# Temperature Anisotropy Effect on the Toroidal Ion Temperature Gradient Mode

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**Abstract**

Using the local-electrostatic kinetic approach, we investigate the effects of the anisotropies in the ion temperature and the ion temperature gradient, and the effect of increasing the ion temperature relative to the fixed electron temperature on the toroidal ion temperature gradient driven mode. Comparisons are given between the new toroidal results and the previous slab results.

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# I. Introduction

In recent tokamak experiments, the use of powerful auxiliary heating schemes such as ion cyclotron heating (ICRF) and neutral beam heating (NBI) can generate a significant magnitude of anisotropy in the velocity distribution of the ions. Thus, it is important to consider the effect of anisotropy on the drift wave instabilities responsible for anomalous transport.

Recently, Migliuolo[1] has investigated the anisotropy effect in the ion temperature (specifically due to a $T_\perp$ increase from ICRF) on the ion temperature gradient drift mode in the slab geometry, and found that the anisotropy in the direction of $T_\perp > T_\parallel$ gives an overall stabilizing effect. On the other hand, Mathey and Sen[2] have investigated, in a similar work, the effect of the anisotropy in the ion temperature gradient in the same slab geometry. They showed that a finite gradient in the parallel temperature is necessary for the instability, and a gradient in the perpendicular temperature can either enhance or diminish the instability. In particular, it was shown that a large gradient in the perpendicular temperature can give a significant increase of the threshold value of the parallel temperature gradient, suggesting a stabilization scheme of this mode.

In this work, we generalize these studies to the toroidal system. We also investigate the effect of a parallel temperature increase ($T_\parallel > T_\perp$), which can be generated by the neutral beam injection (NBI) heating scheme. As is well known in toroidal geometry another branch[3–6] of temperature gradient mode besides the slab mode is generated by the toroidal magnetic curvature drift effect. As shown explicitly in our recent work,[7] this toroidal mode occurs when the toroidal magnetic curvature drift resonance dominates the parallel Landau resonance. For most tokamaks, this toroidal mode is more dangerous than the slab mode because it has a larger growth rate and lower threshold value of the temperature-to-density gradient parameter.

In Sec. II, we introduce the basic equations for the toroidal ion temperature gradient mode in the local limit. Numerical results are discussed with some analytical analysis in

Sec. III. Finally, in Sec. IV, the summary and conclusions are given.

## II. Basic Equations

We assume that the equilibrium ion velocity distribution function can be described by a bi-maxwellian and use a local electrostatic approximation. Then, we can derive the following perturbed ion density from the linearized gyrokinetic equations given in Ref. 8,

$$\widetilde{n}_i = -\frac{en\widetilde{\phi}}{T_{\perp i}}\left\{1 + \alpha\Gamma_0(b_i) - \frac{T_{\perp i}}{T_{\parallel i}}\int \frac{(\omega - \omega_{*i}^T)J_0^2(k_\perp v_\perp/\omega_{ci})F_{Mi}(v_\perp, v_\parallel)d^3v}{\omega - \omega_D - k_\parallel v_\parallel}\right\} \tag{1}$$

where the temperature anisotropy term is $\alpha = T_{\perp i}/T_{\parallel i} - 1$,

$$\omega_{*i}^T = \omega_{*i}\left[1 + \eta_\perp\left(\frac{v_\perp^2}{2v_{T\perp}^2} - 1\right) + \eta_\parallel\left(\frac{v_\parallel^2}{2v_{T\parallel}^2} - \frac{1}{2}\right)\right] ,$$

$$\omega_D = \varepsilon_n\omega_{*i}\left(\frac{v_\perp^2}{2} + v_\parallel^2\right)\Big/ v_{T\parallel}^2 ,$$

$$\eta_{\perp,\parallel} = \frac{d\ln T_{\perp,\parallel}}{d\ln n}$$

$$\Gamma_0(b_i) = I_0(b_i)e^{-b_i} , \; b_i = k_\perp^2\rho_i^2 = (k_x^2 + k_y^2)\rho_i^2 ,$$

and we use $T_{\parallel i}$ to define the ion diamagnetic drift frequency $\omega_{*i} = -k_\perp c T_{\parallel i}/eB\,L_n$, $v_{T\parallel,\perp} = (T_{\parallel,\perp i}/m_i)^{1/2}$ is the parallel (perpendicular) thermal velocity, $\rho_i^2 = T_{\perp i}/m_i\omega_{ci}^2$ with $\omega_{ci} = eB/m_i c$, $\varepsilon_n = L_n/R$ with $L_n = -(d\ln n/dr)^{-1}$, and $J_0$ and $I_0$ are the zeroth-order Bessel and modified Bessel function, respectively.

Now, with the adiabatic response of the perturbed electron density; i.e., $\widetilde{n}_e = \frac{en}{T_e}\widetilde{\phi}$, the quasi-neutrality condition gives following dispersion relation in dimensionless form

$$1 + \tau + \frac{\alpha}{1+\alpha}(\Gamma_0 - 1) = \int_{-\infty}^{\infty} v_\perp dv_\perp \int_0^\infty \frac{dv_\parallel}{\sqrt{2\pi}}$$

$$\times \frac{\left[\omega + \tau k_y\left(1 + \eta_\perp\left(\frac{v_\perp^2}{2} - 1\right) + \eta_\parallel\left(\frac{v_\parallel^2}{2} - \frac{1}{2}\right)\right)\right] J_0^2\left(b_i^{1/2}v_\perp\right)e^{-v^2/2}}{\omega + \tau\varepsilon_n k_y\left((1+\alpha)\frac{v_\perp^2}{2} + v_\parallel^2\right) - \tau^{1/2}k_\parallel v_\parallel} \tag{2}$$

where $\tau = T_{\|i}/T_e$ and $b_i = \tau k_\perp^2(1+\alpha)$.

In Eq. (2), the wavenumbers $k_\perp$ and $k_\|$ have been normalized to $\rho_s = c_s/\omega_{ci}$ and $L_n$, respectively, and the frequencies $\omega$ and $\omega_{*i}$ normalized to $L_n/c_s$ with $c_s = (T_e/m_i)^{1/2}$. With this normalization in terms of $T_e$ the temperature anisotropy effect due to increasing $T_{\perp i}$ can be conveniently investigated through the term $\alpha$ only. Similarly, the effect of increasing $T_{\|i}$ also can be easily studied through the term $\alpha$ only, if we just redefine $\tau_{\text{old}} = \frac{T_{\|i}}{T_e}$ as $\tau_{\text{new}} = \frac{T_{\perp i}}{T_e}$ so that $\tau_{\text{old}}(1+\alpha) = \tau_{\text{new}}$. The effect of the isotropic ion temperature increase compared with the electron temperature also comes through just the $\tau$ term only with this normalization.

It is easy to see that in the slab limit with $\varepsilon_n = 0$, Eq. (2) reduces to

$$1 + \tau + \frac{\alpha}{(1+\alpha)}(\Gamma_0 - 1) + \Gamma_0 \left\{ \left[ 1 - \frac{\omega_{*i}}{\omega} \left( 1 + \eta_\perp b_i \left( \frac{I_1}{I_0} - 1 \right) \right) \right] \xi Z(\xi) \right.$$
$$\left. - \frac{\omega_{*i}}{\omega} \eta_\| \left[ \xi^2 + \left( \xi^2 - \frac{1}{2} \right) \xi Z(\xi) \right] \right\} = 0 \tag{3}$$

where $\omega_{*i} = -\tau k_y$, $\xi = \frac{\omega}{\sqrt{2\tau}|k_\||}$ and $Z$ is the plasma dispersion function. We find that Eq. (3) is the same dispersion relation as that used previous works.[1,2] From Eq. (3) we can see that the effect of anisotropy for the slab mode appears mainly through the finite ion Larmor radius parameter $b_i$ with an additional adiabatic term $\alpha(\Gamma_0 - 1)/(1+\alpha)$. In particular, Eq. (3) shows that the anisotropy effect becomes negligible in the long-wavelength regime with $b_i \ll 1$ where $\Gamma_0 \sim 1$. On the other hand, Eq. (2) for the toroidal mode shows that an additional anisotropy effect comes from the ratio of the toroidal curvature to grad-B drift frequencies.

# III. Numerical Results and Discussions

First, we investigate the effect of the anisotropy in the ion temperature. In Fig. 1, our numerical results for the solutions of Eqs. (2) and (3) are given, where the normalized growth rate is plotted as a function of the normalized wavenumber $k_y \rho_{i\perp}$ for various $\alpha$ values. Here, we note that to obtain more simple figure we have normalized $k_y$ by $\rho_{i\perp}$, which depends on the $T_\perp$. Figure 1(a) is the slab case with $\eta_i = \eta_\perp = \eta_{||} = 3$, $\varepsilon_n = 0$, and $k_{||}L_n = 0.1$, while Fig. 1(b) is the toroidal case with $\eta_i = 3$, $\varepsilon_n = 0.1$, and $k_{||}L_n = 0$. Two cases show the similar behaviour that the increasing $T_\perp/T_{||}$ gives some destabilizing effect on the long-wavelength modes (this behaviour is shown more clearly if we normalize $k_y$ by $\rho_s$, which is independent of $T_\perp$, like the case of Fig. 2), while giving a significant stabilizing effect on the short-wavelength modes. As a result, the mode with the maximum growth rate shifts to the long-wavelength regime with the significant reduction of its magnitude. This result can be partially understood if we note that when the wavenumber $k_y$ increases the saturation of the growth rate occurs mainly due to the finite Larmor radius (FLR) effect given by the term $b_i$ in Eqs. (2) and (3). The increasing $T_\perp/T_{||}$ enlarges this FLR effect by $(1+\alpha)$ for a given wavenumber $k_y$, so that the saturation of the growth rate now occurs at a smaller wavenumber, with the corresponding reduction of the maximum growth rate.

In Fig. 1, besides the $T_\perp > T_{||}$ effect we also show the effect of increasing $T_{||} > T_\perp$ from NBI, for example. In this case, we redefine the parameter $\tau$ as $\tau_{\text{new}} = (1+\alpha)\tau_{\text{old}} = \frac{T_{\perp i}}{T_e}$, as mentioned earlier, to make the parameter $\tau$ independent of the change of $T_{||}$. The parameter $\alpha$ now takes negative values with the minimum value $-1$ in the limit $T_{||}/T_\perp \to \infty$. Unlike the increasing $T_\perp/T_{||}$ case the increase of $T_{||}/T_\perp$ does not affect the FLR term but increases the diamagnetic drift frequency. Also, a significant increase occurs in the adiabatic part of the ion response in the limit $\alpha \Rightarrow -1$. In these aspects, the role of the increasing $T_{||}/T_\perp$ differs significantly with that of the increasing $T_\perp/T_{||}$. In the slab case, Fig. 1(a) shows that the increasing of $T_{||}/T_\perp$ gives a small stabilizing effect to the long-wavelength modes, while a destabilizing effect on the short-wavelength modes with some increase of the maximum growth rate. On the other hand, in the toroidal case the effect of $T_{||} > T_\perp$ is substantially different from the slab case. Increasing $T_{||}/T_\perp$ gives a strong stabilization for the short-wavelength modes with some decrease of the maximum growth rate. This difference between the slab and toroidal cases is found to be mainly due to the anisotropy effect coming from the toroidal magnetic curvature term in the denominator of Eq. (3). Numerical test shows with the magnetic curvature term of the form $\omega_D = 2\varepsilon_n \omega_* \left( A_1 \frac{v_\perp^2}{2} + A_2 v_\parallel^2 \right)$ ($A_1 = A_2 = 1$ is the isotropic case) that the increasing $A_1$, which corresponds to $T_\perp > T_\parallel$, gives a destabilizing effect, while the increase of $A_2$, which corresponds to $T_\parallel > T_\perp$, gives a stabilizing effect. In the fluid limit the $A_1\, v_\perp^2$-term gives the perpendicular compressibility to the ion fluid which is a destabilizing effect.

In Fig. 2 we present the numerical results of the marginal temperature gradient value as a function of $k_y$ for various $\alpha$ values in the slab and toroidal cases. Figure 2(a) is the slab case and Fig. 2(b) is the toroidal case with the same parameters as Fig. 1, respectively. In the slab case, it is shown that the increasing $T_\perp / T_\parallel$ shifts the curve of the marginal temperature gradient $\eta_c(k_y)$ to the long-wavelength regime with the increase of the minimum value of $\eta_c$ in agreement with the result by Migliuolo[1] (whose corresponding figure shows $\eta_c$ as a function of $b_i = k_\perp^2\, \rho_{\perp i}^2$). This result may be understood to be primarily due to the FLR effect in similar with Fig. 1. On the other hand, it is interesting to note that the increase of the parallel temperature $T_\parallel$ gives a significant increase of $\eta_c$ over all $k_y$ even though the growth rate was shown to increase in the short wavelength regime by the same effect in Fig. 1(a). In this aspect, the increase of $T_\parallel$ may be favorable for the stability of the $\eta_i$ mode in the slab limit.

For the toroidal case, Fig. 2(b) shows that the marginal stability value of the temperature gradient just shifts to the long-wavelength regime without the notable increase of the minimum value of $\eta_c$ when $T_\perp / T_\parallel$ increases. The increase of the parallel temperature $T_\parallel$ also shows a similar feature except with a smaller shift of the curve. The threshold value of $\eta_c \approx 2/3$ is nearly same for a range of $\alpha$ values from $-0.8$ to $5$ ($T_\perp = 5T_\parallel$ to $T_\parallel = 6T_\perp$). Thus, even though the growth rate for the short wavelength modes can be reduced significantly as shown in Fig. 1(b), in terms of $\eta_c$ the temperature anisotropy is not so favorable for the stability of the toroidal mode. Rather, the anisotropy can be more dangerous for the confinement of plasma because the long-wavelength modes are now unstable at the lower threshold values of $\eta_i$ ($\eta_c \approx 2/3$) when the ion velocity distribution is anisotropic in either the parallel or the perpendicular direction.

Now, we study the effect of an another anisotropy; i.e, the anisotropy in the temperature gradient. Figure 3 shows our numerical results for the slab and toroidal mode. We find that there is a remarkable difference in the anisotropy effect between the two modes. For the slab mode, the parallel temperature gradient plays an essential role for the instability. From Eq. (3), we can obtain easily the criterion of the parallel temperature gradient for marginal stability

$$\eta_{\|c} = 1 - \eta_\perp b_i(1 - I_1/I_0) + \left\{ [1 - \eta_\perp b_i(1 - I_1/I_0)]^2 + 4\frac{\delta}{\Gamma_0}\left(\frac{\delta}{\Gamma_0} - 1\right)\frac{k_\perp^2}{\tau^2 k_y^2} \right\}^{1/2}$$

where $\delta = 1 + \tau + \frac{\alpha}{1+\alpha}(\Gamma_0 - 1)$. We note that $\eta_{\|c}$ decreases monotonously as $\eta_\perp$ increases since $\frac{\partial \eta_{\|c}}{\partial \eta_\perp} < 0$, so that $\eta_\perp$ gives destabilizing effect. However, if we take the experimentally meaningful marginal growth rate as $\frac{\gamma}{k_y v_i} \simeq 0.07$, we can find a rapid increase of $\eta_{\|c}$ for $\eta_\perp$ above some finite value as shown by Mathey and Sen.[2] On the other hand, for the toroidal mode, we find that the two gradients in the parallel and perpendicular temperatures give a similar effect. The perpendicular temperature gradient can generate the instability and does not give the stabilizing effect unlike the slab case. Of course, this significant difference between the slab and toroidal modes is because the two modes have the different types of resonance for the instability.

Finally, in Fig. 4, we show the effect of an ion temperature increase relative to the fixed electron temperature. As well-known, this large $T_i/T_e$ effect is important for the super-shot regime[9] and other hot ion confinement experiments. Here, we assume that the ion temperature is isotropic and consider only the toroidal case. We find that even though there is some difference in the quantitative aspects, the $\tau(= T_i/T_e)$ variation gives an effect similar to that of the temperature anisotropy, in that, while the maximum growth rate is reduced by the increasing $\tau$, the minimum value of $\eta_c$ does not increase and, in fact, shifts to substantially longer wavelength side. This similarity can be partially understood if we compare the roles of the term $\tau$ and the term $\alpha$ in Eq. (2). For the FLR term $b_i$, $\tau$ gives the same effect as the $T_\perp/T_\parallel$ term. On the other hand, in the adiabatic term and the diamagnetic drift term, we can see that increasing $\tau$ has a similar role as increasing $T_\parallel/T_\perp(\alpha < 0)$. Thus, it is expected that the term $\tau$ will have a similar effect with that of increasing $T_\perp/T_\parallel$ or $T_\parallel/T_\perp$. From this point of view, where the stability at long wavelength ($k_\perp \rho_i < 1/2$) is considered to be most important, it can be argued that the isothermal ($T_i = T_e$), isotropic plasma ($T_i = T_e$) is most stable. Toroidal[10] and slab[11] simulations show that the turbulent state has $k_x \sim k_y$, so that if we estimate the transport by $\chi_i = \gamma_{\max}/k_x^2$, then we find that for $T_i = 3T_e$, $\chi_i \equiv 2.0(\rho_s/L_n)(cT_e/eB)$ which is approximately four times that at $T_i = T_e$ where $\chi_i = 0.54(\rho_s/L_n)(cT_e/eB)$.

# IV. Conclusion

In summary, the anisotropy of the ion temperature in either the parallel or the perpendicular directions gives an overall stabilizing effect in the slab limit. However, in the toroidal regime, even though the anisotropy significantly reduces the growth rate of the short-wavelength modes, the minimum value of the marginal stability value of the temperature gradient $\eta_c$ does not change appreciably ($\eta_c \simeq 2/3$), but the first mode to go unstable above $\eta_c$ shifts to the more dangerous long wavelength regime compared with the isotropic case. For the toroidal mode, the shift to long wavelength is found in all three cases: (i) $T_\perp > T_\parallel$, (ii) $T_\parallel > T_\perp$, and (iii) $T_i > T_e$.

In real tokamaks, the characteristic of the temperature gradient modes can be slab or toroidal according to the magnitudes of the parameters such as toroidicity and safety factor $q$ etc.[7] The effect of the temperature anisotropy will depend somewhat on this characteristic regime of the instability.

On the other hand, the effect of the anisotropy in the temperature gradient ($\eta_\parallel \neq \eta_\perp$)

is significantly different between the slab and toroidal modes. For the slab mode, a large gradient in the perpendicular temperature can give a stabilizing effect, while for the toroidal mode the perpendicular-temperature gradient contributes to only the destabilization like the parallel-temperature gradient.

Thus, we conclude that for the toroidal regime there is no significant reduction of the ion temperature gradient driven transport from temperature anisotropy. There is also no reduction in the mixing-length estimate of $\chi_i$ from increasing $T_i/T_e$. In fact, an example is given where the increasing $T_i/T_e$ from one to three leads to a substantial increase in the $\chi_i$.

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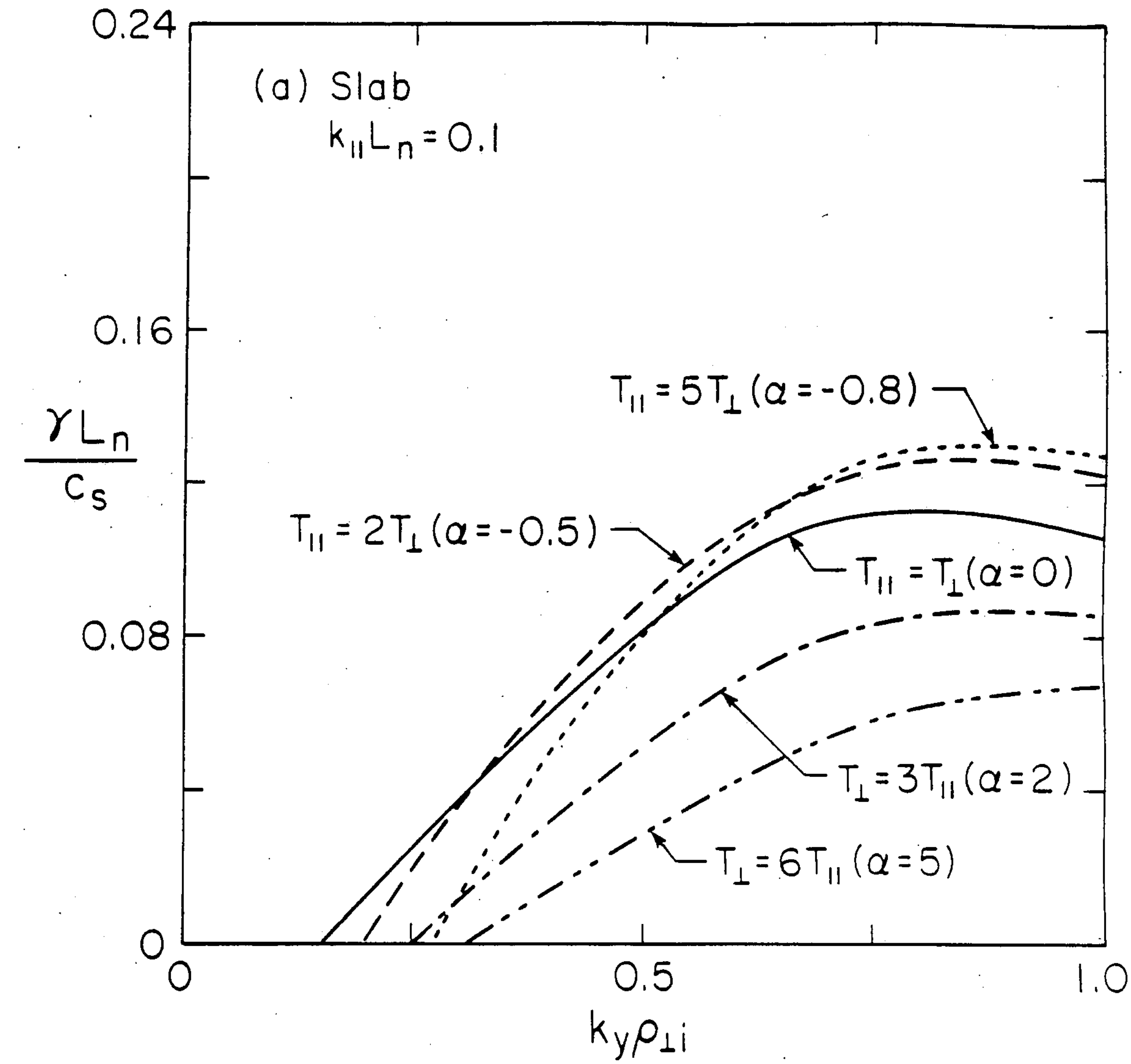
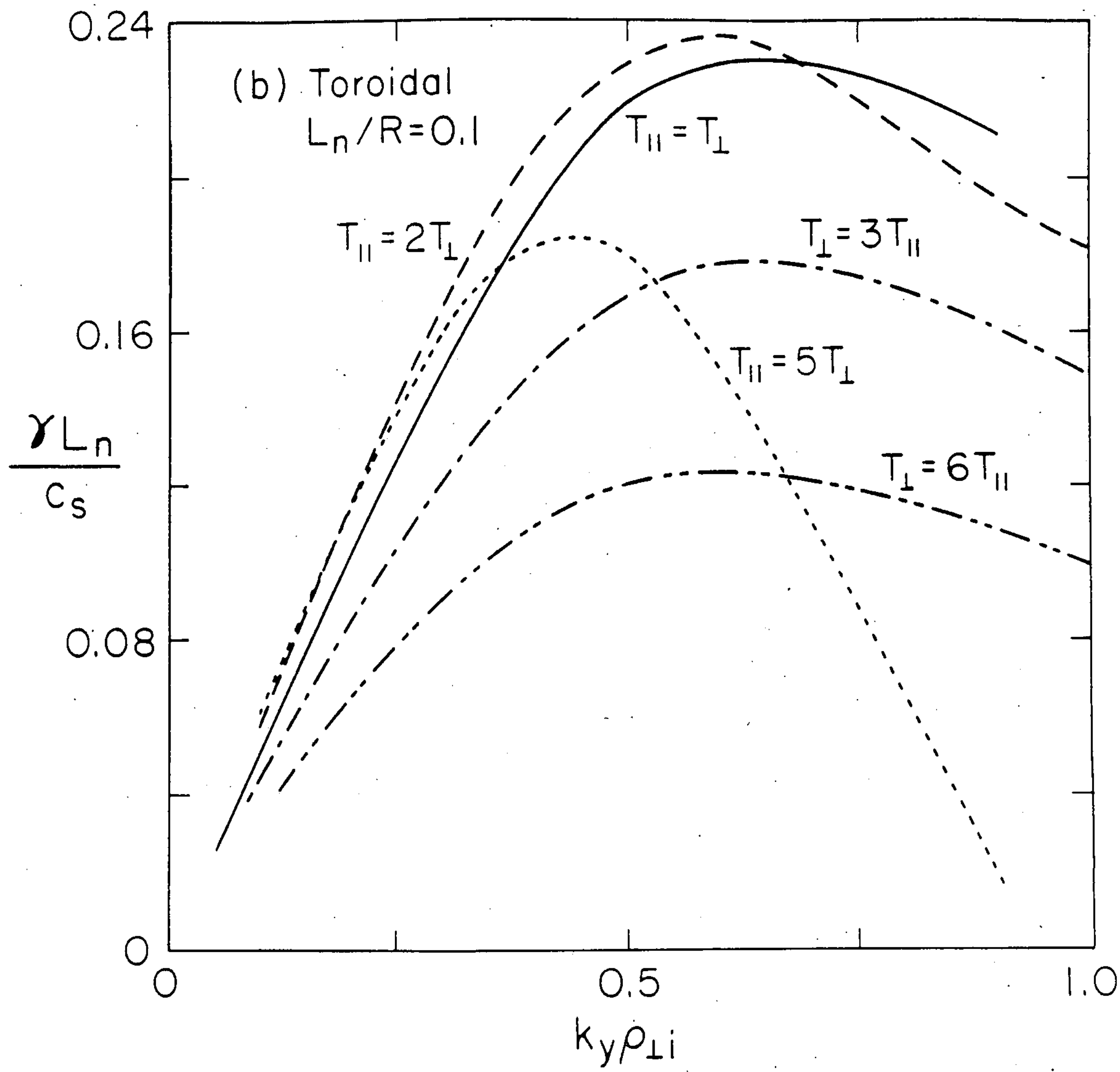
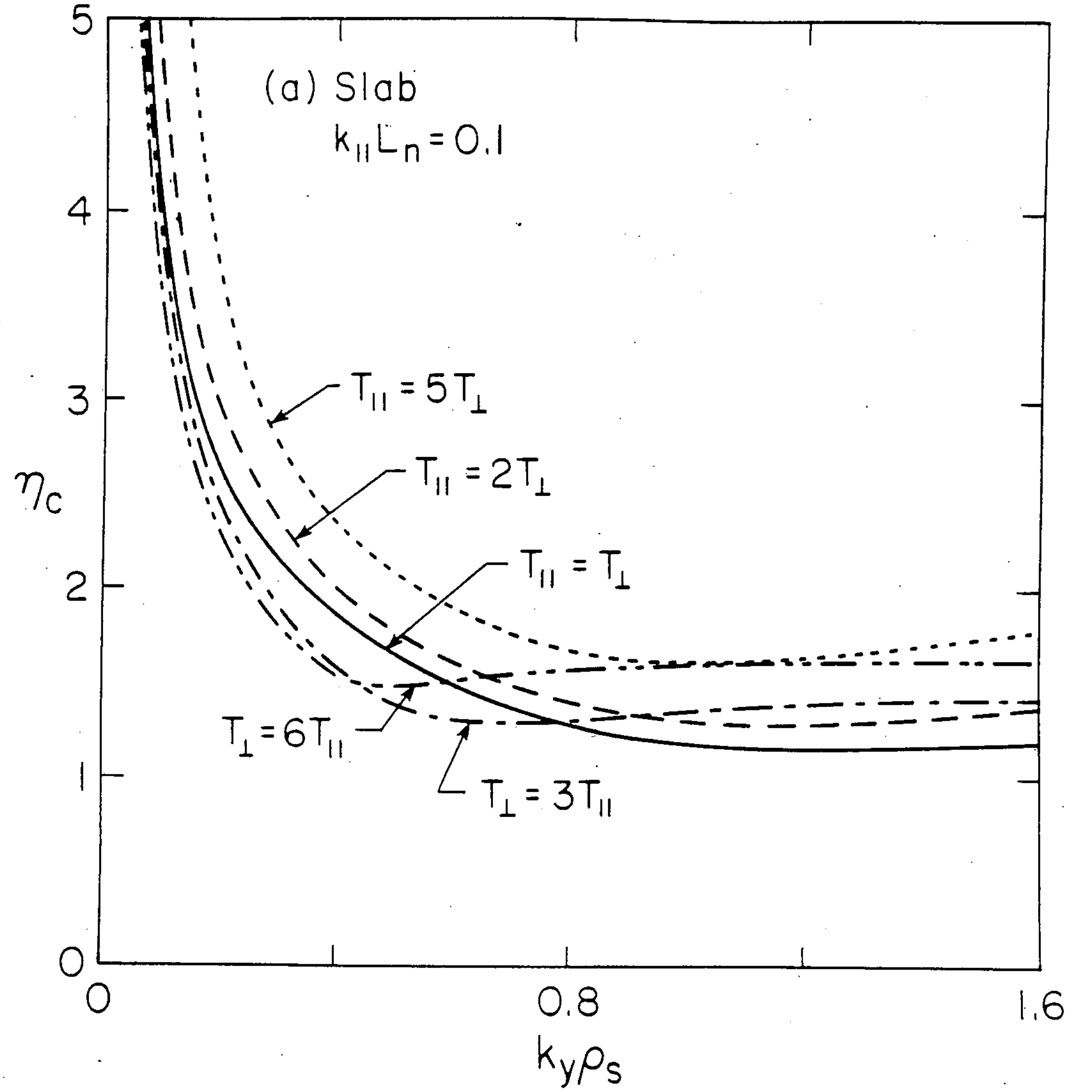

0.24
(a) Slab
$k_{\parallel}L_n = 0.1$
0.16
$\frac{\gamma L_n}{c_s}$
$T_{\parallel} = 5T_{\perp}(\alpha = -0.8)$
$T_{\parallel} = 2T_{\perp}(\alpha = -0.5)$
$T_{\parallel} = T_{\perp}(\alpha = 0)$
0.08
$T_{\perp} = 3T_{\parallel}(\alpha = 2)$
$T_{\perp} = 6T_{\parallel}(\alpha = 5)$
0
0
0.5
1.0
$k_y\rho_{\perp i}$

Fig. 1(a)



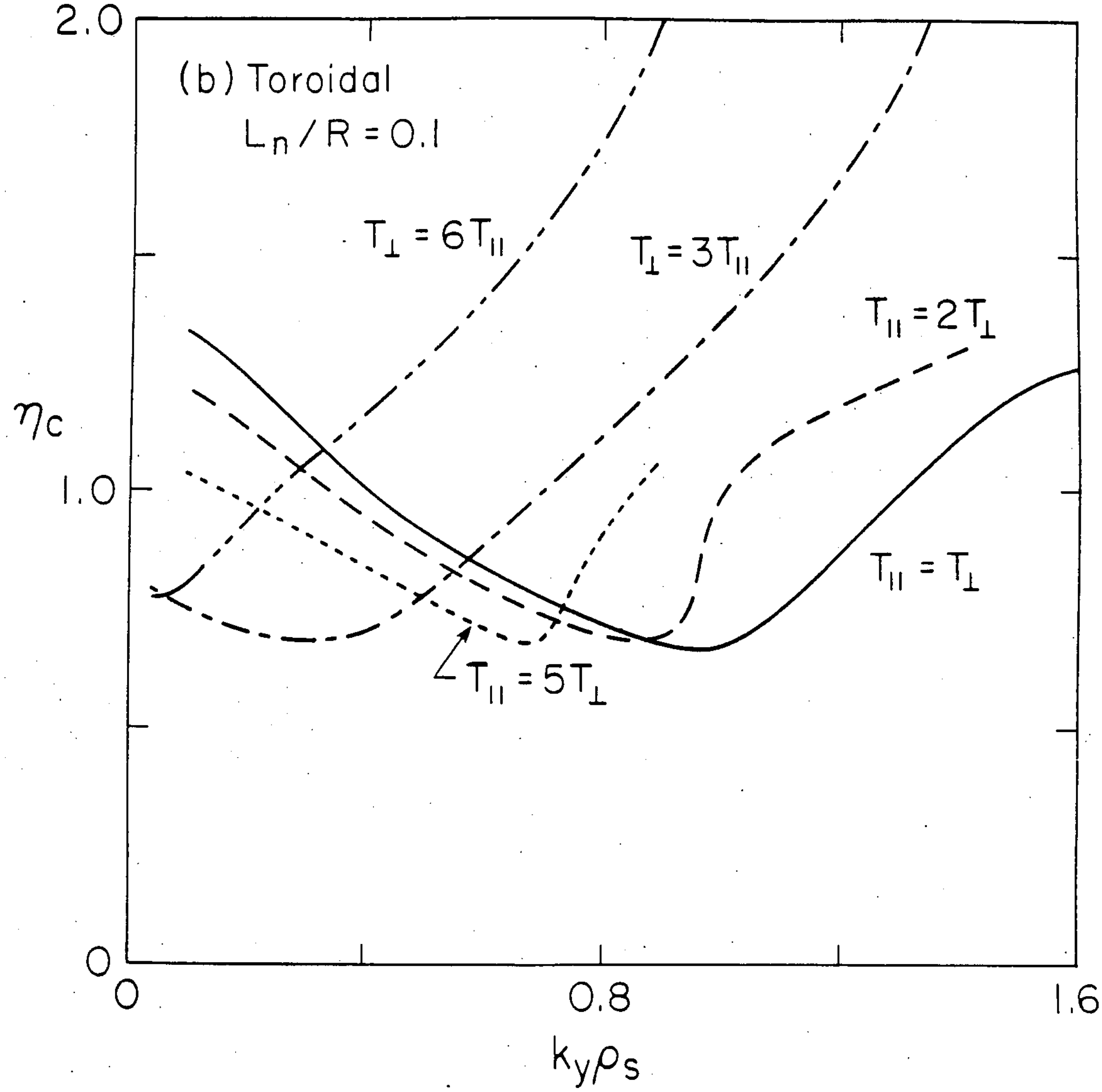
0.24
0.16
0.08
0
(b) Toroidal
$L_n/R=0.1$
$T_{\parallel}=T_{\perp}$
$T_{\parallel}=2T_{\perp}$
$T_{\perp}=3T_{\parallel}$
$T_{\parallel}=5T_{\perp}$
$T_{\perp}=6T_{\parallel}$
$\frac{\gamma L_n}{c_s}$
0
0.5
1.0
$k_y\rho_{\perp i}$


Fig. 1(b)



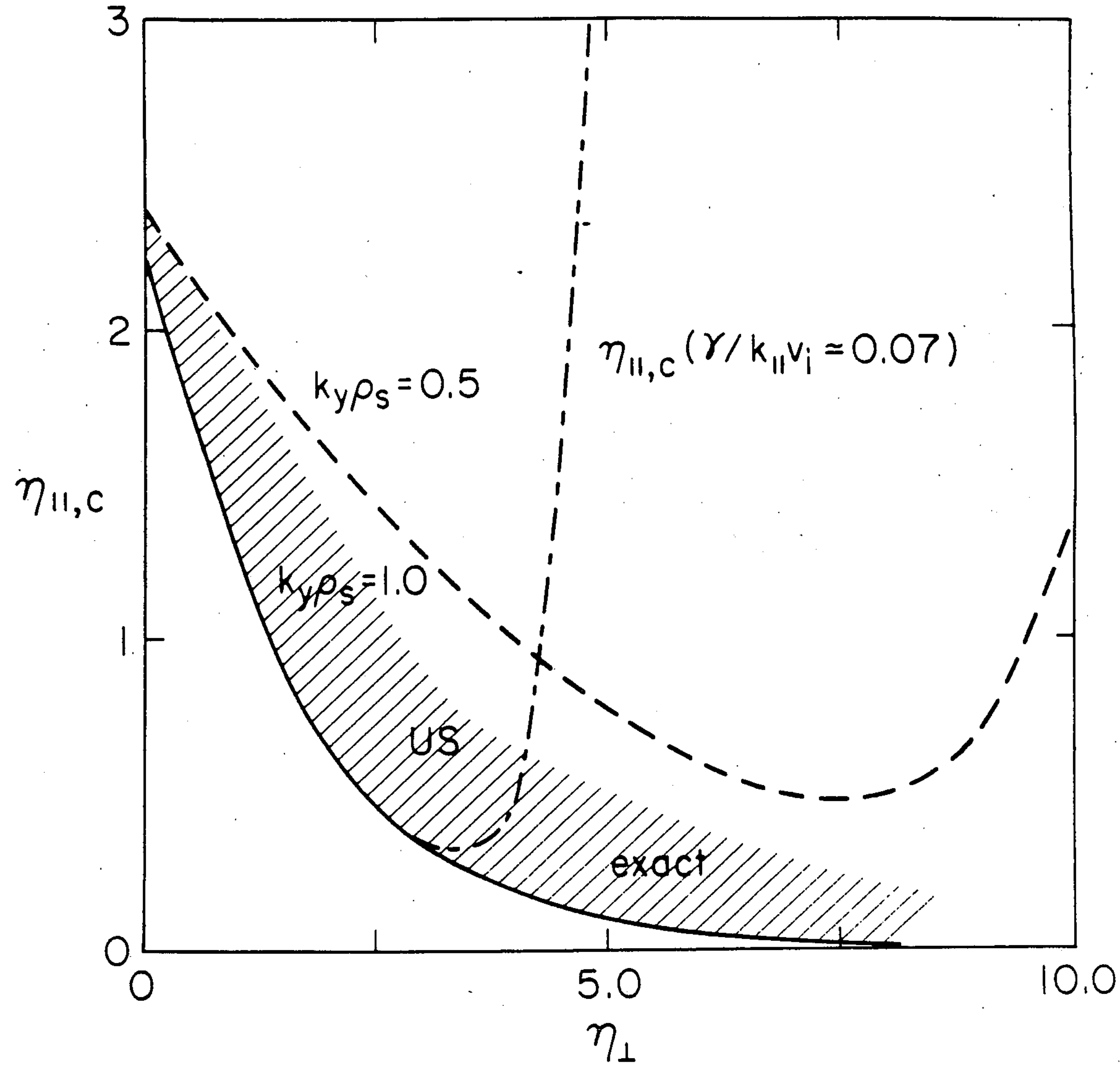
(a) Slab
$k_{\parallel}L_n = 0.1$
$T_{\parallel} = 5T_{\perp}$
$T_{\parallel} = 2T_{\perp}$
$T_{\parallel} = T_{\perp}$
$T_{\perp} = 6T_{\parallel}$
$T_{\perp} = 3T_{\parallel}$
$\eta_c$
$k_y\rho_s$
0
1
2
3
4
5
0.8
1.6


Fig. 2(a)



2.0
(b) Toroidal
$L_n/R = 0.1$
$T_\perp = 6T_\parallel$
$T_\perp = 3T_\parallel$
$T_\parallel = 2T_\perp$
$\eta_c$
1.0
$T_\parallel = T_\perp$
$T_\parallel = 5T_\perp$
0
0
0.8
1.6
$k_y\rho_s$


Fig. 2(b)



3
2
1
0
0
5.0
10.0
$\eta_{\parallel,c}$
$\eta_\perp$
$k_y\rho_s = 0.5$
$k_y\rho_s = 1.0$
$\eta_{\parallel,c}(\gamma/k_\parallel v_i \simeq 0.07)$
US
exact


Fig. 3(a)

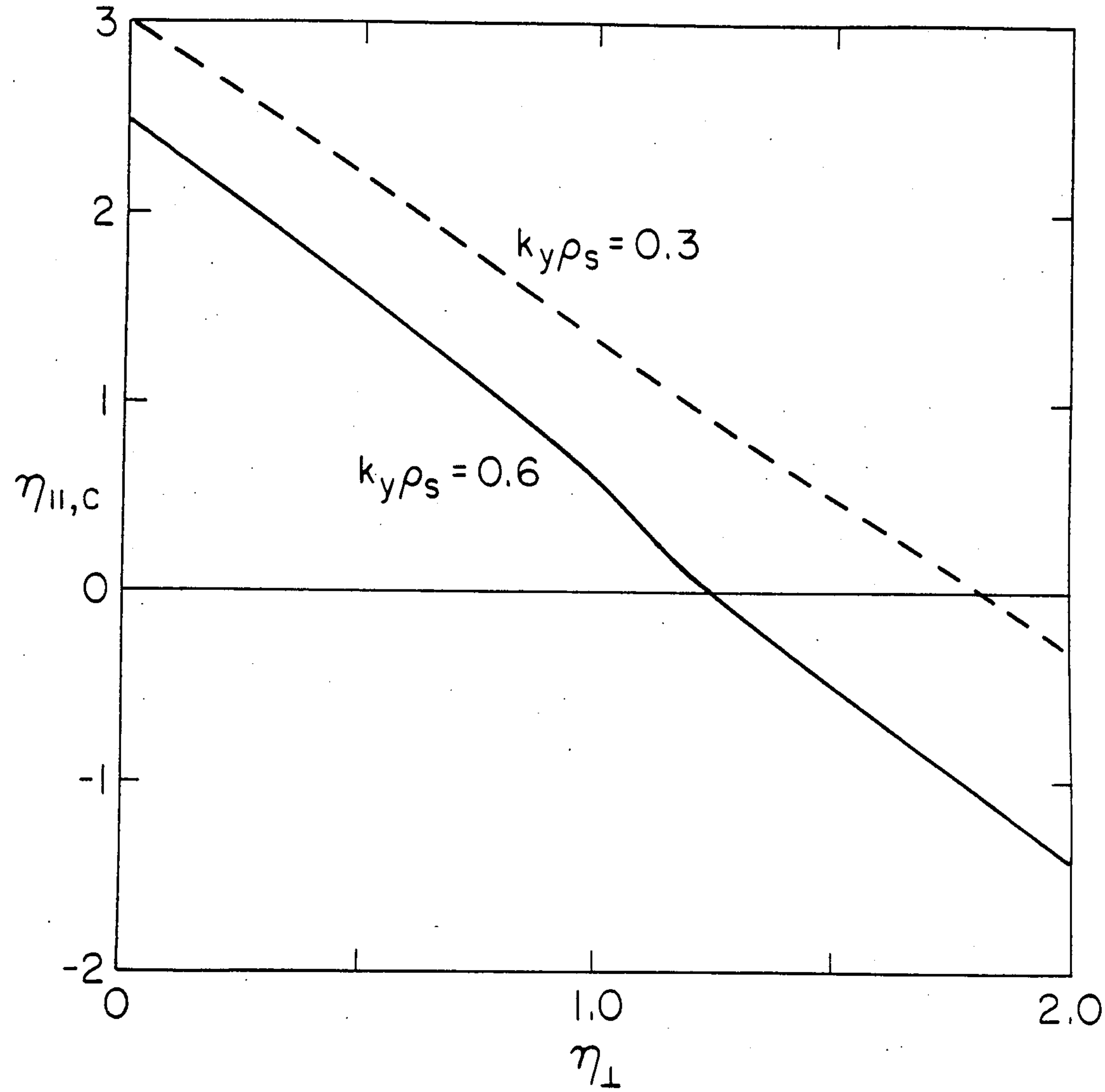



Fig. 3(b)

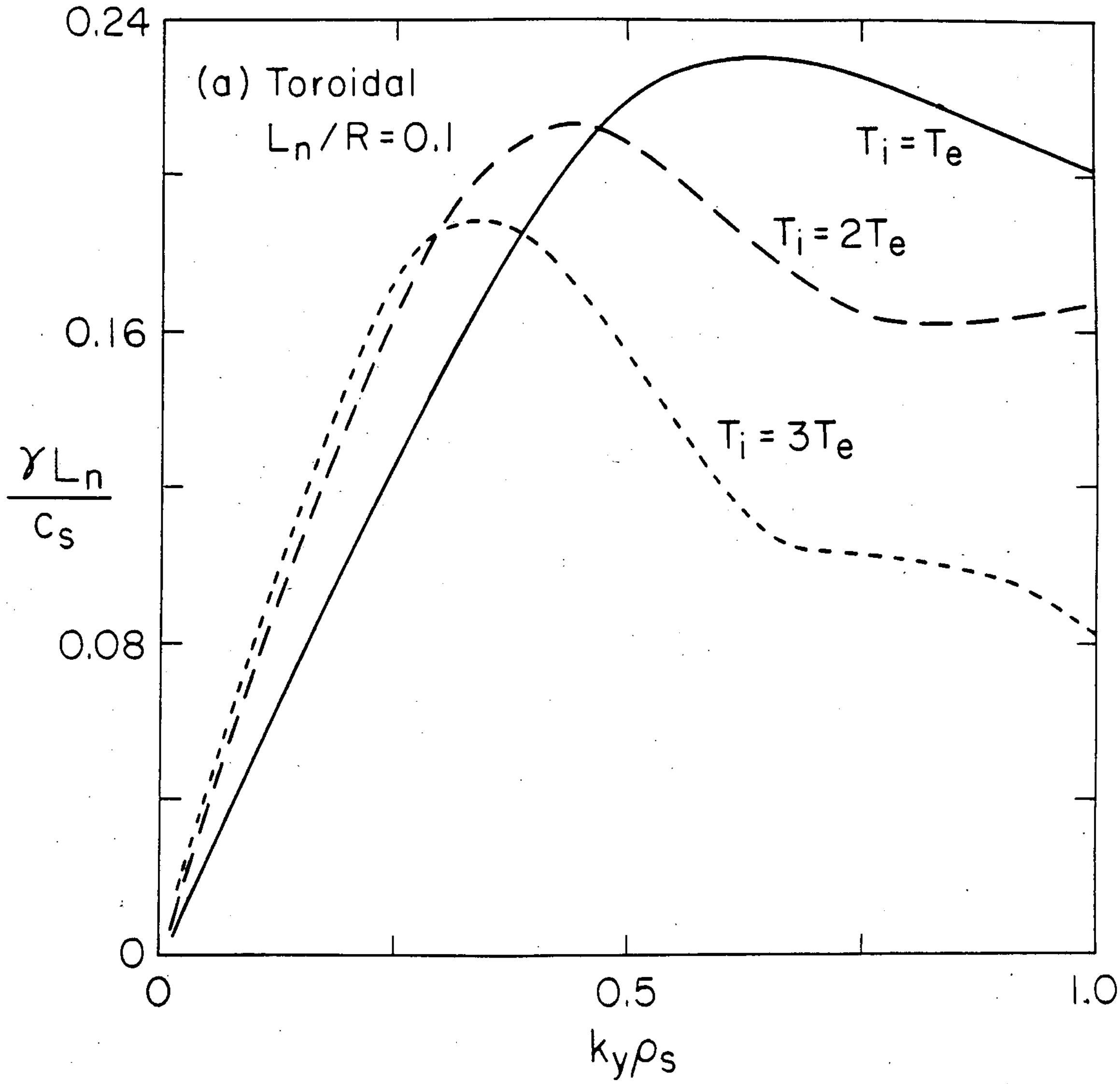

0.24
0.16
0.08
0
(a) Toroidal
$L_n/R = 0.1$
$T_i = T_e$
$T_i = 2T_e$
$T_i = 3T_e$
$\frac{\gamma L_n}{c_s}$
0
0.5
1.0
$k_y \rho_s$

Fig. 4(a)

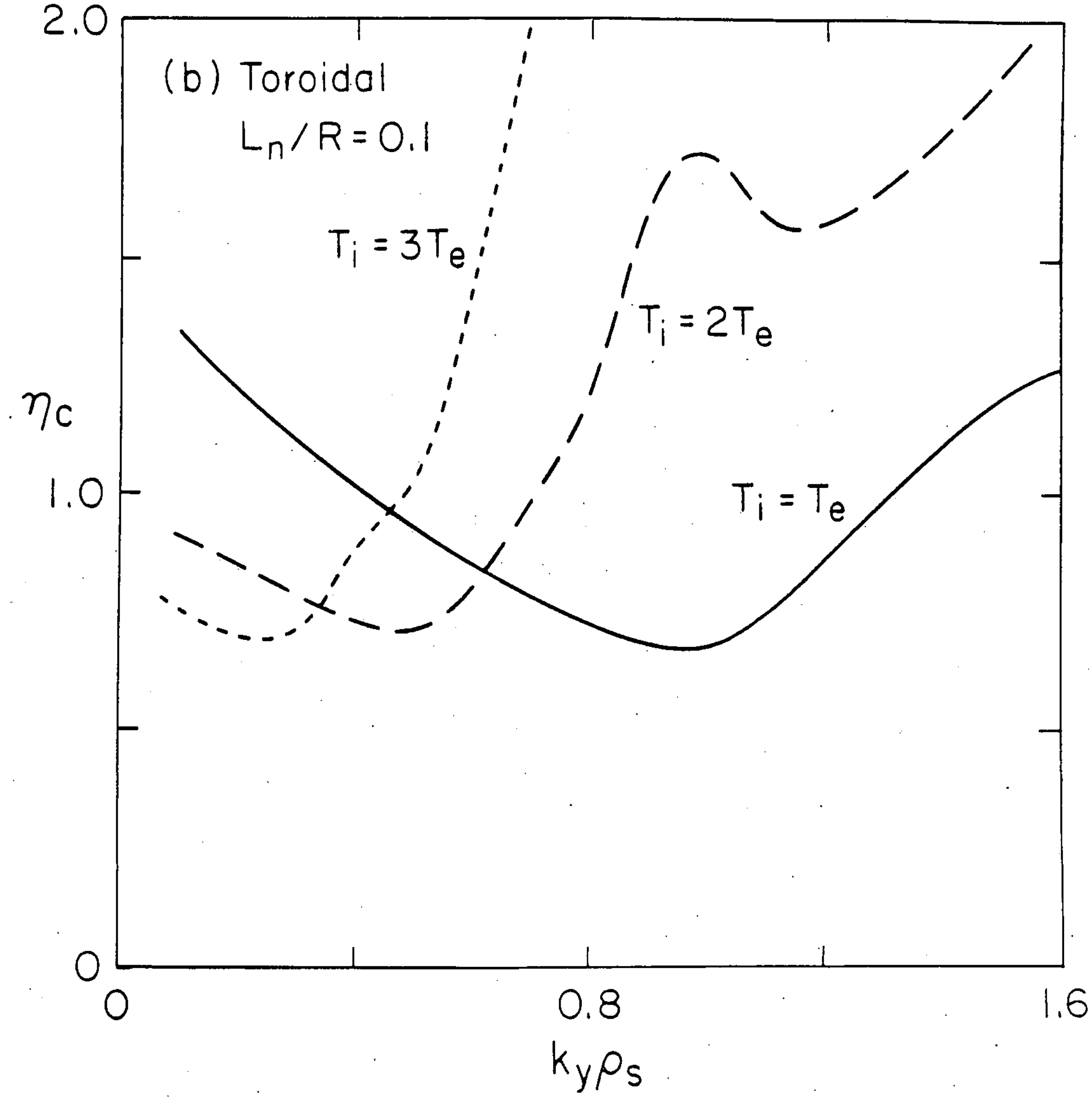



Fig. 4(b)