On Broken Ballooning Symmetry

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A 2-D ballooning transform is devised to investigate ballooning symmetry breaking effects. It is found that there are stringent limitations on the use of 1-D eigenmode equations to describe plasma stability.

The problem of plasma stability in an axisymmetric torus (like a tokamak) is intrinsically two-dimensional (2-D); i.e., the associated eigenvalue equation is a partial differential equation in two variables. For modes with large toroidal mode number n, it is generally believed that the modes are localized about a rational surface. Such a mode is conveniently expressed as

$$\Phi(x,\theta,\zeta) = \exp(in\zeta - im\theta)\sum_{\ell}\exp(-i\ell\theta)\phi(x,\ell) \ , \tag{1}$$

where $\zeta(\theta)$ is the toroidal (poloidal) angle, and $x = \hat{s}k_\theta(r - r_0)$ with $k_\theta = m/r_0$, and the shear parameter $\hat{s} = [r_0/q(r_0)](dq/dr)_{r_0}$, is the normalized local radial variable. Since ℓ characterizes the number of sidebands coupled to the central Fourier mode (n,m), the localization of the total mode to a small region, $(r - r_0)/r_0 \ll 1$, necessarily implies $\ell \ll m$.

The ansatz of a local mode ($\ell \ll m$) immediately reveals the approximate translational symmetry[1] $x \to x+1, \ell \to \ell + 1$ obeyed by the operator for the 2-D eigenmode $\phi(x,\ell)$. It is this translational invariance which we call the 'ballooning symmetry' that reduces the intrinsic 2-D eigenvalue to a 1-D problem. The translational invariance is manifestly displayed

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in the ballooning representation constructed by taking the ballooning transform of the 2-D equation for $\phi(x,\ell)$ converting it into the well-known 1-D ballooning equation.[1,2]

To investigate the appropriateness of the ballooning equations for the description of plasma stability, let us introduce a 2-D ballooning transform

$$\phi(x,\ell) = \oint d\lambda\, dk \exp[ik(x-\ell) - i\lambda\ell]\widehat{\varphi}(k,\lambda) \ , \tag{2}$$

which can be viewed as a generalization of the Lee-Van Dam representation;[1] the latter is the limiting case of Eq. (2) as $\widehat{\varphi}(k,\lambda) \to \widehat{\varphi}(k)\delta(\lambda-\lambda_0)$. It is obvious from this observation that the ballooning equation is meaningful only if the 2-D wave function $\widehat{\varphi}(k,\lambda)$ is localized at a certain λ_0. Making use of Eq. (2), one can easily obtain the transform of the 2-D equation for $\phi(x,\ell)$ by the set of substitutions: $x-\ell \to i\partial/\partial k$, $\ell \to -i\partial/\partial\lambda$, $\partial/\partial x \to ik$, $\phi(x,\ell) \to \widehat{\varphi}(k,\lambda)$, and $\phi(x,\ell\pm 1) \to \exp[\mp i(k+\lambda)]\widehat{\varphi}(k,\lambda)$. In the transformed equation, the symmetry breaking terms (the terms not invariant under the transform $x \to x+1$, $\ell \to \ell+1$, e.g. terms proportional to ℓ/m etc.) will appear as derivatives with respect to λ. Because of the assumed localization in $\lambda(\sim\lambda_0)$, even for large $m = nq(r_0)$, these derivative terms should not be neglected without a proper analysis. In other words, the stability properties determined by the ballooning equation alone would not be consistent with the original assumptions (in particular, $\ell/m \ll 1$), unless one can show that in the presence of the symmetry breaking terms, the quantity $|\Delta\lambda| = |(\partial/\partial\lambda)\ell n\,\widehat{\varphi}(k,\lambda)|^{-1}$, which is the inverse of the effective rate of change of the 2-D mode $\widehat{\varphi}(k,\lambda)$ (localized at λ_0, the parameter of the ballooning equation), is sufficiently large that the contribution of terms containing $(1/n)\partial/\partial\lambda, (1/n^2)\partial^2/\partial\lambda^2 \cdots$ is negligible. More precisely, the inequality $2\pi \gg |\Delta\lambda| \gg 1/n$ must be satisfied.

Without loss of generality, we consider a fluid model for drift waves in an axisymmetric tokamak. The linear equation is

$$\rho_s^2\,\nabla_\perp^2\Phi - \left(1 - i\delta_e - \frac{\widehat{\omega}_e^*}{\omega}\right)\Phi - \frac{c_s^2}{\omega^2}\,\nabla_\parallel^2\Phi - 2\,\frac{\widehat{\omega}_{de}}{\omega}\,\Phi = 0 \ , \tag{3}$$

where $\nabla_{\|} = [q(r)\,\partial/\partial\zeta + \partial/\partial\theta]/qR$, $\nabla_\perp^2 = (1/r)(\partial/\partial r)r(\partial/\partial r) + (1/r^2)(\partial^2/\partial\theta^2)$, $\widehat{\omega}_e^* = -i(T_e c/Ben_0 r)(dn_0/dr)(\partial/\partial\theta)$, $c_s^2 = T_e/m_i$, $\widehat{\omega}_{de} = -i(T_e c/Be\,Rr)(\sin\theta r\,\partial/\partial r + \cos\theta\,\partial/\partial\theta)$, $\rho_s^2 = T_e c/eB\,\omega_{ci}$ with ω the mode frequency, T_e the electron temperature, n_0 the plasma density, B the magnetic field, $e(>0)$ the electron charge, R the major radius of the torus, r the radial position, m_i the ion mass, c the speed of light, $\omega_{ci} = eB/cm_i$ the ion cyclotron frequency, and δ_e stands for the electron dissipation. For simplicity B and R are assumed constant throughout the letter. Defining the variable x by $x = n[q(r) - q(r_0)]$ with r_0 given by $q(r_0) = m/n$, we take the ballooning transform [Eq. (2)] of Eq. (3) to derive the 2-D equations for $\widehat{\varphi}(k,\lambda)$,

$$\left[\mathcal{L}^{(0)} + \mathcal{L}^{(1)}\frac{\partial}{\partial\lambda} + \mathcal{L}^{(2)}\frac{\partial^2}{\partial\lambda^2} + \mathcal{L}^{(\bar{1})}\right]\widehat{\varphi}(k,\lambda) = 0 \tag{4}$$

with $\mathcal{L}^{(i)} = \Pi_1^{(i)}\,\partial^2/\partial k^2 + \Pi_2^{(i)}\,k^2 + \Pi_3^{(i)} + \cos(k+\lambda)\Pi_4^{(i)} + \sin(k+\lambda)\Pi_5^{(i)}k \quad (i = 0, 1, 2, \bar{1})$, where $\Pi_j^{(0)} \sim \mathcal{O}(1)$, $\Pi_j^{(1)} \sim \mathcal{O}(1/n)$, $\Pi_j^{(2)} \sim \mathcal{O}(1/n^2)$ are independent of k, λ, determined completely by the local parameters, and $\Pi_i^{(\bar{1})} = f(k) + g(k)\,\partial/\partial k \sim \mathcal{O}(1/n)$. Expressions for all Π's can be derived in a straightforward manner. For example, $\Pi_1^{(1)} = 2i\Pi_1^{(0)}/nq(r_0)$ with $\Pi_1^{(0)} = (c_s/\omega qR\,\rho_s\,k_\theta\widehat{s})^2_{r_0}$, and $\Pi_2^{(1)} = -(2i/n)(d^2q/dr^2)_{r_0}/(dq/dr)^2_{r_0}$. In Eq. (4) we have assumed that the terms associated with $(1/n^3)(\partial^3/\partial\lambda^3)$, $(1/n^2)(\partial^2/\partial k^2)$, $(1/n^2)(\partial/\partial\lambda)(\partial/\partial k)$ etc., are higher order terms, and are negligible.

Equation (4) can be solved perturbatively. The lowest order equation is the ballooning equation

$$\widetilde{\mathcal{L}}^{(0)}\chi_N(k,\lambda) \equiv \left[\mathcal{L}^{(0)} + \Pi_N(\lambda) - \Pi_3^{(0)}\right]\chi_N(k,\lambda) = 0\ , \tag{5}$$

where $\Pi_N(\lambda)$ is the eigenvalue for an arbitrarily given λ, and N labels the spectrum. In the rest of the letter we deal only with the fundamental eigenmode $\chi \equiv \chi_{N=0}$; we shall drop the subscript N. Since the ballooning equation is invariant under the transform $k \to -k, \lambda \to -\lambda$, $\Pi(\lambda)$ must be an even function of λ. Then, we write $\widehat{\varphi}(k,\lambda) = \psi(\lambda)\chi(k,\lambda) + \varphi_1(k,\lambda) + \varphi_2(k,\lambda)$, with $\varphi_1 \sim \epsilon$, $\varphi_2 \sim \epsilon^2$, treating ϵ as the perturbation parameter. Noting that $(\partial/\partial\lambda)\ell n\, \chi(k,\lambda) \sim \mathcal{O}(1)$, we expect $\psi(\lambda)$ to be a fast varying function in λ, so that $2\pi \gg |\Delta\lambda|$ can be satisfied. The first order equation is

$$\left(\frac{d\psi}{d\lambda}\right) \mathcal{L}^{(1)}\chi + \widetilde{\mathcal{L}}_0\varphi_1 = 0 \ , \tag{6}$$

implying that φ_1 is proportional to $(d\psi/d\lambda)$; i.e., $\varphi_1 = (d\psi/d\lambda)\,\widetilde{\varphi}_1$ which converts Eq. (6) into

$$\widetilde{\mathcal{L}}_0\,\widetilde{\varphi}_1 + \mathcal{L}^{(1)}\chi = 0 \ . \tag{7}$$

The adjointness of $\widetilde{\mathcal{L}}_0$ yields the solvability condition

$$\int dk\, \chi\, \mathcal{L}^{(1)}\chi = 0 \ , \tag{8}$$

which will be examined later. The second order equation, given by

$$\frac{d^2\psi}{d\lambda^2} + Q(\lambda)\,\psi(\lambda) = 0 \ , \tag{9}$$

where $Q(\lambda) = \left[\Pi_3^{(0)} - \Pi(\lambda) + \left\langle\overline{\chi\,\mathcal{L}^{(1)}(\partial\chi/\partial\lambda)}\right\rangle + \left\langle\overline{\chi\,\mathcal{L}^{(\bar{1})}\chi}\right\rangle\right] \Big/ \left[\left\langle\overline{\chi\,\mathcal{L}^{(2)}\chi}\right\rangle + \left\langle\overline{\chi\,\mathcal{L}^{(1)}\,\widetilde{\varphi}_1}\right\rangle\right]$ with $\langle\overline{\cdots}\rangle = \int dk \cdots \Big/\!\!\int dk\,\chi^2$, determines $\psi(\lambda)$, and the 2-D eigenvalue. Equation (9) is a Hill equation; the λ-dependence in $Q(\lambda)$ comes from $\cos\lambda$ and $\sin\lambda$. Notice that $Q(\lambda)$ is proportional to a large number n^2 arising from the smallness of $\mathcal{L}^{(2)}$ and $\mathcal{L}^{(1)}\,\widetilde{\varphi}_1$. If the λ-dependent part of $\Pi(\lambda)$ is not very small (generically it is $\mathcal{O}(1)$), the most localized $\psi(\lambda)$ can be obtained by expanding $\psi(\lambda)$ at its minimum λ_0, which yields a Weber equation. The fundamental solution is $\psi(\lambda) \sim \exp[-\alpha n(\lambda-\lambda_0)^2]$ ($\alpha \sim \mathcal{O}(1)$) leading to the expected ordering $\Delta\lambda \sim 1/\sqrt{n} \sim \epsilon$, that satisfies the requirement $2\pi \gg |\Delta\lambda| \gg 1/n$ for the validity of the ballooning equation. For example, if the λ-dependent part of $\Pi(\lambda) \sim \cos\lambda$ (a good approximation if toroidal coupling is not strong), λ_0 can only be 0, or π, with a correction of $\mathcal{O}(1/n)$, depending on the sign of $\left\langle\overline{\chi\mathcal{L}^{(2)}\chi}\right\rangle + \left\langle\overline{\chi\mathcal{L}^{(1)}\widetilde{\varphi}_1}\right\rangle$. The $\mathcal{O}(1/n)$ shift of λ_0 from 0, or π is caused by the terms $\left\langle\overline{\chi\mathcal{L}^{(1)}(\partial\chi/\partial\lambda)}\right\rangle + \left\langle\overline{\chi\mathcal{L}^{(\bar{1})}\chi}\right\rangle$ in $Q(\lambda)$, which are odd functions in λ. The effective eigenvalue of Eq. (9), $\Pi_3^{(0)} - \Pi(\lambda_0)$, is a quantity of $\mathcal{O}(1/n)$ which necessitates the inclusion of terms like $\left\langle \overline{\chi \mathcal{L}^{(1)}(\partial\chi/\partial\lambda)} \right\rangle$ to obtain an eigenvalue fully correct to $\mathcal{O}(1/n)$. When the toroidal coupling is strong, the λ-dependence in $Q(\lambda)$ can be very different from $\cos\lambda$, and more than one extrema could emerge. This is indeed the behavior for the ideal ballooning mode in some range of parameters; two additional minima are observed about $\lambda \sim \pm\pi/2$.

From the preceding discussion it would appear that the broken ballooning symmetry has only an unimportant effect, an $\mathcal{O}(1/n)$ correction to the eigenvalue predicted by the ballooning equation. However, this may not be true. The broken ballooning symmetry also requires that the solvability condition, Eq. (8), be satisfied; i.e., the localized mode may exist only at some r_0 for a given equilibrium with an envelope width $\Delta r \sim r_0/\sqrt{n}$. When the solvability condition is not satisfied, a fast variation of $\psi(\lambda)$ ($\Delta\lambda \sim 1/n$) is superimposed on the already obtained intermediate variation ($\Delta\lambda \sim 1/\sqrt{n}$), so that all higher order derivatives with respect to λ, such as $(1/n^3)(\partial^3/\partial\lambda^3)$ etc., can not be neglected in Eq. (4), implying $\ell/m \sim \mathcal{O}(1)$, which is inconsistent with the basic assumption of a localized mode.

It is crucial, therefore, to focus our attention on the solvability condition Eq. (8). The solvability condition, in general, will be complex save for some special modes like the 'ideal mode'. The discussion on the real solvability condition will be presented elsewhere. For the complex form, it leads to two real simultaneous equations, which place strong limitations on the existence of localized modes. To illustrate the general situation we concentrate in this letter on drift waves described by the fluid model Eq. (2).

At this point, we would draw the reader's attention to the fact that for analyzing the effects of broken ballooning symmetry, our theory is *quantitatively* different from the theory of Connor *et al.*[2] In the latter theory the symmetry breaking effects arise solely from the variation of equilibrium, while our analysis includes the equally important sideband coupling. The term $(1/r^2)(\partial^2/\partial\theta^2)$ is, for instance, approximated by $(m/r)^2$ in Ref. 2, however, it should be $(m/r)^2(1+\ell/m)^2$, the form used in our 2-D ballooning transform.

For a given equilibrium there are only two adjustable parameters in the solvability condition: r_0 and n, when the mode frequency is expressed in terms of the solution of the ballooning equation. The toroidal mode number n has an upper bound because very large n drift waves are physically uninteresting. We find that for n numbers, in the range of interest, the ensuing complex solvability condition can rarely be satisfied for the fluid drift waves. A typical result illustrating the aforementioned difficulty, is shown in Fig. 1, where we plot Re F and Im F as functions of r_0 for a few values of n. The function $F(\rho)$ in Fig. 1 is defined by $iF(\rho)/nq(r_0) = \left\langle \overline{X\mathcal{L}^{(1)}X} \right\rangle$, where $\rho = r_0/a$ is the radial position normalized to the plasma minor radius a. Required expressions containing X and the eigenvalue are obtained by solving the ballooning equation numerically. For the equilibrium, we choose a constant density scale length L_n, a constant electron dissipation δ_e, a T_e-profile $T_e(\rho) = T_e(0)(1-\rho^2)^2$, and a q-profile $q(\rho) = q_0 + (q_a - 2q_0)\rho^2 + q_0\,\rho^4$. If, for a given equilibrium, $F(\rho)$ is never very close to zero throughout the plasma minor radius, then the localized drift wave predicted by the ballooning equation analysis turns out to be inconsistent with the assumptions, and can not exist within the model.

The plasma stability predicted by the ballooning equation, therefore, is acceptable only if the solvability condition in a given equilibrium has been shown to be satisfied; i.e., there exist a set(s) (n, r_0) for which $F(\rho) = 0$. Evidently one may not expect a solution for any particular n. On the other hand, even if the localized modes indeed exist, they may not be the most unstable modes.

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Appendix A – The Explicit Expressions of $\Pi_j^{(i)}$ for Drift Waves

Defining $x = n[q(r) - q(r_0)]$, and substituting Eq. (1) into Eq. (3), we obtain the 2D-equation for $\phi(x, \ell)$

$$
\begin{aligned}
&\frac{q'}{\tilde{r}\, q_0'} \frac{d}{dx} \tilde{r} \frac{q'}{q_0'} \frac{d}{dx} \phi(x,\ell) + \frac{\omega_s^2}{\omega^2} (x-\ell)^2 \phi(x,\ell) \\
&\quad - \left[\frac{1 - i\delta_e - \tilde{\omega}_e^*}{\rho_s^2\, k_\theta^2\, \hat{s}^2} + \frac{1}{\tilde{r}^2\, \hat{s}^2} \left(1 + \frac{\ell}{m} \right)^2 \right] \phi(x,\ell) \\
&\quad + \frac{T_e\, cm}{Be\, R r \omega} \cdot \frac{1}{\rho_s^2\, k_\theta^2\, \hat{s}^2} \left[\left(1 + \frac{\ell+1}{m} + r \frac{q'}{q_0} \frac{d}{dx} \right) \phi(x, \ell+1) \right. \\
&\quad \left. + \left(1 + \frac{\ell-1}{m} - r \frac{q'}{q_0} \frac{d}{dx} \right) \phi(x, \ell-1) \right] = 0 \,,
\end{aligned}
\tag{A1}
$$

where $\tilde{\omega}_e^* \equiv -[T_e(r) c (m+\ell)/B\, n_0(r) e r]\,(dn_0/dr)$, $\tilde{r} = r/r_0$, $k_\theta = m/r_0$, $q_0 = q(r_0)$, $q' = dq/dr$, $\hat{s} = (rq'/q)_{r=r_0}$, and $\omega_s = (C_s/Rq)/\rho_s\, k_\theta\, \hat{s}$. When we demand $r = r_0$, and $\ell = 0$ in Eq. (A1), and use the 1D ballooning representation, Eq. (A1) is converted into the ballooning equation. To evaluate the effect of broken ballooning symmetry, we should retain the terms proportional to $(r - r_0)/r_0$, and ℓ/n to second order. In accordance with the ballooning symmetry, we expand the function $G[r(q), \ell]$ for small $y/n = (x-\ell)/n$, and ℓ/n

$$
\begin{aligned}
G[r(q), \ell] &= G(r_0, 0) + \left(\frac{\partial G}{\partial r} \right)_{r_0,0} \cdot \frac{y}{n q_0'} + \left(\frac{\partial G}{\partial r} \cdot \frac{1}{q'} + \frac{\partial G}{\partial \ell} \cdot \frac{1}{q} \right)_{r_0,0} \frac{\ell}{n} \\
&+ \frac{1}{2} \left(\frac{\partial^2 G}{\partial r^2} - \frac{q''}{q'} \frac{\partial G}{\partial r} \right)_{r_0,0} \frac{y^2}{n^2\, q_0'^2} + \left[\frac{1}{2q'^2} \left(\frac{\partial^2 G}{\partial r^2} - \frac{q''}{q'} \frac{\partial G}{\partial r} \right) + \frac{\partial^2 G}{\partial r\, \partial \ell} \cdot \frac{1}{qq'} \right]_{r_0,0} \frac{y\ell}{n^2} \\
&+ \left[\frac{1}{2q'^2} \left(\frac{\partial^2 G}{\partial r^2} - \frac{q''}{q'} \frac{\partial G}{\partial r} \right) + \frac{\partial^2 G}{\partial r\, \partial \ell} \cdot \frac{1}{qq'} + \frac{1}{2q^2} \frac{\partial^2 G}{\partial \ell^2} \right]_{r_0,0} \frac{\ell^2}{n^2} + \cdots \,,
\end{aligned}
\tag{A2}
$$

where $q'' = d^2q/dr^2$.

Making use of the 2D ballooning transform Eq. (2), and Eq. (A2), we convert Eq. (A1) into Eq. (4), where the higher order terms are neglected. All relevant $\Pi_j^{(i)}$ of Eq. (4) are given explicitly below

$$\Pi_1^{(0)} = \omega_s^2(r_0)/\omega^2 \ , \tag{A3}$$

$$\Pi_2^{(0)} = 1 \ , \tag{A4}$$

$$\Pi_3^{(0)} = \Omega_*(0) + 1/\widehat{s}^2 \ , \tag{A5}$$

with $\Omega_*(0) \equiv (1 - i\delta_e - \omega_e^*/\omega)/\rho_s^2(r_0)k_\theta^2\,\widehat{s}^2$ with $\omega_e^* = \widetilde{\omega}_e^*\Big|_{r_0,0}$,

$$\Pi_4^{(0)} = -2\omega_{de}/\omega \ , \tag{A6}$$

with $\omega_{de} = (T_e\, cm/Be\, Rr)/\rho_s^2\, k_\theta^2\, \widehat{s}^2\Big|_{r=r_0}$,

$$\Pi_5^{(0)} = \Pi_4^{(0)}\widehat{s} \ , \tag{A7}$$

$$\Pi_1^{(1)} = (2i/nq_0)\Pi_1^{(0)} \ , \tag{A8}$$

$$\Pi_2^{(1)} = (-2i/n)(q''/q'^2)_{r_0} \ , \tag{A9}$$

$$\Pi_3^{(1)} = (i/nq_0)\left[(2/\widehat{s}^2)(1/\widehat{s} - 1) - (\partial\Omega_*/\partial r)q/q' - \partial\Omega_*/\partial\ell\right]_{r_0,0} \ , \tag{A10}$$

with $(\partial\Omega_*/\partial r)_{r_0,0} = -\Omega_*(0)\left\{(dT_e/dr)/T_e + (\partial\widetilde{\omega}_e^*/\partial r)/[\omega(1 - i\delta_e) - \omega_e^*]\right\}_{r_0,0}$,
and $(\partial\Omega_*/\partial\ell)_{r_0,0} = -\Omega_*(0)\omega_e^*/[\omega(1 - i\delta_e) - \omega_e^*]$,

$$\Pi_4^{(1)} = (i/nq_0)(1/\widehat{s} - 1)\Pi_4^{(0)} \ , \tag{A11}$$

$$\Pi_5^{(1)} = -(i/nq_0)(rq''/q')_{r_0}\,\Pi_4^{(0)} \ , \tag{A12}$$

$$\Pi_1^{(2)} = -(3/n^2q_0^2)\Pi_1^{(0)} \ , \tag{A13}$$

$$\Pi_2^{(2)} = -(1/n^2)(q'''/q'^3)_{r_0} \ , \tag{A14}$$

with $q''' = d^3q/dr^3$,

$$\Pi_3^{(2)} = -(1/2n^2q_0^2)\Big\{[\partial^2\Omega_*/\partial r^2 - (q''/q')(\partial\Omega_*/\partial r)](q^2/q'^2)$$

$$+ 2(\partial^2\Omega_*/\partial r\,\partial\ell)(q/q') + 2[(3 + q''r/q')/\widehat{s}^2 - 4/\widehat{s} + 1]/\widehat{s}^2 \Big\}_{r_0,0} \;, \tag{A15}$$

with $(\partial^2\Omega_*/\partial r^2)_{r_0,0} = \Omega_*(0)\Big\{(\partial\Omega_*/\partial r)/\Omega_*(0) - (d^2T_e/dr^2)/T_e + (dT_e/dr)^2/T_e - (\partial^2\widetilde{\omega}_e^*/\partial r^2 + (\partial\widetilde{\omega}_e^*/\partial r)^2/[\omega(1-i\delta_e) - \omega_e^*])/[\omega(1-i\delta_e) - \omega_e^*]\Big\}_{r_0,0}$,
and $(\partial\Omega_*/\partial r\,\partial\ell)_{r_0,0} = \Omega_*(0)\omega_e^*\Big\{(dT_e/dr)/T_e + (\partial\widetilde{\omega}_e^*/\partial r)[1 - (\partial\widetilde{\omega}_e^*/\partial\ell)/\omega_e^*]/[\omega(1-i\delta_e) - \omega_e^*]\Big\}/[\omega(1-i\delta_e) - \omega_e^*]\Big|_{r_0,0}$,

$$\Pi_4^{(2)} = \Pi_4^{(0)}(1 - 1/\widehat{s} - rq''/2\widehat{s}\,q')_{r_0}/n^2\,q_0^2\,\widehat{s}\;, \tag{A16}$$

$$\Pi_5^{(2)} = -\Pi_4^{(0)}(rq'''/q'^2 - rq''^2/q'^3)_{r_0}/2n^2\,q_0\;, \tag{A17}$$

$$\Pi_1^{(\bar{1})} = -(2i/nq_0)\Pi_1^{(0)}\,\partial/\partial k\;, \tag{A18}$$

$$\Pi_2^{(\bar{1})} = (i/nq_0'^2)(q'' - q'/r + 2q''k\,\partial/\partial k)_{r_0}/k\;, \tag{A19}$$

$$\Pi_3^{(\bar{1})} = (i/nq_0)[(\partial\Omega_*/\partial r)q/q' - 2/\widehat{s}^3]_{r_0}\,\partial/\partial k\;, \tag{A20}$$

$$\Pi_4^{(\bar{1})} = -(i/\widehat{s}\,nq_0)\Pi_4^{(0)}\,\partial/\partial k\;, \tag{A21}$$

and

$$\Pi_5^{(\bar{1})} = (i/nq_0)\Pi_4^{(0)}(rq''/q')_{r_0}(\partial/\partial k)\;. \tag{A22}$$

References

[1] Y.C. Lee and J.W. Van Dam, in *Proceedings of the Finite Beta Theory Workshop* (Varenna Summer School of Plasma Physics, September, 1977, Varenna, Italy), eds. B. Coppi and B. Sadowski (U.S. Dept. of Energy, Office of Fusion Energy, CONF-7709167, 1979).

[2] J.W. Connor, R.J. Hastie, and J.B. Taylor, Proc. R. Soc. Lon. A **365**, 1 (1979).

Figure Captions

Fig. 1 Complex $F(\rho)$ versus ρ. Curves a (hollow circles), b (hollow squares), c (dark circles), and d (dark squares) are respectively for normalized $\rho_s^2 k_\theta^2 \equiv T_e(0)cn^2/eB\,\omega_{ci}\,a^2 = 0.35, 0.5, 1.0,$ and 2.5 with $q_0 = 1.0$, $q_a = 3.0$, $\delta_e = 0.8$, $L_n/R = 0.2$ and $\lambda_0 = 0$. The numbers by the curves indicate $\rho = r_0/a$.

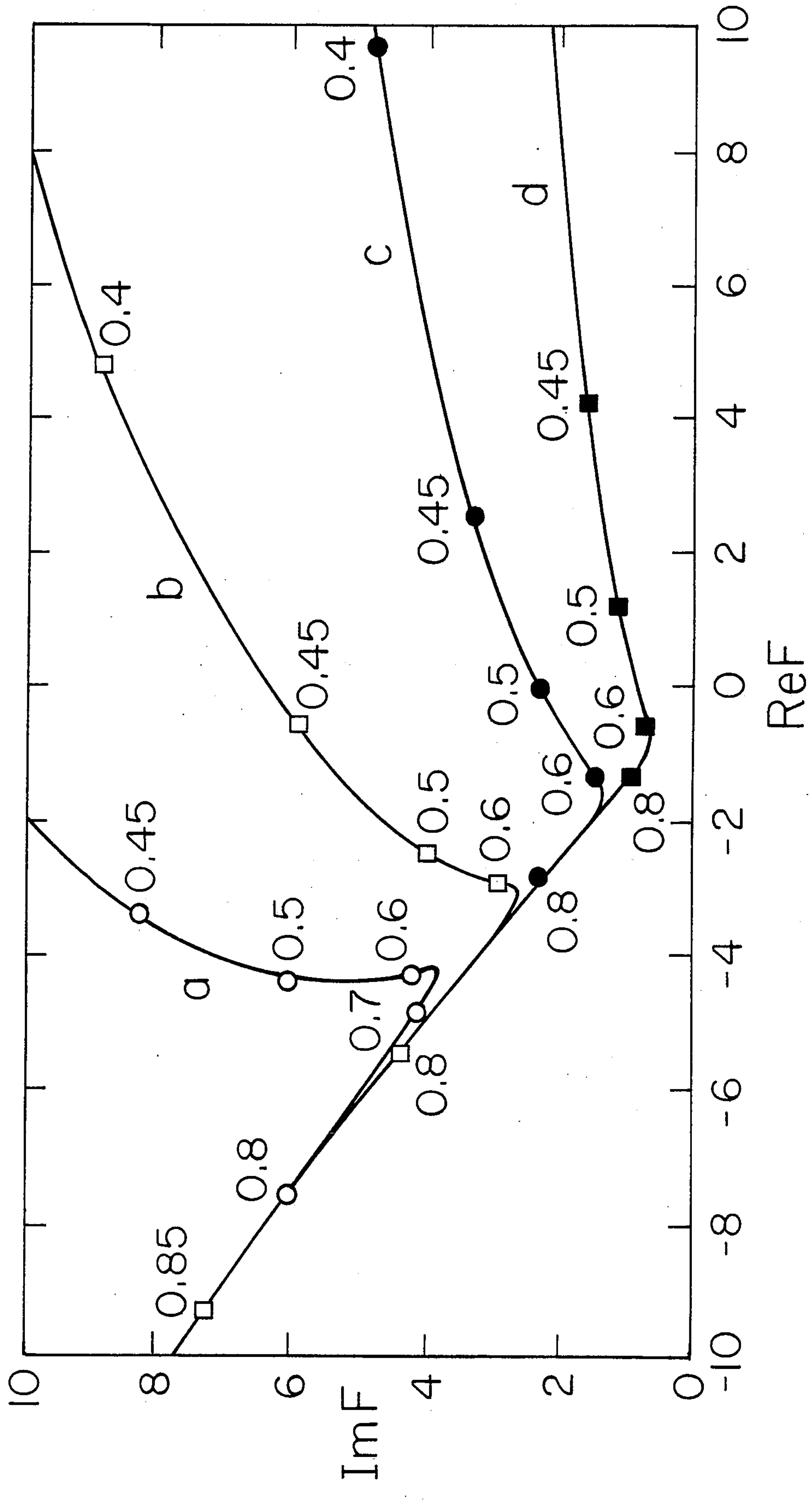

Fig. 1