is the self-inductance of the j^{th} filament; M is the mutual inductance between the active coils and passive conductors; R_j is the resistance of the j^{th} filament; M_{pj} is the mutual inductance between the j^{th} filament and the plasma filament; and V is the voltage acting on the active coil.

The stability of the system is determined by the character of the voltage applied to the feedback control coil. A control law proportional to the plasma displacement and its velocity is given by:

$$
V = -g_1(z + t_1\dot{z}) \,,
$$

where t_1 is the lead time.

The system of equations can be expressed in matrix form

$$
\hat{B}\dot{\vec{x}} = \hat{A}\vec{x} \,, \tag{1}
$$

where $\vec{x} = \{z, \dot{z}, I_1, I_2\}$.

The Laplace transform of Eq.1 is

$$\lambda B\vec{x} = A\vec{x}\,. \tag{2}$$

Denoting the j^{th} eigenvalue and the j^{th} eigenvector by λ_j and $\vec{u}_j$, respectively, the general solution of the system can be written as

$$\vec{x}(t) = \sum_k c_k \vec{u}_k e^{\lambda_k t}\,, \tag{3}$$

where c_k are independent coefficients. The quantities λ_j and $\vec{u}_j$ are usually complex values. For plasma stability it is nesessary that the real parts of all the eigenvalues be negative.

Equation (2) is a polinomial of the fourth order in λ:

$$a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda^1 + a_0 = 0 \tag{4}$$

where the coefficients a_k are:

$$\begin{aligned}
a_4 &= m(L_1L_2 - M^2)\,,\\
a_3 &= m\{L_2R_1 + L_1R_2\}\,,\\
a_2 &= mR_1R_2 + (S_1^M + g_1t_1)(S_1L_2 - S_2M) + S_2^M(L_1S_2 - S_1M) - S_b(L_1L_2 - M^2)\,,\\
a_1 &= S_1^M S_1R_2 + S_2^M S_2R_1 - S_b(L_1R_2 + L_2R_1) + g_1(S_1t_1R_2 + S_1L_2 - S_2M)\,, \quad \text{and}\\
a_0 &= g_1S_1R_2 - S_bR_1R_2\,.
\end{aligned}$$

For stability of the solution the coefficients must satisfy the Routh-Gurvit's criterium [8], i.e, the following values should have the same sign:

$$a_4; \quad a_3; \quad a_2 - \frac{a_4a_1}{a_3}; \quad a_1 - \frac{a_3a_0}{a_1}; \quad a_0\,.$$

Since $a_3a_0/a_1 \sim m \ll a_1$, the fourth term in the sries is $a_1 - a_3a_0/a_1 \approx a_1$.

The first term in the series, a_4 is positive for all values of L_1, L_2, and M, since mutual inductances are smaller than self-inductances. This imposes the requirement that all terms in the series must be positive. We use the notation $b_1 = a_2 - \frac{a_4a_1}{a_3}$. S_j is proportional to S_j^M

so that we can write $S_2 = \beta S_1$ and $S_2^M = \beta S_1^M$. After simple mathematical manipulations we can write:

$$a_4 = m(L_1 L_2 - M^2)\,,$$
$$a_3 = m\{L_2 R_1 + L_1 R_2\}\,,$$
$$b_1 = S_1 S_1^M\{R_1(L_2 - \beta M)^2 + R_2(\beta L_1 - M^2)\} + g_1 S_1\{t_1(L_2 - \beta M - \frac{(L_1 L_2 - M^2)R_2}{R_1 L_2 + R_2 L_1})\}\,,$$
$$a_1 = S_1 S_1^M(\beta^2 R_1 + R_2) - S_b(L_1 L_2 - M^2) + S_1 g_1(t_1 R_2 + L_2 - \beta M)\,, \quad \text{and}$$
$$a_0 = g_1 S_1 R_2 - S_b R_1 R_2\,.$$

In the massless approximation $a_4 = a_3 = 0$. Thus for stability a_2, a_1, and a_0 must be positive. For simplicity, we assume that the passive conductor is closer to the plasma than the active circuit. In this case, $S_1 > S_2$ ($\beta < 1$).

If the passive stabilization of the plasma in the massless approximation is considered, then the coefficients a_k^{pas} are given by

$$a_2^{pas} = S_1 S_1^M(L_2 + \beta^2 L_1 - 2\beta M) - S_b(L_1 L_2 - M^2)\,,$$
$$a_1^{pas} = S_1 S_1^M(\beta^2 R_1 + R_2) - S_b(L_1 L_2 - M^2)\,, \quad \text{and}$$
$$a_0^{pas} = -S_b R_1 R_2\,.$$

When the derivative of the external radial magnetic field is not too large (such that a_2^{pas} and a_1^{pas} are positive) then system has only one unstable positive root. As the value of the external magnetic field (S_b) increases, the relevant time scale of the system approaches Alfven's time scale. When the values of a_2^{pas} and a_1^{pas} are negative, the system has more than one unstable root with large positive real part.

We consider typical tokamak systems which are stabilized in the Alfven's time scale and have only one unstable root with small positive part. The coeficients a_2 and a_1 can be written as

$$a_2 = a_2^{pas} + S_1 g_1 t_1(L_2 - \beta M) \quad \text{and}$$
$$a_1 = a_1^{pas} + S_1 g_1(t_1 R_2 + L_2 - \beta M)\,.$$

For simplicity only positive values of gain and lead time are considered here (i.e., $g_1 > 0$ and $t_1 > 0$). $(L_2 - \beta M) > 0$ (since the mutual inductance is smaller than the self-inductance and $\beta < 1$). In this case the coefficients a_2 and a_1 are positive for all values of g_1 and t_1. Therefore in the massless approximation the coefficient a_0 determines the values of gain required for stabilization. $a_0 > 0$ implies that $g_1 > S_b R_1 / S_1$.

When inertia effects are retained the coeficients a_4, a_3, and a_1 are positive. For stability of the system, the coeficients b_1 and a_0 must also be positive. These two coeficients determine the values of gain and lead time. $a_0 > 0$ implies that , the value of the gain is limited by $g_1 > S_b R_1 / S_1$.

We define the coefficients:

$$C_1 = (L_2 - \beta M) - \frac{R_2(L_1 L_2 - M^2)}{R_1 L_2 + R_2 L_1},$$

$$C_2 = \frac{(L_2 - \beta M)(L_1 L_2 - M^2)}{R_1 L_2 + R_2 L_1}, \quad \text{and}$$

$$C_3 = S_1^M \frac{R_1(L_2 - \beta M)^2 + R_2(\beta L_1 - M)^2}{R_1 L_2 + R_2 L_1},$$

where C_2 and C_3 are positive. Using these notation we can write $b_1 = S_1\{C_3 + g_1(t_1 C_1 - C_2)\}$.

Depending on the sign of C_1, two cases can be considered. When C_1 is negative, the inequality $b_1 > 0$ restricts the gains to

$$g_1 < \frac{C_3}{t_1|C_1| + C_2}$$

for any $t_1 > 0$. The stability diagram in this case is shown in Fig.1. It is seen that if $t_1 > t^*$ then system is unstable for any gain $g_1 > 0$. The value of t^* is determined by the relation:

$$\frac{C_3}{t^*|C_1| + C_2} = \frac{S_b R_1}{S_1}.$$

When C_1 is positive and $t_1 C_1 - C_2 > 0$, then $b_1 > 0$ for any value of gain $g_1 > 0$ and the stability of the system is determined by the same condition $a_0 > 0$ of the massless approximation. If $t_1 C_1 - C_2 < 0$ then

$$g_1 < \frac{C_3}{C_2 - t_1 C_1}.$$

The stability diagram for this case is shown in Fig.2. It is seen that if $g^* < S_b R_1 / S_1$ (where $g^* = C_3 / C_2$) then system is unstable when the voltage control law does not include a derivative term. If derivative term is included, the system is stable for $t_1 > t^*$, where the limiting value t^* is given by the relation:

$$\frac{C_3}{C_2 - t^* C_1} = \frac{S_b R_1}{S_1}.$$

We consider two simple examples.

1. The resistances and the self-inductances of the passive filament and the active circuit are assumed to be equal ($R_1 = R_2$, $L_1 = L_2$), although their z coordinates are different. In this case, $C_1 = (L - M)^2 + 2ML(1 - \beta) > 0$. The limiting lead time is given by $t^* \approx \frac{L+M}{R}$. For $t_1 > t^*$, the same stability region (Fig.2) is obtained with and without inertia effects included.

2. The resistance of the active coil is much smaller than the resistance of the passive conductor ($R_2 \gg R_1$). In this case $C_1 \approx M(\frac{M}{L_1} - \beta)$. Depending on the value of β, C_1 can be either positive or negative. If the z coordinates of the active circuit and the passive conductor have opposite signs, then $C_1 > 0$.

Results

Numerical simulations has been carried out for two cases. In the first, it is assumed that the system consists of one plasma filament, one active circuit, and two passive stabilization coils (with up-down symmetry). In the second, the plasma is represented as a set of filaments, the vacuum vessel is also divided into a set of filaments, and two pairs of active control coils (with up-down symmetry) are used to control the vertical position of the plasma.

When inertia effects are retained, two additional conjugate roots are obtained (with respect to the analysis neglecting mass effects). All the roots, exept the extra pair, are approximately equal to the roots obtained in the massless approximation (this is because the plasma mass is small). As a result of our simulation it is shown that the difference between the two analysis is determined by the behaviour of the two conjugate roots.

In the diagrams that follow the real parts of the extra roots are given as a function of the gain. The root with maximum real part when inertia effects are retained is close to one of the roots in the massless analysis. The real part of the two complex conjugate roots, inroduced by plasma mass effects, is also considered here.

First we consider the symplified system. The passive coils are located closer to the plasma than the active control coil. In Fig.3, the resistivities of the passive conductor and the active coil are 100 times the copper resistivity. The passive growth rate in this case is $\lambda^{pas} = 10.35$. If the voltage in the control law is proportional to the plasma displacement (i.e., $t_1 = 0$), then for values of gain $g_1 > 500\ V/cm$ the system becomes unstable (the real part of conjugate roots is positive) when mass effects are retained. However, the system remains stable (Fig.3-a) when inertia effects are neglected. If the voltage control includes a derivative term ($t_1 = 0.1 sec$), then the real part of the conjugate roots remains negative and stability is determined by the maximum real part of the roots when inertia effects are considered (Fig.3-b). These results are in good agreeement with the analitical treatment of Example 1 before.

When the resistivity of the active coil is the copper resistivity and the resistivity of the passive conductor is 100 times larger ($R_2 \gg R_1$) the roots are shown in Fig.4. The passive growth rate in this case is $\lambda^{pas} = 0.25$. If the voltage applied to active coil is proportional to the plasma displacement ($t_1 = 0$), then the real part of the conjugate roots becomes positive for the $g_1 > 320\ V/cm$. However, the system remains stable in the massless approximation (Fig.4-a). If a derivative term is included with $t_1 = 0.1\ s$, then the system becomes unstable for $g_1 > 6 V/cm$ when inertia effects are retained. The system remains stable in the massless approximation (Fig.4-b). These results are in good agreement with the analitical treatment of Example 2 before.

Next, we consider a second, more realistic model. Here, the plasma and the vacuum vessel filaments are obtained from the solution of the plasma equilibrium problem. The resistive MHD equilibrium and transport code DINA [2] is used to calculate the plasma equilibrium and to find the plasma filament coordinates and currents.

An RDM code has been used to study plasma vertical stability. The code has option of retaining or neglecting inertia effects in the calculation. This code is described in Ref.[7]. The IGNITEX [9-11] tokamak configuration is considered here (Fig.5). The currents in the plasma and the poloidal-field coils are defined in the flat-top regime. The vacuum vessel resistiviy is 100 times larger than the copper resistivity. The passive growth time determined by the vessel is $t = 20\ ms$. Two different values for the resistivity of the active coils are considered.

Fig.6 shows the behaviour of the roots when the resistivity of the active coil is equal to the resistivity of copper. If $t_1 = 0$, then the real part of the conjugate pair of roots remains negative and the stability of the plasma is determined by the maximum root when inertia effects are neglected (Fig.6-a). If a derivative term is added to the voltage ($t_1 = 5\ ms$), then the real part of the conjugate roots becomes positive for gains $g_1 > 1100\ V/cm$; in the massless approximation the system is stable (Fig.6-b).

The roots when the resistivity of the active coil is 100 times larger than the resistivity of copper are shown in Fig.7. With a proportional control law ($t_1 = 0$), the real part of the two conjugate roots is negative (Fig.7-a) and the stability of the plasma is determined by the maximum real part of the roots when inertia effects are neglected. If a derivative term with $t_1 = 2\ ms$ is added to the voltage control law, then for gain values $g_1 > 1500 V/cm$, system becomes unstable. These results are qualitatively described by our previous simple analitical model with $C_1 < 0$.

Conclusions

It has been shown analitically and numerically that the plasma inertia effects should be retained in RDM analysis of tokamak plasmas. A massless approximation when used in the simulation of the active feedback system [4-6] may result in the incorrect determination of the plasma stability region. The RDM system of equations when inertia effects are retained has two extra complex conjugate roots when compared with the massless approximation case. The other roots are similar in both systems (with and without inertia effects), since the plasma mass is small. The behaviour of the real parts of these two extra roots determine the differences between both methods. When the real part is negative, then the stability region is the same in both systems. However, when the real part of these two extra roots is positive, then the system in which inertia effects have been retained becomes unstable while the system in the massless approximation remains stable. In the massless approximation the stability region obtained is wider than the one obtained with inertia effects retained.

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Figure captions

Fig.1. Stability diagram, gain vs lead time. Control law $V = -g_1(z + t_1\dot{z})$; $g^0 = S_b R_1/S_1$. The stable region is S.

Fig.2. Stability diagram, gain vs lead time. Control law $V = -g_1(z + t_1\dot{z})$. $g^0 = S_b R_1/S_1$. If $g^* > g^0$, then region 1 is stable. If $g^* < g^0$, then region 2 is stable. The limiting value t^* is given by the relation: $\frac{C_3}{C_2 - t^* C_1} = \frac{S_b R_1}{S_1}$.

Fig.3. Real part of the eigenvalues as a function of the gain values for a simple tokamak model. The resistivity of the active coil and the passive conductor is 100 times larger than the resistivity of copper.

a) Control input signal with control law $V = -g_1 z$.

b) Control input signal with control law $V = -g_1(z + t_1\dot{z})$, ($t_1 = 0.1$ s).

Line 1: real part of the two complex-conjugate eigenvalues with mass effects retained.

Line 2: maximum eigenvalue in the massless approximation.

Fig.4. Real part of the eigenvalues as a function of the gain values for a simple tokamak model. The resistivity of the active coil is the resistivity of copper and the resistivity of the passive conductor is 100 times larger than the resistivity of copper.

a) Control input signal with law $V = -g_1 z$.

b) Control input signal with law $V = -g_1(z + t_1\dot{z})$, ($t_1 = 0.1$ s).

Line 1: real part of the two complex-conjugate eigenvalues with mass effects retained.

Line 2: maximum eigenvalue in the massless approximation.

Fig.5. Tokamak configuration considered for calculations (Figs.6 and 7).

Fig.6. Real part of the eigenvalues as a function of the gain values. The resistivity of the active coil is the resistivity of copper and the resistivity of the vacuum vessel is 100 times larger than the resistivity of copper.

a) Control input signal with law $V = -g_1 z$.

b) Control input signal with law $V = -g_1(z + t_1\dot{z})$, ($t_1 = 5$ ms).

Line 1: real part of the two complex-conjugate eigenvalues with mass effects retained.

Line 2: maximum eigenvalue in the massless approximation.

Fig.7. Real part of the eigenvalues as a function of the gain values. The resistivities of the active coil and the vaccum vessel are 100 times larger than the resistivity of copper.

a) Control input signal with law $V = -g_1 z$.

b) Control input signal with law $V = -g_1(z + t_1 \dot{z})$, $(t_1 = 2\ ms)$.

Line 1: real part of the two complex-conjugate eigenvalues with mass effects retained.

Line 2: maximum eigenvalue in the massless approximation.

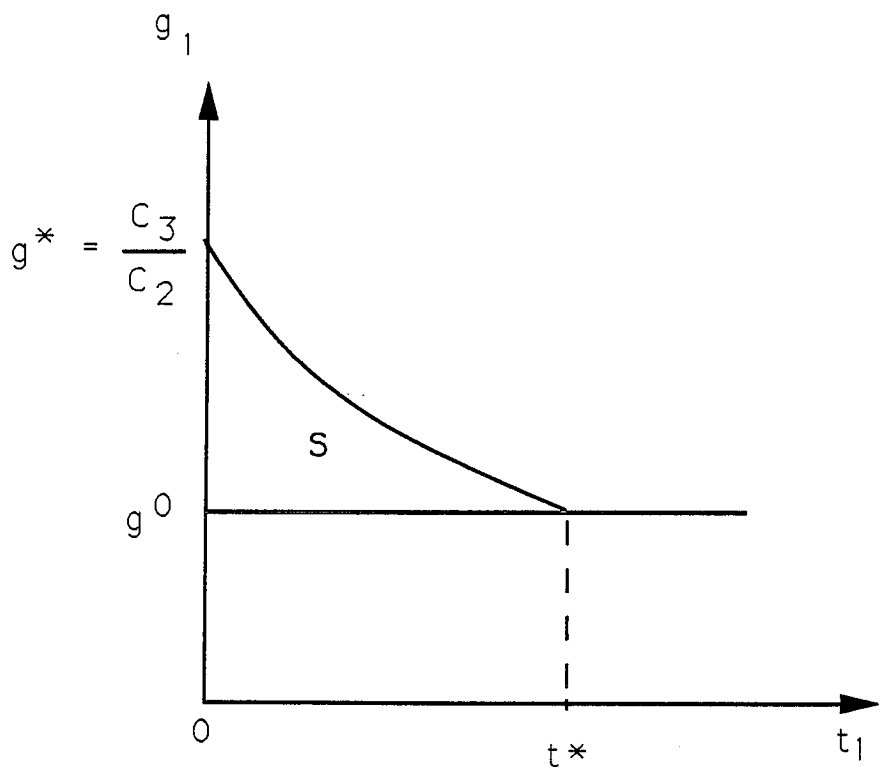


Fig.1

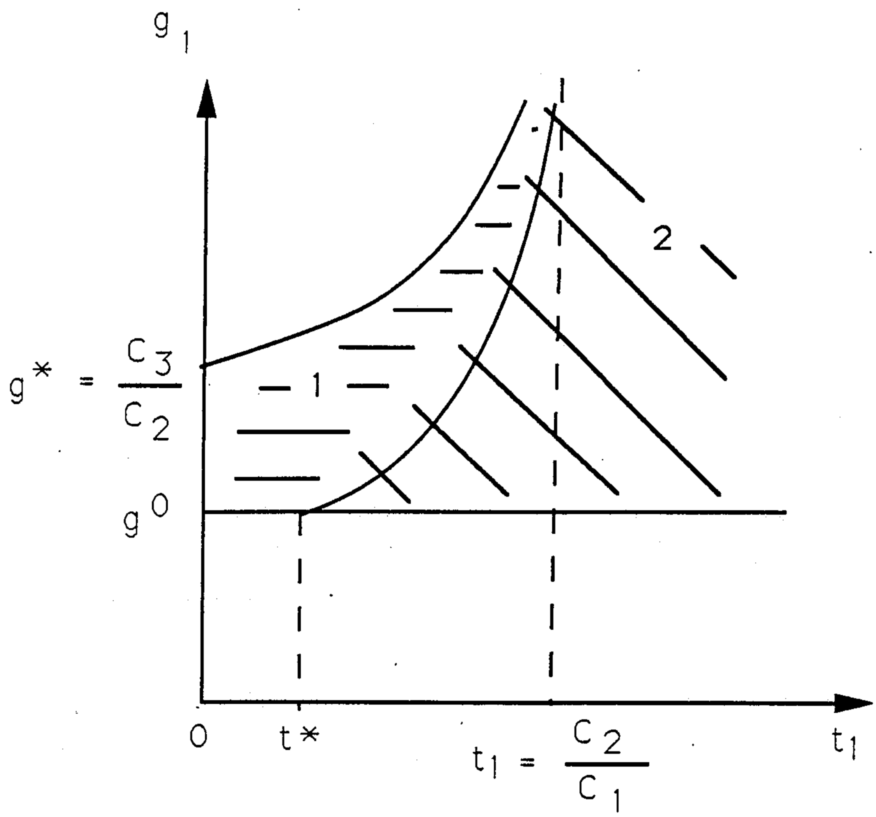


Fig.2

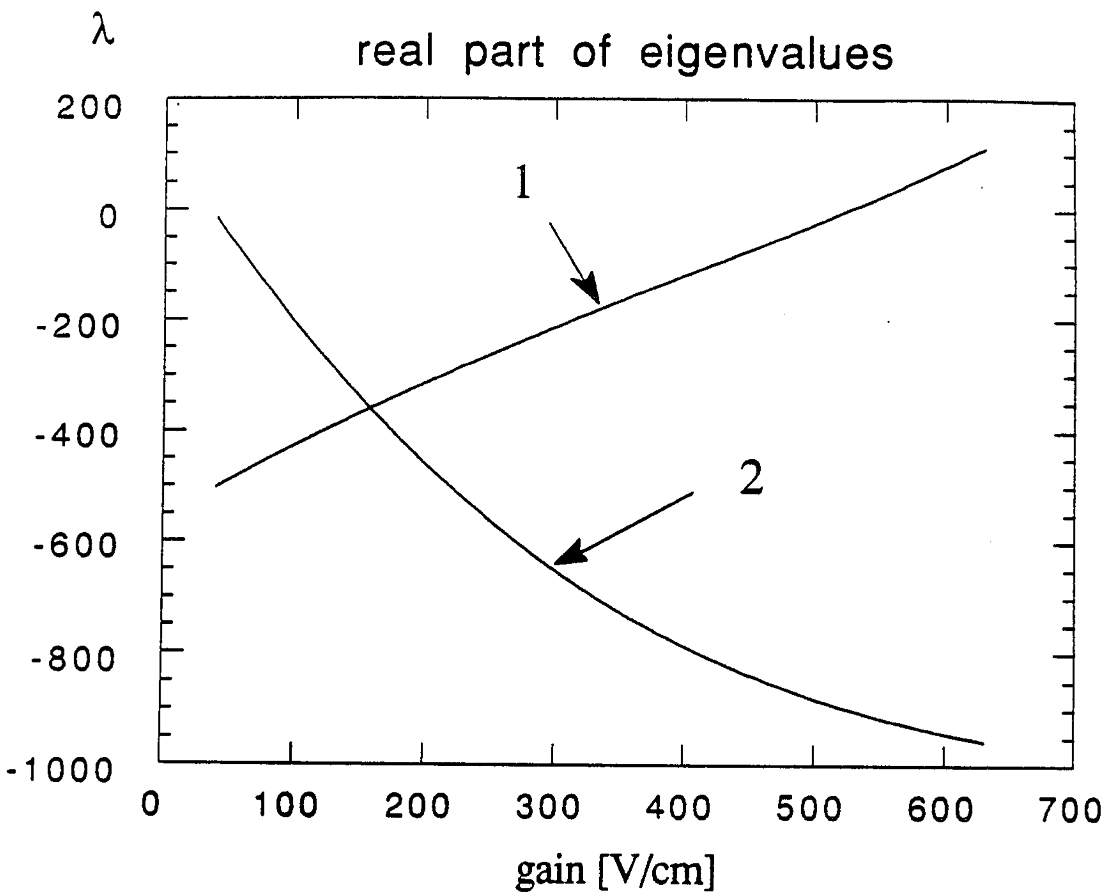

λ
real part of eigenvalues
200
0
-200
-400
-600
-800
-1000
0
100
200
300
400
500
600
700
1
2
gain [V/cm]

Fig.3-a

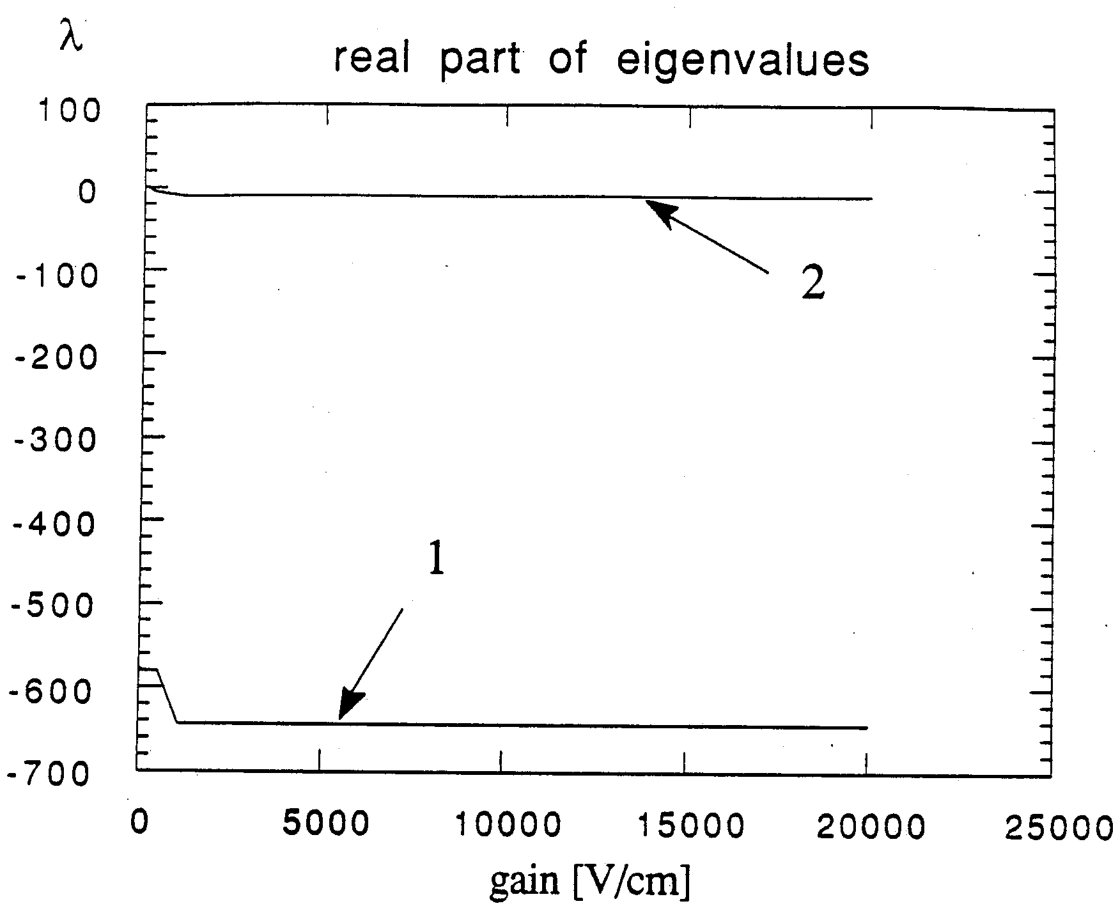
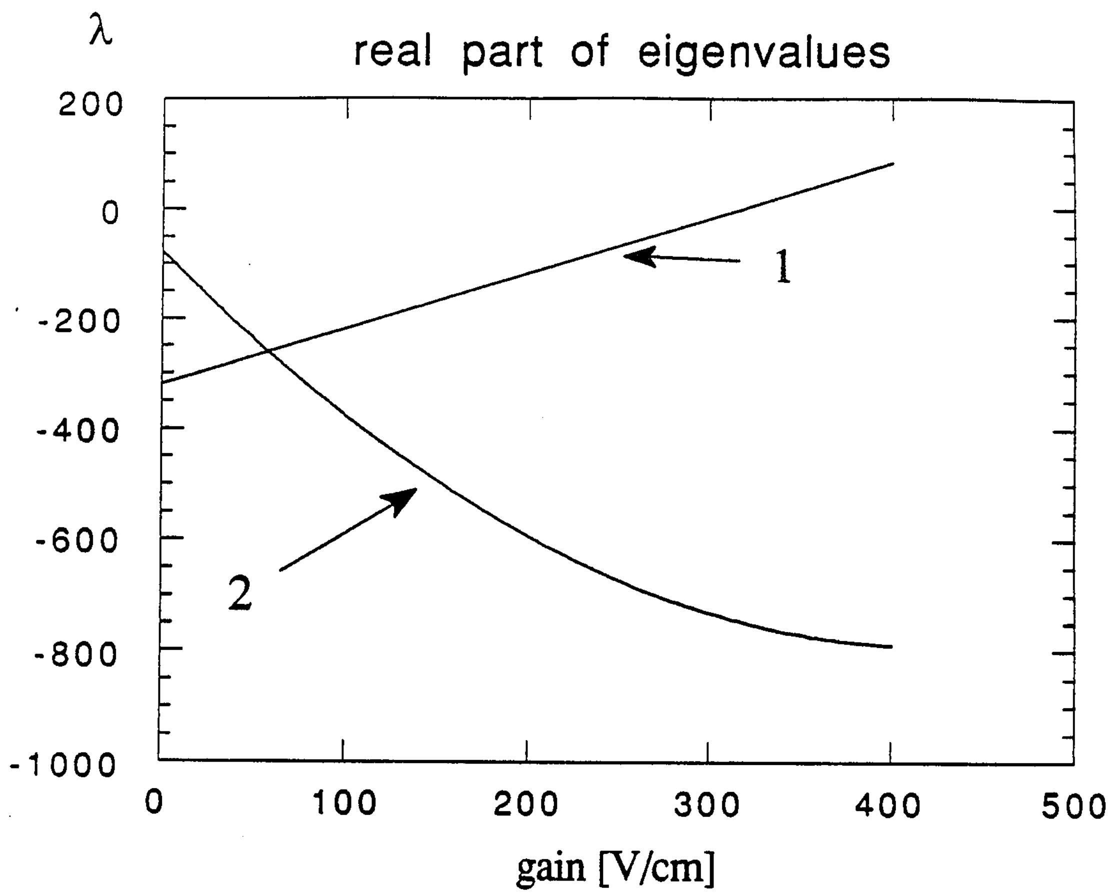
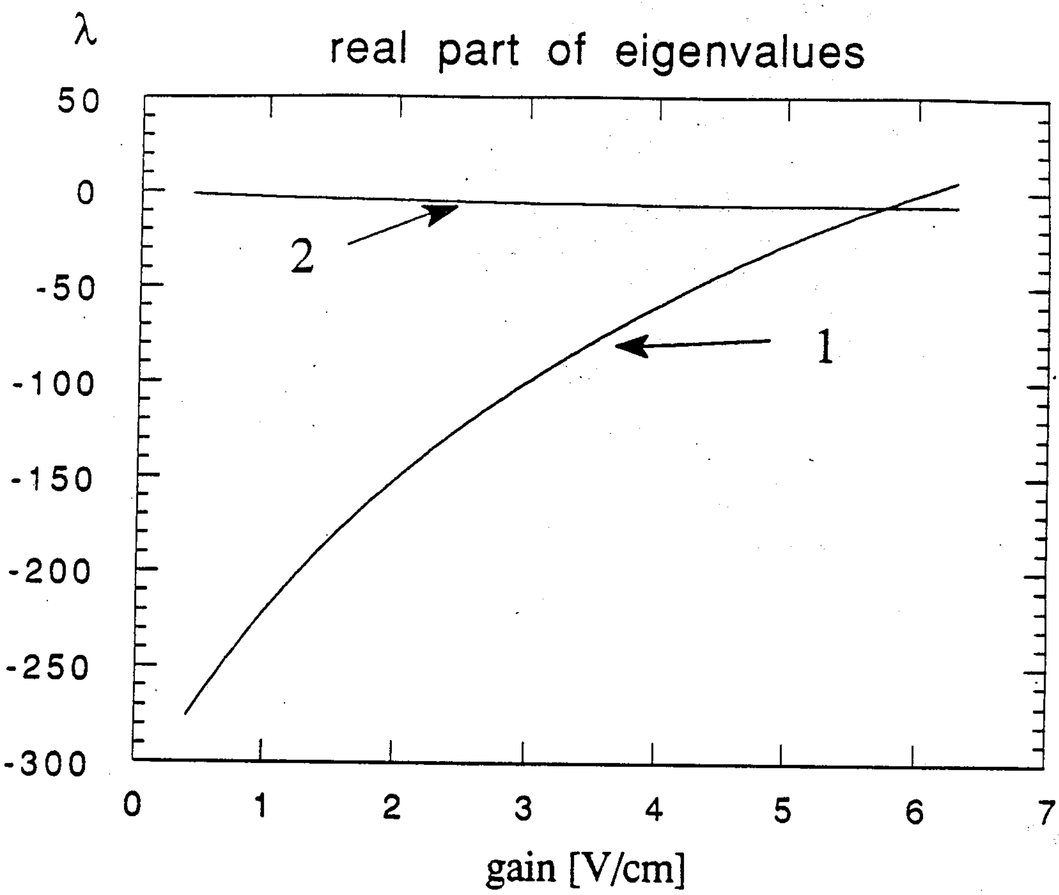

λ
real part of eigenvalues
100
0
-100
-200
-300
-400
-500
-600
-700
0
5000
10000
15000
20000
25000
gain [V/cm]
1
2

Fig.3-b



λ
real part of eigenvalues
200
0
-200
-400
-600
-800
-1000
0
100
200
300
400
500
gain [V/cm]
1
2

Fig.4-a



λ
real part of eigenvalues
50
0
-50
-100
-150
-200
-250
-300
0
1
2
3
4
5
6
7
2
1
gain [V/cm]

Fig.4-b

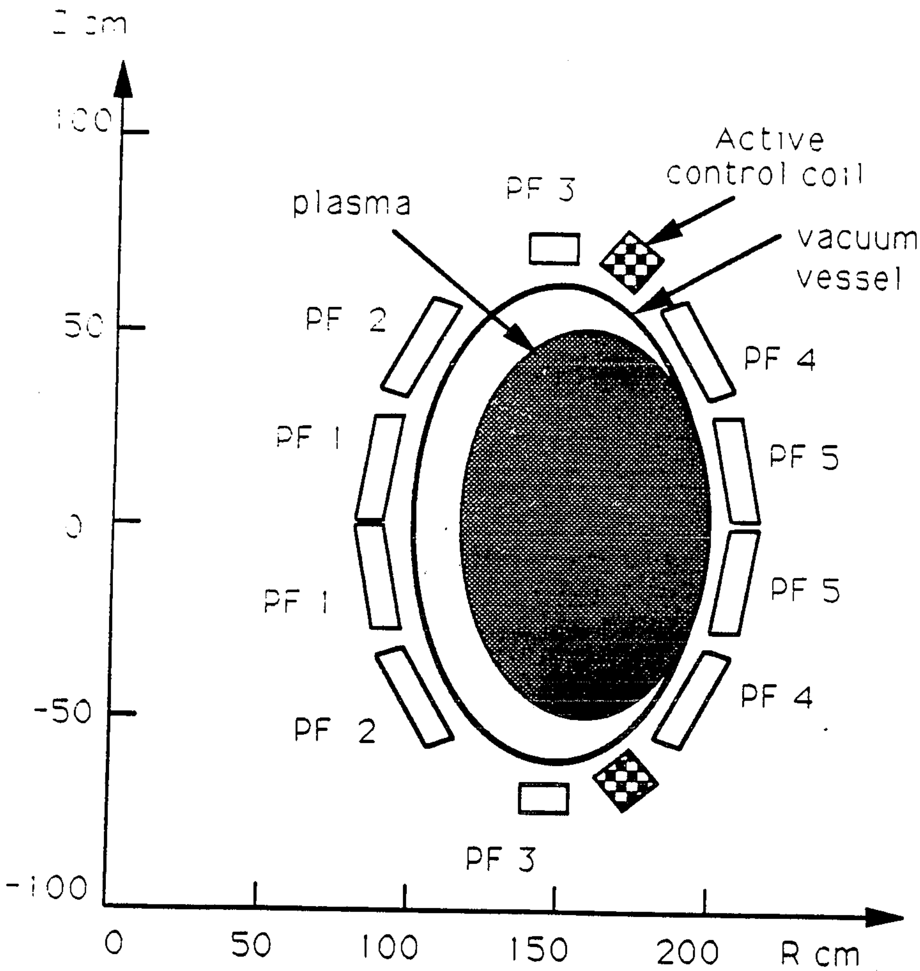

Z cm
100
50
0
-50
-100
0
50
100
150
200
R cm
plasma
PF 3
Active
control coil
vacuum
vessel
PF 2
PF 4
PF 1
PF 5
PF 1
PF 5
PF 4
PF 2
PF 3

Fig.5

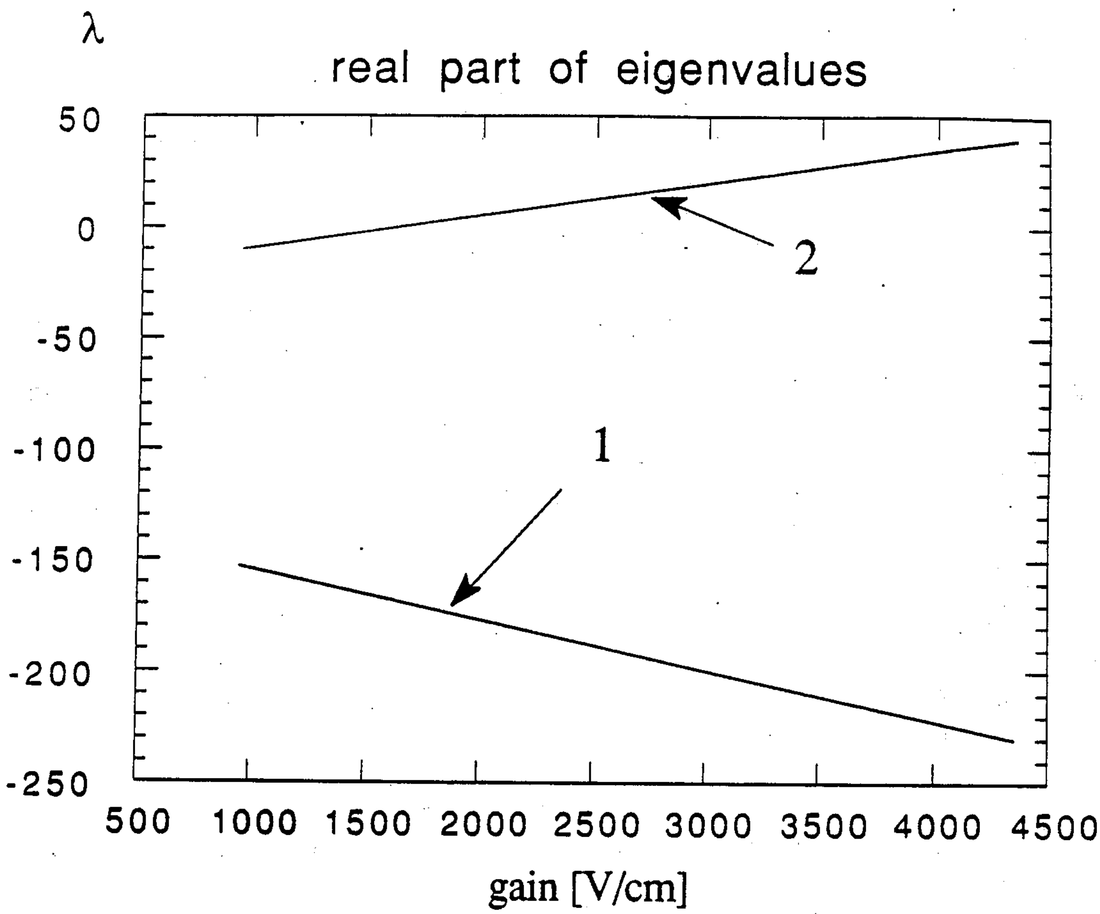

λ
real part of eigenvalues
50
0
-50
-100
-150
-200
-250
500
1000
1500
2000
2500
3000
3500
4000
4500
gain [V/cm]
2
1

Fig.6-a

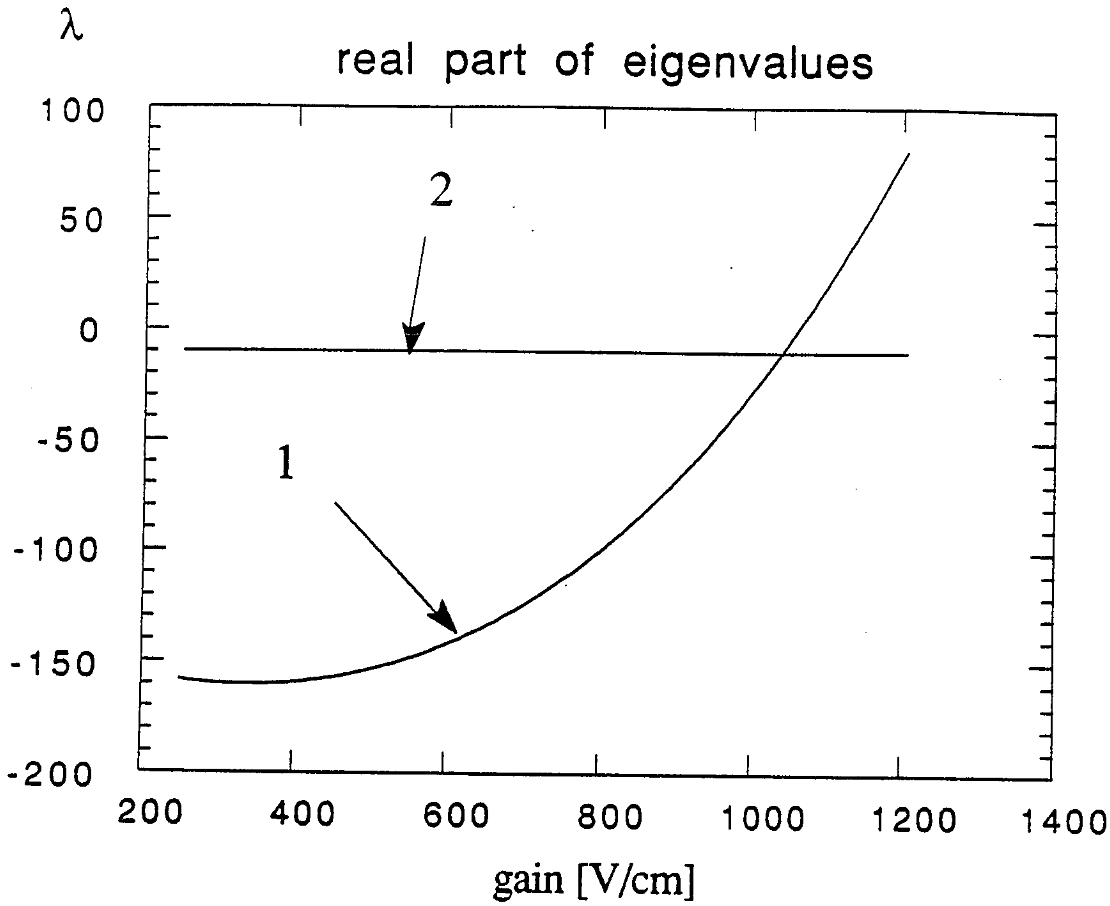

λ
real part of eigenvalues
100
50
0
-50
-100
-150
-200
2
1
200
400
600
800
1000
1200
1400
gain [V/cm]

Fig.6-b