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Scaling Laws of Stochastic E × B Plasma Transport

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Abstract

We discuss two distinct regimes of cross-field test particle diffusion due to electrostatic turbulence. In the *quasi-linear regime*, which takes place for small amplitude ϕ_0 of the potential perturbation, the local turbulent diffusion coefficient scales as $D_T \propto \phi_0^2$. For the turbulence amplitude approaching or exceeding the mixing length level, the $\mathbf{E} \times \mathbf{B}$ drift of particles becomes sensitive to the topology of contours of constant potential $\phi(\mathbf{r}, t)$ due to the longer correlations of drift orbits. In this regime, the percolation theory is used to describe the long flights of particles along the critical level contours. For a single-scale, random distribution of the electric potential, *the percolation scaling* for the diffusivity $D_T \propto \phi_0^{7/10}$ is derived and applied to edge tokamak transport.

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Short-scale drift plasma turbulence is considered as a primary candidate responsible for the anomalous electron transport [1]. There have been undertaken substantial efforts to study the turbulent $\mathbf{E} \times \mathbf{B}$ transport both self-consistently [2, 3] and for a specified turbulence spectrum [4, 5]. Recently, a basic test particle diffusion experiment by McWilliams [6] reported the scaling of diffusion D_T as a linear function of the amplitude of oscillations ϕ_0, which is different form the quadratic quasi-linear dependence. In this letter we take the non-self-consistent approach, which is an unavoidable part of the complete self-consistent problem. In the frame of this approach, using test particle motion arguments, we show that for electrostatic turbulence, at the level of oscillations well above the threshold of the breaking of quasi-linear theory, electron transport can be described analytically with the help of percolation theory [7]. This theory gives the scaling law for the turbulent diffusion $D_T \propto \phi_0^{7/10}$ that differs from previously reported nonlinear regime of the two-wave stochastic Hamiltonian model [5].

Our assertion is based on the recent work [7] that has discussed turbulent diffusion in a two-dimentional incompressible chaotic flow

$$\mathbf{v}(\mathbf{r}, t) = \boldsymbol{\nabla}\psi(x, y, t) \times \widehat{\mathbf{z}}. \tag{1}$$

The stream-function ψ is assumed to be a delocalized, bounded random function possessing a single characteristic spatial scale λ and a characteristic evolution frequency ω considered as a free parameter.

In the case of plasma, the drift motion of a guiding center can be written as

$$d\mathbf{r}/dt = v_{||}\widehat{\mathbf{z}} + (c/B)\widehat{\mathbf{z}} \times \boldsymbol{\nabla}\phi(\mathbf{r}_\perp, z, t)\ , \tag{2}$$

where $x\,, y\,, z$ are local coordinates, $\mathbf{B} = B\widehat{\mathbf{z}}$ is the magnetic field, ϕ the random electric potential ($\mathbf{E} = -\boldsymbol{\nabla}\phi$), and $v_{||}$ the particle velocity along the magnetic field. Let us assume the time scale being much shorter than the electron collision time, so that $v_{||}$ is constant. The

effect associated with toroidal geometry such as the bounce motion in $v_{||}(t)$ are considered in Ref. [3] but are neglected here. Then the cross-field motion of the electron guiding center is described by Eq. (1) with the stream-function

$$\psi(x, y, t) = -(c/B)\phi(x, y, z_0 + v_{||}t, t), \tag{3}$$

where z_0 stands for the initial cordinate of the particle. The effective frequency for the ψ variation is then

$$\omega_{\text{eff}}(v_{||}) \approx \max(\omega_*, k_{||}v_{||}), \tag{4}$$

which is either the fluctuation frequency, $\omega_* \approx \omega_{\mathbf{k}}$, or plainly a frame-of-reference effect from the parallel particle motion, with $k_{||}^{-1}$ being the longitudinal inhomogeneity scale of ϕ typically determined by the confinement geometry as $k_{||} \approx 1/qR$ for passing perticles. In the simple case of a single, travelling wave, $\omega_{\text{eff}} = \omega_{\mathbf{k}} - k_{||}v_{||}$, but in general case of multiple waves the relevant time scale of the coherent motion is $1/\max(\Delta\omega,\ \Delta k_{||}\, v_{||})$. For example, for electrons in the trapped particle mode [8], $k_{||} \approx 1/qR$, for passing electrons, and $k_{||} \approx 0$, for trapped electrons with fractional density $(r/R)^{1/2}$. The spectrum width is assumed to be of the order of the wavenumber, viz. $\Delta k_{||} \approx k_{||}$, $\Delta k_{\perp} \approx k_{\perp}$, implying the correlation lengths of the order of the wavelengths so that the problem is specified by a single, dominant scale length.

In the simplest terms, the turbulent diffusion is determined by the widely adopted assumption that, for a random, zero-mean velocity field $\mathbf{v}$, the chaotic particle motion governed by

$$d\mathbf{r}/dt = \mathbf{v}(\mathbf{r}, t) \tag{5}$$

leads to the asymptotic diffusivity

$$D_T = \lim_{t\to\infty} \left\langle [\mathbf{r}(t) - \mathbf{r}(0)]^2 \right\rangle /4t\ , \tag{6}$$

where the angular brackets in Eq. (6) denote the averaging over the inital conditions $\mathbf{r}(0)$.

The manner of chaotic advection described by Eq. (5) depends strongly on the dimensionless parameter

$$R_v = v_0/(\lambda\omega), \tag{7}$$

where v_0 represents the characteristic amplitude of the velocity field. Expression (7) is the ratio of particle rotation frequency $\Omega_v \approx v_0/\lambda$ and the frequency of the flow non-stationarity ω. At $R_v \ll 1$ (small amplitude, or high frequency limit) one can neglect the dependence of RHS of Eq. (5) on $\mathbf{r}$, hence getting the well known quasi-linear result

$$D_T = v_0^2/\omega\ , \quad R_v \ll 1. \tag{8}$$

In the opposite, strong amplitude limit $R_v \gg 1$ the flow $\mathbf{v}(\mathbf{r}, t)$ evolves much slower than particles rotate around the contours ψ = const. , which represent stream-lines. It means that some test particle lying on anomalously long contours of ψ with diameter $a \gg \lambda$ can contribute to the diffusion strongly, due to the coherent manner of their motion on the distance a, even though the probability to find such contours is relatively small. To describe this process quantitatively, one needs to know the distribution of random contours $\psi(x, y) = h$ over their size. This distribution can be calculated with the help of percolation theory which describes the statistics of clusters, viz. connected aggregates, of randomly occupied sites [9]. The lattice statement of the percolation theory is widely used in the phase transition theory. For the isolines statistics, the continuous percolation theory is relevant, with the "occupancy of a site" meaning a prescribed (say, positive) sign of the expression $\psi(x, y) - h$. Then the contours of constant ψ can be considered as perimeters of the clusters. The application of this theory to the 2D turbulent diffusion [7] yields the result

$$D_T \approx \lambda^2\omega R_v^{7/10}, \quad R_v \gg 1, \tag{9}$$

where the exponent $7/10 = (\nu + 1)/(\nu + 2)$ is expressed through the percolation exponent $\nu = 4/3$ that governs the divergence of the maximum size a of "islands", $a \propto |h - h_c|^{-\nu}$,

near the critical "sea" level $h = h_c$. The diffusion coefficient (9) is basically produced by a small fraction of stream-lines with diameter of the order of

$$a_m = \lambda R_v^{2/5}, \quad R_v \gg 1. \tag{10}$$

The transport correlation length (10) represents the maximum size of "islands" near the critical level a test particle has time to pass around before the contour is destructed due to the non-stationarity of the flow. A similar effect of transport constraining in narrow boundary layers near separarices has been described earlier for steady flows in the presence of a small collisional diffusion in the limit of large Peclet number [10, 11]. In our analysis, the parameter R_v plays a role analogous to the Peclet number.

The nondimensional parameter (7) governing the regimes of $\mathbf{E} \times \mathbf{B}$ stochastic transport, according to Eqs. (3), (4), can be written as

$$R_E = \frac{ck_\perp^2 \phi_0}{B \max(\omega_*, k_\| v_\|)}, \tag{11}$$

where ω_*, $k_\|$, $v_\|$ are the absolute values. Combining Eqs. (8), (9), (11) we obtain the following cross-field turbulent diffusion:

$$D_T(v_\|) \approx \begin{cases} \dfrac{c^2 k_\perp^2 \phi_0^2}{B^2 \max(\omega_*, k_\| v_\|)} , & R_E \ll 1 , \qquad (12a) \\[2ex] \left(\dfrac{\max(\omega_*, k_\| v_\|)}{k_\perp^2} \right)^{3/10} \left(\dfrac{c\phi_0}{B} \right)^{7/10} , & R_E \gg 1 . \qquad (12b) \end{cases}$$

Expression (12a) corresponds to the quasi-linear limit with the well known scaling law $D_T \propto \phi_0^2$ (used extensively in the analysis of fusion experiments [1–5]). In the opposite case (12b), which can be refered to as *the percolation limit*, one reveals the scaling $D_T \propto \phi_0^{7/10}$. This dependence differs from the previously reported scaling $D_T \propto \phi_0$ [5] based on the renormalized propagator $(\omega - k_\| v_\| + ik_\perp^2 D_T)^{-1}$.

A similar classification applies to the regimes of anomalous transport in a stochastic magnetic field $\mathbf{B} + \delta\mathbf{B}$ [12], because the equation of perturbed magnetic field line is mathemati-

cally similar to Eq. (5) when the electric potential ϕ is replaced by the perturbed vector potential A_z, and the time t is replaced by the coordinate z. The quasi-linear regime yields the magnetic line diffusion $D_m \propto (\delta B/B)^2$, while in the percolation regime $D_m \propto (\delta B/B)^{7/10}$.

The mixing length level of turbulence often envoked in theoretical and experimental studies of turbulence is (by definition) the transition point amplitude $R_E = 1$, where rotation around the fluctuation of size $1/k_\perp$ occurs in the correlation time $\omega_{\rm eff}^{-1}$. If this regime is maintained by the turbulence then the distinction between the quasi-linear and strong turbulence is not possible and the fluctuation's scale completely defines the turbulent diffusion as $D_T = \max(\omega_*, k_\| v_\|)/k_\perp^2$, which is commonly used to estimate D_T. However, there is evidence from the 3D fluid turbulence simulation of drift waves [13] that the turbulence saturates in the vortex structures with $R_E > 1$, so that the percolation theory, or strong turbulence regime, applies.

In the TEXT tokamak [14], electrostatic fluctuations with $\phi_0 \leq 10 \div 20$ V and $k_\perp \approx 1-3$ cm^{-1} are measured by probes in the exterior region ($r/a \geq 0.9$) and are known to account for the measured particle losses. Typical parameters are $L_n = 1-4$ cm, $T_e = 20-40$ eV, with $k_\| v_e \leq \omega_{*e} = k_y(cT_e/eBL_n)$. For this regime we find that $R_E = ck_\perp^2\phi_0/B|\omega_{*e}| \approx k_\perp L_n(e\phi_0/T_e) = 1-4$, suggesting the applicability of the strong turbulence limit.

The Constant Current Tokamak [15] also has large amplitude potential fluctuations ($\phi_0 \approx 12-25$ V) with $e\phi_0/T_e = 0.3-0.6 \geq \tilde{n}_e/n_e$ suggesting that the passive convection of plasma may be a valid first approximation. From the direct measurements of the fluctuation power spectrum one finds that $k_\perp \approx 1\ \text{cm}^{-1}$, $f \approx 100$ kHz, $\phi_0 \approx 20$ V leading to the estimate that $R_E \approx 1$. However, if we take into account the large Doppler shift from the rotation, which accounts for the most of the frequency f, by using $\omega_{*e} = k_y v_{de}$ in the calculation for R_E instead of the laboratory frequency $2\pi f$, then we may arrive to the estimate for $R_E = k_\perp L_n(e\phi_0/T_e) = (1\,\text{cm}^{-1}\,5\,\text{cm})(0.3 \text{ to } 0.6) = 1 \text{ to } 3$, which is sufficient to produce the vortex-like convective flights of the fluid elements that are not trapped in the potential

maxima and minima.

For the kinetic description of plasma, the percolation scaling of D_T can be more relevant. Let us discuss the dependence of the turbulent diffusivity D_T on the longitudinal electron velocity $v_{||}$. Due to this dependence, it is in principle possible that fast electrons undergo quasi-linear diffusion, while slow electrons or trapped electrons diffuse percolationally. Generally, the turbulent diffusion can be calculated upon averaging expression (12) over the electron distribution function. Suppose the thermal electrons have the parameter $R_E(v_{||} = v_{Te}) \geq 1$. Then runaway electrons with $v_{||} \gg v_{Te}$ can have $R_E(v_{||}) \ll 1$ and, according to Eq. (12a), will be confined better than thermal electrons. In this case one would have $D_T(v_{||}) \propto v_{||}^{-1}$ that implies the confinement time would scale as the square root of the electron energy. On the other hand, for a stochastic magnetic field, it is known [16] $D_T \approx D_m v_{||}$, and runaway electrons diffuse faster. This suggests the idea [17] that in the experiments with good runaway confinement, anomalous transport is caused by electrostatic turbulence, rather than by magnetic perturbations.

The strict applicability of the above results is restricted to the assumptions made. One can extend the theory to account for multiple scale perturbations, which is a problem under investigation [18]. Another important effect, which affects electron transport in tokamaks, is the radial electric field E_r. Theoretical [5,19] and experimental [14,15] evidence shows that sheared flow from E_r modifies drift orbits in such way that suppresses the radial transport. In this case of a strong E_r, or sheared flow $v_\theta = -cE_r/B$ from E_r, the scaling laws for D_T can be different. Also strong magnetic shear [3,8,13] can reduce the transport.

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